How Important is Human Capital?
A Quantitative Theory Assessment
of World Income Inequality*

Andrés Erosa
IMDEA

Tatyana Koreshkova
Concordia University and CIREQ

Diego Restuccia
University of Toronto

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Abstract
We build a model of heterogeneous individuals — who make investments in schooling quantity and quality — to quantify the importance of differences in human capital versus TFP in explaining the variation in per-capita income across countries. The production of human capital requires expenditures and time inputs; the relative importance of these inputs determines the predictions of the theory for inequality both within and across countries. We discipline our quantitative assessment with a calibration firmly grounded on U.S. micro evidence. Since in our theory human capital production requires expenditures, TFP changes affect disproportionately the benefits and costs of human capital accumulation. Our main finding is that human capital accumulation strongly amplifies TFP differences across countries: To explain a 20-fold difference in the output per worker the model requires a 5-fold difference in the TFP of the tradable sector, versus an 18-fold difference if human capital is fixed across countries.

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1 Introduction

While economists consider human capital a crucial component of aggregate wealth, they have conflicting views on the importance of differences in human capital versus total factor productivity (TFP) in accounting for income differences across countries. In this paper, we develop a quantitative theory of human capital investments to quantify the importance of differences in human capital versus TFP in explaining the variation in per-capita income across countries. Building a quantitative theory allows us to circumvent two major problems faced by growth accounting exercises. First, to date, there are no reliable cross-country measures of the quality of schooling across countries. If this quality is positively associated with the level of economic development, the residual in growth accounting exercises overstates the cross-country differences in TFP. A second problem arises due to the (unobserved) covariance of TFP with measures of physical and human capital, which renders output variance decomposition difficult. Developing a quantitative theory, in turn, is a challenging task due to the lack of conclusive micro-evidence on the parameters of the human capital technology.

In light of these difficulties, our paper provides a novel approach to studying income differences across countries: We build a model of heterogeneous individuals — making investments in schooling quantity and quality — that is consistent with a broad set of microfacts. Motivated by the empirical studies of Neal and Johnson (1996) and Keane and Wolpin (1997), we focus on investments that take place “early” in life of an individual and formulate a dynastic model of parental investments in the human capital of their children. Individuals are heterogeneous in terms of ability, schooling tastes, and parental resources. Individuals across countries face different wage rates and prices for human capital inputs. The model’s main novelty relative to previous work in the area is the inclusion of a production technology for human capital which takes expenditures and time as inputs.\footnote{Manuelli and Seshadri (2005) and Cordoba and Ripoll (2007) follow a similar approach.} The relative importance of these inputs determines the predictions of the theory for inequality both within and across
countries. The intuition is simple: If schooling requires only time inputs, a change in the ability of individuals or in the wage rate affects equally the benefits and the costs of human capital accumulation, leaving the optimal level of human capital unchanged. On the other hand, when schooling requires only input of goods, an increase in ability or in the wage rate raises benefits but not the costs of schooling, hence increasing the optimal human capital stock.

We discipline our quantitative assessment with a calibration firmly grounded on U.S. micro evidence. We exploit the fact that the parameters governing human capital accumulation have important consequences for schooling and earnings inequality and intergenerational mobility within a country. Hence, we use U.S. household data to pin down the key parameters — elasticities of human capital with respect to time and goods inputs — driving the quantitative implications of the theory across countries. Our baseline economy successfully matches a large number of calibration targets on schooling and earnings inequality in the U.S., such as the variances and intergenerational correlations of earnings and schooling, and the slope coefficient and $R^2$ in a Mincer regression. The model economy is also consistent with several dimensions of heterogeneity in the data that were not targeted in the calibration: schooling distribution, evidence on the relationship between schooling attainment of children and resources/background of their parents, and results in the micro literature on the enrollment effects of college tuition changes. Altogether, the paper provides an important contribution to the literature by developing a successful theory of inequality in schooling and earnings in the U.S. economy.

We use the calibrated model economy to quantitatively assess how variations in TFP are amplified through human capital accumulation into large differences in output per worker across countries. We assume that countries are identical in terms of preferences and technolo-

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2Bils and Klenow (2000) point out that the production of human capital is more intensive in time input than the production of output goods. They and Klenow and Rodriguez-Clare (1997) argue that, by using a one-sector growth model, Mankiw, Romer, and Weil (1992) overstate the importance of goods input in the production of human capital and, thus, obtain results that understate TFP differences across countries. The idea that the specification of the human capital technology has important implications for inequality within a country was developed by Erosa and Koreshkova (2007) in the context of a taxation exercise.
gies and only differ in their level of TFP. Following Hsieh and Klenow (2007) and Herrendorf and Akos (2007), we model sectorial productivity differences across countries. Relative to the benchmark economy, we assume that a one percent reduction on the TFP of the manufacturing (tradable) sector is associated with a 0.3 percent reduction on the TFP of the service (nontradable) sector. This assumption allows the model economy to match the cross-country variation on the price of services relative to manufacturing.\(^3\)

Our main finding is that human capital accumulation strongly amplifies TFP differences across countries: The elasticity of output per worker – at PPP prices – with respect to TFP in the tradable sector is 1.94. This implies that a 5-fold difference in TFP explains a 20-fold difference in the output per worker, observed between the top 10 percent and bottom 10 percent of countries in the world income distribution. In contrast, when we solve a version of the model without human capital accumulation, an 18-fold difference in the TFP of the tradable sector is required to account for the income difference between rich and poor countries. Two main channels explain why human capital provides substantial amplification. First, our calibration implies a large share of expenditures in the human capital production function, which means that a reduction in TFP affects disproportionately the benefits and costs of human capital accumulation. Hence, while the benefit of obtaining human capital is proportional to TFP, the cost of education (relative to the price of output) is less than proportional to TFP. This mechanism accounts for the low schooling quantity and quality in poor countries. Second, human capital is an important source of income differences across countries, not only because it directly contributes to cross-country output differences, but also because a lower human capital stock discourages physical capital accumulation by lowering the marginal product of capital.

Finally, we provide several pieces of evidence that support the main predictions of the theory. First, we show that the cross-country differences in schooling implied by our theory

\(^3\)Our baseline experiments minimize the role of human capital in amplifying income differences across countries by assuming that human capital investments only require services. Since poor countries have a comparative advantage in the production of human capital inputs (services are relatively cheap), their low aggregate productivity is not too detrimental to human capital accumulation.
are plausible. Second, we use our model to simulate immigrants in the U.S. and find that some (modest) degree of selection into immigration can reconcile the quality differences in education predicted by our theory with the data on earnings of U.S. immigrants. Third, we show that our theory is consistent with data on relative price variation across countries. In particular, using data from The International Comparisons Programme (ICP) we show that the elasticity of the price of education relative to output is close to zero in the ICP, supporting the driving force in our theory that the cost of education (relative to the price of output) does not rise proportionally with TFP.\(^4\)

The paper proceeds as follows. The next section describes in detail the economic environment. In section 3, we consider a simple version of the model in order to illustrate the main features of our theory driving human capital investments and to motivate our calibration strategy. Section 4 lays out the calibration strategy for the benchmark economy and shows that the model economy is consistent with several dimensions of heterogeneity in the data that were not targeted in the calibration. Furthermore, the predictions of the benchmark economy are tested using results from the micro literature on the enrollment effects of college tuition changes. In section 5, we evaluate the aggregate impact of TFP differences across countries, examine the predictions of the theory for the variation in relative prices across countries, and compare our findings to related papers in the literature. Section 6 concludes.

2 Economic Environment

We consider an economy populated by overlapping generations of people who are altruistic toward their descendants and invest in the human capital of their children.\(^5\) Investments in

\(^4\) As pointed by one referee, the small elasticity of the price of education with respect to output is obtained despite the fact that the ICP cannot take into account unmeasured quality differences in education across countries.

\(^5\) This approach is motivated by some empirical studies of earnings inequality. Keane and Wolpin (1997) find that 90% of the variance of lifetime utility is accounted for by heterogeneity in skills of individuals at age 16, i.e. prior to labor market entry. Similarly, Neal and Johnson (1996) find that parental investments are crucial essential for explaining differences in skill attainments of their children.
human capital involve children’s time and expenditures by parents that affect the quality of
the human capital of their children. Parents cannot borrow to finance investment in human
capital. Since the analysis in this paper focuses on steady states, time subscripts are omitted
in the description of the model and use a prime to indicate the next period value of a given
variable.

Demographic Structure There is a large number of dynasties (mass one). Individuals
live for three periods, so that the model period is set to 20 years. An individual is referred
to as a child in the first period of his life (real age 6-26 years), a young parent in the second
period (real age 26-46 years), and an old parent in the third period of his life (real age 46-66
years).\textsuperscript{6} A household is composed of 3 people: old parent, young parent, and a child.

Production Technologies We assume that production takes place in two sectors — man-
ufacturing and services — with the following technologies:

\begin{align*}
Y_M &= A_M K_M^\alpha H_M^{1-\alpha}, \\
Y_S &= A_S K_S^\alpha H_S^{1-\alpha}
\end{align*}

where $Y_M$ and $Y_S$ denote the output of the manufacturing and service sectors; $K_i$ and $H_i$
represent the services of physical and human capital used in sector $i \in \{M, S\}$. Parameter
$\alpha \in (0, 1)$ is the elasticity of output with respect to physical capital and is assumed to be
equal across sectors. Parameter $A_i$, $i \in \{M, S\}$, represents a sectorial TFP, which is allowed
to vary across sectors.

Manufacturing output can be consumed ($C_M$) or invested in physical capital ($X$). Services

\textsuperscript{6}In an earlier version of the paper, we modeled a retirement period. Since it did not affect the quantitative
implications of the theory, we decided to abstract from retirement in the current version of the paper.
can be either consumed ($C_S$) or invested in human capital ($E_S$). Feasibility requires

$$C_M + X = Y_M,$$

$$C_S + E = Y_S.$$  

Physical capital is accumulated according to $K' = (1 - \delta)K + X$, where investment goods $X$ are produced in the manufacturing sector.

