Sharing aggregate risks under moral hazard

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Abstract

This paper discusses some of the problems associated with the efficient design of insurance schemes in the presence of aggregate shocks and moral hazard. The population is divided into groups, each one composed of ex ante identical individuals who are subject to idiosyncratic shocks. A group may be for example the labour force in a given sector, workers being subject to the risk of unemployment. Without moral hazard, optimality requires (1) full insurance against idiosyncratic shocks, which gives rise to a representative agent for each group and (2) macro-economic risks to be shared between these representative agents. The question investigated in this paper is what remains of this analysis when the presence of moral hazard conflicts with the full insurance of idiosyncratic shocks. In particular, how is the sharing of macro-economic risks across groups affected by the partial insurance against idiosyncratic risks? The design of unemployment insurance schemes in different economic sectors, and the design of pension annuities in an unfunded social security system are two potential applications.

Keywords: moral hazard, insurance, mutuality principle

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1 Introduction

This paper discusses some of the problems associated with the efficient design of insurance schemes in the presence of aggregate shocks and moral hazard. Consider for instance insurance against unemployment risks in different sectors. Without moral hazard, optimal risk sharing requires two properties: first full insurance against idiosyncratic shocks within each sector, and second the pooling of macro-economic risks affecting employment in the different sectors (along the lines first described by Borch 1960, Wilson 1968, and Malinvaud 1972, 1973). As a result, individuals’ consumption levels should vary in the same direction, either all upward or all downward, as the state of the economy varies. Casual observation suggests that neither property holds in practice. Moral hazard explains why idiosyncratic shocks should not be fully insured. The question investigated here is whether this may have a significant impact on the sharing of macro-economic risks.

The setting is one in which individuals cannot freely combine different contracts in various quantities: contracts are exclusive. It describes the current situation in many countries where insurance against pervasive risks, unemployment and illness for instance, is provided at the country level through compulsory schemes. One justification for exclusivity is that effort is better monitored, since otherwise the incentive property of a contract are modified by the other contracts that an individual may buy. Whereas these risks play an important role for individuals, there is relatively little study of the implications of moral hazard at the aggregate level. The design of pension schemes is an important example that motivates this analysis. In several european countries, among them France, Germany, and Italy, the main part of the pension system is a pay as you go system, meaning that the pension benefits to the retirees are paid by the current workers. In the last thirty years, there is some evidence that the well being of retirees has significantly raised relative to that of the rest of the population. Although partly due to active policies, such a relative increase was not anticipated to a large extent. Indeed it resulted from a specific aspect of the design of pension benefits, namely the indexation of annuities on individual wages.\textsuperscript{2} Consequently, annuities increase more than the average income of young

\textsuperscript{2}There is a move to an adjustment with consumer price index. This move is too recent to have been significant yet.
individuals as unemployment increases. This raises the question of why the risks in unemployment and wages are not shared with the retirees. A difficulty is that the welfare system, and the associated moral hazard problem, might play a role in explaining the high level of unemployment. If true, the extent to which retirees should bear the unemployment risk is unclear. The problem can be seen as that of a optimal risk sharing among the whole population, accounting for the moral hazard within the "young" generation.

The paper considers an economy composed of large groups, two for simplicity. Within each group, individuals are \textit{ex ante} identical and subject to idiosyncratic, group-specific shocks. The probability of these shocks are influenced by the "state" of the economy (the macro risk), and also possibly by the non-observable effort expanded by the individual (moral hazard). To insure both types of risks, contracts are defined \textit{ex ante} before the state of the economy and the idiosyncratic risks materialize. We take a second best approach and study the contracts that maximize a weighted sum of utilities. For example, in a problem of pension design as just sketched, the approach amounts to determine how the income of retirees and young individuals, employed and unemployed, should fluctuate in function of the state of the economy, accounting for the impact of unemployment benefits on incentives to find a job.\footnote{The precise contract, in particular the expected level of pension benefits and contributions to social security, is determined by the assigned weights. We focus on the properties of optimal adjustments of income to shocks and the (political) issue of choosing the weights is not addressed.}

If effort is contractible, first best optimality can be obtained. Idiosyncratic risks are mutualized, leading to a representative agent for each group. Aggregate resources, which vary with the macro-economic state, are allocated according to some "sharing rules" that account for the attitudes towards risk of the representative agents. The rules prescribe transfers across groups that are motivated by insurance against the macro economic risks. Crucially, the sharing rules and individuals' consumption levels are increasing in aggregate resources. Sharing rules provide a benchmark to assess the impact of moral hazard. The intuition for why moral hazard affects the transfers \textit{across} groups is the following. If a group is hurt by a shock and receives some transfers from the other group, its budget is eased. This results in a change in the insurance contract \textit{within} the group and the induced
level of effort. The paper derives first some conditions under which moral hazard diminishes or increases the share of aggregate resources received by a group. Also, we show that the monotonicity property may fail under moral hazard, meaning that some may benefit from a change in the state while others are hurt. It may even happen that a group that benefits from an increase in its own resources finally ends up worse off. This may clearly create a difficulty in the implementation of insurance contracts, which may explain why macro-economic risks are poorly pooled, and mainly at a compulsory level.

The paper is organized as follows. Section 2 sets up the model. Section 3 studies the benchmark case of optimal insurance without moral hazard and describes the optimal conditions. Section 4 and 5 compares the expected consumption received by each group with the sharing rules without moral hazard, and Section 6 concludes. Proofs are gathered in the final section.

2 The model

The paper considers an economy with a single good in which individuals face two kinds of risks, at the macro-economic and individual level. The set of individuals is partitioned into observable classes or groups. For simplicity, there are only two groups (or types) of individuals. The following characteristics are assumed.

*Idiosyncratic and aggregate risks.* Within a class, individuals are identical before the realization of an idiosyncratic shock. A \( h \)-individual denotes an individual in group \( h \), \( h = 1, 2 \). The idiosyncratic shock determines the individual’s status. For instance, an individual may be employed or unemployed, be ill or healthy. A level of effort exerted by the individual influences the probability of such his status. The status determines the level of resources or output generated by the individual. There is a moral hazard problem if the levels of effort are not contractible.

The precise environment that individuals in each group are facing, that is the output for each status and the probability of status given a level of effort, is uncertain affected by the (macro-economic) state of the economy. More precisely, denote by \( \theta \) a status, by \( e \) a level of effort and by \( s \) a state of the economy. The state \( s \) determines
- the output\(^4\) \(\omega_h(\theta|s)\) of a \(h\)-individual whose status is \(\theta\),
- the positive probability \(p_h(\theta, e|s)\) for an \(h\)-individual to be in status \(\theta\) if he exerts effort level \(e\).

Thus, a state is characterized by these outputs and probabilities for each group. There is only output risk when the probabilities are constant across states. For each group, the set of possible status, \(\Theta_h\) for \(h\), is finite and can be ordered so that a high value is "good" : for each state \(s\) output \(\omega_h(\theta|s)\) increases with \(\theta\). A simple framework is one where individuals face a binomial risk, success or failure.

The macro-economic state is distributed according to a fixed probability \(\pi\), and may take a finite or infinite number of values.

Preferences. Preferences of a \(h\)-individual are represented by a Von Neumann-Morgenstern utility function, \(u_h\), over income levels \(c\), separable in effort : \(u_h(c) - k(s)e\). The function \(u_h\) is defined over \([c_h, +\infty[\) where \(c_h\) is possibly \(-\infty\), and satisfies an Inada condition. It is concave, strictly increasing, and twice differentiable. In one group at least individuals are strictly risk averse.

Information and timing. The \textit{ex ante} distribution of macro-economic states, \(\pi\) and the distributions \(p_h\) are common knowledge. In most of the paper, the timing of the situation is the following one.

1. insurance contracts are designed,
2. the macro-economic state is revealed,
3. each individual chooses a level of effort, and
4. individuals’ status are observed, and contracts are implemented.

Insurance schemes and insurance contracts.

The timing makes possible for insurance contracts to account for both shocks that affect an individual. An insurance contract specifies the income level that an individual will receive at step 4 contingent on both the macro-economic state \(s\) and his status \(\theta\). I focus on insurance contracts that treat individuals within a group identically. An insurance scheme specifies an insurance contract for each group. A scheme may provide some insurance

\(^4\)The status has no intrinsic value: what matters for an individual is only the distribution of output. Thus one could identify status with output, i.e. take \(\theta\) equal to \(\omega\), and set \(p_h(\omega|c, s)\) to be the probability of receiving \(\omega\). This would however imply to deal with supports for \(\theta\) varying with the state, which is not convenient. Also, distinguishing status and output allows one to interpret more easily the impact of the state on outputs.
across macro-economic shocks through transfers across groups that are state
dependent, as described below.

