Cartel sustainability and cartel stability

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May, 2005

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2I would like to thank my supervisor Ramon Faulí-Oller for his advice and encouragement,
and Lluís Bru and George Symeonidis for his helpful comments. Financial support by the
I.V.I.E, (Instituto Valenciano de Investigaciones Económicas), Ministerio de Ciencia y Tecnologia
through its project BEC2001-0535, the European Commission through its RTN programme
(Research Training Network), the Center for Financial Studies (Frankfurt) and The Department
of Economics of the University of Alicante is gratefully acknowledged. All errors are mine.
Abstract

The paper studies how does the size of a cartel affect the possibility that its members can sustain a collusive agreement. I obtain that collusion is easier to sustain the larger the cartel is. Then, I explore the implications of this result on the incentives of firms to participate in a cartel. Firms will be more willing to participate because otherwise, they risk that collusion completely collapses, as remaining cartel members are unable to sustain collusion.

*JEL Classification*: L11, L13, L41, D43.

*Keywords*: Collusion, Partial Cartels, Trigger strategies, Optimal Punishment.
1 Introduction

For many years it was widely held among economists that firms could not exercise market power collectively without some form of explicit coordination. However the theory of repeated games has cast some doubt on this approach. Stable arrangements may require little coordination between firms, and possibly none at all. This has raised a dilemma for the design of a policy towards collusion. If the legal standard focuses on explicit coordination, a large number of collusive outcomes can fall outside the prohibition, and if it tries to cover collusion without explicit coordination, it will prohibit non-cooperative practises.

Article 81(1) of the Rome Treaty stipulates that agreements or concerted practises between firms which distort competition are prohibited. What is meant by “agreements” and “concerted practises” is not further specified in the treaty. However, decisions recently taken by the Commission show that often firms behavior that do not involve a process of coordination are overlooked although they could mean an exercise of market power.

The literature about collusion, mainly deal with two different approaches. Firstly, there are the papers that investigate cartel stability in static models. They have mainly focused on the incentives of firms to participate in a cartel agreement. These papers focus on firms “participation constraints”. Two different incentives play a role here. Firms face a trade off between participation and nonparticipation in the cartel: firms have an incentive to join the cartel so as to achieve a more collusive outcome, but on the other hand have an incentive to stay out of the cartel to free-ride on the cartel effort to restrict production. By their very nature, in these models cartel members do not cheat on a cartel agreement as it is assumed that agreements are sustained through binding contracts, they may, therefore, be viewed as models of binding collusion. They refer to explicit or overt collusion, that is when firms have engaged in direct communication and obvious coordination. It should include legal cartels, where firms can set up fines and punish cheating using the court system. The seminal papers in this literature are Selten (1983) and d’Aspremont et al. (1983).

A formal collusion agreement among competing firms (mostly oligopolistic firms) in an industry designed to control the market, raise the market price, and otherwise act like a monopoly is frequently also termed explicit collusion. Binding collusion refers, therefore, to an explicit collusive agreement enforceable at law.
There is another strand in the literature on cartel stability, which takes a quite different route. The supergame-theoretic approach to collusion has focused on the problem of enforcement of collusive behavior (see for example Friedman (1971)). In these models, seemingly independent, but parallel actions among competing firms in an industry are driven to achieve higher profits. It is termed tacit or implicit collusion. Implicit or tacit collusion refers to a situation where firms are able to coordinate through some mutual understanding and without the aid of direct communication. How do firms behave when they tacitly collude? This oligopolistic behavior has been modeled assuming that firms recognize their interdependence and design mutually beneficial agreements with punishments for cheating on the agreements. Where courts can not be used to enforce agreements, firms must rely on self-enforcing punishments. This approach focuses on firms “incentive constraints”. Then, what this approach leaves out, are firms’ “participation constraints”: it cannot explain why many real world cartels do not comprise all firms in the industry. Instead, they have studied under which circumstances collusion can be sustained as an equilibrium of the repeated game. Most research on the field has studied symmetric settings and has focused on the sustainability of the most profitable symmetric equilibrium. The reason to select this equilibrium is that it will be the one that firms will agree to play if they secretly meet to discuss their pricing plans (Mas-Colell et al (1996)).

