Earnings Persistence, Homeownership Profile, Residential Mobility and Increasing Mortgage Debt*

Filippo Scoccianti †

Universidad Carlos III, Madrid

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Abstract

In between 1983 to 2004, (i) U.S. average consumer debt, mostly collateral backed, has increased from 124 percent to 206 percent of average labor income; (ii) cross-sectional earnings inequality, as measured by the Gini index, has increased; (iii) housing downpayment requirements have decreased; (iv) the homeownership rate among the 20-35 and 60-75 age groups have increased; (v) and residential mobility rates have decreased. I link those facts through a general equilibrium, overlapping-generations model with a liquid financial asset and both housing and renting markets. Households access debt through collateralized lending. I show that if permanent shocks are the driving force behind earnings inequality then the precautionary saving motive is weak and households are more often at the borrowing limit. Relaxing the collateral constraints then, increases households’ access to mortgage debt and their homeownership rate. High earnings persistence also reduces the need to adjust the housing stock to self-insure against variability in non-durable consumption, thus lowering housing mobility rates.


Keywords: Mortgage Debt, Housing, Heterogeneous Agents, Life-Cycle.

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†Filippo Scoccianti, Department of Economics, Universidad Carlos III de Madrid; fscoccia@eco.uc3m.es.
1 Introduction

In the period going from 1983 to 2004, (i) U.S. average consumer debt, mostly collateral backed\(^1\), has increased from 124 percent to 206 percent of average labor income. This increase in consumer debt has triggered a debate about its determinants. Iacoviello (2007) argues that the reason is the (ii) higher income risk of US consumers today. According to Campbell and Hercowitz (2006), the increase in debt is due to (iii) changes in financial regulation that have facilitated the use of durables as collateral. Over the same time period, two others profound changes in the housing markets have been widely reported: (iv) an increase in the homeownership rate among the 20-35 and 60-75 age groups; (v) and a decrease in households residential mobility rates. To date, no model has studied the evolution of U.S. households debt in a general equilibrium setting by simultaneously restricting the model’s prediction along the life-cycle dimension. This is what this paper does, by building a general equilibrium, overlapping generations model with both housing, renting and non-durable consumption where households are subject to idiosyncratic earnings risk and can self-insure using housing and financial assets. I find that if earnings inequality is mainly driven by differences in permanent income among agents, and with collateral constraints that are exogenous and calibrated to match the observed decrease in equity to loan requirements for housing purchases, the model can account for 75 percent of the increase in consumer debt. This increase goes hand in hand, as in the data, with an increase in the homeownership rate of young and retired households and a decrease in residential mobility rates along the whole life-cycle. The intuition for the result is as follows. With high earnings persistency, young households can forecast their rising life-cycle earnings profile and are thus willing to borrow against their future earnings. Earnings-rich households take then advantage of looser equity to loans requirements to increase their access to homeownership when young, and stabilize, when middle-aged, their housing stock. Earnings stability effectively increases households’ ability to smooth housing consumption without compromising on non-durable consumption. In the model, residential mobility rates are defined as changes in housing assets by households. The model can account for their decrease along the life cycle mainly for two reasons: on the one hand young households reach their optimal housing stock faster, thus their need to trade in the housing market to climb up the property ladder is reduced; on the other hand the persistency of the earnings shock coupled with adjustment costs on housing

\(^1\)According to the Survey of Consumer Finances, homes and vehicles collateralized almost 90 percent of US households’ debt in 2001, see Campbell and Hercowitz (2005).
transactions force middle-aged and retired households to remain more often in the inactive region of their optimal non-convex housing policy function. This last fact also accounts for the lack of decrease in the homeownership rate by households older than 65 year-old. If the 2004 earnings process is not sufficiently persistent, then the observed increase in cross-sectional earnings inequality must come from higher variability in the unpredictable part of earnings. Higher shocks volatility increases precautionary savings: households constitute a buffer stock of savings to insure against volatility in non-durable consumption. As a consequence, the fraction of agents at the borrowing limit decreases and with it, average debt, regardless of how lax collateral constraints are. Housing mobility rises as agents try to smooth non-durable consumption by liquidating or investing in the housing market in response to more variable shocks. The life-cycle homeownership dynamic shows a decrease in homeownership by the young and the retirees, who need to self-insure the high frequency of earnings shocks by trading in the risk-free financial asset. While earnings inequality in the 2004-economy has to be primarily attributed to the persistency of the earnings shocks for the result to hold, the 1983-economy’s earnings process is left unrestricted. In particular, the average debt level in 1983 is unaffected by the combination of persistency and earnings volatility one is willing to assume to replicate the 1983 Gini earnings. As far as downpayment requirements are sufficiently tight, households optimally react to higher earnings persistence by hitting more often the collateral constraints: average debt remains, however, unchanged. I see this result as interesting since it is still an open issue which part of earnings, predictable and unpredictable, has increased more since the 1980s (see Cunha and Heckman (2007)). Beyond households debt, the model is succesful in replicating the total wealth distribution and its two components: financial and housing assets. The lower 99 percent of the wealth distribution is replicated well in both the 1983 and the 2004-economy. I am able to approximate the dispersion in the housing wealth distribution in both years. In the 2004-economy the housing wealth distribution is replicated almost exactly, including the top 1 percent.

This paper is related to Díaz and Luengo-Prado (2006) and Gruber and Martin (2005), who study the role an illiquid durable consumption good plays in determining the level of precautionary savings and the distribution of wealth in a model with heterogeneous agents, idiosyncratic uncertainty, and exogenous borrowing constraints. The overall effect of introducing durable goods on wealth inequality is modest. They find that transactions costs induce an inaction region over which the durable stock and the associated user cost are not adjusted in response to changes in income. This property of the optimal housing policy function - validated in the broader context of durables
goods by Bertola et al. (2005) - is exploited to study mobility rates. This paper is also connected to the literature on consumers debt and earnings instability. Krueger and Perri (2005) consider an economy with complete markets, a financial asset and endogenous solvency constraints: they focus on non-collateralized debt. The authors prove that an increase in earnings instability cause agents to access more often debt markets for consumption smoothing purposes; the result doesn’t hold in an incomplete market model, where the precautionary savings motive makes agents access less often credit markets. Iacoviello (2007) builds an infinite-horizon heterogeneous agents model with collateral backed consumers’ debt. He finds that the rise in debt since the 1980s reflects the increased access of households to the credit market in order to smooth consumption in the face of more volatile incomes. The author assumes that an exogenous fraction of agents is borrowing constrained: I show that treating this share endogenously may alter the results and it is crucial to understand the effect of earnings persistency versus earnings unpredictability on households behavior. Hintermaier and Koeniger (2006) construct an infinite-horizon model with durable and non-durable consumption. They find that neither the increase in earnings volatility, nor the decrease in average downpayment requirements can replicate the increase in households debt: only a decrease in the real interest rate can explain the observed patterns. Their model differs from the one presented here because it considers infinitely-lived households and debt is collateralized by durable goods. The authors specify an endogenous collateral constraints that is different from the exogenous-one used here. I will then discuss in some detail the effect of alternative borrowing constraints specifications. Campbell and Hercowitz (2006) study the impact of lower downpayment requirements in a model with trades between patient savers and impatient borrowers with equity requirements typical of collateralized loan contracts. The authors focus on the transitional dynamics for the decade of the 1980s. They find that lower downpayment requirements increase access to housing equity, increasing by the same token average debt and the equilibrium real interest rate. Since they posit heterogeneous discount factors for savers and borrowers, their result doesn’t arise endogenously as a response to increasing earnings inequality.