A contract for individuals in group $h$ is described by a contingent in-
come plan $\tilde{c}_h = (c_h(\theta|s))$, in which $c_h(\theta|s)$ denotes the income level of a
$h$-individual whose status is $\theta$ if the macro state is $s$. Effort is chosen at
step 3 knowing the state and the contract. Individuals in $h$ will exert an
identical level of effort in state $s$, $e_h(s)$, and we denote $\tilde{e}_h = (e_h(s))$. The
level of effort chosen at step 3 may depend on whether it is contractible or
not. A contractible effort is part of the contract. A non contractible effort
is chosen optimally by each individual as follows. Let $U_h(\tilde{c}_h, e|s)$ be the ex-
pected utility conditional on state $s$ derived by a $h$-agent facing the contract
$\tilde{e}_h$ and choosing level $e$:

$$U_h(\tilde{c}_h, e|s) = \sum_{\theta} p_h(\theta, e|s) u_h(c_h(\theta|s)) - k(s)e.$$  \hspace{1cm} (1)

An optimal effort level in state $s$, $e_h(s)$, satisfies

$$U_h(\tilde{c}_h, e_h(s)|s) \geq U_h(\tilde{c}_h, e|s) \text{ each } e \in E_h.$$  \hspace{1cm} (2)

To focus on the sharing of macro-economic risks across groups, it is
assumed that the frequency of an individual status within a group is exactly
equal to its probability. Therefore, given a contract and effort levels exerted
by members of the group, there is no uncertainty on the levels of the group’s
output and income once the state of the economy is known. More precisely,
given $\tilde{e}_h$ and $\tilde{e}_h$, the aggregate output of group $h$ in state $s$ is risk-less equal
to its mathematical expectation:

$$\Omega_h(s) = \sum_{\theta} p_h(\theta, e_h(s)|s) \omega_h(\theta|s).$$  \hspace{1cm} (3)

and similarly aggregate income is :

$$C_h(s) = \sum_{\theta} p_h(\theta, e_h(s)|s) c_h(\theta|s)$$  \hspace{1cm} (4)

An insurance scheme specifies an insurance contract for each group: $(\bar{c}_h)_{h=1,2}$. The scheme is feasible if, in each state, aggregate income is not larger than aggregate resources, accounting for the chosen level of efforts given effort levels $(\tilde{e}_h)_{h=1,2}$:

$$\sum_h C_h(s) \leq \sum_h \Omega_h(s) \text{ in each state } s.$$  \hspace{1cm} (5)
Note that if each group designs a contract independently, each one is subject to the constraint $C_h(s) \leq \Omega_h(s)$ in each state $s$, thereby preventing macro-economic risk to be shared among the two groups.

**Optimal insurance schemes.** Positive weights are assigned to each group, $\lambda_h$ to $h$. A feasible insurance scheme is said to be optimal if it maximizes the weighted sum of the *ex ante* utilities of the groups over all feasible schemes. Thus, the welfare criteria associated to a scheme $(\tilde{c}_h, \tilde{e}_h)_{h=1,2}$ is equal to

$$
\sum_h \lambda_h E_\pi[U_h(\tilde{c}_h, \tilde{e}_h|s)].
$$

(6)

where $E_\pi$ denotes the expectation with respect to the states under the distribution $\pi$. Exchanging the sums in the welfare function (6), the optimization problem writes as

$$
\text{maximize } E_\pi[\sum_h \lambda_h U_h(\tilde{c}_h, e_h|s)]
$$

(7)

over $(\tilde{c}_h)_{h=1,2}$, and $(e_h)_{h=1,2}$ satisfying for each $s$ the feasibility constraints (5), and the incentives constraints (2) if effort is not contractible. Observe that these constraints are independent across states $s$. Since the objective function (7) is separable, the optimization problem amounts to maximize in each state the weighted sum, $\sum_h \lambda_h U_h(\tilde{c}_h, e_h|s)$, under the probability distribution for the idiosyncratic shocks and the constraints prevailing in that state.\(^5\) Evaluating how macro-economic risks are shared within the groups amounts to study how the solution to this program varies with $s$.

In Section 6, effort has to be exerted before the state is revealed (that is steps 2 and 3 are exchanged). This case makes sense only if effort cannot be adjusted to the state that materializes. An interpretation is that effort is an investment. For example, workers can take on their leisure time to invest say in formation in new technological skills that will be more or less useful depending on the evolution of the economy and technics. The functioning of the model is easily adapted by simply requiring the effort to be independent of $s$ chosen so as to maximize the ex ante utility level. Contracts are still contingent on both $s$ and $\theta$, and subject to the feasibility constraint (5) associated to the chosen effort.

\(^5\)Whereas the distribution of the states $\pi$ does not influence the shape of the optimal scheme, it determines utility levels. Hence if one wants to select a contract that Pareto improves upon a given situation, the distribution matters through the choice of the weights.
Assumptions  It is convenient to take assumptions under which the so-called 'first order' approach to moral hazard problems is valid (see Grossman and Hart 1983 and Rogerson 1985 for example). While these assumptions are not necessary, they allow us to contrast the impact of moral hazard, as explained later on. Recall that outputs are increasing with the status \( \theta \).

The level of effort takes values in an interval, \([0, e^{\text{max}}_h]\). The probabilities \( p_h(\theta, e|s) \) are differentiable with respect to \( e \) continuous with respect to \( s \). Furthermore, in each state \( s \)
- (CDF) the cumulative probability for a status to be smaller than \( t \), \( \sum_{\theta < t} p_h(\theta, e|s) \), is convex in effort \( e \)
- (MLR) the ratio \( (p^{h\prime}_he/p_h)(\theta, e|s) \) is nondecreasing in \( \theta \), where \( p^{h\prime}_h \) is the derivative of \( p_h \) with respect to \( e \).

The roles of the assumptions are the following. Thanks to the convexity of the distribution function (CDF), an effort that satisfies the first order conditions of maximization of \( U(\hat{c}, e|s) \) with respect to \( e \) is optimal provided the contract \( \hat{c} \) is increasing in \( \theta \). The monotone likelihood ratio (MLR) ensures that an optimal contract is indeed increasing in \( \theta \).

With two status levels, "failure" and "success", the assumptions are satisfied when the probability of success is increasing and concave in effort. As an example with more than two status levels, let the distributions be "spanned" by two probabilities : dropping \( s \) and \( h \) to simplify notation, there are distributions \( p \) and \( q \) on \( \Theta \), and a nonnegative function \( \ell \) on \([0,1]\) with \( \ell(0) = 0 \), and \( \ell(1) = 1 \) such that

\[
p(\theta, e) = \ell(e)p(\theta) + (1 - \ell(e))q(\theta) \tag{9}
\]

The assumptions are satisfied if the ratio \( p(\theta)/q(\theta) \) increases with \( \theta \) and \( \ell \) is increasing and concave. When \( \ell(e) = e \), the linear effort model is obtained, which is very close to a model with two values for effort since the optimal effort is either the maximum or the minimum one except in case of indifference.

\footnote{The first order conditions are}

\[
\sum_{\theta} p^{h\prime}_h(\theta, e|s)u(c(\theta)) - k = \begin{cases} 
\leq 0 & \text{if } e = 0 \\
= 0 & \text{if } 0 < e < e^{\text{max}}_h \\
\geq 0 & \text{for } e = e^{\text{max}}_h 
\end{cases} \tag{8}
\]
3 Optimal schemes

As a benchmark, we analyze the basic features of optimal risk sharing in the absence of moral hazard.

3.1 The benchmark case without moral hazard

Assume first no effort. Individuals cannot influence the probability distribution of their status, which only depends on the state. Thus, the resources generated by each group $h$ in a state $s$ are exogenous given by $\Omega_h(s) = \sum_\theta p_h(\theta|s)\omega_h(\theta|s)$ (in which the argument $e$ in $p_h$ has been suppressed). The overall resources $\Omega(s) = \sum_h \Omega_h(s)$ are exogenous as well.

Mutuality principle and sharing rules

To be optimal, the allocation must satisfy the mutuality principle, which asserts that individuals’ income levels are identical across states with identical aggregate resources. Therefore, individuals bear risks only if it is unavoidable. In particular they are fully insured against idiosyncratic shocks. Furthermore, income levels depend on the macro-economic state only through the aggregate resources available in that state: In the terminology of Wilson (1968), they are described by sharing rules, $S_h$ for a $h$-individual, that assigns income in function of the level of $\Omega(s)$ only (since individuals within a group are treated equally, their sharing rules are identical.). Sharing rules depend on the utility functions of the two groups (and the weights), as characterized as follows. For each level of aggregate resources $\Omega$, let $(S_1(\Omega), S_2(\Omega))$ be the unique feasible pair $(c_1, c_2)$ that equalizes the weighted marginal utilities: for some $\alpha$

\[ c_1 + c_2 = \Omega \text{ and } \lambda_h u'_h(c_h) = \alpha, h = 1, 2. \] (10)

In the absence of effort level, the optimal insurance contract allocates income according to the sharing rules $(S_1, S_2)$:

in each state $s$, $c_h(\theta|s) = S_h(\Omega(s))$ for each $h$.