The main point of the paper is that this argument is compelling but it does not take into account that firms may prefer not to attend this meeting in order not to participate in the coordination to a collusive agreement. This takes us back to the literature on the incentive to participate in a cartel, mentioned above. However, now the analysis is richer because one has to study how does the participation incentive interact with the incentive to maintain a collusive agreement. As a first step, I study how does the size of a cartel affect the possibility that its members can sustain a collusive agreement in a supergame theoretical framework. I obtain that collusion is easier to sustain the larger the cartel is. To obtain the result I study the sustainability of partial cartels i.e. cartels that do not include all the firms in a given industry. In practice, there are many cases where collusive agreements do not involve all firms in the industry, being the OPEC the most well known

2“Participation constraints” are firms incentives to join the cartel or the fringe; meanwhile “Incentive constraints” are the incentives to cheat on the cooperative agreement.
The previous result has implications on cartel formation, because it reduces the incentives to free-ride from a cartel by defecting from it. This gives an intuition of what determines the size of the active cartels. I can illustrate the idea with the following extreme example. I find that for some discount factors, the only sustainable cartel is the cartel that comprises all firms in the industry. Then all firms have incentives to participate in the cartel, because otherwise collusion completely collapses. This completely eliminates the gains from free-riding at the participation stage.

One the other hand, one necessarily thinks that firms prefer to tacitly collude as they are not as much at risk of paying penalties. Obviously, in practice it is easier to fight binding collusion than against implicit collusion. In this sense, the present model also highlights that policy measures that induce firms to replace binding with implicit collusion to escape antitrust prosecution may have its costs. Forbidding binding collusion (and forcing firms to collude tacitly) has the positive effect of weaken the incentives to maintain a collusive agreement but the negative effect of making stronger the incentives to participate in a cartel. Therefore the total effect on price is uncertain. In the particular model I analyze price is higher with implicit than with binding collusion. The model also predicts that the size of the cartels enforced can be larger in the implicit collusion model than in the binding collusion model.

We can think of several interesting cases where these results could be of interest. In another significant example is the citric acid industry where eight firms were convicted in United States, Canada and the EU and fined more than one hundred million euros for fixing prices and allocating sales during the period 1991-1995. The joint market share of these eight firms was between 50% and 60%.

...
several European countries, before governments moved to adapt its domestic competition policy to the European regime, agreements restricting competition among firms were not only permitted but also enforceable at law. Namely, Denmark (see OCDE (1993)) where before the Competition Act of 1990 was passed agreements were widespread in several sectors and often took the form of binding agreements and also West Germany where before 1987 hundreds of legalized cartels were enforced through a contract (see Audretsch (1989)). Switzerland and Sweden are other examples of countries where Cartels were sustained for decades by means of enforceable contracts. Therefore, the present model points out a possible consequence of banning binding collusion that perhaps has been unnoticed by antitrust authorities.

The structure of the paper is as follows. In the following section, the central model of the paper is set. The sustainability of the partial cartel is analyzed using the “trigger strategies”. Section 3 presents the participation game where firms decide first whether to join the cartel or not, and afterwards play infinitely a quantity game. The main conclusions are presented in section 4. Finally, in the appendix the model is extended using an optimal punishment and the same results are obtained. All proofs are also relegated to the appendix.
2 The Model: Partial Cartels

Assume that \( n \) firms, where \( n > 2 \), indexed \( i, i = 1, 2, 3, ..., n \) compete in a market whose demand is given by \( P(Q) = 1 - Q \). Cost functions of firms are given by: \( c(q_i) = \frac{q_i^2}{2} \), where \( q_i \) denotes the production of firm \( i \). Assume firms simultaneously choose quantities.\(^6\)

A (partial) cartel will be said to be active in this market if there is a group of firms (cartel members) that maximize joint profits and the remaining firms (nonmembers or fringe firms) maximize individual profits. When a cartel of \( k \) firms is active, members (\( m \)), and nonmembers (\( nm \)), simultaneously produce respectively:

\[
q_{km}^k = \frac{2}{nk - k^2 + 3k + 2 + n} \tag{1}
\]

\[
q_{knm}^k = \frac{k + 1}{nk - k^2 + 3k + 2 + n} \tag{2}
\]

In this situation, profits of members and nonmembers are given respectively by \( \pi_m^k \) and \( \pi_{nm}^k \). Observe that if \( k = 1 \), we have standard Cournot competition and \( q_m^1 = q_{nm}^1 \).

We now seek under which conditions playing (1) and (2) in each period can be sustained as an equilibrium of a game where the one stage game described above is repeated infinite times. Firms will be assumed to discount the future at a factor of \( \delta \). Member firms are denoted with a natural number from 1 to \( k \).

Cartel members will sustain cooperation by using “trigger strategies”, that is, when cheating, firms are punished with infinite reversion to the Nash Cournot equilibrium.

