A Picture of Stock-Flow Unemployment

Melvyn Coles (University of Essex and I.A.E. (ICREA))
Paul Jones (University of Warwick)
Eric Smith (University of Essex)

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Abstract
Stock-flow job matching with heterogeneous agents implies that there are two broad types of unemployed workers - those on the short side of their occupations who can easily find work, and those on the long side who expect an extended spell of unemployment. Using matching data and information on completed and uncompleted unemployment spells for England and Wales, 1986-2000, this paper uses the stock-flow matching hypothesis to identify the fraction (or incidence) of laid off workers who find themselves on the long side of the market and, conditional on being on the long side, their expected duration of unemployment. The average incidence of long-side unemployment is around one half, and increases significantly in recessions. The expected duration of long-side spells is also strongly countercyclical - peaking at 15 months in the 1990-92 recession and falling to a more modest 9 months by January 1999. Cross section estimates also identify a ‘North–South’ divide and a large city effect - the long-side unemployed in the largest cities and in the ‘North’ experience longer spells of unemployment.

Corresponding Author: Eric Smith, Dept. of Economics, University of Essex, Colchester. CO4 3SQ. [essmith@essex.ac.uk]
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1. Introduction

For some job seekers, unemployment is a short-lived transitional step between jobs. For others, escaping from unemployment is a slow process that can last for several months, possibly longer. The purpose of this paper is to develop a new methodology which measures the incidence as well as the duration of the more drawn out spells.\(^1\)

Although the familiar search framework provides a useful benchmark for describing re-employment rates of job seekers, it is not ideally suited for discriminating between relatively fleeting and more prolonged spells of unemployment. Indeed, it seems unlikely that long, uncompleted spells, which typically average more than one year in many OECD countries, arise from the usual notions of matching frictions.\(^2\) For this reason, this paper instead adopts the stock-flow matching approach, which as described below explicitly distinguishes between such unemployment experiences, to analyze short and protracted spells of unemployment.

Transitory employment occurs when job market entrants find themselves on the short side of their particular occupation. With a number of opportunities immediately available, these job seekers experience only short spells as they choose one of these options. On the other hand, non-transitory or enduring unemployment occurs when job seekers find themselves on the long side of their occupations and must wait for suitable new vacancies to enter the market. Given these stock-flow definitions, the paper derives measures of the incidence and expected duration of enduring spells of unemployment.

The stock-flow literature (e.g. Taylor, 1995; Coles and Smith, 1998; Coles and Muthoo, 1998; Coles, 1999; Gregg and Petruntiolo, 1997; Lagos, 2000) assumes that workers do not search randomly for vacancies. Instead, the unemployed have a good idea about where to look for suitable vacancies - they may check help-wanted ads in newspapers and professional journals, contact employment agencies (both private and public) or ask friends and relatives. As in the direct search literature (e.g. Montgomery, 1991; Acemoglu and Shimer, 1999; Burdett

\(^1\)In some cases these enduring spells may become 'long-term unemployment', a significant and persistent problem in many economies. See Nickell (1997) and Machin and Manning (1999).

\(^2\)Machin and Manning (1999) report that in 1995, 44% of those unemployed in the U.K. had been unemployed for more than twelve months (61% for more than six months). Layard et al (1991, p. 223) report that for workers in the unemployment stock (in 1989), the average duration of an uncompleted spell of unemployment was twenty-one months. In contrast, for flow entrants into the pool of unemployment, the average duration of a completed spell of unemployment was a much lower seven months.
et al, 2001), the stock-flow literature assumes the polar case that workers are fully informed about all vacancies currently on the market. Unlike the directed search approach, however, vacancies and workers are assumed to be heterogeneous, and workers can submit multiple job applications.\(^3\) This generates a simple sampling effect: a newly unemployed worker keeps applying for jobs from the current vacancy stock until either the worker is offered (and accepts) a job or the worker fully samples the current stock of vacancies and has found none suitable. Once a job seeker has failed to find a match with existing vacancies, a new phase of unemployment begins - this worker must now wait for suitable vacancies to come onto the market. This process generates “stock-flow” matching where the stock of (long-side) unemployed wait to match with the inflow of new vacancies, while the inflow of newly unemployed workers potentially matches with the current vacancy stock.

Coles and Smith (1998) obtain compelling evidence in favor of this sampling effect. (See also Gregg and Petrongolo, 1999; Andrews and Bradley, 2001; Coles and Petrongolo, 2003. Petrongolo and Pissarides, 2001, provide a recent survey of the matching function.) Using aggregate labor market information for England and Wales, Coles and Smith find that the re-employment probabilities of those workers unemployed for more than one month are highly correlated with the inflow of new vacancies, and uncorrelated with the stock. Their explanation is that such workers have had the time to fully sample the stock of current vacancies on the market, did not find a suitable match, and so wait to match with new vacancies. Coles and Smith also find that recently unemployed workers (those with durations less than one month) experience re-employment rates which are significantly correlated with the stock of vacancies, but that the correlation disappears as the duration of the worker’s unemployment spell increases.

Using this stock-flow matching structure to identify the data, the first part of this paper demonstrates how to decompose information on matching and unemployment spells to provide measures of

(a) the incidence of enduring unemployment spells - defined as the probability of becoming long-side unemployed conditional on entering unemployment, and

(b) the expected duration of unemployment conditional on experiencing a long-side spell of unemployment.

\(^3\)Albrecht et al, (2002) provide an exception. They consider a directed search model with multiple applications by job seekers.
The second part of the paper describes the empirical results based on a monthly panel of local labor market data in England and Wales (from 1986 to 2000). Not surprisingly, the results find that the incidence and expected duration of unemployment are highest in the recession. Averaging across local markets, the estimated duration of non-transient unemployment peaked around January 1992 (the middle of the recession) at approximately fifteen months. Following this peak, the average duration of such unemployment fell gradually over time to around the nine month mark by 1999.

