Fixed Adjustment Costs and Aggregate Fluctuations*

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Abstract

This paper studies the analytics of a canonical model of fixed adjustment costs in the presence of idiosyncratic productivity shocks. We provide a novel analytical characterization of the steady state and dynamics of aggregate outcomes implied by the model. The dynamics are shown to have an intuitive partial-adjustment representation. These results are then used to derive a set of approximations to model outcomes in the presence of a small adjustment cost. Surprisingly, these reveal that both aggregate steady-state outcomes and aggregate dynamics are approximately neutral with respect to a small fixed adjustment cost. We show that this neutrality result emerges from a symmetry property in the distributional dynamics of the model, and arises even in the absence of general equilibrium adjustment of prices. A set of quantitative illustrations confirms these analytical results for parameterizations commonly used in the literature on employment adjustment.

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Inaction in microeconomic adjustment is pervasive. A stylized fact of the empirical dynamics of employment, investment and prices is that they exhibit long periods of inaction punctured by bursts of adjustment (see, among others, Hamermesh, 1989; Doms and Dunne, 1998; Bils and Klenow, 2005). A leading explanation of this phenomenon is that firms face a fixed cost of adjusting.[1] In such an environment, firms will choose not to adjust for some time, with periodic discrete adjustments in response to sufficiently large shocks, consistent with the empirical "lumpiness" of microeconomic dynamics.

In this paper, we analyze the aggregate implications of this lumpiness at the microeconomic level. We do so in the context of a canonical model of fixed adjustment costs in the presence of idiosyncratic shocks that has been used widely in prior literature. For concreteness, we focus on the case of employment adjustment, although the model can be applied equally to investment and price dynamics.

We establish two key results that form the basis of the paper. First, we provide a novel analytical characterization of the aggregate steady state and dynamic outcomes implied by the model, which are shown in general to take on a partial-adjustment representation. Second, analytical approximations to model outcomes reveal that both aggregate steady-state outcomes and aggregate dynamics are neutral with respect to a small fixed adjustment cost.

The remainder of the paper proceeds as follows. In section 1, we describe the basic ingredients of the model. Firms face shocks to labor productivity that induce changes in their desired level of employment. Firms are subject to both aggregate and idiosyncratic shocks. Aggregate shocks drive macroeconomic expansions and recessions; idiosyncratic shocks drive heterogeneity in employment dynamics across firms. Due to the presence of a fixed adjustment cost, however, firms' employment will not adjust in response to all shocks. Instead, employment evolves according to an *Ss* policy at the microeconomic level, remaining constant for intervals of time with occasional jumps to a new level.

Given this environment, Section 2 develops the first of the main results of the paper. Specifically, it takes on the task of aggregating the lumpy microeconomic behavior identified in section 1 up to the macroeconomic level. These aggregate implications are not obvious. Since individual firms follow highly nonlinear *Ss* labor demand policies, and face heterogeneous idiosyncratic productivities, there is no representative firm interpretation of the model.

[1] Of course, fixed adjustment costs are not the only explanation of the observed inaction in microeconomic adjustment. An alternative possibility is that adjustment involves discrete *marginal* costs, such as in the kinked adjustment cost case. See, for example, Bertola and Caballero (1994).

We show how it is possible to infer the dynamics of aggregate employment by solving for the dynamics of a related object, namely the cross-sectional distribution of employment across firms. By applying a simple mass-balance approach, we provide an analytical characterization of the distribution dynamics of employment. We show that these display partial-adjustment type dynamics, continually evolving toward a steady state that varies over time with aggregate shocks. Interestingly, the rate of convergence of this process has the intuitive property of being equal to the probability that a firm adjusts away from its current level of employment. These dynamics of the distribution of employment across firms in turn shape the evolution of aggregate employment, since the latter is simply the mean of that distribution.

Section 3 uses this characterization of aggregate employment dynamics to develop the second main result of the paper—approximate aggregate neutrality. In particular, we use the general results of section 2 to inform analytical approximations to model outcomes in the presence of a small fixed adjustment cost. In the neighborhood of a small adjustment cost, we show that both the steady-state and the dynamics of the firm-size distribution coincide with their frictionless counterparts. It follows that the same approximate neutrality extends to the behavior of aggregate employment in general. Importantly, this result arises despite the existence of substantial inaction in microeconomic adjustment that arises even in the presence of small adjustment costs (for the same reasons noted by Akerlof and Yellen, 1985).

This approximate neutrality result can be traced to a symmetry property that emerges in the distributional dynamics of employment as the adjustment friction becomes small. The mass-balance approach of section 2 makes the intuition for this symmetry particularly transparent. Specifically, the change over time in the mass of firms at a given level of employment can be decomposed into an inflow of firms that adjusts to that level, less an outflow of firms that adjust away from that level of employment. The key is that a fixed adjustment cost reduces both of these flows—fewer firms will adjust away from a given employment level, but in addition fewer firms will find it optimal to adjust to that employment level. For small frictions, these two forces are symmetric, and therefore cancel, leaving the evolution of the distribution of employment approximately equal to its frictionless counterpart.

In section 4 of the paper, we confirm these analytical results in a series of quantitative illustrations. We first calibrate the model using estimates from recent literature on employment adjustment and firm productivity (Bloom, 2009; Cooper, Haltiwanger and Willis, 2005, 2007; Foster, Haltiwanger and Syverson, 2008). We find that this parameterization of the model implies aggregate employment dynamics that are very close to their frictionless analogue, in line with the approximate-neutrality result in section 3. Since there remains a lack of consensus over some of the parameters of the model, however, we also explore the sensitivity of this baseline result to alternative parameterizations. Interestingly, we find that aggregate employment can display more persistent, hump-shaped dynamics in response to aggregate shocks in the case where the adjustment cost is larger relative to the variance of innovations to idiosyncratic productivity, consistent with the general result of section 2.

In the concluding sections of the paper, we discuss how our results dovetail with the large literature on adjustment costs. We highlight an interesting feature of our approximate aggregate invariance result, namely that it does not rely on general equilibrium adjustment of wages. This contrasts with Kahn and Thomas' (2008) recent influential work on investment adjustment costs, who emphasize these general equilibrium forces. In addition, we also revisit two key papers in the recent literature on dynamic labor demand. In simulations of a model without idiosyncratic shocks, King and Thomas (2006) find that the response of aggregate employment to aggregate productivity is persistent, displaying a slight hump-shape. Bachmann (2009) observes a similar result in a model with productive heterogeneity, but with a much lower average adjustment rate. Interestingly, this accords well with our finding that implied aggregate dynamics become more persistent the smaller is idiosyncratic volatility relative to the adjustment friction. These results highlight the importance of obtaining robust estimates of magnitude of adjustment costs and the degree of idiosyncratic risk faced by firms in future empirical work in order to infer the role of lumpy microeconomic adjustment in aggregate employment dynamics.

1 The Firm's Problem

We consider a canonical model of fixed employment adjustment costs. Time is discrete. Firms use labor, n, to produce output according to the production function, $y = pxF(n)$, where p represents the state of aggregate labor demand, x represents shocks that are idiosyncratic to an individual firm, and the function F is increasing and concave, $F_n > 0$ and $F_{nn} < 0$. We assume that the evolution of idiosyncratic shocks is described by the distribution function $G(x'|x)$.

At the beginning of a period, firms observe the realization of their idiosyncratic shocks x, as well as aggregate productivity p. Given this, they then make their employment decision. If the firm chooses to adjust the size of its workforce, it incurs a fixed adjustment cost, denoted C.

It follows that we can characterize the expected present discounted value of a firm's profits recursively as:[2]

$$\Pi\left(n_{-1},x;\Omega\right)\equiv\max_{n}\left\{pxF\left(n\right)-wn-C\mathbf{1}^{\Delta}+\beta\mathbb{E}\left[\Pi\left(n,x';\Omega'\right)|x,\Omega\right]\right\},\tag{1}$$

where $\mathbf{1}^{\Delta}\equiv\mathbf{1}\left[n\neq\tilde{n}_{-1}\right]$ is an indicator that equals one if the firm adjusts and zero otherwise. The wage w is determined in a competitive labor market, and is taken as exogenous from the firm's perspective.[3] The variable Ω summarizes the aggregate state of the economy, including the aggregate shock p, the wage w, and all variables that are informative with respect to their future evolution.

For the analysis that follows, it is helpful to recast the firm's problem in equation (19) into two related underlying Bellman equations. In particular, the value of adjusting (gross of the adjustment cost), $\Pi^{\Delta}(x;\Omega)$, and the value of not adjusting, $\Pi^{0}\left(n_{-1},x;\Omega\right)$, are given by

$$\begin{aligned}\Pi^{\Delta}\left(x;\Omega\right)&\equiv\max_{n}\left\{pxF\left(n\right)-wn+\beta\mathbb{E}\left[\Pi\left(n,x';\Omega'\right)|x,\Omega\right]\right\},\text{ and}&(2)\\ \Pi^{0}\left(n_{-1},x;\Omega\right)&\equiv pxF\left(n_{-1}\right)-wn_{-1}+\beta\mathbb{E}\left[\Pi\left(n_{-1},x';\Omega'\right)|x,\Omega\right].&(3)\end{aligned}$$

Clearly, the value of the firm $\Pi\left(n_{-1},x;\Omega\right)$ is simply the upper envelope of these two regimes,

$$\Pi\left(n_{-1},x;\Omega\right)=\max\left\{\Pi^{\Delta}\left(x;\Omega\right)-C,\Pi^{0}\left(n_{-1},x;\Omega\right)\right\}.\tag{4}$$

In keeping with the literature on fixed adjustment costs, we assume that the optimal labor demand policy takes an Ss form.[4] Figure 1 illustrates such a policy. It is characterized by three functions, $L(n;\Omega)<X(n;\Omega)<U(n;\Omega)$. In the event that the firm chooses to adjust away from n_{-1}, optimal employment is determined by a "reset" function $X(n;\Omega)$

[2] We adopt the convention of denoting lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

[3] The law of one wage can be supported by assuming that workers are perfectly mobile (and thus may seek new job opportunities at any point in time). One might wonder whether a worker is able to hold up a firm and demand a higher wage, since her departure would appear to force the firm to pay an adjustment cost. This strategy is feasible only if the present discounted value of work among firms that are hiring exceeds that at her present firm. We will see in equation (5), however, that firms hire until the value of the marginal worker is zero. Thus, marginal hires yield no rents. It follows that all workers at all firms are paid just enough to make them indifferent between work and non-work. Moreover, if firms cannot commit, this contract must be implemented as a period-by-period, economy-wide wage.