Trigger strategies for a partial cartel can be formulated the following way, where \( q_{t,i} \) denotes the strategy played by firm \( i \) in period \( t \):

\[
\begin{align*}
\text{Firm } i, & \text{ } i = 1, ..k \text{ plays } \\
q_{1,i} & = q_{km}^k \\
q_{t,i} & = q_{km}^k \text{ if } q_{t,j} = q_{km}^k \text{ for any } l < t \text{ for } j = 1, ..., k \\
q_{t,i} & = q_{nm}^1 \text{ otherwise.}
\end{align*}
\]

\[
\begin{align*}
\text{Firm } i, & \text{ } i = k + 1, ..n \text{ plays } \\
q_{1,i} & = q_{km}^k \\
q_{t,i} & = q_{nm}^1 \text{ for any } l < t \text{ for } j = 1, ..., k \\
q_{t,i} & = q_{nm}^1 \text{ otherwise.}
\end{align*}
\]

\(^6\)Shaffer(1995) considers the cartel as a Stackelberg leader because of its power to impose its most preferred timing.
\[
\begin{cases}
q_{1,i} = q_{nm}^k \\
q_{t,i} = q_{nm}^k \text{ if } q_{t,j} = q_{m}^k \text{ for any } l < t \text{ for } j = 1, \ldots, k \\
q_{t,i} = q_{m}^1 \text{ otherwise.}
\end{cases}
\]

Nonmember firms play optimally, because the future play of rivals is independent of how they play today and they maximize current profits. Member firms will behave optimally if the discount factor is high enough. To obtain the conditions on the discount factor such that using trigger strategies is also optimal for member firms, we need to calculate the profits of a member that deviates from the cartel. The deviator will choose:

\[
q_{d}^k = \arg \max \limits_{q} P((k-1)q_{m}^k + (n-k)q_{nm}^k + q) - \frac{q^2}{2}
\]

and will obtain \(\pi_{d}^k\) like the profits obtained in the period of deviation.

Then, trigger strategies are optimal for member firms if:

\[
\frac{1}{1-\delta}\pi_{m}^k \geq \frac{\delta}{1-\delta}\pi_{m}^1 + \pi_{d}^k
\]

If we let \(\delta_k = \frac{\pi_{d}^k - \pi_{m}^k}{\pi_{d}^k - \pi_{m}^1}\), the previous condition can be written in the following way:

If \(\delta_k \geq 1\) the cartel of size \(k\) can not be sustained for any \(\delta\). If \(\delta_k < 1\), the cartel can be sustained for \(\delta \geq \delta_k\).

Although it may be surprising at first sight that some cartel sizes can not be sustained in equilibrium, it comes from the well-known result in the literature that with Cournot competition, mergers (or any other collusive agreement) of a small number of firms reduces profits because non-participating firms react by increasing their production (see Salant et al. (1983)).

Next proposition shows that the previous intuition extends to any cartel size in the sense that whenever a cartel of size \(k\) is sustainable, cartels of larger size are also sustainable.\(^7\)

**Proposition 1** The cutoff discount factor (\(\delta_k\)) that sustain the strategies described above is strictly decreasing in the size of the cartel.

\(^7\)Remark the similarity with the result in Salant et al. (1983) that if a merger of \(k\) firms is profitable, a merger with more firms is also profitable.
\( \delta_k \) can be rewritten like:

\[
\delta_k = \frac{1 - \frac{\pi^k_m}{\pi^k_d}}{1 - \frac{\pi^1_m}{\pi^1_d}}
\]

Then, variations of \( k \) have two different effects. First, \( \frac{\pi^k_m}{\pi^k_d} \) decreases when \( k \) increases because deviation profits increase more than profits from being in the cartel of \( k \) firms. This would increase \( \delta_k \). Second, as \( k \) increases, \( \frac{\pi^1_m}{\pi^1_d} \) also decreases because \( \pi^1_m \) does not depend on \( k \), and deviation profits increase with \( k \). Thus punishment becomes proportionally more painful. This second effect would decrease \( \delta_k \).

The result in Proposition 1 comes from the fact that the second effect dominates the first one.