We model human capital investments that take place ‘early’ in the life of an individual and that include schooling as well as investments outside of formal schooling. Consistent with the view of Becker and Tomes (1986), Haveman and Wolfe (1995), Neal and Johnson (1996), Mulligan (1997), Keane and Wolpin (1997, 2001), among many others, we think that households invest a lot of resources in their children outside of school (health, food, shelter, books, recreational activities and extracurricular educational activities). This view motivates our focus on a broad notion of human capital investments. The human capital of a child is produced with the inputs of schooling time ($s \in [0, 1]$) and expenditures in human capital quality ($e > 0$) according to the following production function:

$$h_c = A_H \ z \ (s^\eta e^{1-\eta})^\xi \ \eta, \xi \in [0, 1],$$  

(3)

A unit of schooling time (quantity of schooling) is produced with one unit of a child’s time and $\bar{l}$ units of market human capital services. In other words, schooling requires own time and human capital purchased in the market.\footnote{Schooling $s$ is a Leontief function of own time $t$ and market human capital services $h_s$: $s = \min\{t, \frac{h_s}{\bar{l}}\}$.} Educational expenditures in quality are assumed to be in terms of services.\footnote{In Erosa, Koreshkova, and Restuccia (2009), we allow for educational expenditures to be a composite of manufacturing goods and services.}

To model heterogeneity across individuals, we follow the micro literature in allowing individuals to differ in terms of their ability $z$ and their taste for schooling $\theta$. We assume that the shocks to $z$ and $\theta$ are idiosyncratic to each dynasty and that they are observed
at the beginning of the period, that is, before human capital investments take place. The ability \( z \) is transmitted across generations according to a discrete Markov transition matrix \( Q(z, z') \), where \( q_{i,j} = Pr(z' = z_i | z = z_j) \). The taste shock \( \theta \) is iid across individuals and, possibly, correlated with the current realization of the ability shock \( z \). The distribution of the taste shock is thus described by a discrete matrix \( Q_\theta(z, \theta) \). The parameter \( A_H \) in the human capital production function (3) is common across all individuals in the economy and is normalized to 1 in our baseline economy.\(^9\)

**Preferences** A per-period utility function of the household is

\[
\frac{1}{1-\sigma} \left( (C_M)^\gamma (C_S)^{1-\gamma} \right)^{1-\sigma} + v(s, \theta),
\]

where \( C_M \) represents consumption of manufacturing goods and \( C_S \) consumption of services. The term \( v(s, \theta) \) represents utility of schooling, where \( \theta \) is a taste shock that varies across individuals. Thus, consistent with the micro literature on schooling (see, for instance, Keane and Wolpin (2001) and Card (2001)), in our model heterogeneity in schooling decisions is driven by variation not only in parental wealth and labor market returns (ability) but also in schooling tastes. We model many sources of schooling variation across individuals because our calibration will target the variance in schooling in the US economy. Had we assumed that individuals only differ in ability, our calibration would have potentially exaggerated the elasticity of schooling decisions to variation in ability. In this case, our results would have likely overestimated the response of human capital investments to TFP, as in our theory differences in ability across individuals operate similarly to differences in TFP across countries. **Is it clear that we mean exogenous heterogeneity since our people also differ in parental resources?**

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\(^9\)In Erosa, Koreshkova, and Restuccia (2009), we consider cross-country variation in the efficiency of the human capital technology by allowing the parameter \( A_H \) to vary across economies (Klenow and Rodriguez-Clare (2001) allow for the possibility that countries differ in the productivity of the education sector.)
Market Structure and Relative Prices  We assume competitive markets for factor inputs and outputs. Profit maximization in the manufacturing and services sectors imply

\[ p_S = \frac{A_M}{A_S}, \]  

where we have set the price of manufacturing goods to 1 (numeraire). COULD PUT THE FORMULA IN THE TEXT. IT IS REFERENCED ONLY ONCE LATER ON, THERE WE COULD USE THE ACTUAL FORMULA INSTEAD OF THE REFERENCE NUMBER.

The optimal allocation of total expenditure \( C \) between consumption of manufacturing goods \( C_M \) and services \( C_S \) solve the static problem

\[
c(C) \equiv \max_{C_M, C_T} (C_M)^{\gamma_j} (C_S)^{1-\gamma_j} \
\text{s.t.} \quad p_SC_S + C_M = C.
\]

Optimal behavior implies that total consumption expenditures \( C' = \frac{(p_S)^{1-\gamma_j}}{\gamma_j (1-\gamma_j)^{1-\gamma_j}} c = p_c c \), where \( p_c \equiv \frac{(p_S)^{1-\gamma_j}}{\gamma_j (1-\gamma_j)^{1-\gamma_j}} \).

Public Education  Since our calibration strategy is to use cross-sectional heterogeneity within a country to restrict the parameters governing human capital accumulation, we cannot abstract from the effects of public education on education and labor market outcomes. We model public education by assuming that education expenditures are subsidized at the rate \( p \) per unit of schooling time. These expenditures are financed with a proportional tax \( \tau \) on households’ income. Public and private expenditures are perfect substitutes in the production of human capital.

Decision Problem of the Household  All decisions of the household are made by the young parent. The state of a young parent is given by a quadruple \((g, h_p, z, \theta)\): resources of
the old parent \( q \), human capital of the young parent \( h_p \), child’s ability \( z \), and child’s taste for schooling \( \theta \). Households face uncertainty over the realization in future periods of ability, school taste and market luck, hence, they maximize the expected discounted lifetime utility of all generations in the dynasty. Young parents choose consumption \( c \), assets \( a' \), time spent in school by their children \( s \) (where \( 1 - s \) is the working time of the children), and resources spent on the quality of education of their children \( e \). A parent who provides his child with \( s \) years of schooling and a quality of education \( e \) incurs expenditures \( P_S e + (w\bar{l} - p)s \), where \( w\bar{l} \) is the cost of market human capital services per year of education, and \( p \) denotes public education expenditures (or subsidies) per year of education.

The decision problem of a young household can be written using the dynamic programming language as follows:

\[
V(q, h_p, z, \theta) = \max_{c, e, s, h'} \left\{ U(c) + \nu(s, \theta) + \beta \sum_{z', \theta', \mu'} Q_z(z, z')Q_{\theta}(z', \theta')Q_{\mu}(\mu')V(q', \mu' h, z', \theta') \right\},
\]

subject to

\[
P_c c + P_S e + (w\bar{l} - p)s + a = (1 - \tau)w[\psi_2 h_p + \psi_1 h_c(1 - s)] + q,
\]
\[
h_c = A_H z \left( s^\eta e^{1-\eta} \right)^\xi,
\]
\[
q' = (1 - \tau)[w\psi_3 h_p + ra] + a
\]
\[
a \geq 0, \quad s \in [0, 1],
\]

where \((\psi_1, \psi_2, \psi_3)\) are life-cycle productivity parameters. The first two terms in the objective function are the current period period utility and the third term is the expected discounted future utility. The expectations of the next period’s value function is taken over the market luck of the current child \( \mu' \) and over the ability \( z' \) and school taste \( \theta' \) of the child born in the next period. The first constraint is the household budget constraint, where the right hand side is given by the sum of the earnings of the young parent and the child upon finishing
school \((1 - \tau)w[\psi_2 h_p + \psi_1 h_c(1 - s)]\) and the resources \(q\) (after tax earnings and gross asset return) brought to the current household by the old parent. The third constraint defines the parental wealth \(q'\) of the next household in the dynasty line.

We emphasize that when young parents make education decisions for their children, they know the ability \(z\) and the taste for schooling \(\theta\). However, they face uncertainty regarding the market luck of their children \(\mu'\), which is realized in the adult stage of the individual’s life cycle. The human capital \(h_c\) of an individual at the end of period 1 evolves stochastically, according to a realization of a market luck shock \(\mu'\): \(h_p = \mu' h_c\), where \(\mu'\) is iid across individuals and time according to a density \(Q_{\mu}(\mu)\) with a mean equal to 1. For tractability reasons and motivated by the empirical cross-country evidence,\(^{10}\) our theory abstracts from on-the-job human capital accumulation. We assume that markets are imperfect in that households cannot perfectly insure against labor-market risk and the human-capital shocks affecting their descendents. Moreover, individuals cannot borrow.

3 Human Capital Investments in a Complete Markets Environment

This section provides some analytical results that shed light on how the parameters of the human capital technology determine the quantitative implications of the theory. To study a simplified version of the model economy, we assume complete markets and abstract from tastes for schooling. As a result, human capital investment decisions are independent of consumption decisions and maximize lifetime income. We show that the quantitative implications of the theory for income inequality – within and across countries – depend crucially

\(^{10}\)Using the coefficients for returns to experience for each country reported in Bils and Klenow (2000), we found that the earnings of a worker with 20 years of experience relative to a worker with 10 years of experience is not systematically related to the level of per-capita income across countries. In fact, we found a small negative correlation between returns to labor market experience (wage growth) and per-capita income across countries, which suggests that on-the-job investments in human capital are not likely to be an important source of income differences across countries.
on the expenditure elasticity of human capital. We also study how cross-country differences in sectorial productivities generate heterogeneity in relative prices and human capital investments.

3.1 Human capital investments across individuals and countries

Consider a world with a large number of countries. Each country is populated by measure 1 of dynasties and by a vector of prices \((w, p_S)\) that varies across countries. Capital markets are assumed to be perfect so that in equilibrium individuals make efficient investments in human capital. The attention is confined to the steady-state analysis. The equilibrium interest rate is given by the individuals’ rate of time preference \(\rho\). Although the theory makes no predictions for the distribution of income, consumption, and wealth, it does have important implications for the variation of schooling and earnings across individuals and countries.

3.1.1 The decision problem

We analyze how variation in wages and variation in ability \((z)\) lead to heterogeneous human capital investments across countries (derive macro elasticities) and across individuals (derive micro elasticities). The goal is to isolate the effects of the parameters of the human capital technology on micro and macro elasticities in our model.