Finally, Barnes and Young (2003) use a calibrated partial equilibrium overlapping generations model, and find that income growth expectations and changing demographics can account for the increase in the debt to income ratio from the 60 to 80’s. However, their model is unable to account for rising indebtedness during the 1980s when high interest rates, lower income growth and an ageing population would have tended to reduce aggregate borrowing. I will show that an increase in the share of the 35-55 age-group at the expenses of the 20-35 age group in total population decreases both the
fraction of households into debt as well as the mass of borrowing constrained households. This two changes have opposite effects on the debt to income ratio, and will be shown to balance out in equilibrium. Hence in the model, a demographic change of the magnitude observed in the U.S. over the last 20 years is shown to have not affected the average debt to income ratio but only the fraction of agents into debt. The rest of the paper is organized as follows. Section 2 gives a overview of data patterns. Section 3 presents the model and Section 4 provides the equilibrium definition. Section 5 explains the calibration of the model parameters. In Section 6 the simulation results are presented and explained. Section 7 concludes.

2 Patterns in the data

2.1 Consumers Debt

U.S. consumer debt has increased substantially during the last decades. The average consumer debt, in the Survey of Consumer Finances 1983 and 2004, has increased from 124% to 206% of average labor income\(^2\). Most of consumer debt, around 90 percent, is collateral backed. The figure below illustrates the boom in home mortgage debt that has been taking place since the beginning of the 1980s\(^3\).

\(^2\)I follow Hintermaier and Koeniger (2006) who normalize by labor earnings and consider only the net financial position of households.

\(^3\)Data are from the Flows of Funds Accounts of the United States.
2.2 Earnings Inequality

The 1983-2004 period has also witnessed a sharp increase in cross-sectional earnings inequality, with an earnings Gini coefficient (SCF data) that have increased from 0.533 in 1983 to 0.60 in 2004. The pattern of increasing earnings inequality has been documented, among others, by Heathcote, Storesletten and Violante (2004), and Krueger and Perri (2005). A consensus has not yet been reached on the share of this increase that is due to greater heterogeneity in the components of earnings that are predictable by agents and the share of the increase that is due to greater uncertainty faced by agents: economists generally agree on the fact that both components have increased. Cunha and Heckman (2007) find that the increase in uncertainty is substantially greater for unskilled workers. For less skilled workers, roughly 60% of the increase in wage variability is due to uncertainty. For more skilled workers, only 8% of the increase in wage variability is due to uncertainty.
2.3 Homeownership Dynamics

Data from the U.S. Bureau of Census show that the aggregate homeownership rate increased from 64.6% in 1983 to 69% in 2004: this increase is mirrored by the increase in the percentage of people with a negative net-financial assets position, which was 40% in 1983 and 43% in 2004. It is instructive to look at the dynamics of homeownership rates along the life-cycle: households under 30 and over 55 year-old are mainly responsible for the overall increase in the homeownership rate. The pick in homeownership is reached at age 55 in 1983; by 2004 the pick is reached at age 70.

![Homeownership Profiles](image)

Disaggregating the homeownership profile by households type, it appears that while the increase is common to all households, single person households, especially female headed, show a comparatively bigger increase.

2.4 Financial Deregulation

Financial markets deregulation that started in the early 1980’s has eased liquidity constraints on households, as reflected by the trend of diminishing average downpayment requirements, reported by Campbell and Hercowitz (2006). While the average equity-value ratio for newly purchased homes in 1983 was 22.6, this ratio has decreased to
16.4 by 2001. As Campbell and Hercowitz (2006) points out, the practice of mortgage refinancings, which by 2001 affected 44.4 percent of first home purchasers against 9.9 percent in 1983, has made it possible to issue second or third mortgages after the first house purchase. This probably means that the reported 16.4 equity-value ratio in 2001 is overestimating the actual average downpayment. According to a survey from the National Association of Realtors, in 2004, 43 percent of first-time buyers financed 100 percent of their homes, up from 28 percent in 2002. Less than two in ten repeat home buyers had no-down-payment loans. The median down payment for first-time buyers was just 2 percent. For all buyers, it was 13 percent.

2.5 Housing Mobility Rates

Wolf and Longino (2005), using data from the U.S. Current Population Survey, report that housing mobility rates have declined over the past four decades. Total moving rates for people ages 30 to 44 have dropped from 20.3 percent in 1953 to 15.5 percent in 2003. Only 7.7 percent of U.S. residents between ages 45 and 64 moved in 2003, compared with 10.5 percent of this group 40 years earlier. And mobility has even declined among young U.S. residents, traditionally considered the most likely group to change residence. Wolf and Longino (2005) calculate that total moving rates for people between the ages of 20 and 29 have declined from 37 percent in 1953 to 29 percent today. Interstate moves among the young also declined during that period, from about 8 percent to about 5 percent. According to the U.S. Census Bureau, among people who changed residence between 2002 and 2003, most (51 percent) moved for housing-related reasons, then family reasons (26 percent) and work-related reasons (16 percent).

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Figure 3: Mobility Rates, from Wolf and Longino (2005)
Moreover, the Census Bureau has collected moving statistics in the decennial census since 1960. Recent movers include those who moved in the 15 months prior to the decennial census. Owner-occupied housing units movers have declined from 11.8% in the 1980 census to 10.3% in the 2000 census.

2.6 Wealth Distribution

Hintermaier and Koeniger (2007), using the Survey of Consumer Finances, find that the wealth distribution only changed slightly during the 1983–2004 period: the distribution in 2004 has a bit more probability mass at the bottom and top of the support. The authors show that this change in the distribution of wealth does not pass conventional tests of statistical significance.

3 The Model

3.1 Environment

I consider an overlapping generations model in which agents face uninsurable idiosyncratic earnings shocks and uncertain life spans. When agents die, transmit after tax liquidated assets and the first earning shock to their immediate successor. I explicitly model housing. Housing has a dual role in the model: it directly provides utility and can be used as a collateral for borrowing. While housing can be at least characterized by tenure, location, size and quality, I consider tenure and size directly, location is indirectly considered through adjustment costs on housing transaction, quality is left out of the picture. Several frictions are present in the model: lack of annuity markets to insure against uncertain lifespan, different specifications of borrowing constraints, transaction costs for trading in housing stock as well as a minimum house purchasing size. The last two features make it possible to talk with sufficient realism of houses, their implied cost being that the resulting non-convexities will complicate the computational task.