[4] It is well-known that it is difficult to prove the optimality of the Ss policy in settings outside the canonical Brownian model (Harrison, Sellke and Taylor, 1983). However, Lemma 2 in the Appendix shows that, in the neighborhood of a small adjustment cost, the optimal labor demand policy is well-approximated by its myopic counterpart. A useful implication of this result is that, since we know the optimal myopic policy takes the Ss form, we also know that the Ss policy is approximately optimal for the context of many of the results in this paper.

which satisfies the first-order condition

$$pX(n;\Omega)F_n(n) - w + \beta\mathbb{E}\left[\Pi_n(n,x',\Omega')\,|x = X(n),\Omega\right] \equiv 0. \tag{5}$$

Due to the adjustment cost, however, the firm will not always choose to adjust: It will decide to adjust only if the value of adjusting, net of the adjustment cost, $\Pi^{\Delta}(x;\Omega) - C$, exceeds the value of not adjusting, $\Pi^{0}(n_{-1},x;\Omega)$. This aspect of the firm's decision rule is characterized by two adjustment "triggers," $L(n_{-1};\Omega)$ and $U(n_{-1};\Omega)$. For sufficiently bad realizations of the idiosyncratic shock, $x < L(n_{-1};\Omega)$, the firm will shed workers; for sufficiently good shocks, $x > U(n_{-1};\Omega)$, it will hire workers. For intermediate values of $x \in [L(n_{-1};\Omega), U(n_{-1};\Omega)]$, the firm will neither hire nor fire, and $n = n_{-1}$. Thus, the adjustment triggers trace out the locus of points for which the firm is indifferent between adjusting and not adjusting. It follows that the triggers therefore satisfy the value-matching conditions

$$\begin{aligned} \Pi^{\Delta}(L(n_{-1};\Omega);\Omega) - C &= \Pi^{0}(n_{-1},L(n_{-1};\Omega);\Omega)\,,\text{ and} \\ \Pi^{\Delta}(U(n_{-1};\Omega);\Omega) - C &= \Pi^{0}(n_{-1},U(n_{-1};\Omega);\Omega)\,. \end{aligned} \tag{6}$$

Firms' optimal labor demand policies clearly depend on the aggregate state Ω. For example, positive shocks to p will cause the *Ss* policy in Figure 1 to shift downward: For any given level of idiosyncratic productivity, a firm will be less likely to fire, and more likely to hire in an aggregate expansion. For notational simplicity, however, we suppress the dependence of the optimal labor demand policy on Ω in what follows.

2 An Aggregation Theorem

In this section, we describe the first key result of the paper, which develops an analytical approach that allows one to aggregate microeconomic behavior to the macroeconomic level when firms face a fixed cost of adjusting the size of their workforces. Aggregation in this context non-trivial. As we have seen, under the optimal *Ss* policy, each individual firm's labor demand depends in a highly nonlinear fashion on both their individual lagged employment n_{-1}, as well the realization of their idiosyncratic shock x. Since firms are heterogeneous in these state variables, it follows that there is no simple representative firm interpretion of the model.

In order to infer the implications of lumpy microeconomic adjustment for aggregate employment, we first obtain a characterization of a related object—the cross-sectional distribution of employment across firms. We denote the density of the latter by $h(n)$, and its associated distribution function by $H(n)$. This aids aggregation since the mean of $h(n)$ reflects aggregate employment, which in turn allows one to infer aggregate labor demand, and thereby aggregate labor market equilibrium.

We construct this distribution by applying a mass-balance approach. The change in the mass of firms with employment below some level n is simply equal to the inflow into the mass $H(n)$ less the outflow from that mass.[5] The results of applying this logic are summarized in Proposition 1.

Proposition 1 *The density of employment across firms evolves according to the difference equation*

$$\Delta h(n) = -\left(1 - \mathcal{G}\left[U(n)|n\right] + \mathcal{G}\left[L(n)|n\right]\right)\left[h_{-1}(n) - \bar{h}(n)\right], \tag{7}$$

where $\bar{h}(n)$ is the associated steady-state density given by

$$\bar{h}(n) = h^*(n)\frac{1 - \mathcal{H}\left[L^{-1}X(n)|X(n)\right] + \mathcal{H}\left[U^{-1}X(n)|X(n)\right]}{1 - \mathcal{G}\left[U(n)|n\right] + \mathcal{G}\left[L(n)|n\right]}, \tag{8}$$

$h^*(n)$ is the frictionless density of employment, $\mathcal{G}(\xi|\nu) \equiv \Pr\left[x \leq \xi | n_{-1} = \nu\right]$, and $\mathcal{H}(\nu|\xi) \equiv \Pr\left[n_{-1} \leq \nu | x = \xi\right]$.

Proposition 1 reveals that the dynamics of the density of employment across firms take on a partial-adjustment representation. The density $h(n)$ is continually evolving toward a (potentially time-varying) steady-state density $\bar{h}(n)$. The latter reflects the distribution of employment that would be attained if the aggregate state p remained at its current level forever. Intuitively, aggregate shocks shift firms' optimal labor demand policy function in Figure 1, which in turn shifts the steady-state density $\bar{h}(n)$ that the labor market converges to. In what follows, we show that each of the components of Proposition 1 admits a very intuitive economic interpretation.

We begin by describing the form of the steady-state density in equation (8). It is instructive to consider first the special case in which the idiosyncratic shocks faced by the firm are i.i.d. across time. In that case, a firm's lagged employment n_{-1} provides no information on the distribution of its current productivity x, and vice versa. It follows that the steady-state

[5] While the use of mass balance is a relatively standard technique in many domains of economics, the application to the fixed adjustment cost model is non-trivial and, to the best of our knowledge, new to the literature.

density of employment in equation (8) takes the simpler form

$$\bar{h}(n) = h^*(n) \frac{1 - H[L^{-1}X(n)] + H[U^{-1}X(n)]}{1 - G[U(n)] + G[L(n)]}. \tag{9}$$

Thus, the density of employment $h(n)$ is defined recursively in this special case—it depends upon the associated distribution function of employment $H(\cdot)$ evaluated at two different points in its domain in the form of a difference-differential equation.

The special case in which idiosyncratic shocks are i.i.d. also clarifies the intuition behind equation (8). The result states that the steady-state density of employment $h(n)$ is proportional to its frictionless counterpart $h^*(n)$. To understand the factor of proportionality, consider first the denominator in equation (9), $1 - G[U(n)] + G[L(n)]$. Inspection of the illustration of the policy function depicted in Figure 1 reveals that this is simply the probability that a firm with employment level n adjusts away from n.

The numerator of equation (9) may also be interpreted in a similar fashion. Consider a firm that draws an idiosyncratic productivity level $x = X(n)$. Absent an adjustment cost, such a firm would adjust to an employment level of n. In the presence of an adjustment cost, however, Figure 1 reveals that the firm's decision to adjust or not will depend on the level of employment the firm has inherited from the past. Firms whose initial employment is either relatively low ($n_{-1} < U^{-1}X(n)$) or relatively high ($n_{-1} > L^{-1}X(n)$) will adjust to n. It follows that the numerator of equation (9) is simply the probability that a firm with productivity $x = X(n)$ adjusts to n.

Returning to the general result reported in equation (8), it is clear that the same interpretation extends to the case with persistent idiosyncratic shocks. To summarize, then, equation (8) may be restated simply as

$$\bar{h}(n) = h^*(n) \frac{\Pr(\text{adjust to } n)}{\Pr(\text{adjust from } n)}. \tag{10}$$

This is a very intuitive result: It simply implies that, if a firm is more likely to adjust to n than it is to adjust away from n, then the distribution of employment will accumulate more mass at n.

These observations in turn provide an intuitive interpretation of the transition dynamics out of steady state in equation (7). As we have noted already, these dynamics take a partial-adjustment-type form. In the light of the discussion above, however, we can interpret the rate at which the density converges to steady state in equation (7), $1 - \mathcal{G}[U(n)|n] + \mathcal{G}[L(n)|n]$, simply as the probability that a firm adjusts away from an employment level of n. The intuitive result that emerges, then, is that the response of the density of employment to aggregate shocks will be more persistent in the presence of fixed adjustment costs relative to its frictionless counterpart, and the degree of this persistence is mediated through the magnitude of a firm's probability of adjusting.

Proposition 1 also aids aggregation of the model, since it provides a clear link from microeconomic behavior in the model to the aggregate outcomes implied by that behavior. Specifically, once one knows the optimal labor demand policy used by individual firms, as summarized by the functions $L(n) < X(n) < U(n)$, the evolution of the distribution of employment in the aggregate is determined according to equations (7) and (8). In turn, this allows one to trace out the evolution of the aggregate demand for labor, which is implied by the mean of $h(n)$. In particular, the optimal labor demand policy analyzed in section 1 is defined implicitly for a given level of the market wage w. Thus, the solution for the steady-state distribution of employment in (8) also is indexed implicitly by w. Making that dependence explicit, one can express the steady-state aggregate demand for labor as

$$N^d(w) = \int n\bar{h}(n; w)\, dn. \tag{11}$$

Thus, when combined with a model of labor supply, our aggregation result allows one to determine aggregate steady-state equilibrium employment and wages.

[Discussion of aggregate dynamics and dynamic aggregate equilibrium.]

3 Approximate Aggregate Neutrality

The previous section provided an analytical characterization of the aggregate dynamics implied by a quite general form of lumpy microeconomic adjustment. In this section, we use these general results to inform analytical approximations to model outcomes in the presence of a small fixed adjustment cost. Such a case is particularly instructive because it is well-known that even small adjustment frictions imply substantial inaction, and hence lumpiness, in microeconomic adjustment (see, for example, Akerlof and Yellen, 1985, and Mankiw, 1985). Lemma 1 in the Appendix reiterates this point by showing that the adjustment triggers can be approximated by

$$L(n) \approx X(n) - \gamma(n)\sqrt{C}, \text{ and } U(n) \approx X(n) + \gamma(n)\sqrt{C} \tag{12}$$

for a given $\gamma(n) > 0$. These analytical approximations give rise to the second key result of the paper:

Proposition 2 *To a first-order approximation around $C = 0$, the evolution of the distribution of employment across firms is given by*

$$\Delta h(n)' \approx -[h(n) - h^*(n)], \tag{13}$$

which is the frictionless law of motion.