3 The participation game

In this central section of the paper, the size of the active cartels is analyzed. The main aim of this section is to study the interaction between incentive and participation constraints. To that purpose, I model the formation of the cartel as a simple, noncooperative game where firms simultaneously decide on their participation to the cartel. This pre-communication is modelled as a stage prior to market competition. Decisions announcing firms participating or not in the cartel do not affect their payoffs and will only be used as a coordination device. If \( k \) firms announce joining the cartel, the future play is only modified if the discount factor allows a cartel of \( k \) firms to be active (\( \delta \geq \delta_k \)). In short, once a cartel of \( k \) firms is formed, it is assumed that discounted payoffs of member and nonmember firms are respectively given by the following expressions:

\[
\Pi^k_m = \begin{cases} 
\frac{1}{1-\delta} \pi^k_m & \text{if } \delta \geq \delta_k \\
\frac{1}{1-\delta} \pi^1_m & \text{otherwise}
\end{cases}
\]

(3)

\[
\Pi^k_{nm} = \begin{cases} 
\frac{1}{1-\delta} \pi^k_{nm} & \text{if } \delta \geq \delta_k \\
\frac{1}{1-\delta} \pi^1_{nm} & \text{otherwise}
\end{cases}
\]

(4)
Assume the standard notion of stability (see d’Aspremont et al. (1983)): a cartel is stable when (i) firms inside do not find it desirable to exit and (ii) firms outside do not find it desirable to enter.\textsuperscript{8}

(i) Internal stability: Either $k = 1$, or:

$$\Pi^k_m \geq \Pi^{k-1}_{nm}$$  \hspace{1cm} (5)

(ii) External stability: Either $k = n$, or:

$$\Pi^{k+1}_m \leq \Pi^k_{nm}$$  \hspace{1cm} (6)

This participation game has been previously analyzed in the literature in the one shot game.\textsuperscript{9} In that case, the static approach compares what Nash equilibrium yields and what is obtained from exogenously imposing some collective preference like joint profit maximization. This approach analyses stability of explicit or overt collusion, that is when firms have engaged in direct communication and obvious coordination. In this case, sustainability of cartels is not at issue as the outcome of the cartel is sustained as if firms could sign binding contracts. Collusion may, therefore, also said to be binding collusion. Then, payoffs of players would be like (3) and (4) when $\delta_k = 0$ and cartels of all sizes can always be sustained. Solving the participation game for the case of binding collusion will be a helpful step and will provide us with a benchmark to compare the results.

The key point in the binding collusion case is that for any cartel size, internal stability is never satisfied. Firms know that the goal of the cartel is to reduce production. Then, firms have incentives to leave the cartel in order to free ride from the output reduction agreed by the remaining cartel members.

**Proposition 2** No cartel is stable when collusion is binding.

We are ready now to determine the Nash equilibrium of the participation game and the main result of the paper. This game has many equilibria in which no cartel is active. For example all firms deciding not to join the cartel is always an equilibrium. For $\delta < \delta_n$\textsuperscript{9}

\textsuperscript{8}It is also assumed that only one cartel is formed and firms hypothesize that no other firm will change its strategy concerning membership in the cartel.

\textsuperscript{9}See Donsimon (1985). The only difference is that she considers the Cartel behaves as a Stackelberg leader while here the cartel and nonmember firms compete à la Cournot.
any choice by firms is an equilibrium because the participation decisions are irrelevant because no cartel can be sustained. To clarify the analysis I focus on the equilibria where cartels are active whenever they exist. It turns out that when they exist, they are unique (except for a permutation of players). I state the results in the following Proposition:

**Proposition 3**  
No cartel is active in equilibrium if $\delta < \delta_n$. Whenever $\delta \in [\delta_k, \delta_{k-1})$ and $\delta_k < 1$, a cartel of $k$ firms is active in equilibrium.

The fact that for $\delta < \delta_n$ no cartel is active comes from Proposition 1. For $\delta_{k-1} > \delta \geq \delta_k$ only cartels of size greater or equal than $k$ can be sustained. Cartels of sizes greater than $k$ are not stable, because the result in Proposition 2 applies: internal stability does not hold. The cartel of size $k$ is internally stable, because firms know that quitting the cartel means that collusion fully collapses and they would be worse off obtaining the Cournot profits. Therefore the cartel of size $k$ is stable. This effect means that the incentives to free-ride from a cartel by defecting from it are reduced. However, only the smallest cartel among those which can be sustained are stable in the present participation game.

Notice the most important implication: although firms have incentives to individually free ride on the cartel agreements (see Proposition 2), when collusion is tacit, the threat of a collusion collapse induces them to stick to the agreement as long as the cartel is sustainable (see Proposition 3).

Once characterized the equilibrium of the participation game, there are two interesting corollaries that can be extracted. Simply comparing Proposition 2 and Proposition 3 leads us to the following conclusion:

**Corollary 1** If $\delta \in [\delta_n, 1)$ the size of active cartels is bigger with implicit collusion than with binding collusion.

