

# Optimal Arrangements for Distribution in Developing Markets: Theory and Evidence\*

William Fuchs<sup>†</sup>

Brett Green<sup>‡</sup>

David I. Levine<sup>§</sup>

September 25, 2017

## Abstract

In addressing the product adoption puzzle, the literature has focused primarily on demand-side barriers. In this paper, we attempt to address frictions on the supply side. We model the relationship between a producer or distributor and its vendors, where credit constraints and contract enforceability present challenges for distribution. We show that providing vendors with an initial endowment of the good and the option to buy additional units at a fixed price is an optimal way in which to overcome these frictions. The arrangement is straightforward to implement and is optimal both for profit-maximizing firms and non-profit organizations with limited resources. We test the arrangement using a field experiment in rural Uganda. We find that the optimal arrangement increases sales by 3-4 times compared to a standard contract. However, the rate of sales growth was lower than predicted by the model. Surveys suggest that an unwillingness to extend credit to their customers and a lack of access to a reliable savings technology were the primary impediments to growth.

---

\*We would like to thank Peter DeMarzo, Andy Skrzypacz, David Sraer, Steve Tadelis, and Rob Townsend for useful suggestions. We have also benefited from seminar participants at Berkeley, Stanford, Kellogg and the Munich Relational Contracts Conference. We gratefully acknowledge financial support from the Clausen Center, the Gartwood Center for Corporate Innovation, the Hellman Family Faculty Fund, the Committee on Research at Berkeley, the European Research Council (Grant 681575), and the generous donors of our Indiegogo funding campaign. Vastinah Kemigisha, Norma Altshuler, and Ross Green provided excellent research assistance.

<sup>†</sup>UT Austin and UC3M, wfuchs@gmail.com

<sup>‡</sup>UC Berkeley, Haas School of Business, greenb@berkeley.edu

<sup>§</sup>UC Berkeley, Haas School of Business, levine@berkeley.edu

# 1 Introduction

Investment in a number of basic technologies (e.g., solar lights, efficient cook stoves, fertilizer, anti-malaria nets, water filters) appear to have large welfare benefits for many households in developing economies. The potential welfare gains from expanding adoption of these products is staggering. To illustrate, consider the case of using solar lights and efficient cook stoves to replace kerosene lamps and cooking over an open flame. As of 2011 the International Energy Agency (IEA) reports that over 1.3 billion people lack access to electricity and 2.7 billion cook over an open flame. The negative health impacts are sobering. The United Nations Development Program and the World Health Organization (WHO) report that 1.6 million deaths per year in developing countries are caused by the indoor air pollution attributed to traditional fuels.<sup>1</sup> Efficiently designed cook-stoves can eliminate up to 94% of the smoke and 91% of the carbon monoxide emissions and have been demonstrated to lead to improved health outcomes. A solar light can altogether eliminate the need for kerosene lamps, which in addition to polluting the air, also pose a serious fire risk.

In addition to the health benefits, these technologies appear to have substantial economic benefits. Consumers spend \$17 billion on kerosene each year to light their homes. The light cast from a kerosene lamp is poorly distributed, has a low intensity, and is expensive. The poor lighting levels from kerosene lamps makes it difficult for children to study, reducing literacy and education, and minimizes the effective working hours for income generating activities. In rural Uganda women are estimated to spend 2 hours a day gathering fuel for cooking. Those in urban areas who purchase their fuel for cooking spend up to 30% of their income on it.<sup>2</sup> The WHO estimates that efficient cook stoves reduce fuel consumption by approximately 50%. The fuel savings alone would pay for the cost of the stoves in less than 3 months. The same is true for solar lights; they have a payback period of 3-4 months while providing brighter light than kerosene without pollution or risk of fire. Despite these seemingly large economic and health benefits, adoption rates are low and markets have been slow to develop.

If indeed these products are so valuable to consumers why is it that their adoption has been so slow? Why have private markets not developed for them? On the demand side, there are a number of well-documented barriers impeding adoption. For instance, poor households face credit constraints (Cole et al., 2013; Tarozzi et al., 2014), lack information about the product benefits or durability (Feder and Slade, 1984; Conley and Udry, 2001; Giné and Yang, 2009), suffer from present bias Duflo et al. (2011), or may be too risk adverse to experiment

---

<sup>1</sup>See Fuel for life: household energy and health World Health Organization, 2006

<sup>2</sup>See the Worldwatch Institute report: Energy for Development: The Potential Role of Renewable Energy in Meeting the Millenium Development Goals 2004.

with a new technology Foster and Rosenzweig (1995); Conley and Udry (2010); Bryan et al. (2014). Product quality can also be hard to assess and, with good reason, consumers are concerned about receiving products of inferior quality.<sup>3</sup> To address these challenges, a variety of retail offers have been suggested. For example, Levine et al. (2013) proposed using a retail offer involving a free-trial period and installment payments. In two different randomized control trials (RCTs) they found the adoption rate increased by more than 40 percentage points (from 5% to 47%) using the offer with a free-trial period and installment payments compared to a standard fixed-price retail contract. Further, repayments rates were extremely high (99%) and 31% of customers completed their installments early.

While this success is encouraging, in order to scale this approach to the firm or market level, supply-side frictions must also be addressed.<sup>4</sup> In particular, in order to reach the final consumer, firms or organizations will generally need to rely on a decentralized sales force (e.g., local vendors and shopkeepers). This is particularly true in rural areas. Based on surveys, we found that the issues faced by consumers are also present with local vendors. Small shopkeepers lack the capital necessary to purchase, transport, and store inventory.<sup>5</sup> Even with sufficient capital, most vendors have little experience with many of these products and may be uncertain, or even pessimistic, about the profitability of retailing these new technologies. To further complicate matters, weak enforcement of contracts and limited liability renders many commonly used incentive schemes for vendors infeasible.

In this paper, we address several of these supply-side frictions. We model the problem of a firm (or organization) who can employ a local vendor to distribute its goods at a lower per unit cost. The cost differential is meant to capture the fact that local vendors have superior information about the local market conditions and a greater ability to recover payments from the final consumer. The vendor is liquidity constrained and therefore cannot purchase inventory outright. Motivated by the limited enforceability of contracts in developing countries, we assume that the firm cannot prevent the local vendor from absconding with inventory. Therefore, the firm must use dynamic incentives in order to make it self-enforcing. The question is how to do so in the most efficient manner.

We characterize the optimal arrangement and show it has an appealing and simple implementation. It entails an initial endowment of the good and a fixed price at which the vendor can buy additional units in the future. The initial endowment helps overcome the

---

<sup>3</sup>For example, Bold et al. (2017) find that 30% of nutrients is missing in fertilizer, and hybrid maize seed is estimated to contain less than 50% authentic seeds.

<sup>4</sup>We developed these ideas in discussions with the world's largest development NGO, BRAC, in an effort to improve the distribution of development goods using their existing network of micro-entrepreneurs called Community Health Promoters (CHPs), and microfinance groups.

<sup>5</sup>See for example de Mel et al. (2008).

liquidity constraint and the opportunity to grow the business induces the vendor to continue reinvesting. We show that this arrangement is optimal both for profit-maximizing producers as well as for non-profit organizations with a limited budget whose objective is to maximize product distribution.

An interesting feature of the optimal arrangement is that it involves “starting small.” That is, the initial endowment is below the vendors’s capacity and only over time does the agent’s business grow to the efficient scale. We provide a closed form solution for the optimal size of the initial endowment and show that starting small is particularly important when profit margins are low and the vendor is relatively impatient.

Using our theoretical framework, we develop several hypothesis that enable us to test assumptions of the model. We then run a field experiment to evaluate the performance of the optimal arrangement implied by the model and study the impact of different contractual arrangements. We recruited vendors in rural Uganda to sell solar lights and randomized over several features of the arrangement that we offered to each vendor.

Our results from the field suggest that vendor liquidity constraints are indeed an important barrier. Vendors that are provided an initial endowment have approximately 4 times the sales of vendors who were not. In addition, consumer uncertainty about product quality appears to be another important factor. Vendors who were given a “loaner light” in order to provide potential customers with a free-trial period also had significantly higher sales. On the other hand, vendor uncertainty did not appear to be a limiting factor: providing the vendors with the right to return any unsold inventory had little effect on total sales.

Perhaps not surprisingly, vendor sales growth was lower and exhibited a different time-series than predicted by the model. This suggest that additional factors outside of the model play an important role in limiting vendors’ ability to grow their business. Exit surveys point to a general inability to save revenues from sales until the next delivery of lights (a period of only a couple weeks) as well as an unwillingness to offer credit (e.g., an installment plan) to customers.

Recent developments in fintech have facilitated ways to overcome these issues (as well as the ones for certain types of products. For example, M-Kopa Solar sells a home solar system with a kill switch using a contract that features time-payments via mobile money. Using mobile money ensures that vendors are not required to handle (and save) cash, while the kill switch makes the product worthless if the customer defaults (thereby increasing repayment rates). M-Kopa has higher production costs because each system must contain many of the capabilities of a mobile phone. Nevertheless, they have sold more than 450,000 units in East Africa. Their success suggests the severity of the frictions is non-trivial.

## 2 Related Literature

Our theoretical results are related to a large literature on optimal contracting and also to the literature on self-enforcing or relational contracts. Within the contracting literature the closest papers are Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007b), Demarzo et al. (2012), Quadrini (2004), Albuquerque and Hopenhayn (2004), Li (2013). They look at the problem of an entrepreneur that needs funding for a new venture and focus on the capital structure that helps deter the agent from stealing cash flows. Although we do not allow for commitment in our model, one feature of the optimal contract that is present in many of these papers and shows up in our setup as well, is the idea that the agent starts consuming only after a sufficiently long stream of positive outcomes.

Starting with DeMarzo and Fishman (2007a) a number of papers in this literature have looked at implementations where the agents get a loan and a credit line. When the cash flows are low the agent draws on the credit line to be able to meet the coupon payments. When the cash flows are high the agent pays back the credit line. If the credit line is repaid and cash is left the agent consumes. If, on the other hand, the agent has reached the limit of the credit line and still cannot meet the coupon payments then the firm is liquidated. Although the dynamics are similar, our implementation is somewhat different.

Within the relational contracts literature the closest paper is Thomas and Worrall (1994).<sup>6</sup> Their model is motivated by the problem of a multinational facing expropriation risk but it is mathematically very similar. As we do, they show that in this case, the agent's continuation value must increase at a rate given by the agent's discounting. A few recent papers have formally studied trade credit in a relational contracting setting. The most related is work by Troya-Martinez (2015).<sup>7</sup> Her implementation involves trade credit being suspended (possibly permanently) when the agent fails to make a full repayment instead. In our model, the adjustment takes place via quantities.

One contribution of this paper is to bridge the gap between the literatures on dynamic and relational contracting and development economics. Research in the areas of both has exploded in the last several decades. Yet, there has been relatively little work that uses the tools developed in dynamic contracting to offers solutions to problems in developing economies. A notable exception is the recent work of Townsend and co-authors.<sup>8</sup> For example, Karaivanov and Townsend (2013) use data from the Townsend Thai surveys to evaluate

---

<sup>6</sup>See also Baker et al. (2002), Levin (2003).

<sup>7</sup>See also Cunat (2007)

<sup>8</sup>Another exception is Dubois et al. (2008), who study both formal and informal mechanisms for risk sharing. There is also an older literature that has focused more on the consumption smoothing problem at the household level. See for example Townsend (1994) and Ethan Ligon and Worrall (2002).

which models best describe the patterns of investment and consumption.<sup>9</sup> Consistent with our hypothesis of liquidity constrained vendors, they document that investment in rural areas is sensitive to cash flows and that a savings-only regime (i.e., no borrowing) provides the best fit with the data. Our approach is complementary in that having documented a set of frictions we do not presume the market has necessary organized optimally in response to them. Instead, we ask if there are possible arrangements that might enhance welfare given the contractual constraints and experiment with these arrangements in a controlled study.

### 3 Illustrative Example

In this section we present a simple example which illustrates the key ideas of the paper. We take as given, that an organization (who we refer to as an NGO) has identified a good (in this case bed nets for malaria prevention) with social benefits for a particular target population and faces the problem of distributing the good throughout the economy without being able to write enforceable contracts with its distributors.

