

Credit Crunches, Asset Prices, and Technological Change

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Abstract

We investigate the role of the credit market in an economy where firms can operate a mature technology or restructure their activity and adopt a new technology. We show that firms' collateral and credit relationships ease firms' access to credit and investment but can also inhibit firms' restructuring. When this occurs, a negative collateral shock and the resulting drop in the price of collateral assets squeeze collateral-poor firms out of the credit market but foster the restructuring of collateral-rich firms. We characterize conditions under which such an increase in firms' restructuring occurs within existing credit relationships or through their breakdown. The analysis reveals that the credit and asset market policies adopted during the recent financial crisis might hinder a process of Shumpeterian restructuring in the credit market.

Keywords: Aggregate Restructuring, Collateral, Credit Relationships, Credit Crunch.

JEL Codes: E44.

1 Introduction

In the last two decades or so, a rich literature has investigated the role of credit in entrepreneurs' investment decisions. In particular, a consensus has formed about the benefits that two key features of the credit market, entrepreneurs' collateral and credit relationships, have for investment. When entrepreneurs have limited ability to commit to repay their lenders, the availability of pledgeable assets eases their access to credit. Lenders can repossess collateral and this may compensate for the limited pledgeability of output. Lenders can also deter entrepreneurs' misbehavior by threatening to repossess collateral. Credit relationships enhance these benefits of collateral. For example, lenders who have established close relationships with entrepreneurs can better monitor their collateral and, hence, obtain more value from its repossession. In an aggregate perspective, an implication of this view is that shocks that erode the value of collateral or break credit relationships can depress investment by hindering entrepreneurs' access to financing.

This view of the credit market is generally confined to technologically static economies. If one allows for technological progress, two questions arise naturally: Do collateral and credit relationships ease entrepreneurs' restructuring activity, meant as the upgrade from mature technologies to new ones? And therefore, in an aggregate perspective, do shocks that erode the value of collateral and break credit relationships depress aggregate restructuring as they allegedly depress investment? In this paper, we address these questions. The intuition can be summed up as follows. In our economy, entrepreneurs operate mature technologies

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or restructure their activity and adopt new technologies. Lenders, in turn, learn information that is crucial for liquidating productive assets pledged as collateral when entrepreneurs default (as in Diamond and Rajan, 2001, for example). Lenders' information on collateral assets eases entrepreneurs' access to credit. However, their information renders lenders conservative towards entrepreneurs' restructuring. In fact, new technologies have less assets pledgeable as collateral. Furthermore, the information on the collateral assets of mature technologies is (partially) specific and non-transferable to the collateral assets of new technologies. Therefore, lenders expect that the value of their information will depreciate if entrepreneurs restructure and may impede their restructuring effort to prevent this.

In this economy, entrepreneurs can form credit relationships with lenders to transfer them more information on collateral assets and obtain cheaper financing. Yet, because of the technological conservatism induced by lenders' information, credit relationships favor technological inertia. When technological inertia arises, entrepreneurs can break their credit relationships, borrow from new lenders and restructure. However, this wastes the information accumulated within the relationships. Hence, these credit relationships and technological inertia can be long-lasting.

The distribution of firms across collateral values replicates salient features of that obtained in previous aggregate models of the credit market (e.g., Holmstrom and Tirole, 1997). Collateral-poor firms lack access to credit because they cannot pledge enough expected returns to lenders, even when these obtain high quality information on collateral. Furthermore, firms with medium collateral value obtain credit from informed (relationship) lenders. The novelty consists of firms' technology adoption. While firms with medium collateral value potentially restructure, collateral-rich firms with credit relationships preserve the mature technology. In fact, the conservatism of their lenders is strong because the lenders expect a large depreciation in the value of their information if the mature technology is abandoned in favor of the new technology.

We perturb this economy with an exogenous contraction in the value of collateral assets that reduces the asset price. Following the drop in the price of collateral assets the credit relationships of collateral-poor firms break down because these firms can no longer pledge enough expected returns to lenders. Consider next collateral-rich firms. The collateral shock erodes the value of the information of their lenders. This mitigates lenders' conservatism, allowing restructuring to occur within the relationships. This also increases the incentive of collateral-rich firms to deliberately break their credit relationships, borrow from new lenders and restructure. Whether the restructuring occurs within existing relationships or through their breakdown depends on the credit regime. There is a credit regime in which lenders' conservatism is weak and/or firms derive large benefits from credit relationships: in this regime, collateral-rich firms restructure within their relationships. There is instead a credit regime in which lenders' conservatism is strong and/or firms derive small benefits from credit relationships: in this regime, collateral-rich firms restructure by breaking their relationships and borrowing from new lenders. Thus, depending on the credit regime, the surge in restructuring activity induced by the shock can entail a moderate or a major breakdown of credit relationships. This is important for the aggregate impact of the shock, because the loss of asset liquidation skills that arises from a breakdown of credit relationships depresses output. Interestingly, we also show that in our model economy the credit rationing of collateral-poor firms may foster the restructuring of collateral-rich firms by bringing down asset demand and values.

The remainder of this paper unfolds as follows. In the next section, we relate the paper to the literature. In Section 3, we outline and discuss the setup. Section 4 solves for the equilibrium. In Section 5, we investigate the effects of a collateral shock. Section 6 concludes. Proofs are relegated to the Appendix.

2 Related Literature

This paper especially relates to two strands of literature. The first investigates the impact of recessions on the financial structure and the consequences for aggregate investment. We have discussed the key elements

we share with Holmstrom and Tirole (1997). We also borrow properties of our modelling strategy from their paper, such as the focus on a highly tractable finite horizon economy. Den Haan, Ramey and Watson (2003) and dell’Ariccia and Garibaldi (2001) are other related papers in this strand of literature. These papers analyze the breakdown of credit relationships that can be caused by a recession in economies with search frictions. While in these studies the breakdown of credit relationships depresses investment, in our economy this breakdown depresses investment but may also foster aggregate restructuring.

The second strand of literature analyzes the impact of recessions on firms’ restructuring. Most of this literature neglects the role of the credit market for aggregate restructuring. Caballero and Hammour (2004), Ramey (2004) and Barlevy (2003) are exceptions. These studies show that credit frictions can become more severe during recessions, hindering aggregate restructuring. Caballero and Hammour (2004) shows that, because of credit frictions, production units can be destroyed at an excessive rate during a recession. Furthermore, during the following recovery, the creation of new production units can be too slow and most of the recovery can occur via a slowdown of destruction. Ramey (2004) endogenizes financial managers’ project selection and shows that, if managers have empire-building incentives, during recessions they can discard efficient projects to preserve the size of their portfolios. Barlevy (2003) finds that credit frictions can reverse the “cleansing effect” of recessions by leading to the disruption of high-surplus production units rather than low-surplus ones. This paper endorses a view opposite to all these studies: while it negatively affects investment, the breakdown of information-intensive credit relationships also mitigates the conservatism of lenders that inhibits restructuring.

3 The Model

This section describes the setup of the model. Table 1 summarizes the notation while Figure 1 illustrates the timing of events.

Agents, Goods and Technology. Consider a four-date economy ($t = 0, 1, 2, 3$) populated by a unit continuum of entrepreneurs (firms) and a continuum of investors of measure larger than one. There is a final consumption good, which can be produced and stored, and productive assets of two vintages, mature and new. Entrepreneurs have no endowment while each investor is initially endowed with an amount i of final good. All agents are risk neutral and consume on date 3.

Each entrepreneur can implement one indivisible project. On date 2, an entrepreneur can experience a technological innovation. If the innovation occurs, the entrepreneur restructures her activity and adopts a new technology; otherwise, she operates a mature, less productive technology. Under the mature (new) technology, on date 3 the entrepreneur transforms an amount i of final good into one unit of mature (new) assets. With probability π the project succeeds and the assets yield an output y ($y(1 + n)$) of final good; otherwise the project fails and the entrepreneur goes out of business. In this case, a fraction a ($\alpha_n a$) of mature (new) assets can be redeployed outside the firm. $a \in [0, 1]$ captures the amount of collateralizable assets of an entrepreneur and is uniformly distributed across entrepreneurs. $\alpha_n \leq 1$ is a parameter that reflects the redeployability of new assets relative to mature assets.

On date 3, each entrepreneur still in business can reuse one unit of liquidated assets, obtaining an amount $\eta\theta$ of final good. θ is uniformly distributed across entrepreneurs over the domain $[0, \bar{\theta}]$; η represents the aggregate productivity of liquidated assets.¹

¹Borrowing from the analysis of the market for collateral assets in Shleifer and Vishny (1992), we generate a downward sloping demand for collateral assets by allowing for heterogeneity in entrepreneurs’ ability to reuse assets. Furthermore, following Shleifer and Vishny (1992), only the entrepreneurs who succeed in production at date 3 have the technology to reuse liquidated assets.

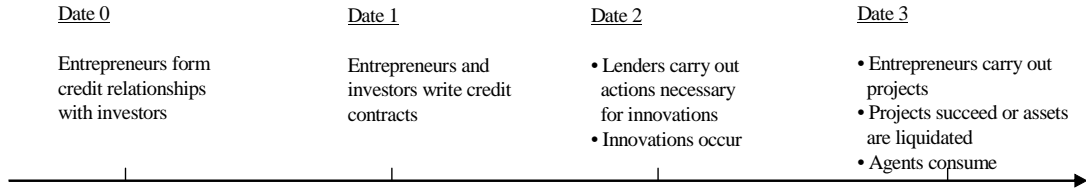


Figure 1: Timing of Events.

Credit Sector. On date 1, each entrepreneur can write a credit contract with one investor. A lender interacts with an entrepreneur along two dimensions besides credit provision: control and information. Following an established literature (e.g., Aghion and Bolton, 1992; Rajan, 1992), we allow the lender to exert control over production opportunities. Precisely, on date 2 the lender can carry out a costless action that affects the probability of the innovation: if she carries out this action, the innovation will occur with probability $1 - \sigma$ ($0 < \sigma < 1$); otherwise, the innovation cannot occur.

The lender also learns information as a by-product of her financing activity. As in Diamond and Rajan (2001) and Habib and Jonsen (1999), information enables her to obtain more than other agents from the liquidation of the entrepreneur's assets, that is, the lender effectively monitors collateralizable assets. Precisely, the share of liquidation value that the lender obtains equals the amount of her information on the assets; the rest of the liquidation value is lost in the form of a (nominal or real) transaction cost. By contrary, we normalize to zero the net amount of final good that any other agent obtains from asset liquidation.

Having characterized the nature of information, we have to specify its amount. We allow the entrepreneur to influence this amount by choosing the type of funding, relationship or transactional. Precisely, on date 0 each entrepreneur can form a credit relationship with one investor. Consider first mature assets: μ ($\sigma\mu$) is the amount of information of a lender if she does (not) carry out the action for the innovation; furthermore, for a relationship lender $\mu = M$ while for a transactional lender $\mu = m$. This specification has two features. First, because a credit relationship entails a long-run tie with the entrepreneur from date 0, a relationship lender learns more information about the entrepreneur's assets ($M > m$). Second, when a lender allows the innovation, she learns less information on mature assets ($\sigma < 1$).² This reflects the idea that the lender has less opportunities - and with endogenous information acquisition, less incentives - to learn information on a technology if the entrepreneur is working to abandon it. Consider next new assets: denoting μ_n the amount of information, we let $\mu_n \alpha_n = 0$. Thus, a lender obtains less value from liquidating new assets than from liquidating mature assets - the normalization to zero is for simplicity. We will discuss this specification shortly.