The approximation result in Proposition 2 implies that both the steady-state and the dynamics of the distribution of employment across firms are second order in the adjustment friction. In the neighborhood of a small adjustment cost, the steady-state firm-size distribution coincides with its frictionless counterpart, $\bar{h}(n) \approx h^*(n)$, and the dynamics of $h(n)$ are approximately jump. As a result, any gap between the distribution of employment and its frictionless counterpart is closed immediately—the aggregate dynamics of the model coincide with frictionless dynamics. Put another way, aggregate outcomes are approximately *neutral* with respect to a small positive adjustment cost.

Such as result is surprising in the light of the partial-adjustment representation of aggregate dynamics in Proposition 1. We show that the invariance of both steady-state and dynamic outcomes can be traced to a symmetry property in the distributional dynamics of $h(n)$.

Consider first the steady-state density of employment across firms. To see intuitively how this symmetry plays out, the special case in which idiosyncratic shocks are i.i.d. over time in equation (9) is again useful. This reveals that the distribution of employment will coincide with its frictionless counterpart, $H(n) = H^*(n) = G[X(n)]$, if the adjustment triggers satisfy the symmetry property: $L^{-1}X(n) = X^{-1}U(n)$ and $U^{-1}X(n) = X^{-1}L(n)$. Intuitively, under this symmetry condition, the probability of adjust to a given level of employment in equation (10) is offset exactly by an identical probability of adjusting away from that employment level.

Figure 2 illustrates how this symmetry property arises in the presence of a small adjustment cost. From equation (12), we know that the adjustment triggers $U(n)$ and $L(n)$ are approximately symmetric around the reset function $X(n)$ in the presence of a small adjustment cost. It seems intuitive from Figure 2 that, by inverting these approximations, the required symmetry for approximate steady-state invariance will hold. The proof of Proposition 2 formalizes this intuition, and shows how the same logic extends to the case with persistent idiosyncratic shocks.

Proposition 2 also suggests that this approximate neutrality extends to the out-of-steady-state dynamics of the density of employment $h(n)$. To understand this, it is helpful to rewrite the law of motion for $h(n)$ in equation (7) more directly in terms of its constituent flows as

$$\Delta h(n)' = \Pr(\text{adjust to } n)\, h^*(n) - \Pr(\text{adjust from } n)\, h(n). \tag{14}$$

Intuitively, the inflow into the density of employment at some level n originates from firms that have a) received an idiosyncratic shock that leads to a desired employment level of n, and b) inherited an employment level sufficiently different from n such that it is optimal to pay the fixed cost and adjust. Thus, the inflow is equal to the density of firms that would choose an employment level of n absent the adjustment cost, $h^*(n)$, times the probability that they will in fact adjust. Likewise, the outflow from the density of employment at n is simply the share of firms at that level of employment that receives a sufficiently large idiosyncratic shock to adjust away from n. Clearly, in steady state, $\Delta h(n)' = 0$, and equation (14) yields the steady-state outcome in equation (10).

To see how this alternative representation informs the approximate neutrality of the dynamics of $h(n)$, imagine a small fixed adjustment cost is introduced into an otherwise frictionless environment. At any instant of time, the adjustment cost reduces the outflow of mass from any given employment level n, but also reduces the mass of firms which find it optimal to adjust to that level of employment. For small frictions, we show that these two forces are symmetric and therefore offset each other almost exactly, leaving the distribution approximately equal to its frictionless counterpart along the transition path.

4 Quantitative Analysis

The preceding sections have highlighted two results on the aggregate outcomes of lumpy microeconomic adjustment. In section 2, we provided an analytical characterization of aggregate dynamics implied by sS employment policies at the firm level. In particular, Proposition 1 revealed that these dynamics take a partial-adjustment form, continually evolving toward a time-varying flow steady state. Section 3 used these analytical results to derive approximations to these outcomes in the presence of a small adjustment cost. Proposition 2 revealed that an approximate neutrality result emerges, with aggregate dynamics coinciding with their frictionless counterparts, despite substantial microeconomic inaction. In this section, we illustrate these results by calibrating the model to estimates of its parameters from recent literature.

4.1 Calibration

The baseline calibration we analyze is summarized in Table 1. The numerical model is cast at a quarterly frequency. We adopt the widespread assumption that the production function takes the Cobb-Douglas form, $F(n) = n^{\alpha}$, with $\alpha < 1$. The returns to scale parameter α is set equal to 0.64 based on estimates reported in Cooper, Haltiwanger and Willis (2005, 2007). This also is similar to the value assumed by King and Thomas (2006) in their analysis of a related fixed-cost model. The discount factor β is set to 0.99, which is the conventional choice for a quarterly model.

The magnitude of the adjustment cost is based on estimates reported in Cooper, Haltiwanger and Willis (2005, 2007) and Bloom (2009). Cooper, Haltiwanger and Willis (2005) estimate a model similar to the one described above using plant-level data from the Census' Longitudinal Research Database. They estimate a fixed cost of adjustment equal to approximately 8 percent of quarterly revenue (see row "Disrupt"in their Table 3A). Using Compustat data, Bloom (2009) finds nearly the same result (see column "All" in his Table 3). Based on this, we calibrate the adjustment cost parameter C to replicate these estimates.

Idiosyncratic and aggregate shocks are assumed respectively to evolve according to the common assumption of geometric AR(1) processes,

$$\log x' = \mu_x + \rho_x \log x + \varepsilon'_x, \text{ and} \tag{15}$$

$$\log p' = \mu_p + \rho_p \log p + \varepsilon'_p, \tag{16}$$

where the innovations are independent normal random variables: $\varepsilon'_x \sim N(0, \sigma_x^2)$, and $\varepsilon'_p \sim N\left(0, \sigma_p^2\right)$. There is less consensus over the parameters of the process of idiosyncratic shocks in equation (15). Cooper, Haltiwanger and Willis (2005) obtain estimates of $\rho_x = 0.39$ and $\sigma_x = 0.5$. In a later paper, Cooper, Haltiwanger and Willis (2007) estimate these parameters within the context of a search-and-matching model using, in part, monthly establishment-level data from the Job Openings and Labor Turnover Survey. These yield estimates of ρ_x that are much smaller than 0.39, and estimates of σ_x nearer to 0.2.[6] Foster, Haltiwanger, and Syverson (2008) report estimates that imply a quarterly persistence rate of $\rho_x \approx 0.95$. We split the difference between these different estimates by setting $\rho_x = 0.7$ and $\sigma_x = 0.35$ for the purposes of the baseline calibration, and examine the sensitivity of the results to alternative calibrations. The baseline calibration is comparable to Bachmann's (2009) analysis of non-

[6] The typical estimate of ρ_x in Cooper, Haltiwanger and Willis (2007) is roughly 0.4 at a monthly frequency. The implied degree of quarterly persistence is therefore $0.4^3 = 0.064$. Likewise, their estimates of the variance of the monthly innovation is in the neighborhood $\sigma_x^2 \approx 0.2^2 = 0.04$. Its counterpart in a quarterly model is therefore $\sqrt{(0.4^4 + 0.4^2 + 1)\, 0.04} \approx 0.22$.

convex adjustment costs.

The parameters of the process of aggregate shocks, ρ_p and σ_p, are calibrated so that the model approximately replicates the persistence and volatility of (de-trended) log aggregate employment. Using time series data on private payroll employment and detrending using the HP filter, we compute an autocorrelation coefficient of 0.96 and a standard deviation of 0.026. Values of $\rho_p = 0.95$ and $\sigma_p = 0.015$ are roughly consistent with these moments (see Table 1). We do this because our goal is not to explain the volatility of aggregate employment, but to compare model outcomes within an environment that is economically relevant. One way of doing that is to generate aggregate outcomes that are comparable to what we observe in the data.

4.2 Labor Supply

In order to solve for the aggregate dynamics and steady state of the model, one must specify the supply side of the market. We consider two assumptions on labor supply. The first is that the elasticity of labor supply is equal to unity. This value is similar to what is implied by Chang and Kim's (2006) structural model of indivisible labor supply, and is also the value advocated by Kimball and Shapiro (2010) based on an analysis of survey evidence.

In addition, however, we also explore the implications of the model under the assumption that labor supply is perfectly elastic, so that the wage may be treated as fixed (and given by its value in Table 1).[7] This case is instructive, as is shuts off any equilibrium adjustment in wages, and thereby allows us to isolate the features of aggregate dynamics solely due to the equilibration of the distribution of employment across firms, $h(n)$, that we emphasize.

Equilibrium wage adjustment We introduce an upward-sloped aggregate labor supply schedule of the simple loglinear form

$$N^s(w) = \psi w^\eta. \tag{17}$$

As noted above, we impose that the labor supply elasticity is one, so $\eta = 1$. The intercept, ψ, is set so that mean equilibrium employment remains near 20, roughly consistent with data from County Business Patterns. Appendix B describes how this labor supply schedule may be derived from the problem of a large household that must allocate its members across

[7] To clarify: we parameterize the process (16) so that it is consistent with the empirical variation in aggregate log employment, conditional on a labor supply elasticity of one. We do not re-calibrate the model when the elasticity is set to infinity. This allows us to compare impulse responses given a fixed stochastic process for p.

market and non-market work in a setup similar to that used in Mulligan (2001). Members of the household obtain different levels of disutility from market work. All members of the household whose disutility falls below a threshold participate in the labor market.[8] The positive slope in (17) reflects the marginal disutility borne by the household when it deploys to the labor market another of its members, who faces a higher disutility from market work on the margin.

It is well-known that, in the presence of (17), individual firms face a difficult prediction problem: Because employment is quasi-fixed, firms must forecast the future path of aggregate wages when they decide on the size of their current workforce. Yet the wage depends on aggregate employment, $N \equiv \int nh\,(n)\,dn$, which in turn depends on the distribution of employment across firms, an infinite-dimensional object.

We adopt Krusell and Smith's (1998) bounded rationality algorithm to solve this problem. Specifically, we assume that firms forecast log aggregate employment using its lag, and the current level of log aggregate productivity,

$$\log N = \theta_0 + \theta_N \log N_{-1} + \theta_p \log p. \tag{18}$$

Using their forecast of aggregate employment implied by equation (18), firms can then forecast future wages.

Figure 3 presents the impulse response of aggregate employment implied by the baseline calibration of the model, and compares it with its frictionless counterpart. Consistent with the result of Proposition 2, the differences between the impulse responses are very small—the adjustment cost does not affect greatly the response of aggregate employment to aggregate shocks. It follows that the forecast equation (18) is very accurate—estimating the equation on model-generated data yields an R^2 in excess of 0.9999. By the same token, the estimated coefficients are very close to what would be expected from the frictionless model, given the calibration of the labor supply elasticity.[9]

[Sensitivity to alternative parameterizations.]