For simplicity, consider a single village in which there are a large number of households. The NGO has raised funds of  $B$  for the purpose of distributing bed nets throughout the village. The NGO can purchase bed nets from a producer at marginal cost  $c$ . Each household within the village is willing to pay up to  $p$  for a bednet.<sup>10</sup>

We assume here that  $p < c$ , and so, in the absence of some form of subsidy, the market for bed nets would remain undeveloped in the village (see Dupas and Cohen, 2010). In order to reach households, the NGO must incur a distribution cost of  $d$ , for the transportation and time involved in delivering each unit. The objective of the NGO is to maximize the discounted sum of all bed nets distributed,  $\sum_{t=0}^{\infty} \delta^t k_t$ , subject to the constraint that the NGO has a limited amount of funds with which to purchase and distribute bed nets.<sup>11</sup> The question we seek to answer in this section is how the NGO should go about distributing the bed nets.

Although the NGO is not concerned with profits, it is constrained by its funding and thereby will find it advantageous to charge households their willingness to pay in order to distribute more bed nets throughout the community.<sup>12</sup> By doing so, the NGO reduces their

---

<sup>9</sup>See for example Chiappori et al. (2014), Karaivanov and Townsend (2013), Kaboski and Townsend (2011), and Townsend and Urzua (2009) among others.

<sup>10</sup>Note that  $p$  may represent household's true value for the good, or the amount that they are able to pay, which might be less than their true value due to credit constraints.

<sup>11</sup>This is the same objective that would obtain if, for example, the NGO ascribes some sufficiently large social value,  $\Delta_s$ , to each bednet distributed and has the objective of maximizing total social surplus.

<sup>12</sup>Charging households for the good may have the additional benefit of selecting those households with the higher willingness to pay and, thus, be more inclined to use the product (Ashraf et al., 2010).

effective marginal cost to  $c + d - p$ . Thus, if the NGO decides to *procure and distribute* bed nets, it can afford to purchase and distribute a total quantity of bed nets equal to

$$K_{pd} = \frac{B}{c + d - p}.$$

Consider now the possibility of forming partnership with a local shopkeeper or vendor in order to assist with the distribution of bed nets. The natural advantage of the partnership is that local vendors can distribute bed nets at a cost of only  $d_A < d$ , based on their knowledge and retail experience within the local community. Thereby, the hope is that by forming this partnership, the NGO will be able to lower its costs and reach more households.<sup>13</sup> As discussed earlier, a difficulty with this approach is that local vendors are financially constrained, and thus, do not have the capital to purchase inventory up front. This is further complicated by the inability to write enforceable contracts and the limited liability of local agents. Thus, in order to sustain a partnership, the NGO will find it necessary to provide local vendors with appropriate incentives. The question then is whether doing so can achieve a better outcome than the procure and distribute strategy described above.

### Maximal First-Period Distribution

One approach is for the NGO to purchase as many bed nets as feasible ( $B/c$ ) and give them to the vendor to distribute. Provided the households' willingness to pay,  $p$ , exceeds the agents distribution cost,  $d_A$ , the agent will find it in her interest to distribute the bed nets to households, from which she derives a net profit of  $(B/c)(p - d_A)$ . If  $d > p$ , then the NGO will have achieved a larger distribution of bed nets using this approach relative to the procure and distribute strategy (otherwise, procure and distribute is preferable). However, because the NGO has used all of its resources, the vendor will have no incentive to continue the partnership. That is, the NGO will be unable to incentivize the agent to use some of her profits to reinvest and distribute more bed nets (recall that  $p < c$ , so without a subsidy, the vendor will not have an incentive to purchase additional units at their marginal cost and distribute them). Hence, the distribution process stops after the first period.<sup>14</sup>

### Starting Small and Building Up

Under certain conditions, the NGO can do better by starting small and facilitating the vendor growing her business over time. In order to provide incentives for the vendor to continue the

---

<sup>13</sup>Naturally, if the potential savings on distribution costs are sufficiently small, the NGO will not find overcoming the agency costs to be worthwhile.

<sup>14</sup>Since it has exhausted its budget, any promises made by the NGO suggesting otherwise are not credible.

partnership beyond the initial period, the NGO cannot exhaust all of its resources in the first period. Instead, the NGO procures an initial quantity of  $k_0 < B/c$ , and provides this to the vendor as “seed” capital. The promise of repeated business then provides the vendor incentives to reinvest. Such incentives can take different forms; one simple way is by offering to sell additional bed nets to the vendor at a subsidized price,  $p_A$ .

As before, provided  $p > d_A$ , the vendor will find it in her interest to sell the initial allocation to households leaving her with a net profit of  $k_0(p - d_A)$ . At this point, the vendor must decide whether to return to the NGO to purchase more bed nets or allocate this profit toward other uses. Assuming that bed nets take one month to sell and the vendor’s (monthly) discount factor is  $\delta_A$ , the vendor will return to the NGO to buy more bed nets provided that

$$\delta_A \left( \frac{k_0(p - d_A)}{p_A} \times (p - d_A) \right) \geq \frac{k_0(p - d_A)}{p_A} \iff p_A \leq \delta_A(p - d_A). \quad (1)$$

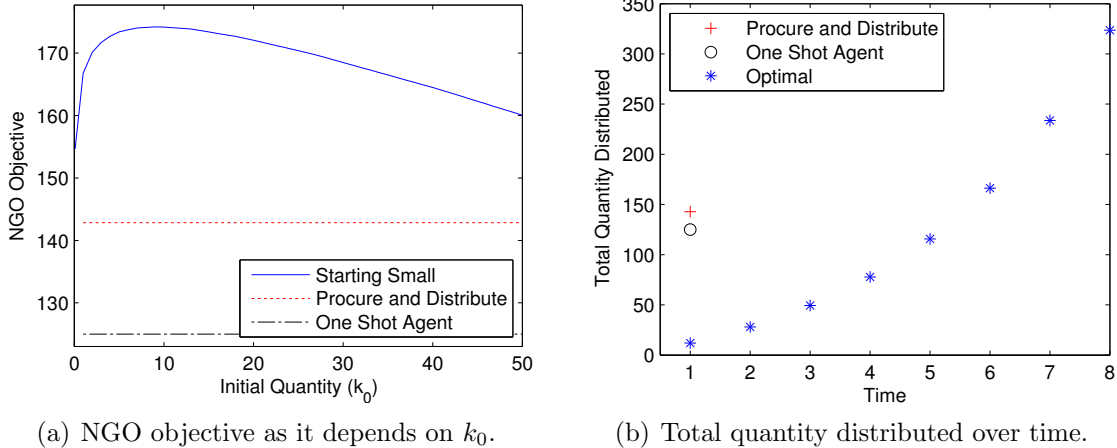
That is, providing appropriate incentives to the vendor, amounts to charging a price low enough that she finds it in her interest to buy and distribute more bed nets from the NGO. Rewriting the equality in (1) as

$$1 \leq \frac{\delta_A(p - d_A)}{p_A},$$

yields a simple interpretation; for each dollar of revenue earned, the vendor must decide whether to consume it (the left-hand side), or reinvest it in the partnership (the right-hand side). By reinvesting in the partnership, the agent can purchase a quantity of  $1/p_A$  bed nets, which can be sold over the next month at profit margin  $p - d_A$ , generating a total revenue of  $\frac{p - d_A}{p_A}$  in the next period. It is straightforward to see that this condition ensures that the vendor will prefer to continue her relationship with the NGO until either the market is saturated or the NGO runs out of money and becomes unable to continue providing bed nets at the subsidized price. Let us now fix  $p_A = \delta_A(p - d_A)$ —it can be shown that this is the optimal price for the NGO to charge the local agent—so that, over time the quantity of bed nets will grow each period at a rate proportional to the agents discount factor (i.e.,  $k_{t+1} = \delta_A^{-1}k_t$ ). Eventually, the NGO will exhaust its resources and the vendor will have distributed a total of  $K^* = \frac{B - ck_0}{c + d_A - p}$  bed nets.

When considering the number of units to allocate initially to the agent,  $k_0$ , the NGO faces a trade-off. A larger initial allocation increases the immediate availability of the good in the market but it reduces the resources of the NGO. Given the vendor can always decide not





**Figure 1:** Illustrates the advantage of starting small in the relationship and building up over time. The figures use the parameters  $p = 4, c = 8, d = 3, d_A = 0.5, B = 10^3, \delta_P = 0.99, \delta_A = 0.75$ .

to return and reinvest her profits, she must receive a continuation value in the relationship proportional to the number of units she is originally allocated. Figure 1(a) shows that the value of the NGO’s objective as a function of  $k_0$ . For this parametrization, it is optimal to use only a small fraction of its total resources in the first period. Although this implies that in the first period distribution will be much lower than with the other strategies (see panel (b)) as the vendor sells the bed nets she can return to the NGO for additional units and in this way slowly grow her business. The NGO’s objective can be increased significantly relative to direct procurement or a one-shot interaction with the agent by forming an arrangement that provides the agent with a small amount of seed capital.

Perhaps the most desirable feature of this arrangement is the simplicity with which the optimal partnership can be implemented; i.e., the NGO provides an initial quantity to the local agent (or seed capital) and charges a fixed (subsidized) price for all subsequent units. The seed capital helps overcome the agent’s liquidity constraints and the subsidized price provides the necessary incentives for reinvestment. In the next section, we present a formal model and demonstrate the optimality of this arrangement.

## 4 Theoretical Framework

We now relabel the NGO as simply the *principal*, which may also refer to a manufacturer or distributor seeking entry to a new market. As in the example, we endow the principal with some initial amount of capital,  $B_0$ , the technology to produce units of the good at a marginal cost,  $c$ , and an additional distribution cost for each unit,  $d$ .

Similarly, we relabel the local vendor as simply the *agent*, who has has a technological

advantage in that her distribution cost per unit is lower than the principal's. We will focus on the case in which the agent's distributional cost advantage is sufficiently large that the principal wants to use the agent to distribute its goods. To simplify notation and without loss in generality we normalize the agent's distribution costs to 0.<sup>15</sup>

The principal and the agent can interact repeatedly over time  $t = 0, 1, \dots, \infty$ . The agent has the capacity to distribute up to  $\bar{k}$  units of the good per period. The goods can be sold on the local market to *households*. We assume there are arbitrarily many potential households in the economy.<sup>16</sup> Each household has unit demand and is willing to pay  $\bar{p}$  for the good.

The agent has no capital and enjoys limited liability. The agent can also walk away from the arrangement in any period. The timing is as follows.

- At the beginning of period  $t$ , the principal gives the agent some amount  $k_t$  of goods for the agent to sell.
- The agent sells the goods and realizes a cash flow of  $\bar{p}k_t$ .<sup>17</sup>
- The agent then makes a transfer  $T_t$  to the principal and consumes the rest.
- Discounting occurs and then period  $t + 1$  begins.

An *arrangement* is a relational contract between the principal and the agent consisting of a sequence of functions,  $\{k_t, T_t\}_{t=0}^{\infty}$ , mapping the relevant histories into quantities produced and delivered by the principal as well as reports and transfers made by the agent.

Both the principal and the agent are risk neutral and care about the expected present value of their per-period payoffs. We consider here a profit maximizing principal and discuss later the connection to the NGO's objective of maximizing distribution. The principal and agent have per period discount factors  $\delta_P$  and  $\delta_A$  respectively and we assume  $\delta_P \geq \delta_A$ . We use  $\Pi$  and  $U$  to denote these values:

$$\Pi = E \left[ \sum_{t=0}^{\infty} \delta_P^t (T_t - ck_t) \right]$$

$$U = E \left[ \sum_{t=0}^{\infty} \delta_A^t (p_t k_t - T_t) \right]$$

---

<sup>15</sup>We could relabel the sale proceeds as being net of agent's costs

<sup>16</sup>This assumption is convenient to preserve stationarity. It is not difficult to extend our results to a setting with a finite number of households.

<sup>17</sup>In Section 4.5, we extend our results to a setting in which sales revenue is risky and privately observed by the agent.

Though we do not explicitly incorporate termination of the arrangement, setting  $k_t = T_t = 0$  for all  $t \geq \tau$  is equivalent to terminating the arrangement at date  $\tau$ .

We assume that the goods can be distributed and sold at an expected profit.

**Assumption 1** (Profitability).  $\delta_A \bar{p} - \delta_P c > 0$

This assumption guarantees the set of equilibria is non-trivial with a profit maximizing firm. This assumption is not necessary when we study the problem of an NGO which is willing to spend resources in order to maximize distribution (see Section 4.4).