Contractual Structure. As in Aghion and Bolton (1992) and Diamond and Rajan (2001), a lender cannot contractually commit to carry out her action because this is non-verifiable; moreover, imperfect enforceability limits agents' commitment to monetary transfers and, hence, the design of monetary incentives for the lender. Specifically, in the event of project success, only a fraction l of the output is verifiable while the rest accrues privately to the entrepreneur. In the event of project failure and asset liquidation, the lender cannot commit the specific liquidation skills tied to her information. Thus, as in Diamond and Rajan (2001), she can threaten to withhold her skills during the liquidation, forcing a renegotiation of the allocation of the asset liquidation proceeds to appropriate them in full.³

²Setting the amount of information learnt by the lender on the mature technology equal to the probability that the mature technology is adopted simplifies the algebra but is not relevant for the results.

³As in Diamond and Rajan (2001), for simplicity the lender has all the bargaining power in the renegotiation.

TABLE I.
Notation of the Model.

► <i>Technology</i>	
Probability of project success	π
Output of mature technology	y
Productivity edge of new technology over mature	n
Investment requirement of project	i
Share of verifiable output	l
Probability of innovation	$1 - \sigma$
► <i>Credit Market</i>	
Information of (share of asset rescued by) relationship lender	M
Information of (share of asset rescued by) transactional lender	m

Note. The table summarizes the symbols used in the model (second column) and their description (first column).

3.1 Discussion

Real Sector. The difference between the two technologies is that the new technology yields more output ($y(1+n) > y$) but its assets have lower liquidation value ($\mu_n \alpha_n p a < \mu p a$, where p denotes the price of assets). We put forward two interpretations. First, new technologies typically have less assets pledgeable as collateral, i.e. $\alpha_n < 1$ (Hall, 2001; Carpenter and Petersen, 2002; Berlin and Butler, 2002; Rajan and Zingales, 2001).⁴ Carpenter and Petersen (2002) argue that “high-tech investments have limited collateral value. R&D investment, which is predominantly salary payments, has little salvage value in the event of failure. Furthermore, physical investments designed to embody R&D results are likely to be firm specific, and therefore may have little collateral value”. Second, for a given liquidation value, lenders typically have less experience in liquidating new vintages of assets than mature ones, i.e., $\mu_n < \mu$.

Credit Sector. We have to discuss three features of the credit sector: the control exerted by a lender; the characterization of information as asset liquidation skills; the amount of information of a lender. We share, for example, with Aghion and Bolton (1992) and Rajan (1992) the assumption that a lender carries out an interim action that affects production opportunities. This action has several real world counterparts. It can consist of providing the entrepreneur with advice or information for expanding the firm’s technological frontier; in an R&D race, it can consist of concealing the findings of the entrepreneur’s internal research from her competitors (Bhattacharya and Chiesa, 1995); if the lender has representatives on the board of the firm, as in the case of German and Japanese banks, it can consist of voting for an innovative strategy. In other circumstances, this action can consist of a refinancing (Rajan, 1992): the need for refinancing is likely for a new technology which generally yields little interim cash flow, especially at the R&D stage (Goodakre and Tonks, 1995). Aghion and Bolton (1992) discuss examples of other actions of lenders which can affect innovation opportunities, such as supporting firms’ mergers and spin-offs.

We borrow the characterization of information as asset liquidation skills from Diamond and Rajan (2001) and Habib and Jonsen (1999). The critical feature is that the lender learns more information than the entrepreneur and the other investors. As for latter, following Diamond and Rajan (2001), our assumption reflects the idea that the lender acquires more information on collateralizable assets through her financing activity. As for the former, “Because he [the entrepreneur] is a specialist at maximizing the value of the asset in its primary use [...] it is reasonable to assume that he lacks the skill even to identify the asset’s next best use or to recognize clearly the occurrence of the bad states, in which case he risks maintaining it in a

⁴In the model, we normalize to zero the value that a lender obtains from the liquidation of a new asset. We could allow a new asset to have positive liquidation value, though lower than that of a mature asset ($0 < \mu_n \alpha_n < 1$). The qualitative results would carry over but the analysis would be more cumbersome.

suboptimal use” (Habib and Johnsen, 1999, p. 145).

The second feature of our information structure is the amount of information. In several models, the distinct characteristic of a relationship lender is her informational advantage over a transactional lender (Berger and Udell, 1998). Indeed, Berger and Udell argue that “banks may acquire private information over the course of a relationship” (1995, p. 352) and “under relationship lending, banks acquire information over time through contact with the firm, its owner, and its local community” (2002, p. 32).

4 Equilibrium

In solving for the equilibrium we start with agents’ decisions taking as given the price of assets p . Next, we solve for the asset price. Finally, we characterize the distribution of firms across collateral values (a) according to whether a firm obtains credit or not, the type of funding it chooses (relationship or transactional) and the technology adopted (mature or new). Throughout this section, we assume that $\alpha_n = 1$, that is we let mature and new assets be homogeneous in the asset market. This means that the difference between the returns that a lender obtains from the liquidation of new assets and the returns she obtains from the liquidation of mature assets is entirely attributable to her lower knowledge about the new assets. Moreover, this implies that the price of assets can be solved independently of the distribution of entrepreneurs between the mature and the new technology. In the next section, we will consider the case in which $\alpha_n < 1$.

4.1 Agents’ Decisions

We focus first on the entrepreneurs who obtain credit and solve for agents’ decision by backward induction: first, we solve for a lender’s action; then, we solve for an entrepreneur’s choice between transactional and relationship funding. Consider the lender’s decision whether to carry out the action on date 2. The contractual structure implies that the contract only determines the loan granted and the repayment to the lender in the event of project success, contingent on the technology adopted. Denote by r (r_n) the repayment contractually due to the lender if the mature (new) technology is successfully adopted. The lender compares her expected return if the innovation can occur with her expected return if the innovation cannot occur. Using simple algebra and assuming that she breaks a tie in favour of inaction, the lender will carry out the action if and only if

$$r_n - r > \frac{(1 + \sigma)(1 - \pi)pa\mu}{\pi}. \quad (1)$$

Inequality (1) is the lender’s incentive compatibility constraint. The left hand side of (1) is the spread between the repayment in the event of successful adoption of the new technology and the repayment in the event of successful adoption of the mature technology. The right hand side of (1) is (a monotonic transformation of) the reduction in her liquidation proceeds that the lender suffers if the entrepreneur restructures and the project fails. This reduction, which is due to the lender’s worse ability to liquidate new assets ($\mu_n = 0$), is positively related to the lender’s information μ and to the liquidation value pa of mature assets.

The lender will allow the innovation if and only if, as in (1), the contract guarantees her a sufficiently higher repayment if the new technology is successfully adopted, compensating her for the reduction in her expected liquidation proceeds. Lemma 1 solves for the lender’s action.

LEMMA 1: *A lender will carry out the action necessary for the innovation if and only if the entrepreneurs’s collateral assets satisfy $a < \bar{a}(p, \mu)$, where*

$$\bar{a}(p, \mu) \equiv \frac{\pi ly(1 + n) - i}{p\sigma(1 - \pi)\mu}. \quad (2)$$

PROOF. In the Appendix.

The intuition behind Lemma 1 is as follows. The spread $r_n - r$ that can be specified in the contract is bounded. On the one hand, the repayment r_n for the new technology is constrained above by the entrepreneur's limited liability constraint $r_n \leq ly(1+n)$. On the other hand, for a given r_n , the repayment r for the mature technology is constrained below by the lender's participation constraint.⁵ Therefore, in the region of the parameter space pinned down by Lemma 1, in (1) the left hand side falls short of the right hand side for any feasible pair (r_n, r) . In this region, the lender impedes the entrepreneur's restructuring activity.

Lemma 1 yields two insights. First, for a given asset price, the technological conservatism of the lender is more likely within a credit relationship, that is, $\bar{a}(p, M) < \bar{a}(p, m)$. In fact, because of her better information ($M > m$), a relationship lender has greater ability to liquidate mature assets than a transactional lender, that is in (1) the right hand side is higher. Second, the conservatism of the lender is more likely when an entrepreneur is rich in collateral (has a high a) and when the asset price p is higher. Intuitively, a lender loses more from the depreciation of her liquidation skills when these skills are better developed ($M > m$) and the value pa of the collateral of the entrepreneur is higher.

We can now solve for an entrepreneur's funding choice. Henceforth, we assume that an entrepreneur breaks a tie in favour of relationship funding.

LEMMA 2: *An entrepreneur will choose transactional funding if and only if $a \in [\bar{a}(p, M), \tilde{a}(p)]$, where*

$$\tilde{a}(p) \equiv \min \left\{ \bar{a}(p, m), \frac{\pi yn(1-\sigma)}{(1-\pi)(M-\sigma^2 m)p} \right\}. \quad (3)$$

PROOF. In the Appendix.

The intuition behind Lemma 2 is as follows. The entrepreneurs who can restructure under both relationship and transactional funding ($a < \bar{a}(p, M)$) and those who cannot restructure under either type of funding ($a \geq \bar{a}(p, m)$) choose relationship funding. In fact, for given innovation opportunities, a relationship lender offers cheaper credit because she is more efficient in liquidating mature assets. The entrepreneurs with $a \in [\bar{a}(p, M), \bar{a}(p, m)]$ face instead a trade-off: on the one hand, only transactional funding allows them to restructure; on the other hand, relationship funding offers them cheaper credit. The extra cost of transactional funding increases with the amount of collateral a and equals the expected cost of the technological inertia induced by relationship funding when $a = \pi yn(1-\sigma)/(1-\pi)(M-\sigma^2 m)p$.

Thus far, we have focused on the entrepreneurs who obtain credit on date 1. In Lemma 3, we establish the conditions under which an entrepreneur has indeed access to credit.

LEMMA 3: *Denote*

$$\underline{a}(p, \mu) \equiv \frac{i - \pi ly(1+n-n\sigma)}{\sigma^2 p(1-\pi)\mu}. \quad (4)$$

Only the entrepreneurs with $a \geq \underline{a}(p, M)$ obtain credit on date 1.

PROOF. In the Appendix.

The intuition behind Lemma 3 is straightforward. The limited verifiability of their output and the low value of their collateral imply that some entrepreneurs cannot pledge enough returns to a lender, even when the lender obtains high quality information on collateral ($\mu = M$). This leads to the exclusion of these collateral-poor entrepreneurs from the credit market.

⁵Note that we do not impose a limited liability constraint for the lender. Implicitly, we are assuming that the lender has more than enough funds to operate transfers to the entrepreneur at date 3 (if needed). Incorporating a limited liability constraint would not alter the results.

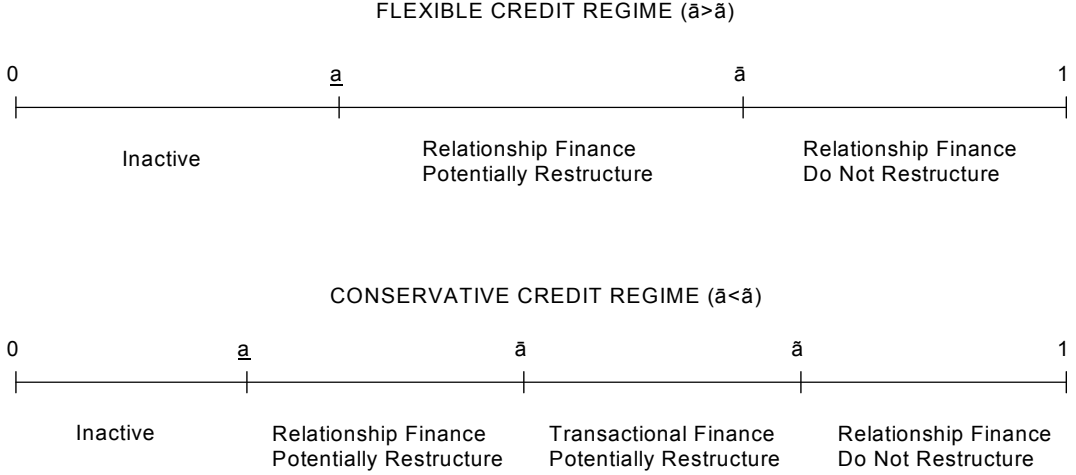


Figure 2: Firm Distribution Across Collateral Values.