Although with binding collusion cartels are always effective, because collusion consists of cartel members committing themselves to produce a certain agreement by signing binding contracts, there are no stable cartels. However with implicit collusion firms do not dispose of any commitment power, but when $\delta > \delta_n$ (see Proposition 3) a cartel of certain size is stable. It is precisely the success of the cartels what reduces the incentive to participate in them in when collusion is explicit or binding.
In the previous Section, we checked that cartels were only active if the discount factor was high enough. Therefore, prices were increasing in the discount factor. In the present Section, the size of the cartel is determined endogenously. Then, price may decrease with the discount factor because it reduces the size of stable cartels. The failure of small cartels to be sustainable when $\delta$ is low, induces firms to create larger cartels. This result is recollected in the following corollary:

**Corollary 2** When the size of the cartel is endogenously determined, if $\delta \in [\delta_n, 1)$ price decreases with the discount factor.

The reason is basically that as the cutoff of the discount factor is decreasing with $k$, when $\delta \geq \delta_n$, the larger the discount factor, the lower the size of the stable cartel. Thus as $\delta$ increases, smaller cartels driving lower prices are active in the market. However, when $\delta$ is very low ($\delta < \delta_n$), as long as no agreement is possible, the price corresponds with the standard competitive Nash equilibrium price.

## 4 Conclusions

Implicit or tacit collusion refers to a situation where firms are able to coordinate through some mutual understanding and without the aid of direct communication. Then, the main aim of the paper has been to analyze a model of partial implicit collusion. With this approach, it was showed that the larger the cartel, the easier is to sustain the cartel. However, this approach usually leaves out firms’ “participation constraints”: it cannot explain why many real world cartels do not comprise all firms in the industry. In practice, there are many cases where collusive agreements do not involve all firms in the industry. To that extent, a participation game has been set to analyze the interaction between the incentive and the participation constraints. The main conclusion is that firms may have incentives to participate in the cartel, because otherwise collusion collapses. This means that, although firms might want to take a free ride on the cartel’s effort to restrict output, the gains from free-riding at the participation stage are compensated by the fear of a collapse in collusion.

Finally, it has been proved that when collusion is binding, the incentives to free ride the cartel play a central role, therefore only very small cartels can be enforced. Hence,
with a simple comparison, it has been shown that implicit collusion can enforce larger cartels than binding collusion, becoming therefore perhaps of greater concern for antitrust authorities, especially in those countries, namely Holland, Denmark, Switzerland, etc., who moving to adapt its domestic competition policy to the European regime banned binding collusion.

In spite of this work, a major lacuna exists in both the understanding of when firms tacitly collude and what are its distinguishing features. Very often tacit collusion is not subject to antitrust penalties. Carlton, D and Perloff (1994, p.207) said, “The current laws have been successful in eliminating overt collusion”. However, we can add: still significant theoretical and econometric advancements are required for uncovering implicit collusion.
Appendix

5.1 Optimal punishment.

Trigger strategies have been used in the first three sections of the model. However, we check here if it is also true when cooperation is sustained by an optimal punishment. Cooperation is sustained now with strategies where cheating firms are punished with the fastest and most severe possible punishment. Abreu (1986) outlines a symmetric, two-phase output path that sustains collusive outcomes for an oligopoly of quantity setting firms. The output path considered by Abreu has a “stick and carrot” pattern. The path begins with a period of low per-firm output for cartel members \(q^k_m\). The strategy calls for all cartel members to continue to produce \(q^k_m\), unless an episode of defection occurs. If some firm cheats on the agreement, all cartel firms expand output for one period \(q^p_m\) (stick stage) and return to the most collusive sustainable output in the following periods, provided that every player of the cartel went along with the first phase of the strategies (carrot stage). As far as fringe firms are concerned, as the future play of the other firms is independent of how they play today, they optimally maximize per period profits. The “stick and carrot” strategies for a partial cartel can be formulated in the following way, where \(q_{t,i}\) denotes the strategy played by firm \(i\) in period \(t\):

\[
\begin{align*}
\text{Firm } i, \ i = 1, \ldots, k \text{ plays:} & \\
(\alpha) & \begin{cases}
q_{1,i} = q^k_m \\
q_{t,i} = q^k_m \text{ if } q_{t-1,j} = q^k_m \text{ for } j = 1, \ldots, k \ \forall t = 2, 3, \ldots, \\
q_{t,i} = q^k_m \text{ if } q_{t-1,j} = q^p_m \text{ for } j = 1, \ldots, k \ \forall t = 2, 3, \ldots, \\
q_{t,i} = q^p_m \text{ otherwise.}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{Firm } i, \ i = k + 1, \ldots, n \text{ plays:} & 
\end{align*}
\]