The history of the game observed by the principal at the start of period  $t$  is:  $h_t^P \equiv \{k_s, T_s\}_{s=0}^{t-1}$ . When choosing an action in period  $t$ , the history of the game for the agent is  $h_t^A \equiv \{h_t^P, k_t\}$ . A pure strategy for the principal is a sequence of functions  $\{\sigma_t^P\}_{t=0}^\infty$  which determine the quantity of goods  $k_t$  to give to the agent as a function of  $h_t^P$ . A pure strategy for the agent is a sequence of functions  $\{\sigma_t^A\}_{t=0}^\infty$  which for each period determine the agent's transfer  $T_t$  as a function of  $h_t^A$ . Mixed strategies are defined in the conventional way and denoted by  $\Sigma^P$  and  $\Sigma^A$ .<sup>18</sup>

Because there is no external enforcement of contracts, the relationship will be governed by self-enforcing arrangements. An arrangement is said to be self-enforcing if the strategy pair that implements it is a Perfect Bayesian Equilibrium (PBE) of the game described above. While there are many PBE of the game, we will focus on *optimal* arrangements (i.e., the set of Pareto efficient PBE), which can be parameterized by the expected continuation utility of the agent (Abreu et al., 1990), which we will denote by  $v$ .

## 4.1 The Principal's Problem

Finding the optimal arrangement can be reduced to solving a dynamic program, which we undertake here. Recalling that we use  $v$  to denote the continuation utility of the agent, the principal's maximization problem can be stated recursively as:

$$\Pi(v) = \sup_{K, T, W} \{T - cK + \delta_P \Pi(W)\} \quad (\text{P})$$

---

<sup>18</sup>e.g.,  $\Sigma_t^A$  is a distributions over pure-strategies as a function of the  $h_t^A$ .

subject to

$$T \in [0, \bar{p}K] \tag{2}$$

$$K \in [0, \bar{k}] \tag{3}$$

$$\delta_A W - T \geq V_{out} \equiv 0 \tag{4}$$

$$\bar{p}K - T + \delta_A W = v \tag{5}$$

The liquidity constraint in (2) implies that neither agent nor principal has access to a borrowing technology. The only mechanism by which the consumption good is created is through selling to households.<sup>19</sup> Equation (4) is the key incentive constraint, which can be interpreted as deriving from the principal’s inability to write an enforceable contract. This constraint ensures that the agent has incentive to actually make the transfer of  $T$  rather than consume it and forego her future continuation value.<sup>20</sup> Finally, (5) is a standard promise keeping constraint which requires the principal deliver  $v$  in utility to the agent.<sup>21</sup>

Due to the linearity of preferences, the solution to the dynamic program is has a “bang-bang” feature. For low  $v$ , the agent is compensated purely with continuation value and does not consume. For high  $v$ , the agent consumes as much as possible.

**Lemma 1.** *There exists a solution to (P). The optimal policy is as follows:*

(i) For  $v \in [0, \bar{v}]$ :

$$K(v) = v/\bar{p}, \quad T(v) = \min\{\bar{p}K(v), \delta_A \bar{v}\}, \quad W(v) = \min\{\delta_A^{-1}v, \bar{v}\}.$$

(ii) For  $v > \bar{v}$ :

$$K(v) = \bar{k}, \quad T(v) = \max\{\bar{v}(1 + \delta_A) - v, 0\}, \quad W(v) = \max\{\bar{v}, \delta_A^{-1}(v - \bar{v})\}.$$

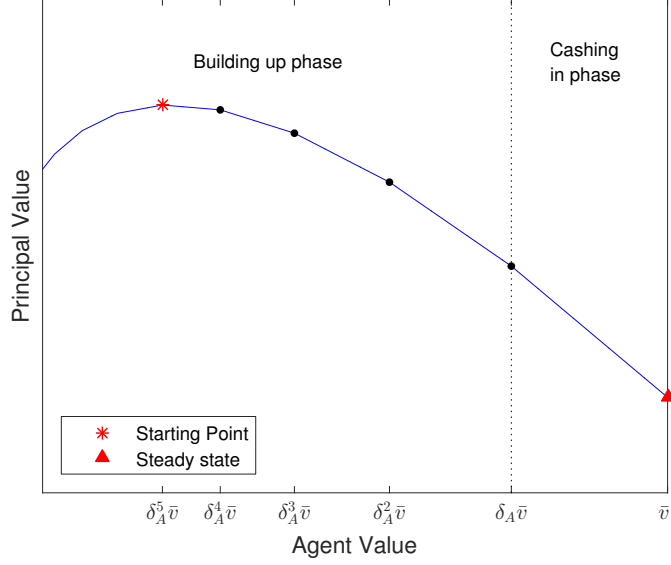
When the agent’s continuation value is low, the principal has to (inefficiently) restrict the amount of inventory in order to prevent the agent from stealing while respecting the promise keeping constraint. The agent then transfers all of the proceeds back to the principal in

---

<sup>19</sup>This constraint eliminates the possibility of dynamic trading gains due to the agent’s relative impatience (Opp and Zhu, 2015).

<sup>20</sup>We have implicitly assumed that it is optimal to use a grim-trigger punishment if the agent deviates from the arrangement. This assumption is not restrictive (Mailath and Samuelson, 2006, Proposition 2.6.1).

<sup>21</sup>To simplify the analysis we solve for the optimal arrangement assuming the agent does not have access to a private savings technology. This assumption is without loss of generality with a risk-neutral agent and thus our optimal arrangement is robust to introducing a private savings technology for the agent. See Kocherlakota (2004) for a discussion of how access to a private saving technology influences the optimal unemployment insurance in a setting with a risk averse agent.



**Figure 2:** This figure illustrates the principal’s value function and the two phases of the arrangement for parameters such that the time-to-capacity is 5 periods (i.e.,  $N^* = 5$ ). The initial phase begins at the red asterisk and moves rightward along the black dots until the agent’s value reaches  $\delta_A \bar{v}$ , at which point investment is efficient and above which the agent begins to consume. The parameters used to generate the figure are  $\delta_A = 0.78, \delta_P = 0.9, \mu = 0.25, \bar{k} = 1$ .

exchange a higher promised utility in the next period. When the agent’s continuation value is sufficiently high, this constraint stops binding and inventory reaches its efficient level. At this point, the agent only transfers a part of her revenues to the principal and consumes the rest.

## 4.2 Implementation

The main insight from this section is that the optimal arrangement can be implemented with a structure that is identical to the one used for the NGO’s problem.

**Proposition 1.** *The optimal arrangement can be implemented by a pair  $(k_0^*, p_A^*) \in \mathbb{R}^2$ . The principal provides the agent with a fixed initial endowment of the good,  $k_0^* < \bar{k}$ , and sets a fixed price,  $p_A^* = \delta_A \bar{p}$  at which the agent can purchase all future units of the good.*

Intuitively, notice that whilst the agent’s liquidity constraint binds, the agent is induced to reinvest all her profits. Thus, she transfers an amount  $T(v) = \bar{p}K(v) = v$  to the principal. In exchange, she obtains a continuation value of  $\delta_A^{-1}v$ , which means that in the next period she will receive a quantity of  $K(\delta_A^{-1}v) = \frac{v}{\delta_A \bar{p}}$ . Thus, the agent is effectively paying a price of  $p_A^* = \delta_A \bar{p}$  for each unit of inventory that she will receive in the next period. Once the

liquidity constraint no longer binds (i.e. for  $v \in (\delta_A \bar{v}, \bar{v}]$ ), the agent makes a transfer of  $\delta_A \bar{v}$  in each period in exchange for  $\bar{k}$  units in the next period. Thus, again, the effective price is  $\delta_A \bar{p}$ .

Initially, the dynamics of the optimal arrangement are familiar with the example from Section 3. A profit-maximizing firm provides the agent with seed capital  $k_0^*$  and a subsidized price, which allows her to grow her business gradually. Once the agent has reached scale (i.e.,  $k_t = \bar{K}$ ), the agent begins to consume and enjoy the profits. The optimal arrangement thus involves two phases. The first is a *building up* phase in which the agent’s business grows at a rate proportional to her discount factor. Although the agent does not consume during this period, her continuation value increases in this region as consumption nears. The second phase begins when the investment reaches its efficient level. At this point, the principal can no longer provide incentives to the agent by promising to grow the agent’s business and thus the agent enters the *cashing in phase* and begins to consume. These dynamics are illustrated in Figure 2.

The two distinct phases arise in part due to the agent’s linear preferences. With strictly concave utility, the optimal arrangement would have similar features, but the distinction between the two regions would be less dramatic as the agent would consume prior to reaching the efficient investment level. We maintain linear preferences so as to preserve the ease with which the arrangement can be implemented, which is particularly critical for our application of interest.

### 4.3 Starting Small

Perhaps the most interesting feature of the optimal arrangement described above is that it involves “starting small”. That is, the initial endowment is below the agent’s capacity and only over time does the agent’s business grow to the efficient scale. In this section we provide a closed-form solution for the optimal initial endowment and several comparative statics.

In order to do so, let  $N^* \equiv \min\{t : k_t = \bar{k}\}$ , which denotes the number of period until the agent reaches capacity (henceforth, the “time-to-capacity”) under the optimal arrangement. Also, let  $\gamma \equiv \frac{\delta_P}{\delta_A}$  denote the agent’s relative impatience and  $\mu \equiv \frac{\bar{p}-c}{\bar{p}}$  denote the profit margin of the good in the absence of any frictions.

**Proposition 2.** *The optimal initial endowment is  $k_0^* = \delta_A^{N^*} \bar{k}$ . If  $\delta_A < \delta_P$  (i.e.,  $\gamma > 1$ ), the time-to-capacity under the optimal arrangement is given by*

$$N^* = \left\lceil \frac{\log\left(\frac{\mu}{1+\gamma(\mu-1)}\right)}{\log(\gamma)} \right\rceil,$$

where  $\lceil x \rceil$  denotes the smallest integer weakly greater than  $x$ . If  $\delta_A = \delta_P$  (i.e.,  $\gamma = 1$ ) then  $N^* = \left\lceil \frac{1-\mu}{\mu} \right\rceil$ .

Notice that  $N^* \geq 1$  (by Assumption (1)) and therefore the initial endowment is always strictly less than  $\bar{k}$ . Intuitively, the endowment is designed to relax the liquidity constraints of the agent, but the principal never recovers the costs of these units. The revenues from first-period sales are the agent's rent.<sup>22</sup> Hence, there is no reason to provide the agent with  $\bar{k}$  units in the first period. Provided  $k_0 \geq \delta_A \bar{k}$ , the revenues from first-period sales will be sufficient to purchase  $\bar{k}$  units for the next period.

Given Proposition 2, it is then straightforward to conduct comparative statics with respect to the two key parameters.

**Corollary 1.** *Under the optimal arrangement:*

- (i) *The initial endowment is increasing in the profit margin ( $\mu$ ) and decreasing in the relative impatience of the agent ( $\gamma$ ).*
- (ii) *The time-to-capacity is decreasing in the profit margin ( $\mu$ ) and increasing in the relative impatience of the agent ( $\gamma$ ).*

This corollary highlights that “starting small” is particularly important when profit margins are low and the agent is relatively impatient, both of which are likely to be important factors in applications of interest as well as our field experiment.

#### 4.4 Relation to the NGO's Problem

When Assumption 1 holds the optimal arrangement for the NGO only differs from what a profit-maximizing firm would do in that the NGO would pick a higher initial  $k_0$ . Indeed, it would pick  $k_0 = \bar{k} > k_0^*$  but would still use  $p_A^*$ . Importantly, the budget constraint would not play a role since this operation is actually profitable.

When Assumption 1 fails, the NGO must take into account the available budget to finance the subsidized sale of the goods. Fortunately, as we show below, this can be handled by recasting the NGO's problem. Assume each unit adopted generates a social surplus of size  $\Delta_S$ . Thus, the NGO objective of maximizing the total (discounted) social surplus can be written as:

$$\sum_{t=0}^{\infty} \delta_P^t (k_t \Delta_S).$$

---

<sup>22</sup>Recall that the transfer in the first-period is for inventory to be received in the beginning of the second period.

The funding constraint requires that the present value of the cost of the operation cannot exceed the NGO's budget.

$$\sum_{t=0}^{\infty} \delta_P^t (ck_t - T_t) \leq B_0$$

Rather than analyzing the problem of maximizing welfare for a given budget, consider the dual problem: minimize the total cost of the operation

$$\min \sum_{t=0}^{\infty} \delta_P^t (ck_t - T_t), \tag{6}$$

subject to achieving a certain level of discounted social surplus,  $S$ :

$$\Delta_S \sum_{t=0}^{\infty} \delta_P^t k_t \geq S.$$

Naturally, the objective in (6) can be rewritten as

$$\max \sum_{t=0}^{\infty} \delta_P^t (T_t - ck_t)$$

which is the same as the profit maximizing firm's objective.