Together, Lemmas 1 and 3 illustrate the dual role of collateral and credit relationships in our model economy. Lemma 3 shows that they both ease entrepreneurs' access to credit: indeed, some entrepreneurs suffer from a shortage of collateral and are excluded from the credit market. Lemma 1 shows instead that an excess of collateral and credit relationships may inhibit restructuring: relationship lenders learn high quality information on collateral and they can impede restructuring to preserve the value of their information.

4.2 The Asset Price

In the previous section, we have solved for agents' decisions taking the asset price p as given. We now solve for the asset price in general equilibrium. The demand \mathcal{D} of liquidated assets satisfies

$$\mathcal{D}(p) = \max \left\{ [1 - \underline{a}(p, M)] \pi \left(1 - \frac{p}{\eta \bar{\theta}} \right), 0 \right\}. \quad (5)$$

Intuitively, a measure $1 - \underline{a}(p, M)$ of entrepreneurs obtain credit and become active. In fact, the threshold value of collateral below which an entrepreneur cannot obtain credit is $\underline{a}(p, M)$. Moreover, a share π of active entrepreneurs is successful and remains in business. Finally, a share $1 - p/\eta \bar{\theta}$ of the entrepreneurs who remain in business obtain an output no lower than p from reusing assets.

The supply \mathcal{S} of assets satisfies

$$\mathcal{S}(p) = \max \left\{ \frac{1}{2} (1 - \pi) \left[1 - (\underline{a}(p, M))^2 \right], 0 \right\}. \quad (6)$$

The supply is given by the probability that an entrepreneur fails times the amount of assets a that are liquidated by a failed entrepreneur, integrated across active entrepreneurs. An equilibrium where neither demand nor supply is rationed obtains when $\mathcal{D}(p) = \mathcal{S}(p)$. In the Appendix, we investigate the properties of the excess collateral demand $\mathcal{D}(p) - \mathcal{S}(p)$. Henceforth, we simplify the notation defining $\bar{a}(p, M) \equiv \bar{a}$ and $\underline{a}(p, M) \equiv \underline{a}$.

4.3 Firm Distribution

We are now in a position to characterize firm distribution across collateral values. A full characterization of all the possible equilibria is provided in the Appendix. Henceforth, we focus on two possible distributions

that can arise in equilibrium under which the mechanisms of interest are at work (these distributions are illustrated in Figure 2).

PROPOSITION 1: *There is a region of the parameter space such that there is a unique equilibrium in which the firms with $a \in [0, \underline{a}]$ do not obtain credit and remain inactive, while the firms with $a \in [\underline{a}, 1]$ choose funding and technology as follows:*

- $a \in [\underline{a}, \bar{a}]$: i) relationship funding, ii) potentially restructure,
 $a \in [\bar{a}, 1]$: i) relationship funding, ii) do not restructure;

There is also a region of the parameter space such that there is a unique equilibrium in which the firms with $a \in [0, \underline{a}]$ do not obtain credit and remain inactive, while the firms with $a \in [\underline{a}, 1]$ choose funding and technology as follows:

- $a \in [\underline{a}, \bar{a}]$: i) relationship funding, ii) potentially restructure,
 $a \in [\bar{a}, \tilde{a}]$: i) transactional funding, ii) potentially restructure,
 $a \in [\tilde{a}, 1]$: i) relationship funding, ii) do not restructure.

PROOF. In the Appendix.

Proposition 1 identifies two credit regimes. The first regime ($\bar{a} \geq \tilde{a}$) arises when lenders' technological conservatism within credit relationships is weak, that is, only firms with large collateral face the conservatism of a relationship lender. Alternatively, this regime arises when entrepreneurs derive a large benefit from credit relationships, that is, they obtain much cheaper financing. In this credit regime, which we call "flexible", all firms form credit relationships and firms only differ in their technology choice. In particular, those with medium collateral ($\underline{a} \leq a < \bar{a}$) potentially restructure whereas those with large collateral ($a \geq \bar{a}$) adopt the mature technology. The second regime ($\bar{a} < \tilde{a}$) arises when the conservatism within credit relationships is strong or entrepreneurs derive a small benefit from relationships. In this regime, which we call "conservative", firms with medium collateral value ($\bar{a} \leq a < \tilde{a}$) avoid the technological conservatism of relationship lenders by choosing transactional funding. As in the flexible regime, firms with medium collateral value ($\underline{a} \leq a < \tilde{a}$) potentially restructure while firms with large collateral ($a \geq \tilde{a}$) adopt the mature technology. In sum, the critical difference between the two credit regimes consists of the role of transactional finance. In the flexible regime, transactional finance is inactive; in the conservative regime, it is active and specializes in the financing of firms that potentially restructure their activity. In both regimes, collateral-rich firms (i.e., those with $a \in [\bar{a}, 1]$ in the flexible regime and with $a \in [\tilde{a}, 1]$ in the conservative regime) form credit relationships with lenders in which they preserve the mature technology.

While Proposition 1 derives a closed form solution, it is useful to consider numerical examples of the two credit regimes.

EXAMPLE 1: (*Flexible Regime*) Consider the parameters in Table II, first column and Panel A (second column). These parameters imply that the probability $1 - \pi$ of failure of a project is 6%; when the innovation occurs, the return from a project $y(1 + n)/i - 1$ in the event of success net of the investment cost equals 19%; in the event of success, the return of the new technology exceeds that of the mature by 9% ($= n$); if the lender carries out the action, the probability $1 - \sigma$ of innovation is 20%; 7% ($1 - l$) of the output is non-verifiable; finally, the share of value lost in liquidation by a relationship (transactional) lender amounts to $1 - M = 10\%$ ($1 - m = 12\%$). With this parameter selection the economy is in the flexible credit regime.

EXAMPLE 2: (*Conservative Regime*) Consider the parameters in Table II, first column and Panel B (second column). These parameters are the same as in Example 1, except that the share of value lost in liquidation by a relationship (transactional) lender amounts to 1% (2%). With this parameter selection the economy is in the conservative credit regime.

TABLE II.
Effects of 1% Collateral Shock

<i>Parameters (Technology)</i>	<i>Parameters (Credit Mkt)</i>	<i>Thresholds</i>		<i>Public Informat.</i>	<i>Private Informat.</i>	<i>Elastic Demand</i>
(Panel A: Flexible Credit Regime)						
	$M = 0.90$	$\underline{a} = 0.74$	$\frac{\Delta CR}{CR}$	-2.28%	-2.28%	-2.97%
$\pi = 0.94$	$m = 0.88$	$\bar{a} = 0.83$	$\frac{\Delta I}{I}$	-2.28%	-2.28%	-2.97%
$y = 0.96$			$\frac{\Delta RF}{RF}$	+1.01%	+1.01%	+1.03%
$n = 0.09$						
$i = 0.88$						
(Panel B: Conservative Credit Regime)						
$\sigma = 0.8$	$M = 0.99$	$\underline{a} = 0.67$	$\frac{\Delta CR}{CR}$	-4.44%	-2.09%	-4.59%
$l = 0.93$	$m = 0.98$	$\bar{a} = 0.75$	$\frac{\Delta I}{I}$	-2.06%	-2.06%	-2.12%
		$\tilde{a} = 0.76$	$\frac{\Delta RF}{RF}$	+1.01%	-2.09%	+1.03%

Note. The table reports a parameter selection (first and second column), implied collateral thresholds (third column), and the effects of a 1% drop in collateral productivity when a has a uniform distribution (fifth to seventh column). Numbers rounded to the second digit.

As the reader may have noticed, in the two numerical examples we have kept the values of the technological parameters fixed and we have allowed only the parameters of the credit market (i.e., M and m) to vary. The purpose of this exercise is to better show the role of lenders' information. Figure 3 helps to further understand this role. The figure illustrates that, for a given level of M , a wider gap between M and m pushes the economy into the flexible credit regime (where transactional finance is inactive). In fact, for a given degree of conservatism, a wider gap $M - m$ implies a larger benefit from credit relationships (i.e., a larger gap between the cost of relationship finance and that of transactional finance). The figure also illustrates that, for a given gap $M - m$, a higher M pushes the economy into the conservative credit regime (where transactional finance is active). In fact, for a given benefit of credit relationships, a higher value of M exacerbates the conservatism of relationship lenders. All in all, this implies that in the $(M, M - m)$ space the frontier between the conservative regime and the flexible regime is concave.⁶

5 Impact of a Shock

We now assume that on date 1, before contracts are written, an unexpected shock to collateral occurs: the aggregate productivity η of liquidated assets drops. We assume that the shock to η is small so we can evaluate its effects with the help of differential calculus. In the real sector, we focus on the effects on total investment (I), on the measure of restructuring firms (RF), on output (Y) and on the asset price (p). In the credit sector, we focus on the effect on the measure of credit relationships (CR). Henceforth, when we refer to such effects, we compare the equilibrium that obtains after the shock with the equilibrium that would obtain in the absence of the shock. For example, when we say that a credit relationship breaks down, we mean that in the absence of the shock the firm would have borrowed from a relationship lender, while after the shock it borrows from a transactional lender or it is inactive.

Proposition 2 summarizes the effects of the collateral shock in the flexible credit regime.

PROPOSITION 2: *In the flexible credit regime, the negative collateral shock reduces the measure of credit relationships and investment while it increases the measure of restructuring firms. The asset price and output drop.*

⁶These properties carry over with parameterizations alternative to that in Table II.

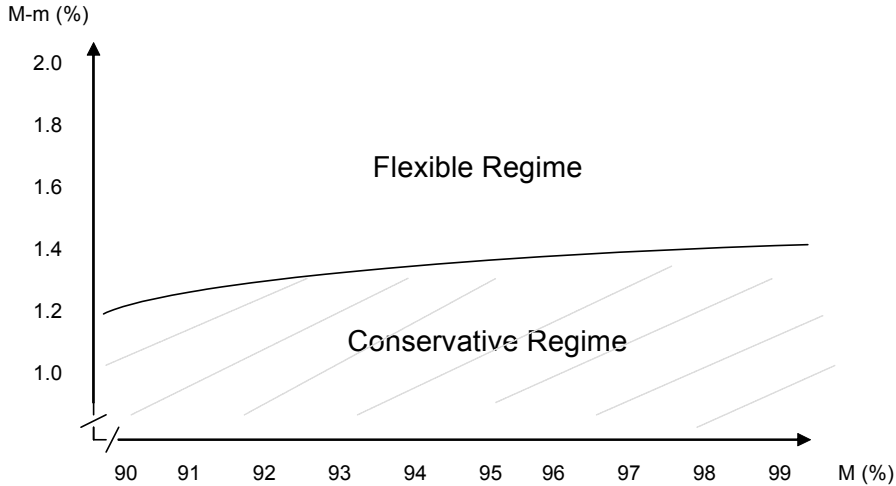


Figure 3: Credit Regimes and Lenders' Information.

PROOF. In the Appendix.