3.2 Demographics

There is a continuum of individuals of measure one at each point in time. Each individual lives at most $J$ periods. In each period $j \leq J$ of his life the conditional probability of surviving and living in period $j + 1$ is denoted by $\alpha_j \in (0, 1)$. Define $\alpha_0 = 1$ and $\alpha_J = 0$. The probability of survival, assumed to be equal across individuals of the same cohort,

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*I will discuss in the conclusion the implications of not considering a quality ladder in housing.
is beyond the control of the individual and independent of other characteristics of the individual (such as income or wealth). I assume that \( \alpha_j \) is not only the probability of survival for a particular individual, but also the (deterministic) fraction of agents that, having survived until age \( j \), will survive to age \( j + 1 \). Annuity markets are assumed to be absent. After death, the individual is replaced by a descendant who inherits its after-tax financial and (liquidated) housing wealth, and part of its permanent productivity according to a stochastic earnings transmission markov matrix. In each period a number

\[
\mu_1 = \left(1 + \sum_{j=1}^{J-1} \prod_{i=1}^{j} \alpha_i \right)^{-1}
\]

of newborns enter the economy, and the fraction of people in the economy of age \( j \) is defined recursively as \( \mu_{j+1} = \alpha_j \mu_j \), with \( \mu_{J+1} = \alpha_J = 0 \). Let \( J = \{0, 1, ..., J\} \) denotes the set of possible ages of an individual.

### 3.3 Technology

#### 3.3.1 The Firm’s Problem

There is one good produced according to the aggregate production function \( F(K_t, L_t) \) where \( K_t \) is the aggregate capital stock and \( L_t \) is the aggregate labor input. I assume that \( F \) is strictly increasing in both inputs, strictly concave, has decreasing marginal products which obey the Inada conditions and is homogeneous of degree one. As usual with constant returns to scale production technologies, in equilibrium the number of firms is indeterminate and without loss of generality I assume that there is a single representative firm. The representative firm solves the following static problem

\[
\max_{K_t, L_t} F(K_t, L_t) - (r + \delta^h)K_t - wL_t
\]

where \( r \) is the rental price of capital net of depreciation and \( w \) is the wage per efficiency unit of labor.

#### 3.3.2 The Financial Institution’s Problem

There is a representative financial institution that in each period receives deposits \( A' \) from households, rents residential services \( F \) to households and rents capital \( K \) to the representative firm. I allow rental units to have a different depreciation rate \( \delta^f \) than owner occupied housing \( \delta^h \). The perfectly competitive financial institution solves the following problem
\[
\Psi(A) = \max_{A', K, F} \left\{ A' - (1+r)A + rK + (i - \delta f)F + \frac{1}{1+r}\Psi(A') \right\}
\]

s.t.

\[K + F \leq A\]

where \(F\) is the stock of rental units and \(i\) is their rental price. The financial institution rents capital and houses in the same period in which it acquires them.

### 3.4 Preferences and Endowments

Individuals are endowed with one unit of time in each period that they supply inelastically in the labor market. Individuals differ in their labor productivity due to differences in age and realizations of idiosyncratic uncertainty. The labor productivity of an individual of age \(j\) is given by \(\varepsilon_j\eta\), where \(\{\varepsilon_j\}_{j=1}^{J}\) denotes the age profile of average labor productivity. The stochastic component of labor productivity, \(\eta\), follows a finite state Markov chain with state space \(\eta \in E = \{\eta_1, \ldots, \eta_N\}\) and transition probabilities given by the matrix \(\pi(\eta' | \eta)\). Let \(\Pi\) denote the unique invariant measure associated with \(\pi\). I assume that all agents, independent of age and other characteristics face the same Markov transition probabilities and that the fraction of the population experiencing a transition from \(\eta\) to \(\eta'\) is also given by \(\pi\). This law of large numbers and the model demographic structure assure that the aggregate labor input is constant. As with lifetime uncertainty I assume that individuals cannot insure against idiosyncratic labor productivity by trading contingent claims. Moral hazard problems may be invoked to justify the absence of these markets. After its death the individual is replaced by a direct descendant who inherits its after-tax financial and (liquidated) housing wealth, if any, and receive its first idiosyncratic shock according to the intergenerational earnings transmission matrix \(\Gamma\) which shares the same states \(\eta \in E = \{\eta_1, \ldots, \eta_N\}\) of the stochastic component of labor productivity. Bequests are accidental in that parents derive no utility from them.

Individuals derive utility from consumption of the nondurable good, \(c\), and from the housing services acquired either through the rental market, \(g(f)\), or through homeownership \(g(h')\). Housing services are a function \(g(\cdot)\) of the housing stock purchased or rented. The choice between homeownership and renting is exclusive at each period, and represented by the indicator function \(I \in \{0,1\}\). Individuals value streams of consumption and housing/renting services \(\{c_j, g(s)_j\}_{j=1}^{J}\), where \(s = (1-I)f + Ih'\), according to
\[
E_0 \left\{ \sum_{j=1}^{J} \beta^{j-1} u(c_j, g(s)_j) \right\}
\]

where \( \beta \) is the time discount factor and \( E_0 \) is the expectation operator, conditional on information available at time 0. The per period utility function \( u(c, g(s)) \) is assumed to be strictly increasing in both arguments and obeying the Inada conditions with respect to nondurable consumption. The instantaneous utility from being dead is normalized to zero and expectations are taken with respect to the stochastic processes governing survival and labor productivity. I assume that the per period utility function is of the CRRA form

\[
u(c, g(s)) = \left( \frac{c^{\gamma} g(s)^{1-\gamma}}{1 - \sigma} \right)^{1-\sigma} - 1
\]

where \( \sigma \) is the coefficient of relative risk aversion and \( \gamma \) is the elasticity of substitution between nondurable and housing services consumption.

### 3.5 Timing and Information

The timing of events in a given period is as follows. Households observe their idiosyncratic labor productivity shock \( \eta \) and, in their first life period, receive net transfers from bequests and their first period labor productivity shock \( \eta \) according to the intergenerational earnings transmission process. Then labor is supplied to the firm and financial assets are supplied to the financial institution. Capital is rented to the firm by the financial institution. Production takes place. Next households receive wages from the firm and interest on their deposits form the financial institution and choose nondurable consumption \( c \), housing \( h' \) or rental consumption \( f \) services and next period asset position \( a' \). A unit of rental housing \( f \) yields consumption services today. A unit of housing stock for tomorrow \( h' \) yields consumption services today. Finally uncertainty about early death is revealed.