Thus, the dual of the NGO problem (as formulated above) is the same as the profit-maximizing firm's problem we analyzed with the additional constraint of achieving a certain level of social surplus. One can then solve the NGO dual for different levels of social surplus,  $S$ , and choose the highest  $S$  for which the total cost satisfies the budget constraint. The dynamics of the relationship will continue to be characterized by a building-up phase where the agent grows her business followed by a cashing-in phase during which the agent consumes.

## 4.5 Risky Cash Flows and Private Information

Thus far, we have assumed that the agent can sell up to  $\bar{k}$  units each period at a fixed price of  $\bar{p}$ . In practice, there is likely to be uncertainty associated with both of these variables. We extend our analysis by allowing the proceeds from the sale of the goods to be stochastic and privately observed by the agent. We demonstrate two main findings.

First, the optimal arrangement in Proposition 1 is robust though it may require the principal to provide the agent with access to a savings technology. Second, the dynamics of the optimal arrangement depend on cash flow realizations. As a result, growth is stochastic and the long run outcome may be a termination of the relationship rather than operation at



full capacity. In what follows, we elaborate further on these two findings.

To do so, let us denote the random variable representing the per unit proceeds or cash flow in period  $t$  by  $p_t$ , which is distributed according to the cumulative distribution function,  $F$ , with support  $[p_{\min}, p_{\max}]$ .  $F$  is i.i.d. over time with mean  $\bar{p} \equiv \mathbb{E}[p_t]$ .

In order for an arrangement to be an equilibrium, it must satisfy constraints similar to the benchmark case with the addition of an incentive compatibility constraint to ensure the agent will report the realized proceeds truthfully.

Given that the agent is risk neutral, we can interpret the promised value  $v$  directly as a favorable money balance the agent has with the principal. The principal gives this value  $v$  to the agent in two forms: 1) units to sell, denoted by  $K(v)$ , and 2) cash to be held in the agent's account during the period, denoted by  $I(v)$ . To fix ideas, we have normalized the intraperiod gross return on this account to one. To satisfy the promise keeping constraint, it must be that

$$\bar{p}K(v) + I(v) = v.$$

Thus, for the units that the agent buys we can think of them as having a beginning of period price of  $\bar{p}$ . At the end of the period  $t$ , the agent will have  $p_t K(v) + I(v)$  dollars that she can choose to reinvest or consume. Note that for every marginal dollar the agent gives the principal she must be promised at least  $\delta_A^{-1}$  dollars in the next period; otherwise the agent would prefer to consume than to transfer cash to the principal. Thus, to incentivize reinvestment, the principal can provide the agent with a savings technology with return  $\delta_A^{-1}$  in addition to a beginning of period price per unit of capital,  $\bar{p}$ . Notice that with these two prices, the agent is indifferent as to how  $v$  is split into  $K(v)$  and  $I(v)$  and also indifferent as to how  $p_t K(v)$  is split into consumption and transfers back to the principal. Therefore, such an arrangement is incentive compatible for any choice of  $K$ ,  $I$ , consumption and transfers.

**Proposition 3.** *When cash flows are risky and privately observed by the agent, the optimal arrangement can be implemented with an initial endowment of units and cash, allowing the agent to buy units in period  $t + 1$  for a price  $p_A^* = \delta_A \bar{p}$  at the end of period  $t$ , and providing the agent with a savings technology that delivers an expected gross return of  $\delta_A^{-1}$ .*

Having determined the prices to satisfy incentive compatibility and promise keeping constraints. We can now think of the principal's problem as a single-person decision problem with two components:

(1) At the beginning of the period: a portfolio choice problem, how much to invest in the risky asset  $K(v)$  and how much in the safe asset  $I(v)$ .

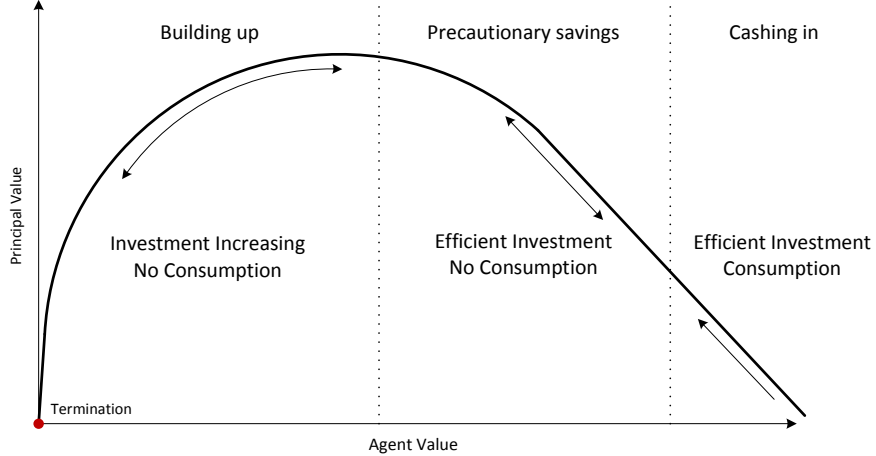
(2) at the end of the period: a consumption-savings decision, deciding how much to allow the agent to consume,  $C(v, p)$  versus how much to save for the future,  $T(p, v)$ .

If the degree of asymmetric information is small, in particular, if  $p_{\min} \geq \delta_A \bar{p}$  then the solution to both these problems is exactly the same we had when output was deterministic. In terms of the portfolio problem (1), when  $p_{\min} \geq \delta_A \bar{p}$  (and therefore  $p_{\min} \geq c$ ), the returns of the risky technology dominate the return on the safe asset. Thus  $I(v) = 0$  if  $K(v) < \bar{k}$ . Regarding the consumption savings decision (2), in this case there will be no need to allow for precautionary savings, this follows since if  $p_{\min} \geq \delta_A \bar{p}$ , upon reaching capacity, the agent will always have sufficient funds to repurchase the full stock in future periods even after a string of the worst possible realizations. Thus, if  $\delta_A \bar{p} < p_{\min}$  the dynamics of the relationship are very similar to the ones for the deterministic case except that the growth is stochastic rather than deterministic. In the long run the agent will always operate at capacity.

When the degree of asymmetric information increases the solution to (1) and (2) may differ, resulting in different dynamics. First note that if  $p_{\min} < c$  then the risky investment no longer dominates the safe investment for all realizations of  $p$  and the solution to the portfolio choice problem may involve  $I(v) > 0$  even for  $K(v) < \bar{k}$ . Even though the principal is risk neutral, once the agent's constraints are taken into account he is effectively risk-averse over the agent's end of period wealth. Hence, the principal may choose to allocate some wealth to the safe investment prior to reaching capacity.

Furthermore, if the difference in discount rates is small, the solution to the consumption-savings decision will delay consumption even beyond when full capacity is reached. This allows the agent to accumulate some precautionary savings that can be used to purchase inventory following a string of negative realizations. In this case, the dynamics of the relationship involve three phases (see Figure 3). There is the initial building up phase in which after high realizations of  $p$  the quantity allocated to the agent will increase and after very low ones it will decrease. Once the agent reaches full capacity then a new phase, the *precautionary savings* phase begins. In this region, the agent does not consume nor does the quantity allocated grow. Rather, the agent deposits precautionary savings into a savings account, which translates into higher continuation values.

The amount of precautionary savings depends both on the relative impatience of the agent and on the distribution of risky cash flows. Regarding the long run outcome, there are two cases: (i) If the savings buffer is sufficiently large (e.g.,  $\delta_A = \delta_P$ ) then the long run outcome is either full capacity or termination depending on the realization of prices, (ii) if the savings buffer is small (i.e.,  $\delta_A \ll \delta_P$ ), then with probability one the relationship will eventually be terminated. The latter case is similar to the immiseration result of Thomas and Worrall (1990).



**Figure 3:** Illustration of the optimal arrangement with risky cash flows when a precautionary savings phase is needed and  $V_L < V_{\bar{k}}$ .

## 5 Testable Hypothesis

In this section, we revisit several assumptions of the model in order to develop hypothesis regarding factors that inhibit adoption and market development. We will later test these hypothesis in our field experiment.

### 5.1 Liquidity Constraints

In the model, we assumed that the agent did not have any wealth to invest in the project nor access to credit markets. Formally, this is represented by the constraint that  $T \leq \bar{p}K$ . As a result, a necessary feature of the optimal contract is to provide the agent with an initial endowment or “seed capital” (see Propositions 1 and 2). If instead, the agent did not face liquidity constraints then this feature would not be necessary (i.e., the optimal  $k_0^*$  would be zero). Instead, the optimal arrangement involves an additional transfer to the principal at  $t = 0$  in the amount  $\bar{v}$  and immediately moving to the steady state (i.e., where the agent continuation value is  $\bar{v}$ ). In this case, the principal can extract the full surplus and hence there is no need to start small. This observation naturally leads us to the following hypothesis.

**Hypothesis 1.** *If the agent has sufficient initial wealth or access to credit markets then the performance of the arrangement should not depend on the size of the initial endowment.*

Thus, by varying the size of the initial endowment, we can evaluate the extent to which the liquidity constraints are relevant.

## 5.2 Risk Aversion or Pessimism

In the previous section, we argued that the optimal arrangement is robust to settings where cash flows are stochastic. There, we maintained the assumption that the agent is risk neutral and has the correct subjective beliefs about the distribution of cash flows.

If, on the other hand, the agent is risk averse or pessimistic about demand for the product then she may be unwilling to reinvest sales revenue to buy more units. One way to overcome such an aversion is to provide the agent with insurance against being unable to sell units at a profit by offering the right to return unsold inventory. Thus, if demand turns out to be sufficiently low, than the agent can simply return the units without losing her investment.

**Hypothesis 2.** *If the agent is risk averse or pessimistic about the ability to sell the good for a profit then providing the “right to return” unsold units should improve the performance of the arrangement.*

## 5.3 Consumer Uncertainty and Learning

As discussed in the introduction, there are a number of demand-side barriers that have been well documented in the literature. Several of these barriers pertain to the information available to the customer. For instance, customers are likely to be skeptical about the benefits and durability of the new technology (Feder and Slade, 1984; Conley and Udry, 2001; Giné and Yang, 2009). And rightly so. There is suggestive evidence that this problem has been caused in part by the proliferation of cheap and unreliable products.<sup>23</sup> This general lack of information or skepticism may lead to an adverse selection problem between the agent and consumer, thereby reducing the number of sales and the amount of revenue the agent is able to generate. Even absent an adverse selection problem, if households are uncertain about the quality of the good and risk averse, then they may be unwilling to invest in the new technology.

As suggested by Levine et al. (2013), providing the customer with a free-trial period may help to overcome these informational barriers. In our setting, the free-trial period also gives customers the chance to experience the financial benefits thereby relaxing the liquidity constraints of households. For both of these reasons, we formulate a third testable hypothesis.

**Hypothesis 3.** *If consumers are uncertain about the quality of the good or liquidity constrained then providing the agent with a “loaner” designated to provide customers with a free-trial period before their purchase should increase the performance of the arrangement.*

---

<sup>23</sup>The Lighting Africa project is well aware of this problem so they have undertaken an effort to field test many of the available products and provide a public certification of their quality.

## 6 Field Experiment

In this section we describe our field experiment. The purpose of the experiment was (1) to evaluate the overall performance of the optimal arrangement implied by the model, and (2) to test the hypothesis formulated in the previous section.

### 6.1 Experimental Design

To conduct the experiment, we partnered with BRAC Uganda, a large non-profit organization.<sup>24</sup> BRAC has a network of community health providers (CHPs) from which we recruited our “agents”. Effectively, there is one CHP per village and, prior to our intervention, these CHP’s worked as vendors of health related consumable goods such as soap, sanitary pads, and malaria pills which they acquired from BRAC.

We visited 8 BRAC branches in rural Uganda. The trial in the first four branches ran from April 2013 to April 2014 and in the second wave of 4 branches from January to June 2014. Each branch was selected based on having low penetration of grid connections and limited distribution of low-cost solar lights.