If individuals are all risk averse, each sharing rule $S_h$ is strictly increasing. If individuals in one group, say 2, are risk neutral ($u_2(c) = c$),

\footnote{In the principal agent literature, the term sharing rule is sometimes used to refer to the share that the agent receives in function of the observable, which corresponds here to the income as a function of the status, given a state $s$.}
without bounds on consumption levels), individuals in group 1 are fully insured against all risks, namely their income level is constant across states \( \alpha = \lambda_2 = \lambda_1 u'(S_1) \).

The optimal scheme can be described as involving two types of risk sharing: a full insurance contract against idiosyncratic shocks within each group, and a contract sharing the macro-economic risks across groups. The second contract can be reached by trading options on the level of aggregate resources. The Arrow-Debreu contingent price of one unit of consumption deliverable in state \( s \) is equal to the probability of the state \( \pi(s) \) multiplied by the value \( \alpha \) in that state as given by (10).\(^8\)

**Contractible effort** The analysis extends easily to the situation in which individuals exert an effort that is contractible.

**Proposition 1** Assume effort to be contractible. An optimal insurance scheme is characterized by

- **optimal risk insurance**: resources are shared according to the sharing rules \( S_h \) as defined by (10):

\[
\text{in each state } s, c_h(\theta|s) = S_h(\Omega(s)) \text{ for each } h.
\]

- **optimal levels of effort**: in each state \( s \), \( e_h(s) \) maximizes the value of output net of effort:

\[
\alpha(s) \sum_{\theta} p_h(\theta, e|s) \omega_h(\theta|s) - \lambda_h k(s)e \text{ in which } \alpha(s) = \lambda_h u'(S_h(\Omega(s))).
\]

Consumption, effort and utility levels are continuous in the state. Furthermore if state \( s' \) is obtained from \( s \) by a uniform change \( \Delta \omega_h \) in the output of group \( h \) (whatever the status \( \theta \)) for each group and \( \Delta \omega_1 + \Delta \omega_2 \geq 0 \) (resp. \( > 0 \)), then both groups are at least as well off in \( s' \) as in state \( s \) (resp. both are strictly better off if both are risk averse).

The mutuality principle still holds, and furthermore each individual consumes the same share of aggregate resources as without effort. To understand this, observe that, given effort levels, the groups face the same

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\(^8\)This is similar to Breeden and Litzenberger (1978) in a stock exchange setup. They show that if all individual risky endowments are issued as securities, an optimal allocation can be reached by trading options on aggregate wealth. Demange and Laroque (1999) extends the result to some situations in which individual risks are not all exchanged.
problem as in the previous section: sharing risky aggregate resources optimally between the groups. The result follows thanks to the separability of preferences in effort and consumption. The monotonicity property is more tricky because an increase in the output of a group results in a change in its effort hence in resources, which in turn affects the resources and effort level of the other group. It turns out that uniform changes in output with $\Delta \omega_1 + \Delta \omega_2 \geq 0$ induce both effort levels to decrease (or stay constant) but the impact is moderate enough so that overall resources nevertheless increase: both effects increase welfare. On the other hand, monotonicity may fail when changes in output are not constant across status. As a simple example, suppose group 2 to be risk neutral, so that both the price $\alpha$ and 1’s consumption are constant across states. Let group 1 be faced with two status, failure and success. An increase in 1’s output in case of success with no change in case of failure induces an increase of his effort level. Since 1’s consumption does not change, 1 is worse off.

**Pensions** As an illustrative example, let group 1 be composed of the working age generation, the "young" and group 2 the retired one, the "old". Young individuals are employed with probability $p(e|s)$, generating output $\omega_1(s)$ (index $\theta$ is unnecessary here and has been dropped) and are unemployed with probability $1 - p(e|s)$ generating no revenue. Retirees’ output is nil. Thus the expected aggregate resources in state $s$ are equal to $\Omega(s) = p(e|s)\omega_1(s)$.

By the mutuality principle, young workers are completely insured against unemployment, and income levels of both young and old are functions of $\Omega(s)$. This result suggests that pension annuities should not be indexed on individual wages, but rather on aggregate wages. In particular benefits should decrease as unemployment raises, and conversely. Such a result supports a *notional* pay as you go system, as has been recently implemented in Sweden, in which annuities are indexed on growth.

Sharing rules are easily computed for some specific utility functions. Under constant risk aversion, with an index of risk tolerance $\tau_h$ in group $h$, each young, worker or unemployed, and each old individual get respectively

$$S_1(\Omega(s)) = \bar{c}_1 + \frac{\tau_1}{\tau_1 + \tau_2} (\Omega(s) - \bar{\Omega}), \quad S_2(\Omega(s)) = \bar{c}_2 + \frac{\tau_2}{\tau_1 + \tau_2} (\Omega(s) - \bar{\Omega})$$

where $\bar{\Omega} = E[\Omega(s)]$ is the *ex ante* expected revenue per worker over macro-
economic states, accounting for the effort levels and $\varpi_1$ and $\varpi_2$ are scalars that sum to the expected wealth $\Omega$ and are determined by the weights $\lambda_h$. Thus, fluctuations in aggregate wealth are allocated to the agents in proportion of their risk tolerance.

With an identical constant relative risk aversion, all income, in particular unemployment and pension benefits are fully indexed on aggregate wealth:

$$S_1(\Omega(s)) = \mu\Omega(s), S_2(\Omega(s)) = (1 - \mu)\Omega(s)$$

in which $\mu$ is a constant determined by the weights.

### 3.2 Optimal schemes under moral hazard

In the presence of moral hazard, individuals’ levels of effort are not observable nor verifiable. Facing a contract that provides full insurance against status risk, the minimal level of effort is exerted. Hence, the optimal scheme described in proposition 1 is typically not feasible without enforcement of effort. Under moral hazard, the optimization problem in state $s$ writes as

$$\begin{align*}
\max_{(\tilde{c}_h, e_h)_{h=1,2}} & \quad \lambda_1 U_1(\tilde{c}_1, e_1|s) + \lambda_2 U_2(\tilde{c}_2, e_2|s) \\
\text{subject to} & \quad \sum_h \sum_\theta p(\theta, e_h|s)[c_h - \omega_h](\theta|s) \leq 0 \quad (\text{feasibility}) \\
& \quad U_h(\tilde{c}_h, e_h(s)|s) \geq U_h(\bar{c}_h, e|s) \text{ each } e \in \mathcal{E}_h \quad (\text{incentives})
\end{align*}$$

The problem decomposes into two sub problems, one intra-group and the other inter-group, thereby making clear the transfers across groups. To see this, given an optimal scheme $(\tilde{c}_h, e_h)_{h=1,2}$ define the net transfer to group $h$ as the expected surplus of its income over outputs accounting for the level of effort:

$$R_h(s) = \sum_\theta p(\theta, e_h(s)|s)[c_h - \omega_h](\theta|s). \quad (11)$$

Surely, the insurance contract $\tilde{c}_h$ for group $h$ is optimal given the net transfer $R_h(s)$ to that group. In other words, if group $h$ is given $R_h(s)$ and can chose a contract freely under the budget constraint (11), it cannot do better than choosing $\tilde{c}_h$. Therefore the optimization problem can be solved by considering

- the intra-group problem that chooses for each group an optimal insurance scheme, given a net transfer $R_h(s)$ to that group:

$$\mathcal{P}_h(R_h(s)) : \max_{\tilde{c}_h, e_h} U_h(\tilde{c}_h, e_h|s) \text{ under the incentives constraints and}$$

12
\[
\sum_{\theta} p(\theta, e_h|s)[c_h - \omega_h](\theta|s) \leq R_h
\]  

(12)

Let us denote by \( V_h(R_h|s) \) the value of the program \( \mathcal{P}_h(R_h|s) \).

- the inter-group problem of allocating net transfers across groups:

find \((R_1(s), R_2(s))\) so as to maximize \(\lambda_1 V_1(R_1|s) + \lambda_2 V_2(R_2|s)\)

over \((R_1, R_2)\) that satisfy the feasibility constraint \(R_1 + R_2 = 0\).