The shock to the productivity η of assets induces a fall of the asset price p because, for a given price, the demand for liquidated assets drops. The measure of firms that have access to credit is positively related to p . Hence, after the shock, the firms that were initially “marginal” in the credit market (that is, with initial collateral in the neighborhood of \underline{a}) are denied credit because they can no longer pledge enough expected returns to a lender. These firms drop out, their investment is lost and their credit relationships break down. Clearly, the exclusion of these firms from the credit market further reduces the demand and the supply of assets and feeds back on the asset price.⁷ This effect of the collateral shock on collateral poor firms replicates that obtained by Holmstrom and Tirole (1997). The new prediction is the impact of the shock on collateral-rich firms. The measure of collateral-rich firms that face conservatism within relationships in the flexible regime is negatively related to p . Thus, the firms with initial collateral in the neighborhood of \bar{a} are now allowed to restructure by their relationship lenders. These firms potentially switch to the new technology within their credit relationships. With a generic distribution, whether the measure of restructuring firms increases or decreases depends on the measure of this group of firms relative to those squeezed out by the shock from the credit market. Under the assumption of a uniform distribution of firms' collateral, the former outweighs the latter and the measure of restructuring firms increases.⁸ In the flexible regime, the output change is the result of two competing forces. The first force is the reduction of output due to the decline of investment; the second force is the increase of output due to the increased restructuring.

We now investigate the effects of the collateral shock in the conservative credit regime.

PROPOSITION 3: *In the conservative credit regime, the negative collateral shock reduces the measure of credit relationships more than in the flexible regime. Investment declines and the measure of restructuring firms rises. The asset price and output drop. If the costs for asset liquidation are real, the output drop is larger than in the flexible regime.*

⁷This means that there is a rich interaction between the exclusion of collateral-poor firms from the credit market and the restructuring of collateral-rich firms after the shock through the price of collateral assets.

⁸Of course, there are right-skewed distributions such that the measure of restructuring firms drops. However, even in this case, the ratio “(restructuring firms)/(total firms)” may increase. Finally, if this ratio declines too, the predictions of the model will resemble those of the studies on the negative effects of credit imperfections on restructuring activity during recessions (e.g., Caballero and Hammour, 2004; Barlevy, 2003).

PROOF. In the Appendix.

As in the flexible regime, the negative shock to the productivity of assets induces a fall of the asset price p . The asset price drop breaks the credit relationships of collateral-poor firms, whose investment is lost. The firms with initial collateral in the neighborhood of \bar{a} would now be allowed to restructure by relationship lenders and would prefer relationship funding. However, since they can no longer form credit relationships, they borrow from transactional lenders. Finally, the firms with initial collateral in the neighborhood of \tilde{a} prefer now borrowing from transactional lenders and (possibly) carrying out a restructuring activity (in fact, the measure of collateral-rich firms that want to preserve inertial credit relationships in the conservative regime is negatively related to p). These firms deliberately break their credit relationships and borrow from new investors.

Propositions 2 and 3 imply that the qualitative effects of the collateral shock on restructuring activity and investment in the real sector are the same across credit regimes. What differs is the way firms' restructuring occurs in the credit market. In the flexible regime, the restructuring of collateral-rich firms occurs within their credit relationships and the breakdown of relationships caused by the shock is entirely attributable to the exclusion of collateral-poor firms from the credit market. In the conservative regime, instead, the restructuring of collateral-rich firms entails the breakdown of their credit relationships. In the conservative regime, the surge in firms' restructuring is thus associated with an additional breakdown of credit relationships besides that induced by the exclusion of collateral-poor firms from the credit market. This is important for the impact of the shock on output, because, when the liquidation costs are real, this additional breakdown of credit relationships induced by firms' restructuring activity implies a surge in liquidation costs. In other words, in the conservative credit regime the output change is the result of three competing forces. Besides the output drop due the decline of investment and the output increase due to the increased restructuring, there is also the increase in transaction costs due to the breakdown of some relationships and the loss of the liquidation skills of relationship lenders. The latter effect would not occur in the flexible regime.

The results above can be further grasped with the help of the two numerical examples (see Table II).

EXAMPLE 1 (continued): *Consider the parameters in Table II, first column and Panel A (second column). Furthermore, let $\bar{\theta} = 1.05$ and $\eta = 1\%$. Then, $\Delta CR/CR = \Delta I/I \cong -2.97\%$, $\Delta RF/RF \cong +1.03\%$.*

EXAMPLE 2 (continued): *Consider the parameters in Table II, first column and Panel B (second column). Furthermore, let $\bar{\theta} = 1.05$ and $\eta = 1\%$. Then, $\Delta CR/CR = -4.59\%$, $\Delta I/I \cong -2.12\%$, $\Delta RF/RF \cong +1.03\%$.*

In the conservative regime the shock causes a 4.44% decline of the measure of credit relationships versus 2.28% in the flexible regime. The percentage loss of output equals [...].

5.1 Asset Heterogeneity

Thus far, we have considered a scenario in which mature and new assets have the same degree of redeployability in the asset market ($\alpha_n = 1$). This implies that the supply of assets does not depend on the distribution of firms between the new and the mature technology, but only on the measure of firms that have access to credit. Hence, when a negative collateral shock hits there is a contraction in the demand and in the supply of assets due to the exit of some firms from the credit market; however, the change in the measure of restructuring firms has no feedback effect on the asset price. In a context in which the redeployability of assets is lower for new technologies ($\alpha_n \leq 1$), the increase in the measure of restructuring firms will reduce the supply of assets. This will tend to increase the asset price dampening the surge in restructuring activity and the decline of investment. In the Appendix, we show that in this alternative scenario the qualitative

results of Proposition 1 remain unchanged, although a dampening effect is at work. We can then prove the following:

PROPOSITION 4: *Both in the flexible credit regime and in the conservative regime, the negative collateral shock has the same effects illustrated in Propositions 2 and 3. However, all the effects of the shock, including the increase in firms' restructuring activity, are smaller.*

PROOF. In the Appendix.

An interesting interpretation of this result is that if the new technologies are radical so that their assets significantly differ from the assets of existing technologies the increase in restructuring activity induced by the shock can be milder. However, the result suggests that the drops in investment, asset price and output can also be smaller than in the case of homogenous assets.

5.2 Robustness Analysis

In this section, we investigate the robustness of the analysis to considering scenarios alternative to that in Proposition 1. In the scenario of Proposition 1, the marginal firms that obtain credit can actually do so only if they implement the new technology (rather than the mature one). This assumption might fail, for example, if the verifiability of output is very low so that for collateral-poor firms it is actually the mature technology that can offer more pledgeable expected returns to a lender. In this alternative scenario, the results would remain essentially unchanged. Indeed, the share of restructuring firms could increase more than in the main scenario. In fact, in this case the distribution across collateral values would feature a group of firms (on the left of the three groups of firms in Figure 2) that adopt the mature technology and obtain relationship finance. Therefore, following the drop in η , some firms initially adopting the mature technology (the marginal firms that have just enough returns to pledge to a lender) would drop out of the credit market.

A second feature of the distribution in Proposition 1 is that there is an upper tail in the distribution of firms' collateral in which lenders' conservatism arises. Whether this assumption is realistic or not, and how fat this upper tail of the distribution of collateral is, depends on the structural characteristics of the economy. In an economy in which very few firms have large collateral, or new technologies have a significant productivity edge over mature ones, this condition will not hold. In this case, the only mechanism at work during a credit crunch will be a traditional mechanism of contraction of investment and reduction of restructuring activity due to tightening collateral constraints.

6 Credit and Asset Market Policy

During the 2008-2009 the Federal Reserve has carried out two types of unconventional policy (Krishnamurty, 2010). First, it has intervened directly in the asset market in order to sustain the price of falling asset values. Indeed, some scholars argue for a formal role of central banks as market makers of last resorts in secondary asset markets (Caballero and Kurat, 2009). A second type of unconventional policy has consisted of providing loans to finance asset holdings at margin requirements lower than the private sector. Our model can help understand the consequences of these two policies on firms' restructuring activity. As we are going to see an unappealing feature of such policies could be that they tend to freeze the increase in restructuring. Following Krishnamurty (2010), we can think of the first policy as [...]

7 Conclusion

We have investigated the role of the credit market in a restructuring economy. In our economy, credit relationships ease information flows between entrepreneurs and lenders and, hence, entrepreneurs' access to

credit. However, relationship lenders inhibit the restructuring of collateral-rich firms to preserve the value of their information on mature technologies. A negative collateral shock squeezes collateral-poor firms out of the credit market but fosters the restructuring of collateral-rich firms, possibly through the breakdown of their credit relationships. We have also investigated the general equilibrium interaction between the credit rationing of collateral-poor firms and the restructuring of collateral-rich firms when collateral values are endogenous.

8 APPENDIX

8.1 PROOF OF LEMMA 1.

The spread $r_n - r$ is maximum when r_n is set at its upper limit. The maximum level of r_n that can be set in the contract is $ly(1+n)$. Given this choice of r_n , the minimum level of r that guarantees non-negative expected profits to the lender if the innovation can occur satisfies

$$\pi r = \frac{i - \pi(1 - \sigma)ly(1 + n)}{\sigma} - (1 - \pi)p\sigma\mu a. \quad (7)$$

By plugging these values of r_n and r into the left hand side of (1), and operating algebraic manipulations, we obtain the condition $a < \bar{a}(\mu)$.

8.2 PROOF OF LEMMA 2.

Here we focus on the formal part of the proof; the rest is in the main text. As we argue in the main text, the non-obvious case is when only a transactional lender allows the entrepreneur to restructure, i.e. $a \in [\bar{a}(p, M), \bar{a}(p, m))$. In this case, it is easy to see that the condition under which an entrepreneur chooses transactional funding is

$$\pi[(1 - \sigma)[y(1 + n) - r_n] + \sigma(y - r)] > \pi(y - r). \quad (8)$$

The repayments r and r_n that guarantee zero profits to a relationship lender satisfy

$$\pi[(1 - \sigma)r_n + \sigma r] = i - (1 - \pi)\sigma^2 pma, \quad (9)$$

whereas the repayment r that guarantees zero profits to a transactional lender satisfies

$$\pi r = i - (1 - \pi)Mpa. \quad (10)$$

Using (10) and (9) to substitute into (22), we obtain

$$a = \frac{\pi y n (1 - \sigma)}{(1 - \pi)(M - \sigma^2 m)p}, \quad (11)$$

i.e. the second term in brackets in (3).

8.3 PROOF OF LEMMA 3.

We first prove that, when the lender allows the innovation, the minimum value of collateral at which the entrepreneur can guarantee zero profits to the lender is lower than when the lender does not allow the innovation. For this to be true it must be that

$$\frac{i - \pi ly [1 + n - n\sigma]}{\sigma^2 p (1 - \pi) \mu} < \frac{i - \pi ly}{p(1 - \pi) \mu}. \quad (12)$$

Operating algebraic manipulations, (12) can be rewritten as

$$\sigma(i - \pi ly) < \pi ly(1 + n) - i, \quad (13)$$

which always holds given the upper bound on i in (??). Furthermore, because of the better information of a relationship lender ($M > m$), it is immediate that relationship funding entails a higher amount of pledgeable returns than transactional funding. All in all, this implies that the minimum value of collateral such that a firm obtains credit is $\underline{a}(p, M)$, where $\underline{a}(p, M) > 0$ because of the lower bound on i in (??).