Consider the decision problem of an individual with ability \(z\) in a country with a wage rate \(w\) and a price of education services \(p_S\), where these prices are expressed in terms of the manufactured good. The human capital investment decision can be formulated as choosing schooling time \((s)\) and expenditures \((e)\) to maximize the present value of the lifetime earnings net of the education costs:

\[
\max_{e,s,h} \left\{ w(1 - s)h\psi + wh\Psi - p_S e - w\bar{I}s \right\}
\]
s.t. \( h = A_H z (s^n e^{1-\eta})^\xi, \)  

(7)

where \( \Psi = \sum_{i=2}^{3} \beta^{i-1} \psi_i \) with \( \psi_i \) representing the life cycle productivity parameters described in the previous section, and \( \beta = \frac{1}{1+r} \) provided \( r = \rho \). The cost of schooling includes expenditures in human capital quality \( (e) \), time-purchases on the market (tuition costs) per unit of schooling time \( (w\bar{l}) \), and foregone earnings in the first period of life \( (sw\psi_0) \).

Assuming an interior solution, the first-order conditions can be expressed as:

\[
A_H z (s^n e^{1-\eta})^\xi \left\{ -\psi_0 + \frac{\eta \xi}{s} [(1-s)\psi_1 + \Psi] \right\} = \bar{l} \tag{8}
\]

\[
e = \left\{ \frac{w}{p_S} A_H z (1-\eta) \xi [(1-s)\psi_1 + \Psi] s^{\eta \xi} \right\}^{1/(1-\eta \xi)}. \tag{9}
\]

In the absence of tuition costs \( (\bar{l} = 0) \), it is easy to solve for \( s \) from (8) and verify that the optimal quantity of schooling does not vary across individuals with different values of \( z \). Intuitively, when there are no tuition costs of schooling \( (\bar{l} = 0) \) a change in \( z \) raises proportionally the benefits and costs of schooling and has no effect on the optimal choice of years of schooling. Moreover, when \( \bar{l} = 0 \), there is no variation in schooling across countries \( (w, p_S) \). We thus maintain \( \bar{l} > 0 \). Similarly, in the absence of education expenditures \( (\eta = 1) \), the quality of schooling does not vary across individuals and countries. On the contrary, when \( 0 < \eta < 1 \), equations (8) and (9) imply that both quantity and quality of schooling vary across individuals \( (z) \) and countries \( (w, p_S) \).

**Proposition 1**: The theory requires \( \bar{l} > 0 \) and \( 0 < \eta < 1 \) in order to generate differences in the quantity and quality of schooling across individuals \( (z) \) and countries \( (w, p_S) \).

### 3.1.2 Micro-elasticities

To gain insights with simple algebra, it is convenient to set \( \psi_1 = 0 \). Combining (8) and (9), taking logs, and differentiating with respect to \( \ln z \), gives an expression for the individual
(ability) elasticity of schooling:

\[ E_{sz} \equiv \frac{\partial \ln(s)}{\partial \ln z} = \frac{1}{1 - \xi}. \]  

(10)

The elasticity of expenditures with respect to ability is obtained by differentiating (9) with respect to \( \ln z \):

\[ E_{ez} \equiv \frac{\partial \ln(e)}{\partial \ln z} = \frac{1}{1 - (1 - \eta)\xi} \left(1 + \eta\xi \frac{\partial \ln(s)}{\partial \ln z}\right) = \frac{1}{1 - \xi}, \]  

(11)

where the last equality used (10). The elasticity of human capital with respect to the ability, is obtained by log-differentiating (7) with respect to \( \ln z \) and by using (10) and (11):

\[ E_{hz} \equiv \frac{1}{1 - \xi}. \]

Taking stock of the above findings, we note that the elasticities of schooling and human capital with respect to ability are the same. The magnitude of this elasticity is determined by the returns to scale in the human capital accumulation technology (parameter \( \xi \)). For a given distribution of \( z \) in the population, the variations in both schooling and human capital increase with the returns-to-scale parameter \( \xi \). Hence this parameter is important for the predictions of the theory on the cross-sectional inequality in schooling and earnings.

### 3.1.3 Macro-elasticities

Combining (8) and (9), taking logs, and differentiating with respect to \( \ln w \), we obtain the cross-country (wage) elasticity of schooling:

\[ E_{sw} \equiv \frac{\partial \ln(s)}{\partial \ln w} = \frac{(1 - \eta)\xi}{1 - \xi}. \]  

(12)
Using (12), the variation in the schooling quality across countries \((w)\) satisfies

\[
E_{ew} \equiv \frac{\partial \ln(e)}{\partial \ln(w)} = \frac{1}{1 - (1 - \eta)\xi} \left(1 + \eta\xi\frac{(1 - \eta)\xi}{1 - \xi}\right).
\] (13)

Log-differentiating (7) with respect to \(\ln(w)\), together with (12) and (13), gives

\[
E_{hw} \equiv \frac{(1 - \eta)\xi}{1 - \xi}.
\] (14)

Since \(E_{hw}\) does not vary across individuals, the aggregation is trivial: If two countries differ in TFP by a ratio \(A_R\), then their ratio of aggregate human capital is: \(H_R = \int (W_R)^{E_hA} dG_z = (W_R)^{E_hA}\). We conclude that \(E_{Hw} = E_{hw} = \frac{(1 - \eta)\xi}{1 - \xi}\). Table 1 summarizes the mapping from the model parameters into the micro and macro elasticities.

Table 1: Elasticities for the Deterministic Model

<table>
<thead>
<tr>
<th>Macro</th>
<th>Micro</th>
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<tbody>
<tr>
<td>(E_{Hw} = E_{hw} = E_{sw})</td>
<td>(E_{sz} = E_{hz} = E_{ez})</td>
</tr>
<tr>
<td>(E_{ew})</td>
<td>(1 - \xi)</td>
</tr>
<tr>
<td>(E_{x,p_e} = -E_{x,w})</td>
<td>(1 - (1 - \eta)\xi)</td>
</tr>
<tr>
<td>(E_{x,w} = E_{x,w})</td>
<td>for (x \in {s, e, h})</td>
</tr>
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We are now ready to explore the sensitivity of the wage elasticity of human capital to the parameters of the human capital technology. Since \(E_{hw}\) increases with the returns-to-scale parameter \(\xi\) and decreases with the time share parameter \(\eta\), \(E_{hw}\) increases with the expenditure elasticity of human capital and is maximized when \(\eta = 0\) and \(\xi = 1\). As the time-share parameter \(\eta\) decreases from 1 to 0, \(E_{Hw}\) takes values in the interval \([0, \frac{\xi}{1 - \xi}]\). For instance, when \(\xi = .9\), \(E_{Hw}\) takes values between 0 and 9, depending on the time-share parameter. In other words, a wage ratio of 3 can generate differences in human capital per
worker anywhere from a factor of 0 to 20 thousand.\footnote{While \( E_{hw} \) is determined both by \( \eta \) and \( \xi \), note that the expenditure elasticity alone provides a lower bound to the amplification effect. This is because \( \eta \geq 0 \) implies \( (1 - \eta) \xi \leq \xi \), which together with (14) and \( \xi < 1 \) implies that \( E_{hw} \in \left[ \frac{(1-\eta)\xi}{1-(1-\eta)\xi}, \infty \right) \). On the other hand, the parameter \( \xi \) implies an upper bound for \( E_{hw} \), since \( E_{hw} \) varies from 0 to \( \frac{\xi}{1-\xi} \) for all feasible values of \( \eta \).}

**Proposition 2:** The amplification effect of human capital, given by (14), depends crucially on the expenditure elasticity of human capital \((1 - \eta) \xi\). In particular, if the expenditure share is zero \((\eta = 1)\), then human capital does not amplify TFP differences across countries, no matter how close \( \xi \) is to 1.

Countries might not only differ in wage rates but on the relative price of education services \((p_S)\). Inspection of the individual’s optimization conditions (8) and (9) implies that the elasticities of the variable \( x \in \{s, e, h\} \) with respect to \( p_S \) are \( E_{x,p_e} = -E_{x,w} \) and \( E_{x,w}^{p_e} = E_{x,w} \). Note that human capital investment decisions are determined by the ratio of the wage rate to the price of education services.

### 3.2 Amplification with cross-country variation in sectorial productivities

We have shown that cross-country differences in relative prices \((w, p_S)\) generate a cross-country variation in human capital investments. The next step is to analyze how cross-country differences in sectorial productivities generate heterogeneity in prices across countries and, hence, in human capital investments. To make progress, assume that

\[
A^j_S = (A^j_M)^\varepsilon, \text{ for all } j, \text{ with } A^US_S = A^US_M = 1 \text{ and } \varepsilon < 1. \tag{15}
\]

This assumption implies that a 1 percent change in the TFP of the manufacturing sector is associated with an \( \varepsilon \)-percentage change in the TFP of the service. Note that the case \( \varepsilon = 1 \) corresponds to the standard one-sector growth model with no variation in relative prices.
across countries. Using (4), it follows that

\[ P^j_s = (A^j_M)^{1-\varepsilon} \]  

so that \( \varepsilon < 1 \) implies that services are cheaper, in terms of manufacturing goods, in poor countries than in rich countries. The findings in Hsieh and Klenow (2007) suggest that \( \varepsilon = 1/3 \) gives a reasonably good approximation of the cross-country data on relative prices.

The firms’ first-order conditions in the service sector imply

\[ w^j = P^j_s (1 - \alpha) A^j_S \left( \frac{K^j_S}{H^j_S} \right)^\alpha, \]  

\[ R^j = \rho + \delta = P^j_s \alpha A^j_S \left( \frac{K^j_S}{H^j_S} \right)^{\alpha - 1}. \]  

Solving for the capital-to-labor ratio in the service sector gives

\[ \frac{K^j_S}{H^j_S} = \left\{ \frac{P^j_s \alpha A^j_S}{\rho + \delta} \right\}^{\frac{1}{1-\alpha}}. \]  

Combine (16), (17), (19) to obtain

\[ w^j = c_w (A^j_M)^{\frac{1}{1-\alpha}}, \]  

where \( c_w = (1 - \alpha)(\frac{\alpha}{\rho^*})^{\frac{\alpha}{1-\alpha}} \) does not depend on \( j \). The real wage rate, measured in terms of manufacturing goods, increases with the TFP of the manufacturing sector.