Optimal contract within a group  

The previous remarks lead us to analyze the intra-group problem faced by a group in a given state \(s\) for various values of transfers, that is \( \mathcal{P}_h(R_h|s) \) when \( R_h \) varies. To simplify notation, state \(s\) and index \(h\) are omitted. It is worth noting that the problem \( \mathcal{P}(R) \) is the dual of a standard principal-agent problem with a risk neutral principal.\(^9\)

**Lemma 1.** At an optimal solution of an intra-group problem \( \mathcal{P}(R) \), there are \(\alpha > 0\), and \(\beta \geq 0\) such that for each \(\theta\):

\[
\frac{1}{\alpha} = \frac{1}{u'(c(\theta))} - \frac{\beta p'(\theta, e)}{\alpha p(\theta, e)}
\]  

(13)

The value \( V(R) \) of the problem \( \mathcal{P}(R) \) satisfies

\[
V'(R) = 1/E_{\theta}[\frac{1}{u'(c(\theta))}]
\]  

(14)

in which expectation is taken over status according to the probability distribution given the chosen effort level.

Conditions (13) are the first order conditions on consumption levels, in which \(\alpha\) and \(\beta\) are the multipliers associated with the budget constraint and the incentive constraint (more precisely the first order condition (8)). These conditions are the same as in a principal-agent problem, as expected from duality. In particular, full insurance against status risk is provided only if \(\beta\) is null.\(^{10}\)

According to (14), the marginal utility for transfer is equal to the reciprocal of the expectation of the reciprocal marginal utility. One may somewhat

\(^9\)In the "standard" problem the principal maximizes the surplus, \( \sum_{\theta} p(\theta)(\omega - c)(\theta) \), under a minimum participation utility level for the agent.

\(^{10}\)An additional condition requires the optimality of the level of effort given the prices of the resource constraint (see the proof.)
understand this as follows. Consider a marginal change in the net transfer $\Delta R$ to the group. If the first order conditions are fulfilled, the marginal change in utility derived by this change is independent of how $\Delta R$ is allocated across the different status.\(^{11}\) In other words, $V'(R)$ can be computed in many different ways (not necessarily the optimal ones). For example, it can be computed by allocating $\Delta R$ so as to equalize the marginal change in utility across various status $\theta$: $u'(c(\theta))\Delta c(\theta)$ is equal to some $\Delta u$ whatever $\theta$. Such a change does not affect the level of effort. Hence probabilities do not change, and the budget equation (12) is satisfied if $\Delta R = \sum_{\theta} p(\theta, e) \Delta c(\theta)$. Using $\Delta c(\theta) = \Delta u / u'(c(\theta))$ gives the marginal change in utility

$$\Delta u = \frac{\Delta R}{E[\frac{1}{u'(c(\theta))}]} | e|,$$

which explains formula (14).

By allocating $\Delta R > 0$ instead uniformly across the various status, the "marginal cost" of moral hazard is derived as follows. A uniform consumption increase, $\Delta c(\theta) = \Delta$, typically reduces effort.\(^{12}\) Hence the increase $\Delta c$ may differ from $\Delta R$. By the envelope theorem, the marginal change of utility is equal to $\Delta E[u']$ so that (14) gives $\Delta E[u'] = \Delta R / E[1/u']$. The extra amount of consumption that can be given to each $\theta$-agent is smaller than $\Delta R$ by Jensen inequality (applied to the reciprocal function $1/x$). More precisely the decrease in the level of effort induces a loss of resources equal to $\Delta R(1 - 1/(E[u']E[1/u']))$.

As far as we know, expression (14) has not been given in the literature. In a repeated moral hazard problem, Rogerson (1985) shows that the reciprocal of the marginal utility plays a role. More precisely, a risk neutral principal always benefits from equalizing the agent’s reciprocal marginal utility in a first period to the discounted expected reciprocal marginal utility of the

\(^{11}\)This is a form of the envelope theorem. The intuition is that if an allocation increases utility more than another one, both being feasible for the transfer $R + \Delta R$ taking into account the impact on the effort level, their difference would yield a direction for improving the standing allocation: if $\tilde{c} + \tilde{\Delta} c$ is preferred to $\tilde{c} + \tilde{\Delta}' c$ and both are feasible for $R + \Delta R$ with changes in effort $\Delta e$ and $\Delta' e$ respectively then $\tilde{c} + \tilde{\Delta} c - \tilde{\Delta}' c$ is feasible for transfer $R$ and change on effort $\Delta c - \Delta' c$ and improves welfare.

\(^{12}\)Assuming $e$ to be interior, the derivation of (8) gives $\Delta [\sum_{\theta} \rho(\theta, c(\theta))] + [\sum_{\theta} \rho'(\theta, c(\theta))] \Delta e = 0$. Both terms in brackets are negative: the first because of increasing likelihood ratio, the second because of the second order condition.
following period. Lemma 1 helps us to understand this result: the agent’s marginal utility for income is equalized across periods.

Optimal schemes

**Proposition 2** Under moral hazard, an optimal insurance scheme satisfies the following conditions. In each state \( s \)

- (optimal contract within a group) : for each \( h \), \((c_h(\theta|s))\) is optimal for group \( h \) given the probability in state \( s \) and the net transfer \( R_h(s) \), that is, solves \( \mathcal{P}_h(R_h(s)|s) \)

- (optimal risk sharing across groups) : weighted marginal groups’ utilities for income are equalized:

\[
\lambda_h \frac{\partial V_h(R_h|s)}{\partial R_h} = \frac{\lambda_h}{E[\frac{1}{u_h}|s]} = \alpha(s) \quad h = 1, 2
\]  

(15)

Condition (15) extends Borch condition (10) to situations with moral hazard. It says that the ratio of the groups’ marginal utilities for income (properly adjusted to account of incentives) is kept constant across the states of the economy. A first natural question is what remains of the mutuality principle. The following weak version holds.

Let state \( s' \) be obtained from \( s \) by a uniform increase \( \Delta \omega_h \) in the output of group \( h \) (whatever the status \( \theta \)) with \( \Delta \omega_1 + \Delta \omega_2 = 0 \). Then, at the optimal contract, both groups get the same contingent contract in states \( s \) and \( s' \)

\[
c_h(\theta|s) = c_h(\theta|s') \text{ for each } \theta, h.
\]

A direct proof is straightforward. The set of feasible allocations and the welfare criteria are identical in the two states changes (through a translation of the transfers). Thus optimal contracts are identical in the two states. The conditions that are required to compare two states are very strong, so the above mutuality principle is weak indeed. One cannot hope much more. As shown by example 5.1 below, a uniform increase in the output of one group may be followed by an unambiguous decrease in consumption and welfare levels of that group, due to a decrease in the level of effort exerted by the members in the other group (who are better off).
4 Impact of moral hazard on consumption levels

This section studies how moral hazard affects the shares received by each group. Consider an outside observer, who observes aggregate levels of income in each group and aggregate resources. Under which conditions should the share of aggregate resource received by a group be smaller or larger than without moral hazard? This involves a comparison of group $h$ income at an optimal scheme, $C_h(s) = E_\theta[c_h(\theta|s)]$, with the share of the same level of aggregate resources the group would receive if there was no moral hazard, namely with $S_h(\Omega(s))$ where $\Omega(s) = C_1(s) + C_2(s)$.

The comparison is easy in the special case where individuals’ relative risk aversion is constant equal to one. For $u(c) = \ln c$

$$\frac{\partial V}{\partial R} = 1/E_\theta[1/u'(c)] = 1/(E_\theta[c]) = u'(E_\theta[c]).$$

Thus, the group's marginal utility for income is equal to the marginal utility of a representative individual who receives the group’s expected income and who is not subject to moral hazard. Suppose now that each group has a log utility function (or is not subject to moral hazard). Then the optimality conditions (15) coincide with the Borch conditions applied to expected consumption levels. Hence an optimal insurance scheme allocates aggregate resources to the two groups as without moral hazard.

$$C_h(s) = S_h(\Omega(s)), \text{ for } h = 1, 2 \text{ each } s$$

The absence of distortion is due to the linearity of the reciprocal of the marginal utility of a log function. It suggests that more generally the concavity, or convexity, of the reciprocal of the marginal utility determines how shares are distorted in the presence of moral hazard. This is indeed true, as stated in the following Proposition.

**Proposition 3** Assume that $1/u'_1$ is convex or group 1 is not subject to moral hazard and that $1/u'_2$ is concave or group 2 is not subject to moral hazard. At the optimal scheme

$$C_1(s) \leq S_1(\Omega(s)) \text{ and } C_2(s) \geq S_2(\Omega(s))$$

(the inequality is strict if $1/u'_1$ is strictly convex and 1’s effort is not minimal or if $1/u'_2$ is strictly concave and 2’s effort is not minimal).
Therefore group 1’s share of aggregate resources is not larger, and group 2’s share is not less, than without moral hazard. An immediate implication of proposition 3 is the following one. Under the stated assumptions, consider two states with identical levels of aggregate resources. If group 1 provides no effort in one state, its share is surely not less than in the other state.