8.4 CHARACTERIZATION OF EQUILIBRIUM.

A number of thresholds matter in the description of the equilibrium. First, a lender does not implement the action that allows innovation if and only if

$$a \geq \bar{a}(m) \equiv \frac{\pi ly(1 + n) - i}{\sigma(1 - \pi)mp}.$$

In turn, a lender can always be induced to implement the action that allows innovation if and only if

$$a < \bar{a}(M) \equiv \frac{\pi ly(1 + n) - i}{\sigma(1 - \pi)Mp}.$$

Finally, if

$$\bar{a}(m) > a \geq \bar{a}(M),$$

a lender can only be induced to implement the action that allows innovation if the borrower chooses transactional funding. We also need to describe the thresholds that allow the borrower to obtain credit. Let

$$\underline{a}(\mu, A) \equiv \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2(1 - \pi)\mu p}.$$

Then, if the lender implements the action that allows innovation, the borrower obtains credit if and only if $a > \underline{a}(\mu, A)$, where $\mu \in \{M, m\}$ denotes the choice between transactional and relational funding. Now, let

$$\underline{a}(\mu, \bar{A}) \equiv \frac{i - \pi ly}{(1 - \pi)\mu p}.$$

If the lender does not implement the action that allows innovation, the borrower obtains credit if and only if $a > \underline{a}(\mu, \bar{A})$, where $\mu \in \{M, m\}$ is as above.

In what follows we assume that, irrespective of $\mu \in \{M, m\}$, borrowers with small enough a do not obtain credit. This implies $\underline{a}(\mu, \bar{A}) > \underline{a}(\mu, A) > 0$, i.e.,

$$A1 : \frac{i}{\pi ly} > 1 + n - n\sigma.$$

We identify outcomes by the pair (μ, α) , where $\mu \in \{M, m\}$ denotes the choice of the borrower between relational and transactional funding, and $\alpha \in \{A, \bar{A}\}$ denotes the behavior of the lender, that is, whether she chooses to implement or not the action that allows innovation.

First, note that if

$$\frac{i}{\pi ly} \geq 1 + n,$$

$\bar{a}(m) \leq 0$ and innovation never occurs. Thus, it must be that the borrower chooses relational funding. Moreover, the borrower obtains credit if and only if $a > \underline{a}(M, \bar{A})$.

Henceforth, we focus on the region of parameters given by

$$\frac{i}{\pi ly} \in (1 + n - n\sigma, 1 + n). \quad (14)$$

We proceed in steps, initially restricting attention to firms with $a \geq \bar{a}(m)$.

8.4.1 $a \geq \bar{a}(m)$: collateral rich firms

If $a \geq \bar{a}(m)$, the lender never implements the action that allows innovation. Thus, the borrower always chooses relational funding. Credit is available for all firms if and only if $\bar{a}(m) > \underline{a}(M, \bar{A})$, that is,

$$\frac{i}{\pi l y} < 1 + n \frac{M}{M + \sigma m}. \quad (15)$$

If (15) does not hold, the borrower does not obtain credit if $a \in [\bar{a}(m), \underline{a}(M, \bar{A})]$ and he obtains credit if $a > \underline{a}(M, \bar{A})$.

8.4.2 $a < \bar{a}(M)$: collateral poor firms

If $a < \bar{a}(M)$, the borrower can always choose a repayment scheme that induces the lender to implement the action that allows innovation. This implies that the borrower always chooses relational funding. The borrower chooses a repayment scheme that induces innovation if and only if

$$(1 - \sigma) \pi (1 + n) y + \sigma [\pi y + (1 - \pi) \sigma M p a] > \pi y + (1 - \pi) M p,$$

that is

$$a < a^* \equiv \frac{\pi n y}{(1 + \sigma)(1 - \pi) M p}.$$

First, if $\bar{a}(M) \leq a^*$, which can be rewritten as

$$\frac{i}{\pi l y} \geq 1 + n - \frac{n \sigma}{l(1 + \sigma)}, \quad (16)$$

the equilibrium outcome if credit is available is (M, A) . Assume that (16) holds. Credit is never available if $\underline{a}(M, A) \geq \bar{a}(M)$, that is,

$$\frac{i}{\pi l y} \geq 1 + \frac{n}{1 + \sigma}. \quad (17)$$

Note that, if (17) holds, the borrower is also not able to obtain credit if the outcome is (M, \bar{A}) . This is so because (17) implies $\underline{a}(M, \bar{A}) \geq \underline{a}(M, A)$. Assume now that (17) does not hold. In this case, the borrower does not obtain credit for all $a \leq \underline{a}(M, A)$, but he obtains credit for all $a \in (\underline{a}(M, A), \bar{a}(M))$. As above, if the borrower does not obtain credit when the outcome is (M, A) , he does not obtain credit when the outcome is (M, \bar{A}) . The reason is that, whenever (17) does not hold, we have $\underline{a}(M, \bar{A}) > \bar{a}(M)$.

It remains to consider the case where (16) does not hold, that is,

$$\frac{i}{\pi l y} < 1 + n - \frac{n \sigma}{l(1 + \sigma)}. \quad (18)$$

Note that (18) is non-empty if and only if

$$l > \frac{1}{1 + \sigma}.$$

In this case, if there is credit, the equilibrium outcome is (M, A) if $a < a^*$, and it is (M, \bar{A}) if $a \in [a^*, \bar{a}(M))$. Note that $a^* > \underline{a}(M, \bar{A})$ if and only if

$$\frac{i}{\pi l y} < 1 + \frac{n}{l(1 + \sigma)}. \quad (19)$$

This inequality is always satisfied since (18) implies (19). Thus, there is always credit in region (18) if the outcome is (M, \bar{A}) . Now, note that $a^* > \underline{a}(M, A)$ if and only if

$$\frac{i}{\pi l y} < 1 + n - n \sigma + \frac{n \sigma^2}{l(1 + \sigma)}, \quad (20)$$

which is always true since (18) implies (20). Thus, the outcome is (M, A) if $a \in [\underline{a}(M, A), a^*)$, but there is no credit if $a < \underline{a}(M, A)$. As above, it is straightforward to show that, if (18) holds, then $\underline{a}(M, \bar{A}) > \underline{a}(M, A)$. In words, if the borrower does not obtain credit under (M, A) , he also does not obtain credit under (M, \bar{A}) .

8.4.3 $a \in [\bar{a}(M), \bar{a}(m)]$: firms with intermediate collateral values

Now, if a borrower wants to have the opportunity to innovate, he must choose transactional funding. This implies that, if there is credit, the equilibrium outcome is either (m, A) or (M, \bar{A}) . We obtain that (m, A) is preferable to (M, \bar{A}) if and only if

$$(1 - \sigma) \pi (1 + n) y + \sigma [\pi y + (1 - \pi) \sigma m p a] > \pi y + (1 - \pi) M p,$$

that is,

$$a < \hat{a} \equiv \frac{(1 - \sigma) \pi n y}{(1 - \pi) p (M - \sigma^2 m)}.$$

We decompose the case of intermediate collateral values in three regions of parameters:

$$\begin{aligned} \text{A} & : 1 + n - n\sigma < \frac{i}{\pi l y} \leq 1 + n - n\sigma \frac{(1 - \sigma) M}{l(M - \sigma^2 m)} \\ \text{B} & : 1 + n - n\sigma \frac{(1 - \sigma) M}{l(M - \sigma^2 m)} < \frac{i}{\pi l y} < 1 + n - n\sigma \frac{(1 - \sigma) m}{l(M - \sigma^2 m)} \\ \text{C} & : 1 + n - n\sigma \frac{(1 - \sigma) m}{l(M - \sigma^2 m)} \leq \frac{i}{\pi l y} < 1 + n \end{aligned}$$

In region A, $\hat{a} \leq \bar{a}(M)$ and, if there is credit, the equilibrium outcome is always (M, \bar{A}) . In region C, $\hat{a} \geq \bar{a}(m)$ and, if there is credit, the equilibrium outcome is always (m, A) . Finally, in region B, if there is credit, the equilibrium outcome is (m, A) if $a \in [\bar{a}(M), \hat{a})$, and (M, \bar{A}) if $a \in [\hat{a}, \bar{a}(m))$.

Region A : $\hat{a} \leq \bar{a}(M)$ First, region A is non-empty if and only if $l > \frac{(1 - \sigma) M}{M - \sigma^2 m}$, which we assume in this subsection.

Credit is available for all borrowers if and only if $\bar{a}(M) > \underline{a}(M, \bar{A})$, that is,

$$\frac{i}{\pi l y} < 1 + \frac{n}{1 + \sigma}. \quad (21)$$

If $l \in \left(\frac{(1 - \sigma) M}{M - \sigma^2 m}, \frac{(1 - \sigma)(M + \sigma M)}{M - \sigma^2 m} \right]$, this is always the case. If $l > \frac{(1 - \sigma)(M + \sigma M)}{M - \sigma^2 m}$, this is the case if and only if

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma, 1 + \frac{n}{1 + \sigma} \right).$$

Credit is never available if and only if $\bar{a}(m) \leq \underline{a}(M, \bar{A})$, that is,

$$\frac{i}{\pi l y} \geq 1 + n \frac{M}{M + \sigma m}. \quad (22)$$

If $l > \frac{M}{m} \frac{(1 - \sigma)(M + \sigma m)}{M - \sigma^2 m}$, this is the case if and only if

$$\frac{i}{\pi l y} \in \left[1 + n \frac{M}{M + \sigma m}, 1 + n - n\sigma \frac{(1 - \sigma) M}{l(M - \sigma^2 m)} \right].$$

If, instead, $l \in \left(\frac{(1 - \sigma) M}{M - \sigma^2 m}, \frac{M}{m} \frac{(1 - \sigma)(M + \sigma m)}{M - \sigma^2 m} \right]$, then it is never the case.

We need to check whether credit would be available under (m, A) conditional on it never being available under (M, \bar{A}) . A necessary condition is $\bar{a}(m) > \underline{a}(m, A)$, that is,

$$\frac{i}{\pi l y} < 1 + \frac{n}{1 + \sigma},$$

which can never be satisfied if (22) holds.

Finally, if $l > \frac{M}{m} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, then in the interval

$$\frac{i}{\pi l y} \in \left[1 + \frac{n}{1+\sigma}, 1 + n \frac{M}{M+\sigma m} \right), \quad (23)$$

there is partial credit, that is, the borrower does not obtain credit if $a \in [\bar{a}(M), \underline{a}(M, \bar{A})]$, but he obtains credit if $a \in (\underline{a}(M, \bar{A}), \bar{a}(m))$. The same is true in the interval

$$\frac{i}{\pi l y} \in \left[1 + \frac{n}{1+\sigma}, 1 + n - n\sigma \frac{(1-\sigma)M}{l(M-\sigma^2 m)} \right], \quad (24)$$

as long as $l \in \left(\frac{(1-\sigma)(M+\sigma M)}{M-\sigma^2 m}, \frac{M}{m} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right]$. If $l \in \left(\frac{(1-\sigma)M}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma M)}{M-\sigma^2 m} \right]$, the region with partial credit is empty.

As above, we need to check whether credit would be available under (m, A) when it is not available under (M, \bar{A}) . A necessary condition is $\underline{a}(M, \bar{A}) > \underline{a}(m, A)$, that is,

$$\frac{i}{\pi l y} < 1 + n \frac{(1-\sigma)M}{M-\sigma^2 m},$$

which can never be satisfied under either (23) or (24).

Region B : $\hat{a} \in ((\bar{a}(M), \bar{a}(m)))$ First, region B is non-empty if and only if $l > \frac{(1-\sigma)m}{M-\sigma^2 m}$, which we assume in this subsection.