### 4 Equilibrium

I will limit attention to stationary equilibria in which prices, wages and interest rates are constant across time. Individual are assumed to be price takers in the goods and factor markets they participate in. In each moment of time individuals are characterized by their position of assets and holdings of housing stock, as well as their age and labor...
productivity status \((a, h, \eta, j)\). Let by \(\Phi(a, h, \eta, j)\) denote the measure of agents of type \((a, h, \eta, j)\), constant in a stationary equilibrium. I normalize the price of the final good to equal one. The price of renting units is denoted by \(i \equiv r + \delta^I\), where \(\delta^I\) is the depreciation rate for renting units. Let \(r\) and \(w\) denote the interest rate and the wage rate per efficiency unit of labor, respectively. The consumer problem can now be formulated recursively as

\[
V(a, h, \eta, j) = \max_{c, a', h', I} u(c, s) + \beta V(a', h', \eta', j + 1)
\]

\[
V(a', h', \eta', j + 1) = \alpha_j \sum_{\eta'} \pi(\eta'|\eta) V(a', h', \eta', j + 1)
\]

\[
s.t.
\]

\[
c + i f + h' + \tau(h, h') + a' = (1 - \tau_l) w \eta_j + (1 + r) a + (1 - \delta^h) h + \Upsilon P
\]

\[
s = (1 - I) f + Ih'
\]

\[
a' \geq b(h', \eta, j)
\]

\[
a_1 = 0, b_1 = 0
\]

\[
\Upsilon = \begin{cases} 
1 & \text{if } j \geq 10 \\
0 & \text{otherwise}
\end{cases}
\]

\[
c \geq 0, h' \in \{0\} \cup [h^{\min}, h^{\max}], I \in \{0, 1\}
\]

Where \(P\) stands for households pension income (which is enjoyed from age 65 on, i.e. when \(j \geq 10\)) which is assumed to be independent of households income history\(^5\). I define \(h^{\min}\) as the minimum house purchasing size while \(\tau(h, h')\) stands for non-convex housing stock’s adjustment costs.

\(^5\)A more realistic assumption is that social security benefit is a concave function of the accumulated contributions. Under this assumption, the total contributions become an additional state variables, which increases the computation time dramatically.
\[ \tau(h, h') = \begin{cases} 
0 & \text{if } h' \in [(1 - \mu)h, (1 + \mu)h] \\
\rho_1 h + \rho_2 h' & \text{otherwise} 
\end{cases} \]

Borrowing constraints \( b(h', \eta, j) \) are specified as being an exogenous fixed fraction of owner-occupied housing services, where households can only borrow up to \((1 - \theta)\) of their desired housing stock

\[ a' \geq -(1 - \theta)h' \]

I am now ready to define a stationary equilibrium. Let \( \mathcal{J} \) and \( \mathcal{E} \) be the power sets of \( \mathcal{J} \) and \( \mathcal{E} \), respectively and \( \mathcal{B} \) be the borel sets of \( \mathbb{R} \). Let \( S = \mathbb{R} \times \mathbb{R} \times \mathcal{E} \times \mathcal{J} \) and \( \mathcal{S} = \mathcal{B} \times \mathcal{B} \times \mathcal{E} \times \mathcal{J} \) and \( M \) be the set of finite measures over the measurable space \((S, \mathcal{S})\).

**Definition 1** A stationary equilibrium is a value function \( V \), policy functions for the households, \((c, a', h', f, I)\), labor and capital demand for the representative firm, \((K, L)\), prices \((w, r)\), and a finite measure \( \Phi \in M \) such that

1. Given \((w, r)\), \( V \) solves the functional equation and \((c, a', h', f, I)\) are the associated policy functions

2. Input prices satisfy

\[ r = F_K(K, L) - \delta^k \]
\[ w = F_L(K, L) \]

3. Rental price is given by

\[ i = r + \delta^f \]
4. Markets clear

\[ \int \eta \, d\Phi = L \text{ (Labor Market)} \]
\[ \int a' \, d\Phi = A' \text{ (Financial Asset Market)} \]
\[ \int f \, d\Phi = F \text{ (Rental Market)} \]
\[ A - F = K \text{ (Capital Market)} \]
\[ \int c(a, h, \eta, j) \, d\Phi = C \text{ (Non-Durable Consumption)} \]
\[ \tau \int a'(a, h, \eta) \, d\Phi = G \text{ (Government Expenditure)} \]

\[ C + \delta K + \int (h' - (1 - \delta^h)h + \tau(h', h)) \, d\Phi + \delta^t F + G = F(K, L) \text{ (Goods Market)} \]

5. The measure \( \Phi \) is stationary and follows

\[ \Phi = T(\Phi) \]

where \( T \) is the law of motion generated by \( \pi \) and the policies \((c, a', h', f, I)\) as described below.

The operator \( T \) maps \( M \) into \( M \) in the following way. Define the transition function \( Q : (S, S) \rightarrow [0, 1] \) by:

**Definition 2**  For all \( S' = R' \times Z' \times E' \times J' \in \mathcal{B} \times \mathcal{B} \times \mathcal{E} \times \mathcal{J} \) and all \( s = (a, h, \eta, j) \in S \)

\[ Q(s, S') = \sum_{\eta' \in E'} \left\{ \begin{array}{ll}
\alpha_j \pi(\eta' | \eta) & \text{if } j + 1 \in J', \ a'(a, h, \eta, j) \in R', \ h'(a, h, \eta, j) \in Z' \\
0 & \text{else}
\end{array} \right\} \]

Then for all \( J' \in \mathcal{J} \) such that \( 0 \notin \mathcal{J} \) we have

\[ T(\Phi)(S') = \int Q(s, S') \, d\Phi \]
For $J' = 0$ we have

$$T(\Phi)(R' \times Z' \times E \times \{0\}) =$$

$$= \sum_{j=1}^{J} \left\{ \begin{array}{ll}
\sum_{\eta' \in \mathcal{E}'} (1 - \alpha_j) \Gamma(\eta' | \eta) & \text{if } j + 1 \in J', \ a'(a, h, \eta, j) \in R', \ h'(a, h, \eta, j) \in Z' \\
0 & \text{else}
\end{array} \right\}$$

Where $T(\Phi)(R' \times Z' \times E \times \{0\})$ describes the stationary distribution for the first generation of individuals, as implied by the net bequests and earnings shocks transmission matrix $\Gamma(\eta' | \eta)$.

5 Calibration

I choose the benchmark parametrization of the economy partly on the basis of microeconomic evidence and partly so that the stationary equilibrium for the economy matches selected long-run averages of US data.

5.1 Endowments

The study will take the form of a steady state comparison between two economies, one in 1983 and the other in 2004. I choose these two reference years because they allow a consistent comparison of households asset positions in the triennial Survey of Consumer Finances (SCF) dataset, as pointed out by Hintemerier and Koeniger (2003). In each period agents supply one unit of time, the productivity of which is given by $\epsilon_j$. The deterministic age profile of the unconditional mean of labor productivity $\{\epsilon_j\}_{j=1}^{J}$ is taken from Hansen (1993). I take $\epsilon_j = 0$ for $j \geq 10$, thus imposing mandatory retirement at the age of 65. The natural logarithm of the stochastic productivity process, $\eta$, is assumed to follow an AR(1) process in logharitm with persistence $\rho_\eta$ and variance $\sigma_\eta^2$, i.e. $\ln \eta_t = \rho_\eta \ln \eta_{t-1} + \mu_t, \mu_t \sim N(0, \sigma_\eta^2)$. I choose the persistency $\rho_\eta$ of the 2004 idiosyncratic earnings process to be 0.9989$^6$, following Storesletten et al. (2004)$^7$, who estimated it from PSID data. To mimic intergenerational earnings transmission, first period shocks are not drawn from the invariant shocks distribution. Instead, when parents die (time $t = t^d$), they transmit their earning shock to their direct successor according to an

$^6$This value is adjusted for the quinquennial time-period that is considered in the model.

