Coarse updating: Bertrand competition

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Abstract

In this paper we propose a theory of Bertrand competition where firms form beliefs by considering only a subset of the available variables. This theory of coarse updating leads to equilibria where firms set prices according to constant mark-up strategies, which is consistent with the evidence from survey data. It also can explain the high price dispersion observed in practice without requiring an unreasonable level of cost dispersion. In general more sophisticated firms who consider all variables have incentives to deviate from coarse equilibria. However we show that when costs are sufficiently correlated, equilibria under coarse updating correspond to ϵ-equilibria under Bayesian updating. Hence in this case even more sophisticated firms have little incentive to deviate from coarse equilibria. Finally in the case of procurement auctions this theory of coarse updating leads to a particularly simple estimation procedure where costs can be recovered from pricing data using fewer observations than standard methods.

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1 Introduction

There is extensive empirical evidence that firms engage in mark-up pricing and set prices by adding a constant percentage mark-up to their marginal cost. For example, when asked how the price of their main UK product was determined, 44% of firms replied that direct cost per unit plus a fixed percentage mark-up was an important or very important component of their price-setting strategy (Greenslade & Parker (2010) and Greenslade & Parker (2012)). Meanwhile a survey of German firms reported that firms take calculated unit costs as a reference and then choose a mark-up based on market and competition conditions (Stahl (2005)). Moreover in a survey of Italian firms 63% reported applying a mark-up to unit variable costs (Fabiani *et al.* (2010)).

These surveys provide substantial evidence that mark-up pricing plays an important role in pricing strategies, in particular when firms have market power. Moreover the mark-up variable is extensively discussed in the business community and there is ample anecdotal evidence that mark-up strategies play an important role in firms' decision making. For instance Rothkopf (1980) writes that *discussions with the preparers of bids in industry do not disclose a policy of systematically varying the multiple of an estimate that is bid as a function of that estimate*, which suggests that mark-up pricing is prevalent. Despite the extensive evidence, the standard pricing model does not predict that firms bid according to mark-up pricing. Indeed the standard model predicts that firms will vary their mark-up according to their cost parameter, tending to bid a lower mark-up when their cost parameter is higher.

In light of this shortcoming of the standard model, we build on the theory of cue competition developed in psychology and apply it to the pricing problem. This theory has several advantages over alternative theories since it is (i) motivated by psychological evidence, (ii) consistent with empirical findings and (iii) easy to recover costs from bids when firms compete in procurement auctions. Building on the theory of cue competition developed in psychology leads us to introduce a new assumption about how firms form beliefs, which we refer to as coarse updating. The theory of coarse updating we consider assumes that firms do not use information about the magnitude of their marginal cost when forming beliefs about the expected quantity they will produce, and results in equilibria where firms use constant mark-up strategies. Meanwhile in the case of procurement auctions the theory leads to a simple procedure for estimating costs when the bids submitted are known. As well as capturing the fact that firms bid according to constant mark-up strategies, this estimation procedure requires less data than existing techniques, such as the procedure proposed by Campo *et al.* (2003).

Generating the price dispersion observed in practice[1] using the standard model typically requires assuming an underlying cost distribution that is unreasonably dispersed and where some firms set unreasonably high-markups. An *et al.* (2010) provide a solution for this problem, by considering an unknown number of bidders, while Krasnokutskaya (2011) addresses these issues by considering unobserved auction heterogeneity. The model proposed here gives an alternative solution. We show that the mark-ups required in the standard model of a procurement auction are higher than under coarse updating. Similarly the cost dispersion required to obtain a certain price dispersion in equilibrium is smaller under coarse updating.

Standard models assume that firms are highly sophisticated and form beliefs by conditioning on all available variables. In particular when forming beliefs about the likely quantity produced y, firms condition on two pieces of information namely the mark-up chosen x and the cost parameter c. The resulting predictions derived from the standard model are often inappropriate because they do not capture the fact that in many situations decision makers have (i) a limited mental capacity for making a decision, (ii) possess only limited information or (iii) need to decide quickly (see for example Arthur (1994) and Rubinstein (1998)).

Several approaches to deal with bounded rationality have been proposed in the literature. Examples include level-k thinking where players go through k-levels of reasoning when trying to make a decision (see Nagel (1995) and Crawford & Irriberri (2007)), or models of framing (Kahnemann & Tversky (1981)), as well as models of bounded recall (see Aumann & Sorin

[1]Examples exist both in the case of symmetric costs, as well as asymmetric costs. See for example Sorensen (2000), Baye & Morgan (2004).

(1989)) and rational inattention (see Sims (2003) and Mackowiak & Wiederholt (2009) among others). The model proposed here fits into the large family of models on bounded rationality: the cognitive constraint modelled in this paper, is that players do not condition their beliefs on all the available variables but rather form their beliefs according to coarse updating. More precisely it is assumed that firms only condition on a subset of the available variables when forming beliefs. In particular when forming beliefs about the likely quantity produced y, firms condition on the mark-up chosen x but not on the cost parameter c.

The assumption of coarse updating introduced here is most directly related to the literature on categorization as done by Mullainathan *et al.* (2008), Peski (2010) and Mohlin (2014) . In these models, players try to predict the unobserved attribute of an item, by considering the corresponding attribute of a similar item. In order to make predictions players group similar items in categories, hence pooling information. Indeed Mullainathan *et al.* (2008) present a different version of coarse updating related to persuasion and argue the following: *individuals do not have separate mental representations for every situation. Instead they have only one representation for all the situations in a category and are effectively unable to differentiate between these situations.* This is also the motivation underlying the version of coarse updating proposed here.