$a \in [\hat{a}, \bar{a}(m)) : (M, \bar{A})$ **dominates** (m, A) If $\underline{a}(M, \bar{A}) < \hat{a}$, credit is always available. This occurs when

$$\frac{i}{\pi l y} \leq 1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)}. \quad (25)$$

If $l \in \left(\frac{(1-\sigma)m}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right]$, this is always the case. If $l > \frac{(1-\sigma)(M+\sigma M)}{M-\sigma^2 m}$, this is never the case. In turn, if $l \in \left(\frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma M)}{M-\sigma^2 m} \right]$, this is the case if and only if

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma \frac{(1-\sigma)M}{l(M-\sigma^2 m)}, 1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)} \right].$$

Credit is never available if $\bar{a}(m) \leq \underline{a}(M, \bar{A})$, that is,

$$\frac{i}{\pi l y} \geq 1 + n \frac{M}{M+\sigma m}. \quad (26)$$

If $l \geq \frac{M}{m} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, this is always the case. If $l \in \left(\frac{(1-\sigma)m}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right]$, this is never the case. In turn, if $l \in \left(\frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{M}{m} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right)$, this is the case if and only if

$$\frac{i}{\pi l y} \in \left(1 + n \frac{M}{M+\sigma m}, 1 + n - n\sigma \frac{(1-\sigma)m}{l(M-\sigma^2 m)} \right].$$

We need to check whether credit would be available under (m, A) conditional on it never being available under (M, \bar{A}) . A necessary condition is $\bar{a}(m) > \underline{a}(m, A)$, that is,

$$\frac{i}{\pi l y} < 1 + \frac{n}{1+\sigma},$$

which can never be satisfied under (26).

Finally, if $l \in \left(\frac{(1-\sigma)(M+\sigma M)}{M-\sigma^2 m}, \frac{M}{m} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right)$, then in the interval

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma \frac{(1-\sigma)M}{l(M-\sigma^2 m)}, 1 + n \frac{M}{M+\sigma m} \right], \quad (27)$$

the borrower does not obtain credit if $a \in [\hat{a}, \underline{a}(M, \bar{A})]$, but he obtains credit if $a \in (\underline{a}(M, \bar{A}), \bar{a}(m))$. The same is true in the interval

$$\frac{i}{\pi l y} \in \left(1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)}, 1 + n \frac{M}{M+\sigma m} \right], \quad (28)$$

as long as $l \in \left(\frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma M)}{M-\sigma^2 m} \right]$. If either $l \geq \frac{M}{m} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$ or $l \in \left(\frac{(1-\sigma)m}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right]$, the region with partial credit is empty.

Again, it remains to check whether credit would be available under (m, A) when it is not available under (M, \bar{A}) . A necessary condition is $\underline{a}(M, \bar{A}) > \underline{a}(m, A)$, that is,

$$\frac{i}{\pi l y} < 1 + n \frac{(1-\sigma)M}{M-\sigma^2 m},$$

which can never be satisfied under either (27) or (28).

$a \in (\bar{a}(M), \hat{a}) : (m, A)$ **dominates** (M, \bar{A}) Credit is always available if and only if $\underline{a}(m, A) < \bar{a}(M)$, that is,

$$\frac{i}{\pi l y} < 1 + n - n\sigma \frac{M}{M+\sigma m}.$$

If $l \in \left(\frac{(1-\sigma)m}{M-\sigma^2 m}, \frac{m}{M} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right]$, then this is always the case. If $l \geq \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, this is never the case. In turn, if $l \in \left(\frac{m}{M} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right)$, this is the case if and only if

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma \frac{(1-\sigma)M}{l(M-\sigma^2 m)}, 1 + n - n\sigma \frac{M}{M+\sigma m} \right).$$

Credit is never available if and only if $\hat{a} \leq \underline{a}(m, A)$, that is,

$$\frac{i}{\pi l y} \geq 1 + n - n\sigma + \frac{n\sigma(1-\sigma)\sigma m}{l(M-\sigma^2 m)}.$$

If $l \geq \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, this is always the case. If $l \in \left(\frac{(1-\sigma)m}{M-\sigma^2 m}, \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m} \right]$, this is never the case. In turn, if $l \in \left(\frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right)$, this is the case if and only if

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma + \frac{n\sigma(1-\sigma)\sigma m}{l(M-\sigma^2 m)}, 1 + n - n\sigma \frac{(1-\sigma)m}{l(M-\sigma^2 m)} \right).$$

We need to check whether credit is available if the outcome is (M, \bar{A}) conditional on it being never available under (m, A) . Note that $\underline{a}(M, \bar{A}) < \hat{a}$ implies

$$\frac{i}{\pi l y} \leq 1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)}.$$

Since

$$1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)} > 1 + n - n\sigma + \frac{n\sigma(1-\sigma)\sigma m}{l(M-\sigma^2 m)},$$

we obtain that whenever

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma + \frac{n\sigma(1-\sigma)\sigma m}{l(M-\sigma^2 m)}, 1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)} \right),$$

credit is not available under (M, \bar{A}) for all $a \in (\bar{a}(M), \underline{a}(M, \bar{A})]$ but it is available under (M, \bar{A}) for all $a \in (\underline{a}(M, \bar{A}), \hat{a})$. In the complementary region, i.e., the region given by

$$\frac{i}{\pi ly} \in \left[1 + n \frac{M(1-\sigma)}{l(M-\sigma^2 m)}, 1 + n - n\sigma \frac{1(1-\sigma)m}{l(M-\sigma^2 m)} \right),$$

credit is never available under (M, \bar{A}) .

Finally, if $l \in \left(\frac{m(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m} \right)$, then in the interval

$$\frac{i}{\pi ly} \in \left[1 + n - n\sigma \frac{M}{M+\sigma m}, 1 + n - n\sigma \frac{1(1-\sigma)m}{l(M-\sigma^2 m)} \right), \quad (29)$$

the borrower does not obtain credit if $a \in [\bar{a}(M), \underline{a}(m, A)]$, but he obtains credit if $a \in (\underline{a}(m, A), \hat{a})$. The same is true in the interval

$$\frac{i}{\pi ly} \in \left[1 + n - n\sigma \frac{M}{M+\sigma m}, 1 + n - n\sigma + \frac{n\sigma(1-\sigma)\sigma m}{l(M-\sigma^2 m)} \right], \quad (30)$$

as long as $l \in \left(\frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} \right)$.

Once more, we need to check whether credit is available if the outcome is (M, \bar{A}) conditional on it being not available under (m, A) . Note that $\bar{a}(M) > \underline{a}(M, \bar{A})$ if and only if

$$\frac{i}{\pi ly} < 1 + \frac{n}{1+\sigma}.$$

If $l < \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}$, this condition is always verified in (29). If $l > \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}$, this condition is always verified in (30). Thus, all borrowers obtain credit under (M, \bar{A}) in the region with partial credit.

Region C : $\hat{a} \geq \bar{a}(m)$ First, note that region C is never empty. If $\underline{a}(m, A) \geq \bar{a}(m)$, credit is never available. This occurs when

$$\frac{i}{\pi ly} \geq 1 + \frac{n}{1+\sigma}. \quad (31)$$

If $l > \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}$, this is always the case. If, instead, $l \leq \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}$, then this is the case if and only if

$$\frac{i}{\pi ly} \in \left(1 + \frac{n}{1+\sigma}, 1 + n \right).$$

We need to check whether the outcome (M, \bar{A}) would allow access to some credit. Note that $\underline{a}(M, \bar{A}) > \bar{a}(M)$ if and only if

$$\frac{i}{\pi ly} < 1 + \frac{n}{1+\sigma},$$

which is inconsistent with (31). Thus, some borrowers who do not obtain credit under (m, A) will also be denied credit under (M, \bar{A}) . Now, note that $\underline{a}(M, \bar{A}) < \bar{a}(m)$ holds if and only if

$$\frac{i}{\pi ly} < 1 + n \frac{M}{M+\sigma m}.$$

Thus, if

$$\frac{i}{\pi ly} \in \left(1 + \frac{n}{1+\sigma}, 1 + n \frac{M}{M+\sigma m} \right), \quad (32)$$

and the outcome is (M, \bar{A}) , the borrower obtains credit if $a \in (\underline{a}(M, \bar{A}), \bar{a}(m))$, but does not obtain credit if $a \in [\bar{a}(M), \underline{a}(M, \bar{A})]$. In turn, if (32) does not hold, i.e., if

$$\frac{i}{\pi ly} > \left(1 + n \frac{M}{M+\sigma m}, 1 + n \right),$$

credit is never available, irrespective of the outcome.

We now move to the case where credit is always available. This occurs when $\underline{a}(m, A) < \bar{a}(M)$, that is,

$$\frac{i}{\pi l y} < 1 + n - n\sigma \frac{M}{M + \sigma m}. \quad (33)$$

If $l \geq \frac{m}{M} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, then this is never the case. If, instead, $l < \frac{m}{M} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, then this is the case if and only if

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma \frac{1}{l} \frac{(1-\sigma)m}{M-\sigma^2 m}, 1 + n - n\sigma \frac{M}{M + \sigma m} \right).$$

Finally, if $l < \frac{m}{M} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$, then in the interval

$$\frac{i}{\pi l y} \in \left[1 + n - n\sigma \frac{M}{M + \sigma m}, 1 + \frac{n}{1 + \sigma} \right], \quad (34)$$

the borrower does not obtain credit for all $a \in [\bar{a}(M), \underline{a}(M, \bar{A})]$, but he obtains credit for all $a \in (\underline{a}(M, \bar{A}), \bar{a}(m))$. The same is true in the interval

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma \frac{1}{l} \frac{(1-\sigma)m}{M-\sigma^2 m}, 1 + \frac{n}{1 + \sigma} \right), \quad (35)$$

as long as $l \in \left[\frac{m}{M} \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m} \right]$. Clearly, if $l > \frac{(1-\sigma)(m+\sigma m)}{M-\sigma^2 m}$, the region with partial credit is empty.

It remains to check whether the borrower gains access to credit if he chooses (M, \bar{A}) when $a \in [\bar{a}(M), \underline{a}(m, A)]$. Note that $\underline{a}(M, \bar{A}) \leq \bar{a}(M)$ if and only if

$$\frac{i}{\pi l y} \leq 1 + \frac{n}{1 + \sigma},$$

which is always the case under either (34) or (35). Thus, all borrowers who do not obtain credit under (m, A) do so under (M, \bar{A}) . This concludes the complete characterization of outcomes in equilibrium.

8.5 PROOF OF PROPOSITION 1.

In the main text, we make some assumptions that allow us to focus on the most interesting cases. As said above, we assume that, irrespective of $\mu \in \{M, m\}$, borrowers with small enough a do not obtain credit, i.e.,

$$A1 : \frac{i}{\pi l y} > 1 + n - n\sigma.$$

We also assume that, irrespective of $\mu \in \{M, m\}$, whenever the lender can be induced to implement the action that allows innovation (i.e., $a < \bar{a}(\mu)$), it must be the case that some borrowers have access to credit (i.e., $a > \underline{a}(\mu, A)$). This requires $\underline{a}(\mu, A) < \bar{a}(\mu)$, which can be rewritten as

$$A2 : \frac{i}{\pi l y} < 1 + \frac{n}{1 + \sigma}.$$

We consider two scenarios, that we discuss in turn.