In the pension example, retirees are not subject to moral hazard. Therefore Proposition 3 applies whenever the reciprocal of the marginal utility function of group 1 is either convex or concave. Convexity is the more plausible assumption: it holds for functions with constant risk aversion or with a constant relative risk-aversion function larger than 1, as is usually assumed (1/u'_1 = c^γ, is convex for γ > 1). Under this convexity assumption, the expected share received by the young generation should be lowered to account for moral hazard.

To better understand the impact of moral hazard, it is also interesting to consider states with identical marginal utilities for income, or equivalently identical values of α. Optimal sharing across groups as given by (15), implies, that the distributions of the reciprocal of marginal utility, 1/u'_h(˜c_h), have constant across the states with identical α. When furthermore these distributions can be compared in the sense of second order stochastic dominance, more can be said. Note that when no effort is exerted by h, the distribution is certain, hence surely dominates any other distribution with the same expectation.

We consider one group, say 1. We have already seen that the convexity of 1/u'_1 plays a role. A related function that is worth considering is (1/u'_1)(u_1^-1), which is the derivative of u^-1.13

**Proposition 4** Consider states with the same price α. Let S_1 be the constant consumption level that group 1 gets if he is not subject to moral hazard (λ_1 u'_1(S_1) = α). Let the distribution of 1/u'_1(˜c_1) given state s be strictly dominated by that in state s'.

If 1/u'_1 is convex, then C_1(s) ≤ C_1(s') ≤ S_1,

the first inequality being strict if 1/u'_1 is strictly convex.

---

13Since (u^{-1})(v) is the consumption level that ensures a utility level equal to v, (1/u'_1)(u_1^{-1})(v)dv is the additional consumption that must be given so as to increase utility from v to v + dv. Note that (1/u'_1)(u_1^{-1}) is surely convex if 1/u'_1 (because (1/u'_1) is increasing and (u_1^{-1}) is convex), but the converse is false. For a function with constant relative risk aversion γ for example, 1/u'_1 is convex for γ > 1 and (1/u'_1)(u_1^{-1}) for γ > 1/2.
If \((1/u'_1)(u_1^{-1})\) is convex then \(E[u_1(\tilde{c}_1)|s] \leq E[u_1(\tilde{c}_1)|s'] \leq u(S_1)\), the first inequality being strict if \((1/u'_1)(u_1^{-1})\) is strictly convex.

It is not always possible to compare contracts along the states with a fixed price \(\alpha\). As can be seen from (13), the distributions of \(1/u'_1(\tilde{c}_1)\), may differ both through the multiplier of the incentives constraint, \(\beta\), and the effort level, hence may not be ordered. In the linear effort model however, the distributions can always be compared. The reason is that optimal effort is (almost always) either at the minimal or maximal value. With the minimum effort, all individuals in group 1 gets the sure level \(S_1\). If effort is maximal, only the value of \(\beta\) can change. As \(\beta\) increases, the distribution stochastically decreases in the sense of second order dominance.\textsuperscript{14}

**A risk neutral group** The proposition readily applies to the situation where individuals in a group, say 2, are risk neutral. Then the price of the resource constraint is constant across states, equal to \(\lambda_2\). With contractible effort, individuals in group 1 (who are strictly risk averse) are fully insured against all shocks, whether on their status or on macro-economic states: they get a constant income level \(S_1\) (even though the level of effort may vary with the state). In the presence of moral hazard, partial insurance against idiosyncratic shocks may be optimal so as to incite individuals to expand effort. The question is whether they are nevertheless insured against macro-economic shocks, that is whether their expected utility over consumption is kept constant across states. From proposition 4 the answer is typically negative except when \((1/u'_1)(u_1^{-1})\) is linear, that is for a constant relative risk aversion equal to \(1/2\).

This result can be translated into the framework of a risk neutral principal facing a risk averse agent in various situations or "states". Assume that a contract is signed before the state is known, and must be honored thereafter whatever the state that materializes. Thus, the agent must be given a minimum ex ante utility level in order to get his participation. The contract that maximizes the profit of the principal under the agent’s participation constraint is optimal (that is there are some \(\lambda_h\) supporting it). From\textsuperscript{14}

\textsuperscript{14}To see this, note that for \(\theta\) for which \(p'_e > 0\), \((1/u'_1(\tilde{c}(\theta)))\) is larger than the mean \(E[1/u'_1(\tilde{c}(\theta))]\) and conversely for \(\theta\) for which \(p'_e > 0\), it is smaller. Thus, as \(\beta\) increases, the values larger than the mean all increase (since \(p'_e > 0\), and those smaller than the mean all decrease: this is a mean preserving spread."
the previous result, the principal is likely to choose a contract in which the agent’s status-contingent income and conditional utility level vary across states.

5 Shares variations

The state of the economy can influence the environment through the distributions of probabilities individuals are facing and their outputs. This section studies how expected shares vary under output risk, the probability of a status given an effort level being constant across states.

5.1 Discontinuities and non monotonicity

Optimal schemes may present discontinuities with respect to the macroeconomic state. Discontinuities can only be due to the endogenous level of effort which induces some form of non convexity.

Consider the (indirect) value of a contract derived under optimal effort in state $s$: $\hat{U}_h(\tilde{c}_h|s) = \max_{e \in \mathcal{E}_h} U(\tilde{c}_h, e|s)$. Note that program $\mathcal{P}_h(R|s)$ amounts to choose the contract that maximizes $\hat{U}_h$ under the feasibility constraint (12). Function $\hat{U}_h$ may not be concave in $\tilde{c}_h$, as identified by Prescott and Townsend (1984), and Arnott and Stiglitz (1991). To see this, note that, by the envelope theorem, the marginal utility for consumption in state $\theta$, is equal to $p(\theta, e)u'(c(\theta))$ where $e$ is the optimal level of effort (dropping index $h$ and $s$). An increase in consumption in a "bad" state $\theta$ results in a lower effort hence in an increase in the probability of the state which may be large enough to increase the product $p(\theta, e)u'(c(\theta))$. For example, in a linear model with two outcomes, success and failure denoted respectively by $\theta_g$ and $\theta_b$, the optimal effort will be typically at the minimum or maximum level. Let $p$ and $q$ be respectively the probabilities of the good state if the maximal and minimum levels are exerted with $p > q$. Starting with a

\[\text{15In a given state, the non concavity of the indirect utility function has been recognized to result into inefficiencies in a competitive insurance market in which individuals can purchase several contracts (Arnott and Stiglitz 1991 or more recently Bisin and Guaitoli 2002). This difficulty does not occur here because contracts are exclusive. Also, from an efficiency viewpoint, the non concavity of $\hat{U}$ may justify the use of lotteries. Even though the discontinuities of contracts might disappear, lotteries do not seem an appealing solution in real life situations.}\]
contract for which $e^{\max}$ is optimal, increase consumption in case of failure $c(\theta_b)$, keeping $c(\theta_g)$ fixed. There is a threshold value $c^*(\theta_b)$ for which the optimal level of effort jumps to the minimum. At this point, the marginal utility for consumption in the bad state jumps from $(1 - p)u'(c^*(\theta_b))$ to $(1 - q)u'(c^*(\theta_b))$, which is larger: $\hat{U}$ is not concave.

As a result, a marginal change in the transfer to a group may induce discontinuous changes in the contract and effort level chosen by the group, and an upward jump in the marginal value for transfer $V'_h(R)$ as given by (15) (the value function $V_h(R|s)$ is not concave in $R$). It follows that transfers across groups at an optimal scheme may be discontinuous as well. Discontinuities and non-monotonicity in contracts are illustrated by the following example.

Only individuals in group 1 are subject to idiosyncratic risk with two values for status and probabilities are linear in effort. The state is characterized by $(\omega_1, \omega_2)$, where $\omega_1$ is the output of a 1-individual in case of success, (0 in case of failure) and $\omega_2$ the output of a 2-individual. Utility functions are log in both groups. Weights are taken to be equal.