The paper also describes the variation in unemployment across local labor markets, based on Travel-To-Work-Areas (TTWAs). The results find much more variation in the expected duration of protracted unemployment across markets than in incidence. Estimates identify not only a large city effect - the expected duration of unemployment is significantly higher in the big cities (London, Manchester, Birmingham) - but also a North-South divide. The expected duration is lowest in the South East and significantly higher in the North.

2. Framework

Stock-flow matching as described by Taylor (1995) and Lagos (2000) assumes a segmented labor market, where submarkets are differentiated by occupation, skill and location. There are two sides to each labor market - a short side and a long side. Let \( p(t) \) denote the proportion of those workers who on becoming unemployed at date \( t \), find they are on the short side of their particular submarket and so quickly rematch. The remaining \( 1 - p(t) \) of entrants are on the long side and are simply referred to here as long-side unemployed (LSU from now on). These workers find there are no suitable vacancies in their submarket and must wait for the arrival of new vacancies. \( 1 - p(t) \) defines the incidence of LSU at date \( t \), conditional on becoming unemployed. Conditional on being LSU, let \( \lambda(t) \) denote the average matching rate of each LSU worker at date \( t \).

The following analysis does not test the stock-flow matching structure on the data but instead uses it as an identifying assumption. It is worth, however, briefly documenting the extent to which stock-flow matching is consistent with the data. In its favor, stock-flow matching is consistent with the observation that roughly a quarter of new vacancies posted in U.K. Job Centers are filled on the first day. The intuition being that there is a large stock of LSU job seekers who frequently check the vacancy boards and quickly snap up good new vacancies. Coles and Petrongolo (2003) look more formally at these U.K. Job Center vacancy data to
test the random matching function approach against stock-flow matching. Their estimates, which strongly support stock-flow matching, find not only that the LSU match with the inflow of new vacancies but they also crowd each other out while doing so. Rather than use Job Center vacancy data, this paper uses unemployment spells data to identify estimates of $p, \lambda$. Furthermore, as vacancy data may be unavailable in many countries (or perceived as too unreliable), the approach described here can be more widely applied.

By assuming two types of workers where those on the short side match quickly while the LSU match slowly, this approach is also potentially consistent with the fact that average re-employment rates decrease with unemployment duration (e.g. Lancaster and Nickell, 1980; Heckman and Singer, 1984; Meyer, 1990; Portugal and Addison, 2000). To obtain a tractable econometric structure, however, this paper simplifies by specifying that workers on the short side match arbitrarily quickly. This is a strong assumption. The results of Coles and Smith (1998) suggest that workers who match with the initial stock of vacancies experience unemployment spells of up to a month. A more plausible alternative would be to complement the stock-flow specification with search frictions that bind on the short side of the market - these workers might take, say, two or three weeks to decide on their preferred employment opportunity before starting work. Unfortunately, outside a steady state such an extension introduces non-linear composition dynamics which, being unobserved, cannot be identified on the data. For tractability and given the focus on the matching experience of the LSU, the paper abstracts from short side frictions by assuming that job seekers on the short side match arbitrarily quickly.

The concluding section reconsiders this assumption and argues that the estimated expected duration of LSU is biased upwards by a few weeks. Given the mean expected duration of this type of unemployment exceeds a year, this bias seems small. The specification that all LSU workers match at the same rate $\lambda(t)$ is also inconsistent with the idea that workers choose different search efforts. It is straightforward to extend the stock-flow approach in this direction but, as discussed in the Appendix, doing so raises identification issues.

Imposing stock-flow matching as an identifying assumption, the next two subsections demonstrate how to infer values of $(p, \lambda)$ directly from the data. These estimates are then used to describe the incidence $(1 - p)$ and expected duration of long-side unemployment, taking into account that these matching parameters

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4See Burdett and Coles (1997) and Shimer and Smith (2001) who describe non-steady state matching equilibria with heterogeneous agents.
vary over the business cycle.

2.1. Matches Per Period

Let \( u(t) \) denote the inflow of newly unemployed workers and \( U(t) \) denote the stock of unemployed workers at time \( t \). Assuming time is continuous, then stock-flow matching implies the match flow, \( M(t) \), is given by

\[
M(t) = p(t)u(t) + \lambda(t)U(t)
\]

where proportion \( p(t) \) of the unemployment inflow \( u(t) \) match immediately (or at least very quickly), while the stock of unemployed workers \( U(t) \) match at rate \( \lambda(t) \). Given data on matching \( (M) \), unemployment stocks \( (U) \) and inflows \( (u) \), this equation provides the first identifying equation for \( p(t) \) and \( \lambda(t) \).

An unfortunate complication, however, is that the identifying relationships are set in continuous time, but the available data are recorded monthly. We deal with this time aggregation problem using the arguments of Gregg and Petrongolo (1999). Let \( t \in [n, n+1) \) denote calendar time in each month \( n \in \mathbb{N} \) and assume that the unemployment inflow and matching parameters are constant within the month; i.e. for each month \( n \), assume

\[
u(t) = u_n, \quad \lambda(t) = \lambda_n, \quad p(t) = p_n \text{ for all } t \in [n, n+1).
\]

At time \( t \), the unemployment stock evolves according to the differential equation

\[
\frac{dU}{dt} = u_n [1 - p_n] - \lambda_n U(t)
\]

where \( u_n [1 - p_n] \) describes the net inflow of entrants into unemployment, while \( \lambda_n U(t) \) describes the outflow. Integrating implies

\[
U(t) = U_n - \frac{u_n}{\lambda_n} (1 - p_n) e^{-\lambda_n (t-n)} + \frac{u_n}{\lambda_n} (1 - p_n)
\]

where \( U_n = U(n) \) denotes the level of unemployment at the start of month \( n \). As the total number of matches during the month, \( M_n \) satisfies

\[
U_{n+1} - U_n = u_n - M_n
\]

