Endogenous Timing in General Rent-Seeking and Conflict Models

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Abstract

This paper examines simultaneous versus sequential choice of effort in a two-player contest with a general contest success function. The timing of moves, determined in a preplay stage prior to the contest-subgame, as well as the value of the prize is allowed to be endogenous. Contrary to endogenous timing models with an exogenously fixed prize the present paper finds the following: (1) Players may decide to choose their effort simultaneously in the subgame perfect equilibrium (SPE) of the extended game. (2) The SPE does not need to be unique, in particular, there is no unique SPE with sequential moves if the direct costs of effort zero. (3) Finally, symmetry among players does not rule out incentives for precommitment to effort locally away from the Cournot-Nash level.

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1 Introduction

By providing a framework for analyzing contests with endogenous timing, an endogenously determined prize and a general contest success function (CSF) the present paper strives to merge two strands of literature. The first group of papers focusses on the distinction between Cournot-Nash equilibria (NE) and Stackelberg equilibria in contest models with an exogenously fixed prize. The second group of papers is broadly concerned with the impact of an endogenously determined prize on the NE of a contest.

Strategic behavior in a two-player contest over a prize of fixed and common value was first explored by Dixit (1987), who uses a logit as well as a probit form of the CSF.[1] He finds that in a symmetric two-player contest there is no local incentive to precommit effort away from the Cournot-Nash level. Moreover, he demonstrates that if two unevenly matched players compete in a sequential manner, it is the favorite (underdog) who has an incentive to overcommit (undercommit) effort compared to the NE.[2] Two decisive factors are responsible for this finding. First, the underdog (favorite) regards efforts as strategic substitutes (complements), i.e., the underdog's best response function is downward sloping in the NE of the game while that of the favorite is upward sloping.[3] Second, efforts exhibit negative externalities, i.e., the payoff of each player is a monotonically decreasing function of the competitor's effort.[4] An important implication of this finding is that sequential play may increase or decrease social costs (compared to the NE) contingent on the leader's win probability in the NE.

In seminal contributions Baik and Shogren (1992) and Leininger (1993) independently extend the Dixit-framework by introducing a preplay stage in which the two players determine the order of their moves prior to the actual choice of effort. They

[1]The logit form of the CSF expresses the probability of winning as a function of the relative effort of players (see Loury (1979) and Tullock (1980)). The probit form CSF is used when players experience some noise components regarding their effective effort (see Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983)).

[2]Dixit (1987) defined the favorite (underdog) as the one player whose probability of winning is greater (smaller) than one-half at the NE. This definition is largely shared by subsequent authors, as for example Nti (1999), Yildirim (2005), or Morgan and Várdy (2007). Following these authors we adopt Dixit's definitions.

[3]For the case of an oligopoly, the issue of strategic complementarity and substitutability has first been examined by Bulow et al. (1985).

[4]The issue of positive vs. negative externalities is imminently important for the analysis of the leader's behavior in a Stackelberg game. See, for instance, Amir (1995) and Eaton (2004).

show that in the unique SPE of the extended game the favorite (underdog) will never (always) move first. Hence, players' voluntary choice of timing leads unambiguously to a sequential move game which contradicts the rational explanation of a contest as a simultaneous move game as originated by Tullock (1980). Moreover, because of the particular order of moves, the unique SPE Pareto-dominates any other sequence of moves.

A limitation of the previous analysis is the fact that it does not address the consequences of an endogenous prize in a contest, a fact which has attracted increasing attention over the last two decades. Basically, there are two ways of endogenizing the value of a prize in a contest. Either (1) the prize itself is a control variable of the players or (2) the players' effort affects the value of the prize.[5]

An example for the first approach is Konrad (2002), where subsequently to the realization of a project, an incumbent decides about his investment in a project as well as about his effort in a contest in which he has to defend his project returns against a challenger. Epstein and Nitzan (2004) analyze in a political competition game the endogenous formation of policies prior to a lobbying contest.[6]

As opposed to this, we provide a framework which uses the second approach, i.e., a framework in which the effort exerted by a player affects the distribution as well as the value of the prize. Depending on whether the direct costs of effort are strictly positive or zero, we distinguish between general and partial equilibrium models, or synonymously, between conflict models and rent-seeking models.[7] A Cournot-Nash type example of a conflict model is Hirshleifer (1991a), where, in a state-of-nature, two players are endowed with an inalienable resource which can be used as an input in a valuable prize (production) or for appropriation. Since effective property rights are absent, the contestants face a trade-off between production and appropriation. He finds that in the NE the richer player, defined with respect to the value of the initial resource, loses his advantage over the poorer player due to the

[5]We do not address the issue of artificially created contest, where a contest designer selects the value of the prize awarded to fulfill a specific goal. See for example Moldovanu and Sela (2001), and Che and Gale (2003).

[6]See also Leidy (1994), who argues that a monopolist whose right is contested in a political market will spend lobbying effort and lower his price to defuse reformist opposition, and Hoffmann (2010), who shows in a two-player conflict model that the anticipation of potential appropriation forces players to engage in trade, since this mutually reduces the gains from appropriation.

[7]Excellent surveys are provided by Corchón (2007), Garfinkel and Skaperdas (2007), and Konrad (2009).

fact that each player uses his comparative advantage. In a comparable framework Skaperdas (1992) finds that contingent on the properties of the CSF, cooperation is not incommensurate with the lack of exogenously enforced property rights in a one-shot contest. In a different conflict model Beviá and Corchón (2010) show that cooperation can be achieved by compensating the poorer player in order to avoid open conflict.[8]

An example of a rent-seeking model with an endogenous prize is Baye et al. (2005), who uses an all-pay auction framework in order to compare different litigation systems. Here, different legal systems are based on different fee-shifting rules, which determine the value of the net-prize of the contest winner and loser contingent on their expenditures on legal representation. Another example is Shaffer (2006) who discusses positive and negative externalities of effort on the value of the prize. An example for the latter are territorial disputes, an example for the former are labor tournaments.[9]

The question we pose is whether the findings of Baik and Shogren (1992) and Leininger (1993) are generalizable beyond fixed prizes. Therefore, in order to unite contests with endogenous timing and with an endogenous prize, we provide a framework of a two-player contest under complete information, given a general production technology of the prize, and a general CSF. The extended game consists of a contest subgame and a preplay stage in which players decide whether to exert effort as soon as or as late as possible. Subsequently, players choose effort in the contest subgame according to their previous decision. Thus, the timing game matches the *extended game with observable delay* by Hamilton and Slutsky (1990) frequently used in games of endogenous timing.[10] No matter when exerted, the players' effort influences not only the win probability of both players but also the value of the prize. We will assume throughout the analysis that effort has a negative impact on the value of

[8]See also Anbarci et al. (2002), who compares various bargaining solutions. Here, bargaining takes place in the shadow of conflict, i.e., players have to make irreversible outlays before the bargain procedure. These investments not only alter a player's disagreement payoff but also the output subject to bargain. Dynamic conflict games are provided by Hirshleifer (1995), Grossman and Kim (1995), Hafer (2006), and Gonzales and Neary (2008).

[9]Alexeev and Leitzel (1996) and Chung (1996) are early contributions to this topic. The former presents a rent-seeking model of hostile take-overs of public companies. Here, anti-takeover strategies, such as the poison pill, diminish the target's stock (the prize). Chung (1996) shows that promotional effort increases the market share of a firm as well as the size of the whole market. Thus, effort-spending does have a positive externality on the combatant.

[10]See for example Amir and Grilo (1999), Normann (2002), Amir and Stepanova (2006), and Kempf and Rota-Graziosi (2010a).

the prize and allow the direct costs of effort to be non-negative. Based on these assumptions we are able to provide solutions for rent-seeking and conflict games. We examine how the endogeneity of the prize will influence the players' timing decision. In particular, we provide a taxonomy of endogenous timing based on the properties of the players' best response functions as well as on the characteristics of the prize-production technology. Hence, in a methodological sense, the paper is close to Kempf and Rota-Graziosi (2010b) who develop an endogenous timing game in which two countries provide public goods with spillovers. Here, a taxonomy is proposed depending on the sign of spillovers among countries and the nature of the strategic interaction between various public goods.

It is found, in line with Baik and Shogren (1992) and Leininger (1993), that a unique SPE of the extended game is Pareto-dominated by no other sequential or simultaneous play payoff; and that, if sequential play emerges in equilibrium, the leader commits less effort than in the NE. However, unlike the aforementioned literature, the present paper finds the following. (1) In the SPE of the extended game, players may decide to choose effort simultaneously, which partly reinforces the argument put forth by Tullock (1980) regarding the rationale of a contest as a simultaneous move game. (2) The SPE of the extended game does not need to be unique. In particular, there is no unique SPE with sequential moves if the direct costs of effort are zero. Hence, in a general equilibrium setting it is impossible to replicate the findings of Baik and Shogren (1992) and Leininger (1993). (3) Finally, we prove that in a symmetric game Cournot-Nash and Stackelberg equilibria typically do not coincide, i.e., there are local commitment incentives for the players. Again, this finding is an artefact of the endogenous prize assumption, since, as has been shown by Dixit (1987), local commitment incentives do not exist in a symmetric fixed-prize contest framework.

The underlying reason for the differences in the strategic incentives in our model compared to the fixed-prize-framework is that in the latter costs of effort are exclusively private costs, i.e., apart from the CSF, there is no additional negative externality stemming from the use of effort. Thus, the marginal payoff of a player does not depend on the marginal costs of his competitor. On the contrary, costs of effort in the present model are at least partially common costs, meaning that they have to be borne by both players. These additional negative externalities arise as a result of the endogenous prize assumption and may represent the opportunity costs of effort measured in terms of foregone production possibilities in a conflict framework. Furthermore, in a rent-seeking framework, they may represent the negative responsiveness of the prize at hand to the effort exerted. Accordingly, common costs reshape the strategic incentives in the NE, compared to the private cost scenario.

Before introducing our model, it should certainly be emphasized that we are not the first to undertake the program of generalizing the findings of Baik and Shogren (1992) and Leininger (1993). However, almost all papers make the assumption of an exogenous prize. For example Yildirim (2005) prescinds from the feature that each player can only move once. Endogenous timing in contests with asymmetric information and a lottery CSF is studied by Fu (2006). Konrad and Leininger (2007) study endogenous sequencing in a n-player all-pay contest with complete information. Finally Kolmar (2008) analyzes the emergence of perfectly secure property rights in a stylized two-player conflict model. Although, as in the present paper, the prize is allowed to be endogenous, its value is not contingent on the players' efforts. Moreover, the paper does not address the question of endogenous timing in a conflict framework and does not provide a taxonomy of endogenous leadership for the case of a general CSF and a general production technology.

The paper proceeds as follows. Section 2 presents the basic model and explores the nature of strategic substitutes vs. complements in our setting and its influence on the players' first-mover and second-mover advantages and incentives. Furthermore, it describes the equilibrium concepts used in the paper. Section 3 provides the equilibria in the full game and the taxonomy of endogenous leadership; we conclude in section 4.

2 The model

Consider a situation in which each of two players exerts effort $x_i \in \mathbb{R}^+$ in order to win a prize of common value, with $i = 1, 2$. The prize is allowed to be endogenous, i.e., its value is contingent on the vector $\mathbf{x} = (x_1, x_2)$. The prize-production technology $V(\mathbf{x}) \in \mathbb{R}^{++}$ has the following properties.[11]

Assumption 1 (Prize-production technology)

[11] The subscript i (j) denotes the partial derivative with respect to x_i (x_j).

For $\mathbf{x} \geq \mathbf{0}$ *we assume that*

$$V_i(\mathbf{x}) \equiv \frac{\partial V(\mathbf{x})}{\partial x_i} \leq 0, \tag{1a}$$

$$V_{ii}(\mathbf{x}) \equiv \frac{\partial^2 V(\mathbf{x})}{\partial x_i^2} \leq 0. \tag{1b}$$

Assumptions (1a) states that an increase in effort has a non-increasing effect on the value of the prize, assumption (1b) that if the effect is negative, it does not decrease in x_i. Note that the marginal productivity with respect to x_i might differ for the two players, i.e., $V_1(\mathbf{x}) \gtreqless V_2(\mathbf{x})$. It is also worth mentioning that we allow for $q-$substitutes and $q-$complements, i.e., we do not restrict the sign of the cross partial derivative of the prize-production technology $\left(V_{12}(\mathbf{x}) \equiv \frac{\partial^2 V(\mathbf{x})}{\partial x_1 \partial x_2}\right)$, which carries important information about the complementarity and substitutability between players' effort.[12] However, we assume that whatever the sign of $V_{12}(\mathbf{x})$, it remains constant for all $\mathbf{x} > 0$.