To illustrate the assumption of coarse updating made in this paper, we now give an example. Suppose a firm has two possible cost parameters $\{c_L, c_H\}$ each of which occur with probability 50% and chooses one of two possible mark-ups $\{x_L, x_H\}$. The table on the left in Figure 1 gives the actual probability of winning a procurement auction given that a firm has a certain cost parameter and chooses a certain mark-up. This actual probability of winning a procurement auction corresponds to the beliefs that the firm would form under Bayesian updating. Meanwhile the table on the right in Figure 1 gives the beliefs that a firm who coarsely updates would assign to the event of winning a procurement auction in each of the different situations:

Hence under coarse updating the firm pools observations and no longer distinguishes between

Full Bayesian updating

	c_L	c_H
x_L	85	75
x_H	25	15

$\Longrightarrow$ Coarsening

Coarse Bayesian updating

	c_L	c_H
x_L	80	80
x_H	20	20

Figure 1: Coarsening of Beliefs

the two marginal costs c_H and c_L in determining its probability of winning when choosing either mark-up. For instance the belief of winning the auction given a mark-up x_L is used is given to be $80\% = (0.5 \times 85\%) + (0.5 \times 75\%)$.

Our primary motivation for considering coarse updating comes from the literature in psychology on cue-competition. This literature examines how decision-makers predict a variable that is stochastically related to multiple variables (or cues). One key finding is that a more valid or more salient cue detracts from the learning of a less valid or less salient cue, and that people do not achieve correct understanding of the correlation structure (Kruschke & Johansen (1999) and Hirshleifer (2001)).

To show how this is relevant to our setting consider a decision-maker who is trying to predict the output y, given that he observes both the mark-up chosen x and his marginal cost c. A fully sophisticated Bayesian learner with prior f forms beliefs based on both variables as follows:

$$\Omega(y|x,c) = \frac{f(y,x,c)}{\sum_y f(y,x,c)}$$

Now suppose that the mark-up variable x is highly correlated with output y, while the marginal cost c is only weakly correlated with y. When a decision maker is cognitively-constrained it seems likely that he will ignore the marginal cost variable c and just focus on the more relevant mark-up variable x. In this case, he might use a coarse version of Bayesian updating and form the following beliefs:

$$\Omega(y|x) = \sum_c \frac{f(y,x,c)}{\sum_y f(y,x,c)}$$

Note that beliefs formed by coarse updating are insensitive to changes in marginal cost, since firms who form beliefs using coarse updating disregard the cost parameter variable when forming their beliefs. This feature of coarse updating leads to an equilibrium where firms bid constant mark-ups because - unlike in the standard model - the beliefs of firms do not depend on their marginal cost. Hence the assumption of coarse updating leads to a model that is consistent with the fact that firms play constant mark-ups.

The motivation for choosing percentage mark-up to be the salient cue is that this variable receives significant attention in the business literature: this makes it likely that it is a variable firms monitor and is an important component of firms' decision making. Moreover considering coarse updating avoids problems associated with the standard model. As well as capturing the fact that firms play according to constant mark-up strategies, a model with coarse updating predicts that firms never set price equal to marginal cost. There is ample evidence that this is indeed the case and that firms always choose prices strictly above their marginal cost - see for example Baye & Morgan (2004). However an immediate implication of the standard model is that the firm with highest marginal cost sets a price equal to its marginal cost: moreover the prices all other firms set are derived from this initial condition. Therefore all the results of the standard model are based on an initial condition, which is not satisfied in reality. A significant advantage of the model proposed here is that it does not suffer from such an objection.

Instead of modelling cognitive constraints directly, an alternative way to ensure players with limited mental capacity choose simple strategies is to carefully design the model in such a way that simple strategies are indeed optimal. This is done by Abreu & Brunnermeier (2003) where investors are fully rational, but in equilibrium it is optimal for investors not to condition on their private information when deciding how long to hold onto the asset. Similarly in Klemperer (1999) bidders are fully rational, but in equilibrium it is optimal for bidders not to condition on their valuation when deciding their mark-down. Eccles & Wegner (2014) generalise these results giving conditions under which players have no incentive to condition on their private information when choosing their action. In this paper we build on this result and show that in certain cases where types are highly correlated, even sophisticated firms who condition on all available variables have little incentive to deviate from a coarse equilibrium.

A more direct way to ensure that firms bid according to mark-up pricing is to impose strategy restrictions as proposed by Compte & Postlewaite (2013). Under strategy restrictions firms submit linear bid functions before learning their cost: this means that after observing their cost firms in general have an incentive to deviate. On the other hand under coarse updating firms choose their bids after learning their costs: this ensures that firms have no incentive to deviate and are behaving optimally at the interim stage. Therefore linear bid functions arise indirectly due to the assumption of coarse updating, whereas they are imposed directly when strategy restrictions are assumed. Finally both theories lead to different results: they are not equivalent ways of looking at the same problem.

The model proposed in this paper can be used for the estimation of procurement auctions, which has received considerable attention in the literature (see for example Guerre *et al.* (2000) and Athey & Haile (2002)). Existing estimation techniques for auctions with bidder specific and hence asymmetric strategies - as proposed by Athey & Haile (2002) and Campo *et al.* (2003) - require a very large number of observations. Our model could be used to obtain player specific estimates for smaller data sets, leading to similar results when the distribution of types approximately has the correlated structure discussed above. On the other hand in settings where the analyst has external evidence that constant mark-up bidding is being used, the estimation procedure proposed here could be used as an alternative to the standard estimation procedure. We propose an estimation method for the model presented in this paper.

The remainder of this paper is structured is follows. In section two we introduce the model for the case of procurement auctions. The analysis of this model when the distribution of costs is known follows in section three. Meanwhile section four investigates the case where the distribution of prices is known and section five considers the estimation of the model. Section six considers the case of more general case of Bertrand competition and finally section seven concludes.