Using \( U_{n+1} = U(n+1) \), the previous expression yields

\[
M_n = U_n (1 - e^{-\lambda_n}) + p_n u_n + u_n (1 - p_n) \left[ 1 - \frac{1 - e^{-\lambda_n}}{\lambda_n} \right].
\]
Equation (2) says that the number of matches within the month includes those who were unemployed at the start of the month and found work, those who entered the market and immediately found work, and those entrants who became LSU but were fortunate enough to find a job before the end of the month. Given data on \((M_n, U_n, u_n)\), (2) describes the first identifying equation for \((p_n, \lambda_n)\).

2.2. Average Durations of Unemployment Spells.

The second piece of information used to estimate \((p, \lambda)\) compares the average duration of uncompleted spells of unemployment against the average duration of completed spells. In continuous time and conditional on a filled vacancy, stock-flow matching implies the average completed spell of unemployment, \(X_c(t)\), is

\[
X_c(t) = \frac{\lambda(t)U(t)}{\lambda(t)U(t) + p(t)u(t)}X^u(t),
\]

where a LSU worker who matches has average uncompleted spell \(X^u(t)\), while a newly unemployed worker on the short side who matches has an (arbitrarily) short unemployment spell. Given information on the ratio of average unemployment spells, \(X_c(t)/X^u(t)\), (3) is a second identifying equation for \(p(t)\) and \(\lambda(t)\). Note in particular that if \(p(t) \equiv 0\), so that all unemployed workers match at the same rate \(\lambda(t)\) (as in the standard random matching model), a matched worker is then a random draw from the current stock of unemployed workers. In this case, the average duration of a completed spell equals the average uncompleted spell. On the other hand, \(p(t) > 0\) implies a significant fraction of those finding employment are those who have been recently laid off and have quickly found work. A higher \(p(t) > 0\) implies a smaller ratio \(X_c(t)/X^u(t) < 1\).

As the data only record average completed spells over the month then, as in the previous subsection, estimates have to correct for temporal aggregation bias. Let \(X^C_n\) denote the mean completed unemployment spell in month \(n\). By definition, \(X^C_n\) is the sum of spell lengths of all workers who match during the month, \(\Sigma_n\), divided by the number of matches, \(M_n\); i.e.

\[
X^C_n = \Sigma_n/M_n
\]

Assuming newly unemployed workers on the short side of the market match (arbitrarily) quickly, \(\Sigma_n\) is determined (almost) entirely by completed spells of unemployment by LSU workers.

These completed spells come from one of two instances. Either
(a) a newly unemployed worker initially fails to match (with probability \(1 - p_n\)),
becomes LSU but is still fortunate enough to match before the end of the
month. The sum of spell lengths from such workers is denoted \(\Sigma^F_n\):

or

(b) a worker who is already in the stock of unemployed at the start of the month
finds work before the end of the month. The sum of spell lengths from such
workers is denoted \(\Sigma^S_n\).

Total spell lengths \(\Sigma_n\) is simply the sum of these two objects

\[
\Sigma_n = \Sigma^F_n + \Sigma^S_n
\]

(a) Calculating \(\Sigma^F_n\): Consider the inflow \(u_n(1 - p_n)\) of workers into the stock of
long-side unemployment. For each entry date \(t \in [n, n + 1)\), these workers match
within the rest of the month according to a Poisson process with parameter \(\lambda_n\).
Over an arbitrarily small time interval \(ds > 0\), proportion \(\left[\lambda_n \exp (-\lambda_n (s - t))\right] ds\)
of these workers find work at date \(s \in (t, n + 1)\) with a corresponding completed
unemployment spell \((s - t)\). Adding up all those spells implies

\[
\Sigma^F_n = \int_{t=n}^{n+1} \left\{ u_n(1 - p_n) dt \left[ \int_{s=t}^{n+1} (s-t) \left[ \lambda_n e^{-\lambda_n (s-t)} \right] ds \right] \right\}
\]

Integration establishes

\[
\Sigma^F_n = \frac{u_n(1 - p_n)}{\lambda_n} \left[ (1 + e^{-\lambda_n}) - 2 \left( \frac{1 - e^{-\lambda_n}}{\lambda_n} \right) \right].
\]

(b) Calculating \(\Sigma^S_n\): Recall that \(U_n\) describes the stock of unemployed workers
at the start of the month, and let \(X^U_{n-1}\) denote the mean uncompleted spell of
unemployment carried over from the previous period. Since LSU workers match
according to a Poisson process with parameter \(\lambda_n\), it follows that

\[
\Sigma^S_n = U_n \int_n^{n+1} \lambda_n e^{-\lambda_n(t-n)} [X^U_{n-1} + t - n] dt
\]
The $\lambda_n \exp (-\lambda_n(t - n)) \, dt$ term describes the proportion of LSU workers from last period's stock who find work (over small time period $dt > 0$) at date $t \in [n, n + 1)$. Their average duration of unemployment is the average uncompleted spell inherited from the previous period, $X_{n-1}^U$, plus the additional time $(t - n)$ spent in unemployment this period. Integration this time gives

$$\Sigma_n^S = U_n \left[ (X_{n-1}^U + \frac{1}{\lambda_n})(1 - e^{-\lambda_n}) - e^{-\lambda_n} \right]$$

Using $X_n^C = [\Sigma_n^F + \Sigma_n^S]/M_n$ implies that the average duration of completed spells satisfies:

$$X_n^C = \frac{u_n}{M_n} \left( \frac{1 - p_n}{\lambda_n} \right) \left[ (1 + e^{-\lambda_n}) - 2 \left( \frac{1 - e^{-\lambda_n}}{\lambda_n} \right) \right] + \frac{U_n}{M_n} \left[ (X_{n-1}^U + \frac{1}{\lambda_n})(1 - e^{-\lambda_n}) - e^{-\lambda_n} \right]$$

(4)

Given data on $(X_n^C, X_n^U, U_n, u_n, M_n)$, equations (2) and (4) identify the underlying matching parameters $(p_n, \lambda_n)$. Although the equations are non-linear, it is straightforward to compute solutions using standard numerical techniques.

