Complements and Substitutes in Sequential Auctions: The Case of Water Auctions*

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[Job Market Paper]

This Version: November 7, 2011

Abstract

We study sequential auctions in which bidders demand multiple units. We model the process as an English auction, where a seller sequentially allocates objects to potential buyers. The buyers have private and uncorrelated valuations for the goods, and pay a participation cost. We use a novel data set on sequential water auctions for the empirical study. Although water units are identical in nature, two features from the empirical setting create a trade-off whereby sequential units of water end up being complements or substitutes. First, since water is delivered to the farmer's plot using a channel that is dug into the ground, there is a water loss that is incurred only for the first unit. This generates a sunk cost. Second, because the amount of land that requires irrigation is fixed, subsequent units of water exhibit decreasing marginal returns. Units of water are complements or substitutes depending on the relative importance of the sunk cost and decreasing returns. Seasonal variations in the average rainfall and water requirements for farming determine the relative importance of the sunk cost and decreasing returns. This provides us with the required variation for our empirical investigation, the first to allow for the goods to be either complements or substitutes. When the goods are complements, one bidder wins all the objects by paying a high price for the first unit, thus deterring other bidders from bidding on subsequent units. A very low price is paid for the remaining units. When the goods are substitutes, different bidders win the objects with positive probability, and they pay prices similar in magnitude. We estimate the proposed model, and recover individual demand that is consistent with this stark pattern of outcomes. We confirm that the observed pattern of outcomes is not collusive, but consistent with non-cooperative behavior. Finally, we show that demand estimates are biased if one ignores participation and sunk costs.

JEL Codes: D44, C13, L10, L40

Keywords: Auctions, Structural Demand Estimation, Market Structure, Competition, Collusion

*We are indebted to our advisors, Joel Mokyr and Robert Porter, for helpful discussions, guidance and support. We are also grateful to the members of our dissertation committee, Meghan Busse, Regina Graffe, Aviv Nevo and Ron Siegel, for advice and encouragement. Discussions with Jia-Young (Mike) Fu, William Rogerson and Florian Zettelmeyer, as well as seminar participants at Northwestern University (Industrial Organization and History Workshops) and Universidad Carlos III have greatly benefited this work. We would also like to express our gratitude to Fernanda Donna and Antonio Espin for superb research assistance, to Pepe and Antonio from the archive of Mula for valuable help in accessing the historical files, and to Maja Butovich and Elizabeth Lenaghan for editorial advice. All errors are our own.

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1 Introduction

This paper investigates sequential auctions in which buyers face sunk costs and decreasing marginal returns due to their preference for multiple units. From an empirical stance, sunk costs and decreasing returns are important because they affect the relationship among the valuation of subsequent units, determining bidder behavior and the dynamics of prices. Price dynamics are central relating observed bids to the underlying distributions that characterize individual demand, which is fundamental to discussing positive and normative questions. Greater variation in prices caused by a high sunk cost, for instance, will affect even relatively simple tasks such as measuring the dispersion in individuals' private valuations. Moreover, a mercilessly competitive environment in such a case could be incorrectly interpreted as collusive.

Depending on the importance of sunk costs relative to decreasing marginal returns of sequential units, goods could be complements (when the marginal utility of subsequent units is greater than the marginal utility of the first unit) or substitutes (when the marginal utility of subsequent units is lower than the marginal utility of the first unit). The existing literature, although abundant, has provided little empirical evidence on the effect that, by creating complementarities or substitutabilities, the valuation of subsequent units has on bidder strategies and price behavior in sequential auctions.[1] From an estimation perspective, the main reason for this lack of evidence is the amount of variation in the degree of complementarity (in our setting, sunk costs relative to decreasing returns) required to perform the empirical investigation. From a theoretical point of view, bidder strategies in this setting rapidly become analytically intractable. The objective of this paper is to address this empirical gap. To that end, we develop an economic model to assess the relative importance sunk costs and decreasing returns have on bidding behavior. This is the first empirical paper where sequential units of the same good may serve as either complements or substitutes within the same market.

Using a novel data set on sequential water auctions, from an institution that was active during eight centuries in southern Spain (Murcia), we estimate the model to recover the structural parameters and distribution of private valuations. Apart from using a remarkably stable market institution that has been active since the 13th century, this study of sequential auctions introduces a unique scenario for analyzing a stark pattern of outcomes not previously documented in the literature. In some seasons, winning prices exhibit a standard competitive pattern. Regardless of whether the same or different bidders (farmers) win the sequential units, the winning prices are similar in magnitude.[2] In others, one farmer wins all the units, and she pays a high price for the first unit, thus deterring other farmers from entering subsequent auctions. A very low price (close to zero) is paid for the remaining units. We call this the *deterrence effect* (Figure 3). We show that this pattern of outcomes is consistent with a non-cooperative equilibrium, where the observed price dynamics are not collusive, but competitive.

The data for our analysis consists of individual winning bids, the bidder characteristics matched to those bids, auction covariates, and, given the nature of the auctioned objects, the amount of rainfall in the town. The basic selling unit is the right to use 15 minutes of water for irrigation ($36,000$ liters). For

[1] In addition to the literature on sequential identical objects (Ashenfelter (1989), Ashenfelter and Genesove (1992)), the relationship between auctioned objects has also been studied by a number of authors for the case of either complementarities (Branco (1997), Gandal (1997)) or substitutabilities (Black and de Meza (1992), Liu (2010)). Albeit more scarce, models encompassing the possibility of both complementarities and substitutabilities have been investigated when analyzing how this feature relates to the auction format (Jeitschko and Wolfstetter (2002), Jofre-Bonet and Pesendorfer (2006)) or, outside the auction literature, when analyzing print and online newspapers to study the valuing of new goods (Gentzkow (2007)). Contrary to this prior literature, we examine, in an English-auction setting, how the degree of complementarity, as determined by the importance of (common) sunk costs relative to (common) decreasing returns, affects price patterns by endogenizing the number of bidders with the introduction of an arbitrary small participation cost (McAfee and McMillan (1987), Engelbrecht-Wiggans (1993), Von der Fehr (1994)).

[2] Declining prices for identical objects in this type of setting is an empirical regularity known as the *declining price anomaly*, which has been broadly studied in the literature and was first documented by Ashenfelter (1989) in his seminal paper.

each weekday, eight units are sold sequentially for each schedule: *four* for daytime (7AM) and *four* for nighttime (7PM) irrigation. The rationale for this structure is related to water requirements in the area: since the irrigation technique used in the region is *flood irrigation*, four units (a *complete hour*) represent the amount of water that can be absorbed by a regular farming plot.[3] Daytime units are not independent due to the presence of sunk costs and decreasing marginal returns. Due to these factors, the relevant unit of analysis for investigating individuals' demand and the pattern of outcomes is *four-unit* auctions. Observing the identity of the winner allows us to implement the data-selection methodology required to estimate the model, which is outlined in Section 4. Weather conditions in the area determine the relevant agricultural irrigation technology and, hence, water demand. Additionally, spring/summer has little rainfall compared to autumn/winter, resulting in the presence of seasonalities, which provide us with the necessary variation in sunk costs relative to decreasing returns (varying the degree of complementarity/substitutability by season) to perform the empirical investigation.

The interpretation of the data through the economic model is fundamental to our structural approach. We model the environment as a sequential (ascending price) English-auction along the lines of Engelbrecht-Wiggans (1993) and Von der Fehr (1994) in which bidders, by incurring a cost of participation, decide whether to attend each sale to maximize their expected utility. Focusing on the symmetric independent private values (IPV) paradigm that has dominated the prior literature (Donald and Paarsch (1996)), a sunk cost is incurred for the first unit bought, and decreasing marginal returns are present for subsequent units. The rationale for this is found in our empirical setting. First, because water flows through a channel dug into the ground, some water is lost when the channel is dry (first unit), but the loss is negligible for subsequent units. Engineers have estimated that between 15% and 40% of the first unit of water that travels through a dry channel is lost (Vera Nicolas (2004)). Second, given that the amount of land that needs to be irrigated is fixed, subsequent units of water exhibit decreasing marginal returns.

The relative importance of the sunk cost and the decreasing marginal returns generates a trade-off, whereby buyers determine their course of action based on whether different units are complements or substitutes. When goods are complements, the same bidder wins all the objects: for the first unit a high price, equal to her valuation for the whole bundle (*four* times the second highest valuation for the first unit, before considering the complementarity effect and participation cost), and she thus deters others from bidding on the remaining three units, for which she pays a very low price (close to zero). Although non-cooperative, the resulting equilibrium price pattern, along with the fact that the same bidder wins all the units, may yield an incorrect collusive interpretation. When goods are substitutes, different bidders win the objects with positive probability, paying prices similar in magnitude (even when the same bidder wins all the objects). Since participation and sunk costs are key features of our model, we provide empirical evidence for them. Because the model is constructed with the purpose of understanding equilibrium price properties in sequential auctions when sunk costs and decreasing returns are present, we argue that buyers are better informed than the seller, who ignores bidder preferences for multiple units. Nevertheless, given that a sequential English auction achieves (*ex-ante*) efficiency, the seller knows that the mechanism allocates water in an efficient way.[4] Not requiring the farmers to reveal their marginal valuations is an additional advantage of the mechanism, whose simplicity reduces costs associated with its implementation.

The price patterns that our model predicts in each *regime* (complements or substitutes) provide us with a straightforward empirical method to determine under which regime the game is being played: when goods are complements, very low prices are paid by the same winner for the second, third and fourth units. Although

[3] This amount is represented by 1-*tahulla*, the surface area used in Murcia. The area encompassed by 1-*tahulla* varies from one city to another because the *tahulla* is defined based on the area that can be irrigated during a certain time. We provide further details in Subsection 4.3.

[4] *Ex-ante* and *ex-post* efficiency are not equivalent in our case due to the presence of participation costs. Neither one implies the other. Moreover, in our model there is no mechanism that achieves *ex-post* efficiency. Stegeman (1996) discusses the single-unit case. In our case, when goods are complements, the proof for efficiency is analog to the single-unit case. The case when goods are substitutes is more cumbersome.

it is clear from the data that there are two different price patterns, given that our estimation is performed conditional on the regime, we show that end-digit preference for these prices (second to fourth units) is consistent with this empirical prediction. Aside from the particular features of our specification, our identification exercise is closest to that of Athey and Haile (2002). Using a parametric distribution specification along with the English structure for the auction, we construct the likelihood of our model and, to secure the needed efficiency, we employ maximum likelihood inference techniques.[5] Consistent with the model, and as a result of the absence of a reservation price, we infer participation costs using data from auctions that were run with some bidders present, but no one placing bids.[6]

By estimating the proposed model, our empirical work establishes three main results. First, we are able to recover the individual demand—characterized by private valuations and the model's structural parameters—which is consistent with the mentioned price patterns, particularly the *deterrence effect*. Second, using the interpretation from the model, we confirm that the equilibrium price dynamics are competitive. In other words, non-cooperative behavior is not only consistent with the *deterrence effect*, but also necessary to generate such price differentials. Since the incentives to deviate from a collusion strategy would be higher in the spring/summer (when the water is more valuable), we provide evidence for seasonal price fluctuations, conditional on other observables. Finally, we show that, should the econometrician ignore the importance of participation and sunk costs, her estimations and inferences will be biased. We test whether price variations, conditioning on covariates, are better explained by the proposed model or a standard English-auction model, using the fact that the latter is encompassed by the former. The insightful approach by Haile and Tamer (2003) to relying on two weak assumptions equips the researcher with a robust structural framework for inference. Given that even these minimal assumptions are not satisfied in the present context, we also discuss how Haile and Tamer's limited structure model can be interpreted in the current setting.

The paper is organized as follows. Section 2 provides a review of the related literature and places this paper within its context. In Section 3 we introduce the theoretical model relevant to our empirical work. Section 4 contains a description of the auction-allocating system and the data selection methodology. We address the empirical regularities that serve as the foundation for the existence of two regimes (the size of sunk costs relative to decreasing marginal returns), and we show how the modeling assumptions fit our context. In Section 5 we show that regime determination is consistent with end-digit preferences. In Section 6 we discuss identification. Section 7 examines the estimation procedure, presents the results, and looks at how these results affect the importance of sunk costs and the interpretation of complementarities. Finally, Section 8 concludes. All tables and figures are displayed at the end of the paper.

2 Literature Review

In this section we describe the related literature and highlight how this paper contributes to the current body of work. Although not very large, there is certain literature on sequential auctions with multiunit demand analyzing the effects that variations in marginal valuations have on biding strategies, and the impact on price dynamics. Contrary to the empirical evidence in our setting, however, most of it do not consider entry costs in their analysis. Weber (1983) finds that, in an stylized sequential auctions model for identical objects, the sequence of prices is a martingale (same expected prices). Relatively slight modifications to Weber's model lead to different conclusions. Including risk-aversion in his model, for instance, generates lower expected price for later objects. In this case, expected utility (not expected price) follows a martingale. The expected utility is lower, of course, for a risk averse bidder. McAfee and Vincent (1993) found that declining prices in equilibrium requires non-decreasing absolute risk aversion. Sequential auctions for objects that are not

[5] Econometrics methods such as simulated non-linear least squares or GMM (Laffont, Ossard, and Vuong (1995), Hansen (1982)) can be applied to our setting as well.

[6] Typically, every Friday, 40 units are offered, and only the first 4 or 8 are sold. Hence, we know that some bidders were present (they bought 4 or 8 units), but also that no one bid on some of the units that were offered (34 or 32 unsold units).

identical but stochastically identical have also been studied (Benhardt and Scoones (1994), Engelbrecht-Wiggans (1994)). In those cases, objects are identical *ex-ante* but not after uncertainty has been resolved. The relationship between early and latter objects is broken because of the stochastic component: the demand for latter objects is smaller than for early ones. Thus, the strategical incentive for bidding lower for early objects disappears. Contrary to our case, these models do not allow for multiunit demand.

While economists have studied price dynamics and the relationship between sequentially auctioned goods, there is no precedent that deals with cases when both complementarities and substitutabilities (sunk cost and decreasing returns), along with participation costs are present.[7] Similar evidence to the one we describe in Subsection 4.2 has been broadly discussed in the empirical auctions literature. Early works have documented declining or downward price trends in sequential wine auctions (Ashenfelter (1989), McAfee and Vincent (1993)) and real-state market (Ashenfelter and Genesove (1992)).

Several authors have studied cases of either complements due to synergies among auctioned goods, or substitutes due to decreasing marginal utility (Black and de Meza (1992), Branco (1997), Liu (2010)). For complements, for instance, Branco finds expected decreasing prices. In that model, individuals bid above their value in early auctions because there is an option value for doing so. In latter auctions, and when early objects have already been assigned, the winner bids aggressively, but the losers will bid their residual value. Thus, the price for later units reflects the willingness to pay from a bidder without synergies. This value is lower than the price paid for earlier units that reflect both the willingness to pay from an individual without synergies and the option value. Selling goods in a bundle increases seller's revenue when goods are complements (Palfrey (1983), Levin (1997), Armstrong (2000)). Our setting differs from these in that we consider sequential auctions instead of simultaneous.[8] Numerous empirical studies have highlighted the importance of complementarities. They arise in different industries such as defense contracts (in Anton and Yao (1987) complementarities emerge in sequential competition due to higher experience that reduces costs), sequential cable television license auctions (Gandal (1997)), sequential electricity auctions (in Wolfram (1998) the start-up price generates complementarities in electricity generation between adjacent time periods), sequential procurement auctions for school milk contracts where, similar to our analysis, complementarities arise due to the presence of sunk investments (Pesendorfer (2000)), or between adjacent school milk contracts (Marshall, Raiff, Richard, and Schulenberg (2006)).

As regards substitutabilities, Liu's model, for instance, shows that competition for earlier goods is lower than for latter ones. Therefore, winning an early auction generates an externality on other bidders (because of the competition they face in the future). In later stages, this effect is smaller because there are less goods in which a loser can take advantage of the resulting competition. Thus, competition (and prices paid by the winners) will be greater. In the last auction the externality effect disappears, and everybody bids her own valuation for that last good. Substitutabilities also arise, as in our case, when the value of sequential goods fall (in the number of acquired units) generally because of decreasing returns and limited capacity. Substitutabilities are a major component in several industries such as sequential highway construction procurement auctions (Jofre-Bonet and Pesendorfer (2003) estimate a repeated auction game under the presence of capacity constraints with bidder asymmetry), sequential timber auctions (List, Millimet, and Price (2004)), or sequential cattle auctions (Zulehner (2009)).

Before we advance to the description of our model, it is important to note that while prior investigations on the relationship between sequential auctions and the complements or substitutes property are more scarce (Jeitschko and Wolfstetter (2002), Jofre-Bonet and Pesendorfer (2006)), their approaches clearly show the importance of this feature. Jeitschko and Wolfstetter analyze optimal sequential auctions in a binary-

[7] Hendricks and Porter (2007) present a comprehensive review on the theoretical and empirical literature on auctions emphasizing their connection, and different approaches followed by researchers in practice. For standard IPV parametric models, Donald and Paarsch (1996) provide an early discussion on identification, estimation and inference under this paradigm, while Athey and Haile (2002) extensively discuss identification of different models, and derive testable restrictions that allow discrimination between models.

[8] See Milgrom (2000) and Ausubel (2004) for recent contributions to this literature.

valuations case. They find that English-auctionss extract more rent than first-price auctions.[9] Our model differs as we consider the class of continuous valuation distributions. Along with a nice literature review that provides a comprehensive guideline on the methods and applications in the subject, Jofre-Bonet and Pesendorfer propose a different set-up in their model of sequential auctions that also encompasses both complementarities and substitubilities among the goods auctioned (in the sense that an object's value increases or decreases with the number of items already acquired). They find that first-price auctions give greater revenue than a second-price (English) auctions when the goods are substitutes. For the case of complementarities, the opposite is true. Both mechanisms are efficient in this model. Their predictions about price trend are consistent with previous findings. Contrary to our analysis with participation costs, and where buyers are better informed than the seller, they examine buyer's procurement auctions in a two period auction game where sellers have private information about their costs. Finally, although different from the strategy used in this paper, an early influential investigation on how interdependencies among the objects auctioned affect auction's outcome has been studied by Hendricks and Porter (1988), where they analyze auctions for drainage leases and show that better informed firms (who hold neighboring tracts to the drainage tracts that were auctioned) earned higher rents than uninformed ones.

We build upon the existing literature on participation costs and entry fees (McAfee and McMillan (1987), Engelbrecht-Wiggans (1993), Von der Fehr (1994)), by constructing our sequential English-auction model along the lines of Von der Fehr. Our set up differs, however, in that bidders are allowed to buy more than one unit of the good. Von der Fehr considers the case when goods are independent, and finds the same equilibrium as in the complementarities case in ours. We are not aware of any study analyzing the presence of both complementarities and substitutabilities when participation costs are present. Moreover, this is the first empirical investigation in the sequential auctions literature that considers the case where sequential units of the same good may either complement or substitute within the same market. To do that we allow the relative size of sunk cost and decreasing returns to vary across auctions.[10]

3 The Model

In this section we present the theoretical model. To do that, we first explain our strategy for using variation in the relative size of sunk costs and decreasing marginal returns to account for complementarities and substitutabilities among different units of the same good (water).

The model we introduce below is based on the reasonable assumption that the seller does not know the preferences from buyers (farmers) concerning multiple demand in a dynamic environment, four-unit auctions. A sunk cost is incurred only for the first unit bought while decreasing marginal returns are present for second to fourth units. These assumptions will be justified for our empirical environment in Subsection 4.3. The relative importance of the sunk cost and decreasing marginal returns generate a trade-off, whereby buyers coordinate their behavior based on whether different units are complements or substitutes.

A simple way to show this settlement of differences by mutual concessions is by assuming that the initial sunk cost is proportional to the value of water, and decreasing marginal returns are linear in the number of units bought. Then, the marginal utility for the farmer of each unit x of water is given by:

$$MU_i\left(x\right) = \left[1 - \alpha_{1\{x=1\}} - \beta\left(x-1\right)\right] v_i$$

where v_i, only known by farmer i, is a scalar that captures the valuation of the (complete) first unit of water (when $\alpha = 0$ we have $MU\left(1\right) = v_i$), α is a parameter that measures the sunk cost of the first unit

[9] Recall that the English-auction is optimal (revenue maximizing) among a general class of sequential auctions (Lopomo (1998)).

[10] The presence of seasonalities in our empirical setting (due to rainfall and water requirements for farming), provides us the required variation for the empirical analysis.

$(MU(1) = (1-\alpha) v_i)$, β is a parameter that measures the slope of decreasing marginal returns of subsequent units (when $\alpha = 0$ we have $MU(x) = [1 - \beta(x-1)] v_i$) and $1\{.\}$ is an indicator function.

In our model the *regime* where buyers will be playing is linked to complementarities and substitutabilities of sequential units of water. Units of water are complements or substitutes depending on the relative size of the two parameters defined above. If the marginal utility of the first unit is below the marginal utility for the remaining units, we will say that units are *pure complements*. If the marginal utility of the first unit is above the marginal utility for the remaining units, we will say that units are *pure substitutes*.

The discrete jump from the first to second unit, generated by the sunk cost, allows us to disentangle both effects. Figure 1 shows several cases than could arise depending on the relative value of the parameter vector (α, β). Panels I and IV exhibit situations where subsequents units are pure complements and pure substitutes, respectively. The proper comparison in our circumstances, it should be noticed, is between second, third and fourth units with respect to the first one. As would become clear in next subsection, in equilibrium, after the first auction everybody learns the type of the second strongest bidder (the bidder with the second highest valuation). Thus, it is common knowledge that the type of the winner is above the one of the second strongest bidder. The valuations of second, third and fourth units reflect, consequently, the value that the winner attaches to these units respectively. For the first unit, however, the valuation reflects the value that the second strongest bidder attaches to it.

3.1 Set Up

We proceed now to present the theoretical model. We derive, for each regime, the equations that will later be used for the structural estimation. When goods are *complements* we compute the unique Symmetric (Pure Strategy Perfect Bayesian) Equilibrium. We also prove that the strategies played by every player in this equilibrium are dominant strategies. This strong result allows us to (partially) identify the parameters of the model without imposing further structure to the distribution of bidders' valuations. When goods are substitutes we compute necessary and sufficient conditions for equilibrium. Technical details and the solution of the model for this case are provided in appendix A. In this case the equilibrium depends on the believes about other player's types, and strategies that each bidder have.

We consider v_i to be independent identically distributed on the interval $[0, W_i]$, according to the increasing distribution function F_i. It is assumed that F_i admits a continuous density $f_i \equiv F_i'$ and has full support. We allow for the possibility that the support of F_i be the non-negative real line, $[0, +\infty]$. It is assumed that $E[v_i] < \infty$. The assumption that the support of F_i is bounded below by 0 is not restrictive, since bidders with negative valuations will not enter the auction. It would be hard to interpret, however, bidders with negative valuations. The private valuation, v_i, is only known by bidder i, and it is learned before entering the first auction.

To allocate K goods, the seller runs an English (ascending) auction for every object. All participating bidders observe the total number of individuals who take part of the auction, N. After every auction, everybody observes the price paid by the winner and its identity, and the seller continues to run subsequent auctions sequentially until all the objects are allocated. We assume that all buyers share the same utility function, $U_i(\cdot)$, and that they are not financially constrained. The primitives of the model, K, N, $F_i(\cdot)$, $U_i(\cdot)$, are common knowledge.

The strategy set for every individual is the vector $S \equiv \left(y_i^k, b_i^k\right)_{i=1,\ldots,N}^{k=1,\ldots,K}$, where $y_i^k = 1$ indicates that bidder i is participating in the auction for object k, and b_i^k is the maximum amount that bidder i is willing to pay for object k. Bidders play sequentially, or stage by stage. This means that they choose $s^k = \left(y_i^k, b_i^k\right)$ after learning the outcome of the previous $(k-1)$ objects. Bidders participating in auction k observe the price at which each bidder is no longer active (bids are observable) except for the winning bid. Importantly, note that this information transmission is consistent with the auction being an English (or ascending) auction rather than a second price auction.

The seller allocates the object to the highest bidder: $x_j^k = 1$, where $j = \underset{i}{argmax}\left(b_i^k\right)$, at a price equal the second highest bidder: $p^k = b_l^k$, where $l = \underset{i \neq j}{argmax}\left(b_i^k\right)$. If nobody participates in a given auction the object is lost. In case only one bidder participates in a specific auction she obtains the object for free. Each object is allocated to one of the N bidders or is lost if all bidders decide not to participate in the auction, hence: $\sum_{i=1}^{N} x_i^k \leq 1,\ \forall k.$[11]

Participation decision in each auction is done simultaneously by all buyers. To take part in auction k, buyers incur a participation cost, c^k, at the beginning of the period. As explained before, if only one bidder participates she obtains the object for free, but she bears the participation cost, c^k, nonetheless. An English-auction for object k is run when at least two bidders decide to participate. The process is then repeated in every sequential period. For simplicity of the argument, and to avoid unnecessary notation, we restrict the analysis to the case where the participation costs are constant across auctions: $c^k = c > 0,\ \forall k$.