\footnote{The latter were firstly set in a seminal paper from Abreu (1986). These strategies became popular in the literature given their optimality and their renegotiation-proofness quality.}
\[
\begin{aligned}
q_{t,i} &= q_{m}^{k} & \text{if } q_{t-1,j} = q_{m}^{k}, \text{ for } j = 1, ..., k \quad \forall t = 2, 3, ... \\
q_{t,i} &= q_{m}^{n} & \text{if } q_{t-1,j} = q_{m}^{n}, \text{ for } j = 1, ..., k \quad \forall t = 2, 3, ... \\
q_{t,i} &= q_{nm}^{p} & \text{otherwise.}
\end{aligned}
\]

Following Abreu (1986), the strategies described above are considered optimal, that is, they sustain the highest range of collusive outcomes among all possible punishment phases, if continuation profits after unilateral deviation in any period, equal the Minimax value of firms. In the present model it is equal to 0, as firms can always decide not to be active and get 0 profits. Therefore, \((\alpha, \beta)\) are optimal when this condition holds:

\[
\pi_{s}^{k}(q_{m}^{p}) + \frac{\delta}{1 - \delta} \pi_{m}^{k} = 0
\]  

(7)

\(\pi_{m}^{k}\) has been defined in section 2 like profits obtained by cartel firms when cartel and fringe firms produce (1) and (2) respectively, and \(\pi_{s}^{k}(q_{m}^{p})\) are cartel firms profits if firms are in a punishment phase (stick stage):

\[
\pi_{s}^{k}(q_{m}^{p}) = (1 - k q_{m}^{p} - (n - k) q_{m}^{n}) q_{m}^{p} - \frac{(q_{m}^{p})^{2}}{2}
\]

We need the conditions for the strategies \((\alpha, \beta)\) to conform a S.P.N.E. On the one hand, we need that if firms are in a collusive phase (carrot stage), profits that a firm would obtain if deviates from collusion should be no bigger (given that the rest of the firms adhere to the strategies described) than the profits obtained colluding. That is given by the next condition:

\[
\pi_{d}^{k} - \pi_{m}^{k} \leq \delta(\pi_{m}^{k} - \pi_{s}^{k}(q_{m}^{p})) \text{ no deviation in the carrot stage}
\]  

(8)

\(\pi_{d}^{k}\) has already been defined in section 2 like the profits that cartel firm obtains when unilaterally deviates from the collusive agreement.

On the other hand, if we are in a punishment phase (stick stage), we need that firms obtain higher profits in the punishment phase than deviating from it. Therefore, firms do not unilaterally deviate in the stick stage if the following condition holds:

\[
\pi_{d}^{s}(q_{m}^{p}) - \pi_{m}^{s}(q_{m}^{p}) \leq \delta(\pi_{m}^{k} - \pi_{s}^{k}(q_{m}^{p})) \text{ no deviation in the stick stage}
\]  

(9)
where the profits that a cartel firm obtains by unilaterally deviating in the stick stage are defined like:

\[ \pi_d^s(q_m^p) = \max_{q_i} (1 - (k - 1)q_m^p - q_i - (n - k)q_m^p)q_i - \frac{(q_i)^2}{2}. \]

We need to check how to obtain \( q_m^p \), and \( q_m^p \), such that if the discount factor is high enough, collusion will be sustained with the strategies \((\alpha, \beta)\), conforming at the same time, an optimal punishment.

Regarding \( q_m^p \), it must be such that (7) holds. From (9) and (7) we obtain that no deviation in the stick stage is only possible if \( \pi_d^s(q_m^p) \leq 0 \), since otherwise a firm can deviate in the first period and keep doing so every time the punishment is reimposed. Hence, the total output produced by \((k - 1)\) firms must be large enough that \( P((k - 1)q_m^p) \leq 0 \). We have:

\[ P((k - 1)q_m^p) \leq 0 \iff q_m^p \geq x, \tag{10} \]

which sets a lower bound on the quantity produced in the stick stage. This also implies that \( q_m^p = 0 \). To obtain the lower bound of the discount factor such that (9) and (7) hold, that we denote like \( \delta_a \), we compute (7) for the lowest value of \( q_m^p \) that satisfies (10):

\[ \pi_m^s(x) + \frac{\delta_a}{1 - \delta} \pi_m^k = 0 \]

it leads us to:

\[ \delta_a = \frac{\pi_m^s(x)}{\pi_m^s(x) - \pi_m^k} \tag{11} \]

When \( \delta > \delta_a \), you need a harsher punishment such that (7) is satisfied.