We are now ready to focus our attention on the price ratio \( \frac{w^j}{p^j_s} \), which drives the variation in human capital investments across countries. Using (16) and (20), gives the following expression:

\[ \frac{w^j}{p^j_s} = c_{wp} (A^j_M)^{\frac{1}{1-\alpha}-(1-\varepsilon)}, \]
where \( c_{wp} \) is constant across countries. Hence, the \( A_M \)-elasticity of \( \frac{w}{p_e} \) is

\[
E_{(w/p_S)A_M} = \frac{1}{1 - \alpha} - (1 - \varepsilon).
\]

In a one-sector growth model (\( \varepsilon = 1 \)), this elasticity is equal to \( \frac{1}{1 - \alpha} \). When \( \varepsilon < 1 \), the cross-country variation in the relative price \( \frac{w_j}{p_j} \) is lower than in the one-sector growth model (\( \varepsilon = 1 \)). This is quite intuitive: When \( \varepsilon < 1 \), poor countries are very inefficient in producing manufacturing goods, but they are not so inefficient in producing services. Services are cheap in poor countries because these countries have a high TFP in this sector relative to manufacturing. The real wage – expressed in terms of education services – does not increase with per-capita income across countries as much as in the one-sector growth model. It is also intuitive that the elasticity \( E_{w/p_S, A_M} \) decreases with \( \varepsilon \). That is, the elasticity is lower the higher the comparative advantage of poor countries is in producing education services.

The amplifier effect of TFP differences in the manufacturing sector on human capital differences across countries is given by

\[
E_{hA_M} = E_{h,w/p_S} E_{w/p_S, A_M} = \frac{(1 - \eta) \xi}{1 - \xi} \left( \frac{1}{1 - \alpha} - (1 - \varepsilon) \right).
\]

The fact that human capital production requires services (rather than goods) makes human capital less sensitive to a reduction of TFP in the manufacturing sector.

In a similar manner, we obtain

\[
E_{sA_M} = E_{s,w/p_S} E_{w/p_S, A_M} = \frac{(1 - \eta) \xi}{1 - \xi} \left( \frac{1}{1 - \alpha} - (1 - \varepsilon) \right),
\]
and

\[ E_{eA_M} = E_{e,w/pS} E_{w/pS,A_M} = \frac{1}{1 - (1 - \eta)\xi} \left( 1 + \eta\xi\frac{(1 - \eta)\xi}{1 - \xi} \right) \left( \frac{1}{1 - \alpha} - (1 - \varepsilon) \right). \]

**Proposition 3:** Assume that countries differ in their relative productivities across sectors (for all \( j: A_j^S = (A_j^M)^\varepsilon \) with \( \varepsilon < 1 \)). The amplification effect of human capital accumulation is driven by the expenditure elasticity of human capital as in the one-sector growth model. The quantitative response to a change in the TFP in the manufacturing sector diminishes with the extent of a comparative advantage in producing services (decreases with \( \varepsilon \)).

### 4 Calibration

#### 4.1 Parameters and Targets

We calibrate our benchmark economy (B.E.) to data for the United States. We normalize the units in which output is measured so that \( A_S = A_M = 1 \). The calibration of the baseline economy does not require taking an explicit stand on the shares of manufactured goods in consumption expenditures (\( \gamma \)). In particular, we calibrate a one-sector economy with no manufacturing sector (\( \gamma = 0 \)). It is easy to show that for any fixed \( \gamma \in (0, 1) \), the two-sector model economy delivers, after an appropriate normalization of the distribution of ability and of the distribution of taste for schooling, the same equilibrium statistics as the calibrated one-sector model economy. The parameter \( \gamma \) will affect the cross-country experiments in the next section of the paper, and its value will be determined later.\(^{12}\)

\(^{12}\)Letting \( c_1 \) and \( e_1 \) denote expenditures in consumption and human capital in a one-sector model, an equivalent two-sector growth model can be constructed as follows: Define the quantity of consumption \( c_2 \) and human capital (composite) input \( e_2 \) so that \( c_1 = p_c c_2 \) and \( e_1 = p_S e_2 \), for \( p_c = \tilde{\gamma} \) and \( p_S = 1 \). Then, normalize the distribution of ability and the taste shock in the two-sector model as follows: \( z_2 = z_1(p_S)^{(1 - \eta)\varepsilon} \) and \( \theta_2 = \theta_1/(p_c)^{1 - \sigma} \). This insures that all the equilibrium statistics are identical across the one-sector and two-sector model economies.
The mapping between model parameters and targeted data moments is multidimensional, and we thus solve for parameter values jointly. The discussion of calibration is divided into two parts: first, we discuss parameters that relate to preferences, demographics, and the production of goods, and second — parameters that relate to human capital accumulation. A summary of parameter values and data targets is provided in Table 2.

Table 2: Parameters and Data Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>U.S.</th>
<th>B.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption Preferences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRRA</td>
<td>$\sigma$</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta^{1/20}$</td>
<td>0.9646</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Goods/Services Technologies</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual depreciation</td>
<td>$\delta$</td>
<td>0.0745</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Human Capital Technology</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Schooling cost</td>
<td>$\bar{l}$</td>
<td>2.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H.C. RTS</td>
<td>$\xi$</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H.C. time share</td>
<td>$\eta$</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tastes for Schooling</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>$\theta_L$</td>
<td>0.00075</td>
<td>12.6</td>
<td>12.6</td>
</tr>
<tr>
<td>High</td>
<td>$\theta_H$</td>
<td>0.01285</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>Ability-taste corr. control</td>
<td>$b$</td>
<td>1.09</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Ability std</td>
<td>$\sigma_z$</td>
<td>0.23</td>
<td>8.5</td>
<td>8.3</td>
</tr>
<tr>
<td>Ability correlation</td>
<td>$\rho_z$</td>
<td>0.78</td>
<td>0.5</td>
<td>0.49</td>
</tr>
<tr>
<td>Market luck std</td>
<td>$\sigma_\mu$</td>
<td>0.375</td>
<td>0.36</td>
<td>0.38</td>
</tr>
<tr>
<td>Tax rate on income</td>
<td>$\tau$</td>
<td>0.044</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Empirical literature</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Interest rate (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Capital income share</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Investment to output ratio</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Educ. inst. salaries (% of GDP)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Variance of fixed effects</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Correlation of schooling</td>
<td>0.46</td>
<td>0.48</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Mean years of schooling</td>
<td>12.6</td>
<td>12.6</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>$R^2$ in Mincer regression</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Mincer return</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Variance of schooling</td>
<td>8.5</td>
<td>8.3</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Correlation of earnings</td>
<td>0.5</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Variance of earnings</td>
<td>0.36</td>
<td>0.38</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td>Public Education (% of GDP)</td>
<td>3.9</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Preferences, Demographics, and Production of Goods  We set the relative-risk-aversion parameter $\sigma$ to 2. There is no direct empirical counterpart for this parameter in the empirical literature since our model period is 20 years, and there is an infinite intertemporal substitution of consumption within a period. However, we consider a value of $\sigma$ that is in the range of values considered in quantitative studies with heterogeneous agents. The discount factor $\beta$ is set to target an annual interest rate of 5 percent, which is roughly the return on capital in the U.S. economy as measured by the average return on non-financial corporate
capital net of taxes in 1990-96 (Poterba (1997)). The capital-share parameter is set to 0.33, consistent with the capital-income share in the U.S. economy from the National Income and Products Accounts. The depreciation rate $\delta$ is selected to match an investment-to-output ratio of 20 percent as documented in the Economic Report of the President (2004).\(^{13}\)

**Human Capital Accumulation** Recall that the human capital technology is given by $h_c = z (s^\eta e^{1-\eta})^\xi$, where $s$ denotes schooling time and $e$ denotes educational expenditures. Thus, we need to specify two elasticity parameters, $\xi$ and $\eta$. Ability follows an $AR(1)$ process (in logs):

$$\ln(z') = \rho z \ln(z) + \epsilon_z, \quad \epsilon_z \sim N(0, \sigma_z^2).$$

In our computations, we approximate this stochastic process with a discrete first-order Markov chain that takes 7 possible values for ability $z$, using the procedure in Tauchen (1986) to compute transition probabilities. The market luck $\mu$ is $iid$ according to $\ln(\mu) \sim N(0, \sigma_\mu^2)$, approximated over 5 values similarly to $z$.

On the cost side, human capital accumulation is affected by the schooling cost $\bar{l}$ and the public education subsidy $p$. The latter is determined by the tax rate on income $\tau$ in equilibrium. On the preferences side, human capital investments depend on the tastes for schooling. The functional form $v(s, \theta)$ specified for the utility of schooling allows for a diminishing marginal utility from schooling and a bounded marginal utility from schooling at zero level of schooling:

$$v(s, \theta) = \theta [1 - \exp\{-s\}]$$

where $\theta \in \{\theta_L, \theta_H\}$. To allow for tastes for schooling to be correlated with ability, we let the probability of the high-taste shock to increase with ability: $\text{Prob}(\theta_H \mid \ln(z)) = \min\{0.5 + b \ln z, 1\}$.\(^{14}\) Note that $b > 0$ implies that taste for schooling and ability are positively corre-

---

\(^{13}\)A similar target is obtained using the average of the investment-to-output ratio in the PWT6.1 for the period 1990 to 1996 (Heston, Summers, and Aten (2002)).

\(^{14}\)Ability is drawn first, then the schooling taste is determined.
related. Thus, three parameters need to be specified for the tastes of schooling: two values for schooling tastes \( \{\theta_L, \theta_H\} \) and parameter \( b \), governing the correlation of the abilities and schooling tastes.

To sum up, there are ten parameters determining human capital accumulation:

\[ \{\xi, \eta, \rho_z, \sigma_z, \sigma_\mu, \bar{l}, \tau, \theta_L, \theta_H, b\}. \]

These parameters are calibrated so that in equilibrium the model economy matches the following ten targets from the U.S. data:

1. Intergenerational correlation of log-earnings of 0.5 (Mulligan (1997)).
2. Variance of log of permanent earnings of 0.36 (Mulligan (1997)).
3. Average years of schooling of 12.63, computed from CPS data for 1990.
5. Public education expenditures on all levels of education as a fraction of GDP of 3.9 percent from the Statistical Abstract of the United States (1999). In computing this statistic in the data, we treat as public expenditures all state and federal expenditures. We view local public expenditures as private because they are closely tied to property values and, therefore, to the incomes of parents.
6. A variance of individual fixed effects accounts for \( \frac{2}{3} \) of the variance of log-earnings (Zimmerman (1992)). In our model, fixed effects are due to heterogeneity in parental resources, abilities, and tastes for schooling. The rest of the variation in earnings is due to market luck. Thus, the variance of fixed effects relative to the variance of earnings (in logs) is given by \( 1 - \frac{\sigma^2}{\text{var}(\ln(h_c))} \).
7. A Mincer return to schooling of 10 percent. Since our theory is about lifetime inequality, we estimate Mincer returns using NLSY to proxy lifetime earnings with 6-year averages
of the earnings of males aged 30-45.\textsuperscript{15} We obtained Mincer returns in the range .09 to .11, depending on the age group considered (see Table 3). In our model, we measure returns to education by regressing individual log-wages, $wh_p$, on years of education, given by the model period times $s$:

$$\ln(wh_{p,i}) = b_0 + b_1 (20s_i) + u_i,$$

where $b_1$ gives the Mincer returns to schooling.