3. Estimation

Using data on local labor market matching, this section identifies how LSU has varied over time and across regions in England and Wales from March 1986 until December 2000. In particular, this section uses equations (2) and (4) to decompose the data into measures of the incidence of LSU, $1 - p_n^i$, and the average re-employment rate of LSU workers, $\lambda_n^i$, for labor market $i$ in month $n$. (The notation here is extended to allow for spatially distinct labor markets.)

3.1. Data

In the UK, the Travel-To-Work-Area (TTWA) is the standard measure of a self-contained labor market. Based on census figures, these are geographical regions that have:

5 All data are available from the National On-line Manpower Information service (NOMIS) located at the University of Durham and homepage http://www.nomisweb.co.uk/
• a minimum of 3500 residents;
• at least 75% of the people working in the area live in it;
• at least 75% of those living in the area also work there.

There are 254 such areas in England and Wales.\textsuperscript{6}

The working population ($\text{POP}_i$ in market $i$) is concentrated in a few, very large markets. From the 1991 census figures, the largest TTWA is London which has a working age population of 3.8 million workers whereas the mean working age population across all TTWAs is a mere 120,000. The median population is an even smaller 64,000 while the standard deviation is 268,000. To characterize the representative experience of a LSU worker in England and Wales, in what follows observations of each TTWA $i$ are weighted by the proportion of total working age adults who live in that market.

For each TTWA $i$ in each month $n$, the Department of Employment reports:

• unemployment inflow ($u_{in}^i$) during the month;
• unemployment outflow ($M_{in}^i$) disaggregated by time spent unemployed;
• unemployed stock ($U_{n+1}^i$) at the end of month $n$, broken down by the duration of the current spell of unemployment.

In the data, unemployment is defined as the number of people claiming unemployment related benefits. All workers claiming state benefit are required to ‘sign on’ at the unemployment benefit office on becoming unemployed and ‘sign off’ on leaving unemployment.\textsuperscript{7} The flow of individuals signing off unemployment benefits is used to capture matched outflow $M_{in}^i$. This outflow is assumed to be into employment thereby ignoring any movements into states of non-participation. Figures relating to time spent unemployed are available for sixteen duration categories: 0-1 week, 1-2 weeks, 2-4 weeks, 4-6 weeks, 6-8 weeks, 8-13 weeks, 13-26\textsuperscript{6}The 2001 census established new boundaries for 243 TTWAs. Data for the 254 TTWAs from 1991 are not available after 2000.
\textsuperscript{7}Unemployment-related benefits are currently: the Jobseekers Allowance (JSA) introduced from October 1996, and National Insurance (NI) credits. A claimant must declare that they are out of work, capable of, available for and actively seeking work during the week in which their claim is made. This count differs very slightly from the alternative count of people who register at the Job Centre seeking work.
Given this definition of unemployment, Figure 1 plots aggregate unemployment in England and Wales from March 1986 until December 2000. Defining a recession as two consecutive quarters of rising unemployment, it follows that the aggregate economy was in recession from March 1990 to January 1993 (as marked by dashed vertical lines). During this recession, total unemployment more than doubled, rising from 1.2 million to 2.6 million.

Figure 2 plots aggregate unemployment inflows and outflows over this period (seasonally detrended and standardized to a four week month). The main feature of the recession is that both the inflow and outflow from unemployment are initially low, and both increase over time during the recession. The difference between these two series is that the inflow increases more rapidly at the beginning of the recession. Taken together, these features generate two important implications for long-side unemployment. First, low outflow rates imply that it takes longer for each worker to find suitable work, which implies a relatively high expected  

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8 The estimates for $p_i$, $\lambda_i$ below are based on the raw, non-seasonally adjusted data.
duration of LSU. Second, the number of LSU workers increases over the recession. With more LSU workers chasing the same new vacancies, the expected duration of LSU increases still further.

Using the data described above, it is straightforward to compute the average duration of completed and uncompleted spells of unemployment. Figure 3 graphs these series for England and Wales. Note the large gap between the two series. This implies that the newly unemployed match much more quickly than the stock of long-side unemployed workers. These data also exhibit a curious property - the average uncompleted and completed spell of unemployment is lowest in the recession. Based on the estimates for \((p(t), \lambda(t))\) obtained below, the time series features of these data will be fully explained later.

For each TTWA \(i\) and each month \(n\), the data described above are used to compute the average duration of completed \((X^C_n)^i\) and uncompleted spells \((X^U_n)^i\), thereby creating the panel of data \((X^C_n, X^U_n, U_n, u_n, M_n)^i\). In addition, for each TTWA \(i\), let the average unemployment rate be \(\pi^i = U^i / POP^i\) where \(U^i = \sum_n U_n^i / N\). Let the average re-employment rate be \(\bar{M}^i / \bar{U}^i\) where \(\bar{M}^i = \sum_n M_n^i / N\). Figure 4 presents a weighted scatter plot of these mean statistics across TTWAs.
Figure 3: Average Duration of Completed and Uncompleted Spells

where the size of a ‘bubble’ reflects the population weight of the TTWA.