As far as deviation in the carrot stage is concerned, from (8) and (7), we obtain that firms do not deviate if:

\[ \frac{1}{1 - \delta} \pi_m^k \geq \pi_d^k \tag{12} \]

This gives us the lower bound of the discount factor such that (8) and (7) are satisfied:

\[ \delta \geq \frac{\pi_d^k - \pi_m^k}{\pi_d^k} = \delta_b \]

\[ \text{11 The price } P(q) \text{ is interpreted as price net of marginal cost at zero.} \]
Finally, (7), (8) and (9) are satisfied if the following condition on the discount factor holds:

\[ \delta \geq \max\{\delta_a, \delta_b\} = \delta_k \]  

(13)

On the one hand, \( \delta_a \) is decreasing in \( k \). When the number of firms in the cartel increases, it is possible to dissuade unilateral deviations without the need of expanding so much total output. Then profits in the stick stage \( (\pi^s_m(x)) \) are increasing in \( k \) what given (11) implies the result.

On the other hand, \( \delta_b \) increases in \( k \), because \( \delta_b = 1 - \frac{\pi^k_m}{\pi^k_d} \) and deviation profits increase in \( k \) more than cartel profits. This makes \( \delta_b \) increase in \( k \).

Analyzing the behavior of \( \delta_k \) in (13), we obtain:

**Proposition 4**: The cutoff discount factor that sustain the strategies \( (\alpha, \beta) \) as a S.P.N.E. and define an optimal punishment, is strictly decreasing in the size of the cartel \( (k) \), if \( k \leq \min\{n, f(n)\} \).

\[
\text{where } f(n) = \frac{13+3n+\sqrt{(9n^2+138n+249)}}{10} \text{ and strictly increasing otherwise.}
\]

If \( k \) is small compared to \( n \), the decreasing effect over \( \delta_a \) dominates the increasing effect over \( \delta_b \) (see fig. 1 for a graphic representation). The result is no so tight as in Proposition 1, because the cutoff is always decreasing only if \( n \leq 8 \) \( (n < f(n)) \). However, the fact that is decreasing for low enough values of \( k \) (observe that \( f(n) > \frac{n^2}{2} \)) will allow us to obtain similar results as far as the participation game is concerned.

We proceed to solve the participation game, as we did in section 3. Firms show their willingness to participate in a collusive agreement in a stage previous to play the “stick and carrot” strategies. The payoffs are given by (3) and (4) where now \( \delta_k \) is the one in (13). For the result, we need to define \( \overline{\delta} = \min_k \delta_k \)

**Proposition 5**: If \( \delta < \overline{\delta} \) or \( \delta \geq \delta_2 \), no cartel is active in equilibrium. Otherwise, a cartel of \( k \) firms is active in equilibrium if \( \delta_{k-1} > \delta \geq \delta_k \).

This result is analogous to the result in Proposition 3. For \( \delta_{k-1} > \delta \geq \delta_k \) only the smallest cartel among those which can be sustained is stable in the participation game. That is because in the smallest sustainable cartel, if firms do not remain in the cartel,
it means that collusion fully collapses and they would obtain the Nash-Cournot profits which is worse for them if the cartel enforced has a size greater than two. If \( \delta < \overline{\delta} \) collusion is not sustainable and if \( \delta \geq \delta_2 \) firms are better off with the Nash-Cournot profits than with the cartel of size two.

Given that for \( \delta \in [\delta, \delta_2) \) cartels are active, similar results to the ones in Corollaries 1 and 2 can be derived from Proposition (7).

**Corollary 3**: If \( \delta \in [\delta, \delta_2) \) the size of active cartels is bigger with implicit collusion than with binding collusion.

**Corollary 4**: When the size of the cartel is endogenously determined, if \( \delta \in [\delta, \delta_2) \) price decreases with the discount factor.

We have exactly the same result we obtained for the case of the “trigger strategies” when \( \delta \) belongs to the interval \([\delta, \delta_2)\).

Again, it is the success of the cartels what reduces the incentive to participate in them with binding collusion. Meanwhile, although firms do not dispose of any commitment power in implicit collusion, it is the threat to the collapse of collusion what provokes the existence of stable cartels.

Corollary 4 says that, if \( \delta \in [\delta, \delta_2) \), the larger the discount factor, the lower the size of the cartel that is stable. Thus, as \( \delta \) increases, smaller cartels associated to lower prices are enforced.