2 The model

We first consider the special case of a single unit private value procurement auction, where the winner is paid his bid. A general model of Bertrand competition with a general demand function is presented in section 6.

There is a set of firms denoted by $I = \{1, \ldots, n\}$. Each firm privately observes its marginal cost $c_i \in \mathbb{R}_{++}$. Having observed their costs, firms then simultaneously submit a price $p_i \in P_i = \mathbb{R}_+$. We use $\mathbf{c} = (c_1, \ldots, c_n)$ and $\mathbf{p} = (p_1, \ldots, p_n)$ to refer to the vector of all firms' costs and prices respectively. In a single unit procurement auction, only the firm setting the lowest price produces, hence the quantity produced by firm i, given the vector of prices submitted $\mathbf{p}$ is denoted by $y_i \in Y_i = \{0, 1\}$ and is given as follows:

$$y_i = \begin{cases} 1 & \text{if } p_i < p_j \text{ for all } j \neq i \\ 0 & \text{otherwise} \end{cases}$$

We assume that in case of a tie, the contract is not awarded.

The profit earned by a firm is given by $y_i(p_i - c_i)$. Moreover it is assumed that marginal costs are distributed according to a cumulative distribution function $F : \mathbb{R}^n_{++} \mapsto [0, 1]$, where $F_i : (0, \infty) \mapsto [0, 1]$ for all $i \in I$.

Define $C_i \subseteq (0, \infty)$ to be the set of possible cost parameters for firm i. A pure strategy for firm i is a mapping from possible cost parameters to prices, such that $b_i : C_i \mapsto P_i$. Applying the bid function to the distribution of costs, creates a distribution of prices $G : \mathbb{R}^n_+ \mapsto [0, 1]$ which can be written as follows:

$$G(\mathbf{p}) = \int_{\mathbb{R}^n_{++}} \mathbf{1}_{\left\{b_i(c_i) \le p_i \text{ for all } i\right\}} f(\mathbf{c}) d\mathbf{c}$$

We complete the description of the model by defining the beliefs held by firms as follows: Each firm i has a belief $\Omega_i : C_i \times P_i \times Y_i \mapsto [0, 1]$. The term $\Omega_i(y_i|p_i, c_i)$ captures the probability firm i assigns to either winning ($y_i = 1$) or losing ($y_i = 0$) the auction given that his marginal cost is c_i and his price is p_i. This belief structure is used to dovetail with the cue competition theory used in psychology: here the player forms beliefs or predictions about y_i based on the variables (p_i, c_i). Coarse updating will involve dividing the space (p_i, c_i) into categories and assigning one belief to each category.

2.1 Equilibrium

It is assumed that an equilibrium is formed by a set of strategy profiles $(b_1, ...b_n)$ and a set of beliefs $(\Omega_1, ..., \Omega_n)$, one for each player. We will define two types of equilibrium: (i) equilibrium under Bayesian updating and (ii) equilibrium under coarse updating, referred to as coarse equilibrium. Equilibrium under Bayesian updating corresponds to the equilibrium concept used in standard models where players form beliefs by conditioning on all available information; meanwhile coarse equilibrium captures situations where players condition on only some of the available information.

Under both equilibrium concepts firms choose their price in order to maximize profits. The following optimality condition, states that no player believes that he has a profitable deviation leading to a higher expected profit.[2]

[2] In the unlikely case that two or more prices maximize profit, it is assumed that the firm chooses the lowest of these prices:

Condition 1 (Optimality). *For all c_i*

$$b_i(c_i) = \min \left\{ \arg\max_{p_i} \left\{ \Omega_i(1|p_i, c_i)c_i - p_i) \right\} \right\}$$

This condition has to be satisfied under any equilibrium definition.

Before turning to the second condition required for the definition of equilibrium we introduce some additional notation. We use b_{-i} to denote the set of bid functions $(b_j)_{j \neq i}$ excluding the bid function of player i and similarly write $b_{-i}(c_{-i})$ to denote the set of prices $(b_j(c_j))_{j \neq i}$. Moreover define $\phi_i(y_i|p_i, c_i, b_{-i})$ to be the probability that firm i is awarded the contract $(y_i = 1)$ or loses the auction $(y_i = 0)$ given (i) firm i submits a price p_i, (ii) firm i has a marginal cost of c_i and (iii) opponents play according to strategies b_{-i}. Hence:

$$\phi_i(1|p_i, c_i, b_{-i}) \;:=\; \int_{\mathbb{R}_+^{n-1}} f(c_{-i}|c_i)\mathbf{1}\left\{p_i < b_j(c_j) \text{ for all } j \neq i\right\}dc_{-i} \tag{1}$$

$$\phi_i(0|p_i, c_i, b_{-i}) \;=\; 1 - \phi_i(1|p_i, c_i, b_{-i}) \tag{2}$$

The second condition for equilibrium depends on the way in which beliefs are formed. We therefore present two equilibrium definitions, one when firms form beliefs according to standard Bayesian updating, as well as one for coarse updating.

2.2 Bayesian updating

Under the first equilibrium concept it is required that firms update their beliefs according to Bayes' rule, conditioning on both their marginal cost c_i and the price chosen p_i. Therefore the following condition must be satisfied:

Condition 2 (Bayesian updating). *Suppose firms are playing according to strategies* $(b_1, ..., b_n)$ *and beliefs* $(\Omega_1, ..., \Omega_n)$ *are formed according to Bayesian updating. Then beliefs are given as* *follows:*

$$\Omega_i(y_i|p_i, c_i) = \phi_i(y_i|p_i, c_i, b_{-i})$$

Using these two conditions we can now define a Bayesian Nash equilibrium:

Definition 1. *The set of strategy profiles* $(b_1, ...b_n)$ *and the set of beliefs* $(\Omega_1, ..., \Omega_n)$ *constitute a Bayesian Nash equilibrium if they satisfy condition 1 optimality and condition 2 Bayesian updating.*

As discussed above, standard models of price competition fail to explain certain findings, in particular (i) the fact that firms often follow constant mark-up strategies and (ii) the fact that firms never bid at marginal cost. In order to capture these findings in a model we consider alternative assumptions on belief formation and in particular do not assume that firms update their beliefs according to Bayesian updating.