Not surprisingly, re-employment rates are negatively correlated with unemployment rates. The largest ‘bubble’ in Figure 4 is London which has a high average unemployment rate and a low average re-employment rate. Aside from London, there is no clear correlation between city size and unemployment - the larger bubbles seem evenly distributed across the scatter plot.

Figure 5 is the (weighted) scatter plot of the average completed and uncompleted spells of unemployment across TTWAs. Two stylized facts are evident. First the scatter plot is approximately linear in that across all TTWAs, the average completed spell of unemployment is roughly half the average uncompleted spell. Figure 5 also demonstrates a strong city size effect - the larger bubbles are concentrated at high completed and uncompleted spells of unemployment.

Given the panel of data \((X^C_n, X^U_n, U_n, u_n, M_n)\)\(^i\), it is straightforward to solve the matching equations (2) and (4) for a panel of estimated matching probabilities \((p^i_n, \lambda^i_n)\). The next subsection describes how those estimates vary over the business cycle. The subsection after that considers how those estimates vary across TTWAs.
3.2. Long-Side Unemployment and the Business Cycle

Recall that $1 - p^i_n$ is defined as the incidence of LSU in market $i$, in month $n$. Aggregating over the TTWAs, define the average incidence of LSU in England and Wales in month $n$ as

$$1 - \bar{p}_n = \sum_i \omega^i (1 - p^i_n)$$

where $\omega^i$ is the working age population weight of TTWA $i$. Similarly, define the average matching rate of the LSU in England and Wales as

$$\bar{\lambda}_n = \sum_i \omega^i \lambda^i_n.$$ 

As these statistics are highly seasonal, Figure 6 plots these statistics seasonally detrended.\(^9\) The top frame of Figure 6 plots average incidence, $1 - \bar{p}_n$, the

\(^9\)An alternative measure can be found by first aggregating the data economy-wide and then estimating. Doing so yields nearly equivalent estimates - there are only very minor differences between the two series.

\(^{10}\)For example, the incidence of LSU is highest in January and lowest in October.
bottom plots the average matching rate, $\bar{\lambda}_n$. To illustrate TTWA dispersion over time, Figure 6 also plots the 10th and 90th percentiles of $1 - p^*_n$ and $\lambda^*_n$ in month $n$ (using population weights $\omega^*_i$ and seasonally detrended).

Figure 6 establishes that on average, the estimated incidence of LSU is around one half; i.e. half of those who become unemployed quickly find work, the others are stuck waiting for something suitable to come onto the market. The average (monthly) matching rate of the LSU is around 0.08 which implies an average unemployment duration of 12.5 months. Together these imply an average completed spell of unemployment equal to 6.2 months and an average uncompleted spell of 12.5 months, both of which are reasonably consistent with the data. (See Figure 3.)

There is substantial business cycle variation in $1 - p_n$ and $\lambda_n$. Consistent with intuition, LSU incidence is highest in the recession - peaking at around 0.65 - and lowest in the boom with a value around 0.4 by mid-1996. The matching rates of the LSU are also strongly countercyclical. Immediately prior to the recession, LSU monthly matching rates $\lambda_n$ were around 0.1 suggesting an expected long-side unemployment spell $1/\lambda_n$ of around 10 months. The onset of the recession
Figure 6: LSU Incidence and Matching Rates
resulted in a collapse of this matching rate to $\lambda_n \approx 0.06$ suggesting an expected unemployment spell $1/\lambda_n$ of 16 months.

There is also substantial persistence - LSU rates did not begin to recover until long after the end of the recession (July 1996 rather than January 1993). This reflects crowding out where the level of unemployment remained above two million up to this point in time.

A useful test of these results is to ask whether these estimates of $1 - \pi_n$ and $\lambda_n$ are consistent with the time series properties of the unemployment spells data $X^c_n, X^u_n$ as described in Figure 3. This is not a trivial exercise. Although the parameter estimates are derived from these data, (1) and (3) imply the estimates for $(p(t), \lambda(t))$ depend only on $X^c, X^u$ through their ratio $X^c/X^u$. Taking temporal aggregation bias into account implies a more complicated estimator but there is no reason to expect that the estimated values of $1/\lambda_n$ will necessarily be consistent with the observed variation in $X^c_n, X^u_n$.

First consider the average uncompleted spell of unemployment data, $X^u_n$. Figure 3 demonstrates that prior to the recession, the average uncompleted spell of unemployment fell from 18 months to around 12 months. The corresponding estimated matching rates are $\lambda_n \approx 0.06$ at the start of this phase (suggesting an expected unemployment spell $1/\lambda_n = 16$ months), rising to $\lambda_n \approx 0.1$ ($1/\lambda_n = 10$ months) by the end. With turnover, where new LSU workers replace LSU workers who match, these estimates of $1/\lambda$ are not only roughly consistent with the mean uncompleted spell averages, they clearly explain the direction of change.

Somewhat surprisingly, the recession implies a further fall in the average uncompleted spell of unemployment, reaching a low value of 10 months. To see why this occurs, note that the recession generates a large increase in the inflow of newly unemployed workers (Figure 2) and an increase in the estimated incidence of long-side unemployment (Figure 6). Together these imply a large increase in newly unemployed workers into the stock of LSU, which lowers the average uncompleted spell of unemployment.