Given a state $(\omega_1, \omega_2)$, the optimal transfer in state $s$ is obtained by maximizing this welfare function $V_1(R|s) + V_2(-R|s)$ in which $R$ is the transfer from group 2 to group 1. One has $V_2(-R|s) = \log(\omega_2 - R)$. As for group 1 easy computation gives $V_1(R|s)$ as:

- $\log(p\omega_1 + R) - a$ if $R \leq R^*$ (1’s effort level is maximal)
- $\log(q\omega_1 + R)$ if $R \geq R^*$ (1’s effort level is minimal)

for two scalars $a$ and $R^*$. The function is continuous at $R^*$ with an upward jump for its derivative. Thus welfare, $V_1(R) + V_2(-R)$ is concave on each interval where 1’s effort is constant, but not overall concave. As a result, welfare may have two local maxima. In Figure 5.1 the welfare function is drawn in two states with identical value for $\omega_1$ but two distinct values for $\omega_2$. For the low value of $\omega_2$, the dashed line, welfare is maximized at a low enough transfer so as to induce maximal effort. For the high value, the plain line, effort is minimal. The second graph depicts the set of states for which effort is maximal, below the line, or minimal, above the line.\footnote{With probabilities linear in effort, it suffices to consider contracts that incite to the minimum or maximum effort level. The former is risk-less: $c_a = c_b = q\omega_1 + R$. The latter satisfies $(p - q)[\ln c_y - \ln c_b] = k$ (8 as an equality) and $p c_y + (1 - p) c_b = R$. This determines $c_b$ and $c_y$ and the utility level $\ln(p\omega + R) - a$ with $a = q\frac{k}{p-q} - \ln[p \exp(\frac{k}{p-q}) + 1 - p]$. The} The shapes
Figure 1: $k = 0.5$, $\pi_g = 0.8$, $\pi_b = 0.3$
1. Welfare as a function of the transfer from group 2 to group 1.
2. Locus of states for which effort is maximal or minimal.
3. Consumption levels as a function of $\omega_1$.
4. Consumption levels as a function of $\omega_2$. 
of the contracts are given for a fixed value of $\omega_2$ as $\omega_1$ increases and similarly for a fixed value of $\omega_1$ as $\omega_2$ increases.

The important point is that group 2 may be hurt by an increase in its output. The reason is that such an increase triggers an increase in the transfer to group 1 that shifts the effort level to its minimum, and as a consequence decrease overall resources. Since the overall welfare is surely continuous in the state (by standard arguments), the jump downward of 2’s utility is exactly compensated by a positive jump of 1’s utility.

In contrast, consider the situation in which there is no insurance against macro-economic risks, that is there are no transfers across groups. Each group ”solves” $\mathcal{P}_h(0|s)$ in each state $s$. Whereas the policies, that is the contract and the effort, may be discontinuous in the state, the value $V_h(R|s)$ is nevertheless continuous and furthermore is increasing in each output. Thus a group is never hurt by an increase in its output. In view of the case with contractible effort and that without insurance across groups, it is the conjunction of moral hazard and insurance transfers that generates discontinuous and non monotone welfare levels.

5.2 Group’s risk aversion

Discontinuities are due to changes in effort levels. To understand better how income is distributed across groups, we study here their variations along states in which an identical level of effort is implemented. The analysis of Wilson extends somewhat by using the indirect utility function $V_h$. In particular the variation in income levels can be appraised through the ”risk aversion” coefficient of $V_h$. Define this coefficient by

$$\rho_h = \left[ -\frac{\partial^2 V_h}{\partial R_h^2} / \frac{\partial V_h}{\partial R_h} \right](R_h|s).$$

**Proposition 5** Consider states that differ in their output and for which each group provides a constant level of effort at an optimal scheme. Then groups’ income levels vary with a marginal change in aggregate resources as

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value of $R^*$ is defined by $\ln(p\omega_1 + R) - a = \ln(q\omega_1 + R)$. On the interval of transfers smaller than $R^*$ total welfare is maximized at $R_t = (\omega_2 - p\omega_1)/2$ if $R_t$ is smaller than $R^*$ and at $R^*$ otherwise. Similarly, the maximum on $\{R \geq R^*\}$ is obtained at $R_0 = (\omega_2 - q\omega_1)/2$ if it is larger than $R^*$. In a state where $R_t < R^* < R_0$, welfare has two local maxima at $R_1$ and $R_0$ with respective values $2\ln((p\omega_1 + \omega_2)/2) - a$ and $2\ln((q\omega_1 + \omega_2)/2)$. This gives a global maximum at $R_1$ with a maximal level of effort if $\omega_2 \leq \omega_1 K$ for some constant $K$.
follows:

\[ dC_1 = \frac{\rho_2}{\rho_1 + \rho_2} \, d\Omega \quad \text{and} \quad dC_2 = \frac{\rho_1}{\rho_1 + \rho_2} \, d\Omega \]  

(16)

where the risk aversion coefficient \( \rho_h \) of \( V_h \) satisfies

\[ \frac{1}{\rho_h} = E\left[ \frac{u_h'}{-u''_h} \right] - b_h \text{cov}\left( \frac{-u'^2_h}{u''_h}, \frac{1}{u''_h} \right)^2 \]  

(17)

for some positive \( b_h \).

In the absence of moral hazard, a marginal change in aggregate resources is allocated to the groups in proportion of their risk tolerance. In the presence of moral hazard a similar expression holds, as given by (16) and (17). The group’s risk tolerance is equal to the expectation of the risk tolerance over the various status diminished by a correcting term.

For a log function, the expected risk tolerance is equal to the risk tolerance at the expected group’s consumption and furthermore the correcting term is null. As was previously shown, the group behaves as a representative agent (whose utility function is log and consumption is the expected group’s consumption). Otherwise moral hazard has two effects on the group’s risk tolerance: the first on the expected risk tolerance, which may go in various directions, and the second one, which is always non positive, hence diminishes group’s risk tolerance. Under additional assumptions, more can be said as in the following corollary.

**Corollary** Assume for group 1 linear risk tolerance and convexity of \( 1/u'_1 \). Group 2 is not subject to moral hazard and risk aversion coefficient non increasing in income. Then along states in which 1 provides a constant level of effort, moral hazard decreases the variation of 1’s share with respect to aggregate resources.

Examples of utility functions with linear risk tolerance and convexity of the inverse of marginal utility are all functions with constant risk aversion or those with constant relative risk aversion larger than 1.

The proof of the corollary is simple. By linearity, the expected tolerance of group 1 is the tolerance at its expected consumption. Under the convexity of \( 1/u'_1 \), we know that 1’s share is diminished by moral hazard. Hence the expected tolerance of group 1 is lower than the tolerance at the share \( S_1(\Omega) \) if aggregate resources are \( \Omega \). Thus moral hazard decreases both terms of the
right hand side of (17) hence increases \( \rho_1 \). At the opposite group 2 receives a higher share under moral hazard, hence its risk aversion is smaller. From (16) group 1’s consumption varies less than without moral hazard.

Corollary can be applied to the pension example. Assume relative risk aversion constant \( \gamma \) larger than 1 and a simple linear model in which it is optimal to incite workers to exert effort (hence constant equal to the maximal one). As a result, whereas pensions are increased (from proposition 3), they should also be more variable with aggregate resources.

If relative risk aversion is constant but not larger than 1, one can say more in the simple case of two outcomes.\(^\text{17}\) Adjusted risk aversion \( \rho_h \) is

\[
\rho_h = \gamma \frac{E[c_h^{2\gamma-1}]}{[Ec_h]^2}.
\]

The equalization of weighted marginal utility with respect to income (15) writes as \( \lambda_1 C_2^\gamma = \lambda_2 E[c_1^\gamma] \), which gives

\[
\frac{\rho_1}{\rho_2} = [\frac{\lambda_2}{\lambda_1}]^{\frac{1}{2}} \frac{E[c_1^{2\gamma-1}]}{[Ec_1]^{2-1/\gamma}} = [\frac{\lambda_2}{\lambda_1}]^{\frac{1}{2}} E \left[ \left( \frac{c_1^\gamma}{E[c_1]} \right)^{2\gamma-1} \right].
\]

The share received by group 1 varies less under moral hazard if the last expectation term is larger than 1, that is if function \( x^{2\gamma-1} \) is convex, and more if it is concave. Thus, the young’s share is less variable when either the coefficient of relative risk aversion \( \gamma \) is larger than 1 (as expected from the corollary) or is less than 1/2. In the case of \( \gamma \) smaller than 1, we know that there are two effects in opposite directions. For \( 1/2 < \gamma < 1 \) the effect due to an increase in share levels that decreases risk aversion dominates. To sum up, moral hazard has the following impact on the share of group 1:

- if \( \gamma < 1/2 \), it increases the share and lowers its variation with resources
- if \( 1/2 < \gamma < 1 \), it increases the share and its variation
- if \( \gamma > 1 \), moral hazard decreases both the share of group 1 and its variation.

\(^\text{17}\)Expression (24) takes the simpler form

\[
\rho_h = E[-\frac{u''}{u_h} \frac{1}{u_h^2}] / E[\frac{1}{u_h}^2]
\]

that is easy to obtain. The incentives constraint and the feasibility constraint fully determine how the optimal contract within the group is adjusted in function of \( R \): Keeping the effort level constant implies that \( u_h(c_h)dc_h \) is constant hence equal to \( dR \).
6 Concluding remarks

We have shown that moral hazard may significantly distort the sharing of aggregate risks. In particular, optimality may lead to contracts that have undesirable properties: individuals may suffer from an increase of their own outputs.