Fixed wages The second case we examine assumes that the elasticity of labor supply is infinite, so that the wage w is effectively fixed. In doing so, we suppress any equilibrium

[8] To ease computational burden, we assume that members' utility is linear in consumption. In that case, the marginal utility of wealth is fixed at one (and hence absent from (17)), and there is only one price we have to track. This restriction may be relaxed, though it is not clear to us why it would materially affect the results.

[9] Specifically, we get $\hat{\theta}_0 = 2.97$, $\hat{\theta}_p = 0.747$, and $\hat{\theta}_N = 0.008$. In the frictionless model, the elasticity with respect to aggregate productivity is 0.735.

adjustment of wages along the transition path. Examining the aggregate dynamics of the model in this environment allows us to isolate features of these dynamics that can be traced to the equilibration of the distribution of employment from those driven by the adjustment of wages in general equilibrium.

With fixed wages, the numerical model is much simpler to solve. In particular, it is no longer necessary to apply the Krusell-Smith bounded rationality algorithm in this context, since the wage no longer needs to be forecasted. Instead, all that needs to be done is to solve for the optimal labor demand policy functions. Since the equilibrium wage is fixed, the aggregate state Ω is summarized completely by aggregate productivity, and the optimal policy functions take the simple form $L(n;p)$, $X(n;p)$, and $U(n;p)$. As we noted in section 1, a positive innovation to aggregate productivity p shifts these functions downward—for a given level of idiosyncratic productivity, a firm is more likely to hire, less likely to fire, and will select a higher level of employment conditional on adjustment. Thus, the evolution of aggregate productivity p induces shifts in the policy function, which in turn traces out the evolution of the distribution of employment and thereby aggregate employment.

The results of this exercise are illustrated in Figure 3. As in the case with general equilibrium adjustment of wages, the dynamic response of aggregate employment in the presence of adjustment costs lies very close to the frictionless response. Thus, the dynamic neutrality observed in Figure 3 appears to derive in large part from the neutrality of the dynamics of the distribution of employment, as opposed to the adjustment of wages.

[Sensitivity to alternative parameters.]

5 Summary and Discussion

This paper has analyzed the aggregate implications of a canonical model of fixed employment adjustment costs. It has established two main results. First, the dynamics of aggregate employment in the presence of an adjustment friction can be inferred simply and intuitively by characterizing the evolution of the distribution of employment across firms. We show that the latter displays partial-adjustment type dynamics which are in turn inherited by law of motion for aggregate employment.

The second main result of the paper is to show that, for a sufficiently small friction, aggregate employment dynamics coincide with their frictionless counterpart. The intuition for this result arises from a form of symmetry in the dynamics of the firm-size distribution that emerges as the adjustment cost becomes small. In the neighborhood of a small adjustment cost, we show that the probability that a firm adjusts to a given employment level is offset by an equal probability that a firm adjusts away from that level, so that the effect on the cross-sectional distribution of employment is negligible.

In set of quantitative illustrations, we show that recent estimates of the parameters of the model tend to imply aggregate dynamics that are close to frictionless outcomes, in line with the approximate neutrality result of section 3. However, the quantitative analysis also illustrates the circumstances under which aggregate employment will be more persistent. In line with our general analytical characterization of macroeconomic dynamics, aggregate employment displays more persistent, hump-shaped dynamics when the fixed adjustment cost is large relative to the magnitude of idiosyncratic shocks facing firms.

Interestingly, these conclusions dovetail with the large literature on adjustment costs. Within the literature on dynamic labor demand, there are two papers in particular that are related to ours. The first is King and Thomas (2006). They find that, in a model without idiosyncratic productivity shocks, the response of aggregate employment displays a slight hump shape, reminiscent of partial-adjustment type dynamics. A second paper of interest is Bachmann (2009), who also finds that a fixed adjustment cost model induces sluggish dynamics in aggregate employment. While his model allows for idiosyncratic risk comparable to that used in this paper, his calibration implies a comparatively low adjustment rate. Thus, the results of both of these papers are consistent with our finding that it is the magnitude of the adjustment cost relative to idiosyncratic risk that determines the aggregate effects of lumpy adjustment at the microeconomic level.

These results highlight a number of interesting avenues for future research. First, given the importance of the magnitude of adjustment costs and the degree of idiosyncratic risk faced by firms in shaping implied aggregate dynamics, it will be fruitful for future empirical work to focus on obtaining robust estimates of these two critical parameters. Second, to the extent that estimates of these parameters line up with the approximate aggregate neutrality we identify in section 3, it is worthwhile to consider other adjustment frictions that can simultaneously account for lumpy microeconomic adjsutment and persistent aggregate employment dynamics. For instance, both fixed and kinked (proportional) adjustment costs induce inaction at the microeconomic level, but may have very different implications for aggregate employment dynamics.

6 References

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A Appendix

For ease of exposition, section 1 presented a simple model of fixed adjustment costs. In this appendix, we describe a more elaborate version of the model that allows for exogenous worker attrition at a rate that may depend on the aggregate state. The latter generalization allows us to extend our results to factor adjustment models with depreciation (the case of a constant worker attrition rate) and to canonical menu costs models (which is isomorphic to a case with aggregate-state-dependent attrition). The proofs of Propositions 1 and 2 in what follows hold for this more general case.

Worker attrition is modeled as follows. At the same time as a firm observes the realization of its idiosyncratic shock x, an exogenous fraction $\delta(\Omega)$ of its workforce separates. It follows that we can characterize the expected present discounted value of a firm's profits recursively as:

$$\Pi(\tilde{n}_{-1}, x; \Omega) \equiv \max_{n} \left\{ pxF(n) - wn - C\mathbf{1}^{\Delta} + \beta\mathbb{E}\left[\Pi(\tilde{n}, x'; \Omega') | x, \Omega\right] \right\}, \tag{19}$$

where $\tilde{n}_{-1} \equiv (1 - \delta(\Omega)) n_{-1}$ denotes employment carried into the period, and all other variables are as defined in section 1. As in section 1 and the literature on fixed adjustment costs, we continue to assume that the optimal labor demand policy takes an Ss form. The"reset" function $X(n)$ satisfies the first-order condition

$$pX(n) F_n(n) - w + \beta(1 - \delta)\mathbb{E}\left[\Pi_1(\tilde{n}, x'; \Omega') | x = X(n), \Omega\right] \equiv 0, \tag{20}$$

and the adjustment triggers satisfy the value-matching conditions

$$\begin{aligned} \Pi^{\Delta}(L(\tilde{n}_{-1}; \Omega); \Omega) - C &= \Pi^{0}(\tilde{n}_{-1}, L(\tilde{n}_{-1}; \Omega); \Omega), \text{ and} \\ \Pi^{\Delta}(U(\tilde{n}_{-1}; \Omega); \Omega) - C &= \Pi^{0}(\tilde{n}_{-1}, U(\tilde{n}_{-1}; \Omega); \Omega). \end{aligned} \tag{21}$$

Proof of Proposition 1. We wish to derive the flows in and out of the mass of firms with employment below some number m, $H(m)$. Consider first the inflow into that mass—i.e. the mass of firms that cuts employment from above m to below m. To derive this flow, first fix a level of lagged employment, n_{-1} and denote the distribution of productivity conditional on lagged employment as $\mathcal{G}(\xi|\nu) \equiv \Pr[x \leq \xi | n_{-1} = \nu]$. We can use Figure 1 to identify four potential sets of inflows corresponding to the following cases:

1) If $m < X^{-1}L(\tilde{n}_{-1})$, so that $n_{-1} > \frac{L^{-1}X(m)}{1-\delta}$, the probability of reducing employment below m will be $\mathcal{G}[X(m) | n_{-1}]$.

2) If $m \in [X^{-1}L(\tilde{n}_{-1}), \tilde{n}_{-1}]$, so that $n_{-1} \in \left[\frac{m}{1-\delta}, \frac{L^{-1}X(m)}{1-\delta}\right]$, the probability of reducing employment below m will be $\mathcal{G}[L(\tilde{n}_{-1}) | n_{-1}]$.

3a) If $n_{-1} < X^{-1}U(\tilde{n}_{-1})$, and $m \in [\tilde{n}_{-1}, n_{-1}]$, so that $n_{-1} \in \left[m, \frac{m}{1-\delta}\right]$, the probability of reducing employment below m will be $\mathcal{G}[U(\tilde{n}_{-1}) | n_{-1}]$.

3b) If $n_{-1} > X^{-1}U(\tilde{n}_{-1})$, and $m \in [\tilde{n}_{-1}, X^{-1}U(\tilde{n}_{-1})]$, so that $n_{-1} \in \left[\frac{U^{-1}X(m)}{1-\delta}, \frac{m}{1-\delta}\right]$, the probability of reducing employment below m will be $\mathcal{G}[U(\tilde{n}_{-1}) | n_{-1}]$.

4b) If $n_{-1} > X^{-1}U(\tilde{n}_{-1})$, and $m \in [X^{-1}U(\tilde{n}_{-1}), n_{-1}]$, so that $n_{-1} \in \left[m, \frac{U^{-1}X(m)}{1-\delta}\right]$, the probability of reducing employment below m will be $\mathcal{G}[X(m) | n_{-1}]$.

It follows that the inflow is given by

$$\begin{aligned}\text{Inflow into } H(m) &= \int_{\frac{L^{-1}X(m)}{1-\delta}}^{\infty} \mathcal{G}[X(m)|n_{-1}]\,dH_{-1}(n_{-1}) + \int_{\frac{m}{1-\delta}}^{\frac{L^{-1}X(m)}{1-\delta}} \mathcal{G}[L(\tilde{n}_{-1})|n_{-1}]\,dH_{-1}(n_{-1}) \\ &\quad + \int_{\max\left\{m,\frac{U^{-1}X(m)}{1-\delta}\right\}}^{\frac{m}{1-\delta}} \mathcal{G}[U(\tilde{n}_{-1})|n_{-1}]\,dH_{-1}(n_{-1}) \\ &\quad + \int_{m}^{\max\left\{m,\frac{U^{-1}X(m)}{1-\delta}\right\}} \mathcal{G}[X(m)|n_{-1}]\,dH_{-1}(n_{-1}), \qquad (22)\end{aligned}$$

Following a similar logic, the outflow from the mass $H(m)$ is given by the mass of firms that raises employment from below m to above m. Similar to above, we can use Figure 1 to identify two potential sets of outflows from $H(m)$:

4a) If $n_{-1} < X^{-1}U(\tilde{n}_{-1})$, and $m \in [n_{-1}, X^{-1}U(\tilde{n}_{-1})]$, so that $n_{-1} \in \left[\frac{U^{-1}X(m)}{1-\delta}, m\right]$, the probability of increasing employment above m will be $1 - \mathcal{G}[U(\tilde{n}_{-1})|n_{-1}]$.