We now introduce some examples.

Example 1 (A conflict framework)

For example in Hirshleifer (1991a) each of two players possesses R_i *units of an inalienable primary resource which can be used to produce one-to-one two kinds of inputs,* x_i *and* y_i*, where the latter will be used in the joint production of a single consumption good representing the prize while the former will be used as an input in the appropriative competition. Suppose that* $R = R_1 = \frac{R_2}{\kappa}$*. Implementing the individual budget-constraint* $(R_i = x_i + y_i)$ *and assuming a CES-type of production function, we get* $V(\mathbf{x}) = \left(\alpha(R - x_1)^\rho + (1-\alpha)(\kappa R - x_2)^\rho\right)^{\frac{1}{\rho}}$*, with* $R, \kappa \in \mathbb{R}^{++}$*,* $0 \neq \rho \leq 1$*, and* $\alpha \in (0,1)$*. For* $\rho = 1$ *this leads to* $V_{12}(\mathbf{x}) = 0$*, while* $\rho \to 0$ *leads to* $V_{12}(\mathbf{x}) > 0$ *for* $x_i < R_i$*.*

Next, we provide an example in a rent-seeking framework.

Example 2 (A rent-seeking framework)

Shaffer (2006) provides a model of a two-player destructive contest, where the effort exerted reduces the value of the prize. If we assume that the value of the basic prize is 1*, the prize-production technology may be given by* $V(\mathbf{x}) = 1 - x_1 x_2$*, with* $V_{12}(\mathbf{x}) < 0$*.*

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[12] The terms $q-$substitutes and $q-$complements have been suggested by Hicks (1956, p. 156). In the contest literature several specifications have been proposed with respect to the prize: Dixit (1987), Nti (1999) and Grossman (2001) consider exogenous prizes ($V(\mathbf{x}) = K$), Shaffer (2006) considers an endogenous prize, with $V_{12}(\mathbf{x}) = 0$, whereas Skaperdas (1992) assumes $q-$complements ($V_{12}(\mathbf{x}) > 0$). Hirshleifer (1991a,b) suggests a CES-production function, and consequently, assumes $V_{12}(\mathbf{x}) \geq 0$. The most general form, i.e., $V_{12}(\mathbf{x}) \gtreqless 0$, can be found in Neary (1997a) and Bös (2004).

Next, we turn to the CSF, $p^i : x_i \times x_j \to [0,1]$, which determines for any given value of the vector $\mathbf{x}$ player i's probability of winning the prize.[13] As a notational simplification we introduce $p(\mathbf{x})$ as the win probability of player 1 and $1 - p(\mathbf{x})$ as player 2's win probability. The function $p(\mathbf{x})$ exhibits the following properties:

Assumption 2 (Contest success function)

For $\mathbf{x} > \mathbf{0}$ we assume that

$$p_1(\mathbf{x}) \equiv \frac{\partial p(\mathbf{x})}{\partial x_1} > 0 \quad \text{and} \quad p_2(\mathbf{x}) \equiv \frac{\partial p(\mathbf{x})}{\partial x_2} < 0, \tag{2a}$$

$$p_{11}(\mathbf{x}) \equiv \frac{\partial^2 p(\mathbf{x})}{\partial x_1^2} < 0 \quad \text{and} \quad p_{22}(\mathbf{x}) \equiv \frac{\partial^2 p(\mathbf{x})}{\partial x_2^2} > 0, \tag{2b}$$

$$p_{12}(\mathbf{x})(1 - p(\mathbf{x}))\, p(\mathbf{x}) - p_2(\mathbf{x})\, p_1(\mathbf{x})(1 - 2p(\mathbf{x})) = 0. \tag{2c}$$

Assumptions (2a) and (2b) show that each player's win probability is an increasing (decreasing) and concave (convex) function of his own (his competitor's) effort. Assumption (2c) is a technical one which, inter alia, allows us to simplify the analysis for the proof of the uniqueness of the NE.[14]

The payoff function of player 1 and 2 are given by

$$\Pi^1(\mathbf{x}) = p(\mathbf{x})\, V(\mathbf{x}) - C^1(x_1), \tag{4.1}$$

$$\Pi^2(\mathbf{x}) = (1 - p(\mathbf{x}))\, V(\mathbf{x}) - C^2(x_2), \tag{4.2}$$

[13] To avoid repetition, we use $i, j = 1, 2$ and $i \neq j$ when it is obvious.

[14] It is similar to assumption (3) in (Skaperdas, 1992, p. 725). Assumption (2c) is fulfilled by any logit form CSF. Assumption (2) as a whole is fulfilled, for example, by the following CSF

$$p(\mathbf{x}) = \frac{f_1(x_1) + \alpha}{f_1(x_1) + 2\,\alpha + f_2(x_2)}, \tag{3}$$

as long as each player's impact function $f : \mathbb{R}^+ \to \mathbb{R}^+$ is a twice differentiable, increasing and concave function and $\alpha > 0$. See Amegashie (2006) and Rai and Sarin (2009) for the CSF in (3) with $f_i(x_i) = x_i$ and Corchón and Dahm (2010) for $f_i(x_i) = q_i\, x_i^r$, with $q_i, r > 0$. This particular form of the CSF avoids an existence problem of the logit form CSF at $\mathbf{x} = \mathbf{0}$ if $f_i(0) = 0$, and which holds in particular for the Tullock CSF (where $f_i(x_i) = x_i^r$). A different approach is to assume that $p(\mathbf{0}) = \frac{1}{2}$ in two-player contests (see Yildirim (2005) and Morgan and Várdy (2007) for the general logit-form, Münster (2006) and Garfinkel and Skaperdas (2007) for $f_i(x_i) = f(x_i)$, and finally Nti (1999) and Beviá and Corchón (2010) for $f_i(x_i) = x_i^r$). This, however, creates a discontinuity at $\mathbf{x} = \mathbf{0}$, which generates technical problems regarding the existence of an equilibrium. Note that the general logit-form CSF can be obtained as the limit of (3) as $\alpha \to 0$ (see Myerson and Wärneryd (2006) for a similar argument regarding the Tullock CSF).

with $C_i^i(x_i) \geq 0$, and $C_{ii}^i(x_i) \geq 0$. Each player maximizes his expected payoff which equals the prize that goes to the sole winner, weighted by the probability that he wins the contest minus the direct effort cost. These effort costs are allowed to be zero.[15] We remark that the players' objective functions have two kinds of properties. First, these functions exhibit *plain substitutes* as defined by Eaton (2004)[16]. Therefore, the cross marginal effect on the payoff function is negative, i.e., we have negative spillovers with respect to the effort invested:

$$\Pi_2^1(\mathbf{x}) \equiv \frac{\partial \Pi^1(\mathbf{x})}{\partial x_2} = p_2(\mathbf{x})\,V(\mathbf{x}) + p(\mathbf{x})\,V_2(\mathbf{x}) < 0, \tag{5.1}$$

$$\Pi_1^2(\mathbf{x}) \equiv \frac{\partial \Pi^2(\mathbf{x})}{\partial x_1} = -p_1(\mathbf{x})V(\mathbf{x}) + (1 - p(\mathbf{x}))V_1(\mathbf{x}) < 0. \tag{5.2}$$

A second property concerns the players' strategic incentives. Following Bulow et al. (1985), we will say that efforts are strategic substitutes (SS) for player i if his marginal payoff decreases in the effort of player j, and they are strategic complements (SC) if player i's marginal payoff increases in player j's effort. Moreover, in the case where player i's marginal payoff is not influenced by player j's strategy choice, we will say that efforts are strategically independent (SI) for player i. Due to the properties of the CSF, a player's marginal payoff depends in a non-monotonic way on the competitor's effort. We will thus define SS, SC and SI in the neighborhood of the NE.

2.1 Efforts in the three basic games

Next, we consider the three basic games; the Cournot-Nash game (Γ^N) and the two Stackelberg games, depending on whether player 1 or player 2 leads (Γ^{S_1} or Γ^{S_2}, respectively). The NE of the contest subgame $\left(\Gamma^N\right)$ is defined by the following system of maximization programs

$$\begin{cases} x_i^N \equiv \underset{x_i}{\operatorname{argmax}} \quad \Pi^i(\mathbf{x}), \quad x_j^N \text{ given}, \\ x_j^N \equiv \underset{x_j}{\operatorname{argmax}} \quad \Pi^j(\mathbf{x}), \quad x_i^N \text{ given}. \end{cases} \tag{6}$$

[15] For $C_i^i(x_i) = 0$ the present model describes a conflict model, i.e., a model in which the direct costs of effort are zero. We remark that $C_i^i(x_i) = 0$ is only applicable if the prize is fully endogenized, i.e., $V_i(\mathbf{x}) \leq V_j(\mathbf{x}) < 0$. Otherwise, at least one player's optimization problem is not well defined.
[16] Bulow et al. (1985) referred to this as *conventional substitutes*.

The FOCs for players 1 and 2 are therefore evaluated at $\mathbf{x}^N$, which denotes the NE values $\left(\mathbf{x}^N \equiv \left(x_i^N, x_j^N\right)\right)$.[17] The FOCs for player 1 and 2 are, therefore,

$$p_1\left(\mathbf{x}^N\right) V\left(\mathbf{x}^N\right) + p\left(\mathbf{x}^N\right) V_1\left(\mathbf{x}^N\right) - C_1^1(x_1^N) = 0, \tag{7.1}$$

$$-p_2\left(\mathbf{x}^N\right) V\left(\mathbf{x}^N\right) + \left(1 - p\left(\mathbf{x}^N\right)\right) V_2\left(\mathbf{x}^N\right) - C_2^2(x_2^N) = 0. \tag{7.2}$$

We make the following assumption.

Assumption 3 (Interior NE)

We assume that $\Pi_i^i(\mathbf{0}) > 0$, *i.e.*

$$\frac{p_1(\mathbf{0})}{p(\mathbf{0})} > -\frac{V_1(\mathbf{0})}{V(\mathbf{0})} + \frac{C_1^1(\mathbf{0})}{p(\mathbf{0})\,V(\mathbf{0})}, \tag{8.1}$$

$$-\frac{p_2(\mathbf{0})}{p(\mathbf{0})} > -\frac{V_2(\mathbf{0})}{V(\mathbf{0})} + \frac{C_2^2(\mathbf{0})}{p(\mathbf{0})\,V(\mathbf{0})}. \tag{8.2}$$

This assumption guarantees that, if a NE exists, it is an interior one.[18] In order to guarantee that the concept of SC, SS or SI is unique for each player we introduce the following assumption.

Assumption 4 (Uniqueness of the NE)

We assume that the one-shot Cournot-Nash equilibrium is unique. In particular, we derive the following sufficient condition for uniqueness if $V_{ii}(\mathbf{x}) < 0$.

1. *If* $C^i(x_i) = 0$ *for* $i = 1, 2$, *we assume that*

$$V_{11}\left(\mathbf{x}^N\right) V_{22}\left(\mathbf{x}^N\right) \geq \left(V_{12}\left(\mathbf{x}^N\right)\right)^2, \tag{9}$$

2. *if* $C^i(x_i) > 0$ *for* $i = 1, 2$, *we assume that*

$$\left(\underline{p}\left(\mathbf{x}^N\right)\right)^2 V_{11}\left(\mathbf{x}^N\right) V_{22}\left(\mathbf{x}^N\right) \geq \left(\bar{p}\left(\mathbf{x}^N\right)\right)^2 \left(V_{12}\left(\mathbf{x}^N\right)\right)^2 \tag{10}$$

with $\bar{p}\left(\mathbf{x}^N\right) = \max\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$ *and* $\underline{p}\left(\mathbf{x}^N\right) = \min\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$.

[17] In a similar way, we will note $\mathbf{x}^{S_i} \equiv \left(x_i^L, x_j^F\left(x_i^L\right)\right)$ the levels of effort at the Stackelberg equilibrium in which player i leads.