2.3 Coarse updating

We now turn to the formation of beliefs under coarse updating. The term mark-up parameter is used to refer to the term $x_i = p_i/c_i$. Hence a mark-up parameter of 1 signifies that a firm sets a price at marginal cost, whereas a mark-up parameter of 1.5 means that a firm sets a price at a level equal to 150% of marginal cost. When firms form their beliefs according to mark-up updating, we assume that they do not differentiate between two instances (x_i, c_i) and (x_i, c_i'). In the terminology of the categorization literature, firms have one category for each choice of mark-up $x_i \in X$. Firms first partition the space $P \times C_i$ into categories $x \in X = \mathbb{R}_+$, and then assign beliefs to each category $x \in X$.

The average of all instances within a category is computed to form the following condition for mark-up updating:

Condition 3 (Coarse updating)**.** *Suppose firms are playing according to strategies* $(b_1, ..., b_n)$ *and beliefs* $(\Omega_1, ..., \Omega_n)$ *are formed according to mark-up updating. Then beliefs are given as follows:*

$$\Omega_i(y_i|x_i c_i, c_i) = \int_0^\infty \phi_i\Big(y_i|x_i\tilde{c}_i, \tilde{c}_i, b_{-i}\Big) f_i(\tilde{c}_i) d\tilde{c}_i$$

Since firms form their beliefs by taking an average over all marginal costs, beliefs do not depend on marginal cost (assuming the mark-up x_i is held constant). This is captured in the following equation:

$$\Omega_i(y_i|x_i c_i, c_i) = \Omega_i(y_i|x_i c_i', c_i') \text{ for all } c_i \ \ c_i' \tag{3}$$

In order to achieve an equilibrium under coarse updating, which we will refer to as coarse equilibrium, it is required that firms form their beliefs conditioning only on the mark-up chosen $x_i = p_i/c_i$. and that the optimality condition is satisfied, with firms having no incentives to deviate. This leads to the following definition:

Definition 2. *The set of strategy profiles* $(b_1, ...b_n)$ *and the set of beliefs* $(\Omega_1, ..., \Omega_n)$ *constitute an equilibrium under mark-up updating if and only if they satisfy condition 1 (optimality) and condition 3 (mark-up updating).*

Hence equilibria under coarse updating also preserve optimality from the firms' point of view: no player believes that he can profit from deviating. In this case only the mark-up variable (x) is considered when forming beliefs.

3 Analysis with known distribution of costs

We now analyse the behaviour of firms under both Bayesian updating and coarse updating, allowing to compare and contrast the outcomes resulting from the two ways of forming beliefs.

3.1 Bayesian updating

The case where firms form their beliefs using Bayesian updating has been extensively studied in the literature. Since firms are fully rational and form their beliefs by conditioning on all variables the following lemma holds:

Lemma 3.1. *Suppose beliefs $(\Omega_i)_{i\in I}$ and strategy profiles $(b_i)_{i\in I}$ form an equilibrium under Bayesian updating. Then for all c_i and for all p_i:*

$$\Omega_i(1|p_i;c_i) = \int_{\mathbb{R}_+^{n-1}} f_{-i}(c_{-i}|c_i)\mathbf{1}\Big\{p_i < b_j(c_j) \text{ for all } j \neq i\Big\}dc_{-i}$$

The maximization problem of firm i can now be written as follows:

$$\max_{p_i} \Big\{\Omega_i(1|p_i;c_i)(p_i - c_i)\Big\} \tag{4}$$

Let $\omega_i(1|p_i;c_i)$ denote the derivative of $\Omega_i(1|p_i;c_i)$. This is the change in the firm's belief about how likely it is to win the contract when raising its price slightly. Differentiating equation 4 with respect to p_i and evaluating at $p_i = b_i(c_i)$ leads to:

$$\omega_i\Big(1|b_i(c_i);c_i\Big)\Big(b_i(c_i) - c_i\Big) + \Omega_i\Big(1|b_i(c_i);c_i\Big) = 0 \tag{5}$$

Re-arranging leads to the following proposition:

Proposition 3.2. *Suppose beliefs (Ω_i) and strategy profiles $(b_i)_{i\in I}$ form an equilibrium under Bayesian updating. Then for all $i \in I$ and $c_i \in C_i$ the following condition is satisfied:*

$$b_i(c_i) = c_i + \frac{\Omega_i(1|b_i(c_i);c_i)}{-\omega_i(1|b_i(c_i);c_i)}$$

Note that this is the standard equilibrium condition found in procurement auctions, written in terms of a firm's expectations. Closed form solutions can be obtained for different cost distributions F, which determine the beliefs.

3.2 Coarse updating

Recall that in the case of coarse updating, firms only condition on the mark-up submitted when predicting expected output. We now show that when players form beliefs according to coarse updating, they bid according to constant mark-up strategies.