Following this low point, the average uncompleted spell of unemployment grows inexorably over time, reaching its highest value of around 18 months by July, 1996. During this phase, the estimated LSU matching rates collapse to $\lambda_n \approx 0.06$ ($1/\lambda_n \approx 16$ months). This not only explains the observed increase in the average uncompleted spell during this phase given the starting average of 10 months, but also lines up with the matching rate $\lambda_n$ recovery after July 1997 (Figure 6) which coincides with the subsequent decline in the average uncompleted spell of unemployment.
Of course $1/\bar{\lambda}_n$ is only a rough approximation of the expected duration of unemployment. Figure 7 below provides better measures of this statistic. Before constructing those estimates however, consider briefly the average completed spell of unemployment data. Figure 3 implies this data series has the same time profile as the uncompleted spell data series, but with roughly half its value (also see Figure 5). Given that, (3) implies $p(t)u(t) \approx \lambda(t)U(t)$ on average over the cycle. The stock-flow matching interpretation is that roughly half of all vacancies filled are on the short side of their markets (and so are filled by the long-side unemployed), the other half are on the long side and wait to be filled by newly unemployed workers. This result is reassuringly consistent with the above estimates of LSU incidence. $1 - \bar{p}_n \approx 0.5$ also implies that approximately half of newly unemployed workers are on the long side of their markets, the others are on the short side.\footnote{Note, a 50-50 split of entrants between the short and long sides is not an implication of a long run, steady state; e.g. Coles (1999).}

Based on the estimated values of $\bar{\lambda}_n$, Figure 7 below plots estimates of the expected duration of long-side unemployment, denoted $ED_n$. This statistic is computed by assuming a LSU worker matches at Poisson rate $\{\lambda_s\}_{s=0}^{\infty}$ in each month $s$. $ED_n$ is then defined recursively by

$$ED_n = \int_n^{n+1} [t - n]e^{-\bar{\lambda}_n(t-n)}\bar{\lambda}_n dt + e^{-\bar{\lambda}_n} [1 + ED_{n+1}].$$

A given LSU worker at date $n$ gets a job at date $t < n + 1$ with probability $e^{-\bar{\lambda}_n(t-n)}\bar{\lambda}_n dt$ and so experiences a further $(t - n)$ spell of unemployment. With probability $e^{-\bar{\lambda}_n}$ the worker remains unemployed at the end of the month $[n, n+1)$ and so has expected duration $[1 + ED_{n+1}]$ in that event. Integration implies

$$ED_n = \frac{1 - e^{-\bar{\lambda}_n}}{\bar{\lambda}_n} + e^{-\bar{\lambda}_n} ED_{n+1}.$$

As the estimates for $\bar{\lambda}_n$ terminate at December 2000, constructing the series $\{ED_n\}_{n=1}^N$ requires a terminal value for $ED_{N+1}$ at January 2001. Robustness checks find that the estimates for $ED_n$ prior to January 1999 are largely unaffected by any reasonable choice of $ED_{N+1}$. This occurs as the probability a LSU worker in January 1999 remains unemployed by January 2001 is very small and hence the choice of $ED_{N+1}$ has only a very small impact on $ED_n$ prior to January 1999.\footnote{Put differently, the difference equation for $ED_n$ is stable when iterating backwards.} The series for $ED_n$ plotted in Figure 7 in fact uses boundary value
$ED_{N+1} = \frac{1}{\lambda_N}$, but reflecting the above robustness issue, we only plot the estimated series $\{ED_n\}$ as far as January 1999. To provide some idea of dispersion across TTWAs over time, Figure 7 also displays the 90th and the 10th percentiles of $ED_n^i$ (individually constructed using estimated values $\lambda_n^i$), again weighted by $\omega_i$.

As $ED_n$ is constructed using realized values of $\{\lambda_n\}_{n=1}^N$, it is not the ‘expected duration of unemployment given period $n$ information’. Instead, conditional on how the economy evolved over time, $ED_n$ is the prediction on how long it took a LSU worker at date $n$ to find employment. Consistent with intuition, the expected duration of LSU peaks in the middle of the recession. Not only were matching rates lowest then, $\lambda_n \approx 0.06$, they remained at that low point for several more years. The stock-flow interpretation is that these workers were on the long side of their markets and were stuck waiting for suitable new vacancies to come onto the market. Further, Figure 2 suggests that the low LSU matching rate occurred not because the inflow of new vacancies was particularly small - the number of

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13 These estimates are not seasonally adjusted - integration largely washes out the seasonal effects.
matches rose steeply during the recession - but because there was a large increase in the number of LSU workers chasing those new vacancies. By the end of the recession, the number unemployed had more than doubled to 2.6 million, and displacement effects led to a collapse in individual LSU re-employment rates.

After the recession, Figure 7 establishes that the expected duration of LSU gradually falls over time, falling to a low of 9 months by 1999. Note, however, that this decline cannot be attributed to a large increase in the outflow of unemployed workers. Figure 2 shows the outflow gradually decreases over this time period. Instead the rise in $\lambda_n$ observed after July 1996 is entirely due to the decrease in the number unemployed. This results in fewer LSU workers chasing each new vacancy, which raises individual re-employment rates.

3.3. Cross Section Dispersion

To see the way in which LSU varies across TTWAs, define the mean incidence for TTWA $i$ as

$$1 - \bar{p} = \frac{\sum (1 - p^i_n)}{N}$$

where $N = 166$ is the total number of months in the data sample, and the mean expected duration as

$$ED^i = \frac{\sum ED^i_n}{N}.$$ 

Table 1 provides the descriptive statistics for these measures.\(^\text{14}\)

\(^\text{14}\)All figures are calculated up to December 1998 since expected duration becomes unreliable after this time.
Table 1

Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>(1 - p^i)</th>
<th>(\lambda^i)</th>
<th>(ED)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.510</td>
<td>0.084</td>
<td>12.35</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.033</td>
<td>0.019</td>
<td>2.56</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.065</td>
<td>0.223</td>
<td>0.21</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>0.464</td>
<td>0.069</td>
<td>8.53</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.553</td>
<td>0.120</td>
<td>14.25</td>
</tr>
<tr>
<td>Number of Observations</td>
<td></td>
<td></td>
<td>254</td>
</tr>
</tbody>
</table>