8. $R^2$ in the Mincer regression of .22. We find that the $R^2$ tends to increase with the age-group considered, taking values between 0.16 and 0.26 (Table 3). Because the average value of $R^2$ over the life cycle is about 0.22, we set this value as a calibration target.

Table 3: Mincer Regression Results, NLSY

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Constant</th>
<th>Mincer Return</th>
<th>$R^2$</th>
<th>Num. Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-35</td>
<td>5.68</td>
<td>0.08</td>
<td>0.16</td>
<td>1857</td>
</tr>
<tr>
<td>35-40</td>
<td>5.56</td>
<td>0.10</td>
<td>0.21</td>
<td>1307</td>
</tr>
<tr>
<td>40-45</td>
<td>5.38</td>
<td>0.12</td>
<td>0.26</td>
<td>427</td>
</tr>
</tbody>
</table>

9. Teacher and staff compensation share in GDP of 0.05. According to the U.S. Department of Education, Digest of Education Statistics, 2007, (public and private) institutional costs for all levels of education amounted to 7 percent of GDP in 1990-1995. Seventy two percent of these expenditures were on teacher and staff compensation (Education at a Glance, OECD, 2007). In the model, this expenditure corresponds to the $\bar{w}ls$ cost of schooling aggregated across households.

10. Intergenerational correlation of schooling of 0.46 as obtained by regressing children’s years of schooling on parental education, where the latter is defined as the average

\textsuperscript{15}Each 6-year age group includes all males who worked full time for at least 3 out of 6 years, with observed wages and hours.
years of schooling among mothers and fathers (see Hertz, Jayasundera, Piraino, Selcuk, Smith, and Verashchagina (2007)).

The calibration solves a rather complicated multidimensional mapping. Nonetheless, it is useful to discuss how model parameters affect some specific targets. Given mean years of schooling, the cost of teachers \( \bar{l} \), and the income tax rate \( \tau \) to finance public education expenditures, are almost directly pinned down by the share of teacher and staff salaries on GDP and by the share of public education expenditures on GDP (that is, the distribution of schooling matters little). The variance of market luck \( \sigma^2_{\mu} \) is set to match the variance of earnings. The parameter \( \xi \) – determining returns to scale in the human capital technology – targets the variance of individual fixed effects. In our theory, the earnings of parents and children are correlated in part due to differences in parental resources (the poor invest less), and in part due to an exogenous correlation of parental and children’s abilities. Thus, the correlation of ability \( \rho_z \) targets the intergenerational correlation of earnings.

Given the parameters just discussed, the five remaining parameters – variance of ability \( (\sigma_z) \), mean and variance of schooling tastes (controlled by \( \theta_H \) and \( \theta_L \)), correlation of ability and schooling taste (controlled by \( b \)), and the time share (\( \eta \)) – jointly determine the mean and variance of schooling, the \( R^2 \) and schooling coefficient in a Mincer regression, and the intergenerational correlation of schooling. The mean value for schooling taste \( 0.5(\theta_L + \theta_H) \) can be targeted to mean years of schooling, as the utility of schooling increases the benefits of schooling time. However, this parameter has important consequences for other targets as well. To develop this point, note that the return to schooling is affected by tastes and ability, where the former determines the utility of schooling and the latter the labor market returns to schooling. By making utility of schooling more prominent in schooling decisions, an increase in the mean of schooling tastes reduces the explanatory power of schooling on earnings. On the other hand, an increase in the variance of ability raises the importance of labor market returns in schooling decisions, hence raising the \( R^2 \) and the schooling coefficient in a Mincer regression. Moreover, while the targets for Mincer returns and for the \( R^2 \) tend to
move together in response to parameter changes, the relative magnitudes of these responses
depend on the parameter being changed.\textsuperscript{16} The variance of schooling increases with a rise
in the heterogeneity in the returns to schooling, which can be attained with an increase in
both the variance of schooling tastes and ability or with a decrease in the time share ($\eta$).
Furthermore, these alternative ways of increasing the variance of schooling have different
implications for a Mincer regression: While the explanatory power of schooling in a Mincer
regression decreases with the time-share parameter, it increases with both the variance of
schooling tastes and ability (the former by increasing the $R^2$ and the latter by increasing
the slope coefficient). Moreover, it also increases with the parameter $b$ – controlling the
correlation of schooling and ability – by making ability more important than taste shocks as a
source of schooling variance. Nonetheless, the parameter $b$ has a distinctive effect: While the
intergenerational correlation of schooling increases with the parameter $b$, by making schooling
tastes correlated across generations in a dynasty, this target is unaffected by the variance
of ability or decreases with the variance of schooling tastes. Altogether, in spite of the
high interdependence of the targeted moments, the parameters have distinctive quantitative
effects on those moments.

\subsection*{4.2 The Benchmark Economy}

The benchmark economy matches all the calibration targets quite closely (see Table 2).
We now show that the model is also consistent with several dimensions of heterogeneity
in the data that were not targeted in the calibration: schooling distribution, evidence on
the relationship between schooling attainment of children and resources/background of their
parents, and results in the micro literature on the enrollment effects of college tuition changes.
Overall, our paper makes an important contribution by developing a successful theory of
inequality in schooling and earnings in the U.S. economy.

\textsuperscript{16}For instance, changes in schooling tastes (mean and variance) tend to have a strong impact on the $R^2$, and changes in the standard deviation of ability tend to have a strong impact on the schooling coefficient of the Mincer regression.
Schooling Distribution  While the calibration only targeted the mean and the variance of schooling, the model economy accounts surprisingly well for the distribution of schooling.\textsuperscript{17} Table 4 reports maximum attained school years by population percentiles obtained from CPS 1990 data and those generated by our model. The model slightly overpredicts educational attainments at the bottom of the distribution and underpredicts them at the top of the distribution.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>12</td>
<td>14.1</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>Model</td>
<td>10</td>
<td>10.2</td>
<td>10.6</td>
<td>11.2</td>
<td>14.1</td>
<td>17.2</td>
<td>19.4</td>
</tr>
</tbody>
</table>

Data source: CPS 1990

Schooling and Parental Background  Although the calibration targeted the intergenerational correlation of schooling, the benchmark economy is consistent with other statistics relating parental background to offspring’s schooling. According to the statistics reported in Keane and Wolpin (2001), the probability that a child attains schooling less than or equal to 12 years conditional on his highest-educated parent having less than or equal to 12 years of schooling is .71 in the data. This probability is .72 in the model. Similarly, the probability that a child attains more than 12 years of schooling conditional on his highest-educated parent having more than 12 years of schooling is .60 in the data. This probability is .67 in the model economy.

In reviewing the literature on children’s educational attainment, Haveman and Wolfe (1995) report that the elasticity of children’s educational attainments with respect to family economic resources varies in the range of .02 to .20. In many of the studies cited in their

\textsuperscript{17}We note, however, that time in school is a continuous variable in our model, making its comparison with the data non-trivial. In particular, the distribution of schooling in the data has clear spikes at levels of education where an educational degree is completed.
survey, family income is recorded only in a single year and hence measures permanent income with an error. Haveman and Wolfe (1995) argue that when income is measured over a long period of time, the estimated impact of income is far greater. Our model economy produces an elasticity of .16, which is well within the range of values in the empirical literature.

**Expenditures on Education**  In a well-known study, Haveman and Wolfe (1995) estimated the annual investment in children in the US economy in the year 1992. Their calculations distinguish the investments made by public institutions from those made by parents, as well as between direct and indirect private costs. They report that direct non-institutional private costs of education of children aged 0-18 accounted for 8 percent of GDP. Private and public institutional costs for all levels of education add 7.5 percent of GDP (U.S. Census Bureau). The total direct cost of education in the U.S. is thus 15.5 percent of GDP. In our model, the calibration did not target the aggregate amount of expenditures in education. Computed as the sum of \((p_e e + \bar{w}ls)\) over all students, the total cost of education accounts for 14.3 percent of GDP, a value slightly below the estimate of Haveman and Wolfe (1995).

**Experiment: Effects of Tuition on College Enrollment.** There is a large literature on enrollment effects of college tuition changes to which the predictions of the model can be compared. This literature is surveyed by Leslie and Brinkman (1987) and discussed by Keane and Wolpin (2001). Typically, the college costs effects on enrollment are identified from time series and cross-state variations in tuition rates and grant levels. To compare results across studies, it has become standard to use the percentage change in the overall enrollment rate of 18-24 year olds in response to a tuition increase of $100 per year, expressed in dollars for the academic year 1982-1983. In a survey covering 25 empirical studies, Leslie and Brinkman conclude that, for national studies including the full range of public and private institutions, estimates of the effects of a $100 increase in 1982-83 dollars tend to tightly pack in the range of a 1.8 to 2.4 percentage decline in the enrollment rate of 18-24 year olds.

To evaluate the response of college enrollment to a price change in the model economy,
we simulate a one-period unanticipated increase in college tuition of $1000 in 1982-1983 dollars. This experiment is done in partial equilibrium so that factor prices are kept fixed. We find that college attendance declines by 1.5% per 100 dollars increase in college tuition, which is close to the consensus estimates in the empirical literature review by Leslie and Brinkman and to the recent estimates in Keane and Wolpin (2001). In a schooling model structurally estimated with NLSY data on white young males, the authors found a decline in the college enrollment rate of 1.2 percent per $100 tuition increase in 1982-83 dollars. Using estimates from Kane (1994), Keane and Wolpin report that a $1000 tuition increase in 1982-1983 dollars leads to declines in the enrollment rate of 34.0, 20.0, 12.3, and 3.0 percent, respectively, for white males whose parents are in the first through fourth income quartiles. In comparison, our model economy predicts declines of 23.6, 21.9, 18.0, and 9.8 percent for individuals with parents in the first through fourth income quartiles. The model is thus consistent with the evidence that tuition effects are much stronger among individuals born in families with a low parental income, although tuition effects decline with parental income more steeply in the data than in the model. Altogether, the model is consistent with the micro evidence on the enrollment effects of college tuition changes.