Note that the assumption that $c^k = c,\ \forall k$ is not restrictive since there are no informational shocks before the decision to enter the first auction. If we consider a case with $c^1 \neq 0$, and given the equilibrium outcome, we can compute the expected utility for every type. Bidders enter the (first and maybe subsequent) auction if, and only if, the expected utility they obtain from the game is positive. The *ex-ante* expected utility is continuous and increasing in the agent's type. Therefore, we can perform the same analysis considering the types distribution:

$$F^*\left(w_i\right) \equiv F\left(v_i \mid EU\left(v_i\right) \geq 0\right)$$

Throughout the paper we make the assumption that $EU\left(v_i\right) \geq 0$ for all v_i in the support. This equation is trivially satisfied when the lower bound in the support is positive and c is arbitrarily small.

The assumption $c^k \neq 0,\ k > 1$, however, is not without loss of generality because there is information transmitted before the decision of entering the second auction. As we further explain in Subsection 4.3, this assumption is consistent with the data in our empirical setting, where we observe no demand for some of the goods, even though the reservation price is zero. The interpretation is that, in those situations where *no-demand* is observed, the utility for all bidders is smaller than the participation cost, c. We later use this information to identify participation costs.

3.2 Two Units

We first analyze the situation where only two goods, $K = 2$, are sold sequentially. In this case, the marginal utility for the first unit is $MU\left(1\right) = \left(1-\alpha\right)v_i$, while the marginal utility for the second one is given by: $MU\left(2\right) = \left[1 - \beta\left(2-1\right)\right]v_i = \left(1-\beta\right)v_i$. Let us normalize the marginal utility of the first unit and define the normalized marginal utility as:

$$NMU\left(x\right) = \frac{MU\left(x\right)}{1-\alpha}$$

With this normalization, $NMU\left(1\right) = v_i$, and $NMU\left(2\right) = \left(\frac{1-\beta}{1-\alpha}\right)v_i = \left(1+\rho\right)v_i$, where $\rho \equiv \frac{\alpha-\beta}{1-\alpha}$. This representation allows us to characterize the utility function in a simple way, whose interpretation, in terms of complementarities or substitubilities, depends on the sign of one parameter, ρ. Of course, the sign of the complementarity parameter (ρ), is linked, ultimately, to the size of sunk cost (α) relative to deceasing marginal returns (β):

- $\rho > 0$ iff $\beta < \alpha$: When $\rho > 0$, we say that goods are *complements*. The reason is that in this case, the marginal utility for the second unit is greater than the marginal utility for the first unit. This is a situation where the sunk cost effect is relatively more important than the decreasing returns effect,

[11] We do not consider here the case where nobody bids for any of the objects, i.e., $y_i^k = 0\ \forall i, k$.

($\beta < \alpha$). In our empirical setting, and for reasons that will become clear in Section 4, this is an instance that will be likely to occur in spring/summer.

- $\rho < 0$ iff $\beta > \alpha$: The opposite takes place when $\rho < 0$, where we say that goods are *substitutes*, since the marginal utility for the second unit is lower than the marginal utility for the first unit. Now the decreasing marginal returns effects is relatively more important than the sunk cost effect ($\beta > \alpha$). This situation is likely to manifest in autumn/winter, as we discuss in Section 4.

The following is the utility of bidder i in the English-auction:

$$U_i = \left(x_i^1 + x_i^2 + \rho x_i^1 x_i^2\right) v_i - y_i^1 c - y_i^2 c \tag{1}$$

where $v_i \sim F\left[0, W_i\right]$ is a scalar (with W_i finite or infinite), $c^k > 0$, is a scalar arbitrarily close to zero, and $y_i^k, x_i^k \in \{0, 1\}$. As discussed above, after normalizing the marginal utility of the first unit to be equal to v_i, the marginal utility of the second unit is $(1 + \rho) v_i$.

In this section, we will only consider Pure-Strategy-Symmetric Equilibrium. Cases where $\rho \leq -1$ and $\rho = 0$ are close to Von der Fehr (1994), in Subsections 3.2 and 3.4, respectively. Uniqueness, however, is not proved by Von der Fehr in any of those cases.

In the remaining of the paper we refer to $v_{N:N}$, as the highest realization of the random variable v_i among N distributions F_i (one draw from each distribution), and $v_{N-1:N}$, as the second highest realization. More generally, $v_{j:N}$ is the jth order statistc for a sample of size N from the distribution F_i.

3.2.1 Goods are Complements: $\rho > 0$

Proposition 1. *The (Pure-Strategy-Symmetric Sequential) Equilibrium in this case is:*

- *First auction:*
  - ⋆ *Participation: Player i always participate in the first auction, i.e., $y_i^1 = 1$*
  - ⋆ *Bidding Strategy: We have two cases:*
    - *(a) If $(1 + \rho) v_i \equiv \tilde{v}_i \leq c$, then $b_i^1(v_i) = b^1(v_i) = v_i$*
    - *(b) If $(1 + \rho) v_i \equiv \tilde{v}_i > c$, then $b_i^1(v_i) = b^1(v_i) = (2 + \rho) v_i - c$*
- *Second auction:*
  - ⋆ *Participation: Here we have two cases:*
    - *(a) If $(1 + \rho) v_i \equiv \tilde{v}_i \leq c$, then do not participate in the second auction*
    - *(b) If $(1 + \rho) v_i \equiv \tilde{v}_i > c$, then participate in the second auction if, and only if, she won the first auction, i.e. $y_i^2 = 1$ iff $x_i^1 = 1$*
  - ⋆ *Bidding Strategy: If player i participates in the second auction ($y_i^2 = 1$), she will continue bidding until the price reaches her own valuation, i.e. $b^2(v_i) = v_i$*

Proof. Notice that the first case will happen when the cost c is relatively high. In this case, the cost of participating in the second auction is so high that bidder i does not want to participate in the second auction, even if she won the first unit. Hence, player i will behave as in the single unit case and $b_i^1(v_i) = b^1(v_i) = v_i$ is a weakly dominant strategy. Notice that because this is an English-auction, if only bidder i is in the second case and all remaining bidders are in the first case, $b_i^1(v_i) = b^1(v_i) = (2 + \rho) v_i - c$ is still a dominant strategy for player i because she will pay $p_1 = v_j < (2 + \rho) v_j - c$, where v_j is the maximum valuation of the remaining bidders. This inequality is a direct consequence of bidder i being in case 1 and $v_i > v_j$. The second case is more interesting.

The first step of the proof consists on proving that in any *revealing* (strictly increasing) equilibrium, i.e. an equilibrium in which it becomes common knowledge after the first round who is the bidder with the highest valuation, only the winner (the bidder with the highest type) will enter the auction, and pay the cost c.

Since both a direct mechanism and the sequential auction will give the same utility to the winner, and both will give the 2 objects to the bidder with the highest valuation, the total utility for the winner should be $W_i = (2+\rho)\, v_i - (2+\rho)\, v_j = (2+\rho)\,(v_i - v_j)$ in both cases, where j is the bidder with the second highest valuation. For the case of a direct mechanism we can assume that there is a cost of communication for each of the auctions, that should be paid by every bidder who wants to get the object.

The second step is to show that the winner will pay $(2+\rho)\, v_j - c$ in the first auction. This payment, together with the utility the winner obtains from both goods ($(2+\rho)\, v_i$) and the cost of entering the second auction (c), will give him the same utility as in the direct mechanism. $(2+\rho)\, v_i - ((2+\rho)\, v_j - c) - c = (2+\rho)\, v_i - (2+\rho)\, v_j = W_i$. This is easy to check. The utility for the winner in the second auction is $(1+\rho)\, v_i - c$ since the equilibrium price in the second auction is zero, i.e. $p^2 = 0$, and the utility for the loser is zero. Then, the total value of winning the auction is $(2+\rho)\, v_i - c$. Let us define $z_i = (2+\rho)\, v_i - c$ and consider this game to be a single object auction in which the valuation for the good of player i is z_i. Now we are back into the standard case (single unit auction) and $b_i = z_i = (2+\rho)\, v_i - c$ is a weakly dominant strategy (see Krishna (2010) for details).

So far, we have proven that in any *revealing* equilibrium only the winner will enter the second auction. The intuition for this result is that due to the cost of participating in an auction it is a necessary condition, for player i to participate in the auction, that her probability of winning is positive. If I am going to lose the auction for sure and it is costly to participate, Why should I enter the auction? Hence, only the winner will enter the auction. Her utility in the second auction is $(1+\rho)\, v_i - c$. This fact, together with her utility in the first auction v_i and the Revenue Equivalence Theorem show that she should bid $b_i^1 = (2+\rho)\, v_i - c$ and pay $p^1 = (2+\rho)\, v_j - c$. Hence, this is a *revealing* equilibrium. We have shown that, given the payoffs in the second auction, there is only one possible payoff and one possible bid for every player in the first auction. Hence, this is also the unique Symmetric Equilibrium in pure strategies. □

Note that, in this case, the allocation is efficient and only one bidder will enter the second auction and pay the cost, c. This is the minimum cost we can expect to be paid if we want the object to be allocated. Thus, the mechanism is also efficient in terms of minimizing entry costs.

3.2.2 Goods are Substitutes: $-1 < \rho < 0$

We proceed now to introduce two propositions that will help us identify the model's primitives. Note that, in this case, we do not need to solve the complete model to obtain the following results.

Proposition 2. *When $-1 < \rho < 0$, the probability that a player different from the winner enters the last auction is decreasing in the participation cost, c. Moreover, this probability goes to 1 when c goes to zero, i.e:* $\underset{c\to 0}{Lim}\left\{Pr\left(y_i^2 = 1 \mid x_j^1 = 1, i \neq j\right)\right\} = 1$.

Proof. It is sufficient to show that one bidder, the bidder with the second highest valuation, will enter with probability approaching 1. If the equilibrium in the first auction is fully revealing, it will become common knowledge that $v_1 \sim \tilde{F}_1(v_1) \equiv F_1(v_1 \mid v_1 > v_2)$, where player 1 is the winner of the first auction and player 2 in the second highest bidder in the first auction. The expected utility of entering the second auction for player 2 is:[12]

$$v_2 Pr\left[v_2 > (1+\rho)\, v_1\right] - c = v_2 \tilde{F}_1\left(\frac{v_2}{1+\rho}\right) - c \tag{2}$$

[12]Note that in the second auction the winner (in the first auction) now has complete information. Hence, a necessary condition for equilibrium is that if player 2 wins, player 1 does not enter the second auction.

Player 2 will enter the second auction if, and only if, her expected utility is positive, i.e: $Pr\left(y_2^2 = 1 \mid x_1^1 = 1\right) = Pr\left(v_2 \tilde{F}_1\left(\frac{v_2}{1+\rho}\right) - c > 0\right)$. Notice that c only appears in the right hand side, and the probability is decreasing in c. The first term in the left hand side is always strictly positive, by construction. This implies that there exist some value for c such that this term is smaller. Formally, we have:

$$\underset{c \to 0}{Lim}\left\{Pr\left(y_i^2 = 1 \mid x_j^1 = 1, i \neq j\right)\right\} \geq \underset{c \to 0}{Lim}\left\{Pr\left(y_1^2 = 1 \mid x_2^1 = 1\right)\right\} = \underset{c \to 0}{Lim}\left\{Pr\left(v_2 \tilde{F}_1\left(\frac{v_2}{1+\rho}\right) - c > 0\right)\right\} = 1$$

Notice that we are considering only the cases where the first auction is fully revealing. If the first auction is only partially revealing (equilibrium is pooling or semi-pooling),[13] then the analysis is less restrictive. In a pooling bidding region bidders with valuation within an interval bid the same amount. Ties are broken randomly. Hence, with positive probability it could be the case that the bidder with the highest valuation losses the lottery and we have $v_2 > v_1$. In this case, player 2 has even greater incentives to enter the second auction. □

Proposition 3. *When goods are substitutes, $-1 < \rho < 0$, it is a dominant strategy for all players to bid their valuations for the unit in the last auction, conditional on entering the auction.*

Proof. Conditional on entering the last auction, there is no informational not dynamic concerns and, hence, all bidders behave as in a standard (single-unit) second price auction. It is a well known result that, in this type of auction, it is a dominant strategy to bid your own valuation. We skip the details of the proof here. □

Proposition 4. *When $\rho = c = 0$ it is weakly dominant strategy for all players to enter all auctions and bid their valuations in every auction, i.e. $y_i^k = 1$ and $b_i^k = v_i$, $\forall i, k$.*

Proof. The proof is straightforward. The situation here is similar to the single-unit case. The difference, of course, is the information that each player has in every stage. Nevertheless, this will not affect the result. Note, first, that it is a weakly dominant strategy for every player to enter every auction because there are no entry costs and the gains are zero or positive. Next, note that, conditional on entering, it is a weakly dominant strategy for every player to bid her own valuation (for the unit) in the last auction. The game played in the last auction is the same as in the single-unit case with the exception that now some of the players' distributions are degenerated or have changed, due to their strategies in the previous stage. However, this does not change the result.

In the first stage, it is also a dominant strategy for all players to bid their valuations because their strategies, and the information that other players can infer from it, will not change the payoffs of any player in the second stage. Hence, the only thing that matters is whether they get the first unit and at what price. □

This is indeed the unique equilibrium in weakly dominated strategies, but not the unique Nash Equilibrium. Since all the valuations are revealed to all types in the first stage, equilibria can be constructed where some players do not enter the second auction, or enter the second auction and bid below their valuations, provided that the two highest types bidders enter both auctions and bid their valuations in both. Note, however, that in any case the bidder with the highest valuation will win both units and will pay a price equal to the valuation of the bidder with the second highest valuation.

This result will also hold when $\rho > 0$. When $\rho < 0$ the model is identical to Black and de Meza (1992) and the result does not longer hold. Hence, there is continuity in strategies only when $\rho > 0$ and $\rho \to 0$ (but not when $c > 0$ and $c \to 0$).

[13] It can be shown that, when $c \to 0$, the equilibrium is semi-pooling with probability 1.

3.3 Four Units

Now we consider the case where four goods are sold, $K = 4$. The marginal utilities in this case are: $MU(1) = (1-\alpha)v_i$, $MU(2) = (1-\beta)v_i$, $MU(3) = (1-2\beta)v_i$ and $MU(4) = (1-3\beta)v_i$.

In the remaining of this section we consider the participation cost c to be arbitrarily small, in order to avoid excessive notation and irrelevant cases. We further assume that $c^1 = c^2 = c^3 = c^4 = c$. We argue that, given the structure of the utility function, which is common to all bidders, if the same bidder ends up winning three or four units, then she is the bidder with the highest type $v_{N:N}$.[14] Although it is not important at this point we will restrict the parameter space to be $\alpha, \beta \in [0,1]$.

3.3.1 Pure Complements

As explained previously in this section, we say that goods are *pure complements* if the marginal utility for the first unit is below the marginal utility for the remaining units. This situation corresponds with a case similar to the one illustrated in panel I in Figure 1. Since marginal utility is decreasing after the second unit, this is analogous to say that the marginal utility of the first unit is below the marginal utility of the last unit, $MU(1) \leq MU(4)$, or, equivalently, $3\beta \leq \alpha$.

Proposition 5. *When goods are pure complements, $\beta \leq \frac{\alpha}{3}$, the (Pure-Strategy-Symmetric Sequential) Equilibrium is:*

- *First auction:*
  - ⋆ *Participation: Player i will always participate in the first auction, i.e. $y_i^1 = 1$*
  - ⋆ *Bidding Strategy: $b_i^1(v_i) = b^1(v_i) = [(1-\alpha)+(1-\beta)+(1-2\beta)+(1-3\beta)]v_i - 3c = [4-\alpha-6\beta]v_i - 3c$*
- *Second, third and fourth auctions:*
  - ⋆ *Participation: participate in all the remaining auctions if, and only if, she won the first auction, i.e. $y_i^2 = y_i^3 = y_i^4 = 1$ iff $x_i^1 = 1$*
  - ⋆ *Bidding Strategy: If player i participates in the second, third or fourth auction ($y_i^k = 1$ for $k = 2,3,4$), she will continue bidding until the price reaches its own valuation, i.e. $b^2(v_i) = b^3(v_i) = b^4(v_i) = (1-\alpha)v_i$.*[15]

Proof. The proof is similar to the one in Proposition 1. The details are in the appendix A. □

In this case the strategies and the outcomes are also analog to the single-unit case: since all second, third and fourth units yield greater marginal utility than the first unit, the equilibrium is fully revealing.

Corollary 6. *When goods are pure complements, $\beta \leq \frac{\alpha}{3}$, the marginal utility of the winner (of all auctions) solves the following equation:*

$$[4-\alpha-6\beta]v_{N:N} - 3c \geq \sum_{k=1}^{4} p^k = [4-\alpha-6\beta]v_{N-1:N} - 3c \tag{3}$$

[14]This will be the case if the bidding functions for each unit are strictly increasing in v_i. Moreover, even if the bidding functions are just weakly increasing in v_i but they are strictly increasing in v_i for some range in the support of F_i, then this will also be the case.

[15]Note that this is an strategy off the equilibrium path.

3.3.2 Non pure complements

The case where $\beta > \frac{\alpha}{3}$ encompass situations similar to the ones depicted in panels II to IV in Figure 1. The equilibrium behavior, however, will be different. Nevertheless, we are still able to use analogous results from propositions 2 and 3 to compute equilibrium prices in this regime. Note, first, that proposition 2 also holds for the case where four units are sold. Thus, there will be multiple entry in the last auction with positive probability, and this probability goes to 1 as c tends to zero. Proposition 3, on the other hand, can be easily extended to the case where four units are sold.

Proposition 7. *When goods are not pure complements, $\beta > \frac{\alpha}{3}$, it is a dominant strategy for all players to bid their valuations in the last auction (for that unit), conditional on entering the auction.*

Proof. The argument for the proof is the same as in proposition 3. □

When four units are sold for the case of not pure complements, there are more possible configurations for the winner, and the second highest bidder, in the last auction. We summarize them in the following corollary.

Corollary 8. *When goods are not pure complements, $\beta > \frac{\alpha}{3}$, the marginal utility of the winner in the last auction solves the following equations, depending on how many units she won:*

If she won all four units:

$$(1-3\beta)v_{N:N} \geq p^4 = (1-\alpha)v_{N-1:N} - c \tag{4}$$

If she won three units, two out of the first three, and the last one:

$$(1-2\beta)v_{N:N} \geq p^4 = (1-\beta)v_{N-1:N} - c \tag{5}$$

Several remarks concerning the behavior in these cases worth discussing. First, note that in both cases, whoever wins three or four units will necessary be the bidder with the highest type. When the same individual buys all four units, we can only claim equality on the price paid for the last sequential unit (equation 4). In this case, we know that all the remaining buyers are bidding in the last auction according to $MU_i(1) = (1-\alpha)v_i$. The reason is that none of them bought any of the first three units. In the case of equation 5, however, we do not know if the second highest bidder for the last unit has already bought one unit, and hence bidding according to $MU_{N-1}(2) = (1-\beta)v_{N-1:N}$; or has not bought any unit, and thus bidding according to $MU_{N-2}(1) = (1-\alpha)v_{N-2:N}$. If the winner of the last unit ($K = 4$) has bought only two units, we are unable to say whether she is the highest or the second highest type without imposing further structure. A similar argument applies to the case where the winner of the last unit only buys one good. We can only tell whether the last winner is bidding according to $MU_i(1) = (1-\alpha)v_i$ or $MU_i(k)$, depending on her previous purchases. Nonetheless, if we do not know her ranking on the valuations, we cannot use this information to infer price behavior. In Section 6 we discuss how to use corollaries 6 and 8 for identification of the distribution of private valuations and structural parameters.[16]

[16] Going back to the illustrative example in Figure 1, in the first case ($\alpha = 0.8$ and $\beta = 0.05$), all the remaining goods (second, third and fourth) have a valuation greater than the first one. Hence, no loser will enter those auctions and the *deterrence effect* will last the whole auction. In the second case ($\alpha = 0.8$ and $\beta = 0.1$), only the second and third goods have greater valuation than the first one. In the third case ($\alpha = 0.85$ and $\beta = 0.1$), only the second good have greater valuation than the first one. Hence, no loser will enter the 2nd auction but some (at least the second highest bidder) may enter the third and fourth sequential auctions. Finally, in the forth case ($\alpha = 0.95$ and $\beta = 0.1$), all the remaining goods have lower valuation than the first one. In this case deterrence is impossible and some bidders might enter every auction with positive probability. See also the discussion in footnote 48.

4 Data Description

In this section we describe the auction allocating system and the data. We then introduce the empirical regularities that support the contention that the size of sunk costs relative to decreasing marginal returns provide the basis for the existence of two regimes. Finally, we justify the modeling assumptions made in Section 3 within our empirical context.

The data in this paper comes from all water-auctions in Mula from January 1954 through August 1966, when the last auction was run.[17] On August 1st, 1966 the allocation system was modified from being an auction allocation system to a two-sided bargaining system. In the bargaining system, the *Heredamiento the Aguas* (water-owners holding) and *Sindicato de Regantes* (land-owners association) arranged a fixed a price (renegotiated at the beginning of each month or every six months) for every *cuarta* of water (the smallest unit auctioned). Gradually, the *Sindicato de Regantes* bought shares in the *Heredamiento the Aguas* association until they finally merged in 1974.

The reasons for focusing on the period from 1954 to 1966 are, first, that it represents the final period of the auction allocating system that was used for at least eight centuries in this region. Second, availability of rainfall and weather data for Mula starts in 1933, which introduces a natural starting date for compiling this information. Finally, given that the government carried out a special census in 1954/55, we are able to obtain a detailed profile information about the farmers that bid in the auctions for this period.

Apart from being a remarkable stable market institution that had been active since the 13th century, the study of sequential auctions in the present scenario introduces a unique circumstance for analyzing an unusual price behavior not documented in the literature before. In certain months, winning prices exhibit a "standard" competitive pattern where regardless of whether the same or different farmers win the sequential units, prices are similar in magnitude (Figure 4).[18] In others, where the same farmer wins all sequential units, she pays an "abnormal" high price for the first unit, deterring other farmers from entering subsequent auctions. This allows her to pay prices close to zero for the remaining (sequential) units. We call this the *deterrence effect* (Figure 3). The relative importance of sunk costs and decreasing marginal returns cause farmers to coordinate, affecting their marginal utility for subsequent units (within four-unit auctions). Should the econometrician not take into account this feature, her estimations and inferences would not be accurate. An alternative hypothesis to the behavior of farmers in the case of complementarities that is discussed in Subsection 7.4, is that bidders might be playing some collusive (non-competitive) strategy. When the former effect is relatively high (as defined in previous section), the *deterrence effect* arises and the "abnormal" price pattern is observed.

4.1 Water Auctions as Allocation System

Although the process of allocating water in Murcia has varied slightly over the years, its basic structure has remained, essentially, unchanged since the 13th century. Land in Murcia is divided into *regadio* (irrigated land) and *secano* (dry land). Irrigation is only permitted in the former. A channel system allows water from the river to reach all *regadio* lands.[19] The fundamental reason for this division is that *regadio* are fertile lands that are close to rivers and, hence, allow a more efficient use of scarce water in the region. Since it is forbidden to irrigate lands categorized as *secano*, only the farmers that own a piece of *regadio* land in Murcia are allowed to participate buy water.

The mechanism to allocate water to those farmers is a sequential English-auction. The auctioneer sells by auction each of the units sequentially and independently of each other. She keeps track of the name of

[17] Data available in the historical archive of Mula goes back until 1801. See Donna and Espin-Sanchez (2011) for further details.

[18] Declining prices for identical objects in these settings is an empirical regularity that has been broadly studied. For a seminal paper see Ashenfelter (1989).

[19] The channel system was expanded from the 13th to 15th century, as a response to the greater demand for land due to the increase in population. The *regadio* land's structure has not change since the 15th century.

the buyer of every unit and the price paid by the winner.

The basic selling unit is a *cuarta* (quarter), the right to use 15 minutes of water for irrigation. Water storage is done in the *De La Cierva* dam. Water flows from the dam through the channels at approximately 40 *l/s*. As a result, one *cuarta* carries 36,000 liters of water. Traditionally, auctions were made every 21 days to complete a *tanda* (quota), which is the basic aggregate unit of irrigation time. During our sample period auctions were carried out once a week, every Friday.

In every session 40 *cuartas* were auctioned: 4 *cuartas* for irrigation during the day (from 7:00 AM to 8:00 AM) and 4 *cuartas* for irrigation during the night (from 7:00 PM to 8:00 PM), every weekday (Monday to Friday). The auctioneer sells, first, 20 *cuartas* corresponding to the night-time and, afterwards, 20 *cuartas* corresponding to the day-time. Within each of these groups (day and night), units are sold beginning with Monday (4 *cuartas*), and finishing with Friday's *cuartas*.

4.2 Data Summary Overview

We combine data from different sources for our analysis. To get a sense of the industry context during the period under analysis, we present a brief description for the region's demographics, agriculture production and weather.[20] Murcia's population share in Spain was around 3% during the period. As a municipality, Mula comprised 2% of Murcia in 1954, ranking Mula 20th in terms of population. The three main citrus fruits produced in the area are apricot, lemon and peach trees. Murcia's share of these crops was 50% (2.3 million), 44% (1.5 million) and 42% (4.3 million), respectively, in terms of total Spain's total production of these fruits for the year 1962. *Regadio* land in Murcia constitutes 4% (70,000 ha.) of Spain's.