Scenario 1: flexible regime Any borrower who would like to optimally choose a repayment scheme that induces the lender to implement the action can do so under relationship lending. This implies $\hat{a} \leq \bar{a}(M)$, i.e.,

$$\frac{i}{\pi l y} \in \left(1 + n - n\sigma, 1 + n - n\sigma \frac{1}{l} \frac{(1-\sigma)M}{M-\sigma^2 m} \right]. \quad (AS1)$$

In the analysis above, this scenario corresponds to region A of firms with intermediate collateral values. Note that this region is non-empty if and only if

$$l > \frac{(1-\sigma)M}{M-\sigma^2 m}.$$

Scenario 2: conservative regime Consider a borrower who would like to optimally choose transactional funding in order to induce the lender to implement the action that allows innovation, i.e., a borrower with collateral $a \in (\bar{a}(M), \hat{a})$. We initially assume that this set is non-empty, that is $\hat{a} > \bar{a}(M)$. This implies

$$\frac{i}{\pi ly} \in \left(1 + n - n\sigma \frac{1}{l} \frac{(1-\sigma)M}{M - \sigma^2 m}, 1 + \frac{n}{1+\sigma} \right).$$

The conservative regime scenario corresponds to regions B and C of firms with intermediate collateral values. Note that this region is non-empty if and only if

$$l < \frac{(1-\sigma^2)M}{M - \sigma^2 m}.$$

We further assume that a borrower with $a \in (\bar{a}(M), \hat{a})$ cannot be forced to choose relationship funding and the mature technology in order to obtain access to credit, i.e., $\underline{a}(m, A) < \bar{a}(M)$. This can be rewritten as

$$\frac{i}{\pi ly} < 1 + n - n\sigma \frac{M}{M + \sigma m}.$$

There is a non-empty intersection between the two regions above if and only if

$$l < \frac{(1-\sigma)(M + \sigma m)}{M - \sigma^2 m}.$$

In this case, we obtain

$$\frac{i}{\pi ly} \in \left(1 + n - n\sigma \frac{1}{l} \frac{(1-\sigma)M}{M - \sigma^2 m}, 1 + n - n\sigma \frac{M}{M + \sigma m} \right) \quad (\text{AS2})$$

In order to make sure that (AS1) and (AS2) are always possible, we assume

$$l \in \left(\frac{(1-\sigma)M}{M - \sigma^2 m}, \frac{(1-\sigma)(M + \sigma m)}{M - \sigma^2 m} \right).$$

We now describe the conditions under which the flexible and conservative regime arises. Given the discussion above, the relevant range of parameters is given by

$$\frac{i}{\pi ly} \in \left(1 + n - n\sigma, 1 + n - n\sigma \frac{M}{M + \sigma m} \right), \quad (\text{AS})$$

and

$$l \in \left(\frac{(1-\sigma)M}{M - \sigma^2 m}, \frac{(1-\sigma)(M + \sigma m)}{M - \sigma^2 m} \right). \quad (\text{AUX 1})$$

Equilibrium under the flexible regime ($\hat{a} \leq \bar{a}(M)$)

$$\frac{i}{\pi ly} \in \left(1 + n - n\sigma, 1 + n - n\sigma \frac{1}{l} \frac{(1-\sigma)M}{M - \sigma^2 m} \right]. \quad (\text{AS1})$$

Note that

$$a^* \equiv \frac{\pi ny}{(1+\sigma)(1-\pi)Mp} > \frac{(1-\sigma)\pi ny}{(1-\pi)p(M - \sigma^2 m)} \equiv \hat{a}.$$

There are two cases in the region where $a < \bar{a}(M)$. First, if $\bar{a}(M) \leq a^*$, that is,

$$\frac{i}{\pi ly} \geq 1 + n - \frac{n\sigma}{l(1+\sigma)},$$

then the equilibrium outcome if credit is available is always (M, A) . Since (AS) implies $\underline{a}(M, A) < \bar{a}(M)$, the borrower does not obtain credit for all $a < \underline{a}(M, A)$, but he obtains credit for all $a \in (\underline{a}(M, A), \bar{a}(M))$. It may also be the case that $\bar{a}(M) > a^*$, that is

$$\frac{i}{\pi l y} < 1 + n - \frac{n\sigma}{l(1+\sigma)}.$$

Assumption (AS) implies that this region is non-empty if and only if $l > \frac{1}{1+\sigma}$. To simplify exposition, we assume that the intersection between $l > \frac{1}{1+\sigma}$ and $l \in \left(\frac{(1-\sigma)M}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}\right)$ is empty. This requires

$$\frac{M-m}{m} > \frac{1-\sigma^2}{\sigma}. \quad (\text{AUX 2})$$

Now consider the region $a \in [\bar{a}(M), \bar{a}(m)]$. Again, (AS) implies that credit is available for all firms. Since $\hat{a} \leq \bar{a}(M)$, all firms with $a \in [\bar{a}(M), \bar{a}(m)]$ choose (M, \bar{A}) .

Finally, consider the region $a \geq \bar{a}(m)$. In this case, the lender never implements the action that allows innovation. Thus, the borrower always chooses relational funding and the outcome, if credit is available, is (M, \bar{A}) . Once more, (AS) ensures that credit is available for all firms.

Summing up, in the flexible regime, equilibrium is as follows: (1) there is no credit for $a \in [0, \bar{a}(M)]$; (2) there is credit and the outcome is (M, A) for $a \in (\underline{a}(M, A), \bar{a}(M)]$; (3) there is credit and the outcome is (M, \bar{A}) for $a > \bar{a}(M)$.

Equilibrium under the conservative regime ($\hat{a} > \bar{a}(M)$) Note that, since $\hat{a} > \bar{a}(M)$ and $a^* > \hat{a}$, then it must be that $a^* > \bar{a}(M)$. Thus, (AS) implies that the borrower does not obtain credit for all $a \leq \underline{a}(M, A)$, but he obtains credit and the outcome is (M, A) for all $a \in (\underline{a}(M, A), \bar{a}(M))$.

Consider now the region $a \in [\bar{a}(M), \bar{a}(m)]$. We start with the case $\hat{a} \in ((\bar{a}(M), \bar{a}(m)))$, which corresponds to region B of firms with intermediate collateral values. First, let $a \in [\hat{a}, \bar{a}(m)]$. In this case, if there is credit, the outcome is (M, \bar{A}) . Assumption (AS) and $(AUX1)$ imply that credit is always available. We now move to the region $a \in [\bar{a}(M), \hat{a}]$. In this case, if there is credit, the outcome is (m, A) . Again, (AS) and $(AUX1)$ imply that credit is always available.

Finally, consider the region $a \geq \bar{a}(m)$. In this case, the lender never implements the action that allows innovation. Thus, the borrower always chooses relational funding and the outcome, if credit is available, is (M, \bar{A}) . Assumption (AS) then implies that this is always the case.

Summing up, in the conservative regime, equilibrium is as follows: (1) there is no credit for $a \in [0, \bar{a}(M)]$; (2) there is credit and the outcome is (M, A) for $a \in (\underline{a}(M, A), \bar{a}(M)]$; (3) there is credit and the outcome is (m, A) for $a \in [\bar{a}(M), \hat{a}]$; (4) there is credit and the outcome is (M, \bar{A}) for $a > \hat{a}$.

8.5.1 Asset price

Now let us solve for the asset price. Remember that firms' collateral a is uniformly distributed in the interval $[0, 1]$ and that a firm's ability to reuse assets is given by $\eta\theta$, where θ is uniformly distributed in the interval $[0, \bar{\theta}]$. Since

$$\underline{a} = \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 p (1 - \pi) M},$$

the demand for assets is

$$D(p) = \max \left\{ [1 - \underline{a}(p)] \pi \left(1 - \frac{p}{\eta \bar{\theta}} \right), 0 \right\},$$

while the supply of assets is

$$S(p) = \max \left\{ \frac{1}{2} (1 - \pi) [1 - (\underline{a}(p))^2], 0 \right\}.$$

Clearly, p such that $\underline{a}(p) \geq 1$ is an equilibrium with zero supply and zero demand. This occurs when

$$p \leq \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 (1 - \pi) M}.$$

In this equilibrium, no firm receives credit since

$$(1 - \sigma)\pi ly(1 + n) + \sigma [\pi ly + (1 - \pi)\sigma Mp] \geq i$$

implies

$$\frac{i}{\pi ly} \leq 1 + n - n\sigma,$$

which is generically not true. Thus, in what follows we consider equilibria where $\underline{a}(p) < 1$, that is,

$$p > \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2(1 - \pi)M}.$$

Note that an equilibrium price must involve $p < \eta\bar{\theta}$, otherwise there would be no demand for assets. Thus, equating demand and supply, we obtain

$$\frac{\pi}{\eta\bar{\theta}}p^2 + \frac{1 - 3\pi}{2}p + \frac{1}{2}(1 - \pi)\frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2(1 - \pi)M} = 0$$

from which

$$p = \frac{-\frac{1-3\pi}{2} \pm \left(\left(\frac{1-3\pi}{2} \right)^2 - \frac{1}{\eta\bar{\theta}} 2\pi(1-\pi)\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M} \right)^{\frac{1}{2}}}{2\frac{\pi}{\eta\bar{\theta}}}.$$

We impose the following conditions

$$p_- < \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2(1 - \pi)M} \Leftrightarrow \frac{i}{\pi ly} < 1 + n - n\sigma + \eta\bar{\theta}\frac{(2\pi - 1)(1 - \pi)\sigma^2 M}{\pi^2 ly}$$

and

$$p_+ > \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2(1 - \pi)M} \Leftrightarrow \frac{i}{\pi ly} < 1 + n - n\sigma + \eta\bar{\theta}\frac{(2\pi - 1)(1 - \pi)\sigma^2 M}{\pi^2 ly}.$$

For these conditions to hold, we need

$$\pi > \frac{1}{2}$$

and $\bar{\theta}$ large enough. Note also that $p_+ < \eta\bar{\theta}$ always holds. Thus, there exists a unique non-trivial equilibrium price, namely

$$p = \frac{1}{\frac{2\pi}{\eta\bar{\theta}}} \left\{ \pi - \frac{1-\pi}{2} + \left[\left(\pi - \frac{1-\pi}{2} \right)^2 - \frac{2\pi}{\eta\bar{\theta}} \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2 M} \right]^{\frac{1}{2}} \right\}.$$

8.6 PROOF OF PROPOSITION 2.

To compute the change in the measure of restructuring firms and in the measure of credit relationships, one has to compare the inflows and outflows of firms from each region of the domain of a identified in Proposition 1. As for the change in investment, this is computed by multiplying i by the change in the measure of active firms. The measure of firms that obtain credit is

$$\mu_{\text{credit}} = 1 - F(\underline{a}),$$

or

$$\mu_{\text{credit}} = 1 - \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2(1 - \pi)\mu\bar{\theta}\frac{1}{\frac{2\pi}{\eta\bar{\theta}}} \left\{ \pi - \frac{1-\pi}{2} + \left[\left(\pi - \frac{1-\pi}{2} \right)^2 - \frac{2\pi}{\eta\bar{\theta}} \frac{i - \pi ly(1 + n - n\sigma)}{\sigma^2 M} \right]^{\frac{1}{2}} \right\}}.$$

In the flexible regime, the measure of firms that restructure is

$$\mu_{\text{restructure}}^{\text{flexible}} = F(\bar{a}) - F(\underline{a}),$$

or

$$\mu_{\text{restructure}}^{\text{flexible}} = \frac{\pi l y (1 + n + \sigma) - i (1 + \sigma)}{\sigma^2 (1 - \pi) M \bar{\theta} \frac{1}{\eta \bar{\theta}} \left\{ \pi - \frac{1 - \pi}{2} + \left[\left(\pi - \frac{1 - \pi}{2} \right)^2 - \frac{2\pi}{\eta \bar{\theta}} \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 M} \right]^{\frac{1}{2}} \right\}}.$$

It is easy to see that $\frac{\partial \mu_{\text{credit}}}{\partial \eta} > 0$ and $\frac{\partial \mu_{\text{restructure}}^{\text{flexible}}}{\partial \eta} > 0$. *PROVE THIS?*