5.2 Proofs

**Proof of Proposition 1**: We have \( \delta(k) = \frac{\pi^d - \pi^k}{\pi^d - \pi^r} \). If we calculate \( \frac{\partial \delta(k)}{\partial k} \), we have that it is the following expressions in our model:

\[
-24 \frac{28n-60k-84nk+48k^2+24nk^2-12nk^3+60nk^2-39n^2k+30n^2-12k^3+2n^4+13n^3-3n^2k^3+3n^3k^2-6n^3k+8}{(9k^3-18nk^2-45k^2+5n^2k+2nk-16k+28+28n+7n^2)^2}
\]

It is tedious but straightforward to show that, as long as \( k \leq n \), we obtain that the derivative is negative. ■

**Proof of Proposition 2**: The conditions for stability are the following:

Internal stability:

\[
2k+1 \geq \frac{3k^2}{(nk-k^2+3k+2+n)^2}
\]

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External stability:
\[(k + 1) \frac{\frac{2k+3}{2k+2}}{(nk-k^2+3k+2+n)^2} \geq 2 \frac{2k+3}{[n(k+1)-(k+1)^2+3k+5+n]^2}\]

We can show that the expression of internal stability is decreasing in \(k\). Therefore showing that the condition does not hold at \(k = 3\) also proves that coalitions of \(k \geq 3\) are not stable. When \(k = n = 2\), cooperation is sustainable. For \(k = 2\), we can see in the internal stability that if \(n \geq 3\) there are incentives to leave the cartel.

Proof of Proposition 3: If \(\delta < \delta_n\), no collusive agreement is sustainable, therefore looking at (3) and (4) we can see that firms profits are the Nash equilibrium profits and no cartel can be active in equilibrium.

When \(\delta \in [\delta_k, \delta_{k-1}]\) only a cartel of size \(k\), is sustainable. Using Proposition 2 we can see that no cartel is stable in the explicit game. However, looking again at (3) and (4), we can see that \(\Pi_m^k \geq \Pi_{nm}^{k-1}\) holds with \(k \geq 3\). This is true as the profits of a cartel of size equal or bigger than 3 are larger than Cournot equilibrium profits. At the same time, \(\Pi_{m}^{k+1} \leq \Pi_{nm}^k\) also holds. This is true for every \(k\) and thus also for every \(\delta \in [\delta_k, \delta_{k-1}]\). Therefore, for every discount factor, only the smallest cartel among those which are sustainable is stable and will be active in equilibrium.

Proof of Corollary 2: This is straightforward to show, only seeing that the price of the market is decreasing with \(k\). Therefore, as the configuration enforced in the market involves smaller cartels, prices decrease.

Proof of Proposition 4: We obtain the cutoff \(\delta\) for both stages of the punishment phase, where the envelope of both will be the significative cutoff that sustain the strategies. It is easy to show that \(\delta_a = 3 \frac{(-nk+k^2-3k-2-n)^2}{16+36k^3+3k^2+12n+36nk+3n^2k^2-6nk^2+12nk^2+6n^2k+3n^2+3k^2-10k^2}\) and \(\delta_b = \frac{k^2-2k+1}{(k+2)^2}\) are respectively strictly decreasing and strictly increasing with \(k\). Therefore the minimum value of the decreasing \(\delta\) will be at \(k = n\). So we just have to calculate up to which value the decreasing part is above the increasing part. Thus the envelope from above of both cutoffs is decreasing with \(k\). If we construct the function \(\delta_a - \delta_b\). We have that this is 0 whenever \(k = \frac{13+3n+\sqrt{(9n^2+138n+249)}}{10}\). We can see that for smaller \(k\), \(\delta_a > \delta_b\) therefore the envelope is decreasing.

Proof of Proposition 5: If we see \(\delta_a\) and \(\delta_b\) we can check that for every \((k, n)\) whenever \(\delta_b = \delta_a = \overline{\delta}\), this \(\in (0, 1)\). That is, the envelope max(\(\delta_b, \delta_a\)) is never increasing with \(k\)
for all range of $k$. We know from Proposition 2 that no cartel is stable in the binding collusion model. Therefore if we look at which are the stable cartels in the implicit collusion model we see that if $\delta < \delta$, no cartel is either stable with implicit collusion because $\delta = \min(\max(\delta_b, \delta_a))$ and represents the minimum discount factor from which a cartel of any size can be sustainable. When $\delta > \delta_2$ all sizes of cartels are sustainable but as no cartel is stable in the participation game, if we apply to the argument of Proposition 3 this fails because Nash-Cournot profits are larger than cartel of size 2 profits. Therefore no cartel configuration is stable. Whenever $\delta \in (\delta, \delta_2)$ if we check the stability of those which are sustainable we can apply exactly the same argument of proposition 3, and the Corollary 1, and the smallest sustainable cartel is stable. ■
References