First note that in the case of mark-up updating firms do not condition on their marginal cost when forming their beliefs about expected output. This leads to the following lemma:

Lemma 3.3. *Suppose beliefs* $(\Omega_i)_{i\in I}$ *and strategy profiles* $(b_i)_{i\in I}$ *form an equilibrium under mark-up updating. Then for all* c_i *and for all* x_i*:*

$$\Omega_i(1|x_i c_i; c_i) = \int_{\mathbb{R}^n_+} f(\mathbf{c})\mathbf{1}\Big\{x_i c_i < b_j(c_j) \textit{ for all } j \neq i\Big\}d\mathbf{c}$$

Hence the belief of firm i about how likely it is to be awarded the contract, depends only on the mark-up parameter x_i:

$$\Omega_i(y_i|x_i c_i; c_i) = \Omega_i(y_i|x_i c_i'; c_i') \quad \text{for all} \quad c_i, c_i' \tag{6}$$

In this case a firm ignores any information his marginal cost gives about relative competitiveness, and assumes that his likelihood of being awarded the contract depends only on the mark-up chosen. Crucially he associates low mark-ups with competitive bidding and high mark-ups with uncompetitive bidding regardless of the absolute price level. This assumption is particularly appropriate when firms experience mostly common shocks that affect the marginal cost of all firms. We now make the following definition:

Definition 3. *Suppose firms are playing according to constant mark-up strategies, where* $b_i(c_i) = x_i c_i$ *for all* i*. Then* $Q_i(x_i|x_{-i})$ *is defined to be:*

$$Q_i(x_i|x_{-i}) = \int_{\mathbb{R}^n_+} f(\mathbf{c})\mathbf{1}\Big\{x_i c_i < x_j c_j \textit{ for all } j \neq i\Big\}d\mathbf{c}$$

Note that in the case of constant mark-up strategies the term $Q_i(x_i, x_{-i})$ captures both (i) the firm's belief about the probability with which it wins the contract $Q_i(x_i, x_{-i}) = \Omega_i(1|x_i c_i; c_i)$. From now on - when constant mark-up strategies are being considered - the term $Q_i(x_i, x_{-i})$ will be referred to as the expected quantity produced.

We now define $q_i(x_i|x_{-i})$ to be the derivative of $Q_i(x_i|x_{-i})$ with respect to x_i. Hence $Q_i(x_i|x_{-i})$ denotes the expected quantity produced, while $q_i(x_i|x_{-i})$ denotes the rate of change in expected quantity as the mark-up is increased and hence the likelihood with which a firm goes from winning the auction to losing the auction by raising its price slightly. Note that in a procurement auction $-q_i(x_i|x_{-i}) \geq 0$. Using this notation we now state the following proposition:

Proposition 3.4. *Suppose $(b_1, ..., b_n)$ is an equilibrium under mark-up updating. Then bidding strategies for all firms $i \in I$ are linear and of the form $b_i(c_i) = x_i^* c_i$. Moreover the following system of equations is satisfied:*

$$x_i^* = 1 + \frac{Q_i(x_i^*|x_{-i}^*)}{-q_i(x_i^*|x_{-i}^*)} \text{ for all } i \in I$$

This proposition states that any coarse equilibrium has a linear form. This stems from the fact that firms disregard their cost parameter when thinking about their choice of mark-up: rather than forming precise beliefs based both on the mark-up parameter x_i and the cost parameter c_i they form beliefs based solely on the mark-up parameter x_i. This removes any incentive for firms to condition their mark-up on their marginal cost, and hence leads to constant mark-up equilibria.

A partial intuition for this result can be given by considering the profit firm i expects to earn given that (i) his marginal cost is c_i, (ii) he submits a price $b_i(c_i) = x_i c_i$ and (iii) other firms play according to constant mark-ups $b_j(c_j) = x_j^* c_j$. We call this the subjective profit function since it relates to the profits that firm i expects to receive, which may differ from the profits a modeller would expect firm i to receive:

$$\Pi_i^S(x_i, x_{-i}^*|c_i) = Q_i(x_i|x_{-i}^*)(x_i - 1)c_i \tag{7}$$

In equilibrium firm i must have no incentive to deviate. Hence differentiating this subjective profit function with respect to the mark-up x_i leads to the following first order condition:

$$\frac{\delta \Pi_i^S(x_i, x_{-i}^*)}{\delta x_i} = \Big[q_i(x_i|x_{-i}^*)(x_i^* - 1) + Q_i(x_i|x_{-i}^*)\Big] c_i \tag{8}$$

Setting the first order condition equal to 0 and rearranging leads to the following:

$$- q_i(x_i|x^*_{-i})(x^*_i - 1) = Q_i(x_i|x^*_{-i}) \tag{9}$$

Here the left-hand side corresponds to the cost of raising the mark-up slightly, since the expected quantity of goods sold will decrease. Meanwhile the right-hand side corresponds to the benefit from raising the mark-up slightly, since goods will be sold at a greater margin. In equilibrium all firms are behaving optimally and hence the benefit from raising the price corresponds exactly to the cost of raising the price. Re-arranging the first order condition this is the expression in proposition 3.4. This explains why any constant mark-up equilibrium $(x^*_1, ..., x^*_n)$ must satisfy the system of equations in proposition 3.4.

3.3 Existence

In the previous section we presented a property that any coarse equilibrium must satisfy. We now introduce two conditions which ensure that such a constant mark-up equilibrium does in fact exist when firms form beliefs according to coarse updating. One way to ensure existence is to put conditions on the primitives of the model, namely the distribution functions $(F_i)_{i \in I}$.

However in order for our conditions to be easily interpretable, we choose instead to place conditions on the subjective profit functions $(\Pi^S_i)_{i \in I}$ that describe the profits firms expect to receive from playing certain constant mark-up strategies. These profit functions are defined in the previous section in terms of the primitives. The first condition ensures that price cuts from rivals never induce a firm to raise its prices:

Assumption 1. *Suppose that for all c_i:*

$$\Pi^S_i(x_i, x_{-i}|c_i) \geq \Pi^S_i(\hat{x}_i, x_{-i}|c_i) \;\; \textit{for all} \;\; \hat{x}_i$$
$$\Pi^S_i(\tilde{x}_i, \tilde{x}_{-i}|c_i) \geq \Pi^S_i(\hat{x}_i, \tilde{x}_{-i}|c_i) \;\; \textit{for all} \;\; \hat{x}_i$$

then if element by element $x_{-i} \geq \tilde{x}_{-i}$, then $x_i \geq \tilde{x}_i$.