A BRAC branch has a few dozen microfinance groups, each with 20 or so women. We divided each branch into 4 zones, each of which typically had 10 or more microfinance meetings. A BRAC credit officer escorted us to four microfinance group meetings per zone. The meetings were geographically dispersed so that each vendor would have a catchment area of 200 or so households. Our goal was that each catchment area would have enough residents to support one vendor.

At these microfinance meetings we presented the solar lights and explained we were recruiting vendors to sell these lights. We provided vendors with several different models of lights. One of these was the Firefly Mobile, produced and distributed by Barefoot Power. This solar light is bright enough for reading and it can also charge a mobile phone. We also introduced the basic Firefly, which is slightly less bright and cannot not charge a phone. The wholesale and suggested retail prices were \$20 and \$26 for the Firefly mobile and \$12 and \$16 for the basic.

We invited one woman per microfinance meeting to a recruitment meeting. If more than one woman at a recruitment meeting was interested, we gave preference to one who had access to SMS text messaging. In a few cases we asked the credit officer privately for a recommendation. If a recommendation from the credit officer was not possible, we selected

---

<sup>24</sup>Vastinah Kemigisha of CIRCODU ran the sales study, assisted by Moses Oundo of BRAC Research. Aisha Nansamba of BRAC Research was instrumental in designing the procedures and connecting us with branch managers. Anne Kayiwa was key in our partnership with Barefoot Power.

the vendor who first expressed interest. During the recruitment meeting, we trained vendors on how to use the light as well as its economic benefits.

Our training involved explaining features of the light, the operation of the light, the terms of the retail sales offer (such as the one-year warranty). We showed vendors how the light can save customers money, where the savings on kerosene can quickly sum up to more than the cost of the light.

We anticipated many customers would be liquidity constrained. Thus, we explained to the vendors the advantages of several sales offers that overcome liquidity constraints: layaway (which Guiteras et al. (2014) found worked well selling water filters in Bangladesh), installments (which Levine et al. (2013) found worked well selling cookstoves in urban and in rural Uganda), and selling via a rotating savings and credit association (ROSCA). In a ROSCA, a group of customers pool their funds each meeting to purchase one light. The group continues until all customers have purchased a light. ROSCAs are common in this part of Uganda.

After completing training, we gave each interested woman a solar light. She was then asked to pay for the light with mobile money over the next several weeks. The purpose was two-fold: first, we hoped to familiarize potential vendors with the type of sales offer to use with their own customers and second, to partially screen out vendors that were unlikely to perform well. Eventually, all vendors completed payments though several of them took longer than was originally specified.

After the field staff made invitations and prior to the recruitment meeting, we randomized half of the zones to receive an arrangement that included a trade-credit line of up to 4 lights and the other half of the zones did not receive a trade-credit line.<sup>25</sup> We offered all vendors the right to purchase lights at the wholesale price during the first meeting or at any point in the future.<sup>26</sup> In order to test Hypothesis 2 and 3, we conducted two additional (and orthogonal) randomizations. First, we randomized over whether the agent was provided the right to return unsold inventory. Second, we randomly selected a subset of the vendors to which we provided a loaner light. The light was clearly labeled as “Property of BRAC” and the vendor was instructed to use the loaner light in order to give potential customers a free-trial period.

We held a separate recruitment for all of the potential vendors who received the same

---

<sup>25</sup>In a setting without enforcement, storage costs, or default costs, a trade-credit line is equivalent to providing the agents with an endowment of the good. We framed the arrangement as a “credit line” in attempt to recover our costs in cases that the vendor was unable to sell lights. About 50% of the vendors who received trade credit and placed one order did not make a second order. Among these vendors, we recovered approximately 30% of the inventory.

<sup>26</sup>Setting the vendor price equal to the wholesale price is roughly consistent with a distribution cost equal to 10% of sales and a monthly discount factor of  $\delta_A = 0.85$ .

vendor arrangement (that is, all the women in the same zone who had the same arrangement). At the recruitment meeting we discussed strategies for how to sell lights. We emphasized offering the customer a free-trial period and time payments (as the vendors were given). We then introduced the vendors to their designated arrangement. At the end of the recruitment meeting we took initial orders. Vendors not allocated an endowment paid cash for their initial orders, while vendors offered the endowment paid cash only if ordering more than 4 lights.

Vendors sold lights throughout the month. Once a month Barefoot sent a text message asking vendors to SMS back with their next order. Barefoot made a delivery a week after the text message requesting orders. Vendors met the delivery driver at the BRAC branch headquarters to make their payment and accept delivery of the lights they had ordered.

## 6.2 Summary of Findings

The 62 vendors with no credit line had average sales of 1.8 lights (SD = 6.1, median = 0).<sup>27</sup> The 67 vendors who received the “optimal” arrangement, which included a credit line sold an average of 6.6 lights (SD = 7.3, median = 4). The difference is highly statistically significant ( $p < 0.01$ ) with both a two-sided t-test and with a nonparametric sign-rank test. The difference becomes even more significant when we omit a single outlier who sold 44 lights without a credit line, as we do in the remainder of our analysis. The difference between are illustrated in Figure 5. A regression of  $\log(1 + \text{sales})$  on the characteristics of the arrangement can be found in Tables 2 and 3. Again, providing vendors with trade credit predicts significantly higher sales across all model specifications.<sup>28</sup>

Qualitative evidence also supports the statistical result that providing trade credit to ease liquidity constraints is important. In interviews, vendors not offered trade credit stated that they would have ordered and sold more lights if they had been given credit for their initial orders. They said that it was challenging to get money in advance from customers to order more lights. Conversely, vendors with trade credit stated they would not have sold as many lights without the initial credit. Thus, while the sample is small, our study provides strong evidence that liquidity constraints are important and providing trade credit can substantially increase sales. Although sales were much lower in the second wave of four branches, the credit line remains important.<sup>29</sup>

---

<sup>27</sup>The actual sale of a light from vendor to household was not observable to us. We therefore refer to “sales” as the total number of lights distributed to the vendor (presumably to sell to customers) during the life of the study.

<sup>28</sup>For the t-test on sales and the regression on  $\log(\text{sales}+1)$  we cluster standard errors by branch, the unit of randomization.

<sup>29</sup>Our qualitative interviews partially explain why sales were so much lower in the second wave. Vendors



**Figure 4: Average Sales by Arrangement.** This graph shows the average total sales of vendors by the characteristics of the arrangement they were offered. The blue bar is the average of total sales across all vendors who received an offer with the corresponding characteristics. The range between the red capped line indicates the 95% confidence interval.

To see if vendor uncertainty or pessimism was a relevant consideration barrier, we gave a random half of the vendors who were not provided trade credit the right to return unsold inventory for a full refund.<sup>30</sup> Perhaps surprisingly, there is little evidence to suggest that the right to return improves the performance of the arrangement (see Figure 6). In the regression estimation the coefficient is (at best) only weakly positive and not statistically significant.

On the other hand, there is convincing evidence that consumer uncertainty is important factor. Within each of the study arms, we provided a randomly selected subset of the vendors a loaner light to rotate among potential consumers. In interviews, vendors with free trials reported they lent it to at least 5 households. The average vendor without a loaner light sold about 3 lights. The vendors allocated a loaner light sold more than 3 times as many (mean=11.2, median=10, SD=9.7). These results are illustrated in Figure 7.

Figure 4 summarizes our main results from the field experiment. Providing agents with either a credit line or a loaner light had a significantly positive impact on sales, while the right to return did not. Perhaps most notable is increase in sales observed via an arrangement

---

suggested sales were low because competitors were selling low-quality lights for lower prices. As most potential customers could not detect the quality difference, demand was low for the Barefoot lights. In addition, two of the branches in the second wave were in areas with higher NGO penetration, so consumers may have become used to receiving free or deeply discounted goods thereby reducing their willingness to pay.

<sup>30</sup>Vendors who received trade credit could return inventory rather than repay their debt, whereas vendors with no credit could return their inventory for cash.



that provided the agent with *both* a credit line and a loaner light.

### 6.3 Discussion

Our results suggest that vendor credit constraints and consumer uncertainty (or lack of information) are important barriers, while vendor uncertainty is not. One plausible explanation for the last finding is that vendors rarely purchased inventory and then sought customers. Instead, despite our encouragement to do otherwise, 70% of vendors reported waiting for a client to pay cash for the light prior to ordering it. Thus, while vendor uncertainty does not appear to be an important factor in our experiment, it may be more crucial in other settings where vendors use different sales techniques.

In the model, vendors retain earnings and increase their inventory and, on average, should grow sales over time at a rate proportional to  $\delta_A^{-1}$  until reaching capacity. Unlike our model, the initial burst of sales in our experiment only rarely led to a sustained growth. Even for vendors with trade credit, by four months after the first recruitment, the average sales was less than 1.5 lights per month. In part this could be simply due to the agents suffering customer defaults or simply not being able to sell the lights at a profitable price. However, as we discuss below, we believe there were two other key reasons why vendor's sales growth was lower than predicted.<sup>31</sup>

#### Factors that Inhibited Growth

*Inability to Save.* Many vendors noted that it was difficult to retain cash from sales until the next order even if it was only a few weeks away. Vendors with cash in hand reported being subject to a lot of demands and found it difficult to avoid using it before the next delivery. Indeed, our model suggests that a savings technology may be necessary when cash flows are risky (see Proposition 3). The difficulty in saving might have been exacerbated by the fact

---

<sup>31</sup>While there are certainly other factors that may have contributed to a lack of growth, we believe most of them can be overcome with proper screening and training. We intentionally did not engage in much screening so as to obtain cross-sectional variation. For instance, vendors reported they intended to work a median of 20 hours a week selling solar lights. At the same time, they reported working about 40 hours a week at their current jobs and having an average of 5 children at home. Thus, it seems unlikely that vendors would have anywhere near the 20 hours a week they forecast to market solar lights. Amplifying this concern, the median vendor reported taking 60 minutes to travel to the BRAC branch office. Thus, most vendors faced meaningful transaction costs. In addition, vendors had imperfect recall of the content of our product training. Almost all (98%) knew to keep the lamp out of the sun when charging the solar panel. A lower share (79%) knew the manufacturer's warranty was for one year, and even fewer (60%) recalled that the solar light should charge for 2 days prior to its first use. Inability to explain product features may also have reduced their sales effectiveness.

that the vendors in our study were all women.<sup>32</sup> There are several RCTs that have studied ways in which to facilitate the commitment to savings.<sup>33</sup> In practical applications, we believe it will be important to find ways in which to build such mechanisms into the arrangement.

One way to avoid the savings problem is to have customers pay the principal directly using an electronic payment technology such as mobile money. With this technology, the vendor is not required to handle and save cash between delivery dates. Instead, the vendor could have an account with the producer whose balance increases whenever customer's make payments. Moreover, the use of both mobile phones and mobile money is already widespread in rural Uganda. For instance, 98% of our vendors reported owning a mobile phone and 83% reported having some experience with mobile money.

*Failure of the Credit Chain.* Existing literature has shown that poor households face credit constraints, which is an important factor limiting adoption rates (Cole et al., 2013; Tarozzi et al., 2014). Therefore, as mentioned earlier, during their training we emphasized several techniques vendors should use to help alleviate these constraints and boost their sales (i.e., installments, lawaway, ROSCA). Despite this encouragement, only 30% of vendors actually employed these techniques according to surveys. Overall, vendors seemed generally unwilling to extend credit to their customers even when the vendors themselves were extended credit and vendors were explicitly encouraged to offer credit to their customers. Post-study interviews suggest vendors' unwillingness to extend credit was due to a fear of customers defaulting on their payments.

## **Is Technology the Answer?**

When vendors are both unwilling to offer consumer credit and cannot easily save to increase their inventory, it is efficient if the producer or distributor can offer credit directly to consumers and receive payments from them. By doing so, the vendor's role is essentially reduced to that of a credit officer. Of course, such an arrangement could lead to potential moral hazard problems of vendors selecting customers carelessly and/or not retrieving products when consumers default.

Some distributors and producers have combined payments via mobile money with a "kill" switch on their products to overcome these challenges as well as the ones emphasized in our model. The advantage of the kill switch is that it makes the product worthless to the customers if they do not make payments and to vendors if they abscond with inventory, thereby increasing customer repayment rates and reducing vendor's ability to extract rents.

---

<sup>32</sup>See Bobonis (2009), Bobonis et al. (2013) and references therein regarding the allocation of resources within households and domestic violence.

<sup>33</sup>See Ashraf Nava and Yin (2006) and Basu (2014).