Auction data, the primary source of data for this study, is obtained from the historical archive of Mula.[21] Based on bidding behavior and water availability, auction data can be divided into three categories: (i) Regular periods, where for each transaction the name of the winner, price paid, date and time of the irrigation for each auction is registered, (ii) No-supply periods, where due to water shortage in the river or dam/channels damages (usually because of intense rain), no auction is carried out, and finally (iii) No-demand periods refer to auctions where no one bids and the registration auction sheet is blank. As we mentioned above, the sample for this study includes almost 13 years of auction data spanning January 1954 to August 1966. Every week, 40 units (corresponding to 40 *cuartas*) are sold, with the exception being when no auction is run (no-supply) or no bids are observed (no-demand). During the period under analysis a total of 17,075 auctions were run.

We complement auctions data with daily rainfall data for Mula and monthly price indexes for Spain, which we obtain from the *Agencia Estatal de Metereologia*, AEMET (which is the National Meteorological Agency), and the *Instituto Nacional de Estadística de España*, INE (which is the National Statistics Institute of Spain), respectively. Mediterranean climate rainfall occurs mainly in spring and autumn. Peak water requirements for the products cultivated in the region are reached in spring and summer, between April and August. During this period more frequent irrigation is advisable because it is in this period where citrus trees are more sensitive (in terms of quality of production) to water deficits. Soaring demand is reflected by the frequency of auctions where the same farmer buys all four consecutive units (4CU), which reaches its peak during these months, as it is depicted in Figure 12. Of course, the frequency of 4CU is not homogenous over time but, as can be seen in Figure 7, related to seasonal rainfall. Weather is important for our analysis as it is a determinant of seasonality. The coastal strip of southeast Spain is the most arid region of all continental Europe due to the *Foehn Effect*[22] and because of its location: right to the west of the mountain chain *Sistema Penibetico*, which includes the *Mulhacen* (the second highest mountain in Europe). Although annual average

[20] These descriptive statistics are obtained from *Population* and *Agricultural Census* from the *National Statistics Institute* of Spain (INE) (available online at http://www.ine.es/inebaseweb/treeNavigation.do?tn=201299&tns=199923#199923).

[21] From the section *Heredamiento de Aguas*, boxes No.: HA 167, HA 168, HA 169 and HA 170.

[22] A *foehn* wind is "a type of dry down-slope wind that occurs in the lee (downwind side) of a mountain range. It is a rain shadow wind that results from the subsequent adiabatic warming of air that has dropped most of its moisture on windward slopes. As a consequence of the different adiabatic lapse rates of moist and dry air, the air on the leeward slopes becomes warmer than equivalent elevations on the windward slopes" (obtained from: http://en.wikipedia.org/wiki/Foehn_wind).

rainfall is 320 mm., rainfall frequency distribution is skewed, making the majority of years dryer than this yearly average. Aridity during the summer is especially acute. Autumn is the only relatively humid season. The number of days when torrential rain occurs is not particularly high, but when such rain occurs it is substantial.[23] Potential evaporation is four or five times higher than rainfall and the number of arid months vary from 7 to 11 in our sample. These arid conditions found in southeast Spain are related to the circular air movement in the occidental Mediterranean area and to the Atlantic-origin storms.

We further augment our data with individual characteristics of the farmers' land, which we obtain from the 1954/55 agricultural census.[24] This census was conducted by the Spanish government to enumerate all cultivating soil, producing crops and agricultural assets available in the country. Individual characteristics for the farmers' land (potential bidders which we match with the names in the auctions data) include the type of land and location, area, number of trees, production and the price at which this production was sold in the census year. Figure 6 shows a sample card for one farmer from the census data.[25] It can be seen in Table 2, that *Land Extension*, *Number of Trees* and *Kg sold* vary considerably across farmers. As we discuss later in Section 7, during the 13-year period under analysis, there are approximately 500 different bidders in our sample. Of course, the number of bidders that win auctions during a specific year is considerably lower (the mean for our sample is around 50) and conditional on showing up, each farmer wins on average 22 units per year (an average of 792 thousand liters per year). This is consistent with the census data we have collected, where mean land extension is 5.5 ha. with an average of 33 trees per ha.[26]

In our theoretical model, it is important to be able to identify, for four-unit auctions, cases where the same farmer wins four or three sequential units, to avoid imposing further structure on the dynamic strategic considerations of the bidders, which are outside the scope of this paper.[27] Thus, we have selected for our estimation, auctions where a single person wins all units or where the last winner also won two out of the first three units (for a total of three units). This represents 54% of the total number of water units sold in our sample as displayed in Table 1. The table also exhibits the frequency distribution of units sold by number of units bought by the same farmer. Overall 42% of the units were sold in 4CU.

Selected summary statistics for the main variables are provided in Table 2. Importantly for our analysis, we only observe the transaction price (winning bid) and the identity of the winner (name). Winning prices range from \$0.05 to \$4,830 being the mean \$374. As expected, winning prices and frequency distribution of 4CU are strongly correlated with past rain (Figure 7). Interestingly, in the sample used in the structural estimation, the counterintuitive positive correlation between average daily winning prices and daily rainfall recovers its "correct" sign (with high significance) once we condition on seasonality. The endogeneity issue arises because both demand (due to the nature of the trees) and supply (rainfall) are high during spring, generating an artificial positive correlation between the variables that is, ultimately, caused by seasonality

[23] As an example, on October 10th 1943, 681 mm. of rain water were measured in Mula, more than twice the yearly average for our sample.

[24] Detailed census data is also obtained from the section *Heredamiento de Aguas* in the historical archive of Mula, box No. 1,210.

[25] One nice feature of this data is that every individual record (card) contains information on any plot owned by the farmer, both in the *huertas* and in other places. Information on whether a farmer owns another plot of land not allowed for irrigation is important as it is the farmer's outside option (for other sources of income) in case she does not buy water at a given auction during a specific period.

[26] Average annual rainfall during the period is 320 mm. Recent irrigation studies on young citrus plantings have shown a water use of 2-5 megalitres per hectare annually. Water savings are possible if irrigations can be allocated to similar units of production such as young trees or reworked sections of a property. In arid regions like Murcia water requirements could be around 20% less and, naturally, they are lower for grown up trees. Note that, as mentioned above, some farmers that are part of water-owner holding use their own water instead of selling it through auctions. Although water stress during draughts affects considerably the quality of production, trees would hardly die as a result. During a normal year without draughts trees could survive the whole year from rainfall. For further details see, for example Chott and Bradley (1997), Wright (2000) and du Preez (2001). Finally, note that although the average number of trees per farmer is 161 (see Table 2), the average number of trees per hectare in our sample is 33 (this number is relatively lower than the conventional spacing for citrus trees of 100 trees per hectare).

[27] A complete characterization of the equilibrium when goods are not pure complements would require further structure on the primitives of the model as the equilibrium depends on the believes about other player's types, and the strategies, that each bidder have.

(we further discuss seasonality below). There is substantial price variation, both within and across four-unit auctions. Table 3 exhibits the distribution of winning prices by both, the number of consecutive units bought by the same individual (1CU, 2CU, 3CU or 4CU) and by sequential auction (1st, 2nd, 3rd or 4th). Interestingly, the greater variation that we observe for 4CU (with respect to non-4CU) has a well defined and consistent pattern. While mean prices for the 1st auction in 4CU are considerably higher than for non-4CU (635.2 against 365.6, 288.3 or 204.6), mean prices for 4th auctions in 4CU are the smallest one. Similar patterns can be seen for the median and maximum prices. Figure 8 decomposes price variation by number of consecutive auctions won by the same individual (left panel) and by sequential auction (right panel). This particular pattern in prices is caused by the above mentioned *deterrence effect* whereby farmers exhibit different behavior based on seasonality and rainfall (i.e., residual demand for water). During high demand and low rainfall months (summer, for example) the same farmer buys all four sequential units paying an abnormally high price for the first unit (with respect to the median or average price conditioning on rain) and prices close to zero for the remaining units. In months where demand is not high (due to farming seasonalities) or where rainfall is high, winning prices for all units are similar in magnitude, regardless of whether the same farmer wins all sequential units (4CU) or different farmers win subsequent units (1CU, 2CU or 3CU).

Aggregate prices over time display consistent trends with the ones found in the empirical literature on sequential auctions. Figure 9 shows that, on average, per unit prices decline by sequential unit (being the first unit of each day considerably higher than second to fourth units, for the reason explained in previous paragraph) and by day of the week (prices decline from Monday to Friday). Figure 9 also shows that per unit prices are also slightly higher during the day than during the night. High water requirements for citrus during summer cause prices to soar during those months (Figure 10). Also as expected, prices are also higher during droughts after conditioning on seasons (Figure 11).[28]

To get a more precise understanding of these patterns and the factors affecting winning prices, Table 4 shows that these correlations are robust after conditioning on past rain, unit, weekday, schedule, week-of-the-year, month and individual fixed effects. Specifically, the table displays the results obtained by regressing daily unit prices on a seven-day-rain moving average (*Rain MA7*), the rain on the day bought in the auction and the mentioned fixed effects. The estimated coefficients on *Rain MA7* have the expected sign and they are statistically significant at 1% level. A 10 millimeter increase in average rain in the previous week is associated with a decrease of 40 pesetas in the equilibrium price paid in the auction. The regression in column (2) adds unit, weekday and schedule fixed effects. The estimated coefficient on *Rain MA7* increases in magnitude and also has the expected sign. It can also be seen from this regression that, as noticed in previous figures, price declines within day and across units (both for daylight and night auctions) and across schedules (price is on average 110 pesetas lower for high auction than for daylight auctions). As regards weekdays average prices, the estimated coefficients show that equilibrium prices decline monotonically within the week. Columns (3) and (4) add week fixed effects and month seasonal dummies to the specification in column (2). The estimated coefficient on *Rain MA7*, though smaller, has again the expected sign and it is statistically different from zero. Column (5) adds both week fixed effects and month seasonal dummies to the specification in column (2). Finally, column (6) adds, additional to the specification in column (5), individual fixed effects (we have 537 different individuals in our sample). Similar qualitative results are obtained; however, the estimated coefficient on *Rain MA7* has increased. Note that the goodness of the fit in this last regression of the table is relatively high, indicating that average (or *ex-ante*) prices can be explained by observables such as rain in the previous week and time of the allocation relatively well (favoring, thus, the idea of common knowledge for the parameters within four-unit auctions). Although not reported, as a robustness exercise, we performed

[28] Drought definition has been studied by several scientific studies. In our analysis, we define a drought as an indicator that is equal to one when average monthly rain during the specific year is below a consensus threshold defined in the literature in terms of the historic annual average (following Gil Olcina (1994) we use a threshold of 40% in 11). For further details, see for example a study from Valiente (2001) where the author presents diverse methodological issues analyzing drought definitions.

an analogue analysis using average daily prices within schedule. Similar results are obtained.

A nice feature of our data is that we are able to observe the identity of the winner. This is crucial to our analysis since it allows us to identify auctions where the same farmer buy all units (4CU). Complementary data from the agricultural census, where we also observe the identity of the land owners, allows us to match these characteristics to each auction's winner. Aside from the variation in these characteristics, which is important to justify our independence assumption across auctions, this data allows us to confirm specialized bidding behavior from certain outliers who own a great amount of land and, therefore, bid and win more often. This is depicted in Figure 13. We also use these characteristics in Section 7.4 to analyze an alternative hypothesis to our model.

4.3 Fitting the Model to the Data

We now proceed to show how the above described data fits the model and we rationalize the modeling assumptions in our context. Validating the model requires to show we can justify, in our empirical setting, the following assumptions: (i) conditioning on auction-specific covariates farmers have IPV, (ii) the relevant unit of analysis for individual bidders' demand are four-unit auctions, (iii) conditioning on auction-specific covariates accounts for possible dynamic strategic behavior relevant to our analysis, (iv) sunk cost and decreasing marginal returns are important features in this market and not a mechanical result, and (v) participation costs affect bidders in the market.

As emphasized in the theoretical model we assume that bidders draw new IPV at every four-unit auction. The independence assumption allows us to model the equilibrium as Bayesian equilibrium. For the estimation we assume that, conditional on observed covariates, farmers at each four-unit auction have IPV. The first justification for IPV is that each bidding farmer (who may or may not be a water-owner) has her own land, her own extension and her own mixture of trees and crops. This eliminates a pure common value scenario. Second, the products being sold are units of water. Assuming that farmers have private (from other farmers) information about the characteristic of this product, would not be in line with the homogeneous nature of water units. Finally, the independence assumption is the most sensible in our context given the varying nature of farming products and soil conditions across farmers. To understand why, recall that in the water market the sellers are a holding formed by the water owners and the buyers are farmers that own fertile land. Around 500 different farmers are observed to win auctions in our sample. Of course, not all these farmers show up at every auction nor decide to participate in the case they show up. Farming products cultivated in the area are mainly fruit and citrus trees (lemon, orange, peach, mandarin and apricot) and vegetables (tomato, lettuce and onion). The amount of water required by the trees depends on the time of the year and type of crop (citrus trees should not be irrigated daily). Moreover, and given that we condition on seasonality, water requirements vary across products. For example, water needs for grapefruit and lemons are about 20% higher than those for oranges, while water requirements for mandarins are about 10% less. Ground conditions (which also vary across areas where different farmers have their own land) also affect water necessity. Table 5 displays appropriate intervals for watering citrus. These variations across farmers generated by these factors, provide support for the fact that, given that each day the market is quite specific and since we work with data for four-consecutive auctions as a unit of analysis (sequential auctions), the independence assumption seems satisfied.[29]

The most comprehensive independent unit of analysis that could be considered are weekly auctions, encompassing all 40 units sold per week. This would be the relevant definition to answer questions related to demand fluctuations generated by supply shocks (such as no-auctions due to draught or excessive rain) on an aggregate level. Alternatively, the narrowest possible unit observed (bought) is a *cuarta* (1 of the 40

[29] Our justification the IPV paradigm is in line with the literature on empirical auctions. For first price descending auctions see, for example, Laffont, Ossard, and Vuong (1995) in an application to agricultural products (greenhouse eggplants in Marmande, France) where the number of bidders vary between 11 and 18. For English-auctionss, Haile and Tamer (2003) apply their limited structure model to U.S. Forest Service timber auctions where the number of bidders vary from 2 to 12.

weekly units). As we explain below in this subsection, the presence of sunk costs and decreasing marginal effects indicate *cuartas* within a day-schedule are not independent. Moreover, they are not the relevant unit of analysis to investigate individual farmers' demand nor the price pattern described above.

Our original question is motivated by the particular price behavior caused by the *deterrence effect*. This particular behavior is observed within four-unit auction and, consistent with this empirical observation, is the unit of analysis in the theoretical model. This is an immediate implication of the way the auction is structured: one hour of water (subdivided into four *cuartas*) during daylight and one hour of water during night, each weekday. The logic behind this structure is related to water requirements in the area. First, water scarcity in the region made water accountability crucial. The standard unit used to measure surface area in Murcia is called *tahulla*. One *tahulla* is, by definition, the surface area which can be irrigated in such a way that water level rises 1-foot high in 1 minute.[30] Needless to say, depending on soil conditions, the surface area from 1-*tahulla* varies from one town to another.[31] A four-consecutive units auction (a *complete hour* of irrigation) is, in that sense, the amount of water that can be absorbed by a regular *parcela* (individual piece of land). Of course, water requirements could and actually do differ (a) across farmers depending on farming trees and land extension, and (b) for the same farmer over time depending on past rainfall.

Second, the irrigation technique used in Mula is *flood irrigation*. This is an ancient method of irrigating crops that it is still one of the most common methods of irrigation used today. The farmer builds small embankments in her *parcela* and water is delivered to the land by the channel system that simply flows over the ground through the crop. Flood irrigation requires a minimum of water delivery that, for a regular *parcela*, is captured by one *tahulla*.

Finally, a supply-side consideration also plays its role. The reason to supply water every 12 hours (one hour during daylight and one during night) is to guarantee a particular and homogenous quantity for each *cuarta* (which depends water crucially on water pressure since units are defined in minutes). The fact that the *De La Cierva* dam is continuously filled with water from the river ensures, by spreading supply provision across the week, homogeneity of water units.

Our data confirms these three points, validating the fact that the relevant unit of analysis for individual demand with the IPV paradigm are four-consecutive units: the most frequent quantity purchased by farmers is a *complete hour* of water (42% of sold units in Table 1).

Of course, the way in which the auction system is carried out every week raises the question of the importance of dynamic strategic considerations between four-unit auctions both among days (Monday to Friday for a specific schedule) and between schedules (Day vs Night for a specifc day). Table 4 shows that, consistent with the literature on empirical sequential auctions, winning prices decline across days (for a given schedule) and at night (for a given day). Although interesting, these dynamic strategic considerations are outside the scope of the present investigation, and we abstract from them in our model in Section 3 (for a broader discussion see Donna and Espin-Sanchez (2011)). It is important, however, to not that, even if present, dynamic behavior considerations do not invalidate our model's assumptions. As emphasized above, the independent unit of analysis are four-unit auctions (not day-auctions of 8 units nor week-auctions of 40 units) which, conditional on covariates, are homogeneous goods. As is evident from the correlations presented in Table 4, previous patterns are consistent along the whole sample and robust to the inclusion of a whole set of fixed effects and covariates. Moreover, residuals from these reduced-form regressions do not show dynamic patterns. This is not surprising as the principal difference between these four-unit auctions is related to the uncertainty of future rain. As it is explained later in the estimation section, we will include covariates for schedule, day-of-the-week and past rain in our estimation that would capture any technological or strategic effect. Future rain, on the other hand, will also be included as a proxy for farmer's beliefs to account

[30] The traditional Murcian measure of foot is not exactly the same to the American foot.

[31] The surface area of 1-*tahulla* is 1,118 square meters in Murcia and 1,185 square meters in the old *Kingdom of Aragón*, with the exception of the region of *Pías Fundaciones*. The *tahulla* is used in *regadio* lands since Charles IV (king of Spain from 14 December 1788 until his abdication on 19 March 1808). In *secano* lands the surface area measure that is used is the *fanega* and the *celemín*. For further details see Vera Nicolas (2004).

for these (possible) strategic behaviors not accounted by previous covariates. In that sense, our estimates should be interpreted as four-unit day-schedule specific auctions, conditioning on past rain and seasonality. It seems highly implausible that after accounting for these observables and unobservables,[32] and given that the relevant unit of analysis are four-unit auctions, dynamic behavior would not affect our results concerning individual demand.[33] Nevertheless, we further investigate the residuals after our estimation in Section 7.2 to analyze specific patterns related to this concern.

There are two main specific features from the model that need to be justified in our empirical setting. First, the sunk cost (SC) that farmers incur when they buy their first unit. Water is allocated during the auction and is distributed on the specific day and time of the irrigation accordingly. Water stored in the dam is delivered to the farmer's plot on this date using the channel system. With the exception of the main canal, all channels are dug into the ground. On the date of the irrigation, a guard opens the corresponding gates in order to allow the water to flow to the appropriate farmer's land. These channels are land-specific in the sense that different areas and lands have their own system of channels which only carry water when the corresponding gates are opened.[34] There is a water loss that is incurred due to the fact that water flows over a dry channel. Engineers have estimated this loss to be between 15% and 40% of the water carried by one *cuarta* when the channel is completely dry (see Vera Nicolas (2004)). This is the SC incurred by the bidder for her first unit. Of course, the SC is only incurred once, for the first unit, since water losses associated with a wet channel are negligible. In the model, the SC effect is captured by the parameter α, whose interpretation is the percentage of water loss from the first unit due to the fact that water is flowing through a dry channel (hence proportional to the valuation of the bidder for the unit of water). One would expect that, conditioning on rain, water loss would be constant within season with relatively more importance (higher α) in summer.[35]

The second feature refers to the decreasing marginal returns (DMR) effect. The classic textbook case for DMR is appropriate for our empirical application: given that the amount of land owned by each farmer is fixed, marginal productivity of subsequent units of water is decreasing. When assessing the relative importance of DMR, the impact in summer would generally be greater than in autumn. More generally, one would expect DMR to be affected by season and rain. When water requirements are high, the slope of the marginal productivity function will be relatively flat, as in the left panel in Figure 2. This is likely to occur in spring and summer. On the other hand, when water requirements are low, the slope of the marginal productivity function will be steeper, as in the right panel in Figure 2. This is likely to happen in autumn or winter. In the model, the DMR effect is captured by the parameter β. In Section 6.1 we discuss how we model DMR for the estimation to take into account the points addressed in this paragraph.

There are several reasons to believe that farmers face participation costs in this market. First, the most obvious component are opportunity costs. Farmers who value their time may prefer not to participate in the whole auction session. Auctions were run on Friday evenings during work time. Attending the auction entailed alternative use of working time for the farmer. To contextualize this component in our empirical setting, it is worth noting that it is not uncommon to observe the farmer's wife, a son or other relative substitute for the farmer on certain occasions.[36] Another component of participation costs are the various hassle costs associated with active bidding. Actually, only a fraction of the individuals who attend a Friday

[32] While farmers' use their (reasonable good) predictions in their decisions, we use *actual future rainfall* in our estiamtion.

[33] Once we have conditioned on these covariates, the concern that bidders' outside option would vary according to the day of the week (or schedule), can be addressed by redefining the idiosyncratic individual valuation in such a way that the new one be the original valuation net of the outside option. By normalizing Friday-night's outside option to zero the model's assumptions remain valid even in such scenario.

[34] An obvious concern is that farmers whose lands lie next to each other may be buying different sequential units for the same auction. In this case, the SC would only be incurred by the first farmer for her first unit but not for the second farmer for her first unit. Fortunately, we have data on the specific location of the farmers that we are able to match to auctions' winners to analyze these situations as we do in 7.4.

[35] We discuss variation of SC across auctions (conditioning on covariates) in Subsection 6.1.

[36] We are able to identify these cases by matching census land data (with information on the person who registered the property) with auctions data (where the winning bid names that is written is a relative of the name that is written in the census card). In other situations is even more evident since the winning name in the auction's sheet just states *Wife (or son) of* the farmer who owns the land.

auction are actively engaged in the bidding for a particular sequential auction of water and not everyone who is present participates in every auction. As Von der Fehr (1994) points out, a reasonable assumption for why not all attendees participate may be that they consider it so unlikely to that they will win at a price below what they will be willing to pay, that they are not willing to bother to engage in bidding. Finally, and related to the previous argument, participation costs may also be interpreted as the cost of preparing the bid and the cost of learning what the item is worth in the specific environment (i.e., conditional on the specific covariates of the auction).

Empirical evidence from our data is consistent with the assertion that farmers dislike participation, facing positive entry costs. In particular we observe multiple weeks per year where auctions are run, farmers show up and buy the first units of water, but where nobody bids for the last units. Since there is no reservation price and the minimum bid increment are cents, they could potentially win all the remaining units bidding this amount. On the contrary, they decide not to bid and leave the auction. For example, on January 22, 1954 units 1 to 12 were sold to 9 different farmers but nobody bid for units 13 to 20 (Figure 5). In 1954 we observe similar behavior for 14 weeks,[37] and this is consistent along the remaining years in our sample. Of course, the interpretation is that, conditioning on covariates, the utility for all bidders is smaller than the participation cost. We later use this information to identify these costs.

5 End-Digit Preferences and Regime Determination

When goods are *pure complements*, very low prices (symbolic prices) are paid by the same winner for second, third and fourth units (Figure 3). Although predicted price pattern by our model for each each *regime* (*pure complements* and *non pure complements*) provides us with a straightforward empirical method to determine them, we show in this section that end-digit preference for these prices is consistent with the empirical prediction, without specifying further assumptions on the model's primitives.[38] When goods are pure complements a key prediction from proposition 5 is that the same bidder will win all units, pay her valuation for the whole bundle in the first auction, and pay a price of zero for the second, third and fourth units. The baseline model implicitly assumes that the seller will not require a reserve price for the second, third and fourth objects. As we discuss further in this section, we do not observe zero prices (for second, third and four units in the data), but prices very low (relatively to the first price), that is, a symbolic price. This can be seen in light color in Figure 14, that displays the histogram (for 4CU auctions) of the percentage change of the first winning price against the median of the second to fourth prices. The baseline model can be easily modified to take into account this phenomenon by introducing a minimum price for the second to fourth objects that is a function of the first price. Although there is no reserve price in the real auctions, the minimum price can be interpreted as a general agreement to bid a symbolic price in subsequent auctions. We further discuss this issue in the rest of this section. A commonly found effect in our data is that farmers bid certain preferred end-digits prices substantially more often than others. This provides us with valuable information to determine both regimes.

Kandel, Sarig, and Wohl (2001) use Israeli IPO auctions to present evidence that investors have end-digit preference for round numbers (prices that end with 0 or 5) and that prices that end with 0 are used more often than those ending with 5. To the best of our knowledge there has not been an examination of the effect of end-digit preferences on inference in the auctions literature. Digit preference and systematic age misreporting are important and broadly studied issues in demography, particularly in survey and census data when respondents inaccurately report ages or dates of birth (Myers (1940), Das Gupta (1975), Coale and

[37] Weeks of January 22, February 5, April 5, May 1, May 8, May 15, May 22, May 29, June 5, June 12, July 3, July 10, November 26 and December 3.