We conclude by noting that it is not moral hazard per se that generates these difficulties, but rather the timing at which effort is exerted. To see this, assume an “investment” effort that has to be exerted before the state is revealed. Contracts are still contingent on both $s$ and $\theta$, and subject to feasibility constraint (5). An optimal effort level, $e_h$, satisfies

$$E_\pi U_h(c_h, e_h|s) \geq E_\pi U_h(c_h, e'|s) \text{ each } e' \in \mathcal{E}_h,$$

in which $E_\pi$ is the expectation over all states under probability $\pi$.

Optimal conditions are easily adapted. Let $q(\theta|e)$ be the prior (before knowing $s$) of reaching status $\theta$ given the effort $e$, that is $q(\theta|e) = \int p(\theta, e|s) d\pi(s)$. At an optimal scheme, there are $\beta_h \geq 0$ and $\alpha(s) > 0$ such that for each $h$, each $\theta$ in $\Theta_h$:

$$\frac{1}{\alpha(s)} = \frac{1}{u'_h(c_h(\theta|s))} - \frac{\beta_h}{\alpha(s)} \frac{q'_{eh}(\theta, e_h)}{q_h(\theta, e_h)}$$

When effort is positive, the “price” of effort ($\beta_h$) is positive as well, which implies that consumption levels depend on status. Thus, hidden effort may still prevent full insurance against idiosyncratic shocks. The key difference is that the effort level and the price of effort do not vary with the state. As a result, optimal consumption levels, which are solutions of (19), are continuous with respect to the state. Furthermore they all vary in the same direction, either up or down, for each status and group.\footnote{If the optimal effort $e$ hence $\beta$ is null this is obvious. Otherwise, rewrite (19) as $u'_h(c_h(\theta|s))[1 + \beta_h \frac{q'_{eh}(\theta, e_h)}{q_h(\theta, e_h)}] = \alpha(s)$. Surely, $e_h$ and $\beta_h$ are such that $[1 + \frac{q'_{eh}(\theta, e_h)}{q_h(\theta, e_h)}]$ for each $\theta$. The consumption level solution to (19) as a function of $\alpha$ is decreasing, for each $\theta$ and each $h$. Hence, given $s$ the net surplus $\sum_h E[\omega_h - c_h]$ increases with $\alpha$ (since effort levels are constant): There is a unique value $\alpha(s)$ for which the feasibility constraint is satisfied as an equality. This implies that the price $\alpha(s)$ varies continuously with the state, hence also individuals’ consumption levels.}

References


7 Proofs

Proof of Proposition 1  Drop index s. Denoting by α the multiplier
associated with the feasibility constraint (5), the Lagrangean of the problem writes:

$$\sum_h \lambda_h \left[ E_u \theta \bar{c}_h - ke_h \right] + \alpha \left( \sum_h \sum_{\theta \in \Theta} p_h(\theta, e) [\omega_h - c_h(\theta)] \right).$$

The maximization with respect to consumption straightforwardly gives the first order conditions as given by (10), hence the sharing rules. Since $\bar{c}_h$ is certain $Eu_h(\tilde{c}_h)$ and $E\tilde{c}_h$ do not depend on $e_h$. Hence the optimal level of effort $e_h$ maximizes

$$F(\alpha, \omega_h, e) = \alpha \sum_{\theta} p_h(\theta, e_h) \omega_h(\theta) - k\lambda_h e_h. \quad (20)$$

Let us consider continuity and monotonicity properties. Under the assumptions, the problem is concave in $c$ and $e$, hence the solution is continuous. Let us consider a uniform change in $h$’s output, $\Delta \omega_h$. This will induce a change in $\alpha$ and effort. To simplify, let us consider only the case where $e_h$ is interior. The function $F$ given by (20) is twice differentiable in $e$ and concave thanks to CDC (Write $\sum_{\theta} p_h(\theta, e_h) \omega_h(\theta)$ as the sum $\sum_{\theta} P_h(\theta, e_h)[\omega_h(\theta_j) - \omega_h(\theta_{j-1})]$ where $P$ is the cumulative probability. Under CDC it is a sum of convex functions multiplied by negative terms since resources are increasing in status). The first order condition on effort, which is sufficient, is $F_e(\alpha, \omega_h, e) = \alpha \sum_{\theta} p^\prime_{he}(\theta, e_h) \omega_h(\theta) - k\lambda_h = 0$. By the implicit function theorem, the marginal change in $\alpha$ and effort satisfy

$$\sum_{\theta} p^\prime_{he}(\theta, e_h) \omega_h(\theta) \Delta \alpha + \alpha F_{ee} \Delta e_h + \alpha \sum_{\theta} p^\prime_{he}(\theta, e_h) \Delta \omega_h(\theta) = 0.$$  

Since $\Delta \omega_h(\theta)$ is independent of $\theta$ the last term is null. Also thanks to MLR the term multiplying $\Delta \alpha$ is positive and $F_{ee}$ is nonpositive by concavity of $F$. Hence $\Delta e_h$ and $\Delta \alpha$ are of the same sign: if the price of the feasibility constraint decreases both groups exert less effort.

To assess the impact on consumption and welfare, we distinguish two cases depending on whether one group is risk neutral or not. In the first case, the price $\alpha$ does not change, hence effort levels and the consumption of the risk averse group do not change either. The risk neutral group benefits entirely of the increase in outputs. If both groups are risk averse, both shares strictly increase when the price of the feasibility constraint decreases by
Borch condition. This means that individuals consume more and exert less effort: they are all better off (also even though individuals exert less effort thanks to the good shock, the impact does not offset the exogenous increase: overall resources increase). In the opposite case where $\Delta \alpha$ is positive, both would be worse off. It suffices now to notice that by the envelope theorem the marginal change of the weighted sum of the utilities is equal to $\alpha (\Delta \omega_1 + \Delta \omega_2)$. Hence if the exogenous change in output is positive, each group is better off (and if it is negative, each group is worse off).

**Proof of Lemma 1.** The proof follows similar arguments as for a principal agent model. Index $s$ and $h$ are dropped to simplify notation. Thanks to the assumptions, problem $P(R)$ writes as max $U(\tilde{c}, e)$ over $(\tilde{c} = c(\theta))$ and $e$ that satisfy

$$\sum_{\theta} p(\theta, e)[c - \omega](\theta) \leq R \quad \text{(21)}$$

and

$$\sum_{\theta} p'_e(\theta, e)u(c(\theta)) - k = \begin{cases} 
\leq 0 & \text{if } e = 0 \\
0 & \text{if } 0 < e < e_{\text{max}} \\
great 0 & \text{for } e = e_{\text{max}} 
\end{cases} \quad \text{(22)}$$

in which (21) is the feasibility constraint and (22) the first order condition of the incentives constraint. Denote by $\alpha$ and $\beta$ the multipliers associated with (21) and (22). The lagrangean of the problem writes:

$$L = U + \alpha (R + \sum_{\theta} p(\theta, e)[c - \omega](\theta)) + \beta (\sum_{\theta} p'_e(\theta, e)u(c(\theta)) - k).$$

An optimal allocation satisfies the first order conditions on consumption $c(\theta)$ for each $\theta$ and on $e$. The conditions on $c(\theta)$, $p(\theta, e)u'(c(\theta)) - \alpha p(\theta, e) + \beta p'_e(\theta, e)u(c(\theta)) = 0$, can be written as:

$$1 - \frac{\alpha}{u'(c(\theta))} + \beta \frac{p'_e(\theta, e)}{p(\theta, e)} = 0. \quad (13)$$

By the envelope theorem $\alpha$ is equal to the marginal utility for revenue. Since $\sum_{\theta} p(\theta, e) = 1$, one has $\sum_{\theta} p'_e(\theta, e) = 0$. Multiplying (13) by $p(\theta, e)$ and summing over $\theta$ gives (14):

$$\frac{1}{V'(R)} = \frac{1}{\alpha} = E_{\theta}[\frac{1}{u'(c(\theta))}].$$
The first order condition with respect to $e$ is that the derivative:
\[
L_e = \left[ \sum_{\theta} p'_e(\theta, e)u(c(\theta)) - k \right] + \alpha \left[ \sum_{\theta} p'_e(\theta, e)(\omega - c(\theta)) \right] + \beta \left[ \sum_{\theta} p''_{ee}(\theta, e)u(c(\theta)) \right]
\]
is $\leq 0$ if $e = 0$, null if $0 < e < e^{max}$ and $\geq 0$ if $e = e^{max}$.