For example, M-Kopa Solar sells a home solar system with a solar panel, three small lights, a phone charger, and a radio for an initial \$35 deposit, followed by 365 [daily] payments of 45 cents paid by mobile money.<sup>34</sup> A comparable system would retail for a (cash) price of roughly \$75.

M-Kopa has higher production costs because each system must contain many of the capabilities of a mobile phone. The benefits are apparently large; the present value of payments is about \$153 at a discount rate of 100% per year and \$185 at a discount rate of 20% per year. Thus, a consumer could purchase a similar system for about half the present value of payments if paying in cash. Nevertheless, M-Kopa has sold more than 450,000 systems in East Africa and is growing at a rate of 500 new systems per day.<sup>35</sup> The success of M-Kopa and its peers suggests the severity of the credit constraints, savings constraints, and moral hazard emphasized in our model is non-trivial.

It is important to note that not all products lend themselves to a kill switch (e.g., cook stoves, water filters, malaria nets). In this regard, solar lights are unique because they operate on electricity whereas many of the products that lack widespread adoption in developing countries do not. Thus, M-Kopa's technological strategy is likely to be prohibitively expensive to incorporate into many of the other products to which our model applies.

## 7 Concluding Remarks

Markets for new technologies in emerging economies develop slowly due to a variety of economic frictions. Products that would enhance the welfare of many poor households are not adopted as fast as socially desirable. Most of the literature has focused on addressing demand-side barriers to product adoption. In this paper, we have developed a theory to address supply-side barriers. Two important issues to be overcome are the liquidity constraints and lack of enforceable contracts. The optimal arrangement involves providing the agent with a “small” amount of seed capital as well as the option to more units in the future at a fixed price. Interestingly such a solution is optimal for both a profit maximizing firm or a non-profit organization with a limited budget.

We conducted a field experiment to test our theory. The evidence clearly indicates that liquidity constraints are an important factor and that, in some cases, dynamic incentives work to get the agents to desire to continue doing business and return for additional units. Growth, however, was lower than expected. As we learned from interviewing the agents, this lack of growth was largely due to the inability to save even for the short periods of time between

---

<sup>34</sup><http://www.m-kopa.com/products>

<sup>35</sup><http://solar.m-kopa.com/about/company-overview>, date accessed: April 12, 2017.

orders and a failure of the “credit chain” (i.e., vendors were unwilling to offer credit to their customers). Therefore, we believe it is critical for researchers to incorporate reliable savings technologies in future experiments and encourage the flow of credit. Emerging technologies, such as those used by M-Kopa, have developed to help overcome these obstacles, but they come at a non-trivial increase in cost and are likely to be prohibitively expensive for many products. An interesting direction for future work is to explore the extent to which these obstacles can be partially overcome using appropriately designed incentive schemes rather than expensive hardware.

## References

- Abreu, Dilip, David Pearce, and Ennio Stacchetti**, “Toward a Thoery of Discounted Repeated Games with Imperfect Monitoring,” *Econometrica*, 1990, *58*(5), 1041–1063.
- Abuquerque, Rui and Hugo A Hopenhayn**, “Optimal Lending Contracts and Firm Dynamics,” *The Review of Economic Studies*, 2004, *71*, 285–315.
- Ashraf Nava, Dean Karlan and Wesley Yin**, “Tying Odysseus to the mast: Evidence from a commitment savings product in the Philippines,” *The Quarterly Journal of Economics*, 2006, *121* (2), 635–672.
- Ashraf, Nava, James Berry, and Jesse M Shapiro**, “Can Higher Prices Stiumulate Product Use? Evidence from a Field Experiment in Zambia,” *American Economic Review*, 2010, *100*, 2383–2413.
- Baker, George, Robert Gibbons, and Kevin J. Murphy**, “Relational Contracts and the Theory of the Firm,” *The Quarterly Journal of Economics*, 2002, *117* (1), 39–84.
- Basu, Karna**, “Commitment savings in informal banking markets,” *Journal of Development Economics*, 2014, *107*, 97–111.
- Bobonis, Gustavo**, “Is the Allocation of Resources within the Household Efficient? New Evidence from a Randomized Experiment,” *Journal of Political Economy*, 2009, *117* (3), 453–503.
- , **Melissa Gonzalez-Brenes, and Roberto Castro**, “Public Transfers and Domestic Violence: The Roles of Private Information and Spousal Control,” *American Economic Journal: Economic Policy*, 2013, *5* (1), 179–205.
- Bold, Tessa, Kayuki C. Kaizzi, Jakob Svensson, and David Yanagizawa-Drott**, “Lemon Technologies and Adoption: Measurement, Theory and Evidence from Agricultural Markets in Uganda\*,” *The Quarterly Journal of Economics*, mar 2017, *103*, 76–84.
- Bryan, Gharad, Shyamal Chowdhury, and Ahmed Mushfiq Mobarak**, “Underinvestment in a Profitable Technology: The Case of Seasonal Migration in Bangladesh,” *Econometrica*, 2014, *82* (5), 1671–1748.
- Chiappori, Pierre-André, Krislert Samphantharak, Sam Schulhofer-Wohl, and Robert M. Townsend**, “Heterogeneity and risk sharing in village economies,” *Quantitative Economics*, 2014, *5* (1), 1–27.
- Clementi, Gian Luca and Hugo A. Hopenhayn**, “A Theory of Financing Constraints and Firm Dynamics,” *Quarterly Journal of Economics*, 2006, *121*, 229–265.
- Cole, Shawn, Xavier Giné, Jeremy Tobacman, Petia B Topalova, Robert M Townsend, and James I Vickery**, “Barriers to household risk management: evidence

- from India,” *American Economic Journal: Applied Economics*, 2013, 5 (1), 104–135.
- Conley, Timothy and Christopher Udry**, “Social Learning through Networks: The Adoption of New Agricultural Technologies in Ghana,” *American Journal of Agricultural Economics*, 2001, 83 (3), 668–673.
- Conley, Timothy G and Christopher R Udry**, “Learning About a New Technology : Pineapple,” *American Economic Review*, 2010, 100 (1), 35–69.
- Cunat, Vincent**, “Trade Credit: Suppliers as Debt Collectors and Insurance Providers,” *Review of Financial Studies*, 2007, 20 (2), 491–527.
- de Mel, Suresh, David McKenzie, and Christopher Woodruff**, “Returns to Capital in Microenterprises: Evidence from a Field Experiment,” *The Quarterly Journal of Economics*, 2008, 123 (4), 1329–72.
- DeMarzo, Peter M and Michael J Fishman**, “Agency and Optimal Investment Dynamics,” *Review of Financial Studies*, 2007, 20, 151–188.
- DeMarzo, Peter M. and Michael J. Fishman**, “Optimal long-term financial contracting,” *Review of Financial Studies*, 2007, 20, 2079–2128.
- Demarzo, Peter M., Michael J. Fishman, Zhiguo He, and Neng Wang**, “Dynamic Agency and the q Theory of Investment,” *Journal of Finance*, 2012, 67 (6), 2295–2340.
- Dubois, P, B Jullien, and T Magnac**, “Formal and informal risk sharing in LDCs: Theory and empirical evidence,” *Econometrica*, 2008, 76 (4), 679–725.
- Duflo, Esther, Michael Kremer, and Jonathan Robinson**, “Nudging farmers to use fertilizer: Theory and experimental evidence from Kenya,” *American Economic Review*, 2011, 101 (6), 2350–2390.
- Dupas, Pascaline and Jessica Cohen**, “Free Distribution or Cost-Sharing? Evidence from a Randomized Malaria Prevention Experiment,” *The Quarterly Journal of Economics*, 2010, 125, 1–45.
- Ethan Ligon, Jonathan P Thomas and Tim Worrall**, “Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies,” *The Review of Economic Studies*, 2002, 69 (1), 209–244.
- Feder, Gershon and Roger Slade**, “The Acquisition of Information and the Adoption of New Technology,” *American Journal of Agricultural Economics*, 1984, 66 (3), 312–320.
- Foster, Andrew D. and Mark R. Rosenzweig**, “Learning by Doing and Learning from Others: Human Capital and Technical Change in Agriculture,” *The Journal of Political Economy*, 1995, 103 (6), 1176–1209.
- Giné, Xavier and Dean Yang**, “Insurance, credit, and technology adoption: Field experimental evidence from Malawi,” *Journal of Development Economics*, 2009, 89 (1), 1–11.

- Guiteras, R, David I Levine, T Polley, and B Quistorff**, “Credit Constraints, Present Bias and Investment in Health: Evidence from Micropayments for Clean Water in Dhaka,” *Unpublished*, 2014.
- Kaboski, Joseph P. and Robert M. Townsend**, “A Structural Evaluation of a Large-Scale Quasi Experimental Microfinance Initiative,” *Econometrica*, 2011, 79 (5), 1357–1406.
- Karaivanov, Alexander and Robert Townsend**, “Dynamic Financial Constraints: Distinguishing Mechanism Design from Exogenously Incomplete Regimes,” *Econometrica*, 2013, 82 (3), 887–959.
- Kocherlakota, Naryana R**, “Figuring out the impact of hidden savings on optimal unemployment insurance,” *Review of Economic Dynamics*, 2004, pp. 541–554.
- Levin, Jonathan**, “Relational Incentive Contracts,” *American Economic Review*, 2003, 93, 835–857.
- Levine, David I, Theresa Beltramo, Garrick Blalock, and Carolyn Cotterman**, “What Impedes Efficient Adoption of Products? Evidence from Randomized Variation in Sales Offers for Improved Cookstoves in Uganda,” *IRLE working paper*, 2013.
- Li, May**, “Optimal Lending Contracts with Long Run Borrowing Constraints,” *Journal of Economic Dynamics and Control*, 2013, 37 (5), 964–983.
- Mailath, George J and Larry Samuelson**, *Repeated Games and Reputations*, Oxford University Press, 2006.
- Opp, Marcus M and John Y Zhu**, “Impatience Versus Incentives,” *Econometrica*, 2015, 83 (5), 1601–1617.
- Quadrini, Vincenzo**, “Investment and liquidation in renegotiation-proof contracts with moral hazard,” *Journal of Monetary Economics*, 2004, 51, 713–751.
- Tarozzi, Alessandro, Aprajit Mahajan, Brian Blackburn, Dan Kopf, Lakshmi Krishnan, and Joanne Yoong**, “Micro-loans, insecticide-treated bednets, and malaria: Evidence from a randomized controlled trial in Orissa, India,” *American Economic Review*, 2014, 104 (7), 1909–1941.
- Thomas, Jonathan and Tim Worrall**, “Income fluctuation and asymmetric information: An example of a repeated principal-agent problem,” *Journal of Economic Theory*, 1990, 51 (2), 367–390.
- and –, “Foreign Direct Investment and the Risk of Expropriation,” *The Review of Economic Studies*, 1994, 61(1), 81–108.
- Townsend, Robert and Sergio Urzua**, “Measuring the Impact of Financial Intermediation: Linking Contract Theory to Econometric Policy Evaluation,” *Macroeconomic Dynamics*, 2009, 13 (Suppl S2), 268–316.

**Townsend, Robert M**, “Risk and Insurance in Village India,” *Econometrica*, 1994, 62 (3), 539–591.

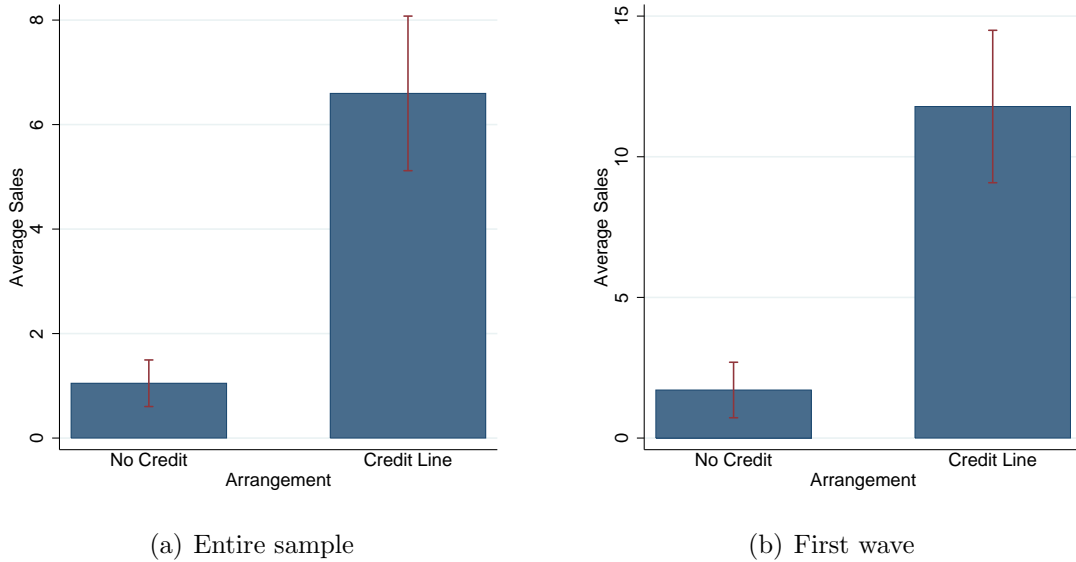
**Troya-Martinez, Marta**, “Self-Enforcing Trade Credit,” *Working Paper*, 2015.