5 Quantitative Results

This section uses the theory developed to quantitatively assess the consequences of TFP differences across countries. We assume that countries are identical in terms of preferences and technologies but only differ in their level of TFP. We asked the following questions: What cross-country differences in TFP are required for the model economy in order to account for a 20-fold income ratio between rich and poor countries? Does human capital play an important role in amplifying income differences across countries?

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18 More precisely, we found that a $1000 increase in tuition raises the enrollment rate by 15%. Following the literature, we divide by 10 to obtain the response to a change in tuition of 100 dollars. We obtained quite similar results when we simulated an increase in tuition of 500 dollars. In this case, the decrease in tuition was 7.3 percent which implies a decline of 1.45 percent in enrollment per $100 increase in tuition.
5.1 The experiment

To assess the magnitude of the TFP differences needed to account for the observed disparity in per-capita income across countries, we first need to take a stand on the values of three key parameters ($\varepsilon, \gamma$). The first parameter, $\varepsilon$, determines the elasticity of the TFP in the service sector to a change of the TFP in the manufacturing sector. The other parameter, $\gamma$, pins down the share of manufacturing goods in consumption. Intuitively, $\varepsilon$ determines the importance of cross-country heterogeneity in relative prices while $\gamma$ affects how the variation in relative prices impact on investment decisions and output per worker across countries.

The quantitative experiments below assume that the data counterpart to the service and manufacturing sectors in the model economy are the nontradable and tradable sectors in the data analyzed by Hsieh and Klenow (2007). In a cross-country study, these authors find that a one-percent variation in the TFP of the tradable sector is associated with a .3-percent variation in the TFP of the nontradable sector in 1996, and that this elasticity was about .4 in 1985 (see Table 7 on page 581). We thus consider experiments with $\varepsilon = .3$ and .4. To evaluate the sensitivity of the results, we also consider a 'low' and a 'high' value for the parameter $\varepsilon$ by setting $\varepsilon = .1$ and $\varepsilon = 1$.

Recall that the parameter $\gamma$ — determining the share of manufacturing goods in consumption — does not affect equilibrium statistics in the benchmark economy. With no loss of generality, we can then set $\gamma = .27$ in the Benchmark Economy so that this economy is consistent with the share of services in consumption expenditures in the US. Note that in the data the share of services in aggregate consumption expenditures increases with per-capita income across countries, suggesting that the parameter $\gamma$ varies with the level of economic

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$^{19}$Hsieh and Klenow report that the elasticity of TFP with respect to PPP-output is about one third lower in the nontradable sector than in the tradable sector in the year 1996 (see Table 7 on page 581). This ratio is .5 in 1980 and .4 in 1985.

$^{20}$Note that in the data the education services provided by private non-profit institutions and government are included in the final consumption of households at their cost (see the Handbook of the International Comparison Programme). To be consistent, we define total consumption in our model as the sum of household consumption $c$ and expenditures in education $e$. Hence, in the baseline economy the parameter $\gamma$ determining the share of tradable in aggregate consumption is set so that $\frac{\gamma c}{c + e} = 0.23$ (author's estimate using PWT data), which implies $\gamma = .27 > .23$. 

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28
development. To match the variation in the share of services in consumption expenditures across countries, the experiments below assume that the parameter $\gamma$ varies with TFP in the manufacturing sector with constant elasticity. For each of the values of $\varepsilon$ considered, we calibrate the value of this elasticity so that the theory is consistent with the fact that the elasticity of the share of tradables in consumption expenditures with PPP output is $-0.13$.

5.2 Measurement

COMMENT: I AM NOT SURE WHAT TO DO WITH THIS SECTION. PERHAPS WE SHOULD CUT IT (OR REDUCE ITS SIZE SIGNIFICANTLY) AND LEAVE MATERIAL FOR ANOTHER PAPER. I GUESS IT DEPENDS ON WHAT WE GET ON THE TESTS TO BE MADE ON SCHOOLING AND PRICE OF EDUCATION. NEED TO WRITE AN INTRODUCTION TO GUIDE THE READER ABOUT THIS SECTION. To measure GDP at PPP prices, a set of ‘international prices’ needs to be chosen in a manner consistent with the methodology in the PWT. The set of ‘international prices’ in the PWT is constructed by averaging prices among all countries, according to the procedures established by the International Comparison Program (ICP) of the United Nations. In order to calculate the average price for a product across countries, each with its own currency, the prices in the individual countries are converted into a common numeraire currency using PPP exchange rates. The average price for good $i$ is defined as:

$$P_i = \sum_j \frac{p_i^j \cdot q_i^j}{E_j \sum_j q_i^j} \quad \text{for } i = 1, \ldots, n,$$

(21)

where $p_i^j$ and $q_i^j$ represent the price and quantity of product $i$ in country $j$. Each national price is converted into a common numeraire currency by dividing by the country’s PPP exchange rate.

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21 Since we have assumed that all education inputs are produced by the service sector, our results will minimize the role of human capital accumulation in accounting for income differences across countries. In Erosa, Koreshkova, and Restuccia (2009), we investigate how the quantitative findings change when a fraction of educational expenses are in the form of tradable goods (such as pencils, paper, books, and computers) and when we allow countries to differ in their efficiency at producing human capital.
exchange rate $E^j$, and then averaged across all countries. The resulting price $P_i$ is a weighted arithmetic average of the converted national price using the quantity shares as weights. Thus, $P_i$ is the total value of the world transactions for good $i$, expressed in terms of PPP exchange rates, divided by the total quantity of the good.

Note that the set of international prices and the PPP exchange rates in the PWT are jointly determined as the solution to a system of equations involving prices for all goods and PPP exchange rates for all countries. Solving such a system of equations in our model economy is a very demanding task as it involves simulating a set of model economies. The simulated model economies should mimic the world distribution of countries in terms of their population sizes and income distribution. In this way, the distribution of quantities transacted across countries for various commodities can be aggregated as in the PWT data. To circumvent this difficult problem, researchers typically calibrate the baseline economy to the US and set international prices equal to the prices in the baseline economy. This approach is motivated by the fact that, because the PWT use aggregate quantities to aggregate country prices, rich country prices are weighted more than poor-country prices (see the discussion in Hsieh and Klenow). Below, we argue that this approach has some serious drawbacks when applied to a model of schooling, such as the one in this paper.

As discussed in the Handbook of the ICP, there is very little basis for comparing education prices across countries using tuition or fees because they usually do not cover full cost and are not market-prices due to heavy government subsidies. It is thus not possible to use (21) to determine an international schooling price that can be used to value schooling output across countries. As a result, the ICP uses an “indirect approach” which involves using data on the PPP prices of inputs to aggregate at international prices expenditures on education inputs. Since the salaries of teachers are a major schooling cost, the international salary for teachers is a crucial determinant of schooling expenditures at international prices. However, the U.S. wage badly approximates the real wage used in the PWT to value education costs across

\[\text{The price of education is then obtained as the ratio of education expenditures at domestic prices to expenditures at international prices.}\]
countries. While for most products, such as cars and airline tickets, the US and similarly rich economies account for most of the world transactions, this is not the case for schooling, for two reasons. First, the variation in average years of schooling across rich and poor countries is easily an order of magnitude smaller than the variation in the consumption of cars and airline tickets. Second, poor countries account for the bulk of the world population of school-aged individuals. Thus, (21) is likely to put a significant mass on the salaries of teachers in poor countries.\footnote{While it is obviously important to aggregate all international prices in the model economy in a manner consistent with the PWT, this issue is of a first-order importance when it comes to aggregating teachers’ wages. Because the cross-country variation in real wages is very large, incorrect weights can lead to an international salary for teachers in the model economy that is grossly at odds with the PWT. To deal with this problem, one approach would be to calibrate the model economy to replicate the world population distribution across rich and poor economies and use (21) to jointly solve for the set of international prices and countries’ PPP exchange rates. This is a daunting task. Moreover, there is no guarantee that our simulations can mimic the world distribution of years of schooling because our calibration only targets average years of schooling in the baseline economy.}

The above discussion stresses a novel point: The choice of an international wage rate to value teachers’ services across countries in a schooling model is a delicate issue. Moreover, as we have verified in our computational experiments, the results for output inequality across countries critically depend on the choice made. To circumvent these difficulties, we compute GDP at international prices net of teacher output or, equivalently, we measure national income net of the salary of teachers. The advantage of this approach is that we avoid taking a stand on how to set the international real wage for teachers. Moreover, using data from the PWT on institutional expenditures in education, we have made a similar adjustment to the GDP data in the PWT by computing GDP at international prices net of institutional expenditures. We have verified that all statistics of interest (such as the dispersion in income per capita and the income elasticity of schooling) are not affected in a significant manner by this adjustment. To sum up, the experiments below measure GDP at international prices as follows:

\[ GDP^*_j = Y_M + Y_s, \]
where the price of services is set as in the baseline economy \((p^s_s = p^{US}_s = 1)\).\(^{24}\)

### 5.3 Amplification Effect

Unlike the results in Section 3, the amplification effect in the benchmark economy cannot be characterized with an analytical expression. However, there is a simple way of measuring the amplification effect of TFP in the calibrated model economy. For each value of \(\varepsilon\), we simulate the model economy for different values of \(A_M\) and run the following regression in the simulated data

\[
\ln Y = a_1 + a_2 \ln A_M + u_i,
\]

where \(Y\) denotes GDP. The values considered for \(A_M\) are 1.5, 25, 125. We run the regression for GDP measured at domestic prices and PPP prices. The fact that the \(R^2\) in all the regressions are close to 1 implies that the estimated regressions represent a good description of how \(A_M\) and \(Y\) covary in the simulated data. The coefficient \(a_2\) can then safely be interpreted as the elasticity of output with respect to \(A_M\).