[18] Note that, given the CSF in (3), the left hand side (LHS) of (8.1) is equal to $\frac{f_1'(0)}{8\,\alpha}$ >0, which becomes arbitrarily big as $\alpha \to 0$. The denominators of the right-hand side (RHS) of (8.1) are finite, so that the inequality holds as long as $0 \leq C_i^i(0), -V_1(\mathbf{0}) < \infty$. By symmetry the same argument can be applied to (8.2).

Now we can establish the following lemma.

Lemma 1
Under assumptions (1), (2), (3), *and* (4) *a unique and interior one-shot Cournot-Nash equilibrium* $\left(\mathbf{x}^N\right)$ *exists.*

Proof. See Appendix A.1. ■

We now turn to the issue of strategic incentives in the NE of the contest subgame. Applying the envelope theorem to (7), it is easy to show that

$$\frac{dx_j}{dx_i} = -\frac{\Pi^j_{ij}(\mathbf{x})}{\Pi^j_{jj}(\mathbf{x})} \lesseqqgtr 0 \Leftrightarrow \Pi^j_{ij}(\mathbf{x}) \lesseqqgtr 0, \tag{11}$$

for $\mathbf{x} > \mathbf{0}$. Therefore, the sign of the slope of a player's best response function at a point in the strategy space is solely determined by the sign of the cross partial derivative of the same player's payoff function which - as was said earlier - may vary. However, uniqueness of the NE implicates that our definition of strategic interaction (SS, SC or SI) is unique for each player. In particular, $\Pi^i_{ij}(\mathbf{x})$ is given by

$$\Pi^1_{12}(\mathbf{x}) = p_{12}(\mathbf{x})\, V(\mathbf{x}) + p_1(\mathbf{x})\, V_2(\mathbf{x}) + p_2(\mathbf{x})\, V_1(\mathbf{x}) + p(\mathbf{x})\, V_{12} \tag{12.1}$$

$$\Pi^2_{12}(\mathbf{x}) = -\, p_{12}(\mathbf{x})\, V(\mathbf{x}) - p_2(\mathbf{x})\, V_1(\mathbf{x}) - p_1(\mathbf{x})\, V_2(\mathbf{x}) + (1-p)(\mathbf{x})\, V_{12}(\mathbf{x}). \tag{12.2}$$

2.1.1 The case of a not fully endogenous prize

We start by analyzing the rent-seeking framework where the effort of at least one player has no impact on the value of the prize, i.e., $V_i\left(\mathbf{x}\right) \leq V_j\left(\mathbf{x}\right) = 0$. In this case the strategic incentives at the NE are

$$\Pi^1_{12}\left(\mathbf{x}^N\right) = -\Pi^2_{12}\left(\mathbf{x}^N\right) = p_{12}(\mathbf{x}^N) V\left(\mathbf{x}^N\right) + p_j\left(\mathbf{x}^N\right)\, V_i\left(\mathbf{x}^N\right), \tag{13}$$

so that $\Pi^1_{12}(\mathbf{x}^N) + \Pi^2_{12}(\mathbf{x}^N) \equiv V_{12}(\mathbf{x}^N) = 0$. Accordingly, either $p_{12}\left(\mathbf{x}^N\right)\, V\left(\mathbf{x}^N\right) = -p_j\left(\mathbf{x}^N\right)\, V_i\left(\mathbf{x}^N\right)$ and both players regard efforts as SI, or the strategic incentives are directly opposed. Apparently, the Dixit-framework where $V\left(\mathbf{x}\right) = K$ is a limiting case of our model. More precisely, for $V(\mathbf{x}) = K$ equation (13) becomes $\Pi^1_{12}\left(\mathbf{x}^N\right) = -\Pi^2_{12}\left(\mathbf{x}^N\right) = p_{12}(\mathbf{x}^N)\, K$, so that the strategic incentives depend only on the sign of $p_{12}\left(\mathbf{x}^N\right)$, which, interestingly, also determines the favorite and underdog of the game. The following equivalence holds for any logit type CSF:[19]

$$p\left(\mathbf{x}^N\right)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\frac{1}{2} \Leftrightarrow p_{12}\left(\mathbf{x}^N\right)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0. \tag{14}$$

Hence, given the terminology adopted from Bulow et al. (1985), we conclude that for $V\left(\mathbf{x}\right) = K$ the favorite (underdog) regards efforts as SC (SS). Moreover, players regard efforts as SI if and only if $p_{12}\left(\mathbf{x}^N\right) = 0$. We will use this specific correlation between win-probability and strategic incentives ($\omega\sigma$-correlation) as a reference point in our further analysis.

Focusing on the case where $V_i(\mathbf{x}) = 0$ leads us to the question *what determines whether a player is a favorite or an underdog ?*. Two sources of asymmetries between players may exist. First, players may be unequal with respect to their cost function, in particular their marginal costs.[20] Second, players' effort may have different impact on the value of the CSF. As has been shown by Nitzan (1994) the relative cost efficiency of a player may be compensated, undercompensated or overcompensated by the same player's lack in the relative impact of effort on the value of the CSF. By endogenizing the value of the prize we now introduce a third source of asymmetry between players which stems from the negative impact of a player's effort on the value of the prize.

Example 3 (A different rent-seeking framework)

Suppose that the prize-production function is given by $V\left(\mathbf{x}\right) = 1 - \theta\, x_1$, with $\theta \geq 0$. The CSF is represented by a simple logit-type CSF, $p\left(\mathbf{x}\right) = \frac{x_1}{x_1+\mu\, x_2}$, with $\mu \in (0,1)$. Given that $C_1^1(x_1) = \gamma\, C_2^2(x_2) = \gamma$, with $\gamma > 1$, we get $\Pi_{12}^1\left(\mathbf{x}^N\right) = -\Pi_{12}^2\left(\mathbf{x}^N\right) = \frac{(1-\gamma\,\mu)\sqrt{(1+\gamma\,\mu)^2+4\mu\,\theta}}{\mu}$, so that player 1 regards efforts as SC (SS) as long as $\mu\,\gamma < 1$ (> 1). For $\theta = 0$ the $\omega\sigma$-correlation holds, since then $\Pi_{12}^1\left(\mathbf{x}^N\right) = -\Pi_{12}^2\left(\mathbf{x}^N\right) = p_{12}(\mathbf{x}^N)$, so that player 2's relative cost efficiency is overcompensated (undercompensated) by player 1's relative impact of effort on the CSF for $\mu < \frac{1}{\gamma}$ $(\mu > \frac{1}{\gamma})$, so that $p_{12}\left(\mathbf{x}^N\right) > 0$ $(p_{12}\left(\mathbf{x}^N\right) < 0)$. However, for $\theta > 0$ the $\omega\sigma$-correlation only holds as long as player 1's marginal impact on the prize is sufficiently small, namely as long as $\theta < \frac{2(1-\gamma\mu)}{\mu}$. Indeed, an increase in θ raises

[19] See Dixit (1987). For the case of a CSF described by (3), this also holds, since then

$$p_{12}\left(\mathbf{x}^N\right) = \frac{f_1'\left(x_1^N\right) f_2'\left(x_2^N\right)}{\left(f_1\left(x_1^N\right) + 2\,\alpha + f_2\left(x_2^N\right)\right)^3}\left(f_1\left(x_1^N\right) - f_2\left(x_2^N\right)\right).$$

[20] It is a well known fact that this is equivalent to having different valuations for the prize (see Konrad (2009, p. 70)).

player 1's opportunity costs of effort, while it does not affect player 2's. This increases the equilibrium effort of player 2 with respect to player 1's effort. The strategic incentives, however, remain unaltered.

2.1.2 The case of a fully endogenous prize

Next, we turn to the case where $V_i(\mathbf{x}) \leq V_j(\mathbf{x}) < 0$. Then, implementing the FOCs in each player's cross partial derivative of the payoff function and utilizing assumption (2c) yields

$$\Pi_{12}^1\left(\mathbf{x}^N\right) = p\left(\mathbf{x}^N\right) V_{12}\left(\mathbf{x}^N\right) + \Omega\left(\mathbf{x}^N\right), \tag{15.1}$$

$$\Pi_{12}^2\left(\mathbf{x}^N\right) = \left(1 - p\left(\mathbf{x}^N\right)\right) V_{12}\left(\mathbf{x}^N\right) - \Omega\left(\mathbf{x}^N\right), \tag{15.2}$$

with

$$\Omega\left(\mathbf{x}^N\right) = \frac{p_1\left(\mathbf{x}^N\right) C_2^2(x_2^N)}{1 - p\left(\mathbf{x}^N\right)} + \frac{p_2\left(\mathbf{x}^N\right) C_1^1(x_1^N)}{p\left(\mathbf{x}^N\right)}.$$

The particular form of $\Pi_{ij}^i(\mathbf{x}^N)$ stems from the fact that the absolute value of the first three terms on the RHS of (12) are equal to $|\Omega(\mathbf{x}^N)|$ at $\mathbf{x}^N$, which is an artefact of assumption (2c). Eq. (15) state that the sum of the cross partial derivative of each player's payoff function equals the cross partial derivative of the prize-production function, i.e., $\Pi_{12}^1(\mathbf{x}^N) + \Pi_{12}^2(\mathbf{x}^N) \equiv V_{12}(\mathbf{x}^N)$. Since $V_{12}(\mathbf{x}) \gtreqless 0$, we will now distinguish between the following cases: First, there is a group of cases in which the players' strategic incentives are aligned. Here, we find that we either have a game of SC $\left(\Pi_{12}^i\left(\mathbf{x}^N\right) \geq \Pi_{12}^j\left(\mathbf{x}^N\right) > 0\right)$, which is only consistent with q-complements $(V_{12}(\mathbf{x}) > 0)$, or a game of SS $\left(\Pi_{12}^i\left(\mathbf{x}^N\right) \leq \Pi_{12}^j\left(\mathbf{x}^N\right) < 0\right)$, which is only consistent with q-substitutes $(V_{12}(\mathbf{x}) < 0)$, or efforts are SI for both players $\left(\Pi_{12}^i\left(\mathbf{x}^N\right) = \Pi_{12}^j\left(\mathbf{x}^N\right) = 0\right)$, which is only consistent with $V_{12}(\mathbf{x}) = 0$.[21] Note that $\Omega\left(\mathbf{x}^N\right) = 0$ if $C_i^i(x_i) = 0$, i.e., in a conflict model the strategic incentives of both players are always aligned and depend solely on $V_{12}(\mathbf{x})$. Hence, given a symmetric game and $V_{12}(\mathbf{x}) \neq 0$, there are local commitment incentives, which, in a fixed-prize framework, cannot emerge, as has been shown by (Dixit, 1987, p. 893).

Example 1 (A conflict framework - continued)

[21] Games of SC can be found, for example, in Hirshleifer (1991a,b) for $s > 1$, and in Skaperdas (1992), Neary (1997b), and Skaperdas and Syropoulos (1997). Players regard efforts as SI in Hirshleifer (1991a,b) for $s = 1$, in Fabella (1996), Garfinkel and Skaperdas (2000), Shaffer (2006), and Beviá and Corchón (2010). Games of SS can be found in Neary (1997a) and Bös (2004).