5a) If $n_{-1} < X^{-1}U(\tilde{n}_{-1})$, and $m > X^{-1}U(\tilde{n}_{-1})$, so that $n_{-1} < \frac{U^{-1}X(m)}{1-\delta}$, the probability of increasing employment above m will be $1 - \mathcal{G}[X(m)|n_{-1}]$.

5b) If $n_{-1} > X^{-1}U(\tilde{n}_{-1})$, and $m > n_{-1}$, so that $n_{-1} < m$, the probability of increasing employment above m will be $1 - \mathcal{G}[X(m)|n_{-1}]$.

It follows that the outflow is given by

$$\begin{aligned}\text{Outflow from } H(m) &= \int_{\min\left\{m,\frac{U^{-1}X(m)}{1-\delta}\right\}}^{m} \left(1 - \mathcal{G}[U(\tilde{n}_{-1})|n_{-1}]\right) dH_{-1}(n_{-1}) \\ &\quad + \int_{0}^{\min\left\{m,\frac{U^{-1}X(m)}{1-\delta}\right\}} \left(1 - \mathcal{G}[X(m)|n_{-1}]\right) dH_{-1}(n_{-1}). \qquad (23)\end{aligned}$$

The mass of firms with employment below some level n this period is equal to the mass below n in the previous period plus inflows into the mass less outflows from the mass. Thus, using equations (22) and (23) we can express the evolution of the distribution function $H(n)$ as

$$\begin{aligned}\Delta H(m) &= G[X(m)] - H_{-1}(m) - \int_{\frac{U^{-1}X(m)}{1-\delta}}^{\frac{L^{-1}X(m)}{1-\delta}} \mathcal{G}[X(m)|n_{-1}]\,dH_{-1}(n_{-1}) \qquad (24) \\ &\quad + \int_{\frac{m}{1-\delta}}^{\frac{L^{-1}X(m)}{1-\delta}} \mathcal{G}[L(\tilde{n}_{-1})|n_{-1}]\,dH_{-1}(n_{-1}) + \int_{\frac{U^{-1}X(m)}{1-\delta}}^{\frac{m}{1-\delta}} \mathcal{G}[U(\tilde{n}_{-1})|n_{-1}]\,dH_{-1}(n_{-1}).\end{aligned}$$

Differentiating, denoting the frictionless density of employment as $h^*(m) \equiv g[X(m)]X'(m)$, using Bayes' rule to write the distribution of lagged employment conditional on current productivity as $\mathcal{H}(\nu|\xi) \equiv \Pr[n_{-1} \leq \nu | x = \xi] = \int_0^{\nu} \frac{\mathcal{G}'(\xi|\tilde{\nu})}{g(\xi)} dH_{-1}(\tilde{\nu})$, and defining $\tilde{m} \equiv (1-\delta)m$

yields the stated result,

$$\Delta h\left(\tilde{m}\right)=-\left[h_{-1}\left(\tilde{m}\right)-\bar{h}\left(\tilde{m}\right)\right]+\frac{1}{1-\delta}\left(\mathcal{G}\left[U\left(\tilde{m}\right)|m\right]-\mathcal{G}\left[L\left(\tilde{m}\right)|m\right]\right)\left[h_{-1}\left(m\right)-\bar{h}\left(m\right)\right], \tag{25}$$

where the steady-state density satisfies the recursion

$$\begin{aligned}\bar{h}\left(\tilde{m}\right) &= \left(1-\mathcal{H}\left[\frac{L^{-1}X\left(\tilde{m}\right)}{1-\delta}|X\left(\tilde{m}\right)\right]+\mathcal{H}\left[\frac{U^{-1}X\left(\tilde{m}\right)}{1-\delta}|X\left(\tilde{m}\right)\right]\right)h^{*}\left(\tilde{m}\right)\\ &\quad+\frac{1}{1-\delta}\left(\mathcal{G}\left[U\left(\tilde{m}\right)|m\right]-\mathcal{G}\left[L\left(\tilde{m}\right)|m\right]\right)\bar{h}\left(m\right).\end{aligned} \tag{26}$$

Setting $\delta=0$ yields the result stated in Proposition 1. ■

Lemma 1 *In the presence of a small fixed adjustment cost, the adjustment triggers and their inverses are approximately equal to*

$$L\left(\tilde{n}\right) \approx X\left(\tilde{n}\right)-\gamma\left(\tilde{n}\right)\sqrt{C}, \qquad U\left(\tilde{n}\right)\approx X\left(\tilde{n}\right)+\gamma\left(\tilde{n}\right)\sqrt{C}, \text{ and} \tag{27}$$

$$L^{-1}\left(x\right) \approx X^{-1}\left(x\right)+\bar{\gamma}\left(x\right)\sqrt{C},\ U^{-1}\left(x\right)\approx X^{-1}\left(x\right)-\bar{\gamma}\left(x\right)\sqrt{C}, \tag{28}$$

where $\gamma\left(\tilde{n}\right)\equiv\sqrt{2/\Delta_{xx}\left(\tilde{n},X(\tilde{n})\right)}$, *and* $\bar{\gamma}\left(x\right)\equiv\sqrt{2/\Delta_{11}\left(X^{-1}(x),x\right)}$. *In addition,* $\gamma\left(\tilde{n}\right)=X'\left(\tilde{n}\right)\bar{\gamma}\left(X\left(\tilde{n}\right)\right)$.

Proof of Lemma 1. The proof holds for any given aggregate state Ω, and so for transparency we suppress dependence on Ω in what follows. Recall that the adjustment triggers satisfy the value matching condition, $\Delta\left(\tilde{n},x'\right)\equiv\Pi^{\Delta}\left(x'\right)-\Pi^{0}\left(\tilde{n},x'\right)=C$. In the presence of $C\approx0$, we may restrict our focus to a second-order approximation to $\Delta\left(\tilde{n},x'\right)$ around $x'=X(\tilde{n})$:

$$\begin{aligned}\Delta\left(\tilde{n}_{-1},x\right) &\approx \Delta\left(\tilde{n}_{-1},X(\tilde{n}_{-1})\right)+\Delta_{x}\left(\tilde{n}_{-1},X(\tilde{n}_{-1})\right)\left(x-X\left(\tilde{n}_{-1}\right)\right)\\ &\quad+\frac{1}{2}\Delta_{xx}\left(\tilde{n}_{-1},X(\tilde{n}_{-1})\right)\left(x-X\left(\tilde{n}_{-1}\right)\right)^{2}.\end{aligned} \tag{29}$$

The first and second terms on the right side are zero by optimality. Setting $\Delta\left(\tilde{n}_{-1},x\right)=C$, it follows that the triggers are as stated in (27).

The inverse triggers may be derived symmetrically by approximating $\Delta\left(\tilde{n}_{-1},x\right)$ around $\tilde{n}_{-1}=X^{-1}(x)$:

$$\begin{aligned}\Delta\left(\tilde{n}_{-1},x\right) &\approx \Delta\left(X^{-1}(x),x\right)+\Delta_{1}\left(X^{-1}(x),x\right)\left(\tilde{n}_{-1}-X^{-1}(x)\right)\\ &\quad+\frac{1}{2}\Delta_{11}\left(X^{-1}(x),x\right)\left(\tilde{n}_{-1}-X^{-1}\left(x\right)\right)^{2}.\end{aligned} \tag{30}$$

Again, optimality implies the first two terms in the expansion are zero. Setting $\Delta\left(\tilde{n}_{-1},x\right)=C$ yields the stated inverse triggers in (28).

To complete the proof, define the firm's objective function, gross of the adjustment cost, by $\Theta(n,x) \equiv pxF(n) - wn + \beta \int \Pi(\tilde{n}, x')\, dG(x'|x)$. Note that $\Delta(\tilde{n}_{-1}, x) = \Theta(X^{-1}(x), x) - \Theta(\tilde{n}_{-1}, x)$. It follows that $\Delta_{11}(\tilde{n}_{-1}, x) = -\Theta_{nn}(\tilde{n}_{-1}, x)$, and that

$$\begin{aligned} \Delta_{xx}(\tilde{n}_{-1}, x) &= \Theta_n\left(X^{-1}(x), x\right) \frac{\partial^2 X^{-1}(x)}{\partial x^2} + \Theta_{nn}\left(X^{-1}(x), x\right)\left[\frac{\partial X^{-1}(x)}{\partial x}\right]^2 \\ &\quad + 2\Theta_{nx}\left(X^{-1}(x), x\right)\frac{\partial X^{-1}(x)}{\partial x} + \Theta_{xx}\left(X^{-1}(x), x\right) - \Theta_{xx}(\tilde{n}_{-1}, x). \quad (31) \end{aligned}$$

By optimality, we know that $\Theta_n(X^{-1}(x), x) \equiv 0$. It follows that $\Theta_{nn}(X^{-1}(x), x)\frac{\partial X^{-1}(x)}{\partial x} + \Theta_{nx}(X^{-1}(x), x) = 0$. Thus, we can rewrite (31) as

$$\Delta_{xx}(\tilde{n}_{-1}, x) = -\Theta_{nn}\left(X^{-1}(x), x\right)\left[\frac{\partial X^{-1}(x)}{\partial x}\right]^2 + \Theta_{xx}\left(X^{-1}(x), x\right) - \Theta_{xx}(\tilde{n}_{-1}, x). \quad (32)$$

Recalling from above that $\Delta_{11}(\tilde{n}_{-1}, X(\tilde{n}_{-1})) = -\Theta_{nn}(\tilde{n}_{-1}, X(\tilde{n}_{-1}))$, noting that $\frac{\partial X^{-1}(x)}{\partial x} = [X'(X^{-1}(x))]^{-1}$, and evaluating at $x = X(\tilde{n}_{-1})$ yields

$$\Delta_{xx}(\tilde{n}_{-1}, X(\tilde{n}_{-1})) = \Delta_{11}(\tilde{n}_{-1}, X(\tilde{n}_{-1}))\left[\frac{1}{X'(\tilde{n}_{-1})}\right]^2, \quad (33)$$

which implies that $\gamma(\tilde{n}) = X'(\tilde{n})\,\bar{\gamma}(X(\tilde{n}))$, as required. ■

Lemma 2 *To a first-order approximation around $C = 0$, the optimal labor demand policy coincides with its myopic ($\beta = 0$) counterpart.*