Table 5 reports the main results in the paper. The elasticity of GDP – at PPP prices – with respect to \(A_M\) is 1.94 when \(\varepsilon\) is .3 and 2.08 when \(\varepsilon\) is .4. To assess what the estimated elasticities imply for understanding the observed income differences across countries, we compute the TFP ratio in the manufacturing sector needed to generate a ratio of aggregate income at PPP prices of 20. This ratio is roughly the PPP-income ratio between the 10% richest countries to the 10% poorest countries in the world income distribution. The ratio of TFP in tradables to explain a PPP-income ratio of a factor of 20 is 4.7 when \(\varepsilon = .3\) and 4.0 when \(\varepsilon = .4\). These findings imply a substantial amplification of TFP differences across countries. The mechanism generating a large income disparity is that a low TFP leads individuals in poor countries to invest few resources in accumulating both physical and

\(^{24}\)In Erosa, Koreshkova, and Restuccia (2009) we evaluate how the results change when the US wage rate is used as an international price to value teachers’ services in GDP. We show that this procedure has highly counterfactual implications.
Table 5: Amplification

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>.1</th>
<th>.3</th>
<th>.4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Human Capital Model</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>TFP elasticity of GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPP prices</td>
<td>1.53</td>
<td>1.94</td>
<td>2.08</td>
<td>2.8</td>
</tr>
<tr>
<td>Domestic prices</td>
<td>1.98</td>
<td>2.16</td>
<td>2.26</td>
<td>2.8</td>
</tr>
<tr>
<td>$A_M$ ratio for GDP, PPP, ratio of 20</td>
<td>7.1</td>
<td>4.7</td>
<td>4.0</td>
<td>2.9</td>
</tr>
<tr>
<td><strong>TFP elasticity of Human Capital</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP elasticity of Physical Capital</td>
<td>1.97</td>
<td>2.15</td>
<td>2.23</td>
<td>2.8</td>
</tr>
<tr>
<td>TFP elasticity of Human Capital</td>
<td>0.46</td>
<td>.63</td>
<td>.70</td>
<td>1.24</td>
</tr>
<tr>
<td><strong>Exogenous Human Capital Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TFP elasticity of GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPP prices</td>
<td>.856</td>
<td>1.046</td>
<td>1.12</td>
<td>1.49</td>
</tr>
<tr>
<td>Domestic prices</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
</tr>
<tr>
<td>$A_M$ ratio for GDP, PPP, ratio of 20</td>
<td>33.1</td>
<td>17.5</td>
<td>14.5</td>
<td>7.5</td>
</tr>
<tr>
<td><strong>TFP elasticity of Physical Capital</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
</tr>
</tbody>
</table>

human capital relative to individuals in rich countries.

One way of assessing the amplification results in our paper is to compare them with the findings in Hsieh and Klenow (2007). These authors perform a development accounting exercise using a growth model with no human capital accumulation and find that a 1-percent change in the TFP of the tradable sector leads to an increase in output per worker of 1.04. To show that the much larger amplification in our theory is due to human capital accumulation, we calibrate a version of the model economy with no investments in human capital. When $\varepsilon = .3$ and human capital is exogenous, the $A_M$ elasticity of output at PPP prices is 1.05, which is quite close to the 1.04 estimated by Hsieh and Klenow. The ratio of TFP in manufacturing to explain a PPP-income ratio of a factor of 20 is now 17.5, which is much higher than the 4.7 ratio in the economy with investments in human capital.

Human capital is an important source of amplification of income differences across countries for two reasons: First, human capital directly contributes to cross-country output dif-

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25They report an elasticity of TFP in the tradable sector with respect to income per capita in 1996 of .962, which implies a TFP elasticity of income of $1/.962 = 1.04$.

26Labor productivity is fixed at $h = z \eta \xi$, where $s$ is set at its average value in the benchmark economy.
ferences because poor countries accumulate less human capital than rich countries. Second, a higher human capital stock stimulates more physical capital accumulation by raising the marginal product of capital. As a result, human capital accumulation amplifies the effects of TFP differences on physical capital: While the $A_M$ elasticity of physical capital is 2.15 in the economy with human capital accumulation, it is only 1.49 in the model with no human capital investments (as documented in Table 5 for the economy with $\varepsilon = .3$ ). Note that the strength of these effects increases when sectorial productivity differences across countries are small ($\varepsilon$ is high). The TFP elasticities of human and physical capital rise with $\varepsilon$ as the higher the value of this parameter the lower the comparative advantage of poor countries at producing services (see Figure 1).

The results in Table 5 indicate that it is important to model sectorial productivity differences for assessing the role of human capital in amplifying income differences. When $\varepsilon$ is set at a low value, poor countries exhibit a high relative TFP in the service sector, which, in turn, leads to a low price of services relative to rich countries. Since services are a key input in the production of human capital, cheap services in poor countries operate as a force towards reducing income inequality. Nevertheless, the amplification role of human capital is large even for implausibly low values of $\varepsilon$. When $\varepsilon = .1$ the TFP elasticity of PPP output is 1.53 in the model with human capital accumulation and .86 in the economy with exogenous human capital. This differential in TFP elasticities across model economies is not minor: To generate an income ratio of 20 the economy with endogenous human capital requires an $A_M$ ratio of 7.1 while the economy with exogenous human capital requires an $A_M$ ratio of 33.1 (see Table 5).

At the theoretical level, it is interesting to answer the following question: How does a one-percent change in TFP in all sectors in the economy affect output per worker? The answer is provided by the one-sector version of the model economy ($\varepsilon = 1$), and it is startling: The amplification effect is now 2.8. In a world were TFP varies uniformly across sectors, the TFP ratio needed to generate a PPP-income ratio of a factor of 20 would be only 2.9,
which is about two thirds of what is implied by the two-sector model with \( \varepsilon \in [0.3, 0.4] \).

Nevertheless, from a development accounting view, the relevant amplification effect is the one estimated with the two-sector model as the evidence suggests that TFP does not vary uniformly across sectors. We conclude that it is important to model both human capital and sectorial productivity differences for assessing the cross-country variation in productivity.

5.4 Discussion on Relative Prices and Human Capital

We have shown that human capital is an important source of amplification and that the TFP elasticity of output depends critically on the parameter \( \varepsilon \). Since \( \varepsilon \) determines the variation in relative prices across countries, it is important to examine the implications of the theory for relative prices and test them with the evidence from the Penn World Tables (Heston, Summers, and Aten (2002), hereafter, PWT). Moreover, to address the concern that our quantitative theory may be exaggerating the TFP elasticity of human capital investments, we examine evidence on the variation in human capital investments across countries.

5.4.1 Theory and Evidence on Relative Price Variation

We can construct a proxy for the price of education inputs across countries using data on the price of services from the PWT. This seems a reasonable proxy in the view that educational inputs are mostly provided by the service industry. Figure 2 (top) plots cross-country data on the price of services versus per-capita income from the PWT for the year 1996 as well as the simulated model data. Note that the elasticity of the relative price of services with respect to PPP output is 0.30 in the PWT data, which is quite close to the 0.29 value predicted by the economy with \( \varepsilon = 0.4 \) and to the 0.36 value obtained in the economy with \( \varepsilon = 0.3 \). The economies with \( \varepsilon = 1 \) and \( \varepsilon = 0.1 \) have counterfactual predictions for the variation in the price of services across countries: The former implies no variation in relative prices across countries while the latter grossly overpredicts the variation in the data (2, top). We conclude that the evidence supports values of \( \varepsilon \) within the range \([0.3, 0.4]\), with the best fit of the data
obtained when $\varepsilon$ is parameterized with values close to .4.

The PWT define PPP exchange rates for education as education expenditures in national currency divided by their real value in international dollars. The education PPP exchange rates are computed with data on expenditures by educational institutions, for there are no cross-country data on educational expenditures at the level of the household. Figure 2 (middle) plots the PWT data on the price of education, normalized by the PPP price of GDP, and the PPP output per capita. For comparison, we plot model-simulated data on the prices of two key schooling inputs normalized by the PPP price of GDP: the wage rate and the price of services (nontradables) (Figure 2, middle and bottom). By focusing on the prices of these two schooling inputs, we avoid taking a stand on how to aggregate and value at international prices expenditures in our model economy (see the discussion in the Measurement section).

Figure 2 (middle) documents that the price of institutional expenditures in education tends to increase with income across countries albeit the relationship is not very strong. While the income elasticity of the education price is .046, in our model economy the income elasticity of the real wage rate — in terms of PPP output — takes values above .60 when $\varepsilon = .3$ and .4. The one-sector economy ($\varepsilon = 1$) exhibits the smallest elasticity, .53, which is still well above the value in the data. Figure 2 (bottom) shows that the income elasticity of the relative price of services — in terms of the PPP price of output — takes values of .25 and .21 when $\varepsilon = .3$ and .4. Hence, the findings suggest that in our simulations the price of education inputs rises too fast with the level of economic development relative to the data, suggesting that the quantitative results understate human capital differences across countries.

5.4.2 Theory and Evidence on Variation in Human Capital Investments

Next, we turn to the question: Are the cross-country differences in human capital investments implied by the theory plausible? For each value of $\varepsilon$, we obtain observations for average years
of schooling and output per worker by simulating economies that vary in their relative levels of TFP. In Figure 3 (top), we plot cross-country data on schooling and income, taken from Cohen and Soto (2007) and Heston, Summers, and Aten (2002), together with the simulated data from the model economy. The figure reveals that the income semi-elasticity of schooling in the cross-country data is 2.56. All the simulated model economies generate a lower schooling-income semi-elasticity than in the data. The highest value of the semielasticity, 1.86, is generated by the economy with \( \varepsilon = 1 \). Given that our theory does not overstate schooling differences across countries, we then ask if the the cross-country differences in the quality of schooling across similar schooling levels are reasonable.