All figures weighted by population

The coefficient of variation implies there is much more variation in LSU matching rates \(\lambda^i\) (and expected durations) across local labor markets, than in incidence \(1 - p^i\). For example, the average duration of LSU, \(ED\), was 12.3 months over this sample period, but 10% of the population mass (recall, cities are weighted by population) had average LSU durations exceeding 14.2 months, while another 10% had LSU durations less than 8.5 months. In contrast, the incidence of LSU ranged only from 0.46 to 0.55. This result is consistent with the scatter plot of the unemployment spells data, \(X^c, X^u\), in Figure 3. The scatter is tightly distributed around the line \(X^c = 0.5X^u\), suggesting little variation in the incidence of LSU across TTWAs, while the larger cities clearly have longer uncompleted unemployment spells.

Figure 8 presents a (weighted) scatter plot of the incidence of unemployment against expected duration. Two main features are evident: (a) the larger cities are correlated with high expected durations of LSU, and (b) cities with longer expected durations of unemployment have lower LSU incidence (the raw correlation being \(-0.278\)). Using a log linear regression equation and weighted least squares, Table 2 describes how these variables are correlated with city size (population and geographical area) and region across England and Wales.
Table 2
Weighted Least Squares Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\log(1 - p_i)$</th>
<th>$\log(ED_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.487* (.078)</td>
<td>1.143* (.228)</td>
</tr>
<tr>
<td>log(Population)</td>
<td>-0.011* (.004)</td>
<td>0.127* (.013)</td>
</tr>
<tr>
<td>log(Area)</td>
<td>0.003 (.007)</td>
<td>-0.036 (.020)</td>
</tr>
<tr>
<td>REGION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Anglia</td>
<td>-0.148 (.022)</td>
<td>0.194* (.066)</td>
</tr>
<tr>
<td>East Midlands</td>
<td>-0.113* (.019)</td>
<td>0.244* (.055)</td>
</tr>
<tr>
<td>North</td>
<td>-0.188* (.019)</td>
<td>0.325* (.056)</td>
</tr>
<tr>
<td>North West</td>
<td>-0.107* (.015)</td>
<td>0.158* (.047)</td>
</tr>
<tr>
<td>South East</td>
<td>-0.083* (.015)</td>
<td>0.026 (.044)</td>
</tr>
<tr>
<td>South West</td>
<td>-0.118* (.019)</td>
<td>0.188* (.056)</td>
</tr>
<tr>
<td>Wales</td>
<td>-0.124* (.021)</td>
<td>0.287* (.062)</td>
</tr>
<tr>
<td>West Midlands</td>
<td>-0.111* (.016)</td>
<td>0.240* (.046)</td>
</tr>
<tr>
<td>Yorkshire &amp; H/side</td>
<td>-0.148* (.017)</td>
<td>0.262* (.050)</td>
</tr>
<tr>
<td>$\bar{r}^2$</td>
<td>0.40</td>
<td>0.47</td>
</tr>
<tr>
<td>$N$</td>
<td>254</td>
<td></td>
</tr>
</tbody>
</table>
As suggested by the scatter plots, there are significant city size effects. The largest cities tend to have a slightly smaller incidence of LSU, but each LSU worker expects much longer unemployment spells. A doubling of the city size leads to a 1% fall in incidence, and a 13% increase in the expected durations of unemployment. The overall effect implies that LSU is much more of a problem in the largest cities.\(^{15}\)

There is also strong evidence of a North-South divide on LSU. The omitted region variable is the ‘London Area’. The regressions find that the incidence of LSU in the ‘northern’ regions is lower but the expected durations of LSU are much higher. Evaluated at the means, expected durations on average rise by more than 25%, or approximately three months, in the eight regions outside southeastern England. In contrast, the average incidence of becoming LSU in these regions is 12% lower. This implies that the north is characterized by a smaller minority of workers experience much longer spells of long-side unemployment.

4. Conclusion

This paper has shown how the stock-flow approach to job matching decomposes unemployment flows and spells data into easily computed measures of the incidence \((1 - p)\) and average matching rate \((\lambda)\) of long-side unemployment. It has also identified estimates of the expected duration of long-side unemployment. The estimates are consistent not only with intuition - the incidence and expected duration of LSU is highest in the recession and lowest in the boom - but also with the way in which average completed and uncompleted spells of unemployment evolve over the cycle.\(^{16}\) The paper further finds a significant city size effect and a ‘North-South’ divide where

(i) LSU is a bigger problem in the larger cities (higher expected durations) and

(ii) the ‘North’ is characterized by a smaller minority of workers experiencing much longer spells of unemployment

\(^{15}\)Other regressions not reported here find that demographic, industrial and occupational characteristics of the TTWAs from the 1991 census do not correlate strongly with either risk or duration.

\(^{16}\)The results also corroborate the findings of Coles and Petrongolo (2003). Using vacancy data rather than unemployment spells data, they estimate sample averages \(1 - p \approx 0.5\) and \(\lambda \approx 0.07\) for this data period. They also find support for the econometric specification \(\lambda = \lambda(v/U)\), which implies that the long term unemployed are job rationed and chase new vacancies as they come onto the market.
These estimates have potentially important implications for labor market policies.\textsuperscript{17} Identifying the incidence and average matching rate of long-side unemployment provides policy makers with the underlying parameters of interest for assessing the impact of public programs. In contrast, compositional effects compromise the standard measures of the related phenomenon of long-term unemployment—the proportion of those unemployed with spells exceeding 6 or 12 months. As seen in Figure 9, these measures fall at the onset of the recession which, naively interpreted, suggests that long-term unemployment is less of a problem in recessions. Our estimates demonstrate why this outcome is misleading. The recession is characterized by an increased inflow of workers into the pool of unemployment. As these workers initially have short unemployment spells, the proportion of workers with durations exceeding 6 or 12 months falls. The conventional measures of long-term unemployment are thus not interesting numbers to look at. Unlike the measures developed here, they fail to distinguish between workers who have short durations and workers who expect to be unemployed into the long-term.