Proof of Lemma 2. Fix next period's aggregate state Ω'. In what follows, we show that the stated result holds for any given realization of Ω'. We seek to show that the expected future value of the firm $\int \Pi(\tilde{n}, x'; \Omega')\, dG(x'|x)$ is independent of n to a first-order approximation around $C = 0$ for all Ω'. To begin, partition the forward value into parts associated with each of the three continuation regimes—firing, inaction, and hiring:

$$\begin{aligned} \int \Pi(\tilde{n}, x'; \Omega')\, dG(x'|x) &= \int_0^{L(\tilde{n};\Omega')} \left[\Pi^{\Delta}(x'; \Omega') - C\right] dG(x'|x) \\ &\quad + \int_{L(\tilde{n};\Omega')}^{U(\tilde{n};\Omega')} \Pi^0(\tilde{n}, x'; \Omega')\, dG(x'|x) \\ &\quad + \int_{U(\tilde{n};\Omega')}^{\infty} \left[\Pi^{\Delta}(x'; \Omega') - C\right] dG(x'|x). \quad (34) \end{aligned}$$

For ease of exposition, the remainder of the proof suppresses the dependence on Ω'. Differentiating, using the value-matching conditions in (6) to eliminate the derivatives of the limits of integration, (3) to substitute for $\Pi_1^0(\tilde{n}, x')$, and denoting $\mathcal{D}(\tilde{n}, x) \equiv \int \Pi_1(\tilde{n}, x')\, dG(x'|x)$

yields the recursion:

$$\mathcal{D}(\tilde{n},x)=\int_{L(\tilde{n})}^{U(\tilde{n})}\left[px'F_n(\tilde{n})-w\right]dG(x'|x)+\beta(1-\delta)\int_{L(\tilde{n})}^{U(\tilde{n})}\mathcal{D}((1-\delta)\tilde{n},x')\,dG(x'|x)\,. \tag{35}$$

It is straightforward to show that the latter is a contraction mapping in $\mathcal{D}$, and therefore that there exists a unique fixed point. The mapping depends implicitly on the adjustment cost C via the adjustment triggers, $L(\tilde{n})$ and $U(\tilde{n})$.

Consider a first-order approximation to $\mathcal{D}(\tilde{n},x)$ around the frictionless ($C=0$) case,

$$\mathcal{D}(\tilde{n},x)\approx\mathcal{D}(\tilde{n},x)\big|_{C=0}+\mathcal{D}_C(\tilde{n},x)\big|_{C=0}\cdot C. \tag{36}$$

The leading term $\mathcal{D}(\tilde{n},x)\big|_{C=0}=0$—in the absence of an adjustment friction, the firm's problem is static. From equation (35) the derivative, $\mathcal{D}_C(\tilde{n},x)$, is given by the recursion

$$\begin{aligned}\mathcal{D}_C(\tilde{n},x) &= \left[pU(\tilde{n})F_n(\tilde{n})-w+\beta(1-\delta)\mathcal{D}((1-\delta)\tilde{n},U(\tilde{n}))\right]g(U(\tilde{n})|x)\frac{\partial U(\tilde{n})}{\partial C}\\ &\quad-\left[pL(\tilde{n})F_n(\tilde{n})-w+\beta(1-\delta)\mathcal{D}((1-\delta)\tilde{n},L(\tilde{n}))\right]g(L(\tilde{n})|x)\frac{\partial L(\tilde{n})}{\partial C}\\ &\quad+\beta(1-\delta)\int_{L(\tilde{n})}^{U(\tilde{n})}\mathcal{D}_C((1-\delta)\tilde{n},x')\,dG(x'|x)\,.\end{aligned} \tag{37}$$

We conjecture that $\mathcal{D}_C(\tilde{n},x)\big|_{C=0}=0$, and confirm that the latter recursion (37) verifies this conjecture when evaluated at $C=0$. Under the conjecture, the approximation (36) implies that $\mathcal{D}(\tilde{n},x)\approx 0$. From the first-order condition (5) for an adjusting firm, this in turn implies the simple approximate labor demand rule, $pxF_n(n)\approx w$. Therefore, the policy function is given by $X(n)\approx w/[pF_n(n)]$. Substitution into the recursion for $\mathcal{D}_C(\tilde{n},x)$ (37) then implies

$$\begin{aligned}\mathcal{D}_C(\tilde{n},x) &\approx pF_n(\tilde{n})\left[U(\tilde{n})-X(\tilde{n})\right]g(U(\tilde{n})|x)\frac{\partial U(\tilde{n})}{\partial C}\\ &\quad-pF_n(\tilde{n})\left[L(\tilde{n})-X(\tilde{n})\right]g(L(\tilde{n})|x)\frac{\partial L(\tilde{n})}{\partial C}\\ &\quad+\beta(1-\delta)\int_{L(\tilde{n})}^{U(\tilde{n})}\mathcal{D}_C((1-\delta)\tilde{n},x')\,dG(x'|x)\,.\end{aligned} \tag{38}$$

Now consider the triggers, $L(\tilde{n})$ and $U(\tilde{n})$. Using equation (27) from Lemma 1, and their derivatives, we can write[10]

$$\begin{aligned}\mathcal{D}_C(\tilde{n},x) &\approx \frac{1}{2}\gamma(\tilde{n})^2 pF_n(\tilde{n})\left[g\left(X(\tilde{n})+\gamma(\tilde{n})\sqrt{C}|x\right)-g\left(X(\tilde{n})-\gamma(\tilde{n})\sqrt{C}|x\right)\right] \\ &\quad +\beta(1-\delta)\int_{L(\tilde{n})}^{U(\tilde{n})}\mathcal{D}_C\left((1-\delta)\tilde{n},x'\right)dG(x'|x). \end{aligned} \tag{39}$$

Continuity of the density of idiosyncratic shocks $g(\cdot|x)$ implies that the first line converges to zero as $C \to 0$. It follows that the conjecture is confirmed, $\mathcal{D}_C(\tilde{n},x)|_{C=0} = 0$. ■

Lemma 3 *The evolution of the density of idiosyncratic productivity conditional on lagged employment satisfies the dynamic equation*

$$\mathcal{G}'(x|n_{-1}) = \pi_{-1}\frac{\int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})} g(x|x_{-1})\,\mathcal{G}'_{-1}\left(x_{-1}|\frac{n_{-1}}{1-\delta}\right)dx_{-1}}{\mathcal{G}_{-1}\left[U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]-\mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]}+(1-\pi_{-1})\,g(x|X(n_{-1})). \tag{40}$$

where $\pi_{-1} \equiv \Pr[n_{-1}=\tilde{n}_{-2}]$ is the probability of not adjusting last period. If g is analytic, the law of motion preserves analyticity of $\mathcal{G}'$.

Proof of Lemma 3. First note that we may write $\mathcal{G}(x|n_{-1}) = \int G(x|x_{-1})\,d\mathbb{G}(x_{-1}|n_{-1})$, where

$$\begin{aligned}\mathbb{G}(\xi|n_{-1}) &\equiv \Pr[x_{-1}\le\xi|n_{-1}] \\ &= \Pr[x_{-1}\le\xi|n_{-1},n_{-1}=\tilde{n}_{-2}]\Pr[n_{-1}=\tilde{n}_{-2}] \\ &\quad +\Pr[x_{-1}\le\xi|n_{-1},n_{-1}\ne\tilde{n}_{-2}]\Pr[n_{-1}\ne\tilde{n}_{-2}]. \end{aligned} \tag{41}$$

In the event that the firm adjusted last period, $n_{-1} \ne \tilde{n}_{-2}$, we know that the firm would have adjusted so that $x_{-1} = X(n_{-1})$. Thus,

$$\Pr[x_{-1}\le\xi|n_{-1},n_{-1}\ne\tilde{n}_{-2}] = \mathbf{1}[\xi\ge X(n_{-1})]. \tag{42}$$

In the case in which the firm did not adjust last period, we know n_{-2}. That information alone implies that x_{-1} will be distributed according to the c.d.f of $x_{-1}|n_{-2}$, which denote by $\mathcal{G}_{-1}$, the lagged counterpart of $\mathcal{G}$. In addition, however, we also know n_{-1}. This implies that $x_{-1}\in[L(\tilde{n}_{-1}),U(\tilde{n}_{-1})]$, but is otherwise uninformative on the distribution of x_{-1}. Thus,

$$\Pr[x_{-1}\le\xi|n_{-1},n_{-1}=\tilde{n}_{-2}] = \frac{\mathcal{G}_{-1}\left(\xi|\frac{n_{-1}}{1-\delta}\right)-\mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]}{\mathcal{G}_{-1}\left[U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]-\mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]}. \tag{43}$$

Defining

$$\pi_{-1}\equiv\Pr[n_{-1}=\tilde{n}_{-2}] = \int\left(\mathcal{G}_{-1}[U(\tilde{n}_{-2})|n_{-2}]-\mathcal{G}_{-1}[L(\tilde{n}_{-2})|n_{-2}]\right)dH_{-2}(n_{-2}), \tag{44}$$

[10] When we take these derivatives, recall that, since $X(n) = w/[pF_n(n)]$ under the conjecture, it is therefore independent of C.

we can therefore write

$$\mathbb{G}(x_{-1}|n_{-1}) = \pi_{-1}\frac{\mathcal{G}_{-1}\left(x_{-1}|\frac{n_{-1}}{1-\delta}\right) - \mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]}{\mathcal{G}_{-1}\left[U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right] - \mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]} + (1-\pi_{-1})\mathbf{1}[x_{-1} \geq X(n_{-1})]. \tag{45}$$

Substituting into the definition of $\mathcal{G}(x|n_{-1})$ yields its dynamic update equation,

$$\mathcal{G}(x|n_{-1}) = \pi_{-1}\frac{\int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})} G(x|x_{-1})\,\mathcal{G}'_{-1}\left(x_{-1}|\frac{n_{-1}}{1-\delta}\right)dx_{-1}}{\mathcal{G}_{-1}\left[U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right] - \mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]} + (1-\pi_{-1})\,G(x|X(n_{-1})). \tag{46}$$

That the latter preserves analyticity of $\mathcal{G}$ follows from analyticity of G, the fact that sums, products and integrals of analytic functions are themselves analytic, and that quotients of analytic functions with a non-zero denominator are also analytic. ■