[38] Alternatively, price data in these water auctions could be interpreted as indirect observations of a latent distribution subject to digital preference, that is, with tendency to round prices to pleasing digits (as we discuss below). See for example, Camarda, Eilers, and Gampe (2008) for a recent study showing how digit preferences can me modeled combining a composite link model and penalizing the likelihood function to uncover the latent distribution.

Li (1991) and Siegel and Swanson (2004)). The concern in these cases is, typically, heaping on particular ages such as those ending in 0 or 5. Crayen and Baten (2009) use some of these techniques to investigate the phenomenon of age heaping, and to test the hypothesis that an unequal distribution of human capital reduces welfare growth. Baker (1992) focuses on digit preferences in CPS unemployment duration data, where he raises the question of what can be said without making any specific assumptions concerning the true nature of digit preference in the CPS, and shows that employment duration is quite sensitive to the choice of a corrective for digit preference. Finally, digit preference has also been studied in medical literature pertaining to individual report body weight and height, blood pressure and cigarette consumption (Bopp and Faeh (2008)). Our set up is different from all these studies as we do not (nor do we have a compelling argument to) a priori impose a uniform (nor a particular) distribution on end-digit preference.

Table 6 shows in columns 4 and 5 the frequency distribution by the last digit of price for first-unit prices.[39] We observe strong preferences for 0 and 1, and somewhat weaker preferences for 5. In 33.6% of the cases we see an exact multiple of 10, in 34.1% we see a price ending in 1, while 8.6% report a multiple of 5. The frequencies also show some preference for 2 and 6, but not a marked one. After taking into account these effects we find that 48% of the first-unit price observations are inconsistent with a uniform distribution in each digit.[40] That is, we would need to reclassify 48% of the cases to obtain a uniform distribution by digit. This is clearly not the case for our underlying distribution. We interpret these results as strong end-digit preference for 0 and weak end-digit preference for 5.

This is perhaps not surprising because initial bids are often approximations taken in a relatively arbitrary fashion. An important feature that is captured in our model is that, unlike sealed bid auctions, English-auctionss reveal the identity of all bidders. The highest bidder at any given moment is considered to have the standing bid, which can only be displaced by a higher bid from a competing buyer. If bidder i's valuation is sufficiently above the standing bid, it seems natural to skip certain prices and to bid directly the next round price. In that sense, the bidders may informally round their marginal utility to the nearest end-digit bid, according to our data ending in 0 or 5.

We can replicate this end-digit behavior by assigning a simple rule to the bidding strategy. At each bidding round, one of the remaining bidders is assigned to bid in a predetermined manner (the bidder assignment could be deterministic or random). Then, this bidder can drop the auction or make a bid greater than the existing highest bid. If she decides to bid, she will bid a number ending in 0 with probability q_0, a number ending in 5 with probability q_5 and a number just one *peseta* above the existing bid with probability $(1-q_0-q_5)$. This bidding behavior is characterized by parameters q_0 and q_5, and the probability of dropping at any given point. With the proper restrictions in these elements, such as $q_0 > q_5$, we can replicate the pattern observed on the data, that displays peaks at 0 and 5, as well as the exponential decline following 0 and 5.[41]

This behavior can be rationalized in a model where $(q_0 + q_5)\%$ of the bidders have preferences for round numbers (0 and 5), while the remaining bidders $(1 - q_0 - q_5)\%$ do not. If the private valuations from both types of bidders are drawn from the same distribution, then the assumption that bidders drop at random is satisfied. Alternatively, a different way to rationalize this behavior, is to assume that bidders incur a cost each time they make a bid. These models are known as *penny auctions*: open ascending auctions in which it is costly to make every bid. In these models, a necessary condition for equilibrium is to to randomize between bidding one *penny* above the actual winning bid, or to jump bidding. If the equilibrium jumps take the form of a round number, this would also fit the behavior described above.

Strong preference for digit 1 is not, in general, an indication of a preference for this digit *per se* but,

[39]We obtain similar patterns if we restrict the sample by season, month and/or schedule (daylight or night).

[40]This number corresponds to the value of the Myer's blended index, used to obtain a *blended population* which helps control for the mentioned trends. In the absence of digit preference one would expect 10% in each terminal digit.

[41]If the probability of dropping the auction is increasing (decreasing) in the existing highest bid, then we will observe a positive (negative) relationship between the frequency of 0 and 5, and the price paid by the winner. This implies a relationship between the hazard rate of the distribution and end-digit preferences.

instead, a sign of competition. According to our model, first-unit prices are always competitive (in the sense that all N bidders will enter the auction as no information has yet been revealed), regardless of the regime. Nevertheless, second to fourth-unit prices are not competitive in the pure complements regime (competition in this regime takes part in the first unit where they bid for the whole bundle, and then pay a symbolic price for the second to fourth units since it is optimal for the remaining $N-1$ bidders not to enter in these sequential auctions). Hence, end-digit preference for 1 in second to fourth-unit prices, as a sign of competition, are indicative of a *non pure complements* regime. Alternatively, end-digit preference for 0 for second to fourth-unit prices are indicative of a *pure complement* regime. Moreover, in the pure complement regime the model predicts that all second, third and fourth consecutive prices will simultaneously behave in this fashion. Columns 6 and 7 in Table 6 display the frequency distribution by the last digit of price for the second to fourth units. It can be seen that prices exhibit a pattern consistent with this description.

This behavior provides us with a natural lower bound for the *pure complements* regime, namely, second, third and fourth unit prices within the same four-unit auction show a strong end-digit preference for 0. This behavior, that comes from the empirical prediction of the model, will allow us to identify the two regimes. We could also use weaker or stronger definition of end-digit or round-number preferences to obtain different bounds for the empirical distributions of prices in each regime.[42]

Specifically, the strongest version of the empirical prediction would be that all second, third and fourth prices display, for the same individual, an end-digit preference for zero in any given four-unit auction. A weaker version would be that two out of the three (among second to fourth) prices show an end-digit preference for 0. The weakest version is that just one of these three prices exhibit an end-digit preference for zero. Clearly, the last (weakest) specification only provides us an upper bound for *pure complements* regime identification since, as shown in Table 6, the underlying distribution displays an end-digit preference for zero even for the *non pure complements* regime. Figure 14 displays the histogram of the percentage change of first price against the median of second to fourth price, by regime.[43] It can be seen in the figure that end-digit preference behavior (as defined above) also captures, in general, the other empirical prediction from the model, namely, that prices are *competitive* in the *non pure complements* regime but exhibit the *deterrence effect* in the *pure complements* one (the way to see this in the figure is that percentage change from the first to the second, third or fourth prices is high when goods are *pure complements*). This is remarkable as the end-digit preference behavior used to identify the regime is unrelated *a priori* to this second empirical prediction. This provides further evidence in favor of our model.[44] Regime identification is done by using the strongest version of the empirical prediction to identify the *pure complements* case, i.e. the case in the left panel in Figure 14.

A final robustness check further shows that the approach in this section consistently identifies both regimes in terms of our model. Columns 2 to 5 (first unit) in Table 7 display, by regime, the frequency distribution in terms of end-digit prices for first-unit, among each four-unit auction. As emphasized above, both regimes should exhibit competition for first-unit prices according to our model. This competition is captured by the same distribution among ending digits in both regimes. This is what we observe in columns 2 to 5 (first unit). The last two columns in Table 7 (fourth unit) show that, as predicted by the model, fourth-unit prices for *non pure complements* are also competitive (the percentage of preference is 29.3 for 0 vs. 39.4 for 1).[45]

[42] We could instead assume that in the *pure complements* regime second, third and fourth unit prices within the same four-unit auction show simultaneously a strong end-digit preference for 0 or 5. Although our baseline identification assumption focuses only on end-digit preferences for 0 for sake of transparency (in the way we define this preference), our results are robust to include end-digit preference for 5 as well.

[43] The figure looks similar if we use the second, or the third, or the fourth, or the average of second to fourth prices.

[44] Using a modified version of this assumption that differentiates end-digit preference for prices ending in 0 that exhibit more frequency (e.g. prices like 100 are more frequent than 150) yields almost identical results.

[45] These results are consistent under different specifications of end-digit preference behavior.

6 Identification

In this section we discuss identification of the distribution of private valuations and structural parameters conditioning on the specific regime being played. The algorithm described in previous sections allow us to separate data into four categories:

a) Same bidder wins all four units and goods are *pure complements*, $\beta \leq \frac{\alpha}{3}$.

b) Same bidder wins all four units and goods are *non pure complements*, $\beta > \frac{\alpha}{3}$.

c) Last winner also bought two out of the first three units, three units in total, $\beta > \frac{\alpha}{3}$.

d) Otherwise.

According to our model, categories c and d are only consistent with a regime where goods are *non pure complements*.

Our model does not require symmetry in the distribution of private valuations. However, in our empirical application we only observe the identity of the winner who pays the valuation of the runner up (second highest valuation). Identification in this case requires stronger assumptions on F_i (with respect to the symmetric case, where $F_i = F \ \forall i$). Alternatively, if we maintain the (arguably strongest) assumption of homogeneous bidders, we can let the distributions of valuations be more flexible.

The foundations for identification come from corollaries 6 and 8 (equations 3, 4 and 5). Note that the system provides us with three restrictions in the parameter space, not six. For the rest of our analysis, we focus in the symmetric case (the rationale for identification for the non symmetric case is, however, similar once we specify a parametric distribution for valuations).

6.1 Parametric Identification

We now discuss identification by using a specific parametric functional form assumption for the distribution of private valuations. The question is whether the joint distribution of private valuations can be recovered from the distribution of (observed) winning bids alone. The distribution of the Jth order statistic from an *i.i.d.* sample of size N from an arbitrary distribution $F_V(.)$ is given by:[46]

$$F_{J:N}(x) = \frac{N!}{(N-J)!(J-1)!} \int_0^{F_V(x)} t^{J-1}(1-t)^{N-J} dt \ \ \forall x$$

Since the right-hand side is strictly increasing in $F_V(x)$, for any J and N we can define the function $\phi(F_{K:N}(v); K, N) : [0,1] \to [0,1]$ implicitly by:

$$F(x) = \frac{N!}{(N-J)!(J-1)!} \int_0^{\phi(x)} t^{J-1}(1-t)^{N-J} dt \ \ \forall x$$

This result is immediately useful in our sequential English-auction model where bids are independent draws from a distribution $F_V(.)$ and the equilibrium (observed) transaction price is a function of the second highest valuation, $v_{N-1:N}$.[47] Thus, conditioning on goods being *pure complements*, the distribution of valuations is identified up to the the multiplying constant using equation 3 and the previous result. To identify the remaining structural parameters, α and β, we use Corollary 8. Equations 3 to 5 jointly identify the distribution of private values and the structural parameters α and β.

For our estimation we will allow decreasing marginal returns, β_t, to vary across auctions. Specifically, we are going to model them as a function of farmers' expectations of rain that we are going to proxy by actual (observed) future rain:

[46] Arnold and Nagaraja (1992) provide an extensive discussion on order statistics, while Athey and Haile (2002) discuss its application for the case of empirical auctions.

[47] Also note that this result extends immediately to cases where valuations are affected by auction-specific covariates, Z_t. In this case, $F_V(.|Z_t)$ is uniquely identified by $F_{N-1:N}(.|Z_t) \ \forall Z_t$.

$$\beta_t = \beta_0 + \beta_1 R_t^F + \eta_t$$

where R_t^F refers to future rain and η_t is a seasonal (monthly) fixed effect. Additionally, we further let β_t have different intercepts in each regime:

$$\begin{aligned} \beta_{t,s} &= \beta_s + \beta_1 R_t^F + \eta_t \\ \beta_{t,c} &= \beta_c + \beta_1 R_t^F + \eta_t \end{aligned} \tag{6}$$

An heuristic argument to understand the reasons behind this behavioral equation is given in Table 8. This table presents probit regressions of a dummy variable identifying the regime (*pure complements* and *non pure complements*) on future rain and other covariates. As noted above, we interpret future rain in these regressions as a proxy for (aggregate) expected future rain for the farmers. Table 8 shows that low expected rain and high demand months (May to August) significantly increase the likelihood of being in a *pure complement* regime. Of course, the correct interpretation is the opposite. Farmers have some information about future rain. While the idiosyncratic part of this information is captured by their type v_i, the common part is captured by β_t. When farmers expect, on aggregate, no rain in a given day, they will coordinate to play in the *pure complements* regime. Seasonality also affects the demand for water and affects the position of a farmer in the curve in Figure 2. The results in Table 8 reinforce our hypothesis that it is the slope on the marginal returns effect that drives the change of regime.

Finally, let $v_i \sim F(v;\theta)$, where θ is a parameter characterizing the distribution of valuations, F. Equations 3, 4, 5 and 6 jointly identify the parameter vector $(\theta, \alpha, \beta_s, \beta_c, \beta_1)$, conditioning on regime identification (that we identify non parametrically as described above) and (exogenous) covariates. The full system of equations is given by:[48]

$$\begin{gathered} \left[4-\alpha-6\beta_{t,c}\right] v_{N:N} - 3c \geq \sum_{k=1}^{4} p_a^k = \left[4-\alpha-6\beta_{t,c}\right] v_{N-1:N} - 3c \\ (1-3\beta_{t,s}) v_{N:N} \geq p_b^4 = (1-\alpha) v_{N-1:N} - c \\ (1-2\beta_{t,s}) v_{N:N} \geq p_c^4 = (1-\beta_{t,s}) v_{N-1:N} - c \\ \beta_{t,s} = \beta_s + \beta_1 R_t^F + \eta_t \\ \beta_{t,c} = \beta_c + \beta_1 R_t^F + \eta_t \end{gathered} \tag{7}$$

Finally, note that in our parameterization we fix α across auctions (and seasons, of course) but we allow β_t to vary. This is necessary to identify them separately but, ultimately (and as evident in the sequential auction model), it is the relative magnitude of the effects that matters. The reason why we let β_t vary (instead of α) is related to the fact that the reason behind a regime switch is given by the (residual) demand for water by the farmers, which is determined by rain and seasonal effects. Of course, we expect α to vary across auctions and seasons as well. Given that this variation is not separately identified from the variation on β_t we should interpret the estimated changes in β_t as relative changes with respect to α.[49]

[48] The third equation in the system should be in general equal to $p_b^4 = Max\left\{(1-\alpha)v_{N-1:N}, (1-\beta)\, v_{N-2:N}\right\}$ since we do not know whether the runner-up in the last auction was the bidder that already won one unit or a bidder without previous purchases. However, when N is large we have $v_{N:N} \simeq v_{N-1:N} \simeq v_{N-2:N}$ so $(1-\alpha)v_{N-1:N} < (1-\beta)\, v_{N-2:N}$ will only happens if $\beta \simeq \alpha$. But in this case we will not expect that the same bidder to win three out of four units. In other words, in an auction where N is large and the same bidder wins three out of four units, we expect β to be greater than α, thus $p_b^4 = (1-\alpha)v_{N-1:N}$.

[49] Alternatively, if the parameter α_t is also allowed to vary across auctions, our estimate for β_t could be thought as an upper bound for the real β.

7 Estimation

In this section we discuss the estimation procedure with homogeneous bidders and how we handle the difficulties that arise from it.

The econometric problem consists of finding the common distribution of valuations F and structural parameters that best rationalize the bidding data. As discussed in the previous section, the bid levels at which bidders drop out of the auctions are not observed with the exception of the bidder with the second-highest valuation. Our model allows us to identify, however, the regime where bidders are playing before performing the estimation, as discussed in the non-parametric regime identification section. Let T denote the total number of four-unit auctions and t the subscript for all relevant variables to the tth auction.

A second concern arises from a likely heterogeneity across auctions. Observed heterogeneity arises due to seasonal effects, rain, and the day and time of the week when the auction occurs. This means that the distribution of private values for the tth auction, $F_t(\cdot)$, is not constant across auctions. In our estimation we recover the family of distributions $F(\cdot|Z_t, \gamma)$. That is, we assume that for every four-unit auction, $F_t(\cdot) = F(\cdot|Z_t, \gamma)$, where $\gamma \in R^k$ is a parameter vector and Z_t is a vector of fully observed characteristics describing the environment of the tth auction. We describe the inclusion of these covariates in next subsection.

The number of potential bidders, N_t, is not observed. We assume that it is constant for every four-unit auction, $N_t = N$, and use the names of auction winners to infer it. Table 9 displays the timing structure for different bidders in our sample. These numbers should be interpreted as an upper bound on the actual number of auction winners. The reason is that many names correspond to members of the same family that owns only one farming land (as it is clear from the agricultural census data), which is the relevant unit for water use. For our estimation, we let the number of potential bidders in each auction be the yearly average of different farmers that win auctions in our sample. We perform a sensitivity analysis on this assumption.

An additional concern that, ultimately, maintains an important relation with the empirical setting is that the econometrician may be less informed than the bidders. Throughout, we have assumed that the vector Z_t of covariates is fully observed by the econometrician. In our environment, unobserved heterogeneity implies that the distribution of bids may not independent across t. All farmers may, for example, observe some factor (unobservable by the researcher) that shifts the location of the distribution values. This unobserved heterogeneity could lead to correlation among bidders' valuations, causing an identification problem and inconsistent estimates to arise. From the agricultural census data we observe individual characteristics of the farmers that we are able to link to the winning bids. Given the structure of the agricultural water market we are modeling it, does not appear to be an important concern once we consider the homogeneity of the selling good and the observed characteristics we introduce in our estimations (seasonality, past and future rain, among others). Modeling unobserved heterogeneity requires additional assumptions on the behavior of unobservables (independence, separability, strict monotonicity) and is outside the scope of this paper.[50]

Unlike in Donald and Paarsch (1993) and Laffont, Ossard, and Vuong (1995), the support of the distribution of the winning bid as defined in the previous section does not depend on the parameter vector. Maximum likelihood estimation is possible even in our setting where losing bids are not observed.[51] [52]Let $\delta \equiv (\alpha, \beta_0^C, \beta_0^S, \beta_1)$ and let v_i be independent draws from a parametric distribution $F(\cdot|\theta, \delta, Z_t, \gamma)$ where $\theta \in \Theta^k$ is the parameter of certain family of distributions. Then, the likelihood function is given by:

[50]For a discussion on this issue see, among others, Athey, Levin, and Seira (2004) for an application to timber auctions and Krasnokutskaya (2004) for a semi-parametric approach to Michigan highway procurement contracts. In a recent investigation, Roberts (2009) use information contained in reserve prices to allow bidders' private signals to depend on the realization of the unobserved heterogeneity.

[51]Alternatively, we could use econometrics methods like simulated non linear least squares (SNLLS) as in Laffont, Ossard, and Vuong (1995)or GMM (Hansen (1982)).

[52]Another potential concern with the likelihood approach in this context is the assumption that the winning bid equals the second highest valuation (after considering sunk cost and decreasing marginal returns' effect). This assumption is likely to be violated when bids rise in discrete steps and, specially, in cases where bids increase faster than the required minimum (i.e., jumping bidding). This is clearly not the case in our setting. Winner prices exhibit great variation (see summary statistics) and cents bids are frequently observed.

$$L(\theta,\delta,\gamma \mid p_t^i, R_t^F, Z_t, \{D_t^j\}_{j\in\{a,b,c\}}, N) = \prod_{t=1}^{T} f_{N-1:N}\left(\frac{\sum_{k=1}^4 p_t^k}{4-\alpha-6(\beta_c+\beta_1 R_t^F)}, \theta \mid \delta,\gamma,R_t^F,Z_t,\{D_t^j\}_{j\in\{a,b,c\}},N\right)^{D_t^a} \mathbf{x}$$

$$f_{N-1:N}\left(\frac{p_t^4}{1-\alpha}, \theta \mid \delta,\gamma,R_t^F,Z_t,\{D_t^j\}_{j\in\{a,b,c\}},N\right)^{D_t^b} \mathbf{x}$$

$$f_{N-1:N}\left(\frac{p_t^4}{1-\beta_s+\beta_1 R_t^F}, \theta \mid \delta,\gamma,R_t^F,Z_t,\{D_t^j\}_{j\in\{a,b,c\}},N\right)^{D_t^c}$$

where $f_{N-1:N}(v,\theta)$ is the probability density function (PDF) of the $N-1$th order statistic from a sample of N from the distribution of valuations F, $D_t^a + D_t^b + D_t^c = 1\ \forall t$, and D_t^a, D_t^b, D_t^c are respectively indicator variables for *pure complements* regime, *non pure complements* where the same bidder wins all units and *non pure complements* where the same bidder wins three units, two out of the first three units and the last unit.

7.1 Distribution of Private Values

Our model and the context of the market under analysis provide insight on how farmers and auctions' characteristics should affect private values, but it offers little guidance on the functional form of this distribution. We assume valuations follow an Exponentiated Gamma (EG) distribution for each four-unit auction.[53] The cumulative distribution function (CDF), PDF and moments of the EG distribution are given, respectively, by the following equations:

$$F(v;\theta) = \left[1-(v+1)e^{-v}\right]^{\theta}, \qquad v>0, \quad \theta>0 \tag{8}$$

$$f(v;\theta) = \theta v e^{-v}\left[1-(v+1)e^{-v}\right]^{\theta-1}, \qquad v>0, \quad \theta>0 \tag{9}$$

$$\mathbb{E}(V^r) = \theta\int_0^\infty v^{r+1}e^{-v}\sum_{i=0}^{\infty}\binom{\theta-1}{i}(-1)^i e^{-iv}(1+v)^i dv \tag{10}$$

The reasons to use an EG distribution are related to tractability and the empirical distribution of the $N-1$th order statistic in our data. In the first place, the EG distribution gives us a close form solution for the distribution of the jth order statistic that is characterized by a single parameter. Additionally, the density distribution of the jth order statistic is a weighted average of EG densities. This is useful in view of the computational difficulties of the MLE approach. Second, it has positive support. We are considering v_i to be the valuation that a farmer has for 15 minutes of water, which we expect to be always positive. Finally, and most important, the PDF distribution of the jth order statistic of the EG distribution has a convenient 'bell' shape form. Although not exclusive from the EG distribution, this feature fits well the empirical distribution we observe in our data. Figure 15 depicts the PDF of both the EG distribution and the corresponding $N-1$th order statistic for different configurations of the parameter θ. Using the EG distribution, we can further expand the previous likelihood function plugging the distribution of the second highest order statistic, $f_{N-1:N}(v,\theta)$:[54]

[53] See Shawky and Bakoban (2008) and Shadrokh and Pazira (2011) for further details on this distribution. Note also that when the shape parameter $\theta = 1$, the EG distribution is identical to a gamma distribution $\Gamma(2,1)$.

[54] We perform a Monte Carlo study in Appendix B to evaluate how well the proposed estimation procedure performs in our setting.

$$L(\theta,\delta,\gamma \mid p_t^i, R_t^F, Z_t, \{D_t^j\}_{j\in\{a,b,c\}}, N) = \prod_{t=1}^{T}\left[\sum_{i=0}^{1} d_i(N,N-1) f\left(\frac{\sum_{k=1}^4 p_t^k}{4-\alpha-6(\beta_c+\beta_1 R_t^F)}, \theta(N+1+i) \mid \delta,\gamma,R_t^F,Z_t,\{D_t^j\}_{j\in\{a,b,c\}},N\right)\right]^{D_t^a} \times$$

$$\left[\sum_{i=0}^{1} d_i(N,N-1) f\left(\frac{p_t^4}{1-\alpha}, \theta(N+1+i) \mid \delta,\gamma,R_t^F,Z_t,\{D_t^j\}_{j\in\{a,b,c\}},N\right)\right]^{D_t^b} \times$$

$$\left[\sum_{i=0}^{1} d_i(N,N-1) f\left(\frac{p_t^4}{1-\beta_s+\beta_1 R_t^F}, \theta(N+1+i) \mid \delta,\gamma,R_t^F,Z_t,\{D_t^j\}_{j\in\{a,b,c\}},N\right)\right]^{D_t^c}$$

where $f(v,\theta)$ is the PDF of the EG(θ) and $d_i(n,r) = (-1)^i\ n\frac{\binom{n-1}{r-1}\binom{n-r}{i}}{r+i}$.

To recover both, the structural parameters and the distribution of values conditioning on the vector of covariates we let the distribution of valuations, depend on various characteristics that are drawn from our data (observed heterogeneity), Z_t, as discussed above. We assume, specifically, that observed prices follow a linear function of the following exogenous variables and estimate all parameters using the likelihood function:[55]

$$p_t^i = Z_t'\gamma = \gamma_0 + \gamma_1 R_t^P + \gamma_2 Night_t + \sum_{i=2}^{4}\gamma_{1+i} Day_t^i + \sum_{i=2}^{12}\gamma_{4+i} Month_t^i$$

The first exogenous variable, R_t^P, refers to *Past Rain*, a moving average of the rain beginning seven days prior to the date of the auction. The second variable is a dummy variable that equals one if the water was bought for night use. The next four variables are a set of dummy variables for each weekday. Finally, the last eleven variables are a complete set of month dummy variables to condition on seasonality. The nature of the trees cultivated in the area makes water prices soar in our market during summer and drop in winter. We accommodate these shocks to demand with seasonal monthly dummy variables.