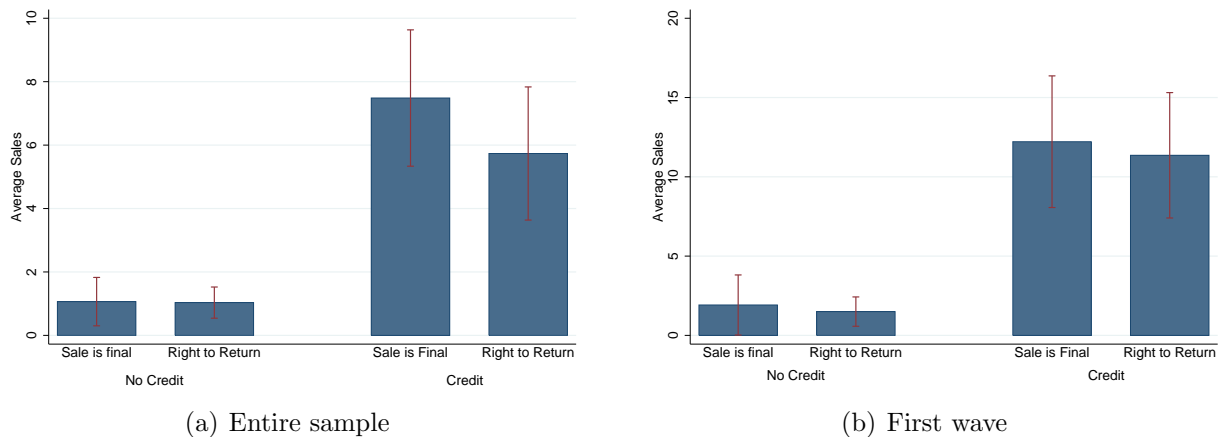
## A Tables and Figures

Education	None 28%	Primary 60%	Secondary 12%	
Occupation	Retail 51%	Agriculture 40%	Livestock 13%	Other 40%
	Mean	Median	Std Dev	
Age	39.1	38	9.55	
Experience	9.4	6	10.2	
Work hours/day	8.46	8	3.98	
Children	4.91	5	2.74	
Residency	17.8	15	13.3	
Travel time to BRAC	58.3	60	37.7	
Kerosene expenditure				
Pre-solar	3,264	3,000	2,799	
Post-solar	305	0	826	
Female	100%			
Married	70%			
Own mobile phone	98%			
Use SMS	47%			
Use Mobile Money	83%			

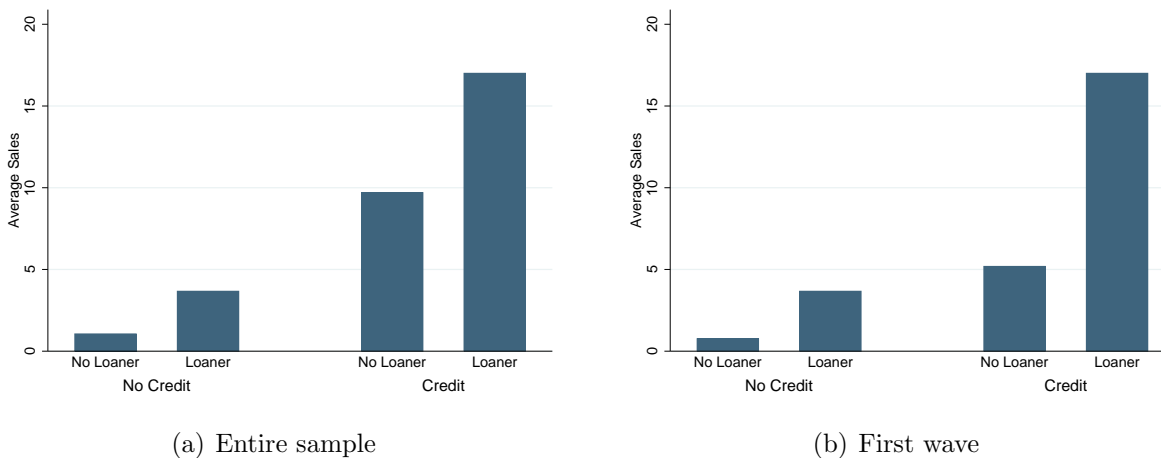
**Table 1:** Summary of Vendor Characteristics



**Figure 5: Credit Line versus No Credit:** This graph shows the effect of including a line of trade credit in the arrangement for the entire sample (left-panel) and for the first-wave only (right panel). The blue bar is the average of total sales across all vendors who received an offer with the corresponding characteristic. The range between the red capped line indicates the 95% confidence interval.



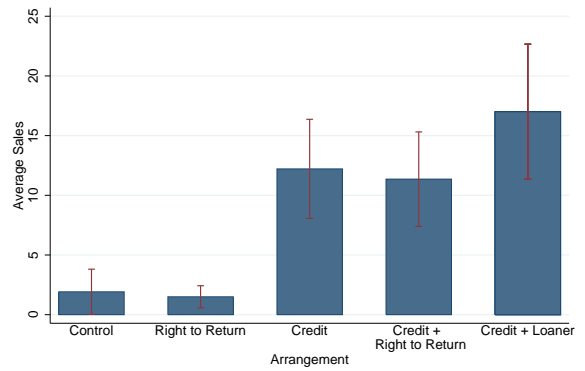
**Figure 6: Addressing Vendor Uncertainty with the Right to Return.** This graph shows the effect of including the right to return unsold inventory in the arrangement for the entire sample (left-panel) and for the first-wave only (right panel). The blue bar is the average of total sales across all vendors who received an offer with the corresponding characteristics. The range between the red capped line indicates the 95% confidence interval.



**Figure 7: Addressing Consumer Uncertainty with a Loaner Light.** This graph shows the effect of including the right to return unsold inventory in the arrangement for the entire sample (left-panel) and for only the first wave (right panel). The bar denotes the average of total sales across all vendors who received an offer with the corresponding characteristics.



(a) Entire sample



(b) First wave

**Figure 8: Average Sales by Offer:** This graph shows the average total sales of vendors by the characteristics of the arrangement they were offered the entire sample (left-panel) and for the first-wave only (right panel). The blue bar is the average of total sales across all vendors who received an offer with the corresponding characteristics. The range between the red capped line indicates the 95% confidence interval.

**Table 2: Regression Estimates for Full Sample.** This table gives the results of the estimation of the regression equation

$$\log(1 + \text{total sales}_i) = \alpha + \beta \text{Arrangement Characteristics}_i + \epsilon_i, \quad (7)$$

where the unit of observation ( $i$ ) is the vendor and  $\text{Arrangement Characteristics}_i$  is a vector of attributes associated with the arrangement offered to that vendor. For instance, column (2) contains the estimates from the model in which  $\text{Arrangement Characteristics}_i$  is a pair of dummy variables ( $\text{Credit}_i, \text{Loaner}_i$ ), where the first dummy indicates whether vendor  $i$ 's arrangement included trade credit and the second dummy indicates whether vendor  $i$ 's arrangement included a loaner light. Standard errors are in parenthesis below the estimated coefficients and are clustered at the branch level. Standard errors are in parenthesis below the estimated coefficients and are clustered at the branch level. The data used for this estimation includes the full sample (i.e., both first and second wave).

	(1)	(2)	(3)	(4)	(5)
Credit	1.086*** (5.11)	1.078*** (5.03)	1.087*** (5.13)	1.078*** (5.05)	1.071*** (4.86)
Loaner		1.232** (2.77)		1.227** (2.75)	0.760 (1.57)
Right to Return			-0.122 (-1.45)	-0.100 (-1.15)	-0.115 (-1.30)
First wave					0.701** (2.90)
Constant	0.500*** (3.88)	0.361*** (3.90)	0.561*** (5.53)	0.411*** (6.33)	0.189 (1.24)
R <sup>2</sup>	0.266	0.407	0.269	0.409	0.496
N	129	129	129	129	129

*t*-statistics in parenthesis

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 3: Regression Estimates for First Wave.** This table gives the results of the estimation of the regression equation

$$\log(1 + \text{total sales}_i) = \alpha + \beta \text{Arrangement Characteristics}_i + \epsilon_i, \quad (8)$$

where the unit of observation ( $i$ ) is the vendor and  $\text{Arrangement Characteristics}_i$  is a vector of attributes associated with the arrangement offered to that vendor. For instance, column (2) contains the estimates from the model in which  $\text{Arrangement Characteristics}_i$  is a pair of dummy variables ( $\text{Credit}_i, \text{Loaner}_i$ ), where the first dummy indicates whether vendor  $i$ 's arrangement included trade credit and the second dummy indicates whether vendor  $i$ 's arrangement included a loaner light. Standard errors are in parenthesis below the estimated coefficients and are clustered at the branch level. The data used for this estimation includes only the first wave.

	(1)	(2)	(3)	(4)
Credit	1.541*** (6.47)	1.537*** (6.73)	1.543*** (6.40)	1.539*** (6.66)
Loaner		0.763 (1.39)		0.770 (1.39)
Right to Return			0.0815 (0.88)	0.119 (1.92)
Constant	0.798** (3.77)	0.584** (3.27)	0.756** (3.50)	0.521** (3.28)
R <sup>2</sup>	0.473	0.567	0.474	0.570
N	53	53	53	53

*t*-statistics in parenthesis

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## B Proofs

*Proof of Lemma 1.* It is straightforward to verify that the stated policy satisfies (2)-(5). In particular, (2) binds from above for  $v \leq \delta_A \bar{v}$ , from below for  $v \geq (1 + \delta_A)\bar{v}$  and is interior for  $v \in (\delta_A \bar{v}, (1 + \delta_A)\bar{v})$ ; (3) is slack for  $v \in (0, \bar{v})$  and binds for  $v \geq \bar{v}$ ; (4) binds for  $v \leq \bar{v}$  and is slack otherwise. The rest of the proof is by construction. We will first construct the principal's value function under the stated policy and then verify that it indeed solves (P).