Proof of Proposition 2. Recall the law of motion for the density of employment $h(n)$, which we rewrite here as

$$\begin{aligned}\Delta h(\tilde{n}) \;&=\; \left(1 - \mathcal{H}\left[\frac{L^{-1}X(\tilde{n})}{1-\delta}|X(\tilde{n})\right] + \mathcal{H}\left[\frac{U^{-1}X(\tilde{n})}{1-\delta}|X(\tilde{n})\right]\right)h^*(\tilde{n}) \\ &\quad + \frac{1}{1-\delta}\left(\mathcal{G}[U(\tilde{n})|n] - \mathcal{G}[L(\tilde{n})|n]\right)h_{-1}(n) - h_{-1}(\tilde{n}).\end{aligned} \tag{47}$$

We conjecture that $\partial h/\partial C|_{C=0} = 0 = \partial\mathcal{G}/\partial C|_{C=0}$, and confirm that this is verified by the law of motion. Note from the proof of Proposition 1 that, since $\mathcal{H}(\nu|\xi) = \int_0^\nu \frac{\mathcal{G}'(\xi|\tilde{\nu})}{g(\xi)}dH_{-1}(\tilde{\nu})$, it follows that $\partial\mathcal{H}/\partial C|_{C=0} = 0$ under the conjecture. In addition, from Lemma 2 we know that $\partial X/\partial C|_{C=0} = 0$. Given this, under the conjecture we can write the derivative of the law of motion (47) with respect to the adjustment cost in the neighborhood of $C = 0$ as

$$\begin{aligned}\frac{\partial[\Delta h(\tilde{n})]}{\partial C} \;&\approx\; -\left[\mathcal{H}'\left[\frac{L^{-1}X(\tilde{n})}{1-\delta}|X(\tilde{n})\right]\frac{\partial L^{-1}X(\tilde{n})}{\partial C} - \mathcal{H}'\left[\frac{U^{-1}X(\tilde{n})}{1-\delta}|X(\tilde{n})\right]\frac{\partial U^{-1}X(\tilde{n})}{\partial C}\right]\frac{h^*(\tilde{n})}{1-\delta} \\ &\quad + \left[\mathcal{G}'[U(\tilde{n})|n]\frac{\partial U(\tilde{n})}{\partial C} - \mathcal{G}'[L(\tilde{n})|n]\frac{\partial L(\tilde{n})}{\partial C}\right]\frac{h_{-1}(n)}{1-\delta}.\end{aligned} \tag{48}$$

Noting that $C = 0$ implies that $\frac{L^{-1}X(\tilde{n})}{1-\delta} = n = \frac{U^{-1}X(\tilde{n})}{1-\delta}$, $U(\tilde{n}) = X(\tilde{n}) = L(\tilde{n})$, $\mathcal{G}'(\xi|\nu) = g[\xi|X(\nu)]$ and $\mathcal{H}'(\nu|\xi) = g[\xi|X(\nu)]\frac{h^*_{-1}(\nu)}{g(\xi)}$, and recalling that $h^*(n) = g[X(n)]X'(n)$ yields

$$\frac{\partial[\Delta h(\tilde{n})]}{\partial C} \to \left\{-\left[\frac{\partial L^{-1}X(\tilde{n})}{\partial C} - \frac{\partial U^{-1}X(\tilde{n})}{\partial C}\right]X'(\tilde{n}) + \left[\frac{\partial U(\tilde{n})}{\partial C} - \frac{\partial L(\tilde{n})}{\partial C}\right]\right\}g[X(\tilde{n})|X(n)]\frac{h^*_{-1}(n)}{1-\delta}, \tag{49}$$

as $C \to 0$. Lemma 1 implies that $\frac{\partial L^{-1}X(n)}{\partial C} \approx \frac{\bar{\gamma}(X(n))}{2\sqrt{C}} \approx -\frac{\partial U^{-1}X(n)}{\partial C}$ and that $\frac{\partial U(n)}{\partial C} \approx \frac{\gamma(n)}{2\sqrt{C}} \approx -\frac{\partial L(n)}{\partial C}$ in the neighborhood of $C = 0$. Thus, we can write

$$\left.\frac{\partial[\Delta h(\tilde{n})]}{\partial C}\right|_{C=0} = \lim_{C\to 0}\left\{-\frac{\bar{\gamma}(X(\tilde{n}))}{\sqrt{C}}X'(\tilde{n}) + \frac{\gamma(\tilde{n})}{\sqrt{C}}\right\}g[X(\tilde{n})|X(n)]\frac{h^*_{-1}(n)}{1-\delta}. \tag{50}$$

Finally, recalling from Lemma 1 that $\gamma(n) = X'(n)\bar{\gamma}(X(n))$ for all C, we find that $\partial[\Delta h(n)]/\partial C|_{C=0} = 0$, as required.

It remains to verify that $\partial\mathcal{G}/\partial C|_{C=0} = 0$. First, rewrite the law of motion for $\mathcal{G}$ as

$$\mathcal{G}'(x|n_{-1}) = \pi_{-1}\frac{\int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})}[g(x|x_{-1}) - g(x|X(n_{-1}))]\,\mathcal{G}'_{-1}\left(x_{-1}|\frac{n_{-1}}{1-\delta}\right)dx_{-1}}{\mathcal{G}_{-1}\left[U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right] - \mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right]} + g(x|X(n_{-1})), \tag{51}$$

and recall the probability of not adjusting last period π_{-1} in equation (44). Using Lemma 1 and the conjecture, it follows that $\pi_{-1} = O\left(C^{1/2}\right)$ and $\mathcal{G}_{-1}\left[U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right] - \mathcal{G}_{-1}\left[L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right] = O\left(C^{1/2}\right)$. Thus, the limiting behavior of $\mathcal{G}$ as $C \to 0$ is determined by the limiting behavior of the remaining term, $A \equiv \int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})}[g(x|x_{-1}) - g(x|X(n_{-1}))]\,\mathcal{G}'_{-1}\left(x_{-1}|\frac{n_{-1}}{1-\delta}\right)dx_{-1}$. Under the conjecture,

$$\begin{aligned}\frac{\partial A}{\partial C} \approx\; & [g(x|U(\tilde{n}_{-1})) - g(x|X(n_{-1}))]\,\mathcal{G}'_{-1}\left(U(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right)\frac{\partial U(\tilde{n}_{-1})}{\partial C} \\ & - [g(x|L(\tilde{n}_{-1})) - g(x|X(n_{-1}))]\,\mathcal{G}'_{-1}\left(L(\tilde{n}_{-1})|\frac{n_{-1}}{1-\delta}\right)\frac{\partial L(\tilde{n}_{-1})}{\partial C}.\end{aligned} \tag{52}$$

From Lemmas 1 and 3, in the neighborhood of $C = 0$ we can write the latter as

$$\frac{\partial A}{\partial C} \approx g_2(x|X(\tilde{n}_{-1}))\,g'\left[X(\tilde{n}_{-1})|X\left(\frac{n_{-1}}{1-\delta}\right)\right]\gamma(\tilde{n}_{-1})^3\sqrt{C} \to 0 \text{ as } C \to 0. \tag{53}$$

It follows that $\partial\mathcal{G}/\partial C|_{C=0} = 0$, as required. ■

B Labor supply

Imagine a large household with a continuum of identical members. Let L be the size of the household and N the number of employed members. Each member i derives felicity from consumption equal to $\frac{\sigma}{\sigma-1}c_i^{\frac{\sigma-1}{\sigma}}$. If member i works, he suffers disutility denoted by ζ_i. We follow Mulligan (2001) and assume that disutilities differ across members. Thus, household utility is given by

$$\frac{\sigma}{\sigma-1}\int_0^L c_i^{\frac{\sigma-1}{\sigma}}di - \int_0^N \zeta_i di. \tag{54}$$

The household's decision problem with respect to N may be recast as the choice of a threshold, $\bar{\zeta}$, such that households with disutility in excess of $\bar{\zeta}$ are not sent to work. It follows that the employment rate is given by $\frac{N}{L} \equiv n = Z\left(\bar{\zeta}\right)$, where Z is the distribution function of ζ over household members.[11]

[11] If one interprets disutility as a fixed attribute of a worker, then workers with high disutility generally do not work but receive the same consumption as everyone else. As a result, the non-employed are always better off. This seems unpalatable, but it is not the only interpretation of the model. Instead, assume that there is a fixed distribution of disutilities, and each worker takes an i.i.d. draw from this distribution each period. That is, there are some periods where the marginal value of time is particularly high for some workers (and

The first-order condition is given by

$$\bar{\zeta} = Z^{-1}(n) = \lambda w, \tag{55}$$

where w is the real wage rate and λ is the marginal value of wealth. To obtain a closed-form solution, assume the probability density of disutility takes the form $z(\zeta) = A\zeta^b$, where $A > 0$, in which case we can write

$$n = Z(\bar{\zeta}) = A\frac{\bar{\zeta}^{1+b}}{1+b} \implies \bar{\zeta} = Z^{-1}(n) = \left[A^{-1}(1+b)\,n\right]^{\frac{1}{1+b}}. \tag{56}$$

Since we have a free parameter, it is convenient to set $A \equiv \frac{1+b}{a^{1+b}}$, where $a > 0$. Putting all this together, the first-order condition becomes

$$an^{\frac{1}{1+b}} = \lambda w. \tag{57}$$

Note that as $b \to -1$, the Frisch elasticity of labor supply goes to zero. As $b \to \infty$, the elasticity goes to infinity. Also, note that if $\sigma = \infty$, then the first-order condition for consumption implies $\lambda = 1$. In that case, the labor supply schedule simplifies further to $an^{\frac{1}{1+b}} = w$, which is of the form given in the main text. Specifically, in the main text, we set $\eta \equiv \frac{1}{1+b}$ and $\psi = aL^{-\frac{1}{1+b}}$, which leaves us with $w = \psi N^{\eta}$.

so they want to remain at home), and there are other periods where the disutility of work is relatively low for those same workers. In this environment, members are equally well-off on average over their (infinite) lives.