Although throughout the previous estimation procedure, participation costs, c, have been fixed at an arbitrary small magnitude, they can be recovered from our data. To do that we use our model and data where auctions were run, no bids observed and farmers were present, along with the ML estimates. Participation costs are identified by the necessary condition for a bidder to bid in the first auction that is given by:

$$(1-\alpha)v_{N:N} < c$$

More generally, a condition that additionally involves second, third and fourth marginal utilities for the case where the bidder also enters the individual auctions for 2, 3 or 4 units should be considered. In these cases, participation costs are also greater than the average marginal utility for second, third and fourth units. Formally:

$$Max\left\{(1-\alpha), \frac{(2-\alpha-\beta_t)}{2}, \frac{(3-\alpha-2\beta_t)}{3}, \frac{(4-\alpha-3\beta_t)}{4}\right\} v_{N:N} < c \tag{11}$$

Note that when $\alpha < \beta_t$ that the former condition is sufficient, implying the latter. In our econometric specification the structural parameter α is fixed while the parameter β_t varies according to the farmers' expectations of (exogenous) future rain. One would expect to observe auctions without bids when farmers's expectations for rain, as captured by actual future rain, are high (which in the model is represented by a relatively high β_t). This is what we observe in the data. Therefore, absence of bids will only be present in a situation where $\alpha < \beta_t$, also making the former the sufficient identification restriction.

Analogously, using the model and the remaining data not used in the structural estimation, an upper bound can can be obtained using the fact that participation cost are lower than the minimum registered price (conditioning on covariates, SC and DMR).

[55] Laffont, Ossard, and Vuong (1995) assume that private values follow a log-normal distribution and let the mean of the logarithm of the valuations be a linear function of exogenous characteristics. Haile and Tamer (2003) condition on covariates by constructing the conditional empirical distribution functions using Gaussian kernels.

7.2 Estimation Results

In this section we present the estimation of the structural model associated with the theoretical benchmark of Section 3 under various econometrics specifications. We present ML estimates obtained by maximizing the likelihood function presented above. We use a tolerance level of 1.0e-09, let private valuations for each four-unit auction follow an EG distribution, and follow the procedure described in previous subsections. As discussed in Subsection 4.2, the number of bidders, N, is determined by the mean number of bidders that won auctions during a specific year. In our 13-year sample, the average number of active bidders is slightly below 50. It is worth mentioning that each of these farmers regularly won auctions (it is reasonable, therefore, to assume that they attended the auctions). Table 10 presents our estimation results. Columns 1 and 3 present the estimates for $N = 50$, while columns 2 and 3 do it for $N = 40$.[56]

The top part of the table contains the estimate of the model's structural parameters. All parameters have the expected signs. We use the estimate of the parameter θ (that characterizes the distribution of private valuations), to compute the mean valuation of the complete first unit of water. In the case of column 4, the value of the complete first unit of water is 365 *pesetas*. As expected, in the specification in column 3 (with 50 different bidders), the mean value of the complete first unit of water is slightly lower, 324 *pesetas*.

The parameter β_1 captures the effect of future rain. As farmers expectations of future rain increase, DMR are more severe ($\beta_1 > 0$), increasing the likelihood of coordinating in a *non pure complements* regime and, therefore, reducing their valuation of subsequent units of water ($\frac{\partial p_t^i}{\partial R^F} < 0$). One way to evaluate the economic significance of the parameter, is to compute the implied price elasticity, $\frac{\partial p_t^i}{\partial R^F} \frac{R^F}{p_t^i}$. In particular, the value of this elasticity evaluated at the mean price and future rain from the sample is -4.7% (we compute the derivative numerically using the system 7). Predicted DMR are obtained by adding the estimates of intercepts β_c and β_s to the one of β_1, conditioning on the rain in the day of the auction. When evaluated at the average future rain from each regime, the null hypothesis that overall DMR are lower in the *pure complements* regime cannot be rejected (*p-value* above 10%):

$$H_0: \quad \hat{\beta}_s + \hat{\beta}_1 \mathbb{E}_s(R_t^F) > \hat{\beta}_c + \hat{\beta}_1 \mathbb{E}_c(R_t^F)$$

where $\mathbb{E}_s(R_t^F) = \frac{1}{T_s} \sum_{t:D_t^a=0} R_t^F$, $\mathbb{E}_c(R_t^F) = \frac{1}{T_c} \sum_{t:D_t^a=1} R_t^F$, T_s and T_c are the number of auctions in *non pure complements* and *pure complements* regimes, respectively.

The estimates of sunk cost parameters, α, are statistically significant in all specifications. Given the choice of parameterization for sunk costs, the parameter estimates can be interpreted as the percentage loss in terms of the a complete unit of water. For our estimate in column 4 this represents *12* pesetas loss (using the value of 365 *pesetas* for a complete unit). To get a sense of the magnitude in terms of water loss, an alternative interpretation can be obtained by determining the expected rain increase required to compensate the sunk cost loss (so that the average price remains unchanged, conditioning on covariates, regimes and parameters estimations). For our estimates in column 4 this would correspond to 0.1656 liters per square meter (obtained by multiplying mean average rain in the sample, 4.87 mm, by the increase in R^F required to lower prices in the same magnitude as sunk costs, 3.39%, obtained by solving for this magnitude numerically using system 7). For an average *parcela* (5.5 ha in our sample), this is equivalent to $9,106$ liters of water or 25.3% of a complete unit of water ($36,000$ liters).

As regards its relative size with respect to DMR in each regime, we fail to reject the following null hypothesis at the 5% level:[57]

[56] In their simulated NLLS estimation, Laffont, Ossard, and Vuong (1995) search for the best value of N by minimizing a lack-of-fit- criterion (proposition 4). Note that, as discussed in Subsection 6, identification of the distribution of valuations and structural parameters of our model requires observation of the total number of bidders. The rationale for this is straightforward: whether second highest realization of the random variable v_i is from a sample of size $N = 10$, or from a sample of size $N = 100$, is crucial to interpret the second highest bid (observable in our data). Although observation of an additional order statistic can eliminate this requirement (Song (2004)), this would require imposing further structure on the distribution of beliefs in our model (to interpret auctions where, for example, 3 different farmers win auctions), which is outside the scope of this investigation.

[57] Instead of testing the validity of the restrictions at the average rain in each regime, an alternative approach would be to

$$H_0: \ \frac{\hat{\alpha}}{3} > \hat{\beta}_c + \hat{\beta}_1 \mathbb{E}_c(R_t^F)$$

$$H_0: \ \frac{\hat{\alpha}}{3} < \hat{\beta}_s + \hat{\beta}_1 \mathbb{E}_s(R_t^F)$$

The estimated coefficient for covariates have the expected sign. To evaluate the economic significance of the seasonal dummy variables it is convenient to use the estimates to compute the percentage change in the equilibrium price of the auctioned unit of water, that is: $\frac{\partial p_t^i}{\partial Z_t} \frac{1}{p_t^i}$. For specification 4, for instance, prices in September (February) are significantly 20% higher (6% lower) than on January. This is consistent with the conventional wisdom that water is more (less) valuable during these months because of high (low) water demand. Also as expected, past rain decreases equilibrium prices. A 10% increase in the average rainfall from the previous week (with respect to the day of irrigation), decreases average price of a unit of water by 8.6%.

Participation cost can be recovered using data where auctions were run, no one bid and farmers were present in the auction, along with the identifying restriction 11 that holds in such cases. Out of the 3,203 auctions where no on bid (Table 1), we use those 2,423 where some bidders where present (auctions similar to the one in Figure 5). Although the estimates we obtain using specification 4 ($0.0094 < c < 0.1565$) are in line both, with the intuition from the model (hassle/opportunity costs due to the fact that farmers value their time) and the value used in the econometric specification (0.01), our interval estimate suggests caution in its interpretation.

In comparison of columns 1-2 and 3-4, it is clear that the model with covariates outperforms the model without them, as shown by the significance of past rain and seasonal dummy estimates, as well as the increase in the likelihood function and the improvement in the goodness of the fit. The main reason is the dependence of prices on seasonal factors that we capture in our specification with seasonal dummy variables. From the residual analysis we find no evidence that the increase in the log likelihood function is due to the parametric misspecification of the value distribution itself, and the models pass the Kolmogorov-Smirnov test, so that EG distribution of private valuations is not rejected (for the specification in column 4 the *p-value* of the test is 37%).[58] Also as a robustness check, column labeled *Residuals* shows that residuals from the estimated model in column 4 display no correlation with any of the covariates.

As regards the goodness-of-fit, our specification in column 4 performs relatively well, being the $pseudo - R^2 = 83.3\%$ (obtained by computing predicted prices by our model: $pseudo - R^2 = 1 - \frac{\sum_{t=1}^{T}(p_t - \hat{p_t})^2}{\sum_{t=1}^{T}(p_t - \bar{p})^2}$, where $\hat{p_t}$ are prices predicted by the model and $\bar{p}$ is the mean of prices). These results are in line with the R^2 obtained in the reduced-form regressions. Although not directly comparable given the distribution assumptions in the structural approach, the $R^2 = 66.1\%$ in the reduced-form specification with all covariates and without individual fixed effects (column 5 in Table 4) can be heuristically interpreted as the proportion of variability in the data set that is accounted for by the covariates, while the proportion accounted by the model without covariates in column 2 in table 10 is $R^2 = 26.2\%$. As can be seen in Figure 16, our model allows us to follow winning prices accurately. The figure displays real prices against (i) predicted prices using specification 4 (Table 10), and (ii) a reduced-form regression with *Past Rain* and multiple fixed effects as regressors (similar to Table 4).

estimate the model imposing both restrictions for each auction.

[58] We perform the nonparametric test to evaluate the equality of two distributions of valuations: our sample of private values with a reference from an EG distribution.

7.3 Understanding the Importance of the Model

We proceed now to analyze our model's implications with respect to the importance of sunk costs and decreasing marginal returns. Suppose that the researcher neglects the dynamics that arise from the model and, instead, estimates a standard (button) English-auction model. Suppose, for instance, that we are in the *pure complements regime* (where $\frac{\alpha}{3} > \beta > 0$), and that valuations follow a distribution with mean μ_v and σ_v. Then, using the result from proposition 4, we can see that the estimated mean of the distribution of valuations using the standard model, will be underestimated: $\hat{\mu}_v^{SM} < \mu_v$. Similarly, the estimation of the standard deviation of valuations will overestimated: $\hat{\sigma}_v^{SM} > \sigma_v$.[59] The same is true, of course, in the *non pure complements* case.

The overestimation of the variance of the distribution is caused by attributing the variation in prices (among different units) to a relative more disperse underlying distribution, while the farmer is actually paying for the whole bundle in the first unit deterring, hence, entrance of other bidders in the remaining three auctions. Not taking into account the (common) SC and DMR in the estimation is the cause for the underestimation of the mean. In the case of the EG distribution that it is used in our specifications, this will translate into an underestimation of the parameter θ.[60]

Columns 5 and 6 in Table 10 present the estimates from a standard English (button) auction (see proposition 4). Aside from the mentioned bias in the parameter that characterizes the distribution, the results in these columns unequivocally indicate that taking SC and DMR into account significantly contributes to the explanatory power. Figure 17 replicates figure 16 using predicted prices by the standard (button) auction model instead the ones predicted by the reduced-form model. That is, it compares winning prices with (i) predicted prices using specification 4 (Table 10), and (ii) predicted prices using specification 6 (Table 10). Consistent with these results, the *p-value* for the null hypothesis that $\hat{\alpha} = \hat{\beta}_s = \hat{\beta}_c = \hat{\beta}_1 = \hat{c} = 0$ is less than 10^{-4}.

An alternative approach would be to ask how the incomplete model approach from Haile and Tamer (2003) could be adapted to the present case. They rely on two assumptions that have intuitive appeal: (i) bidders do not bid more than they are willing to pay for a unit, and (ii) bidders do not allow an opponent to win at a price they are willing to beat. Of course, in our case with SC and DMR, these two simple assumptions are violated. In the *pure complements* regime bidders bid according to $b_i^1(v_i) = [4 - \alpha - 6\beta]\, v_i - 3c > v_i$, violating (i), and no bidder (except the highest type) participates in second to four unit auctions, violating thus (ii) (see 5). In the *non pure complements* regime, both assumptions are also violated as well. The intuition is different, however. In this case, the equilibrium is not fully, but partially revealing: bidders strategies are step functions, so the equilibrium is semi-pooling. When α is greater than β but very close to β bidders will bid above their valuations to "intimidate" other bidders and deter entry in the second auction (but, maybe, not in the third and fourth auctions). Thus, (i) is violated. Additionally, the same argument as in Black and de Meza (1992) and Liu (2010), when goods are substitutes, the winner of the first auction imposes a negative externality to herself. Her willingness to pay for the second unit is lower than it was for the first unit, making her a weaker bidder in such situation. Given that all bidders will internalize this effect, they will bid below their marginal utility for the object. This effect will be greater, the greater are DMR, β. Even with entry costs this effect is still present.

Although an adaptation of these assumptions to the whole four-units bundle may sound as the natural extension of Haile and Tamer's approach to our case, it is unlikely to produce informative bounds since

[59] In the *pure complements* case and given a fixed number of potential bidders N, the (true) mean and variance of the $N-1$ order statistic will be greater than the estimated using the standard model because the (true) price paid will be $[4 - \alpha - 6\beta_{t,c}]\, v_{N-1:N} - 3c$ and not $4v_{N-1:N}$ (predicted by the standard model).

[60] The mean from both an exponentiated gamma distributed random variable an its $N-1$ order statistic are a positive function of θ. Using equation 10 we obtain that:

$\mathbb{E}(V) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \binom{i}{j} \binom{\theta-1}{i} (-1)^i \frac{\theta(j+2)!}{(i+1)^{j+3}}$

$\mathbb{E}(V^2) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \binom{i}{j} \binom{\theta-1}{i} (-1)^i \frac{\theta(j+3)!}{(i+1)^{j+4}}$

marginal valuations of the units differ according to the regime and the number of different winners per four-unit auction. In an important sense, bundling four-unit or even the application of Haile and Tamer's approach separately for each regime, would require the model in Section 3 as an interpretation of the underlaying behavior.[61] Of course, the purpose of the authors' approach is to provide a robust structural framework for inference.

7.4 Complementarities are not Collusion

An alternative hypothesis to farmers' behavior in the *pure complements* regime is that bidders might be playing some collusive (non-competitive) strategy. As emphasized in Section 4, the demand side of this market for water is composed by potentially hundreds of farmers (Table 9). Even when they attend the auction and do not bid, the observed number of different winners is relatively high (Figure 5). Because of weather conditions, farmers are competing for water that, ultimately, will determine the quality and quantity of their crop, and in some cases, even the survival of their trees. It is unlikely that in this situation farmers can made credible collusive commitments. Contemporaries emphasize the opposite situation: farmers compete aggressively and mercilessly for water (specially during droughts), and water owners are reluctant to lower the price of the water to meet the needs of the poorer farmers.[62]

An argument against collusion is the high number of non-collusive auctions. Farmers meet every week, hence the discount rate from one week to the next one is close to 1. If we focus on two consecutive 4-unit auctions, the discount rate is virtually 1. Thus, any collusive agreement would be easy to sustain and we will observe no "price-wars", that is, no deviations from collusion's strategies. If the collusion hypothesis were true, all auctions will look collusive with the exception, may be, during certain periods where we will observe price-wars. We observe in many cases, however, that both regimes are present for the same week. Unlike Baldwin, Marshall, and Richard (1997) this is not a formal test.[63]

Nevertheless, taking the analysis one step forward, if the collusion hypothesis were true (instead of the *sunk-and-entry-cost* hypothesis), we would expect more collusion in autumn-winter and less collusion in spring-summer. Incentives to deviate from the collusion strategy are higher in spring-summer because the value of the water is higher due to seasonalities (Figures 7, 10, 11 and 12). Punishment is about the same in any season: maximum punishment would be to play the competitive equilibrium forever. Future discounted earnings in this case are very similar in summer and in winter. Hence, deviating from the collusive strategy is more profitable in summer than in winter. However, what we see in the data is exactly the opposite pattern. Figure 18 displays the distribution of auctions in the complementarities regime by month: complementarities are more likely to be observed in summer than in winter, where water requirements (and, hence, equilibrium prices) soar. This is in line with our interpretation according to the model with sunk and entry costs described above.

[61] Note also that not considering the effect of the structural parameters (SC and DMR) explicitly, would introduce further difficulties.

[62] These opinions along with an interesting qualitative analysis can be found in Vera Nicolas (2004).

[63] A discussion on how to detect collusion can be found on Porter (2005).

A "competitive" collusion?

When we introduced the model and explained how it fitted the data we implicitly assumed that farmers plots were sufficiently far away from each other. Specifically, we assumed that no other farmer could use the same sub-channel that was (just) used by her neighbor. This is not true in reality in many cases. Because the cost of "watering" the sub-channel is sunk, if the plots of two farmers are located next to each other and they share the same sub-channel, then one farmer could free-ride and outbid the first winner in the second auction. Knowing this, the first winner would bid lower in the first auction. This situation would reduce the revenue of the auction and would create inefficiencies. Since farmers are not internalizing this free-riding effect, they would take into account the equilibrium outcome for the reaming auction, and lower their bid in the first auction. Then, they will try to outbid their neighbors in latter auctions.

In a situation like the one described above, it would be relatively easy to sustain a collusive agreement among neighbors farmers. The number of members of the coalition will be small (say 3 or 4 farmers), and because they are neighbors, they know each other very well and might even share animals or machinery for agricultural purposes. Each farmer in the coalition would compete in the auction for the first unit, but will not enter the remaining auctions if one member of the coalition won the first unit. With this agreement they will achieve efficiency by solving the free riding problem. With the resulting increase in efficiency, the revenue of the auction will also increase, and the auctioneer will be happy with the "collusion".

7.5 Efficiency

The model displayed in Section 3 assumes that it is costly for the bidders to enter the auction. To compare this mechanism (sequential ascending-price auction) with other possible ones, this cost has to be taken into account. In this context, and following Stegeman (1996), we can interpret this cost as the cost that the farmers have to incur when they send a message to the auctioneer (or to some other farmers). In this case, the notions of *ex-ante* and *ex-post* efficiency are no longer equivalent. The reason is simple, while it might be *ex-ante* efficient that more than one player send a message, it is always *ex-post* efficient that at most one player send a message.

For this case where it is costly to send messages to the coordinator Stegeman has showed that the ascending-price auction has an equilibrium that is *ex-ante* efficient. In contrast, the first-price auction may have no efficient equilibrium. The author only considers the single-unit case. In our sequential unit case, we have shown that when goods are *pure complements* ($\rho \geq 0$ or $\alpha \geq 3\beta$) the analysis is identical as in the single unit case. Hence, the result applies here as well. When goods are *not pure complements*, however, the result would only apply to the last auction. Although outside the scope of this paper, further work in this area to investigate whether a sequential ascending-price auction is *ex-post* efficient when the coordinator has to allocate several objects to players that face SC, DMR and costly messages, will be useful for an adequate comparison among other mechanism allocations.

8 Conclusions

By affecting bidder behavior in sequential auctions, the presence of sunk costs and decreasing marginal returns (along with participation costs) may generate very different price dynamics within the same market. This difference in price dynamics can be attributed to the varying extent to which the value of sequential goods complements or falls relative to previous units. The *deterrence effect*, where the same bidder pays a high price for the first unit (deterring others from entering subsequent auctions), and a low price for the remaining units, arises when sunk costs are relatively high compared to the decreasing marginal returns, thus creating complementarities among the goods. Substitutability arises due to decreasing returns when sunk costs are relatively small. In this case, equilibrium prices are similar in magnitude, regardless of whether the same or different bidders win the objects. Careful consideration of these features is fundamental to demand characterization, a cornerstone of many positive and normative questions in economics.

Using a novel data set from a decentralized market institution that was operating privately during eight centuries in southern Spain, the current paper documents these price dynamics and develops a model to recover the underlying structural parameters and distribution of valuations. In the model, although the buyers are better informed than the sellers, the latter know that the sequential English-auction allocates water (*ex-ante*) efficiently. Not requiring that farmers reveal their marginal valuations is an advantage of the mechanism, whose simplicity reduces costs associated with its implementation and helps explain its extraordinary stability. We address three main questions. Are water units complements or substitutes, and why? Is the *deterrence effect* consistent with a competitive market structure or a consequence of collusive behavior among farmers? And what would happen to the estimates in this setting if the researcher, by ignoring the importance of participation and sunk costs, failed to account for the complementarity feature of the sequential goods?

We document, first, that in the period under study, both complementarities and substitutabilities are observed in the data, generating two different equilibrium price dynamics. Seasonality, related to the products' water requirements and the expected rainfall, affect the relative importance of sunk costs and decreasing returns, causing bidders to coordinate their actions into these regimes. Second, the apparent collusive behavior, where the same bidder wins all the goods, paying very low prices (close to zero) for all the units following the first unit, is actually competitive (non-cooperative). Contrary to the collusion hypothesis, this behavior (caused by complementarities) is observed when the value of water (as well as the average price paid per unit and, thus, the incentive to deviate from a collusion strategy) increases relative to the standard competitive pattern registered in the *non-complements* regime. This shows the importance of interpreting the data through the economic model. Finally, by estimating the model, we confirm the relevance of participation and sunk costs in our empirical environment. By testing the performance of our model relative to a standard English-auction model, we confirm that, in our case, estimations using the latter are not accurate. Aside from the obvious bias generated by ignoring sunk costs and decreasing returns, price dynamics play an important role, since it would not be appropriate to attribute the variation in prices among sequential units (when the goods are complements) to a relatively more disperse underlying distribution of valuations.

Appendix

A Technical Proofs and Extensions

A.1 Equilibrium when $-1 < \rho < 0$

Assumption A1:

$$\overline{v} \geq \frac{2c-\rho W}{2+\rho}$$

Assumption A2:

$$\{(1+\rho)W - E[v_l|v_l > \overline{v}]\}F[(1+\rho)W|v_l > \overline{v}] < c$$

Assumption A2 ensures that the winner never enters the second auction.

Theorem 9. *Under assumptions A1 & A2, the unique symmetric equilibrium satisfies the following properties:*

(a) First Auction

(a) All players participate in the first auction $y_i^1 = 1$ for all i

(b) The bidding strategy for player i is

$$b_i^1(v_i) = b^1(v_i) = \begin{cases} v_i & \text{if } \tilde{v}_i \leq c \\ (2+\rho)v_i - c & \text{if } c < \tilde{v}_i < v_i \leq \overline{v} \\ (2+\rho)\overline{v} - c & \text{if } v_i \geq \overline{v} \end{cases}.$$

(b) Second Auction

(a) Case 1: No pooling in the first auction (at most one bidder has valuation above $\overline{v}$). Player i's participation function ($y_{i-np}^2(v_i, x_i^1)$) is

$$y_{i-np}^2(v_i, 0) = 0$$

$$y_i^2(v_{i-np}, 1) = \begin{cases} 0 & \text{if } \tilde{v}_i \leq c \\ 1 & \tilde{v}_i > c \end{cases}$$

(b) Case 2: Pooling in the first auction (both bidders above $\overline{v}$). Player i's participation function ($y_{i-p}^2(v_i, x_i^1)$) is:

$$y_{i-p}^2(v_i, 0) = 1$$

$$y_{i-p}^2(v_i, 1) = 0$$

(c) If player i participates in the second auction, their bidding strategy is:

$$b^2(v_i, x_i^1) = \begin{cases} v_i & \text{if } x_i^1 = 0 \\ \tilde{v}_i & \text{if } x_i^1 = 1 \end{cases}$$

Proof. The proof will proceed by backwards induction. In all that follows, player 1 is the player with the highest valuation, $v_1 > v_2$. Notice that when $c \to 0$ the probability that we are in the pooling region goes to one. This is so because $c \to 0$ implies $\overline{v} \to 0$. When the number of players increases this probability also increases.

2-(c): From the players perspective, once the participation decision has been made, the final auction is a standard English-auction. As a result, it is a dominant strategy of this stage game for each player to bid their valuations.

2-(b)-Case-2: This is the case after pooling in the first auction, that is $v_1, v_2 \geq \overline{v}$. We will write down the relevant expected utilities from entering and argue that any deviation (in pure strategies) results in negative expected utility. In this case, since the winner of the auction is determined by a random probability lottery, we will refer to the winner and loser valuations as v_w and v_l, respectively. This is simply to emphasize that in this pooling region, the entry decision only depends on the outcome of the first auction, not the relative valuations. First, consider the loser of the first auction. The strategy says that they enter the second auction. The only possible deviation in pure strategies is not to enter the second auction, but given the winner's strategy (not to enter), we get:

$$\begin{aligned} EU[y^2_{i-p}(v_l, 0) = 1 | y^2_{i-p}(v_i, 1) = 0] &= v_l - c \geq \overline{v} - c \\ \geq P\left[v_w < \frac{\overline{v} + c}{1+\rho} \middle| v_w > \overline{v}\right]\overline{v} - c &= 0 = EU[y^2_{i-p}(v_l, 0) = 0 | y^2_{i-p}(v_i, 1) = 0] \end{aligned}$$

Hence, there is no profitable deviation. Now consider the winner of the first auction. By following the strategy $y^2_{i-p}(\cdot, \cdot)$, this player will not enter the auction and get an expected utility of zero. Now, the only possible pure strategy deviation is to enter the auction, which gives her an expected utility of:

$$\begin{aligned} P[v_l < (1+\rho)v_w | v_l > \overline{v}][(1+\rho)v_w - & E[v_l | v_l > \overline{v}] - c] - (1 - P[v_l < (1+\rho)v_w | v_l > \overline{v}])c \\ &= P[v_l < (1+\rho)v_w | v_l > \overline{v}](v_w(1+\rho) - E[v_l | v_l > \overline{v}] - c \\ &\leq P[v_l < (1+\rho)v_w | v_l > \overline{v}](W(1+\rho) - E[v_l | v_l > \overline{v}] - c \\ \text{(by assumption 2)} \quad &\leq 0 \end{aligned}$$

Thus, by entering, the winner will now get negative utility, and, therefore, she has no profitable deviation. Notice that this result depends exclusively on A2. If A2 does not hold, but A1 does, then there will be an equilibrium in which the winner also enter the second auction if her valuation is high enough.