$^7$Storesletten et al. (2004) process is more general than the AR(1) process I am considering here.
AR(1) process ln $\eta_1 = \rho_{\eta h} \ln \eta_{t-1} + \nu_1, \nu_1 \sim N(0, \sigma_{\eta h}^2)$. The inheritance earnings shocks parameters are taken from Zimmerman (1992), so as to match an intergenerational earnings correlation of 0.4. The persistency of the earnings shock in the 1983 economy is chosen to be the same as in the 2004 economy. The five-year earnings shocks volatility in both reference years is chosen to match an earnings Gini coefficient of 0.53 and 0.60 in the 1983 and 2004 equilibrium respectively: they are 0.0980 and 0.1230 respectively. Using the method proposed by Tauchen (1986), I approximate both continuous AR(1) processes with a six states Markov chain, which results in the following earnings shocks, for the 2004 and 1983 economy, respectively

$$E^{2004} = \{0.0655, 0.1692, 0.4368, 1.1275, 2.9100, 7.5106\}$$

$$E^{1983} = \{0.1270, 0.2569, 0.5469, 1.1642, 2.4780, 5.2745\}$$

where mean earnings are normalized to 1. In the table below I show how well the chosen earnings processes approximate the U.S. earnings distribution in both reference years:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.60</td>
<td>-0.16</td>
<td>4.19</td>
<td>13.06</td>
<td>22.93</td>
</tr>
<tr>
<td>Model</td>
<td>0.596</td>
<td>0.00</td>
<td>3.19</td>
<td>11.02</td>
<td>25.84</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Earnings Gini (1983)</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.533</td>
<td>0.00</td>
<td>3.52</td>
<td>17.22</td>
<td>26.36</td>
</tr>
<tr>
<td>Model</td>
<td>0.533</td>
<td>0.00</td>
<td>4.68</td>
<td>14.09</td>
<td>27.75</td>
</tr>
</tbody>
</table>

### 5.2 Preferences and Technology

The utility function is of the constant relative risk aversion class with a Cobb-Douglas aggregator between housing services and non-housing consumption. Housing services are assumed to be proportional to the housing stock, i.e. $g(s) = s$. The coefficient of risk aversion $\sigma$ is set to 2, well within the range of commonly used values.

$$u(c, s) = \frac{(c^{\gamma}s^{1-\gamma})^{1-\sigma}}{1 - \sigma}$$

The Cobb-Douglas aggregator can be considered as a special case of the constant elasticity of substitution (CES) function when the elasticity parameter is equal to zero.
Villaverde and Krueger (2001) report that according to the literature that estimates the degree of elasticity between housing and non-housing consumption, assuming this one to be zero is a reasonable choice. I select a Cobb-Douglas production function 

\[ F(K_t, L_t) = NK_t^\alpha L_t^{1-\alpha} \]

as a representation of the technology that produces the final good. I normalize \( N = 1 \). I closely follow the construction of measures of output, capital and stock of houses from Díaz and Luengo-Prado (2006), who use data from the National Income and Product Account and the Fixed Asset Tables. I define capital as the sum of non-residential private fixed assets plus the stock of inventories plus consumer durables. Investment in capital is defined accordingly. \( H \) is private residential stock. Finally I need a measure of output. Output is defined as GDP minus housing services. I proceed as Cooley and Prescott (1995) to calculate the capital share of the economy. I do not make any imputation to output for government owned capital since my focus is on privately held wealth. The implied share of capital in output \( \alpha \) is 0.26.

### 5.3 Market Arrangements

I calibrate the minimum downpayment requirement so as to achieve the observed average equity to loan ratios for first time buyers, which are 16% in 2004 and 24% in 1983, respectively (see Campbell and Hercowitz (2006)). In equilibrium, the (minimum) downpayment parameter turns out to be 93 percent in 2004, i.e. individuals can borrow up to 93 percent of the value of a house. The downpayment parameter is 81 percent in 1983. I consider non-convex costs of adjustment in the housing market, which results in infrequent adjustment of the housing stock. Martin (2002) finds that the monetary costs of buying a new home, which include agent fees, transfer fees, appraisal and inspection fees, range on average from 7 to 11 percent of purchase price of a home, I set the cost of purchasing a house at 10 percent and cost of selling homes at 6 percent in both the 1983 and the 2004 economies. The depreciation rate of owner occupied housing and renting units, \( \delta^h \), is set to match a housing investment-stock ratio of 0.043. I use the exactly-identified Generalized Method of Moments (GMM) to jointly choose the capital depreciation rate \( \delta \), the discount factor \( \beta \), the aggregation parameter \( \gamma \), the depreciation rate of renting units, \( \delta^f \) to match the following set of statistics: a capital-output ratio of 1.64, a total (owned and rented) housing stock-output ratio of 1.07, an annual real interest rate of 4.15 percent and an economy-wide ownership rate of 0.69 and 0.64 in 2004 and 1983 respectively. The table below shows all parameters.
### Table 1: Parameters

<table>
<thead>
<tr>
<th>Calibration</th>
<th>1983</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_j$ survival probability</td>
<td>Faber(1982)</td>
<td>Faber(1982)</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ capital share in National Income</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>$\delta$ depreciation rate of capital</td>
<td>0.105</td>
<td>0.105</td>
</tr>
<tr>
<td>$\delta^h$ housing depreciation</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta^f$ renting depreciation</td>
<td>0.008</td>
<td>0.005</td>
</tr>
<tr>
<td>Endowment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho^e$ earnings persistency</td>
<td>0.9946</td>
<td>0.9946</td>
</tr>
<tr>
<td>$\sigma^2_{\eta}$ earnings volatility</td>
<td>0.0980</td>
<td>0.1230</td>
</tr>
<tr>
<td>$\rho_{gh}$ inheritance persistency</td>
<td>0.677</td>
<td>0.677</td>
</tr>
<tr>
<td>$\sigma^2_{\eta h}$ inheritance volatility</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Government policy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$ social security tax</td>
<td>0.083</td>
<td>0.083</td>
</tr>
<tr>
<td>$P$ social security replacement rate</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>$\tau_e$ estate tax rate</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Housing market</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$ minimum down payment</td>
<td>19%</td>
<td>7%</td>
</tr>
<tr>
<td>$\rho_1$ housing selling transaction cost</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td>$\rho_2$ housing buying transaction cost</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>$l_{min}$ minimum house size</td>
<td>10% of E(w)</td>
<td>10% of E(w)</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ risk aversion coefficient</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$ weight of non-housing</td>
<td>0.936</td>
<td>0.947</td>
</tr>
<tr>
<td>$\beta$ discount factor</td>
<td>0.945</td>
<td>0.962</td>
</tr>
</tbody>
</table>

### 6 Results

Table 2 shows the result, where in the column for the US economy average debt refers to mortgage debt. The initial 1983 steady state accounts for 65 percent of U.S. total consumer debt level and 88 percent of U.S. mortgage debt level. The percentage of
households with a negative net-financial asset position is 35 percent against 40 percent in the data. The percentage of households at the borrowing limit is 12.5 percent, while Jappelli (1990) estimates that 20 percent of U.S. consumers are borrowing constrained. The 2004 steady state, characterized by a higher cross-sectional earnings inequality (i.e. the Gini index for earnings goes up from 0.533 to 0.60) and a lower average downpayment requirement, accounts for 75 percent of the increase in households’ debt position. The percentage of households in debt increases slightly to 37 percent, while 13.5 percent of households are at the borrowing limit.