This condition says the following. Suppose playing a mark-up x_i is an optimal mark-up when opponents are playing mark-ups x_{-i}. Now if opponents lower their mark-ups to some $\tilde{x}_{-i} \leq \tilde{x}_{-i}$, then any new optimal mark-up $\tilde{x}_i$ will be weakly below the previous mark-up x_i. In other words it is never optimal to increase the mark-up in response to opponents decreasing their mark-up. When the mark up played by opponents is higher, player i's optimal mark-up is also weakly higher. This captures most real-life situations, where more competitive bidding from one firm is typically met with more competitive bidding from others. The second condition effectively puts a ceiling on the level that mark-ups can reach:

Assumption 2. *There exists $\overline{x}_i$ such that whenever $x_i \geq \overline{x}_i$ and $x_i \geq x_j$ for all j then:*

$$\Pi_i^S(x_i.x_{-i}|c_i) \geq \Pi_i^S(\hat{x}_i, x_{-i}|c_i) \quad \textit{whenever} \quad \hat{x}_i > x_i$$

This condition says that for every firm i there exists a ceiling $\overline{x}_i$ with the following property. Whenever firm i is submitting a mark-up which is both (i) above the ceiling $\overline{x}_i$ and (ii) above the mark-up of every other player x_j then firm i weakly prefers not to raise its mark-up further. This condition captures the fact that when firms are bidding sufficiently high, the benefits of undercutting and submitting a mark-up $x_i < x_j$ outweigh the benefits of continuing to bid a high mark-up. This captures most real-life situations, where excessively high mark-ups give the firm choosing the highest mark-up an incentive to cut prices in order to increase the probability of winning.

Proposition 3.5. *Suppose assumption 1 and assumption 2 hold. Then there exists a coarse equilibrium of the form $b_i(c_i) = x_i^* c_i$ for all $i \in I$.*

This proposition shows that existence of equilibrium can be guaranteed by adding two mild assumptions which are often satisfied in applications.

3.4 Efficiency

In this section we investigate the efficiency of a procurement auction when players use coarse updating. Given the existence conditions above hold, in the case of two symmetric firms we can give a closed form solution for the mark-up that firms use:

Proposition 3.6. *Consider two symmetric firms where $F(c_1, c_2) = F(c_2, c_1)$. Suppose assumptions 1 and 2 are satisfied. Then there exists a symmetric equilibrium where firms bid $b_i(c_i) = x^* c_i$ where x^* is given by:*

$$x^* = \frac{2\int_{\underline{c}}^{\overline{c}} cf(c,c)dc}{2\int_{\underline{c}}^{\overline{c}} cf(c,c)dc - 1}$$

Propositions 3.4 and 3.6 immediately imply that when two symmetric firms play according to the symmetric equilibrium in a procurement auction, the auction is efficient. The firm with the lowest cost submits the lowest bid, since in this equilibrium all firms multiply their cost by the same constant mark-up. Therefore the firm with the lowest cost wins the auction. Note however that in general the outcome of procurement auctions under mark-up updating need not be efficient. In particular when firms' costs are drawn from asymmetric distributions, they may set different mark-ups. In this case if the firm with the lowest cost is also setting a higher mark-up than some of its competitors, the firm's bid may be higher than that of it's opponents. This leads to an inefficient outcome.

3.5 Correlated costs

We now investigate particular cases where costs are correlated. These cases of cost correlation are important because a higher marginal cost for one firm is often associated with a higher marginal cost for other firms. Indeed using independent types as a benchmark seems inadequate in the case of procurement auctions, where costs tend to be highly positively correlated. This section shows that when the distribution structure of firms' costs is correlated in a certain way, there is little incentive for firms to deviate from a coarse equilibrium.

We restrict attention to case of a procurement auction with two firms, although results could easily be generalised to cover n firms. The parameter θ represents some common component that affects the cost of both firms, while z_i and z_j represent firm specific elements of the cost. In particular the cost of player i is given to be $c_i = \theta(1 + \tau z_i)$. The parameter τ is a measure of the degree of correlation of firms' costs. When $\tau = 0$ both firms have the same cost and hence costs are perfectly correlated. Here we consider the case where $\tau \to 0$. It is assumed that θ is drawn from some cumulative probability distribution $G(\theta)$ with derivative $g(\theta)$. Furthermore the parameters z_i and z_j are independently drawn from some distributions $H_i(z_i)$ and $H_j(z_j)$, with supports $[\underline{z}_i, \overline{z}_i]$ and $[\underline{z}_j, \overline{z}_j]$ respectively. It is further assumed that $\underline{z} = \min\{\underline{z}_i, \underline{z_j}\} > 0$ and $\overline{z} = \max\{\overline{z}_i, \overline{z_j}\} < \infty$.

In order to make comparisons between coarse equilibrium and Bayesian equilibrium we introduce the concept of ϵ-Bayesian equilibrium. Under an ϵ-Bayesian equilibrium firms cannot increase their expected profits by more than a fraction ϵ by deviating to another strategy.