2-(a)-Case-1: Now consider the participation decision when no pooling occurred in the first auction, i.e.: $v_2 \leq \overline{v} < v_1$. Consider player 1. His strategy is simply a threshold strategy, as long as her valuation for the second good is larger than the cost of entry, she will enter since player 2 will not enter, and player 1 will win the object and pay a price of 0. So, assuming player 2 follows $y^2_{i-np}(\cdot, 0)$, player 1's strategy is clearly optimal. However, we must show that given this strategy by player 1, player 2's strategy is in fact optimal. This is not immediately obvious, since player 2's threshold $\overline{v}$ is the solution in terms of v_2 to:

$$P[(1+\rho)v_1 < v_2 + c | v_1 > v_2]v_2 = c$$

That is, since player 2's valuation will become public after the first auction when there is not any pooling, player 1 will enter no matter what if her valuation is above $v_2 + c$. We have to show that this threshold is still binding when player 2 knows that player 1 will enter with any valuation above c. In this case, $v_2 < \overline{v}$, the only deviation to consider is that player 2 enters after losing for some valuation less than $\overline{v}$. First, note that if $v_2 < c$, it clearly makes no sense to enter, since regardless of player 1's entry choice, player 2 will get a lower utility. There are now 2 cases to analyze.

(a) Suppose $v_2(1+\rho) < c$. In this case, it is possible that $(1+\rho)v_1 < c$, i.e., that agent 1 will not enter.

Then, the expected utility from entering is:

$$
\begin{aligned}
P[(1+\rho)v_1 < c | v_1 > v_2](v_2 - c) + P[c < (1+\rho)v_1 < v_2 | v_1 > v_2](v_2 - c)& \\
+ P[(1+\rho)v_1 > v_2 | v_1 > v_2](-c)& \\
= P[(1+\rho)v_1 < v_2 | v_1 > v_2]v_2 - c& \\
\leq P[(1+\rho)v_1 < v_2 + c | v_1 > v_2]v_2 - c < 0,& \quad \forall\, v_2 < \overline{v}
\end{aligned}
$$

Since by not entering, player 2 can ensure 0 expected utility, this deviation is not profitable.

(b) Suppose $v_2(1+\rho) \geq c$. When this is true, player 1 will always enter since $v_1 \geq v_2$. Then, by deviating and entering, player 2 gets:

$$
\begin{aligned}
P[\tilde{v}_1 < v_2 | v_1 > v_2](v_2 - c) + P[\tilde{v}_1 \geq v_2 | v_1 > v_2](-c)& \\
= P[\tilde{v}_1 < v_2 | v_1 > v_2]v_2 - c& \\
\leq P[\tilde{v}_1 < v_2 + c | v_1 > v_2] - c \leq 0,& \quad \forall\, v_2 < \overline{v}
\end{aligned}
$$

Again, since not entering gives 0 expected utility, this is not a profitable deviation.

Thus, there is no profitable deviation from the second auction participation strategy.

1-(b): We need to show that if players are following the bidding strategy for the first auction as described in Theorem 9, they have no profitable deviation. We will consider each part of the function separately and show that any deviation results in lower utility for the players.

$\tilde{\mathbf{v}}_\mathbf{i} \leq \mathbf{c}$: Consider an upwards deviation, $\tilde{b}^1(v_i) = v_i + \varepsilon$, for $\varepsilon > 0$. For player 1, since she is the highest type, this will not change anything, she will still win for sure and still pay v_2. In the case where $v_1 = v_2$, any upward deviation will still result in a utility of 0. Finally consider player 2, if she follows $\tilde{b}^1(\cdot)$, she will now win with positive probability and get utility:

$$U_2 = v_2 - v_1 < 0$$

Thus, no player will gain from deviating from the prescribed strategy upwards. Now, consider a downwards deviation. Now, if player 2 deviates, she will still lose for sure, and get utility of 0. The same for the case where $v_1 = v_2$, either deviation still results in utility of 0. However, if player 1 plays $\tilde{b}(v_1) = v_1 - \varepsilon$, with positive probability she will lose when she is the highest type, thus getting 0 utility when previously she was getting positive utility $v_1 - v_2 > 0$. Thus, downward deviations are also not profitable, so when $\tilde{v}_i < c$, the strategy in Theorem 9 has no profitable deviations.

$\mathbf{c} < \tilde{\mathbf{v}}_\mathbf{i} < \mathbf{v} < \overline{\mathbf{v}}$: Here, agents bid above their valuation for the first object, because if they win, they will also get the second object for free. Since they get the second auction for free, the effective valuation of the first object (since winning it implies they get they second one) is $(2+\rho)v_i - c$. Again consider an upwards deviation, $\tilde{b}^1(v_i) = (2+\rho)v_i + c + \varepsilon$. For player 1, an upwards deviation has no effect, as she will still win the object for sure, and the price payed does not change. For player 2, bidding $\tilde{b}^1(v_2)$ means that she will win the 1st auction with positive probability, giving utility of:

$$v_2 - [(2+\rho)v_1 - c + (1+\rho)v_2 - c = (2+\rho)(v_2 - v_1) < 0$$

So an upwards deviation makes no player better off, and some strictly worse off. Now, consider a downwards deviation, $\tilde{b}^1(v_i) = (2+\rho)v_i + c - \varepsilon$. Player 2 will still lose with probability 1, so no change. But now, player 1 may lose when she is the highest type, and thus obtains utility of 0 from deviating, while playing the prescribed strategy would give utility

$$v_1 - b(v_2) + (1+\rho)v_1 - c \geq v_1 - (2+\rho)v_2 - c + (1+\rho)v_1 - c = (2+\rho)(v_1 - v_2) > 0$$

Thus, no downwards deviation is profitable. So when $c < \tilde{v}_i < v < \overline{v}$, deviating gives all players lower utility in some cases, and never improves upon the strategy in Theorem 9.

$\tilde{\mathbf{v}}_{\mathbf{i}} > \overline{\mathbf{v}}$: The first step is to show that no jump in bidding occurs at $\overline{v}$. Then, we will show that agents will pool for all values between $\overline{v}$ and W. The difference between this and the previous case, is that the loser of the first auction now gets positive utility from entering the second auction. Consider the case where $v_i = \overline{v} + \varepsilon$. We now show that such agents will bid $(2+\rho)\overline{v} - c$. Assuming ties are broken with an equal probability, the utility to agent i bidding $b^1(v_i)$ is:

$$\begin{aligned} U_i &= \frac{1}{2}[v_i - [(2+\rho)\overline{v} - c]] + \frac{1}{2}[v_i - c] \\ &= v_i - \frac{2+\rho}{2}\overline{v} \\ &= (v_i - \overline{v}) - \frac{\rho}{2}\overline{v} \\ &> 0 \end{aligned} \tag{12}$$

This is true for all $\varepsilon > 0$. We will now show that deviating from this results in lower utility. First, consider an upward deviation, $\tilde{b}^1(v_i) = (2+\rho)\overline{v} - c + \varepsilon$. This will give:

$$\begin{aligned} U_i &= v_i - [(2+\rho)\overline{v} - c] + 0 \\ &= (v_i - \overline{v}) - (1+\rho)\overline{v} + c \\ &\xrightarrow{\varepsilon \to 0} c - (1+\rho)\overline{v} \\ &< 0 \end{aligned}$$

by assumption. Thus, players will not bid strictly higher at $\overline{v}$.

Now consider a downward deviation, say for player i, with $v_i > \overline{v}$, $\tilde{b}^1(v_i) = (2+\rho)\overline{v} - c - \varepsilon$. There are two possibilities, either $v_{-i} \geq \overline{v}$ or $v_{-i} \leq \overline{v}$. Such a deviation, will mean that player i never reaches the pooling region, and that there exists a set of v_{-i}'s such that $b^1(v_{-i}) > \tilde{b}^1(v_i)$. We will write down the payoff from playing the candidate equilibrium strategy as well as that from the deviation. The original payoff is:

$$\begin{aligned} &\frac{1}{2}P[v_{-i} > \overline{v}](v_i - b(v_{-i}) + \tilde{v}_i - c) + P[v_{-i} \leq \overline{v}](v_i - b(v_{-i}) + \tilde{v}_1 - c) \\ &\quad = \frac{1}{2}P[v_{-i} > \overline{v}](v_i - (2+\rho)\overline{v} + \tilde{v}_i) + (1 - P[v_{-i} > \overline{v}])(v_i - (2+\rho)\overline{v} + \tilde{v}_1) \\ &\quad = (2+\rho)\left(\frac{1}{2}P[v_{-i} > \overline{v}](v_i - \overline{v}) + (1 - P[v_{-i} > \overline{v}])(v_i - v_{-i})\right) \end{aligned} \tag{13}$$

Note that both terms are positive. Define $\hat{v}_{-i}$ so that $b^1(\hat{v}_{-i}) = \tilde{b}^1(v_i)$. Now, the payoff from deviating to $\tilde{b}^1(\cdot)$, is:

$$\begin{aligned} P[v_{-i} \geq \overline{v}] * 0 + &P[v_{-i} \in [\hat{v}_{-i}, \overline{v}] * 0 + P[v_{-i} < \hat{v}_{-i}](v_i - (2+\rho) + c + \tilde{v}_i - c) \\ &= P[v_{-i} < \hat{v}_{-i}](2+\rho)(v_i - v_{-i}) \\ &\leq P[v_{-i} < \hat{v}_{-i}](2+\rho)(v_i - v_{-i}) + P[v_{-i} \in [\hat{v}_{-i}, \overline{v}]](2+\rho)(v_i - v_{-i}) \end{aligned}$$

Now, this is just equal to the second term of (13), and since the first term of (13) is also positive, the payoff from deviating is strictly less than the payoff from following the strategy. So in any neighborhood to the right of $\overline{v}$, there is no incentive to deviate.

What remains to show, is that all agents will pool, including those with the highest possible valuation W. An agent with valuation W will not deviate down for the same reason as above. So we need only show that there is no incentive to deviate upwards. From (12) we know that any agent with valuation above $\overline{v}$ has positive expected utility by following the prescribed bidding strategy. We will show that deviating upwards will result in negative utility even for type W. If type W bids $\tilde{b}^1(W) = (2+\rho)\overline{v} - c + \varepsilon$, the expected utility is:[64]

$$P[v_2 \leq \overline{v}](2+\rho)(W - v_2) + P[v_2 > \overline{v}](W - (2+\rho)\overline{v} + c)$$

If instead, the player follows the original strategy, she will get payoff of:

$$P[v_2 \leq \overline{v}](2+\rho)(W - v_2) + \frac{1}{2}P[v_2 > \overline{v}](2+\rho)(W - \overline{v})$$

The first terms of these expressions are the same, so we need only compare the second terms. To complete the proof, we need only show that

$$\frac{(2+\rho)}{2}(W - \overline{v}) > W - (2+\rho)\overline{v} + c$$

This is the same as showing that

$$\overline{v} > \frac{2c}{2+\rho} - \frac{\rho}{2+\rho}W,$$

which is exactly A1.

$\square$

A.2 Proposition 5

Proof. Following the same argument as in the proof of Proposition 1, the first step of the proof consists on proving that in any *revealing* (strictly increasing) equilibrium, i.e. an equilibrium in which it becomes common knowledge after the first round who is the bidder with the highest valuation, only the winner (the bidder with the highest type) will enter the remaining auctions, and pay the cost c.

Since both a direct mechanism and the sequential auction will give the same utility to the winner, and both will give the 4 objects to the bidder with the highest valuation, the total utility for the winner should be $W_i = (4-\alpha-6\beta)\, v_i - (4-\alpha-6\beta)\, v_j = (4-\alpha-6\beta)\,(v_i - v_j)$ in both cases, where j is the bidder with the second highest valuation. For the case of a direct mechanism we can assume that there is a cost of communication for each of the auctions, that should be paid by every bidder that want to get the object.

The second step is to show that the winner will pay $(4-\alpha-6\beta)\, v_j - 3c$ in the first auction. This payment, together with the utility the winner get from both goods $((4-\alpha-6\beta)\, v_i)$ and the cost of entering the remaining auctions $(3c)$, will give him the same utility as in the direct mechanism. $(4-\alpha-6\beta)\, v_i - ((4-\alpha-6\beta)\, v_j - 3c) - 3c = (4-\alpha-6\beta)\, v_i - (4-\alpha-6\beta)\, v_j = W_i$. The utility for the winner in the second auction is $(1-\beta)\, v_i - c$ since the equilibrium price in the second auction is zero, i.e. $p^2 = 0$, and the utility for the loser is zero. Similarly, the utility for the loser in the third auction is $(1-2\beta)\, v_i - c$ and the utility in the fourth auction is $(1-3\beta)\, v_i - c$. Then, the total value of winning the auction is $(4-\alpha-6\beta)\, v_i - 3c$. Now lets define $z_i = (4-\alpha-6\beta)\, v_i - 3c$ and consider this game to be a single object auction in which the valuation for the good of player i is z_i. Now we are back into the standard case (single unit auction) and $b_i = z_i = (4-\alpha-6\beta)\, v_i - 3c$ is a weakly dominant strategy.

[64] From the equilibrium, the participation decision of type W will be not to enter the second auction if she wins the first

We have proven that in any *revealing* equilibrium only the winner will enter the remaining auctions. The intuition for this result is that, as in Proposition 1, because to the cost of participating in an auction, it is a necessary condition for player i to participate in the auction, that her probability of winning is positive. Thus, only the winner will enter the remaining auctions. Her utility in the second, third and fourth auctions is $(1-\beta)\, v_i - c$, $(1-2\beta)\, v_i - c$ and $(1-\beta)\, v_i - c$ respectively. This fact, together with her utility in the first auction being $(1-\alpha)v_i$ and the Revenue Equivalence Theorem show that she will bid $b_i^1 = (4-\alpha-6\beta)\, v_i - c$ and pay $p^1 = (4-\alpha-6\beta)\, v_j - c$. Hence, this is a *revealing* equilibrium. Therefore, we have shown that, given the payoffs in the second, third and fourth auctions, there is only one possible payoff and one possible bid for every player in the first auction. Hence, this is also the unique Symmetric Equilibrium in pure strategies. □

B Monte Carlo Study

In this section we perform a Monte Carlo study to test the estimation procedure from Section 7. We are interested in evaluating how well the proposed estimation procedure outlined in the text is performing. We simulate T English-auctionss in which for any auction t there are N bidders whose values are distributed according to an exponentiated gamma distribution. Prices are simulated using the model in Section 3. Then, conditioning on the regime and assuming only that we have data on the transaction price from these auctions (and rain), we attempt to recover the structural parameters via the maximum likelihood procedure outlined in Section 7. Specifically, first, we generate data that mimics rain in each regime. As explained in the text, rain behavior is characterized by high frequency of non rain days and mean rain significantly differs across regimes. To generate rain data we first generate a random indicator variable equals to one if rain is positive on a given day, and zero otherwise. We let this indicator $I = 1$ with probability p , and $I = 0$ with probability $1-p$, and choose a value of p using the frequency of days with positive rain in our data. For those days with positive rain we let amount of rain be a uniform random variable with mean equal to the observed mean in the data. Given that MLE is performed conditional on regime determination, we let the proportion of auctions in *non pure complements* and *pure complements* regimes, $\frac{T_s}{T}$ and $\frac{T_c}{T}$ respectively, match the proportion from our data (Section 5). Moreover, we let the proportion farmers that win three units, two out of the first three, and the last one (equation 5), match the proportion we observe in our data. We then draw T values from $f_{N-1:N}(v,\theta)$, where $f(v,\theta)$ is the PDF of the EG(θ) and $f_{N-1:N}(v,\theta)$ is its $N-1$-order statistic. To do this we let $N = \{40, 50\}$, as in our estimation in Section 7. Then, given specific values for the structural parameters, θ and $\delta \equiv (\alpha, \beta_0^C, \beta_0^S, \beta_1)$, we construct observed prices according to the structural equations from our model (system in 7). We then repeat this process S times. The results displayed in Table 11, where the estimators perform relatively well, gives further support and confidence to the results discussed in the text.

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Table 1: Frequency distribution of units bought by the same farmer

	# of goods	Frequency	Total
1CU	3,527	.21	3,527
2CU	2,838	.17	6,365
3CU	1,683	.10	8,048
4CU	5,824	.34	13,872
No bids	3,203	.19	17,075
Total	17,075	1.00	17,075

Table 2: Summary Statistics of Selected Variables

Variable	Mean	SD	Min	Max	Obs
Rain	8.53	46.33	.00	980.00	3,834
Price	271.61	374	.05	4,830	13,872
Land Extension	5.54	32.24	.25	900	819
Selling Price	15.07	222.52	.02	5,700	964
Kg sold	5,569.70	10,003.76	0	110,000	1,000
Number of Trees	161.49	493.45	1	12,300	946

Table 3: Distribution of winning prices: by # of CU and sequential auction

Price distribution by number of consecutive bids

	Median	Mean	SD	Max	Min	Obs
1CU	100	208.4	303.9	3000	0.05	3527
2CU	121	243.2	330.4	2700	0.05	2838
3CU	161	291.7	369.2	2850	0.05	1683
4CU	161	318.2	424.1	4830	0.05	5824

Price distribution by number of consecutive bids: 1st Auction

	Median	Mean	SD	Max	Min	Obs
1CU	100	204.6	288.3	2921	0.05	979
2CU	146	288.3	384.7	2700	0.05	666
3CU	204	365.6	433.9	2809	0.05	371
4CU	444.5	635.2	607.4	4830	0.05	1456

Price distribution by number of consecutive bids: 2nd Auction

	Median	Mean	SD	Max	Min	Obs
1CU	95	216.4	360.4	3000	0.10	629
2CU	102	217.4	293.4	2685	0.05	857
3CU	161	268.8	326.8	2850	0.05	528
4CU	101	226.2	280.2	2605	0.05	1456

Price distribution by number of consecutive bids: 3rd Auction

	Median	Mean	SD	Max	Min	Obs
1CU	93	191.3	284.9	2357	0.10	713
2CU	125	245.1	328.8	2601	0.10	773
3CU	150	262.8	346.9	2801	0.05	527
4CU	100	214.0	269.8	2625	0.05	1456

Price distribution by number of consecutive bids: 4th Auction

	Median	Mean	SD	Max	Min	Obs
1CU	110.5	217.5	294.7	2600	0.05	1206
2CU	111	225.9	310.6	2601	0.10	542
3CU	160.5	292.0	382.4	2630	0.05	257
4CU	100	197.5	255.1	2935	0.05	1456

Price distribution for 4-consecutive bids

Auction	Median	Mean	SD	Max	Min	Obs
1st to 4th	161.0	318.2	424.1	4830	0.05	5824
1st	444.5	635.2	607.4	4830	0.05	1456
2nd	101.0	226.2	280.2	2605	0.05	1456
3rd	100.0	214.0	269.8	2625	0.05	1456
4th	100.0	197.5	255.1	2935	0.05	1456
1st and 2nd	250.0	430.8	515.3	4830	0.05	2912
2nd and 3rd	100.0	220.1	275.1	2625	0.05	2912
3rd and 4th	100.0	205.7	262.6	2935	0.05	2912
1st to 3rd	200.0	358.5	460.1	4830	0.05	4368
2nd to 4th	100.0	212.5	268.8	2935	0.05	4368

Table 4: Correlation between winning prices and covariates

Variables	(1)	(2)	(3)	(4)	(5)	(6)
Rain MA7	-4.0543***	-4.1117***	-0.2505***	-2.9911***	-4.6681**	-5.1535**
	(0.6742)	(0.6894)	(0.0784)	(0.5580)	(2.0885)	(2.0363)
Rain Day Bought	-0.2346	-0.1853	0.0064	0.0519	0.0050	0.0576
	(0.1434)	(0.1416)	(0.0732)	(0.1558)	(0.0639)	(0.0642)
Unit 2 Day		-167.9547***	-167.9118***	-167.8286***	-167.9125***	-179.3032***
		(19.4659)	(19.7591)	(19.4542)	(19.7671)	(22.2753)
Unit 3 Day		-173.0328***	-172.9898***	-172.9066***	-172.9905***	-190.7941***
		(19.9287)	(20.2295)	(19.9165)	(20.2378)	(23.1855)
Unit 4 Day		-176.5446***	-177.2624***	-176.5451***	-177.2450***	-195.7469***
		(20.4404)	(20.7581)	(20.4387)	(20.7678)	(23.7353)
Unit 2 Night		-237.5795***	-238.2013***	-237.8597***	-238.2003***	-250.8435***
		(24.9031)	(25.3139)	(24.9367)	(25.3245)	(27.8536)
Unit 3 Night		-243.3244***	-243.8726***	-243.5220***	-243.8720***	-258.5112***
		(25.4507)	(25.8682)	(25.4838)	(25.8790)	(28.8682)
Unit 4 Night		-254.8376***	-255.8049***	-255.1867***	-255.7997***	-271.2714***
		(25.7254)	(26.1506)	(25.7817)	(26.1619)	(29.3638)
Tuesday		26.0232***	-31.9125***	32.1906***	-32.4870***	-35.3381***
		(7.4927)	(5.5094)	(8.2359)	(5.6156)	(7.0576)
Wednesday		-34.5756***	-94.7660***	-29.3838**	-95.3776***	-88.4682***
		(10.6269)	(11.1828)	(11.9714)	(11.0480)	(10.8098)
Thursday		-59.6530***	-121.7359***	-55.4057***	-121.3586***	-104.5693***
		(10.7704)	(11.9630)	(12.3518)	(11.8223)	(10.6204)
Friday		-94.9538***	-158.8524***	-95.5421***	-159.2511***	-147.2448***
		(12.7055)	(14.3726)	(14.4995)	(14.4039)	(13.7166)
Night		-110.0908***	-124.5486***	-111.0406***	-124.6170***	-126.7151***
		(11.3642)	(10.2793)	(10.9544)	(10.2805)	(11.9505)
UnitFE	NO	YES	YES	YES	YES	YES
Weekday FE	NO	YES	YES	YES	YES	YES
Schedule FE	NO	YES	YES	YES	YES	YES
Week FE	NO	NO	YES	NO	YES	YES
Month FE	NO	NO	NO	YES	YES	YES
Individual FE	NO	NO	NO	NO	NO	YES
R^2	0.016	0.083	0.659	0.230	0.661	0.688
Observations	13,801	13,801	13,801	13,801	13,801	13,801

All columns are OLS regressions. Dependent variable is the winning price in each auction (one *cuarta*). Robust standard errors in parentheses. FE stands for *Fixed Effects*. *Individual FE* refers to a set of dummy variables identifying different winners (names) in our sample. *Week FE* refers to a set of dummy variables identifying (52 or 53) weeks of the corresponding year. *** p<0.01, ** p<0.05, * p<0.1. Sample restricted to auctions with positive bids during the period January 1954 to August 1966.

Table 5: Irrigation requirements for Citrus Trees

Timing after planting	Month				
	Dec. - Feb.	Mar. - Apr.	May - Jun.	Jul. - Sep.	Oct. - Nov.
0 - 1 month	2 to 3 days				
2 - 3 months	3 to 5 days				
4 months to 1 year	14 days	7 to 10 days	5 to 7 days	2 to 5 days	5 to 10 days
1 to 2 years	14 to 21 days	10 to 14 days	7 to 10 days	7 to 10 days	10 to 14 days
3 years or older	21 to 30 days	14 to 21 days	14 days	10 to 14 days	14 to 21 days

Obtained from Table 2 in Wright (2000), modified from Chott and Bradley (1997).

Table 6: End-Price Preferences

	All units		First Unit		Non First Unit		Second Unit		Third Unit		Fourth Unit	
Last Digit	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%
0	2584	53.7	404	33.6	2,180	60.4	724	60.2	727	60.4	729	60.5
1	1265	26.3	411	34.1	854	23.7	298	24.8	291	24.2	265	22.0
2	141	2.9	87	7.2	54	1.5	21	1.7	17	1.4	16	1.3
3	88	1.8	43	3.6	45	1.2	20	1.7	10	0.8	15	1.2
4	46	1.0	32	2.7	14	0.4	7	0.6	3	0.2	4	0.3
5	448	9.3	104	8.6	344	9.5	87	7.2	124	10.3	133	11.0
6	133	2.8	61	5.1	72	2.0	21	1.7	21	1.7	30	2.5
7	50	1.0	30	2.5	20	0.6	13	1.1	5	0.4	2	0.2
8	31	0.6	19	1.6	12	0.3	3	0.2	4	0.3	5	0.4
9	28	0.6	13	1.1	15	0.4	9	0.7	1	0.1	5	0.4
Total	4,814	100	1,204	100	3,610	100	1,203	100	1,203	100	1,204	100

Notes: Sample restricted to 4CU auctions. *Last Digit* refers to the end-digit winning price. Non integer winning prices are excluded.