Let us assume that both players have identical initial endowments ($\kappa = 1$)*,* $\alpha = \frac{1}{2}$*, and that we have a Tullock CSF, with* $p(\mathbf{x}) = \dfrac{q\, x_1^r}{q\, x_1^r + x_2^r}$*,* $q > 0$ *and* $r \in (0, 1]$*. In the symmetric NE* ($q = 1$)*, we have the following: In both cases* ($\rho \to 0$ *and* $\rho = 1$) *we get* $x_1^N = x_2^N = \frac{r\,R}{1+r}$*. This leads to* $\Pi_{12}^1\left(\mathbf{x}^N\right) = \Pi_{12}^2\left(\mathbf{x}^N\right) = 0$ *if* $\rho = 1$*, so that both players regard efforts as SI, i.e., there are no local commitment incentives for both players. The reason for this is that in a conflict game the net effect of an increase of player* j*'s effort on player* i*'s marginal payoff is contingent only on the value of* $V_{12}(\mathbf{x})$*, since* $\Omega\left(\mathbf{x}^N\right) = 0$*. Hence, if an increase of* x_j *leaves the (negative) marginal impact of* x_i *on the prize* ($V_i(\mathbf{x})$) *unaffected, no player has an incentive to commit to a higher of lower level of effort. Note that the* $\omega\,\sigma$*-correlation holds for* $q = 1$*, since* $p(\mathbf{x}^N) = \frac{1}{2}$*. Because* $\Omega\left(\mathbf{x}^N\right) = 0$ *players still regard efforts as SI if* $q \neq 1$*. However, the win probability reacts sensitively to changes of* q *away from unity. Hence,* $p_{12}(\mathbf{x}^N) \neq 0$ *if* $q \neq 1$*, and the* $\omega\sigma$*-correlation does not hold.*

If $\rho \to 0$*, then* $\Pi_{12}^1\left(\mathbf{x}^N\right) = \Pi_{12}^2\left(\mathbf{x}^N\right) = \frac{1+r}{8\,R} > 0$*, so that both players regard efforts as SC. By increasing his effort, player* j *decreases the (negative) marginal impact of* x_i *on the prize due to* $V_{12}(\mathbf{x}) > 0$*. Or, to put it differently, player* j *decreases the opportunity costs of effort for player* i*. Again,* $p(\mathbf{x}^N) = \frac{1}{2}$ *for* $q = 1$*, and therefore the* $\omega\sigma$*-correlation never holds if* $\rho \to 0$*.*

In the second group of cases the strategic incentives are not aligned. More precisely, we may find that $\Pi_{ij}^i\left(\mathbf{x}^N\right) = 0 < \Pi_{ij}^j\left(\mathbf{x}^N\right)$, which is only consistent with $V_{12}(\mathbf{x}) > 0$, or that $\Pi_{ij}^i\left(\mathbf{x}^N\right) = 0 > \Pi_{ij}^j\left(\mathbf{x}^N\right)$, which is only consistent with $V_{12}(\mathbf{x}) < 0$. Finally, it may be that $\left(\Pi_{ij}^i\left(\mathbf{x}^N\right) < 0 < \Pi_{ij}^j\left(\mathbf{x}^N\right)\right)$, which is consistent with $V_{12}(\mathbf{x}) \gtreqless 0$.[22] As already noted, in conflict frameworks strategic incentives are always aligned. In a rent-seeking framework, however, the strategic incentives depend on the value of $V_{12}\left(\mathbf{x}^N\right)$ as well as on the value of $\Omega\left(\mathbf{x}^N\right)$. It is thus possible, that strategic incentives alter within a framework if one changes, for example, the impact of a player's effort on the value of the CSF.

Example 2 (A rent-seeking framework - continued)

Let us assume that the CSF is of the logit type, with $f_1(x_1) = \lambda\, x_1$ *and* $f_2(x_2) = x_2$*. In the symmetric case* ($\lambda = 1$) *we have a game of SS, since* $\Pi_{12}^1\left(\mathbf{x}^N\right) = \Pi_{12}^2\left(\mathbf{x}^N\right) = -\frac{1}{2}$*. Thus, if* $\lambda = 1$*, an increase in player* i*'s effort increases the (negative) marginal impact of player* j*'s effort on* $V(\mathbf{x})$*, or, synonymously, increases the opportunity costs of effort for player* j*. Due to the continuity of the functions involved the direction of strategic incentives*

[22] This case must emerge if players are asymmetric rent-seeking games where $V_{12}(\mathbf{x}) = 0$. If, for example, one adds an asymmetry, either regarding the impact function, or the cost function in Fabella (1996) or Shaffer (2006), then the strategic incentives are directly opposed.

remain unaltered for sufficiently small changes in λ. The sign of $p_{12}\left(\mathbf{x}^N\right)$, however, reacts sensitively to small changes of λ away from unity. Accordingly, for $\lambda \neq 1$, $p_{12}\left(\mathbf{x}^N\right) \neq 0$ and favorite and underdog regard efforts as SS, so that the $\omega\sigma$-correlation does not hold. If player 2 is sufficiently relatively effective (for example if $\lambda = \frac{3}{4}$), then $\Pi^2_{12}\left(\mathbf{x}^N\right) \approx 1.04686 > 0 > \Pi^1_{12}\left(\mathbf{x}^N\right) \approx -2.04686$. Consequently, player 1 (2) regards efforts as SS (SC). Since now $f_1(\cdot) < f_2(\cdot)$ player 2 needs less effort compared to the symmetric case in order to increase his own win probability by the same amount. The marginal impact of x_2 on the prize, however, remains unaltered. Thus, now a marginal increase of player 1's effort leads to an increase of player 2's effort due to the lower opportunity costs, compared with the symmetric case. Hence, for $\lambda = \frac{3}{4}$, $p_{12}\left(\mathbf{x}^N\right) < 0$, and the $\omega\sigma$-correlation holds. It is worth noting that due to continuity a level of relative efficiency exists so that player 2 regards efforts as SI ($\Pi^2_{12}\left(\mathbf{x}^N\right) = 0$). This is the case for $\lambda \approx 0.78922$.

Next, we turn to the sequential move games.[23] The subgame perfect equilibrium of the contest subgame (the Stackelberg equilibrium) is determined by applying backward induction. Thus, in the game where player i leads (Γ^{S_i}), we first focus on the follower's (F) maximization program which is $x_j^F(x_i) \equiv \underset{x_j}{\operatorname{argmax}}\, \Pi^j(\mathbf{x})$. This yields

$$\Pi^j_j\left(x_j^F(x_i), x_i\right) = 0. \tag{16}$$

We assume that the second order condition of the leader's maximization program holds. In particular, we assume that

Assumption 5

$$\frac{d^2\Pi^i\left(x_i, x_j^F(x_i)\right)}{dx_i^2} < 0.$$

This assumption is crucial since it assures the existence and uniqueness of the Stackelberg equilibrium where the latter property guarantees that the sign of the slope of a player's best response function at the NE is equal to the sign of the slope of the same player's best response function once he becomes a Stackelberg follower in a sequential move game.

[23]We are aware of the fact that given assumptions (1) and (2) we cannot rule out corner solutions for the sequential move games. This topic has been analyzed, for example, by Grossman and Kim (1995), Kolmar (2008), and Hoffmann (2010). However, we will assume only interior solutions for the sequential move games.

2.2 Effort ranking

Given the optimizing behavior in the basic games, we are now in the position to establish the rankings of the levels of effort in the different equilibria.

Lemma 2
Under assumptions (1), (2), (4), *and* (5) *the level of effort for the Nash and Stackelberg games are such that if*

1. $\Pi^j_{ij}\left(\mathbf{x}^N\right) > 0 \;\Rightarrow\; x_i^N > x_i^L \wedge x_j^N > x_j^F,$

2. $\Pi^j_{ij}\left(\mathbf{x}^N\right) < 0 \;\Rightarrow\; x_i^N < x_i^L \wedge x_j^N > x_j^F,$

3. $\Pi^j_{ij}\left(\mathbf{x}^N\right) = 0 \;\Rightarrow\; x_i^N = x_i^L \wedge x_j^N = x_j^F.$

Proof. See Appendix (A.2). ■

Our second lemma compares the effort exerted by the Stackelberg leader and follower with the one exerted in the NE of the contest subgame. If efforts are SC (SS) for player j, the Stackelberg-leader i reduces (increases) his effort compared to the NE-level in order to decrease the follower's effort. Because of that we always find that $x_j^F < x_j^N$ in these cases. Finally, if efforts are SI for player j, the leader (player i) has no local commitment incentives and provides the same level of effort as in the NE. Consequently, the follower's effort also equals his NE-level ($x_j^F = x_j^N$).

2.3 First-mover/Second-mover advantage and incentive

Given these rankings, we can now compare the payoffs in the three basic games (Γ^N, Γ^{S_1} and Γ^{S_2}). This will give us the opportunity of detecting potential first-mover (second-mover) advantages or incentives, which will be defined in accordance with Gal-Or (1985) and van Damme and Hurkens (1996), respectively. First, we compare the payoffs of player i in the SPE of the two different sequential move subgames.

Definition 1 (First-mover (second-mover) advantage)
Player i has a

$$\left\{\begin{array}{l}\textit{first-mover advantage}\\ \textit{second-mover advantage}\end{array}\right\} \Leftrightarrow \Pi^i\left(\mathbf{x}^{S_i}\right)\left\{\begin{array}{c}>\\<\end{array}\right\}\Pi^i\left(\mathbf{x}^{S_j}\right).$$

Next, we compare the payoffs in the NE to the one obtained in the Stackelberg equilibrium.

Definition 2 (First-mover (second-mover) incentive)
Player i has a

$$\left\{ \begin{array}{l} \textit{first-mover incentive} \\ \textit{second-mover incentive} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \Pi^i\left(\mathbf{x}^{S_i}\right) \\ \Pi^i\left(\mathbf{x}^{S_j}\right) \end{array} \right\} \geq \Pi^i\left(\mathbf{x}^N\right).$$

It is worth noting that whatever the nature of strategic interactions (SC, SS or SI) might be, players always have a first-mover incentive, that is, they weakly prefer their leader payoff over their payoff in the NE $\left(\Pi^i\left(\mathbf{x}^{S_i}\right) \geq \Pi^i\left(\mathbf{x}^N\right)\right)$. This result holds for a continuous strategy spaces and follows from the definition of the leader's maximization program. From lemma (2) follows the last lemma.

Lemma 3
Under assumptions (1), (2), (4), *and* (5) *we have:*

1. *If efforts are strategic complements for player i $\left(\Pi^i_{ij}(\mathbf{x}^N) > 0\right)$, then player i has a strong form of a second-mover incentive, i.e.,* $\Pi^i\left(\mathbf{x}^{S_j}\right) > \Pi^i\left(\mathbf{x}^N\right)$.
2. *If efforts are strategic substitutes for player i $\left(\Pi^i_{ij}(\mathbf{x}^N) < 0\right)$, then player i has a first-mover advantage and no second-mover incentive, i.e.,* $\Pi^i\left(\mathbf{x}^{S_i}\right) \geq \Pi^i\left(\mathbf{x}^N\right) > \Pi^i\left(\mathbf{x}^{S_j}\right)$.
3. *If efforts are strategically independent for player i $\left(\Pi^i_{ij}(\mathbf{x}^N) = 0\right)$, then player i has a weak form of a second-mover incentive, i.e.,* $\Pi^i\left(\mathbf{x}^{S_j}\right) = \Pi^i\left(\mathbf{x}^N\right)$.
4. *If efforts are strategically independent for player j $\left(\Pi^j_{ij}(\mathbf{x}^N) = 0\right)$, then player i has a weak form of a first-mover incentive* $(\Pi^i\left(\mathbf{x}^{S_i}\right) = \Pi^i\left(\mathbf{x}^N\right))$. *If* $\Pi^j_{ij}(\mathbf{x}^N) \neq 0$ *he has a strong form of a first-mover incentive* $(\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^N\right))$.

Proof. see Appendix A.3. ■

If efforts are SC for player i, player j reduces his level of effort at the Stackelberg equilibrium in which he leads, compared to the NE (see lemma (2.1)). This increases the payoff of player i due to the property of plain substitute and induces the second-mover incentive. If efforts are SS for player i, we unambiguously have $x^L_j > x^N_j$ (cf. lemma (2.2)), and then player i prefers leading over following due to the negative externality of player j's effort. If efforts are SI for player i, then $x^L_j = x^N_j$ and $x^F_i = x^N_i$ (cf. lemma (2.3)). Consequently, player i's NE and follower-payoff, as well as player j's NE and leader-payoff are equivalent. Finally, if efforts are not SI for player i, then a leader j will always deviate from is NE-level of effort, which, given the assumptions of the model, means he must have a strong form of first-mover incentive.

An interesting point of the preceding lemma is that we establish a second-mover incentive or a first-mover advantage for player i depending only on the concept of strategic complementarity or strategic substitutability of efforts for player j at the NE; that is without assuming monotonicity of the best response function.