Table 2: Debt Statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Model</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt 1983</td>
<td>80</td>
<td>91</td>
</tr>
<tr>
<td>Average Debt 2004</td>
<td>120</td>
<td>157</td>
</tr>
<tr>
<td>% Households in Debt 1983</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>% Households in Debt 2004</td>
<td>37</td>
<td>43</td>
</tr>
<tr>
<td>% Borrowing Constrained Households 1983</td>
<td>12.5%</td>
<td>20%</td>
</tr>
<tr>
<td>% Borrowing Constrained Households 2004</td>
<td>13.5%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Inspection of the housing and financial asset policy functions (see Appendix) reveals that the increase in debt is primarily driven by high-earnings households. The set of housing and financial wealth levels for which households access debt is greater for households in the middle of their life-cycle than for households at the beginning of their life-cycle. Older households have reached the pick of their deterministic life-cycle earnings profile: they are effectively richer than young households in terms of earnings. Focusing on the 1983 equilibrium first, agents younger than 30 years are majoritarily renting -as in the data- then they progressively build up their housing stock until reaching maximum at age 50, against 55 in the data. Retired households accelerate the decumulation of their housing stock only after age 65, as in the data. The model replicates the increase in the homeownership rate for the 20-35 and over 60 age groups in the 2004 equilibrium, and actually overpredicts it. This is due to the assumption of a uniform downpayment requirement for all ages. The highly correlated earnings process allows, together with a lower downpayment requirement, young agents with good shocks to borrow more and earlier in their life cycle. Once the housing stock is built up and households are approaching retirement age, transaction costs in houses purchases make the downsizing process be extremely slow. However, young households are not primarily
driving the increase in debt, as they make up for a smaller fraction of total population than the 35-65 age group, and the percentage of households in the 20-35 age group who has a negative net-financial asset position is almost half that of the 35-65 age group.

The 2004 homeownership profile shows that the pick in housing homeownership is found 10 years later than in the 1983 equilibrium, at age 60. Also, old households hold on to their housing equity much more than in the 1983 equilibrium: there is almost no decumulation of housing assets until age 90.

I finally study mobility patterns. I define mobility as any change in housing equity that entails an adjustment cost. In that way I can identify mobility rates even when the average housing stock is stable along the life cycle.

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>1983 Mobility Rates</th>
<th>2004 Mobility Rates</th>
<th>2000 U.S. Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-35</td>
<td>28</td>
<td>15</td>
<td>82</td>
</tr>
<tr>
<td>40-65</td>
<td>10</td>
<td>5</td>
<td>41</td>
</tr>
<tr>
<td>&gt;70</td>
<td>25</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
Moving rates by age in the data are taken from Schachter (2001) and are aggregated for different age groups. Mobility rates in the model economy are considerably lower than in U.S. data, except for retired households. This stems from the fact that households move for reasons other than income shocks and aging. However, the model captures the 5 percent decrease in mobility rates since the 1980s, as observed in the data. The result is due to the increased predictability of earnings shocks: on the one hand, young agents with good shocks are willing to immediately access the mortgage market to build up their optimal housing stock. Once the optimal housing stock is reached, the relatively small unpredictability of the earnings shocks coupled with transaction costs on housing purchases make agents stay more often in the inactive region of their optimal non-convex policy function for housing.

6.1 Wealth Distribution and Life-Cycle Profiles

The 2004 model attains a wealth Gini coefficient of 0.75 (0.80 in U.S. data, from the 1998 Survey of Consumer Finances) and replicates quite well the lower 99 percent of the distribution. The model, thanks to the renting option, does quite well with respect to the housing wealth distribution, reaching a Gini coefficient of 0.62, thus approaching the level of concentration found in the data, 0.63. Moreover, the model can match quite closely the whole Lorenz curve for housing wealth. The 1983 model comes also close to replicate the lower 99 percent of the wealth distribution. It is also successful in accounting for inequality in housing wealth.
Table 4: Wealth Distribution

<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.80</td>
<td>-0.29</td>
<td>1.35</td>
<td>5.14</td>
<td>12.38</td>
<td>81.42</td>
</tr>
<tr>
<td>2004</td>
<td>0.75</td>
<td>0.09</td>
<td>1.41</td>
<td>4.73</td>
<td>13.91</td>
<td>79.85</td>
</tr>
<tr>
<td>1983</td>
<td>0.73</td>
<td>0.0</td>
<td>1.49</td>
<td>5.01</td>
<td>18.74</td>
<td>74.85</td>
</tr>
<tr>
<td><strong>Housing Wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.63</td>
<td>0.0</td>
<td>1.40</td>
<td>12.31</td>
<td>22.08</td>
<td>64.21</td>
</tr>
<tr>
<td>2004</td>
<td>0.64</td>
<td>0.0</td>
<td>1.58</td>
<td>10.97</td>
<td>24.15</td>
<td>64.88</td>
</tr>
<tr>
<td>1983</td>
<td>0.60</td>
<td>0.0</td>
<td>1.12</td>
<td>13.53</td>
<td>31.24</td>
<td>55.21</td>
</tr>
<tr>
<td><strong>Financial Wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.96</td>
<td>-7.27</td>
<td>-0.25</td>
<td>1.14</td>
<td>6.92</td>
<td>99.45</td>
</tr>
<tr>
<td>2004</td>
<td>0.94</td>
<td>-11.53</td>
<td>-1.24</td>
<td>3.01</td>
<td>15.33</td>
<td>94.43</td>
</tr>
<tr>
<td>1983</td>
<td>0.87</td>
<td>-5.48</td>
<td>-1.36</td>
<td>3.69</td>
<td>18.50</td>
<td>84.65</td>
</tr>
</tbody>
</table>

Turning to life-cycle profiles of consumption, using data from the CEX (1986-2001), and controlling for cohort and time effects, non-housing consumption shows a hump, increasing from around $10,500 to nearly $18,200, then decreasing later in life to about $9,400. The peak is reached at age 45, and the size of the peak measured by the ratio of age 45 to age 20 non-housing consumption is 1.74.

The model is consistent with the observed life cycle profile of consumption: the peak in consumption is reached at age 45 in the 1983 economy and at age 40 in the 2004 economy. The ratio of peak consumption to age 20 consumption is 2.16 in the 1983 economy and 1.32 in the 2004 economy.