8.7 PROOF OF PROPOSITION 3.

This proof is analogous to that of Proposition 2. In the conservative regime, the measure of firms that restructure is (where $\tilde{a}(p) \equiv \min \left\{ \bar{a}(p, m), \frac{\pi y n (1 - \sigma)}{(1 - \pi)(M - \sigma^2 m)p} \right\}$)

$$\mu_{\text{restructure}}^{\text{conservative}} = F(\tilde{a}) - F(\underline{a}).$$

If $\bar{a}(m) \leq a(m, M)$, we have

$$\mu_{\text{restructure}}^{\text{conservative}} = \frac{\pi l y (1 + n) - i}{p \sigma (1 - \pi) \mu} - ,$$

while, if $\bar{a}(m) > a(m, M)$, we have

$$\mu_{\text{restructure}}^{\text{conservative}} = \frac{2}{\bar{\theta}} \frac{\pi l y (M - \sigma^2 m) \left[1 + n - n\sigma \frac{(M - \sigma^2 m)l - (1 - \sigma)\sigma M}{(M - \sigma^2 m)l} - \frac{i}{\pi l y} \right]}{\frac{3\pi - 1}{2\pi} + \left[\left(\frac{3\pi - 1}{2\pi} \right)^2 - \frac{2}{\bar{\theta}} \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 \pi M (1 - \eta)} \right]^{\frac{1}{2}}}.$$

In turn, in the conservative regime, the measure of firms choosing relational funding is

$$\mu_{\text{relational}}^{\text{conservative}} = F\left(\frac{\bar{a}}{1 - \eta p} \frac{1}{p}\right) - F\left(\frac{\underline{a}}{1 - \eta p} \frac{1}{p}\right) + 1 - F\left(\frac{\tilde{a}}{1 - \eta p} \frac{1}{p}\right) = \frac{\bar{a}}{1 - \eta p} \frac{1}{p} - \frac{\underline{a}}{1 - \eta p} \frac{1}{p} + 1 - \frac{\tilde{a}}{1 - \eta p} \frac{1}{p}.$$

If $\bar{a}(m) \leq a(m, M)$, we have

$$\mu_{\text{relational}}^{\text{conservative}} = 1 + \frac{2}{\bar{\theta}} \frac{\pi l y (\sigma m + m - \sigma M) \left[1 + n - n\sigma \frac{m}{m - \sigma(M - m)} - \frac{i}{\pi l y} \right]}{\frac{3\pi - 1}{2\pi} + \left[\left(\frac{3\pi - 1}{2\pi} \right)^2 - \frac{2}{\bar{\theta}} \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 \pi M (1 - \eta)} \right]^{\frac{1}{2}}},$$

while, if $\bar{a}(m) > a(m, M)$, we have

$$\mu_{\text{relational}}^{\text{conservative}} = 1 + \frac{2}{\bar{\theta}} \frac{\pi l y \frac{1 + \sigma}{\sigma^2 M} \left[1 + n - n\sigma + \frac{n\sigma(M - \sigma^2 m)l - (1 - \sigma)\sigma^2 M}{(1 + \sigma)l(M - \sigma^2 m)} - \frac{i}{\pi l y} \right] \frac{1}{(1 - \pi)(1 - \eta)}}{\frac{3\pi - 1}{2\pi} + \left[\left(\frac{3\pi - 1}{2\pi} \right)^2 - \frac{2}{\bar{\theta}} \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 \pi M (1 - \eta)} \right]^{\frac{1}{2}}}.$$

8.8 PROOF OF PROPOSITION 4.

Under the assumption that collateral a is uniformly distributed in the interval $[0, 1]$ and that a firm's ability to reuse assets is given by $\eta\theta$ (with θ uniformly distributed in the interval $[0, \bar{\theta}]$), we have

$$\underline{a} = \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 p \left(\frac{1}{2} - \pi \right) M},$$

and the demand for assets is

$$D(p) = \max \left\{ [1 - \underline{a}(p)] \pi \left(1 - \frac{p}{\eta\bar{\theta}} \right), 0 \right\}.$$

The supply of assets now depends on the degree of restructuring. There is potential restructuring as long as

$$a \in \left[\underline{a}(p), \underbrace{\max \{ \bar{a}(p), \tilde{a}(p) \}}_{\hat{a}(p)} \right], \text{ where}$$

$$\bar{a}(p) = \frac{1}{p} \frac{\pi l y (1+n) - i}{\sigma(1-\pi)M}$$

and

$$\tilde{a}(p) = \min \left\{ \frac{1}{p} \frac{\pi l y (1+n) - i}{\sigma(1-\pi)m}, \frac{1}{p} \frac{\pi y n (1-\sigma)}{(1-\pi)(M - \sigma^2 m)} \right\}.$$

In this case, the supply of assets is

$$S(p) = \int_{\bar{a}(p)}^1 a da + \int_{\underline{a}(p)}^{\min\{\hat{a}(p), 1\}} \left[\underbrace{(1-\sigma)\alpha_n + \sigma}_{\hat{\alpha}_n} \right] a da = \frac{1-\pi}{2} \left\{ 1 - (\hat{a}(p))^2 + \alpha_n \min \left[(\hat{a}(p))^2 - (\underline{a}(p))^2, 1 - (\underline{a}(p))^2 \right] \right\}.$$

As before, we restrict attention to non-trivial equilibria in which

$$p > \underbrace{\frac{i - \pi l y (1+n - n\sigma)}{\sigma^2(1-\pi)M}}_A \text{ and } p < \eta\bar{\theta}.$$

Equating demand and supply, we obtain

$$\begin{aligned} [1 - \underline{a}(p)] \pi \left(1 - \frac{p}{\eta\bar{\theta}} \right) &= \frac{1-\pi}{2} \left\{ 1 - (\hat{a}(p))^2 + \hat{\alpha}_n \left[(\hat{a}(p))^2 - (\underline{a}(p))^2 \right] \right\} \\ \left(1 - \frac{1}{p} \underline{a} \right) \pi \left(1 - \frac{p}{\eta\bar{\theta}} \right) &= \frac{1-\pi}{2} \left\{ 1 - \left(\frac{1}{p} \hat{a} \right)^2 + \hat{\alpha}_n \left[\left(\frac{1}{p} \hat{a} \right)^2 - \left(\frac{1}{p} \underline{a} \right)^2 \right] \right\} \\ \pi \left(1 - \frac{1}{p} \underline{a} \right) \left(1 - \frac{p}{\eta\bar{\theta}} \right) &= \frac{1-\pi}{2} \left[1 - \frac{1}{p^2} \hat{a}^2 + \hat{\alpha}_n \left(\frac{1}{p^2} \hat{a}^2 - \frac{1}{p^2} \underline{a}^2 \right) \right] \\ \pi \left(1 - \frac{1}{p} \underline{a} \right) \left(1 - \frac{p}{\eta\bar{\theta}} \right) &= \frac{1-\pi}{2} \left\{ 1 - \frac{1}{p^2} \left[\hat{\alpha}_n \underline{a}^2 + (1 - \hat{\alpha}_n) \hat{a}^2 \right] \right\} \end{aligned}$$

$$\begin{aligned} D(\underline{a}) &= D(\eta\bar{\theta}) = 0 \\ D'(p) &= \pi \left(\frac{\underline{a}}{p^2} - \frac{1}{\eta\bar{\theta}} \right) = \frac{\pi}{p} \left(\frac{\underline{a}}{p} - \frac{p}{\eta\bar{\theta}} \right) \\ D''(p) &= -\frac{2\pi\underline{a}}{p^3} < 0 \end{aligned}$$

$$\begin{aligned} S(\underline{a}) &= 0 \\ S'(p) &= \frac{(1-\pi) \left[\hat{\alpha}_n \underline{a}^2 + (1 - \hat{\alpha}_n) \hat{a}^2 \right]}{p^3} \\ S''(p) &= -\frac{3(1-\pi) \left[\hat{\alpha}_n \underline{a}^2 + (1 - \hat{\alpha}_n) \hat{a}^2 \right]}{p^4} < 0 \end{aligned}$$

We assume

$$\begin{aligned}
S'(\underline{a}) &< D'(\underline{a}) \\
\frac{(1-\pi) [\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2]}{\underline{a}^3} &< \frac{\pi}{\underline{a}} \left(1 - \frac{\underline{a}}{\eta\theta}\right) \\
\frac{(1-\pi) [\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2]}{\underline{a}} &< \frac{\pi}{\underline{a}} \left(1 - \frac{\underline{a}}{\eta\theta}\right) \\
(1-\pi) \frac{\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2}{\underline{a}^2} &< \pi \left(1 - \frac{\underline{a}}{\eta\theta}\right) \\
\frac{\pi}{1-\pi} &> \frac{\frac{\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2}{\underline{a}^2}}{1 - \frac{\underline{a}}{\eta\theta}}
\end{aligned}$$

We also assume

$$\begin{aligned}
S((\underline{a}\eta\bar{\theta})^{\frac{1}{2}}) &< D((\underline{a}\eta\bar{\theta})^{\frac{1}{2}}) \\
\frac{1-\pi}{2} \left\{1 - \frac{1}{\underline{a}\eta\bar{\theta}} [\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2]\right\} &< \pi \left(1 - \frac{1}{(\underline{a}\eta\bar{\theta})^{\frac{1}{2}}} \underline{a}\right) \left(1 - \frac{(\underline{a}\eta\bar{\theta})^{\frac{1}{2}}}{\eta\bar{\theta}}\right) \\
(1-\pi) \left\{\frac{1}{2} - \frac{\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2}{2\underline{a}\eta\bar{\theta}}\right\} &< \pi \left(1 - \frac{\underline{a}^{\frac{1}{2}}}{(\eta\bar{\theta})^{\frac{1}{2}}}\right)^2 \\
\frac{\pi}{1-\pi} &> \frac{\frac{1}{2} \frac{1 - \frac{\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2}{\underline{a}\eta\bar{\theta}}}{\left(1 - \frac{\underline{a}^{\frac{1}{2}}}{(\eta\bar{\theta})^{\frac{1}{2}}}\right)^2}}
\end{aligned}$$

Together, $S'(\underline{a}) < D'(\underline{a})$ and $S((\underline{a}\eta\bar{\theta})^{\frac{1}{2}}) < D((\underline{a}\eta\bar{\theta})^{\frac{1}{2}})$ imply that there is a unique equilibrium, and that the price in equilibrium is above $(\underline{a}\eta\bar{\theta})^{\frac{1}{2}}$.

$$\begin{aligned}
\frac{\partial p}{\partial \eta} &= \frac{\pi \left(1 - \frac{\underline{a}}{p}\right) \frac{p}{\eta^2 \bar{\theta}}}{\pi \left[\left(1 - \frac{\underline{a}}{p}\right) \frac{1}{\eta\bar{\theta}} - \left(1 - \frac{p}{\eta\bar{\theta}}\right) \frac{\underline{a}}{p^2}\right] + (1-\pi) \frac{[\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2]}{p^3}} \\
\frac{\partial p}{\partial \eta} &= \frac{\pi \left(1 - \frac{\underline{a}}{p}\right) \frac{p}{\eta^2 \bar{\theta}}}{\pi \left(\frac{1}{\eta\bar{\theta}} - \frac{\underline{a}}{p^2}\right) + (1-\pi) \frac{[\widehat{\alpha}_n \underline{a}^2 + (1-\widehat{\alpha}_n) \widehat{a}^2]}{p^3}}
\end{aligned}$$

Note that

$$\frac{1}{\eta\bar{\theta}} - \frac{\underline{a}}{p^2} > 0 \Leftrightarrow p > (\underline{a}\eta\bar{\theta})^{\frac{1}{2}},$$

which always holds. Hence, $\frac{\partial p}{\partial \eta} > 0$.

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