3 Selecting a leader through a timing game

The issue of endogenous timing is examined according to the concept proposed by Hamilton and Slutsky (1990) in their *extended game with observable delay*. This extended game $\widetilde{\Gamma}$ allows players to choose non-cooperatively and simultaneously when to exert effort in a preplay stage. The set of possible pure strategies of player i is $a_i \equiv \{e, l\}$, where $e \equiv$ early and $l \equiv$ late. Their decision is announced by the players subsequently. In the consecutive *basic game* (Γ^k, with $k = \{N, S_1, S_2\}$) the players choose their effort according to their timing decision to which they are committed.[24] Thus, the *basic game* consists of three different constituent games: Γ^N if the strategy profile $\mathbf{a} = (a_1, a_2) = (l, l)$ or $\mathbf{a} = (e, e)$, Γ^{S_1} for $\mathbf{a} = (e, l)$, and Γ^{S_2} for $\mathbf{a} = (l, e)$. Thus, if players decide to choose effort at different times, the player who chooses to move late observes the effort exerted by the player who chooses to move *early* and acts accordingly.[25] It is worth noting that the order of moves does not affect the payoffs which are conditional only on the players' strategies.

The normal form representation of the preplay stage is shown in table 1. The

Player 2

		e	**l**
Player 1	e	$\Pi^1\left(\mathbf{x}^N\right), \Pi^2\left(\mathbf{x}^N\right)$	$\Pi^1(x^{S_1}), \Pi^2(x^{S_1})$
	l	$\Pi^1(\mathbf{x}^{\mathbf{S_2}}), \Pi^2(\mathbf{x}^{\mathbf{S_2}})$	$\Pi^1\left(\mathbf{x}^N\right), \Pi^2\left(\mathbf{x}^N\right)$

Table 1
Normal form representation of $\widetilde{\Gamma}$

[24]This assumption, as has been shown by Hamilton and Slutsky (1990, p. 32) is not restrictive, i.e., no player can gain by deviating from a chosen strategy in the preplay stage.

[25]Following Hamilton and Slutsky (1990) and Amir and Stepanova (2006), we restrict our attention to the SPE of $\widetilde{\Gamma}$.

solution to this reduced form game is equivalent to characterizing the solution to the leadership problem. There is no leader if both players choose the same action; a leader emerges when they choose complementary roles. Following Amir and Grilo (1999), $\mathcal{E}$ denotes the set of SPE of $\widetilde{\Gamma}$, where each element of $\mathcal{E}$ is a pair $\{(a_i, a_j), \mathbf{x}^k\}$. Hence, each element of $\mathcal{E}$ represents the equilibrium timing decision in the preplay stage, as well as the Nash equilibrium in the basic game. In case $\mathbf{a}$ implies a sequential choice of effort $\mathbf{x}^k$ must be subgame perfect. We obtain the following proposition:

Proposition 4

Under assumptions (1), (2), (4), *and* (5) *we find the following:*

1. *If efforts are strategic complements for both players* $\left(\Pi^i_{ij}\left(\mathbf{x}^N\right) \geq \Pi^j_{ij}\left(\mathbf{x}^N\right) > 0\right)$, *then* $\mathcal{E} = \left\{(e,l), \mathbf{x}^{S_i}\right\} \cup \left\{(l,e), \mathbf{x}^{S_j}\right\}$.

2. *If efforts are strategic substitutes for both players* $\left(\Pi^i_{ij}\left(\mathbf{x}^N\right) \leq \Pi^j_{ij}\left(\mathbf{x}^N\right) < 0\right)$, *then* $\mathcal{E} = \left\{(e,e), \mathbf{x}^N\right\}$.

3. *If efforts are strategically independent for both players* $\left(\Pi^i_{ij}\left(\mathbf{x}^N\right) = \Pi^j_{ij}\left(\mathbf{x}^N\right) = 0\right)$, *then* $\mathcal{E} = \left\{(e,l), \mathbf{x}^{S_i}\right\} \cup \left\{(l,e), \mathbf{x}^{S_j}\right\} \cup \left\{(e,e), \mathbf{x}^N\right\} \cup \left\{(l,l), \mathbf{x}^N\right\}$, *with* $\mathbf{x}^N = \mathbf{x}^{S_i} = \mathbf{x}^{S_j}$.

4. *If efforts are strategic substitutes for player* i *and strategic complements for player* j $\left(\Pi^i_{ij}\left(\mathbf{x}^N\right) < 0 < \Pi^j_{ij}\left(\mathbf{x}^N\right)\right)$, *then* $\mathcal{E} = \left\{(e,l), \mathbf{x}^{S_i}\right\}$.

5. *If efforts are strategically independent for player* i *and strategic complements for player* j $\left(\Pi^i_{ij}\left(\mathbf{x}^N\right) = 0 < \Pi^j_{ij}\left(\mathbf{x}^N\right)\right)$, *then* $\mathcal{E} = \left\{(e,l), \mathbf{x}^{S_i}\right\} \cup \left\{(l,e), \mathbf{x}^{S_j}\right\}$.

6. *If efforts are strategically independent for player* i *and strategic substitutes for player* j $\left(\Pi^i_{ij}\left(\mathbf{x}^N\right) = 0 > \Pi^j_{ij}\left(\mathbf{x}^N\right)\right)$, *then* $\mathcal{E} = \left\{(e,e), \mathbf{x}^N\right\} \cup \left\{(l,e), \mathbf{x}^{S_j}\right\}$, *with* $\mathbf{x}^N = \mathbf{x}^{S_j}$.

From Proposition (4) we are able to determine under which conditions a leader emerges at the SPE(s). Its identity does not depend on his probability of winning at the NE of the static game but on the nature of strategic interactions among players. As shown in example 3, the favorite, in Dixit's terminology, may lead at the SPE if we introduce an endogenous prize, so that the $\omega\sigma$-correlation does not hold.

In proposition (4.1) both players have a strong form of a first as well as second-mover incentive in the basic game. Thus, a coordination game results in the preplay stage, with two pure strategy Nash equilibria, (e, l) and (l, e). To solve this issue we may utilize the equilibrium selection concepts of *payoff dominance* or *risk dominance* introduced by Harsanyi and Selten (1988).

Example 1 (A conflict framework - continued)
Assume that $\alpha = \frac{1}{2}$, $R = r = 1$ *and* $\kappa = 2$. *According to proposition (4.1) we have a game of coordination in the preplay stage, which is confirmed by the payoffs in the three different games given by table (2). Figure (1) represents the strategy space in this case.*

$1\setminus 2$	**e**	**l**
e	0.27539, 0.43628	0.28333, 0.53879
l	0.33983, 0.44325	0.27539, 0.43628

Table 2
Payoffs in the 1st example

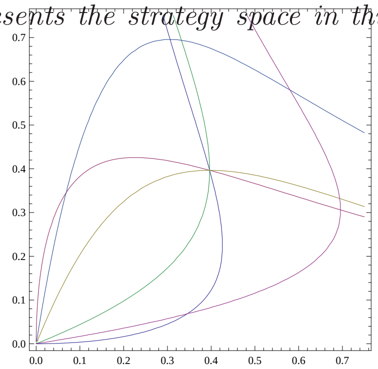

Figure 1
Strategy space in the 1st example

The solid convex (concave) curve represents the best response function of player 1 (2), *and the dashed concave (convex) curve the iso-payoff curve of player* 1 (2) *in the NE of the game. The grey surface represents the set of strategy profiles which Pareto-dominate the NE (Pareto-superior set). Obviously, both players have a strong form of first-mover and second-mover incentive, and the payoffs resulting from* (e, l) *and* (l, e) *cannot be ranked in a Pareto sense. We will therefore utilize the concept of risk dominance. In our framework, the SPE* $\mathbf{x}^{S_2}$ *risk-dominates* $\mathbf{x}^{S_1}$ *if the former is associated with a higher (Nash) product of deviation losses. More formally,* $(e, l) \underset{risk}{\succ} (l, e) \Leftrightarrow \Delta < 0$, *with*

$$\begin{aligned}\Delta \equiv & \left(\Pi^1\left(\mathbf{x}^{S_2}\right) - \Pi^1\left(\mathbf{x}^N\right)\right)\left(\Pi^2\left(\mathbf{x}^{S_2}\right) - \Pi^2\left(\mathbf{x}^N\right)\right) \\ & - \left(\Pi^1\left(\mathbf{x}^{S_1}\right) - \Pi^1\left(\mathbf{x}^N\right)\right)\left(\Pi^2\left(\mathbf{x}^{S_1}\right) - \Pi^2\left(\mathbf{x}^N\right)\right).\end{aligned}$$

Since $\Delta \approx -0.00036$, *we find that* (e, l) *risk-dominates* (l, e).

In proposition (4.2) both players have a first-mover advantage and no second-mover incentive so that both players have the dominant strategy in the timing game (e) which leads to a Cournot-Nash game $(\mathbf{x}^N)$. In proposition (4.3) both players are indifferent between e and l, since $\mathbf{x}^{S_i} = \mathbf{x}^{S_j} = \mathbf{x}^N$. This case is represented by example (1) for $\rho = 1$. Note that this particular case may also be represented by an exogenous-prize rent-seeking game, where $p_{12}(\mathbf{x}^N) = 0$. Proposition (4.4) represents the case which is strategically equivalent to the endogenous timing game examined by Baik and Shogren (1992) and Leininger (1993) if players are unevenly matched: Both players' strategic incentives are directly opposed, so that player i has a dominant strategy in the preplay stage (e). Given this player j's best response is $a_j = l$ and the unique SPE of $\widetilde{\Gamma}$ is $\{(e,l), \mathbf{x}^{S_i}\}$, which corresponds to the leadership of the underdog in a fixed prize scenario.

In the two remaining cases (proposition (4.5) and (4.6)) efforts are SI for player i and are not SI for his competitor. Consequently, player i's follower-payoff (player j's leader payoff) equals his NE-payoff and player i has a first-mover advantage (see lemma (3.3) and (3.4)).

Moreover, in proposition (4.5) player j regards efforts as SC, so that he has a strong form of a second-mover incentive (see lemma (3.1)). Accordingly, $\mathbf{a} = (e,l)$ as well as $\mathbf{a} = (l,e)$ are NE in the preplay stage and a game of coordination results. However, unlike the case in proposition (4.1), we can now use the concept of payoff dominance in order to select an equilibrium. In particular, (e,l) dominates (l,e) in a Pareto sense, as should be clear from the previous analysis.[26]

Finally, in proposition (4.6) player j regards efforts as SS, so that he has no second-mover incentive (see lemma (3.2)) and therefore a dominant strategy in the preplay stage (e). Given this, player i is indifferent between all his pure strategies, since, as was already pointed out, he has a weak form of second-mover incentive. Consequently, $\mathbf{a} = (e,e)$ as well as $\mathbf{a} = (l,e)$ is a NE in the timing game. Since player j has a weak from of first-mover incentive, both SPEs yield the same payoff for both players and hence neither risk nor payoff dominates the other.[27] Propositions (4.2), (4.4) and (4.6) are represented by example 2.

Example 2 (A rent-seeking framework - continued)

Below are the payoff matrices for $\lambda = 1$ (so that $0 > \Pi_{12}^2\left(\mathbf{x}^N\right) = \Pi_{12}^1\left(\mathbf{x}^N\right)$), for $\lambda \approx$

[26] It is worth noting that in this case (e,l) also risk dominates (l,e), since

$$\Pi^i\left(\mathbf{x}^{S_j}\right) - \Pi^i\left(\mathbf{x}^N\right) = \Pi^j\left(\mathbf{x}^{S_j}\right) - \Pi^j\left(\mathbf{x}^N\right) = 0 \text{ and } \left(\Pi^i\left(\mathbf{x}^{S_i}\right) - \Pi^i\left(\mathbf{x}^N\right)\right)\left(\Pi^j\left(\mathbf{x}^{S_i}\right) - \Pi^j\left(\mathbf{x}^N\right)\right) > 0.$$

This finding is in line with the analysis of Matsumura and Ogawa (2009).

[27] Here, one finds that

$$\Pi^i\left(\mathbf{x}^N\right) - \Pi^i\left(\mathbf{x}^{S_j}\right) = \Pi^j\left(\mathbf{x}^{S_j}\right) - \Pi^j\left(\mathbf{x}^N\right) = 0,$$

so that $(e,e) \underset{risk}{\sim} (l,e)$.