Table 7: End-Price Preferences by Regime

	First Unit				Fourth Unit			
	Pure Compl		Not Pure Compl		Pure Compl		Not Pure Compl	
Last Digit	Freq	%	Freq	%	Freq	%	Freq	%
0	202	38.0	202	30.1	532	100	197	29.3
1	174	32.7	237	35.3	0	0	265	39.4
2	38	7.1	49	7.3	0	0	16	2.4
3	17	3.2	26	3.9	0	0	15	2.2
4	15	2.8	17	2.5	0	0	4	0.6
5	41	7.7	63	9.4	0	0	133	19.8
6	27	5.1	34	5.1	0	0	30	4.5
7	9	1.7	21	3.1	0	0	2	0.3
8	7	1.3	12	1.8	0	0	5	0.7
9	2	0.4	11	1.6	0	0	5	0.7
Total	532	100	672	100	532	100	672	100

Notes: Sample restricted to 4CU auctions. *Last Digit* refers to the end-digit winning price. Non integer winning prices are excluded. Regime is determined by assuming all second, third and four unit prices paid for the same winner within the same four-unit auction display end-price preference for 0.

Table 8: Rain expectations and regime coordination

Variables	(1)	(2)	(3)
Future Rain	-0.0052***	-0.0055***	-0.0061***
	(0.0013)	(0.0013)	(0.0014)
Tuesday		0.0303	0.0324
		(0.0439)	(0.0444)
Wednesday		-0.0332	-0.0378
		(0.0437)	(0.0440)
Thursday		0.0220	0.0237
		(0.0449)	(0.0453)
Friday		-0.0347	-0.0423
		(0.0476)	(0.0481)
Night		-0.0615**	-0.0653**
		(0.0294)	(0.0298)
February			0.0668
			(0.1016)
March			0.0268
			(0.0974)
April			0.1885**
			(0.0867)
May			0.2387***
			(0.0828)
June			0.2319***
			(0.0835)
July			0.1561*
			(0.0867)
August			0.2188**
			(0.0855)
September			0.0592
			(0.1011)
October			-0.0749
			(0.0959)
November			0.0927
			(0.0950)
December			0.1147
			(0.1024)
Weekday FE	NO	YES	YES
Schedule FE	NO	YES	YES
Month FE	NO	NO	YES

Notes: Same restricted to the used in the structural estimation in Table 10 (see Section 4). Almost identical results are obtained using the whole sample. All specifications are probit regressions. Marginal effects are reported. Robust standard errors in parenthesis. Dependent variable is a dummy variable equal to one if the regime is *pure complements* (see Section 6). *Future Rain* is a moving average of rain in Mula for seven days after the corresponding date of the auction (*Future Rain* is a proxy variable for farmers' rain expectations for the day where they are buying water). *Past Rain* (a moving average of rain in Mula for seven days before the corresponding date of the auction) and *Actual Rain* (the amount of rain in Mula in the day of the auction) are not statistically significant in any of the above regressions. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 9: Timing Structure of different winners: Estimation Sample

Month	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	Total
1	4	6	0	3	1	11	3	0	0	0	28	0	11	61
2	4	4	0	2	4	2	3	0	0	0	19	0	21	57
3	5	3	0	9	0	1	2	0	0	10	29	8	23	79
4	0	2	0	6	2	5	4	6	0	17	28	38	28	121
5	5	7	0	6	1	13	9	6	9	4	32	30	31	130
6	3	7	0	7	0	8	10	7	10	14	23	25	29	119
7	2	3	0	6	9	26	8	5	13	15	17	21	23	117
8	9	3	0	3	4	10	7	14	18	15	21	16	3	102
9	8	8	0	3	8	10	5	13	0	8	35	19	0	97
10	8	7	3	2	11	2	0	9	0	10	16	19	0	78
11	7	2	3	0	8	2	0	4	0	21	29	23	0	82
12	1	0	2	2	3	1	0	0	0	36	18	12	0	69
Total	48	43	8	43	48	80	47	54	44	106	179	147	128	537

Notes: *Total* in the last line refers to the total number of *different winners* for the *specific year*. Given that, within a year, the same bidders win multiple units in several months, this number is below the sum over months by year. Similarly for the last column, where *Total* is the number of *different bidders* for the *specific month* during the 13-year sample. Finally, 537, refers to the total number of different bidders in the whole sample.

Table 10: Estimation Results

Structural parameters	Specifications						
	Sequential Auction Model					Standard Model	
	(1)	(2)	(3)	(4)	Residuals	(5)	(6)
Mean Valuation [$\mathbb{E}(V)$]	389.06	547.43	324.91	365.38		136.13	178.67
	[0.0411]	[0.0001]	[0.0001]	[0.0001]		[0.0005]	[0.0001]
Sunk Cost (α)	0.0113	0.0377	0.0126	0.0342			
	[0.0211]	[0.0001]	[0.0002]	[0.0001]			
β_c	-0.1134	-0.0575	-0.2139	-0.1153			
	[0.3608]	[0.6105]	[0.1788]	[0.5416]			
β_s	0.0123	0.01671	0.3103	0.1024			
	[0.6413]	[0.6684]	[0.0186]	[0.3457]			
Marginal Effects on Price							
Future Rain (β_1)	-0.1081	-0.0938	-0.0705	-0.0471			
	[0.0192]	[0.0025]	[0.0119]	[0.0516]			
Selected Covariates							
Past Rain (γ_1)			-0.0394	-0.0474	1.7687	0.0131	0.0119
			[0.0773]	[0.0424]	[0.0001]	[0.0858]	[0.1628]
Feb (γ_6)			-0.0498	-0.0623	26.4326	0.1062	0.0024
			[0.1396]	[0.0533]	[0.7778]	[0.3506]	[0.9770]
Mar (γ_7)			0.0381	0.0404	-16.0550	0.1625	0.1918
			[0.2539]	[0.3909]	[0.9991]	[0.1137]	[0.0557]
Apr (γ_8)			0.0753	0.0593	8.0710	0.1490	0.3107
			[0.0315]	[0.1599]	[0.9176]	[0.1218]	[0.0006]
May (γ_9)			0.0602	0.0665	8.3087	0.1349	0.1090
			[0.1110]	[0.2241]	[0.9138]	[0.1646]	[0.2408]
Jun (γ_{10})			0.0737	-0.0422	-21.5523	0.1370	0.1016
			[0.1082]	[0.3482]	[0.7826]	[0.1253]	[0.2276]
Jul (γ_{11})			0.2225	0.1975	-126.9779	0.3260	0.2023
			[0.0513]	[0.1229]	[0.1054]	[0.0293]	[0.3236]
Aug (γ_{12})			0.1954	0.1462	-76.7241	0.4184	0.2424
			[0.0246]	[0.0529]	[0.9238]	[0.0717]	[0.2336]
Sep (γ_{13})			0.1842	0.2010	-33.1308	0.2836	-0.0399
			[0.0837]	[0.0350]	[0.6857]	[0.2360]	[0.8539]
Oct (γ_{14})			0.1912	0.2171	18.6739	0.1881	0.7153
			[0.0356]	[0.0264]	[0.8283]	[0.4702]	[0.0004]
Nov (γ_{15})			0.0223	0.0603	4.1315	0.0898	0.2176
			[0.5330]	[0.2102]	[0.9616]	[0.3679]	[0.0021]
Dec (γ_{16})			0.0720	0.0344	-9.7702	0.0570	0.0907
			[0.1014]	[0.4173]	[0.9186]	[0.5347]	[0.2564]
Mean ρ							
- *Pure complements*	0.3707	0.2319	0.6725	0.4003			
- *Non-complements*	-0.0221	-0.0084	-0.9267	-0.2804			
N	50	40	50	40	40	50	40
Schedule FE	No	No	Yes	Yes	Yes	Yes	Yes
Weekday FE	No	No	Yes	Yes	Yes	Yes	Yes
Pseudo R^2	0.2861	0.2620	0.8504	0.8326	0.0235	0.1779	0.1634
Log likelihood	-1,352.53	-1,353.11	-1,262.47	-1,264.91		-3,506.15	-3,625.79
# of Auctions	5,964	5,964	5,964	5,964	5,964	5,964	5,964

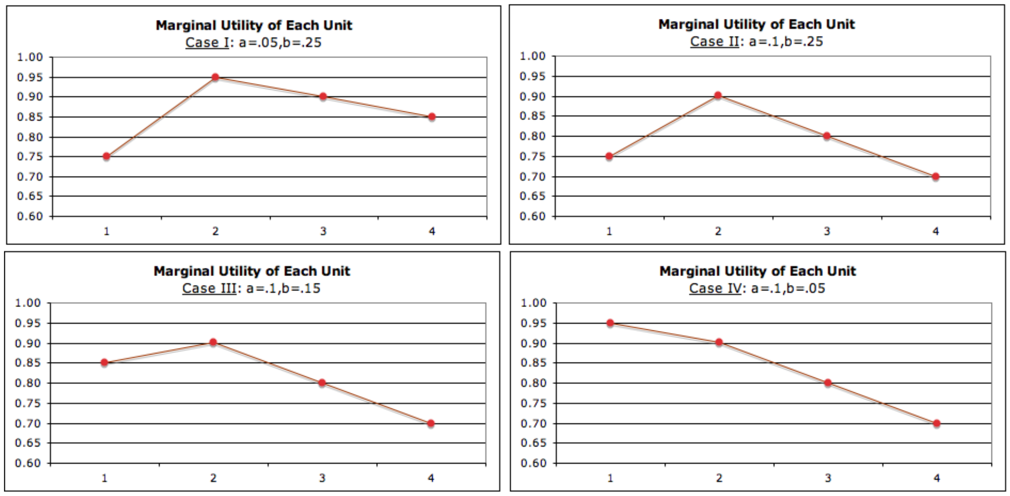
Notes: $P >\mid z \mid$ is reported in squared brackets (for the *Mean Valuation* it corresponds to the *p-value* corresponding to the estimate of the parameter θ). Estimates in columns 1 to 4 (sequential auction model) are MLE obtained by maximizing the likelihood function in Subsection 7.1 using a tolerance level of 1.0e-09, and the procedure described in Section 7. For the distribution of private values and inclusion of covariates, we use the parametric specification described in Subsection 7.1. Estimates in specifications 5 and 6 (standard model) are MLE obtained by maximizing the likelihood function from a standard English-auction model, without fixed costs nor decreasing marginal returns (the sample is the same as the one in columns 1 to 4, including in this cases all sequential prices in the estimation). Estimates in the column labeled *Residuals* are OLS regressions using the residuals from the model in specification 4. Although not reported in the table, weekday, year and schedule fixed effect are included when indicated. Number of years in the sample is 13. Number of months in the sample is 119. The number of different winners (across all 13 years) is 537. Marginal effects are reported for future and past rain, as well as for months covariates (see subsection 7.2 for further details). The complementarity parameter, ρ, is computed as detailed in the theoretical model in Section 3. When the goods are *pure complements* is given by $\rho_c = \frac{\alpha-3\beta_c}{1-\alpha} = \frac{\alpha-3(\beta_c+\beta_1 R_t^F)}{1-\alpha}$. The table reports, for each specification of the sequential auction model in columns 1 to 4: $\bar{\rho}_c = \frac{\hat{\alpha}-3(\hat{\beta}_c+\hat{\beta}_1\mathbb{E}_c(R_t^F))}{1-\hat{\alpha}}$. Similarly, when the goods are *non-complements*, the table reports, for each specification: $\bar{\rho}_s = \frac{\hat{\alpha}-3(\hat{\beta}_s+\hat{\beta}_1\mathbb{E}_s(R_t^F))}{1-\hat{\alpha}}$. See subsection 7.2 for details.

Table 11: Monte Carlo Design and Results for Testing Performance of the Estimators

Parameter	Bias	Variance	MSE
θ	-2.94E-09	6.03574E-07	6.03574E-07
α	4.62E-09	0.024073353	0.024073353
β_c	2.59E-08	0.057346695	0.057346695
β_s	-3.35E-10	0.014881658	0.014881658
β_1	-3.02E-09	0.073328361	0.073328361

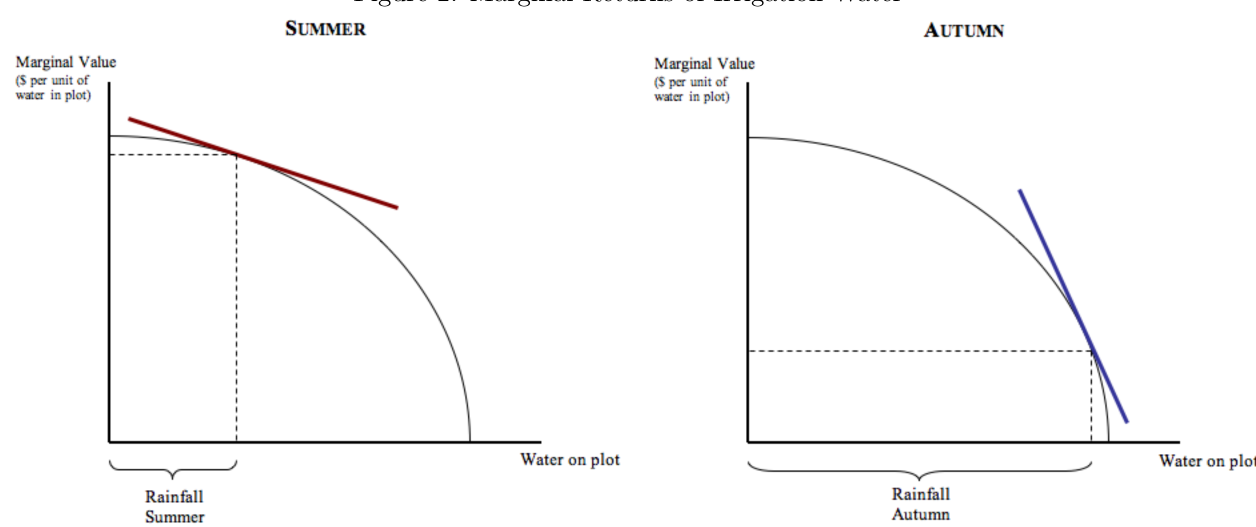
Notes: The table displays the results from a Monte Carlo study to test the performance of the estimators in Section 7. The design is explained in Appendix B. The specifics of the design that appear in the Table are the following: $T = 10,000$, $S = 200$, $N_t = 40$, $\theta = 0.3$, $\alpha = 0.1$, $\beta_c = -0.1$, $\beta_s = 0.1$, $\beta_1 = 0.25$. Similar results were obtained with $N_t = 50$ (not reported).

Figure 1: Sunk Cost and Decreasing Returns Effect



The figure displays the actual utility derived by the first, second, third and forth goods consumed by a bidder with type $v_i = 1$. In case I (top-left), all the remaining goods (second, third and forth units) have a valuation greater than the first one (*pure complements*). Case II (top-right) shows a situation where only second and third units are valued above the first one. In case III (bottom-left), only the second unit is above the first one. Finally, case IV (bottom-right) plots a situation where all goods valuations are below the one of the first unit.

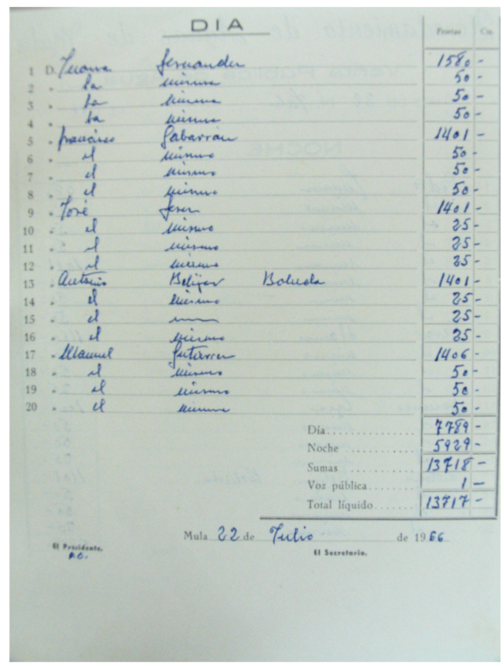
Figure 2: Marginal Returns of Irrigation Water



Marginal returns of water in summer (left) and autumn (right).

Figure 3: Auction Sample: Goods are complements

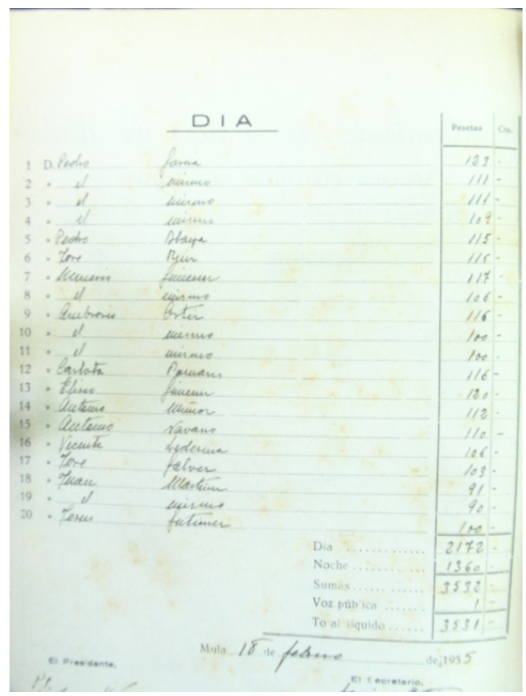
Auction #	Name	Price
1	Juana Fernandez	1580
2	Juana Fernandez	50
3	Juana Fernandez	50
4	Juana Fernandez	50
5	Francisco Gabarron	1401
6	Francisco Gabarron	50
7	Francisco Gabarron	50
8	Francisco Gabarron	50
9	Jose Ferez	1401
10	Jose Ferez	25
11	Jose Ferez	25
12	Jose Ferez	25
13	Antonio Belijar Boluda	1401
14	Antonio Belijar Boluda	25
15	Antonio Belijar Boluda	25
16	Antonio Belijar Boluda	25
17	Manuel Gutierrez	1406
18	Manuel Gutierrez	50
19	Manuel Gutierrez	50
20	Manuel Gutierrez	50



Sample from original data obtained from the historical archive: Goods are complements (Summer - July 22, 1966, Day).

Figure 4: Auction Sample: Goods are Substitutes

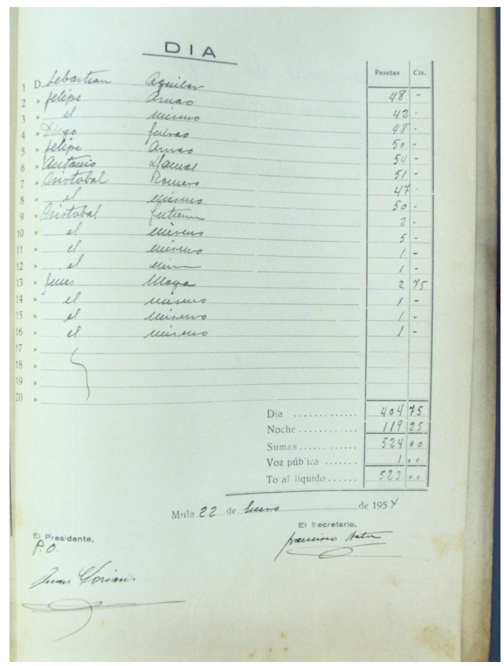
Auction #	Name	Price
1	Pedro Foma	123
2	Pedro Foma	111
3	Pedro Foma	111
4	Pedro Foma	109
5	Pedro Blayas	115
6	Jose Ruiz	116
7	Mauricio Guimenez	117
8	Mauricio Guimenez	106
9	Ambrosio Ortiz	116
10	Ambrosio Ortiz	100
11	Ambrosio Ortiz	100
12	Carlota Pomares	116
13	Eliseo Guimenez	120
14	Antonio Muñoz	112
15	Antonio Navarro	110
16	Vicente Ledesma	106
17	Jose Galvez	103
18	Juan Martinez	91
19	Juan Martinez	90
20	Jesus Gutierrez	100



Sample from original data obtained from the historical archive: Goods are Substitutes (Winter - February 18, 1955, Day).

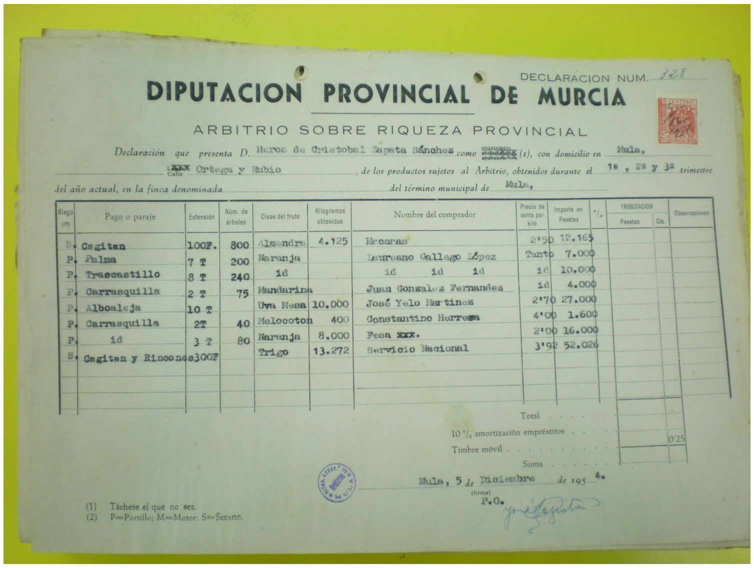
Figure 5: Auction Sample: Auction where farmers are present and no one bids

Auction #	Name	Price
1	Sebastian Aguilar	48
2	Felipe Amaro	42
3	Felipe Amaro	48
4	Diego Guivao	50
5	Felipe Amaro	54
6	Antonio Llamas	51
7	Cristobal Romero	47
8	Cristobal Romero	50
9	Cristobal Futam	2
10	Cristobal Futam	5
11	Cristobal Futam	1
12	Cristobal Futam	1
13	Luis Moya	2.75
14	Luis Moya	1
15	Luis Moya	1
16	Luis Moya	1
17		
18		
19		
20		



Sample from original data obtained from the historical archive: Auction where farmers are present and no one bids (Winter - January 22, 1954, Day).

Figure 6: Sample of individual data obtained from the Agricultural Census


DECLARACION NUM. 328

DIPUTACION PROVINCIAL DE MURCIA

ARBITRIO SOBRE RIQUEZA PROVINCIAL

Declaración que presenta D. Heros de Cristobal Zapata Sánchez como propietario (1), con domicilio en Mula, Calle Ortega y Rubio, de los productos sujetos al Arbitrio, obtenidos durante el 1º, 2º y 3º trimestre del año actual, en la finca denominada ... del término municipal de Mula,

Riego (2)	Pago o paraje	Extensión	Núm. de árboles	Clase del fruto	Kilogramos obtenidos	Nombre del comprador	Precio de venta por kilo	Importe en Pesetas	°/o	TRIBUTACION Pesetas	Cts.	Observaciones
S.	Cagitan	100F.	800	Almendra	4.125	Mecaras	2'50	12.165				
P.	Palma	7 T	200	Naranja		Laureano Gallego López	Tanto	7.000				
P.	Trascastillo	8 T	240	id		id id id	id	10.000				
P.	Carrasquilla	2 T	75	Mandarina		Juan Gonzalez Fernandez	id	4.000				
P.	Alboaleja	10 T		Uva Mesa	10.000	José Yelo Martinez	2'70	27.000				
P.	Carrasquilla	2T	40	Melocoton	400	Constantino Herrera	4'00	1.600				
P.	id	3 T	80	Naranja	8.000	Fesa xxx.	2'00	16.000				
S.	Cagitan y Rincones	300F		Trigo	13.272	Servicio Nacional	3'92	52.026				

Total

10 °/o amortización empréstitos

Timbre móvil ... 0'25

Suma

Mula, 5 de Diciembre de 1954.

(firma)

P.O.

(1) Táchese el que no sea.
(2) P=Portillo; M=Motor; S=Secano.