The estimated housing value, from SCF data, increases until age 65 and then flattens out until the end of the life cycle. The model is also consistent with this fact: the peak is found at age 60 in both reference years and then it flattens out until age 80. Renting consumption units show the opposite pattern, decreasing monotonically as households age. The model accounts for the increase in mortgage debt while being consistent with the basic features of wealth accumulation along the life cycle. In particular, Figure 7 in the appendix plots mean net worth, housing stock, and financial assets against age. Those profiles are in line with the data: young agents tend to hold little wealth. Early in life households borrow to buy houses, and thus save in the form of housing. As time goes by, agents have built stocks of houses and start to increase their holding of financial assets. The profiles of financial assets and housing assets intersect in their early
40’s. Wealth holding peaks at age 70. However, in the data we do not observe a quick decumulation of wealth later in the life cycle. Instead, households continue to hold large amount of wealth. This problem is common to many

7 Decomposition

The purpose of this section is to isolate the effect of higher cross-sectional earnings inequality and lower downpayment requirements on increasing households debt and the life-cycle homeownership profile. I will first change, form their 1983 value, the downpayment requirement leaving unchanged all others parameters; I will then contemporaneously change the downpayment requirement and the earnings process, setting both to their 2004 values. This two exercises show that the increase in mortgage debt is not due to changes in exogenous parameters in the utility function.

7.1 Changing the Downpayment Requirement

The first experiment consists in lowering to its 2004 level the downpayment requirement, leaving all other parameters unchanged. The level of debt is 128 percent of average labor income, close to the value of 120 percent of average labor income found in the benchmark model. The increase in debt is achieved through an increase in average homeownership, which goes up to 90 percent: households are able to completely smooth their housing services consumption. Indeed, the level of homeownership is constant all over the life-cycle. The very high level of homeownership implies a bigger share of households into debt, 47 percent, and at the borrowing limit, 21.8 percent. This experiment shows that as far as the unpredictable part of earnings is very persistent, lowering downpayment requirements is enough to increase debt.

7.2 Changing Both the Downpayment Requirement and Earnings inequality

When the decrease in downpayment requirements is accompanied by an increase in earnings dispersion, the debt level increases to 129 percent of average labor income. Average homeownership rate now reaches 94 percent. The percentage of households with a negative net financial assets position is stable at 47 percent, while slightly more households are at the borrowing limit, 23.4 percent. This experiment shows that higher volatility of earnings, starting from a very high level of persistency, doesn’t set in a precautionary saving motive big enough to overturn the increase in debt.
8 Sensitivity Analysis

In this section I study the robustness of the result to different changes in parameter values. I will consider in turn an earnings process characterized by smaller persistency, a higher minimum size requirement and the effect of adopting a different definition of borrowing constraints.

8.1 The Earnings Process

The results for the 1983 economy are not sensitive to the specification of the earnings process in that year. In particular, the debt level doesn’t change when the same earnings Gini coefficient (0.533) is obtained by increasing the autocorrelation of earnings shocks. The reason for this result lies in the tightness of exogenous borrowing constraints. A highly persistent process in the 1983 economy increases by a factor of three the percentage of households at the borrowing limit. But since the borrowing limit is unchanged, agents cannot borrow more and average debt hardly increases. Hence I don’t have to make any assumption on which part of earnings, predictable or unpredictable, has increased more since the 1980s. The benchmark debt level in the 1983 economy doesn’t change. I see this result as interesting, considering that a consensus has not yet been reached in the literature regarding which earnings components have changed more over the last 20 years. The persistency of the 2004-economy earnings process is instead key to increase households access to mortgage debt: the more persistent is the unpredictable part of the earnings process, the weaker the precautionary saving motive. The 2004 autocorrelation coefficient has been chosen to coincide with the upper bound of the window of plausible autocorrelations coefficients estimated by Storesletten et al (2004) using PSID data. What happens if the 2004 earnings process is instead characterized by an autocorrelation coefficient of 0.92, the lower bound in the window of estimates reported by Storesletten et al (2004)? With lower shock persistency the unpredictable part of the earnings process has to increase much more to match the 2004 earnings Gini coefficient. Higher unpredictable variability increases housing mobility rates along the entire life cycle, because agents are hit more frequently by high or low shocks that force them to liquidate their housing stock more often to smooth non-durable consumption. The mass of households at the borrowing limit is smaller, due to the increased strength of the precautionary savings motive. Since the mass of households with a negative net financial assets position remains unchanged, a smaller fraction of borrowing constrained agents
implies a smaller level of debt in the economy. Indeed, despite the lower downpayment requirement, average debt doesn’t increase from its 1983 level.

### 8.2 Minimum House Size

Precautionary savings are increasing in the minimum house size, since households have to save more up-front in order to buy a house: this tends to reduce access to debt. On the other hand, the higher the minimum house size, the higher the average size of a purchased house and the higher the average mortgage debt: this tends to increase the level of debt. In equilibrium, the second effect dominates the first and a higher minimum house size, while reducing the mass of agents at the borrowing limit, increases considerably the average debt level in the economy. The initial 1983 steady state goes quite close to replicate the average debt over earnings ratio: 120 percent against 124 percent in U.S. data. The percentage of households that are in debt is 35 percent against 40 percent in the data. The percentage of households at the borrowing limit is 9.5 percent. The 2004 steady state now accounts for 50 percent of the actual increase in the debt position of households, hence less than in the benchmark economy with a lower minimum house size.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Model</th>
<th>U.S. economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt 1983</td>
<td>120</td>
<td>124</td>
</tr>
<tr>
<td>Average Debt 2004</td>
<td>164</td>
<td>206</td>
</tr>
<tr>
<td>% Households in Debt 1983</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>% Households in Debt 2004</td>
<td>37</td>
<td>43</td>
</tr>
<tr>
<td>% Borrowing Constrained Households 1983</td>
<td>9.5%</td>
<td>20%</td>
</tr>
<tr>
<td>% Borrowing Constrained Households 2004</td>
<td>13.5%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Focusing on mobility rates, the increase in the minimum size has the same qualitative effect of increasing transaction costs: buying and selling housing assets, in expected terms, becomes more costly. Consequently, as the table below shows, mobility rates decrease in both reference years.
Table 6: Mobility rates by age (in percentage)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>1983 Mobility Rates</th>
<th>2004 Mobility Rates</th>
<th>2000 US data</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-35</td>
<td>20</td>
<td>17</td>
<td>82</td>
</tr>
<tr>
<td>40-65</td>
<td>2</td>
<td>1</td>
<td>41</td>
</tr>
<tr>
<td>&gt;70</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

The high minimum size model substantially overestimates the homeownership rate for households younger than 35 in both reference years.