0.78922 *(and therefore* $\Pi_{12}^2\left(\mathbf{x}^N\right) = 0 > \Pi_{12}^1\left(\mathbf{x}^N\right)$*), and finally for* $\lambda = \frac{3}{4}$ *(so that* $\Pi_{12}^2\left(\mathbf{x}^N\right) > 0 > \Pi_{12}^1\left(\mathbf{x}^N\right)$*). Moreover, figure (2) - (4) represent the different cases in the strategy space.*

1\2	**e**	**l**
e	0.26158, 0.26158	0.26379, 0.21422
l	0.21422, 0.26379	0.26158, 0.26158

Table 3
Payoff matrix in the 2nd example for $\lambda = 1$

1\2	**e**	**l**
e	0.21078, 0.31737	0.21078, 0.31737
l	0.11846, 0.32799	0.21078, 0.31737

Table 4
Payoff matrix in the 2nd example for $\lambda \approx 0.78922$

1\2	**e**	**l**
e	0.15907, 0.38697	0.16110, 0.43759
l	0.02884, 0.42071	0.15907, 0.38697

Table 5
Payoff matrix in the 2nd example for $\lambda = \frac{3}{5}$

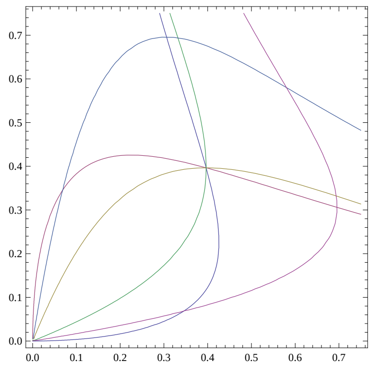

Figure 2
Strategy space in the 2nd example for $\lambda = 1$

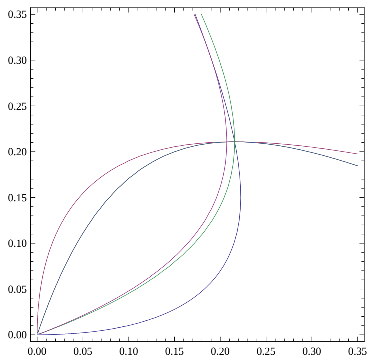

Figure 3
Strategy space in the 2nd example for $\lambda \approx 0.78922$

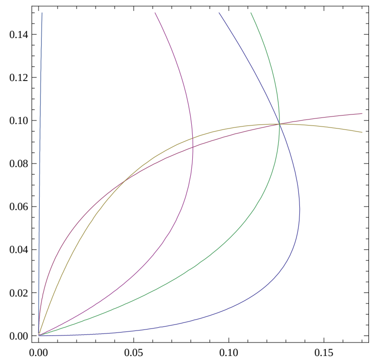

Figure 4
Strategy space in the 2nd example for $\lambda = \frac{3}{4}$

Again, the solid curves represent the best response functions while the dashed curves represent the iso-payoff curves of players in the NE of the game. For $\lambda = 1$ *both players prefer their NE payoff over their follower payoff and therefore neither of the best response functions enters the Pareto-superior set, represented by the grey surface. The same holds for* $\lambda \approx 0.78922$*. However, since* $\mathbf{x}^{S_1} = \mathbf{x}^N$*, we find that both SPEs of* $\widetilde{\Gamma}$ *are payoff-equivalent. For* $\lambda = \frac{3}{4}$ *only player 2 prefers his NE payoff over his follower payoff. Thus, only player 2's best response function enters the Pareto-superior set. Moreover, player 1 undercommits effort, so that the SPE Pareto-dominates* $\mathbf{x}^N$ *as well as* $\mathbf{x}^{S_2}$*.*

Applying proposition (4) we now provide a taxonomy of SPE in $\widetilde{\Gamma}$ based on the properties of the prize-production technology (in particular, the sign of $V_{12}\left(\mathbf{x}^N\right)$) as well as on the sign of the slope of players' best response functions in the NE,

	$V_{12}(\mathbf{x}^N) > 0$	$V_{12}(\mathbf{x}^N) < 0$	$V_{12}(\mathbf{x}^N) = 0$
$\Pi_{12}^1\left(\mathbf{x}^N\right) > 0$	$\mathbf{x}^{S_1}$ or $\mathbf{x}^{S_2}$ if $V_{12}(\mathbf{x}^N) \geq \Pi_{12}^1(\mathbf{x}^N)$ $\mathbf{x}^{S_2}$ if $V_{12}(\mathbf{x}^N) < \Pi_{12}^1(\mathbf{x}^N)$	$\mathbf{x}^{S_2}$	$\mathbf{x}^{S_2}$
$\Pi_{12}^1\left(\mathbf{x}^N\right) < 0$	$\mathbf{x}^{S_1}$	$\mathbf{x}^N$ if $V_{12}(\mathbf{x}^N) < \Pi_{12}^1(\mathbf{x}^N)$ $\mathbf{x}^N$ or $\mathbf{x}^{S_1}$ if $V_{12}(\mathbf{x}^N) = \Pi_{12}^1(\mathbf{x}^N)$ $\mathbf{x}^{S_1}$ if $V_{12}(\mathbf{x}^N) > \Pi_{12}^1(\mathbf{x}^N)$	$\mathbf{x}^{S_1}$
$\Pi_{12}^1\left(\mathbf{x}^N\right) = 0$	$\mathbf{x}^{S_1}$	$\mathbf{x}^{S_2}$ or $\mathbf{x}^N$	$\mathbf{x}^{S_1}$, $\mathbf{x}^{S_2}$ or $\mathbf{x}^N$

Table 6
A taxonomy of SPE in $\widetilde{\Gamma}$

presented in table 6. For simplicity, we only display the equilibrium strategies in the resulting basic game.

From proposition (4), we may deduce the following corollary.

Corollary 5

With the exception of one case every SPE of the extended game $\widetilde{\Gamma}$ is Pareto-undominated.[28]
More precisely, we have:

1. *If $\Pi_{ij}^i\left(\mathbf{x}^N\right) \geq \Pi_{ij}^j\left(\mathbf{x}^N\right) > 0$, both SPEs Pareto-dominate $\mathbf{x}^N$.*

2. *If $\Pi_{ij}^i\left(\mathbf{x}^N\right) \leq \Pi_{ij}^j\left(\mathbf{x}^N\right) < 0$, the equilibria in the three basic games $\mathbf{x}^N$, $\mathbf{x}^{S_i}$ and $\mathbf{x}^{S_j}$ are not Pareto-rankable.*

3. *If $\Pi_{ij}^i\left(\mathbf{x}^N\right) = \Pi_{ij}^j\left(\mathbf{x}^N\right) = 0$, the three SPEs $\mathbf{x}^N$, $\mathbf{x}^{S_i}$ and $\mathbf{x}^{S_j}$ are payoff-equivalent.*

4. *If $\Pi_{ij}^i\left(\mathbf{x}^N\right) < 0 < \Pi_{ij}^j\left(\mathbf{x}^N\right)$, the SPE Pareto-dominates $\mathbf{x}^N$ as well as $\mathbf{x}^{S_j}$.*

5. *If $\Pi_{ij}^i\left(\mathbf{x}^N\right) = 0 < \Pi_{ij}^j\left(\mathbf{x}^N\right)$, the SPEs of $\widetilde{\Gamma}$ are Pareto-rankable. In particular, $\mathbf{x}^{S_i}$ Parteo-dominates $\mathbf{x}^{S_j}$ as well as $\mathbf{x}^N$.*

[28]In what follows we only concentrate on the strategies in the subgames, i.e., $\mathbf{x}^k$, since these exclusively determine the payoff of each player.

6. *If* $\Pi_{ij}^{i}\left(\mathbf{x}^{N}\right)=0>\Pi_{ij}^{j}\left(\mathbf{x}^{N}\right)$*, the payoff-equivalent SPEs (*$\mathbf{x}^{N}$ *and* $\mathbf{x}^{S_i}$*) and the non-SPE (*$\mathbf{x}^{S_j}$*) are not Pareto-rankable.*

Proof. Immediate. ■

The assumptions underlying Corollaries (5.1), (5.2) and (5.4), specifically the fact that the three basic games have a unique equilibrium that differ from one another ($\mathbf{x}^{S_i} \neq \mathbf{x}^{N} \neq \mathbf{x}^{S_j}$), match the assumptions made by Hamilton and Slutsky (1990).[29] As a consequence, the above findings are consistent with the results of the latter, notably theorem V. They show that players' voluntary choice of timing leads to a second-best efficient outcome, just as in the fixed-prize framework. These findings are based on the following facts: If we observe sequential play in the SPE, the leader always *undercommits* effort compared to the NE. If we observe simultaneous play in equilibrium, both players' efforts are - ceteris paribus - lower than their Stackelberg leader effort.

In corollary (5.1) both players' best response functions enter the Pareto-superior set. This case is represented by example 1 (cf. figure (1)). In corollary (5.2) both players prefer their NE payoff over their follower payoff and therefore neither of the best response functions enters the Pareto-superior set (cf. figure (2) of example 2). That is why $\mathbf{x}^{N}$, $\mathbf{x}^{S_1}$ and $\mathbf{x}^{S_2}$ cannot be ranked in a Pareto sense in this case. In corollary (5.4) only player j prefers his NE payoff over his follower payoff (cf. figure (4) in example 2, with $i=1$ and $j=2$). Thus, only player j's best response function enters the Pareto-superior set, and the unique SPE ($\mathbf{x}^{S_i}$) Pareto-dominates $\mathbf{x}^{N}$ as well as $\mathbf{x}^{S_j}$.

In the remaining cases, at least one player (player j) regards efforts as SI. Consequently, $\mathbf{x}^{S_i}=\mathbf{x}^{N}$, which is no longer consistent with the assumptions of Hamilton and Slutsky (1990). Accordingly, we may find that a SPE of $\widetilde{\Gamma}$ is Pareto-dominated. This is indeed the case for $\Pi_{ij}^{i}\left(\mathbf{x}^{N}\right)=0<\Pi_{ij}^{j}\left(\mathbf{x}^{N}\right)$ (cf. corollary (5.5)). Here, $\mathbf{x}^{S_i} \underset{Pareto}{\succ} \mathbf{x}^{S_j} \underset{Pareto}{\sim} \mathbf{x}^{N}$, where $\mathbf{x}^{S_i}$ and $\mathbf{x}^{S_j}$ are both SPEs of $\widetilde{\Gamma}$. In corollary (5.6) we find that both SPEs are payoff-equivalent and that these SPEs and the non-SPE are not Pareto-rankable. This case is represented by figure (3) in example 2. In the trivial case (cf. corollary (5.3)) all equilibria yield the same payoff, so that $\mathbf{x}^{S_i} \underset{Pareto}{\sim} \mathbf{x}^{S_j} \underset{Pareto}{\sim} \mathbf{x}^{N}$.

[29]See footnote 1, p. 31 of Hamilton and Slutsky (1990).

4 Conclusion

Based on the endogenous timing game by Hamilton and Slutsky (1990), we have provided a framework for the analysis of endogenous leadership in contests with an endogenously determined prize. In a stage prior to the contest subgame, the players decided whether they will exert effort as soon as or as late as possible; and their decision, to which they are committed, is announced to the other player subsequently. In this model we have provided a taxonomy of endogenous leadership, based on the properties of the players' best response functions as well as on the characteristics of the prize-production technology. Thus, we were able to generalize the findings of Baik and Shogren (1992) and Leininger (1993) regarding the behavior of the Stackelberg-leader. However, there are differences compared to the aforementioned literature. In particular, we were able to establish that the SPE of the extended game may be represented by a simultaneous move game, and that in a sequential move SPE the leader might be the favorite of the Cournot-Nash game.

Our work can be extended in various ways:

Regarding the previous work of Yildirim (2005) and Romano and Yildirim (2005) it would be interesting to establish in which way the findings of the present paper would be modified if one abstains from the assumption that each player is allowed to exert effort only once. For instance, in the case were players are evenly matched, Yildirim (2005) finds that the outcome of the game is equivalent to a game where players move simultaneously, although effort might be exerted *early* and *late*. Therefore, allowing the players in our framework to exert effort twice might eliminate the coordination issue in a game of strategic complements.