Definition 4. *The strategy profile (b_i, b_j) is an ϵ-Bayesian equilibrium if for all c_i and for all p_i*

$$\Big(b_i(c_i) - c_i\Big) \int_{\mathbb{R}_+} \mathbf{1}\Big\{b_i(c_i) < b_j(c_j)\Big\} dc_j \geq \frac{1}{1+\epsilon}\Big(p_i - c_i\Big) \int_{\mathbb{R}_+} \mathbf{1}\Big\{p_i < b_j(c_j)\Big\} dc_j$$

For a fixed $G(\theta)$ as $\tau \to 0$ and costs become perfectly correlated, the equilibria reached by coarse updating are also ϵ-Bayesian equilibria:

Proposition 3.7. *Suppose there exists a K such that $\left|\frac{\theta g'(\theta)}{g(\theta)}\right| < K$ for all $\theta \in [0, \infty)$. Then for all ϵ there exists a $\tau^*(\epsilon)$ such that whenever $0 < \tau < \tau^*(\epsilon)$, any coarse equilibrium is an ϵ-Bayesian equilibrium.*

Therefore as long as G is sufficiently smooth and τ is sufficiently small, any coarse equilibrium is also an ϵ-Bayesian equilibrium. This result stems from the fact that when τ is small, firms do not know whether their cost is relatively low or high compared to that of their opponents. This makes it optimal for firms not to condition their mark-up on their marginal cost. The next result focuses on a particular distribution of G in order to tackle the case where τ is not necessarily small:

Proposition 3.8. *Suppose*

$$g(\theta) = \begin{cases} \frac{\delta\theta^{\delta}}{2\theta} & \textit{if} \quad \theta \leq 1 \\ \frac{\delta\theta^{-\delta}}{2\theta} & \textit{if} \quad \theta > 1 \end{cases}$$

For all $\epsilon > 0$ for all $\tau > 0$ there exists a $\delta^*(\epsilon, \tau)$ such that whenever $0 < \delta < \delta^*(\epsilon, \tau)$, any coarse equilibrium is an ϵ-Bayesian equilibrium.

This shows that whenever $g(\theta) \approx \frac{1}{\theta}$, then any coarse equilibrium is approximately a Bayesian equilibrium. This results stems from the fact that when θ is drawn from an uninformative prior, firms do not know whether their realisation of the cost is likely to be relatively high or low compared to that of the other firm.

This section shows that when firms' costs are correlated in one of two particular ways, coarse updating leads to equilibria that are approximately Bayesian equilibria. First when τ is small and costs are highly correlated firms cannot deduce their position in the distribution from their marginal cost, and coarse equilibria are ϵ-Bayesian equilibria. Secondly when $g(\theta)$ is approximately distributed according to the uninformative prior, then similarly firms do not receive information about how their marginal cost compares to that of the other firm. As is shown in Eccles & Wegner (2014), in this case there exist Bayesian equilibria where firms do not condition on their marginal cost. To summarise, coarse updating are ϵ-Bayesian equilibria in those settings where firms' costs provide them little rank information.

4 Analysis with known distribution of prices

In this section we show how the firms' costs and the equilibrium mark-ups can be recovered from the distribution of prices. We now assume that firms play according to a constant mark up equilibrium $b_i(c_i) = c_i x_i^*$ as generated by the model and that the resulting distribution of prices is given by $G(\mathbf{p})$ Note that from differentiability of F, it follows that $G(\mathbf{p})$ is differentiable and hence we can use $g(\mathbf{p})$ to denote the corresponding pdf. From now on we restrict attention to procurement auctions, though similar results could be obtained whenever the demand function is known.

The term $Q_i(x_i^*|x_{-i}^*)$ denotes the expected quantity firm i produces in equilibrium and - given the demand function is known - can directly be calculated from the distribution of prices:

$$Q_i(x_i|x_{-i}) = \int_{\mathbb{R}_+^n} \mathbf{1}_{\left\{p_i<p_j \text{ for all } j\neq i\right\}} g(\mathbf{p})d\mathbf{p} \tag{10}$$

Secondly the term $Q_i(x_i^*(1+\epsilon)|x_{-i}^*)$ captures the expected quantity firm i would have received had he increased his mark-up parameter by a proportion ϵ and hence set prices using the bid function $\hat{b}(c_i) = c_i x_i^*(1+\epsilon)$. This term can be calculated by inflating the price submitted by a proportion ϵ and observing the new winner of the auction. Hence the term $Q_i(x_i^*(1+\epsilon)|x_{-i}^*)$ can also be observed directly, and is given as follows:

$$Q_i(x_i^*(1+\epsilon)|x_{-i}^*) = \int_{\mathbb{R}_+^n} \mathbf{1}_{\left\{p_i(1+\epsilon)<p_j \text{ for all } j\neq i\right\}} g(\mathbf{p})dp \tag{11}$$

Finally by taking the limit as $\epsilon \to 0$ the term $-x_i^* q_i(x_i^*|x_{-i}^*)$ can be deduced:

$$-x_i^* q_i(x_i^*|x_{-i}^*) = \lim_{\epsilon\to 0} \frac{Q_i(x_i^*|x_{-i}^*) - Q_i(x_i^*(1+\epsilon)|x_{-i}^*)}{\epsilon} \tag{12}$$

We now return to the original formula for calculating mark-ups:

$$x_i^* = 1 + \frac{Q_i(x_i^*|x_{-i}^*)}{-q_i(x_i^*|x_{-i}^*)} \tag{13}$$

Although $Q_i(x_i^*|x_{-i}^*)$ is easy to calculate, it is difficult for the econometrician to recover the term $q_i(x_i^*|x_{-i}^*)$ directly: the effects of slight variations in prices can be observed, but since x^* is not known, the econometrician cannot evaluate the effect of a slight change in the mark-up. In order to deal with this problem, we re-arrange equation 13 to give:

$$x_i^* = \frac{-x_i^* q_i(x_i^*|x_{-i}^*)}{-x_i^* q_i(x_i^*|x_{-i}^*) - Q_i(x_i^*|x_{-i}^*)} \tag{14}$$