Further work, of course, is required. An obvious but difficult direction for future research is to relax the assumption that workers on the short side of the market match arbitrarily quickly. A natural extension would be to assume search frictions bind for these workers—it takes time to locate and take up the most preferred job opportunity in the current vacancy stock. The results of Coles and Smith (1998) suggest this search process may take up to a month in time. The results presented here find that roughly half of all newly unemployed workers are on the short side of their submarket. If instead such workers match, on average, in say two or three weeks (rather than immediately), and given our matching parameters fit the observed outflow rate $M$, then our results overstate the average duration of long-side unemployment by a similar two or three weeks. This bias is small relative to the estimated durations of long-side unemployment.\textsuperscript{18} Such an extension would nevertheless provide a natural bridge between stock-flow...
Proportion of unemployed workers out of work for greater than 6 months and 12 months

Figure 9: Standard Measures of Long-Term Unemployment

matching and the standard search literature. The interpretation instead being that the short-term unemployed are on the short side of their skills market but matching frictions delay their immediate re-employment, while the LSU have to wait for something suitable to come onto the market.

APPENDIX

An important criticism of the above approach is that it assumes all LSU job seekers match at the same rate. This appendix briefly considers introducing search heterogeneity across the LSU. To keep things simple, suppose all LSU workers can match with each new vacancy (e.g., an efficiency wage story as in Shapiro and Stiglitz, 1984). In addition, assume that the first job seeker on the long side who contacts a new vacancy gets the job.

Stock-flow matching implies that the LSU chase new vacancies as they come onto the market (see Coles and Petrongolo, 2003, for evidence). The frequency
with which a LSU worker checks the vacancy boards affects that worker’s probability of winning the next vacancy chase. It does not, however, affect the aggregate outcome - a vacancy on the short side is always quickly filled by someone.\textsuperscript{19}

More precisely, suppose a LSU worker \( j \) checks vacancies with frequency \( k_j \), and let \( K = \sum_j k_j \) denote aggregate search effort by the long-side unemployed, \( j = 1, \ldots, U \) where \( U \) denotes the number of LSU workers. Job rationing implies individual \( j \)'s re-employment rate is

\[
\lambda_j = \frac{k_j}{K} v,
\]

where \( v \) describes the inflow of new vacancies. A worker who checks the ads twice as often is twice as likely to win the next vacancy chase. Note, however, that if all job seekers double their search efforts, no change in matching probabilities occurs. There are only pure displacement effects - the worker who wins the next vacancy chase does so at the cost of the other workers. Moreover, the aggregate matching rate of all LSU workers, \( \sum_j \lambda_j \), is given by \( \sum_j \lambda_j = v \), which is independent of \( \{k_j\}_{j=1}^U \). The number of LSU workers who match depends entirely on the inflow of new vacancies.

Should some LSU workers check the vacancy boards more frequently than others, the stock-flow matching framework is consistent with negative duration dependence in the hazard function. But most importantly, dispersion in \( k_j \) does not affect the aggregate outcome. A new vacancy on the short side is quickly filled by a LSU worker. The critical point is that identifying matching behavior using micro level data is problematic when individual search efforts \( k_j \) are unobserved (and potentially time varying). In contrast at the aggregate level, the representative LSU worker who chooses average search effort \( \bar{k} = K/U \) matches at rate \( \lambda = v/U \). Assuming pure job displacement effects, the average matching rate is independent of the unobserved \( \{k_j\} \) and so can be identified using aggregate data.

Nevertheless, dispersion in \( \{k_j\} \) generates interesting composition effects. The estimates here find that the expected duration of long-side unemployment (computed using average matching rates) peaks at fifteen months (see Figure 7). This seems too low given that the average uncompleted spell of unemployment reaches 18 months for this data set (see Figure 3). Heterogeneity in \( \{k_j\} \) can account for

\textsuperscript{19}Given the data is recorded as a monthly time series, it is immaterial whether a short-side vacancy is filled the same day or within a few days of being posted.
this discrepancy. In a steady state, the average expected duration of unemploy-
ment for all LSU workers is

\[
\overline{ED} = \frac{1}{U} \sum_{j=1}^{U} \frac{1}{\lambda_j} = \frac{1}{v} \sum_{j=1}^{U} \frac{k}{k_j}.
\]

If some choose zero search effort, \(k_j = 0\), then \(\overline{ED}\) must be arbitrarily large
and so the mean uncompleted spell of unemployment in the stock of unemployed
workers can also be arbitrarily large. More interestingly, note that \(\overline{ED}\) is a convex
function of \(k_j\). A mean preserving spread in \(k_j\) implies an increase in \(\overline{ED}\). The
average duration of unemployment for all LSU workers is therefore minimized
when all choose the same search effort; i.e. \(k_j = \overline{k}\) in which case \(\lambda_j = \lambda = v/U\).

By identifying the average matching rate of the LSU, estimates of the expected
duration of unemployment suggest a lower bound for the average duration of un-
completed unemployment spells. Of course, variations in \(\{k_j\}\) do not have any
implications for aggregate matching in (1), as matching by the LSU is driven by
the arrival of suitable new vacancies. Variations in \(k_j\), however, affect the distribu-
tion of unemployment spells across the LSU. Equation (3) requires modification.

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