Finally, in a rent-seeking framework one may allow for a prize which *in*creases in the effort of the players. Previous papers dealing with this topic include Cohen et al. (2008) and Gershkov et al. (2009). Although the prize is assumed to depend in a positive manner on the effort exerted, the issue of endogenous timing has not yet been analyzed. Contingent on the properties of the prize-production technology, this might lead to a game in which the payoff of a player does not react in a monotonic manner on the effort of his competitor. Hence, one might find in the NE that the effort of each player has a positive effect on each player's payoff, which would reshape the commitment incentives in the sequential move games.

These extensions are the subject of current research.

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A Appendix - Proofs

A.1 Proof of lemma 1

Here, we prove the existence and uniqueness of the Nash equilibrium.

A.1.1 Existence of the Nash equilibrium

Given assumptions (1) and (2), the payoff function $\Pi^i(\mathbf{x})$, described by equations (4), is continuous in (x_i, x_j). We now show that each player's payoff function is strictly concave in his own strategy. The second derivative of the payoff function yields

$$\Pi^1_{11}(\mathbf{x}) = p_{11}(\mathbf{x}) V(\mathbf{x}) + 2p_1(\mathbf{x}) V_1(\mathbf{x}) + p(\mathbf{x}) V_{11}(\mathbf{x}) - C^1_{11}(x_1), \tag{A.1}$$

$$\Pi^2_{22}(\mathbf{x}) = -p_{22}(\mathbf{x}) V(\mathbf{x}) - 2p_2(\mathbf{x}) V_2(\mathbf{x}) + (1 - p(\mathbf{x})) V_{22}(\mathbf{x}) - C^2_{22}(x_2). \tag{A.2}$$

Assumptions (1) and (2) together with $C^i_{ii}(x_i) \geq 0$ imply that

$$\Pi^1_{11}(\mathbf{x}) < 0 \quad \text{and} \quad \Pi^2_{22}(\mathbf{x}) < 0. \tag{A.3}$$

Therefore the solution to the maximization problem (cf. eq. 6) is unique. Moreover, the payoff-function is also continuous. We can thus conclude that best response function $BR_i(x_j)$ is single-valued and continuous.

The strategy space of player i ($X_i = \mathbb{R}^+$) is convex. Next, we eliminate some strategies so that the set of the remaining strategies is compact. Define $\bar{x}_1 > 0$ such that

$$\Pi^1(\bar{x}_1, 0) = 0.$$

Thus, since $\Pi^1_2(\mathbf{x}) < 0$

$$\Pi^1(\bar{x}_1, x_2) > \Pi^1(x_1, x_2),$$

for any $x_1 > \bar{x}_1$ and for all $x_2 \in \mathbb{R}^+$. Therefore for all $x_1 > \bar{x}_1$, $\bar{x}_1$ strictly dominates x_1. Hence, after elimination of those strictly dominated strategies the strategy space of player 1 becomes $[0, \bar{x}_1]$ which is a compact, convex and non-empty set. By symmetry the same argument can be applied to player 2.

A Nash-equilibrium satisfies the following equations:

$$BR_1(x_2) = x_1, \tag{A.4}$$

$$BR_2(x_1) = x_2. \tag{A.5}$$

By substituting (A.5) into (A.4), or vice versa, we see that the NE is given by a fixed point of the composite function $\mathcal{BR}_i := BR_i \circ BR_j : [0, \bar{x}_i] \to [0, \bar{x}_i]$, where the composite function $\mathcal{BR}_i(\cdot)$ is a continuous and single valued mapping of a non-empty, convex and compact set into itself. Hence, the existence of a fixed point directly follows from Brouwer's Fixed Point Theorem. Finally, notice that the one-shot NE is interior, i.e., $\mathbf{x}^N > \mathbf{0}$. In particular, $\mathbf{x} = \mathbf{0}$ can not be an equilibrium due to assumption (3). In addition, $\mathbf{x} = (x_i, 0)$, with $x_i > 0$ can not be an equilibrium, since player i, given assumption (2) can always deviate from any $x_i > 0$ in a strictly profitable manner.

A.1.2 Uniqueness of the Nash equilibrium

We now prove the uniqueness of the NE if $V_{ii}(\mathbf{x}) < 0$, i.e., we prove that $\mathcal{BR}_i(\cdot)$ has a unique fixed point.[30] For this we will utilize the *index theory approach*, (see Kolstad and Mathiesen (1987) and

[30] Uniqueness of the NE for $V_i(\mathbf{x}) = 0$ follows easily from the negative quasi-definiteness of the Jacobian of the marginal payoffs (see Rosen (1965)).

Vives (2001), p. 48), which, in the case of two players, requires the determinant of the Jacobian of the marginal payoffs, evaluated at $\mathbf{x}^N$, to be positive, i.e.,

$$|J| = \begin{vmatrix} \Pi^1_{11}\left(\mathbf{x}^N\right) & \Pi^2_{12}\left(\mathbf{x}^N\right) \\ \Pi^1_{12}\left(\mathbf{x}^N\right) & \Pi^2_{22}\left(\mathbf{x}^N\right) \end{vmatrix} > 0. \tag{A.6}$$

From this it follows that the multiplied slope of both players' best response functions must be smaller than one, i.e.,

$$\frac{\Pi^1_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{11}\left(\mathbf{x}^N\right)} \frac{\Pi^2_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{22}\left(\mathbf{x}^N\right)} < 1. \tag{A.7}$$

We will now split cases.

- <u>Case 1</u>: **Efforts are strategic complements (substitutes) for both players**
  We first explore the case where efforts are strategic complements (substitutes) for both players, i.e., either $\Pi^i_{ij}\left(\mathbf{x}^N\right) \geq \Pi^j_{12}\left(\mathbf{x}^N\right) > 0$ or $\Pi^i_{ij}\left(\mathbf{x}^N\right) \leq \Pi^j_{12}\left(\mathbf{x}^N\right) < 0$.[31]
  We will now distinguish between rent-seeking games ($\Omega(\mathbf{x}) \neq 0$) and conflict games ($\Omega(\mathbf{x}) = 0$).

  a. <u>Rent-seeking games</u>
  For $\Omega(\mathbf{x}) \neq 0$ we deduce from eq. (15)

  $$\Pi^1_{12}\left(\mathbf{x}^N\right)\Pi^2_{12}\left(\mathbf{x}^N\right) = \left(\Omega\left(\mathbf{x}^N\right) + p\left(\mathbf{x}^N\right)V_{12}\left(\mathbf{x}^N\right)\right)\left(-\Omega\left(\mathbf{x}^N\right) + \left(1 - p\left(\mathbf{x}^N\right)\right)V_{12}\left(\mathbf{x}^N\right)\right)$$

  Implementing $\bar{p}\left(\mathbf{x}^N\right) = \max\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$ leads to

  $$\begin{aligned} \Pi^1_{12}\left(\mathbf{x}^N\right)\ \Pi^2_{12}\left(\mathbf{x}^N\right) &\leq \left(\Omega\left(\mathbf{x}^N\right) + \bar{p}\left(\mathbf{x}^N\right)V_{12}\left(\mathbf{x}^N\right)\right)\left(-\Omega\left(\mathbf{x}^N\right) + \bar{p}\left(\mathbf{x}^N\right)V_{12}\left(\mathbf{x}^N\right)\right) \\ &= \left(\bar{p}\left(\mathbf{x}^N\right)\ V_{12}\left(\mathbf{x}^N\right)\right)^2 - \left(\Omega\left(\mathbf{x}^N\right)\right)^2 \\ &< \left(\bar{p}\left(\mathbf{x}^N\right)\ V_{12}\left(\mathbf{x}^N\right)\right)^2. \end{aligned} \tag{A.8}$$

  Using (A.1) and (A.2), and implementing $\underline{p}\left(\mathbf{x}^N\right) = \min\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$, we deduce

  $$\Pi^1_{11}\left(\mathbf{x}^N\right) < p\left(\mathbf{x}^N\right)V_{11}\left(\mathbf{x}^N\right) \leq \underline{p}\left(\mathbf{x}^N\right)\ V_{11}\left(\mathbf{x}^N\right) < 0, \tag{A.9}$$

  and

  $$\Pi^2_{22}\left(\mathbf{x}^N\right) < \left(1 - p\left(\mathbf{x}^N\right)\right)V_{22}\left(\mathbf{x}^N\right) \leq \underline{p}\left(\mathbf{x}^N\right)\ V_{22}\left(\mathbf{x}^N\right) < 0. \tag{A.10}$$

  Thus, combining eq. (A.8), (A.9) and (A.10) as well as assumption (4) yields

  $$\frac{\Pi^1_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{11}\left(\mathbf{x}^N\right)} \frac{\Pi^2_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{22}\left(\mathbf{x}^N\right)} < \left(\frac{\bar{p}\left(\mathbf{x}^N\right)}{\underline{p}\left(\mathbf{x}^N\right)}\right)^2 \frac{\left(V_{12}\left(\mathbf{x}^N\right)\right)^2}{V_{11}\left(\mathbf{x}^N\right)V_{22}\left(\mathbf{x}^N\right)} \leq 1. \tag{A.11}$$

  b. <u>Conflict games</u>
  For $\Omega(\mathbf{x}) = 0$ we deduce from eq. (15)

  $$\Pi^1_{12}\left(\mathbf{x}^N\right)\Pi^2_{12}\left(\mathbf{x}^N\right) = p\left(\mathbf{x}^N\right)\ \left(1 - p\left(\mathbf{x}^N\right)\right)\left(V_{12}\left(\mathbf{x}^N\right)\right)^2 \tag{A.12}$$

  Using (A.1) and (A.2) we deduce

  $$\Pi^1_{11}\left(\mathbf{x}^N\right) < p\left(\mathbf{x}^N\right)V_{11}\left(\mathbf{x}^N\right) < 0, \tag{A.13}$$

[31] Since in this case $sign\left(\Pi^1_{12}(\mathbf{x})\right) = sign\left(\Pi^2_{12}(\mathbf{x})\right)$ it follows that $\frac{\Pi^1_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{11}\left(\mathbf{x}^N\right)} \frac{\Pi^2_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{22}\left(\mathbf{x}^N\right)} > 0$. Thus, condition (A.7) is equal to the condition for *local stability* of the NE (see Vives (2001), p. 51).

and

$$\Pi_{22}^{2}\left(\mathbf{x}^{N}\right) < \left(1 - p\left(\mathbf{x}^{N}\right)\right) V_{22}\left(\mathbf{x}^{N}\right) < 0. \tag{A.14}$$

Combining eq. (A.12), (A.13) and (A.14) as well as assumption (4) yields

$$\frac{\Pi_{12}^{1}\left(\mathbf{x}^{N}\right)}{\Pi_{11}^{1}\left(\mathbf{x}^{N}\right)} \frac{\Pi_{12}^{2}\left(\mathbf{x}^{N}\right)}{\Pi_{22}^{1}\left(\mathbf{x}^{N}\right)} < \frac{\left(V_{12}\left(\mathbf{x}^{N}\right)\right)^{2}}{V_{11}\left(\mathbf{x}^{N}\right) V_{22}\left(\mathbf{x}^{N}\right)} \leq 1. \tag{A.15}$$

- **<u>Case 2</u>: Residual case.**
  Next, we turn to the residual case where $\Pi_{ij}^{i}\left(\mathbf{x}^{N}\right) \leq 0 \leq \Pi_{ij}^{j}\left(\mathbf{x}^{N}\right)$. In this case condition (A.7) can easily be established. In particular,

$$\frac{\Pi_{12}^{1}\left(\mathbf{x}^{N}\right)}{\Pi_{11}^{1}\left(\mathbf{x}^{N}\right)} \frac{\Pi_{12}^{2}\left(\mathbf{x}^{N}\right)}{\Pi_{22}^{2}\left(\mathbf{x}^{N}\right)} \leq 0. \tag{A.16}$$