In an influential paper, Hendricks (2002) measures cross-country differences in schooling quality using data on relative earnings (adjusted by schooling levels) of immigrants in the United States. Table 6 provides summary statistics on the data analyzed by Hendricks. The population of U.S. immigrants is divided into 4 groups according to the income-percentile of the country of origin relative to the United States. The country groups considered are the 20th, 30th, 40th and 50th to 65th percentiles of the U.S. per-capita income. The average years of schooling among immigrants in these country groups are, respectively, 12.5, 12.8, 12.4, 14.3, and the average earnings of these immigrants are 97%, 92%, 94% and 107% of the earnings of similarly educated workers in the United States (see Figure 4). While Mexico and Puerto Rico\(^{27}\) have a per-capita income of roughly 45% of the U.S. level, we did not include these two countries in the 40th income percentile group because immigrants born in Mexico and Puerto Rico have on average 7.4 years of schooling — a schooling level well below that of all other immigrants in Hendricks’ sample. Nevertheless, we examine the data for Mexico and Puerto Rico separately in a fifth country group.

We simulate immigrants from five potential source countries differing with respect to their TFP in order to generate comparable statistics from the model economy. Immigrants are selected so that they have an average level of schooling consistent with the data reported in

\(^{27}\)To simplify the terminology here, we slightly abuse the language and refer to Puerto Rico as a ‘country’ rather than an incorporated U.S. territory and call migrants from Puerto Rico in the U.S. ‘immigrants’
Table 6: U.S. Immigrant Characteristics (computed from Hendricks (2002))

<table>
<thead>
<tr>
<th>GDP, PPP, percentile</th>
<th>20 − 30%</th>
<th>30 − 40%</th>
<th>40 − 50%</th>
<th>50 − 65%</th>
<th>MEX-PRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>11</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Relative home GDP, PPP</td>
<td>24.4</td>
<td>33.3</td>
<td>44.8</td>
<td>58.5</td>
<td>45.75</td>
</tr>
<tr>
<td>Years of schooling</td>
<td>12.5</td>
<td>12.8</td>
<td>12.4</td>
<td>14.3</td>
<td>7.45</td>
</tr>
<tr>
<td>Relative earnings</td>
<td>0.97</td>
<td>0.92</td>
<td>0.94</td>
<td>1.07</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Hendricks. We then compute the ratio of earnings between immigrants and equally schooled workers in the Benchmark Economy and compare these results to Hendricks’ data.

To proceed, we need to take a stand on how immigrants are selected from the population in the source country. In our model economy, equally schooled individuals can be heterogeneous in many characteristics (taste for schooling, ability, parental human capital, and wealth) and, thus, in their earnings. As a result, selection into emigration from the distribution of these characteristics has important consequences for their average earnings. We now study in detail how different forms of selection by wealth affect the results. For each source country, we compute the wealth distribution for individuals within a given schooling bracket, and we entertain two possibilities on how immigrants are selected from these populations. First, as a benchmark, we assume that immigrants are randomly drawn from the entire wealth distribution. Second, we examine selection into emigration based on the household wealth and show that this type of selection successfully reconciles the relative earnings of immigrants predicted by the model with those obtained from the data. Figure 4 presents results for economies with $\varepsilon = .3$ and $\varepsilon = .4$. When immigrants are a random draw from the entire wealth distribution (100th percentile on Figure 4), our model tends to overpredict the earnings gap between workers of the same schooling level in rich and poor countries. While the earnings gap for immigrants born in

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28 We can also investigate the sensitivity of Hendrick’s findings to his assumption of nonrandom selection into emigration.

29 Note that selection by wealth matters because wealth is correlated with ability and schooling expenditures. Alternatively, we could directly select immigrants in terms of their ability and schooling expenditures but this would not affect our main conclusion that selection can reconcile the predictions of the theory with the data.
countries below the 50th percentile are about .80 in the model, it is above .90 in the data. On the other hand, the model overpredicts the the relative earnings for immigrants born in Mexico and Puerto Rico, producing a ratio of 1.04 relative to a .93 in the data. Moreover, the model cannot account for the fact that immigrants from countries in the 50th percentile earn about 7% more than Americans, an observation suggesting that schooling quality is higher in this group of countries than in the U.S.

We now show that the model can account well for these non-trivial patterns of the immigrant earnings data, provided that selection of immigrants by wealth is allowed to play a role. We assume that immigrants are a random draw from the bottom tail of the wealth distribution in a given schooling bracket. Then, we right-censor this distribution at different wealth percentiles and assume that immigrants are a random draw from the resulting wealth distribution. Notice that selection is more important the more truncated the wealth distribution is. The case of no selection corresponds to the assumption that immigrants are drawn from the bottom 100% of the wealth distribution (e.g. no truncation). Figure 4 graphs, for \( \varepsilon = 0.3 \) and \( \varepsilon = 0.4 \), how average earnings vary as immigrants are increasingly drawn from more wealthy backgrounds. We find that immigrants’ human capital tends to decline with parental wealth for the first four country groups. On the other hand, earnings increase with parental wealth in the fifth country group representing Mexico and Puerto Rico.

Why does the relationship between parental wealth and immigrants’ human capital switch signs across countries? The key is that while immigrants from the first four country groups have on average more than 12 years of schooling, immigrants in the country group representing Mexico and Puerto Rico have on average only about 7 years of schooling. As a result, immigrants from the first four country groups exhibit high average years of schooling relative to their source-country population, and the opposite is true for the case of Mexico and Puerto Rico. When immigrants are positively selected from the schooling distribution, they tend to exhibit a relatively high taste for schooling. In this case, the rate of return
to schooling is a relatively less important determinant of schooling decisions. Moreover, the importance of returns to education for schooling decisions diminishes with parental wealth: Wealthy individuals tend to care more about the utility of schooling as they have a low marginal utility of consumption. As a result, conditional on a level of schooling, individuals with wealthy backgrounds tend to be of a relatively low ability and to spend little on education. This accounts for the negative relationship between earnings and parental wealth among highly-schooled immigrants. Our findings point that the case of Mexico and Puerto Rico is quite different. Immigrants from these countries are relatively low-schooled and care little about the utility of schooling. The rate of return to schooling is the main driving force behind their schooling decisions. A more favourable parental background is associated with more human capital expenditures and, hence, higher human capital.

Figure 4 (top) indicates with a dot, for each country group, the amount of selection needed to account for the immigrants’ earnings data for the economy with $\varepsilon = .3$. The earnings data for the first three country groups can be rationalized if immigrants are drawn from the bottom 20th, 30th, and 50th percent of the wealth distribution. Moreover, assuming that immigrants are drawn from the bottom 40 percent of the wealth distribution accounts for the observed earnings ratio of 1.07 among immigrants born in countries in the 50th to 65th percentile group. The average earnings ratio observed among immigrants born in Mexico and Puerto Rico can be accounted for if immigrants from these countries were drawn from the bottom 60% of the wealth distribution. Figure 4 (bottom) illustrates similar findings for the economy with $\varepsilon = .4$. We conclude that the model can account well for Hendricks’ data.

Our paper supports Hendricks’s conclusion that accounting for cross-country income differences requires a theory of total factor productivity. Our results, however, imply a more important role for factor accumulation. Hendricks estimates that for the five poorest countries in his sample — with an output per capita of 5.8 percent of the US level — low factor accumulation reduces income per capita to 47% of the US level. In our paper, the reduction in output per capita accounted for by low factor accumulation is 25% — almost
twice as big as estimated by Hendricks. Moreover, while the cross-country differences in schooling quantity and quality are taken as given by Hendricks, in our paper they are the result of TFP differences across countries. Nevertheless, both papers imply important TFP differences across countries: To account for a 17-fold difference in incomes across countries, Hendricks estimates that an 8-fold difference in TFP is needed while our paper implies a difference in the TFP of the tradable sector of a factor of 4.3.

6 Conclusions

We build a model of heterogeneous individuals — who make investments in schooling quantity and quality — to quantify the importance of differences in human capital versus TFP in explaining the variation in per-capita income across countries. We use U.S. household data to pin down the key parameters — elasticities of human capital with respect to time and goods inputs — driving the quantitative implications of the theory across countries. Our baseline economy successfully matches a wide set of calibration targets on schooling and earnings inequality in the U.S., such as the variances and intergenerational correlations of earnings and schooling, and the slope coefficient and $R^2$ in a Mincer regression. The model economy is also consistent with several dimensions of heterogeneity in the data that were not targeted in the calibration: schooling distribution, evidence on the relationship between schooling attainment of children and resources/background of their parents, and results in the micro literature on the enrollment effects of college tuition changes. Altogether, the paper develops a succesful theory of inequality in schooling and earnings in the U.S. economy. We also provide several pieces of evidence supporting the cross-country predictions of the theory. Our main finding is that human capital accumulation strongly amplifies TFP differences across countries: To explain a 20-fold difference in the output per worker the model requires a 5-fold difference in the TFP of the tradable sector, versus an 18-fold difference if human

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30Our quantitative findings are quite close to the ones in Schoellman (2009), who uses the returns to schooling of immigrants to the U.S. as a measure of their source-country education quality.
capital is fixed across countries.

We leave for future work two important extensions of our analysis. First, we plan to explore the distributional implications of cross-country differences in TFP, fiscal policies and support for public education. Second, we would like to model heterogeneity in the marginal returns to schooling across individuals, as emphasized by a recent micro literature (see Card (2001)). In this case, the impact of any public-policy reform on schooling is driven by the marginal returns to schooling of the individuals affected by the reform and not by the average return to schooling in the population.
References


Model PPP income is normalized by the 1990 U.S. GDP at PPP prices. Solid points represent simulated economies; lines are OLS regressions with log-TFP in manufacturing sector as an explanatory variable. TFP elasticities (regression coefficients on TFP) are indicated in square brackets.
Model PPP income is normalized by the 1990 U.S. GDP at PPP prices. Solid points represent simulated economies; lines are OLS regressions with per-capita income at PPP prices as an explanatory variable. Income elasticities (regression coefficients on GDP, PPP) are indicated in square brackets. Data on prices of non-tradables and education are from Heston, Summers, and Aten (2002).
Model PPP income is normalized by the 1990 U.S. GDP at PPP prices. Solid points represent simulated economies; lines are OLS regressions with per-capita income at PPP prices as an explanatory variable. Income semi-elasticities (regression coefficients on GDP, PPP) are indicated in square brackets. Data is from Cohen and Soto (2007).

Data on relative earnings of immigrants across countries is from Hendricks (2002), adjusted by the level of schooling of the immigrant population. Each curve corresponds to an economy with a per-capita PPP income relative to that of the U.S. in the percentile indicated. Wealth percentiles are for populations with similar schooling.