Figure 1. An *Ss* labor demand policy

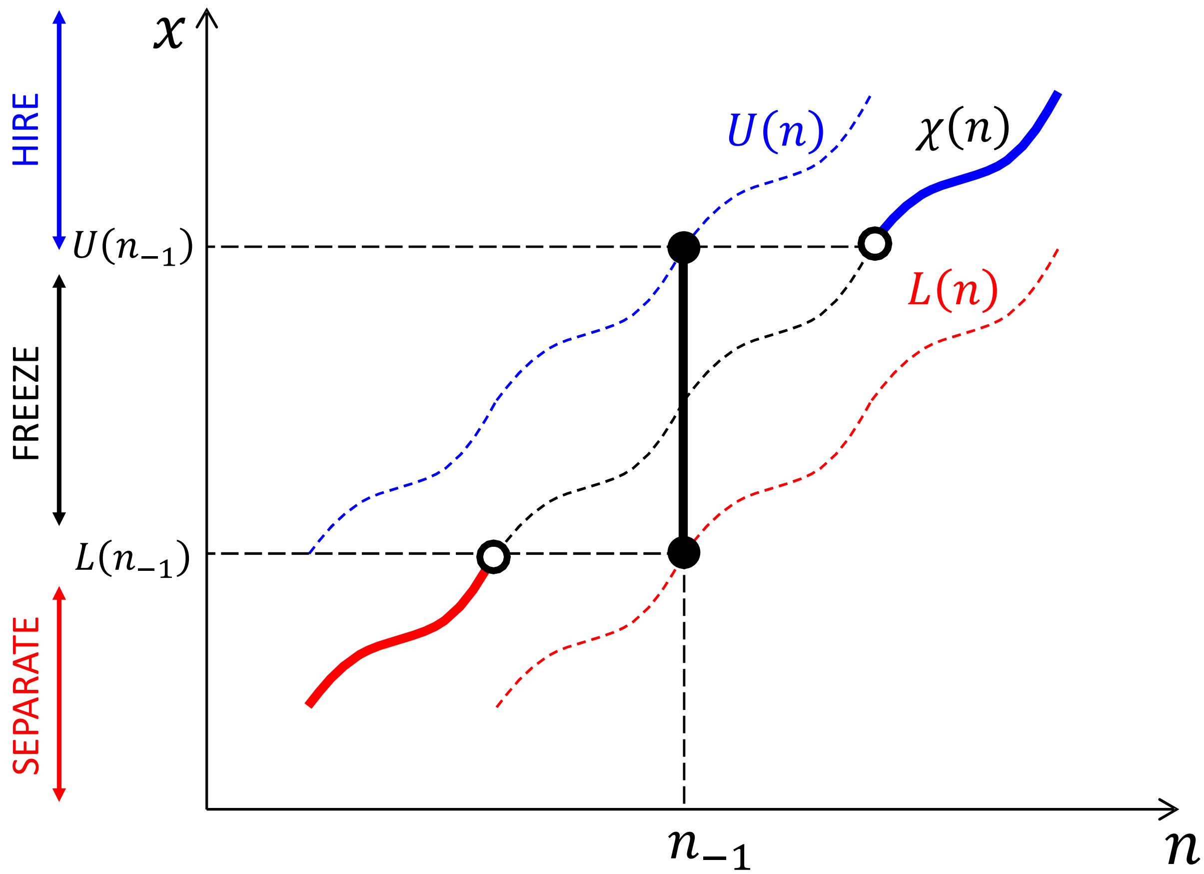

Figure 2. Intuition for the role of symmetry in aggregate neutrality

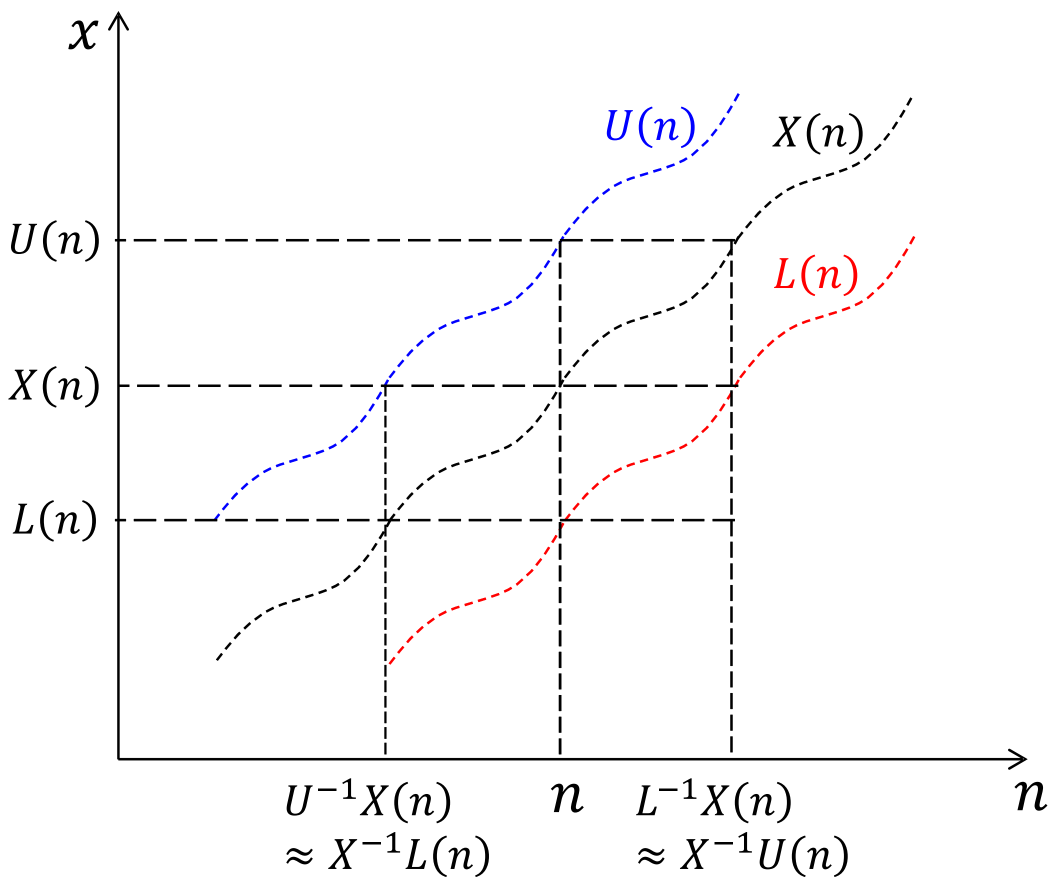

Table 1. Baseline Calibration

Parameter	Meaning	Value	Reason
α	Returns to scale	0.64	Cooper, Haltiwanger and Willis (2005)
β	Discount factor	0.99	Quarterly real interest rate = 1%
$C/\overline{Revenue}$	Adj. Cost / Avg. Revenue	0.08	Cooper, Haltiwanger and Willis (2005); Bloom (2009)
ρ_x	Persistence of x	0.7	Cooper, Haltiwanger and Willis (2005); Foster, Haltiwanger and Syverson (2008)
σ_x	Std. dev. of innovation to x	0.35	Cooper, Haltiwanger and Willis (2005, 2007)
ρ_p	Persistence of p	0.95	Autocorrelation of detrended log N
σ_p	Std. dev. of innovation to p	0.015	Std. dev. of detrended log N

Figure 3. Dynamic response of aggregate employment to a 1% innovation to aggregate productivity: Baseline calibration

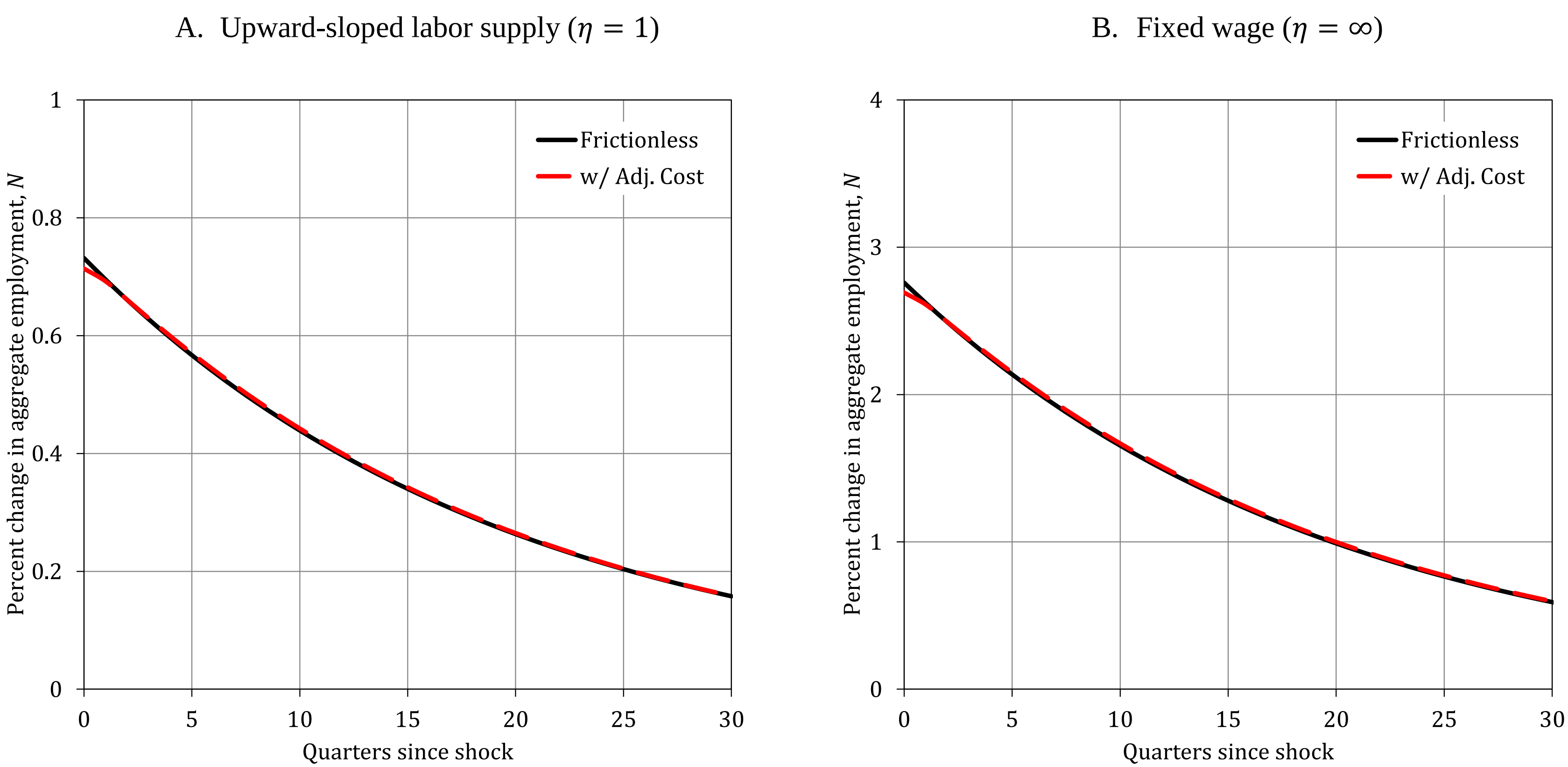

Figure 4. Dynamic response of aggregate employment to a 1% innovation to aggregate productivity: Sensitivity, Fixed wage case