The multiplier $\beta$ is surely nonnegative since $-\beta$ is the derivative of the value function with respect to the cost of effort $k$. Two types of consumption plans are obtained depending on whether the price of the incentives constraint, $\beta$, is nul or positive. In the case where $\beta = 0$, consumption levels are equalized across individual states and effort is minimal: $c(\theta)$ is constant equal to $c$ by (13) and $\sum_{\theta} p'_e(\theta, e)u(c) = 0$ (because $\sum_{\theta} p'_e(\theta, e) = 0$) implies $e = 0$ by (22). Also since $\sum_{\theta} p'_e(\theta, e)c = 0$, $L_e \leq 0$ gives $-k + \alpha [\sum_{\theta} p'_e(\theta, 0)\omega(\theta)] \leq 0$. In the second case where $\beta > 0$, the level of effort is not minimal. Furthermore, the incentive constraint (22) is binding: $\sum_{\theta} p'_e(\theta, e)u(c) = k$. The first term in $L_e$ is null, which gives that the value of $\beta$ satisfies:
\[
\alpha [\sum_{\theta} p'_e(\theta, e)(\omega - c(\theta))] + \beta [\sum_{\theta} p_{ee}(\theta, e)u(c(\theta))] \geq 0 \text{ with } = \text{ if } e < e^{max}.
\]

The fact that (22) is satisfied as an equality if $e = e^{max}$ is a standard result in incentives theory: if a contract is designed so as to incite the agent to provide the maximal level of effort, there is no gain in inciting him more by making the contract more risky.

**Proof of Proposition 3.** Let us rewrite the optimality condition (15):
\[
\frac{1}{u'_1(c_1(\theta|s))} \left[ \frac{1}{E_\theta[1]} \right] = \frac{1}{u'_2(c_2(\theta|s))} \left[ \frac{1}{E_\theta[-]} \right].
\]
Assume $1/u'_1$ to be convex. Then
\[
E_\theta[\frac{1}{u'_1(c_1(\theta|s))}] \geq \frac{1}{u'_1(E_\theta[c_1(\theta|s)] \right]} = \frac{1}{u'_1(C_1(s))}.
\]

Note also that if the group is not subject to moral hazard in state $s$, the just above inequality holds (as an equality), independently of the convexity of $1/u'_1$. Assuming $1/u'_2$ is concave or group 2 is not subject to moral hazard, a similar argument yields
\[
E_\theta[\frac{1}{u'_2(c_2(\theta, s))}] \leq \frac{1}{u'_2(C_2(s))}.
\]
Thus the optimality condition (15) implies \( \lambda_1 u'_1(C_1(s)) \geq \lambda_2 u'_2(C_2(s)) \). Now letting \( \Omega(s) = C_1(s) + C_2(s) \), by definition of the sharing rules \( S_1(\Omega(s)) + S_2(\Omega(s)) = \Omega(s) \). Assume by contradiction \( S_1(\Omega(s)) < C_1(s) \). Surely, \( S_2(\Omega(s)) > C_2(s) \) and the successive inequalities hold:

\[
\lambda_1 u'_1(S_1(\Omega(s))) > \lambda_1 u'_1(C_1(s)) \geq \lambda_2 u'_2(C_2(s)) > \lambda_2 u'_2(S_2(\Omega(s))),
\]

which contradicts \( \lambda_1 u'_1(S_1(\Omega(s))) = \lambda_2 u'_2(S_2(\Omega(s))) \), the Borch conditions (10). The cases of strict concavity or convexity are treated similarly so as to obtain strict inequalities.

**Proof of Proposition 4.** Let the distribution of \((1/u'_1)(\tilde{c}_1)\) given state \(s\) be strictly dominated by that in state \(s'\). This implies that for any concave function \(f\):

\[
E[f(1/u'_1)(\tilde{c}_1)|s] \leq E[f(1/u'_1)(\tilde{c}_1)|s'] \leq f(1/\alpha).
\]

If \(1/u'_1\) is convex, then its inverse is concave. Hence applying the inequality to \(f = (1/u'_1)^{-1}\) gives \(E[\tilde{c}_1|s] \leq E[\tilde{c}_1|s'] \leq S_1\), the desired result. Similarly the inverse of \((1/u'_1)(u^{-1})\) is \(u_1(1/u'_1)^{-1}\). Hence if the former is convex, one gets \(E[u_1(\tilde{c}_1)|s] \leq E[u_1(\tilde{c}_1)|s'] \leq u(S_1)\).

**Proof of Proposition 5.** We consider here states that differ only with respect to resources and for which the level of effort in each group is constant. Thus the probability of status is constant as well. We first show that the share of aggregate income for each group is related to the risk aversion coefficient of the group’s utility for income.

Recall that at an optimal scheme the conditions

\[
\lambda_1 \frac{\partial V_1}{\partial R_1}(R_1|s) = \lambda_2 \frac{\partial V_2}{\partial R_2}(R_2|s) = \alpha \text{ with } C_1 + C_2 = \Omega(s)(15)
\]

hold where \(V_h\) is the value of \(P_h(R_h|s)\). Consider marginal variations of \(R_h\) and of the resources for which the level of effort is kept constant. The induced marginal variation of \(\frac{\partial V_h}{\partial R_h}(R_h|s)\) satisfies

\[
\sum \frac{\partial^2 V_h}{\partial \omega_h(\theta) \partial R}(R_h|s) d\omega_h(\theta) + \frac{\partial^2 V_h}{\partial R^2}(R_h|s) dR_h = \frac{\partial^2 V_h}{\partial R^2}(R_h|s) dC_h. \quad (23)
\]
To see this, note that the envelope theorem applied to $P_h(R_h|s)$ gives

$$\frac{\partial V_h}{\partial \omega_h(\theta)}(R_h|s) = p_h(\theta, e_h(s)) \frac{\partial V_h}{\partial R_h}(R_h|s),$$

which yields (23). Thus, differentiation of (15) gives

$$\lambda_h \frac{\partial^2 V_h}{\partial^2 R_h}(R_h|s) dC_h = d\alpha$$

with $dC_1 + dC_2 = d\Omega$.

Using (15) to eliminate the value of $\lambda_h$ yields:

$$dC_h = \frac{1}{\rho_h} \frac{-d\alpha}{\alpha}$$

where $\rho_h = -\frac{\partial^2 V_h}{\partial^2 R_h} \frac{\partial V_h}{\partial R_h}(R_h|s)$.

and finally thanks to the feasibility constraint

$$dC_1 = \frac{\rho_2}{\rho_1 + \rho_2} d\Omega, \; dC_2 = \frac{\rho_1}{\rho_1 + \rho_2} d\Omega$$

It remains to evaluate $\rho_h$. To simplify notation, index $h$ is dropped. The contracts $c$ for which a given positive level of effort $e$ is optimal are characterized by $(\alpha, \beta)$ that satisfy the first order conditions on consumptions (13) and on incentives (8). Let us denote by $C$ the set of values $(c, \alpha, \beta)$ that satisfy these constraints. For each element in $C$, values for $R$ and $\omega$ for which this contract together with the choice $e$ is indeed optimal can be derived: they satisfy the feasibility constraint (12) and the optimality condition on $e$ (20). The set $C$ is parameterized by $\alpha$, the marginal utility for income: given $\alpha$, there is a unique contract and $\beta$ that satisfy both (13) and (8).

From (24), the variations of $dC_h$ in function of variation of $d\alpha$ along this set gives $\rho_h$.

Without effort, we fall back on Wilson’s formula with $V(R)$ equal to $u(C)$. Assume $e > 0$ so that (8) is satisfied as an equality. Consider marginal variation of contingent income ($dc(\theta)$), and multipliers $d\alpha$ and $d\beta$. Differentiating the constraints (8) and (13) give (using $de = 0$)

$$-\frac{u''}{u} dc(\theta) = \frac{-d\alpha}{\alpha} + \frac{p'_e}{p} \frac{d\beta}{1 + \beta p'_e/p} = \frac{-d\alpha}{\alpha} + \frac{p'_e d\beta}{p \alpha} u' \text{ each } \theta$$

and

$$\sum_{\theta} p'_e u'(c(\theta)) dc(\theta) = 0$$
Multiplying each equation (25) by \(-pu'/u''\) and summing over \(\theta\) gives the expression of \(dC = E[dc]\) in function of \(d\alpha\) and \(d\beta\)

\[
dC = \frac{-d\alpha}{\alpha} [E_\theta \frac{u'}{u''}] + \frac{d\beta}{\alpha} \left( \sum_\theta p'_e \frac{u''^2}{-u''} \right)
\]  

(27)

The value of \(d\beta\) as a function of \(d\alpha\) can be found by plugging the expression of \(dc\) given by (25) into (26):

\[
\left( \sum_\theta \frac{u'^3}{-u''} \frac{p'_e}{p} \right) d\beta = \left( \sum_\theta p'_e \frac{u''^2}{-u''} \right) d\alpha
\]  

(28)

and finally

\[
dC = \frac{-d\alpha}{\alpha} [E_\theta \frac{u'}{u''}] - \frac{\left( \sum_\theta p'_e \frac{u''^2}{-u''} \right)^2}{\sum_\theta \frac{u'^2}{-u''} \frac{p'^2}{p}}
\]

Comparison with (24) gives that the value of \(\rho\) is equal to the term in bracket, which proves (16) for appropriate \(b\).