Figure 5: Homeownership Rates

8.3 The Real Interest Rate and Borrowing Constraint Specification

Since the 1980s, the real interest rate has fallen by 2 percentage points. In contrast to Hintermaier and Koeniger (2006), I don’t find evidence that a lower real interest rate increases average debt in the economy. On the one hand, lowering the interest rate increases the share of households with a negative net-financial assets position, since repaying debt has become cheaper in terms of next-period consumption. On the other hand, the fraction of borrowing constrained households remains unchanged, so that average debt, if anything, diminishes. In particular, with exogenous borrowing constraints, a
lower interest rate affects only the demand for borrowing, not the exogenous fixed supply limit. In Hintermaier and Koeniger (2006) the specification of the borrowing constraints is endogenous and equal to the lowest attainable income level that guarantees full repayment. With this specification, a lower real interest rate increases the equilibrium debt level by affecting both the demand for debt - its price with respect to next period consumption decreases - and the borrowing limit, which is inversely correlated to the amount to be repayed. Moreover, the borrowing limit also depends on the lowest earnings shock, while I showed that in an exogenous borrowing constraints framework the increase in debt is primarily driven by high-earnings households who are facing persistent shocks, independently of their lowest shock realization. To further illustrate the effect of changing the borrowing constraints specification, I consider the implications of introducing endogenous no-default constraints, who specify a punishment for defaulting households. I follow Krueger and Fernández-Villaverde (2005) and define borrowing limit \( b(h^0, \eta, j) \) as the smallest number to satisfy

\[
V(b(h', \eta, j), h', \eta', j + 1) \geq V(0, h^\text{exemp}, \eta', j + 1) \quad \text{for all } \eta' \in \mathcal{E}
\]

that is, households can borrow up to the point at which, for all possible realizations of the stochastic labor productivity shock tomorrow, they have an incentive to repay their debt rather than to default, with the default consequence being specified as losing their debt, but also their housing stock up to an exemption level \( h^\text{exemp} \). In the U.S., consumers filing for bankruptcy can keep basic assets considered necessary for debtors' "fresh start" after bankruptcy, chief among them, part of their housing equity up to a homestead exemption. The exemption level varies considerably from state to state but on average is about $12,500 on a primary residence. In 2006, the median annual household income according to the US Census Bureau was determined to be $48,201 (median earnings was $42,300), hence the homestead exemption represents about a fourth of the annual median income. I set the fixed exemption level accordingly. With endogenous no-default borrowing constraints, an increase in earnings volatility leads to a worse au-

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8The assumption here is that the lender, who lends at the risk-free rate, knows households assets position at the minimum of the support of the income distribution.

9Since I am modelling households, I refer to Chapter 7 procedure of the U.S. bankruptcy code.

10I performed sensitivity analysis with respect to the exemption level. For numbers twice as high as the one used here, model statistics where not significantly affected. This result comes from the share of housing equity in total capital being anyway fixed at its steady-state level. Results are available from the authors upon request.
tarkic option, and thus to more borrowing being enforceable. Using a very persistent earnings process reduces average borrowing\textsuperscript{11}. This specification of endogenous borrowing constraints has three major shortcomings. On the one hand, it can justify the increase in mortgage debt only by explicitly assuming that the increase in cross-sectional inequality has come from higher earnings volatility, something that has not yet been empirically established. On the other hand, higher earnings volatility causes mobility rates to increase, contrary to what we observe in the data. And finally, the resulting wealth distribution is considerably less skewed - Gini coefficient 0.65 - that in the benchmark model with exogenous borrowing constraints.

8.4 Demographic Change

The steady state comparison provided in the paper assumes that the demographic structure of the economy is the same in both 1983 and 2004. Demographic change such as the ‘baby boom’ might have affected total borrowing by altering the numbers of those most likely to borrow, i.e. the young, in the economy. In particular, Figure 7 in the appendix\textsuperscript{12} shows that the fraction of total population accounted for by the 35-55 age group has increased over the last 20 years, at the expenses of the fraction accounted for by the 20-35 age group. I recalibrate both steady states by including the correct demographics in each year. The compositional effect of changing demographics makes the 2004-economy equilibrium have a smaller fraction of households into debt. Also, the fraction of households at the borrowing limit decreases. While the former effect tends to increase average debt, everything else equal, the latter tends to diminish debt. In equilibrium, the two effects balance out so that the average level of debt in the economy equals that of the benchmark model. In the model then, the demographic change observed in the US over the last 20 years would have not influenced average debt, but would have reduced the mass of agents with a negative net-financial assets position.

9 Conclusion

I have built a model with overlapping generations of households who receive idiosyncratic shocks and self-insure through financial and housing assets. Agents have access to debt through collateralized lending on the housing asset. I have found that a highly persistent

\textsuperscript{11} Results are available from the author upon request.
\textsuperscript{12} Demographic data are taken from the U.S. Census Bureau.
earnings process, coupled with a low downpayment requirement for house purchases, can contemporaneously account for the flattening of the homeownership rate along the life-cycle, 75 percent of the increase in households debt since the 1980s, and the observed decrease in households residential mobility rates. The persistency of the earnings process decreases precautionary savings and make it possible for households with high earnings shocks to increase their access to mortgage debt when they are young. Low precautionary saving implies low housing mobility rates, as households don’t need to adjust often their housing stock to self-insure against volatility in non-durable consumption. I have shown that the result is sensible to the definitions of borrowing constraints. I have argued that exogenous borrowing constraints have the advantage of not imposing any restriction on the part of the earnings process -predictable or unpredictable- that has increased most since the 1980s. The model can be improved. In particular, cross sectional data show that the rise in household debt has been driven primarily by middle aged, and higher income, households. Thus, while the build up of household debt is often portrayed as being driven by young couples trying to buy their first home, a more accurate description is that it is mainly being driven by older, higher income households that are trading up to higher quality or better located houses. It would be interesting to study the impact of quality and location characteristics on the access to housing and mortgage debt. This is left for future research.

10 Appendix

10.1 Model’s computation

Non-convex adjustment costs to housing expenditure and a minimum purchasing house size break the smoothness of the optimization problem: first-order conditions could not be used to simulate the model. I resorted instead to discretization of the state space and value function iteration, which is computationally costly but very robust. The upper bounds on the grids for financial assets and housing are chosen large enough so that they are not binding on the optimization problem.

The choice of housing stock and renting units is found by grid search, where the renting units grid includes choice points not included in the housing stock grid, because of the imposed minimum housing purchasing size. The choice of financial assets is found by one-dimensional optimization that doesn’t use differentiability of the value function.

I use 180 points for both the housing and financial assets grids, which combined with a 6 states earnings Markov matrix makes for 194,400 possible states, for each age.
I solve for the steady state equilibrium as follows:

1. Guess \( r \) and use the equilibrium conditions in the factor markets to obtain \( w \).
2. Solve for the value function in the last period of life, then solve recursively for all other ages.
3. Compute the associated stationary distribution of households \( \Phi \). To do that, I need to iterate on the stationary distribution to find the appropriate first period invariant distribution, given by the accidental bequests and first earning shock left over by people dying at all ages.
4. Given the stationary distribution and prices, compute factor inputs demand and supplies and check market clearing.
5. If all markets clear, I found an equilibrium. If not, go to step 1 and update \( r \).

All the programs needed for the computation of the model were programed in Fortran 95 and compiled in Absoft Pro-Fortran 9 to run on Windows PC.
10.2 Policy Functions

Policy Functions at Age 30, 2004-economy

Policy Functions at Age 50, 2004-economy
Figure 6: Share of Total Population by Age, 1980-2000

Figure 7: Wealth Accumulation Along the Life-Cycle
References