Since

$$\mathcal{BR}_i'(x_i^N) \equiv (BR_i \circ BR_j)'(x_1^N) = BR_i'\left(BR_j(x_i^N)\right) BR_j'(x_i^N) = \frac{\Pi_{12}^{1}\left(\mathbf{x}^{N}\right)}{\Pi_{11}^{1}\left(\mathbf{x}^{N}\right)} \frac{\Pi_{12}^{2}\left(\mathbf{x}^{N}\right)}{\Pi_{22}^{2}\left(\mathbf{x}^{N}\right)} \tag{A.17}$$

it follows from (A.11), (A.15) and (A.16) that

$$\mathcal{BR}_i'(x_i^N) \ < \ 1 \tag{A.18}$$

in each case, i.e., we found a bound for the slope of $\mathcal{BR}_i(\cdot)$ at x_i^N. From (A.18) it follows that we can rule out the existence of equilibria which are limit points of other equilibria.[32] That is, there are finitely many equilibria which are isolated. Hence, if (x_1^N, x_2^N) is an equilibrium, then there is an $\varepsilon = \varepsilon(x_1^N, x_2^N)$ such that for all $\hat{x}_i \in [x_i^N - \varepsilon, x_i^N + \varepsilon]$, with $i = 1, 2$, $(\hat{x}_1, \hat{x}_2)$ is not an equilibrium. Using the above results, we can now rule out in either case the existence of a second equilibrium.
Suppose that (x_1^a, x_2^a) and (x_1^b, x_2^b) are two isolated equilibria. Let $x_1^a < x_1 < x_1^b$. Then starting from x_1^a and using the *mean value theorem* we have, since $\mathcal{BR}_1(x_1^a) = x_1^a$,

$$\mathcal{BR}_1(x_1) - x_1^a \quad = \quad \mathcal{BR}_1'(y)(x_1 - x_1^a),$$

for some $y \in (x_1^a, x_1)$. Note that, by assumption, the term in brackets on the right hand side is unambigiously positive. Assuming that $0 < \mathcal{BR}_1'(y) < 1$ we get

$$\mathcal{BR}_1(x_1) < x_1, \tag{A.19}$$

while assuming $\mathcal{BR}_1'(y) \leq 0$ leads to

$$\mathcal{BR}_1(x_1) \leq x_1^a. \tag{A.20}$$

Starting from x_1^b, we have, since $\mathcal{BR}_1(x_1^b) = x_1^b$,

$$x_1^b - \mathcal{BR}_1(x_1) \quad = \quad \mathcal{BR}_1'(z)(x_1^b - x_1),$$

for some $z \in (x_1, x_1^b)$. Now, the term in brackets on the right hand side is unambigiously negative. Assuming that $0 < \mathcal{BR}_1'(z) < 1$, we get

$$\mathcal{BR}_1(x_1) > x_1, \tag{A.21}$$

which contradicts (A.19) as well as (A.20). Assuming that $\mathcal{BR}_1'(z) \leq 0$ leads to

$$\mathcal{BR}_1(x_1) \geq x_1^b, \tag{A.22}$$

[32] See Skaperdas (1992, p. 737) for a game of SC.

which also contradicts (A.19) and (A.20). Thus, there exists a unique NE.
□

A.2 Proof of lemma 2 (Comparison of the levels of effort)

Let the function $\Psi_i(x_i)$ be

$$\Psi_i(x_i) = \Pi_i^i\left(x_i, x_j^F(x_i)\right) + \Pi_j^i\left(x_i, x_j^F(x_i)\right) \frac{dx_j^F(x_i)}{dx_i}. \tag{A.23}$$

This function corresponds to the first derivative of the leader payoff function. For x_i^L, we obtain the FOC of the leader, that is $\Psi_i\left(x_i^L\right) = 0$. Since $\Psi_i'(x_i) < 0$ we find that the Stackelberg equilibrium exists and is unique. Next, we split cases.

- If $\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0$ efforts are strategic complements for the player j at the Nash equilibrium. We deduce that $\frac{dx_j^F(x_i)}{dx_i} > 0$ at x_i^N, i.e., the best response function of player j is increasing at the Nash equilibrium. We thus have

$$\begin{aligned}\Psi_i\left(x_i^N\right) &= \Pi_i^i\left(x_i^N, x_j^F(x_i^N)\right) + \Pi_j^i\left(x_i^N, x_j^F(x_i^N)\right)\frac{dx_j^F(x_i)}{dx_i} \\ &= \Pi_j^i\left(x_i^N, x_j^F(x_i^N)\right)\frac{dx_j^F(x_i)}{dx_i} < 0 = \Psi_i\left(x_i^L\right),\end{aligned}$$

  since by definition $\Pi_i^i\left(x_i^N, x_j^F(x_i^N)\right) = 0$, $\Pi_j^i\left(\mathbf{x}^N\right) < 0$ and $\frac{dx_j^F(x_i)}{dx_i} > 0$. The decreasing of $\Psi_i\left(\mathbf{x}^N\right)$ in x_i involves

$$\Psi_i\left(x_i^N\right) < \Psi_i\left(x_i^L\right) \Leftrightarrow x_i^N > x_i^L. \tag{A.24}$$

  Since $\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0$ this involves that $x_j^F < x_j^N$.

- If $\Pi_{ij}^j\left(\mathbf{x}^N\right) < 0$, then we have $\frac{dx_j^F(x_i)}{dx_i} < 0$ at the Nash equilibrium, and consequently

$$\Psi_i\left(x_i^N\right) > 0 \Leftrightarrow x_i^N < x_i^L. \tag{A.25}$$

  Since $\Pi_{ij}^j\left(\mathbf{x}^N\right) < 0$ this involves that $x_j^F < x_j^N$.

- If $\Pi_{ij}^j\left(\mathbf{x}^N\right) = 0$, then we have $\frac{dx_j^F(x_i)}{dx_i} = 0$ at the Nash equilibrium, and consequently

$$\Psi_i\left(x_i^N\right) = 0 \Leftrightarrow x_i^N = x_i^L. \tag{A.26}$$

  Since $\Pi_{ij}^j\left(\mathbf{x}^N\right) = 0$ this involves that $x_j^F = x_j^N$.
  □

A.3 Proof of lemma 3 (First-mover advantage and second-mover incentive)

We have to consider three different cases:

- If $\Pi_{ij}^i\left(\mathbf{x}^N\right) > 0$, the rankings are: $x_i^F < x_i^N$ and $x_j^N > x_j^L$ (see lemma 2). We have

$$\Pi^i\left(x_i^F, x_j^L\right) = \max_{x_i} \Pi^i\left(x_i, x_j^L\right) \geq \Pi^i\left(x_i^N, x_j^L\right) > \Pi^i\left(x_i^N, x_j^N\right), \tag{A.27}$$

  where the first inequality results from the follower's maximization program, and the second from the fact that $x_j^L < x_j^N$ and $\Pi_j^i(\mathbf{x}) < 0$.

- If $\Pi^i_{ij}\left(\mathbf{x}^N\right) < 0$, the ranking are: $x_i^F < x_i^N$ and $x_j^N < x_j^L$. We have

$$\Pi^i\left(x_i^N, x_j^N\right) = \max_{x_i} \Pi^i\left(x_i, x_j^N\right) \geq \Pi^i\left(x_i^F, x_j^N\right) > \Pi^i\left(x_i^F, x_j^L\right), \tag{A.28}$$

  where the first inequality results from the definition of the Nash maximization program, and the second from the fact that $x_j^N < x_j^L$ and $\Pi^i_j(\mathbf{x}) < 0$.

- If $\Pi^i_{ij}\left(\mathbf{x}^N\right) = 0$, the rankings are: $x_i^F = x_i^N$ and $x_j^N = x_j^L$. Thus,

$$\Pi^i\left(x_i^F, x_j^L\right) = \Pi^i\left(x_i^N, x_j^N\right), \text{ and } \Pi^j(x_i^F, x_j^L) = \Pi^j(x_i^N, x_j^N) \tag{A.29}$$

  follows immediately.

Moreover, the following holds for $\Pi^i_{ij}\left(\mathbf{x}^N\right) \gtreqless 0$.

- If $\Pi^j_{ij}\left(\mathbf{x}^N\right) \neq 0$, then

$$\Pi^i(x_i^L, x_j^F) = \max_{x_i} \Pi^i(x_i, x_j^F(x_i)) > \Pi^i(x_i^N, x_j^F(x_i^N)) = \Pi^i(x_i^N, x_j^N), \tag{A.30}$$

  since $x_i^L \neq x_i^N$ and $\Psi'(x_i) < 0$.

- If $\Pi^j_{ij}\left(\mathbf{x}^N\right) = 0$, then

$$\Pi^i(x_i^L, x_j^F) = \max_{x_i} \Pi^i(x_i, x_j^F(x_i)) = \Pi^i(x_i^N, x_j^F(x_i^N)) = \Pi^i(x_i^N, x_j^N), \tag{A.31}$$

  since $x_i^L = x_i^N$ and $\Psi'(x_i) < 0$.

□

A.4 Proof of proposition 4 (SPE)

1. $\Pi^i_{ij}\left(\mathbf{x}^N\right) \geq \Pi^j_{ij}\left(\mathbf{x}^N\right) > 0$. In this case

$$\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^N\right) \text{ and } \Pi^i\left(\mathbf{x}^{S_j}\right) > \Pi^i\left(\mathbf{x}^N\right) \tag{A.32}$$

   holds for both players. For the first relation see (A.30) for the second (A.27). Hence, $(a_i, a_j) = (e, e)$ or (l, l) cannot be a NE of the timing game.

2. $\Pi^i_{ij}\left(\mathbf{x}^N\right) \leq \Pi^j_{ij}\left(\mathbf{x}^N\right) < 0$. In this case

$$\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^N\right) > \Pi^i\left(\mathbf{x}^{S_j}\right) \tag{A.33}$$

   holds for both players. See (A.30) for the first inequality and (A.28) for the second. Accordingly, both players have a dominant strategy (e).

3. $\Pi^i_{ij}\left(\mathbf{x}^N\right) = \Pi^j_{ij}\left(\mathbf{x}^N\right) = 0$. In this case

$$\Pi^i\left(\mathbf{x}^{S_i}\right) = \Pi^i\left(\mathbf{x}^N\right) = \Pi^i\left(\mathbf{x}^{S_j}\right) \tag{A.34}$$

   holds for both players. See (A.31) for the first equality and (A.29) for the second. Thus, each possible strategy profile constitutes a NE of the timing game.

4. $\Pi^i_{ij}\left(\mathbf{x}^N\right) < 0 < \Pi^j_{ij}\left(\mathbf{x}^N\right)$. In this case

$$\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^N\right) \text{ and } \Pi^i\left(\mathbf{x}^{S_j}\right) > \Pi^i\left(\mathbf{x}^N\right) \tag{A.35}$$

   holds for player i and

$$\Pi^j\left(\mathbf{x}^{S_j}\right) > \Pi^j\left(\mathbf{x}^N\right) \text{ and } \Pi^j\left(\mathbf{x}^{S_i}\right) > \Pi^j\left(\mathbf{x}^N\right) \tag{A.36}$$

holds for player j. Hence, player j's best response to player i's dominant strategy ($a_i = e$) is $a_j = l$.

5. $\Pi^i_{ij}\left(\mathbf{x}^N\right) = 0 < \Pi^j_{ij}\left(\mathbf{x}^N\right)$. In this case

$$\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^N\right) = \Pi^i\left(\mathbf{x}^{S_j}\right) \tag{A.37}$$

holds for player i (see (A.30) for the inequality and (A.29) for the equality) and

$$\Pi^j\left(\mathbf{x}^{S_i}\right) > \Pi^j\left(\mathbf{x}^N\right) = \Pi^j\left(\mathbf{x}^{S_j}\right) \tag{A.38}$$

for player j (see (A.30 for the inequality and (A.29) for the equality). Thus, $(a_i, a_j) = (e, l)$ as well as (l, e) is a NE of the timing game.

6. $\Pi^i_{ij}\left(\mathbf{x}^N\right) = 0 > \Pi^j_{ij}\left(\mathbf{x}^N\right)$. Again, in this case

$$\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^N\right) = \Pi^i\left(\mathbf{x}^{S_j}\right) \tag{A.39}$$

holds for player i. For player j we get

$$\Pi^j\left(\mathbf{x}^{S_j}\right) = \Pi^j\left(\mathbf{x}^N\right) > \Pi^j\left(\mathbf{x}^{S_i}\right), \tag{A.40}$$

where the equality stems from (A.29) and the inequality from (A.28). So player j has a dominant strategy (e), and player i, given the dominant strategy of player j is indifferent. Accordingly, $(a_i, a_j) = (e, e)$ as well as $(a_i, a_j) = (l, e)$ is a NE of the timing game.