Notice that  $W(\bar{v}) = \bar{v}$ , hence  $\bar{v}$  is the steady state. In the steady state, each period,  $K(\bar{v}) = \bar{k}$ , the agent consumes  $(1 - \delta_A)\bar{v}$  and therefore transfers  $\delta_A \bar{v}$  to the principal. The principal's value function at  $\bar{v}$  is therefore

$$\begin{aligned} \Pi(\bar{v}) &= \delta_A \bar{v} - c\bar{k} + \delta_P \Pi(\bar{v}) \\ &= \frac{(\delta_A \bar{p} - c)\bar{k}}{1 - \delta_P} \end{aligned}$$

Next, consider any  $v \in (\delta_A \bar{v}, \bar{v})$ , so that there is one period until the steady state is reached (i.e.,  $W(v) = \bar{v}$ ). Under the stated policy, we have that

$$\Pi_0(v) = \delta_A \bar{v} - c \frac{v}{\bar{p}} + \delta_P \Pi(\bar{v}) \quad (9)$$

Now, fix any integer  $N \geq 0$ , we claim that for  $v \in (\delta_A^{N+1} \bar{v}, \delta_A^N \bar{v})$ , the principal's value function under the stated policy is given by

$$\Pi_N(v) = v\mu \sum_{t=0}^{N-1} \left( \frac{\delta_P}{\delta_A} \right)^t + \delta_P^N \Pi_0 \left( \frac{v}{\delta_A^N} \right) \quad (10)$$

We have already demonstrated the base case (i.e.,  $N = 0$ ) in (9). Suppose that (10) holds for some  $n \geq 1$  and consider any  $v \in (\delta_A^{n+2} \bar{v}, \delta_A^{n+1} \bar{v})$ , under the stated policy we have that  $K(v) = v/\bar{p}$ ,  $T(v) = v$  and  $W(v) = \delta_A^{-1}v$ , therefore the principal's value function is given by

$$\begin{aligned} \Pi_{n+1}(v) &= \left( v - \frac{cv}{\bar{p}} \right) + \delta_P \Pi_n(\delta_A^{-1}v) \\ &= \mu v + \delta_P \left( \frac{v}{\delta_A} \mu \sum_{t=0}^{n-1} \left( \frac{\delta_P}{\delta_A} \right)^t + \delta_P^n \Pi_0 \left( \frac{v}{\delta_A^{n+1}} \right) \right) \\ &= v\mu \sum_{t=0}^n \left( \frac{\delta_P}{\delta_A} \right)^t + \delta_P^{n+1} \Pi_0 \left( \frac{v}{\delta_A^{n+1}} \right), \end{aligned}$$

which is of the form in (10) and thus verifying the claim. Next, for  $v \in (\bar{v}, (1 + \delta_A)\bar{v})$ , we

have that

$$\Pi(v) = g_0(v) = \bar{v}(1 + \delta_A) - v - c\bar{k} + \delta_P \Pi(\bar{v}).$$

Using an induction argument similar argument to the one above, for any integer  $k \geq 0$  and  $v \in \left(\bar{v} \sum_{t=0}^k \delta_A^t, \bar{v} \sum_{t=0}^{k+1} \delta_A^t\right)$

$$\Pi(v) = g_k(v) \equiv -c\bar{k} \sum_{t=0}^k \delta_P^t + \delta_P^k g_0 \left( \delta_A^{-k} \left( v - \bar{v} \sum_{t=0}^k \delta_A^t \right) \right).$$

We have thus constructed the principal's value function under the stated policy for all  $v > 0$ .

Before verifying optimality of the policy, it is useful to observe several properties of the value function. First notice that  $\Pi$  is piecewise linear and concave in  $v$ . Next, notice for  $\gamma \neq 1$  that (10) can be written as

$$\Pi_N(v) = v\mu \left( \frac{\gamma^N - 1}{\gamma - 1} \right) - \gamma^N \frac{c}{p} v + \delta_P^N (\delta_A \bar{v} + \delta_P \Pi(\bar{v})), \quad (11)$$

which is differentiable with respect to  $v$  for  $v \in (\delta_A^{N+1} \bar{v}, \delta_A^N \bar{v})$ ,  $N \geq 0$  with a slope given by

$$\Pi'_N(v) = \mu \left( \frac{\gamma^N - 1}{\gamma - 1} \right) - (1 - \mu) \gamma^N. \quad (12)$$

When  $\gamma = 1$ , (12) becomes  $\Pi'_N(v) = \mu N - (1 - \mu)$ . Finally, for  $v \in \left(\bar{v} \sum_{t=0}^k \delta_A^t, \bar{v} \sum_{t=0}^{k+1} \delta_A^t\right)$ ,  $k \geq 0$ , the slope of the value function is  $\gamma^k$ .

To verify that the stated policy is indeed optimal, notice that by substituting the promise-keeping constraint into the objective and (4), the problem can be restated as:

$$\sup_{K, T} \left\{ T - cK + \delta_P \Pi \left( \frac{v + T - \bar{p}K}{\delta_A} \right) \right\}$$

subject to  $T \in [0, \bar{p}K]$ ,  $K \in [0, \min\{\bar{k}, v/\bar{p}\}]$ . Since  $\Pi$  is concave, it is enough check local deviations are not profitable. The second constraint always binds at the top under the stated policy, so we only need to consider a reduction in  $K$  of  $\epsilon$ . If (2) also binds at the top (i.e.,  $v \leq \delta_A \bar{v}$ ) then this deviation also requires a small reduction in  $T$  to satisfy the first constraint, which leaves the continuation value unchanged, therefore reducing the objective by  $(\bar{p} - c)\epsilon$ . If (2) does not bind from above (i.e.,  $v > \delta_A \bar{v}$ ), then the marginal benefit of this local deviation is  $c + \delta_P \Pi'(W(v)) \frac{\bar{p}}{\delta_A}$ , which is negative provided that  $\Pi'(W(v)) \leq -\frac{c}{\bar{p} \delta_P}$ . Noting that the value function constructed above has a slope of  $-c/\bar{p}$  for  $v \in (\delta_A \bar{v}, \bar{v})$  and is concave verifies that such a deviation is not profitable.



Next, consider a deviation from the stated policy  $T(v)$ . Note that the objective is increasing in  $T$  if  $\Pi'(W(v)^+) \geq -\delta_A/\delta_P$ , which holds if and only if  $v < \delta_A\bar{v}$ , in which case an increase in  $T$  violates the first constraint. For  $v \in [\delta_A\bar{v}, (1 + \delta_A)\bar{v}]$ ,  $\Pi'(W(v)^+) = \Pi'(\bar{v}^+) = -1 \leq -\delta_A/\delta_P$  and  $\Pi'(W(v)^-) = \Pi'(\bar{v}^-) = -c/p > -\delta_A/\delta_P$ . Therefore, neither increasing nor decreasing  $T(v)$  is profitable. Finally, for  $v > (1 + \delta_A)\bar{v}$ ,  $\Pi'(W(v)^+) \leq \Pi'(W(v)^-) \leq -1 \leq -\delta_A/\delta_P$ . Hence, the principal does strictly worse by increasing  $T(v)$  in this region and a reduction in  $T(v)$  violates the first constraint. Thus, we have shown that no profitable deviations from the stated policy exists, which completes the proof.  $\square$

*Proof of Proposition 1.* Fix an arbitrary  $k_0 \leq \bar{k}$ . We first claim the the arrangement  $(k_0, p_A^*)$  implements the policy in Lemma 1 where the initial continuation value of the agent is  $v_0 \equiv k_0\bar{p}$ . To see this, notice that  $K(v_0) = K(k_0\bar{p}) = k_0$  and the agent's revenue in the first period equals  $k_0\bar{p}$ . Clearly, the agent will never optimally purchase more than  $\bar{k}$  units. If  $v_0 > \delta_A\bar{v}$ , then the agent will optimally choose to purchase exactly  $\bar{k}$  (i.e.,  $T(v_0) = \delta_A\bar{p}\bar{k}$ ) and consume the rest. If  $v_0 < \delta_A\bar{v}$ , then the agent will optimally choose to purchase  $k_0/\delta_A$  units and consume nothing in the initial period. In either case, the number of units the agent will have in the next period is  $k_1 = \min\{k_0/\delta_A, \bar{k}\} = K(W(v_0))$  when the transfer will be  $t_1 = \min\{\bar{p}K(W(v_0)), \delta_A\bar{v}\} = T(W(v_0))$ . Therefore, the policy in the next period is also replicated. Since  $k_0$  was chosen arbitrarily, this completes the proof of the claim.

What remains is to prove that  $k_0^* < \bar{k}$ . For this, it suffices to show that  $v_0^* \in \arg \max_0 \Pi(v) < \bar{v}$ . This follows immediately from the fact that the principal's value function is strictly decreasing on  $(\delta_A\bar{v}, \bar{v})$  (see (9)) and the concavity of the value function (see proof of Lemma 1). Thus, the optimal initial endowment is strictly less  $\bar{k}$ .  $\square$

*Proof of Proposition 2.* Recall that the principal's value function is concave and for  $\gamma > 1$  has a slope given by (12) for  $v \in (\delta_A^{N+1}\bar{v}, \delta_A^N\bar{v})$ ,  $N \geq 0$ . Setting the slope equal to zero and solving yields  $N_1 = \frac{\log(\frac{\mu}{1+\gamma(\mu-1)})}{\log(\gamma)}$ . If  $N_1$  is an integer, then  $N^* = N_1$  and the principal's value function has slope zero over the interval  $v \in (\delta_A^{N_1+1}\bar{v}, \delta_A^{N_1}\bar{v})$ , is upward sloping to the left and downward sloping to the right. Therefore  $\delta_A^{N^*}\bar{v} \in \arg \max_v \Pi(v)$ . If  $N_1$  is not an integer, then  $\Pi'(\delta_A^{N^*}\bar{v}^-) > 0 > \Pi'(\delta_A^{N^*}\bar{v}^+)$ . Therefore,  $\delta_A^{N^*}\bar{v} = \arg \max_v \Pi(v)$ . By Proposition 1, the principal can achieve this value by providing the agent with an initial endowment of  $k_0^* = \delta_A^{N^*}\bar{v}/\bar{p} = \delta_A^{N^*}\bar{k}$ , in which case it will take the agent  $N^*$  periods to reach capacity. For  $\gamma = 1$ , the result can be shown using the same technique by substituting  $\Pi'_N(v) = \mu N - (1 - \mu)$  and hence  $N_1 = \frac{1-\mu}{\mu}$ .  $\square$

*Proof of Proposition 3.* Consider a candidate optimal solution for  $K(v)$  and  $T(v, p)$ . First note that any candidate solution for  $K(v)$  and  $T(v, p)$  must satisfy the following con-

straints<sup>36</sup>:

$$K(v)\bar{p} + E[\delta_A W(v, p) - T(v, p)] = v \quad (\text{PK})$$

$$T(v, p) \in [0, K(v)p] \quad \forall p \quad (\text{Liq})$$

$$K(v)p - T(v, p) + \delta_A W(v, p) \geq K(v)\hat{p} - T(v, \hat{p}) + \delta_A W(v, \hat{p}) \quad \forall p > \hat{p} \quad (\text{IC})$$

$$K(v) \leq \bar{K} \quad (\text{Capacity})$$

$$K(v)p - T(v, p) + \delta_A W(v, p) \geq K(v)p \quad (\text{IR}_p)$$

$$\delta_A W(v, p) \geq T(v, p) \quad \forall p$$

Suppose first that the contract satisfies  $E[\delta_A W(v, p) - T(v, p)] = 0$ , since  $\delta_A W(v, p) \geq T(v, p) \quad \forall p$  this implies  $\delta_A W(v, p) = T(v, p) \quad \forall p$  thus, if we think of  $T(v, p)$  as deposits the agent makes in the principal's "bank", these deposits must a rate of return  $1/\delta_A$ . Also, since  $K(v) = \frac{v}{\bar{p}}$  we can think of  $\bar{p}$  as the price per unit of capital. Thus, while  $E[\delta_A W(v, p) - T(v, p)] = 0$  we can implement the contract with these two instruments.

Next suppose  $E[\delta_A W(v, p) - T(v, p)] > 0$ , we can still think of  $\bar{p}$  as the beginning of period price of a unit of capital. But we now need to understand the endowment to mean either a cash amount of  $v$  or some capital  $K(v)$  + some cash, where for each unit of capital, the cash is reduced at a rate  $\bar{p}$ . Let  $I(v, p) = \delta_A W(v, p) - T(v, p)$  and  $I(v) = E[I(v, p)]$ . Note that if  $I(v, p) = I(v)$  i.e. it is not a function of  $p$ . Then all *IC* constraints hold with equality and  $(I(v) + T(v, p))/\delta_A = W(v, p)$ . Which we can then interpret as having an intra-period cash balance of  $I(v)$  which also receives the same inter-period return of  $1/\delta$ .

What remains to be shown is that  $I(v, p) = I(v)$ . To establish this, note first that if  $\Pi(v)$  were not weakly concave in  $v$ , then the principal could always offer to mix over  $W$  (or  $v$ ) in a way that all the constraints are still satisfied. Thus, we assume there is some random variable  $x$  with which we can create random contracts and let  $W(v, p) = E_x[W(v, p, x)]$ . Next, let  $p' > p$  and suppose  $I(v, p') > I(v, p)$ , since  $W(v, p) = (I(v, p) + T(v, p))/\delta_A$  making  $I(v, p)$  increasing in  $p$  increases the variance of  $W(v, p)$  which cannot lead to an improvement since  $\Pi(W(v, p))$  is weakly concave. Note as well that  $I(v, p)$  cannot be decreasing. Otherwise, the agent would have an incentive to under-report and consume the difference and the *IC* would be violated. Hence, the best the principal can do is to set  $I(v, p) = I(v)$  for all  $p$ .

The only thing to note, is that we allow for the continuation value to be stochastic, we must interpret  $\frac{1}{\delta_A}$  as the expected return on the savings account.  $\square$

---

<sup>36</sup>PK: Promise keeping constraint, LIQ: Liquidity constraint, IC: Incentive Compatibility constraints, IR: Participation constraint.