Figure 7: Rain and frequency distribution of 4CU over the sample period

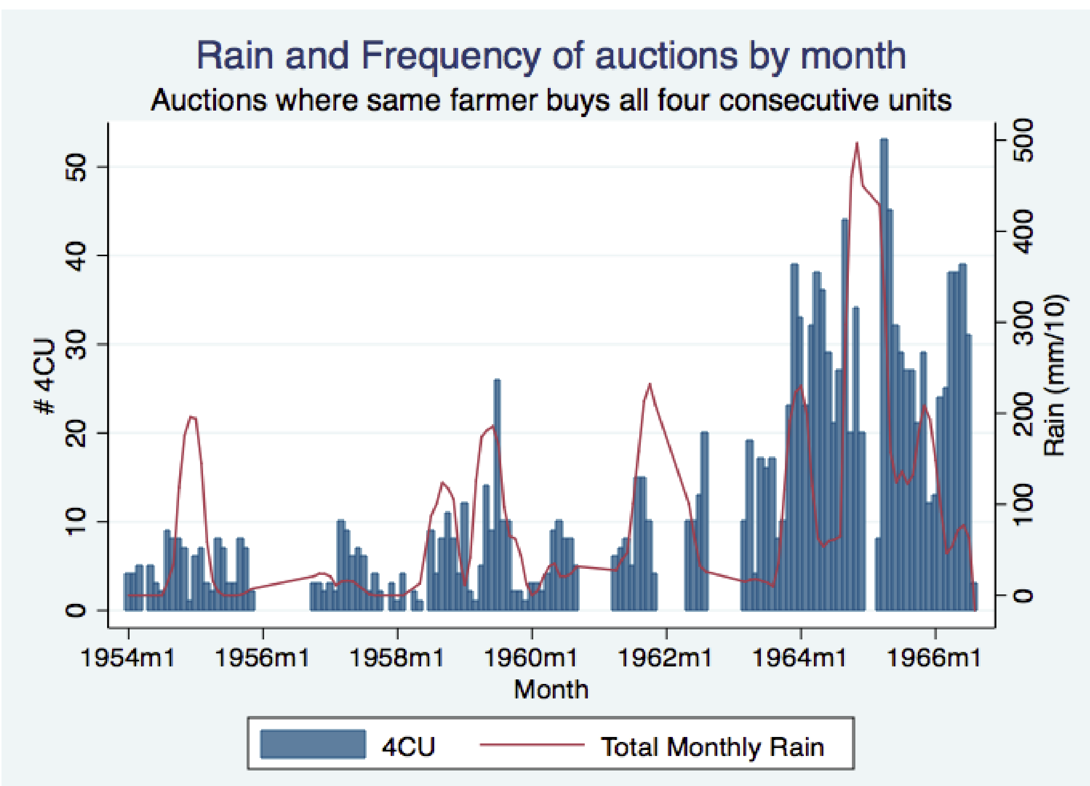


Figure 8: Winning Prices: by # of consecutive units bought, by the same farmer and by unit

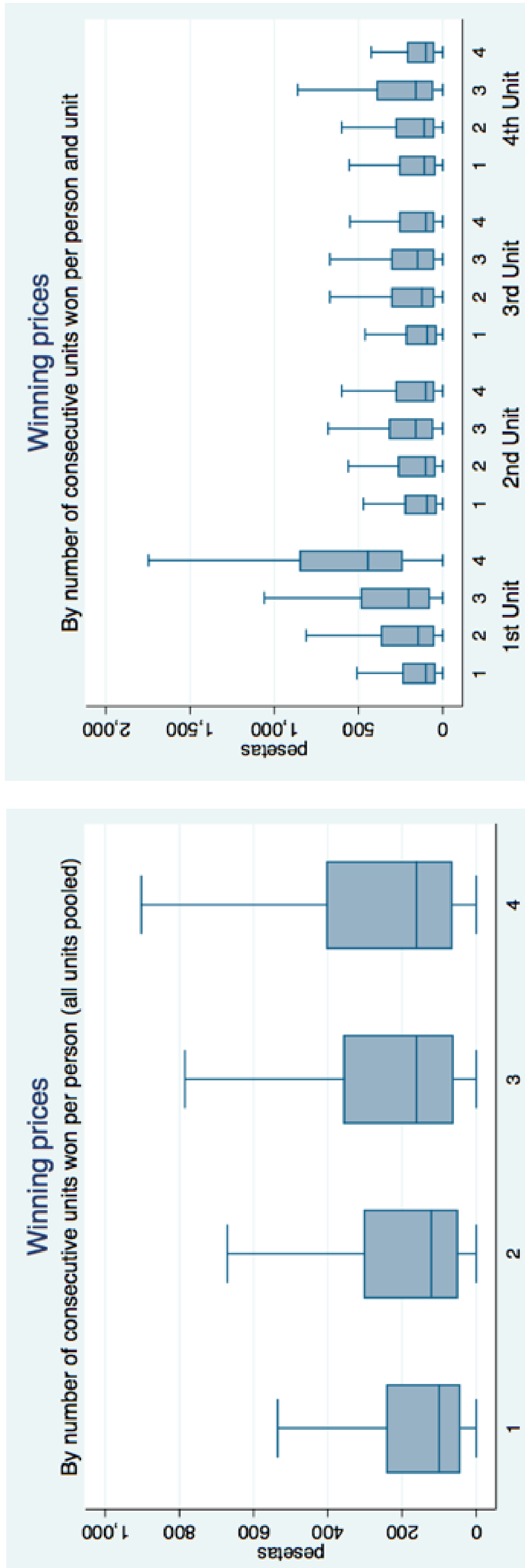


Figure 9: Winning Prices: by weekday, hour and schedule

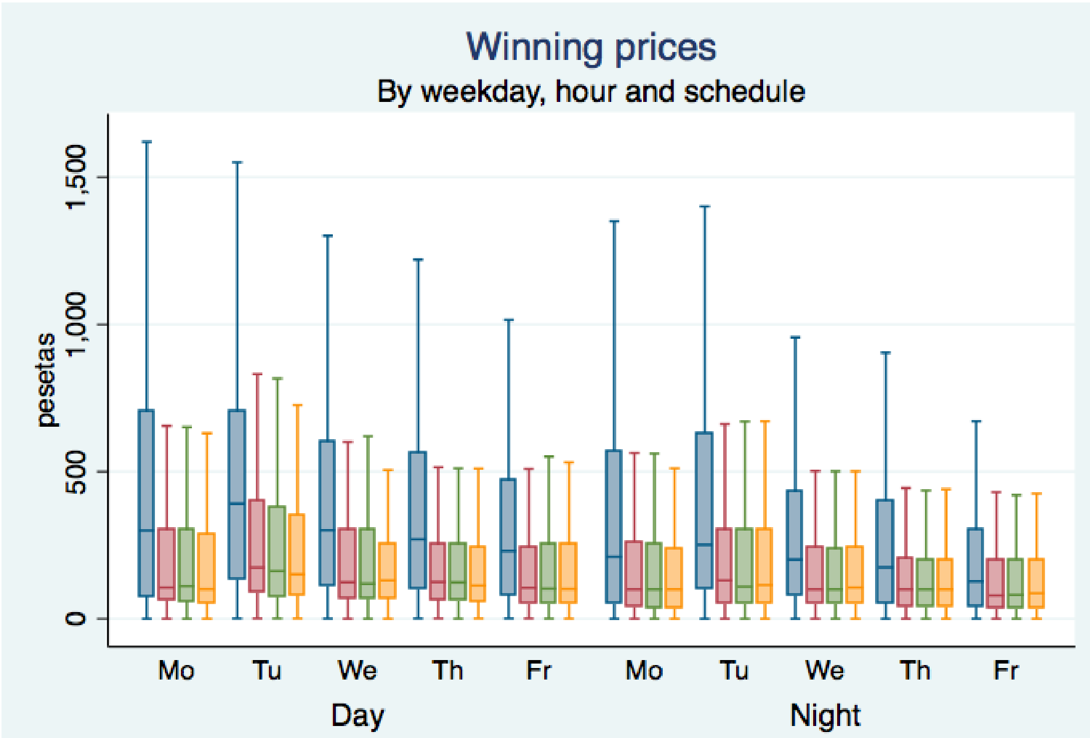


Figure 10: Price and seasonality

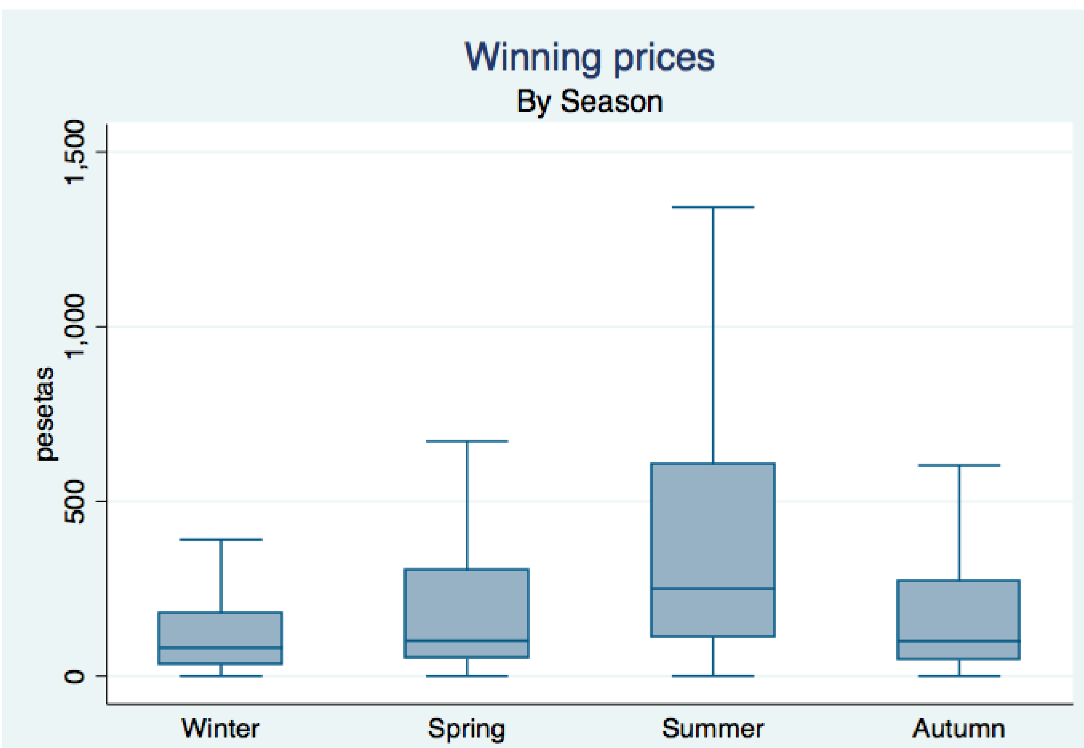


Figure 11: Winning prices: by season and drought

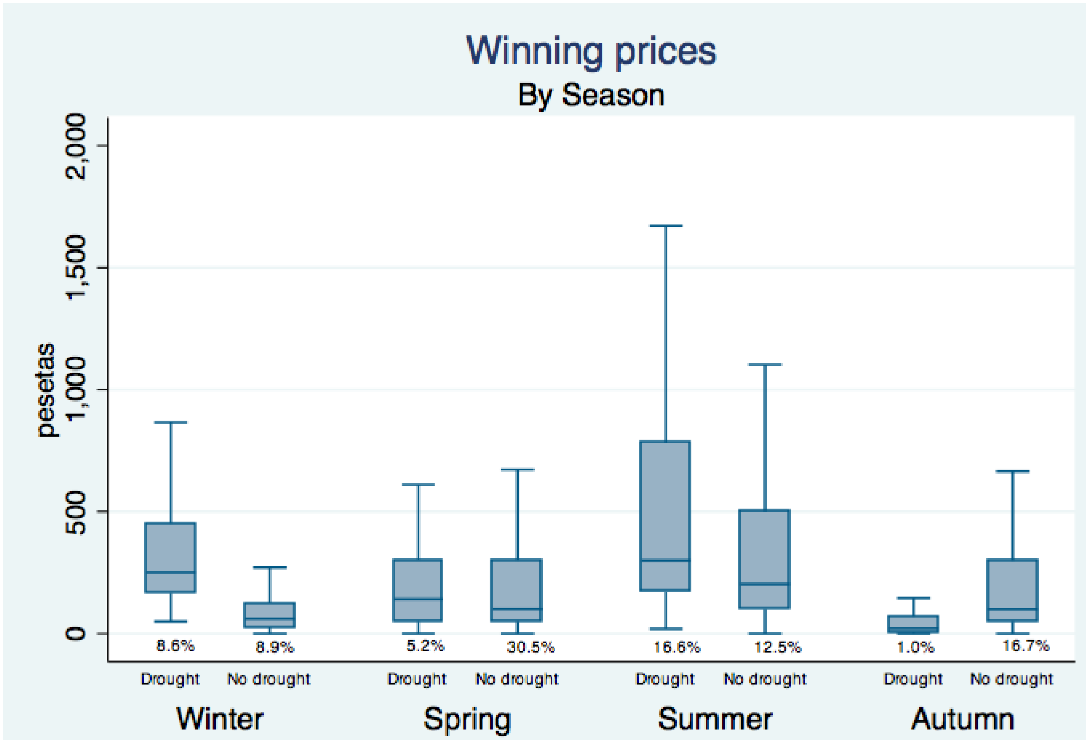


Figure 12: Frequency distribution of auctions by month

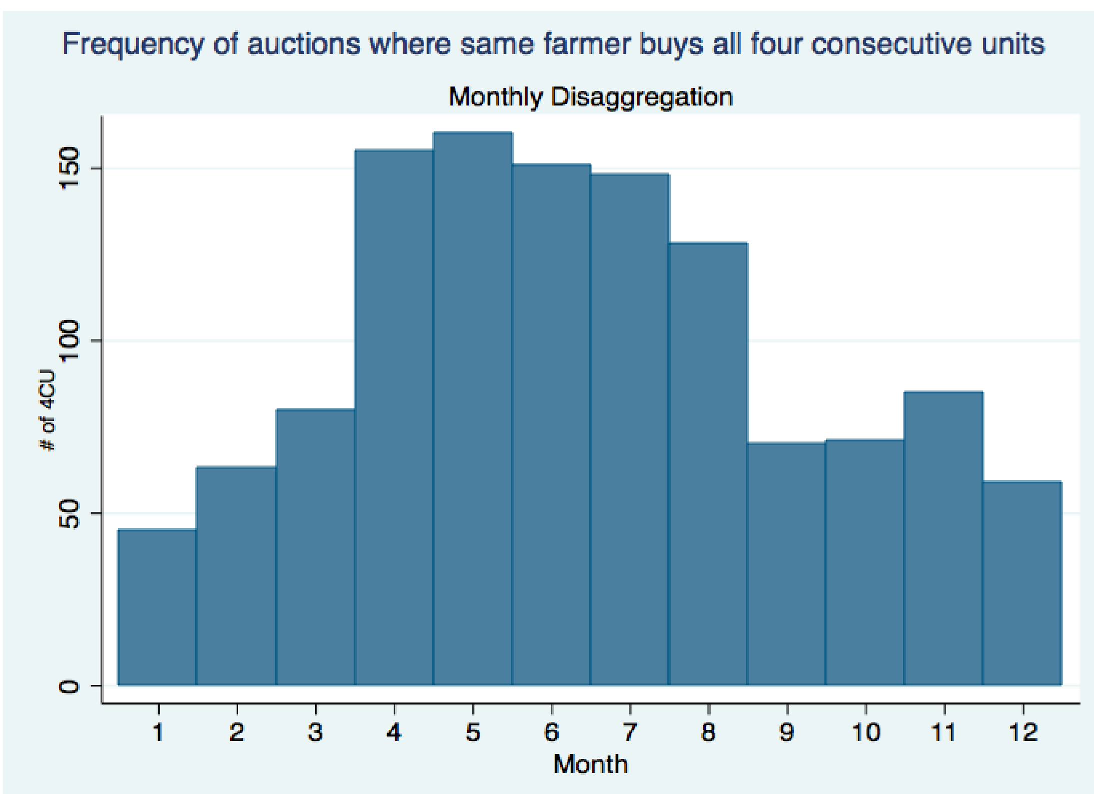


Figure 13: Winning farmers and winning prices

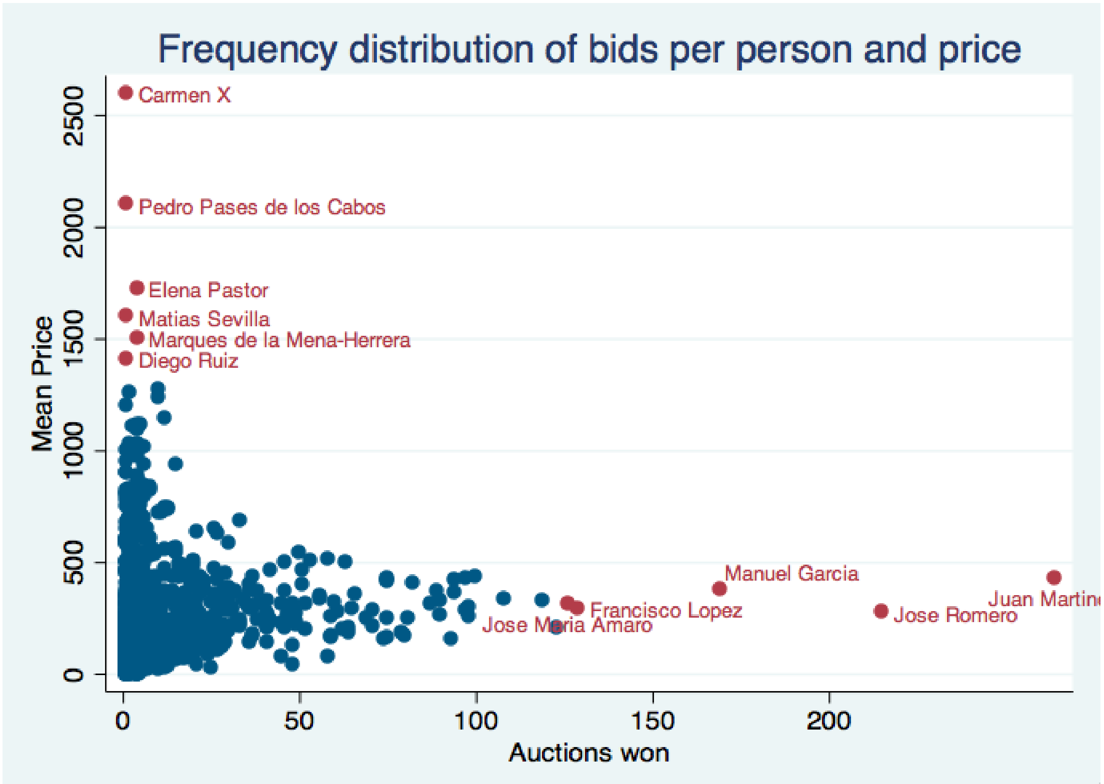
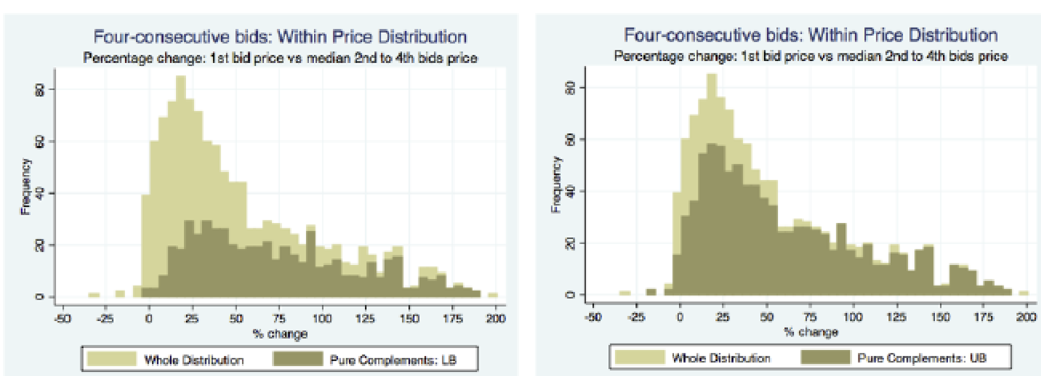


Figure 14: Within Price Distribution by Regime (1st price vs median 2nd to 4th prices)



Notes: Both figures display, for 4CU, the percentage change of the first winning price against the median of the second to fourth winning prices. In the figure on the left, LB is computed by assuming that all second, third and fourth unit prices paid by the (same) farmer within the same four-unit auction display end-price preference for 0. In the figure on the right, UB is computed by assuming that only one among second, third or fourth unit prices paid by the (same) farmer within the same four-unit auction display end-price preference for 0.

Figure 15: Exponentiated Gamma (EG) Distribution: PDF vs $N-1$ order statistic

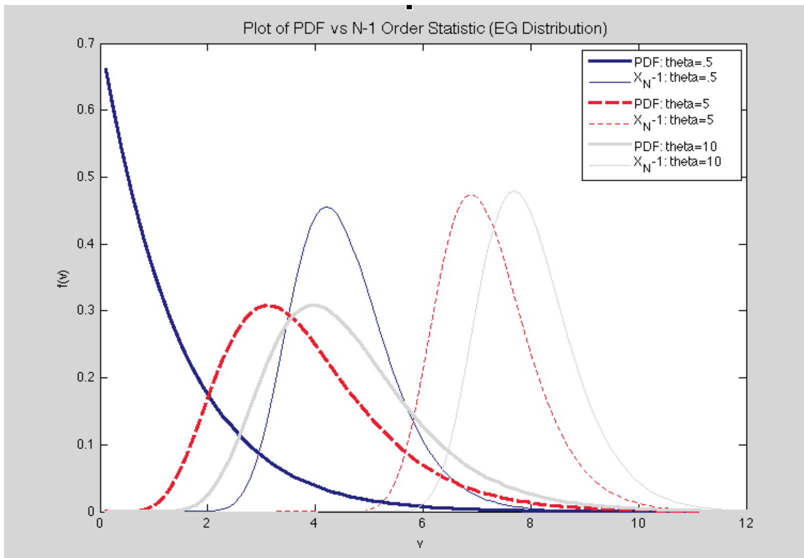
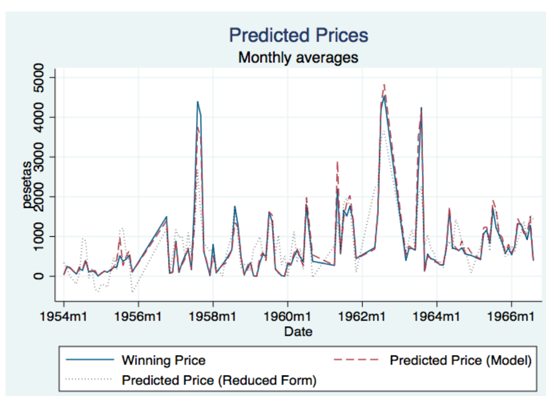
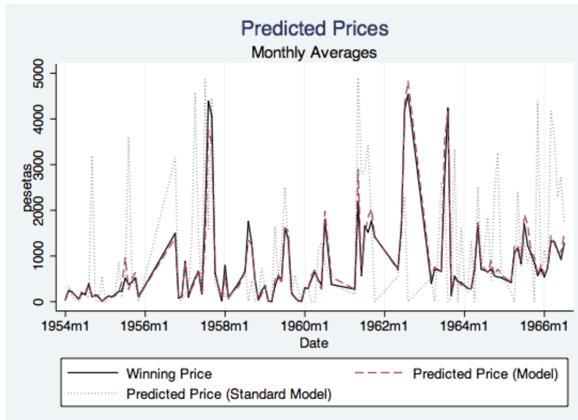


Figure 16: Winning and Estimated Prices I



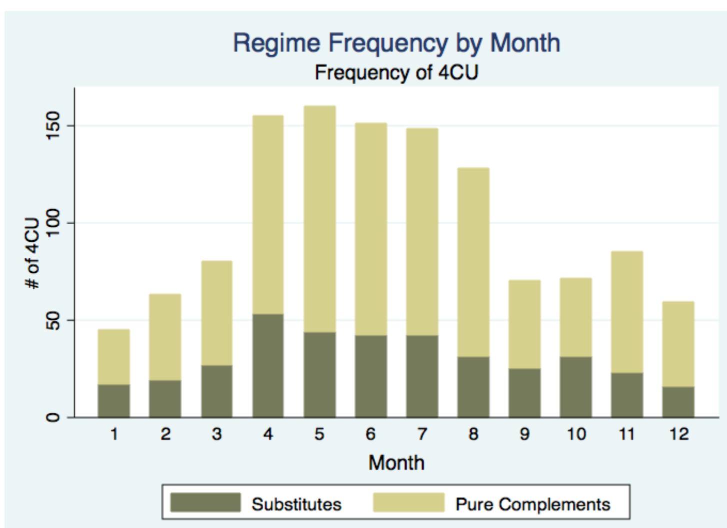
Notes: The figures display winning prices against: (i) predicted prices according to the sequential auction model presented in the text (specification 4 in Table 10), and (ii) predicted prices according to a reduced-form model (for the sample) using as regressors: *Past Rain*, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), week (51 dummy variables) fixed effects, additional to a constant (see Table 4). The graph plots mean monthly values of the prices. Similar results are obtained using a spline.

Figure 17: Winning and Estimated Prices II



Notes: The figure displays winning prices against: (i) predicted prices according to the sequential auction model presented in the text (specification 4 in Table 10), and (ii) predicted prices according to the standard (button) auction model (specification 6 in the same table). The graph plots mean monthly values of the prices. Similar results are obtained using a spline.

Figure 18: Regime Frequency Dissagreagation by Month



Notes: The figure depicts the frequency of 4CU by regime (see Section 5) and month. It can be seen that complementarities are more likely to be observed in summer than in winter, where water requirements (and, hence, equilibrium prices) soar. We interpret this as evidence in favor of the competition hypothesis (according to our model with entry and sunk costs) and against the collusion hypothesis.