As shown above both $Q_i(x_i^*|x_{-i}^*)$ and $-x_i^* q_i(x_i^*|x_{-i}^*)$ can be calulated from the distribution of prices. This means that the mark-up of each player can be calculated from the distribution of prices. This result is summarised in the following proposition:

Proposition 4.1. *Suppose firms are playing according to a coarse equilibrium and the distribution of prices is given by G. Then there is a unique distribution of costs F that generates this distribution of prices. Moreover in the case of a two player procurement auction, the mark-up of each firm is given by:*

$$x_i^* = \frac{\int_0^\infty p_i g(p_i, p_i) dp_i}{\int_0^\infty p_i g(p_i, p_i) dp_i - \int_0^\infty \int_{p_i}^\infty g(p_i, p_j) dp_j dp_i}$$

This proposition shows that the model is identified: every distribution G of outcomes determines a unique distribution F of primitives. Moreover when the distribution G of prices is known then the mark-up parameters $(x_1^*, ..., x_n^*)$ chosen by firms are uniquely determined.

4.1 Cost Dispersion

Theoretical models often struggle to explain the high variation of prices observed in reality, as doing so typically requires some firms to set unreasonably high mark-ups and a large dispersion of costs. A notable exception is Krasnokutskaya (2011) who significantly reduces mark-ups required to create a price dispersion of a magnitude observed in practice, by considering unobserved auction heterogeneity. We now take the distribution of prices G is fixed and determine the corresponding distribution of costs under coarse updating and under Bayesian updating respectively. We restrict attention to the case where players are symmetric, although we allow for costs to be correlated.[3]

We say that a distribution F generates G under a coarse (Bayesian) equilibrium if there exists a coarse (Bayesian) equilibrium (b_1, b_2) such that $F(c_1, c_2) = G(b_1(c_1), b_2(c_2))$. With this terminology in mind, we now state the following proposition:

[3]We believe similar results would hold for the asymmetric case.

Proposition 4.2. *Consider a two-player procurement auction with distribution of prices* $G : [\underline{p}, \overline{p}]^2 \mapsto [0,1]$ *where* $G(p_1, p_2) = G(p_2, p_1)$. *Suppose the distribution* $F^{(1)} : [\underline{c}_1.\overline{c}_1] \mapsto [0,1]$ *generates* G *under a coarse equilibrium and* $F^{(2)} : [\underline{c}_2.\overline{c}_2] \mapsto [0,1]$ *generates* G *under a Bayesian equilibrium. Then:*

1. $[\underline{c}_1, \overline{c}_1] \subset [\underline{c}_2, \overline{c}_2]$

   if and only if

2. $2 \int_{\mathbb{R}_+} p_i g(p_i, p_i) dp_i > \underline{p} g(\underline{p}, \underline{p}) + 1$

This proposition gives a necessary and sufficient condition for the cost distribution obtained when considering coarse updating to be a strict subset of the cost distribution obtained when considering Bayesian updating. This condition is satisfied for many distributions, such as when prices are drawn independently from a uniform distribution on $[a, b]$ (with $2a < b$). This shows that for a wide range of price distributions coarse updating leads to less cost dispersion than Bayesian updating. This suggests that the standard model may predict too much cost dispersion because it assumes firms are more sophisticated than they actually are. The less cognitively demanding theory of coarse updating predicts less cost dispersion, which is in line with the empirical evidence. This lends further credibility to the theory of coarse updating.

5 Estimation with a finite sample of prices

Although the estimation method proposed does not apply exclusively to procurement auctions, we assume throughout that the demand function $(D_i)_{i \in I}$ is known by the econometrician. In procurement auctions this is typically the case. Moreover estimation of procurement auctions has been dealt with in a large number of papers under varying assumptions. For example, Guerre *et al.* (2000) provide a non parametric estimation technique for the case of independent costs, while Athey & Haile (2002) allow for asymmetries as well as common values. Moreover Campo *et al.* (2003) propose a structural estimation method for private value auctions with affiliated values allowing for asymmetric firms and Krasnokutskaya (2011) considers unobserved auction heterogeneity in procurement auctions. The estimation procedure proposed in this paper provides a way to estimate the mark-ups chosen by asymmetric firms. Since in this model estimating the bid function reduces to estimating a single parameter, the mark-up, the number of observations required to estimate firm specific and possibly asymmetric bid functions is significantly smaller than in models previously considered.

5.1 Estimation procedure

Let N be the total number of auctions observed. First define the demand function $D_i(p_i, p_{-i})$ as follows:[4]

$$D_i(p_i, p_{-i}) = \begin{cases} 1 & \text{if} \quad p_i < p_j \text{ for all } j \neq i \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

Now let $p_{i,k}$ denote the bid of firm i in round k and $p_{-i,k}$ denote the bids of the firms other than i in round k. Define $y_{i,k} = D_i(p_{i,k}, p_{-i,k})$. Let $b^+_{i,k,\epsilon} = p_{i,k}(1+\epsilon)$ and $b^-_{i,k,\epsilon} = p_{i,k}(1-\epsilon)$ represent the bid function where all prices are ϵ higher and respectively ϵ lower than the actual price observed. Finally let $y^+_{i,k,\epsilon} = D_i(p^-_{i,k,\epsilon}, p_{-i,k})$ and $y^-_{i,k,\epsilon} = D_i(p^+_{i,k}, p_{-i,k})$ be the hypothetical quantities firm i would have produced had it always bid ϵ lower or ϵ higher.

First note that $Q_i(x_i^*|x_{-i}^*)$ denotes the expected quantity that firm i produces, assuming that all firms play according to the strategy profile $b_i(c_i) = c_i x_i^*$. Hence a consistent estimator $\hat{Q}_i$ for $Q_i(x_i^*|x_i^*)$ can be given as follows:

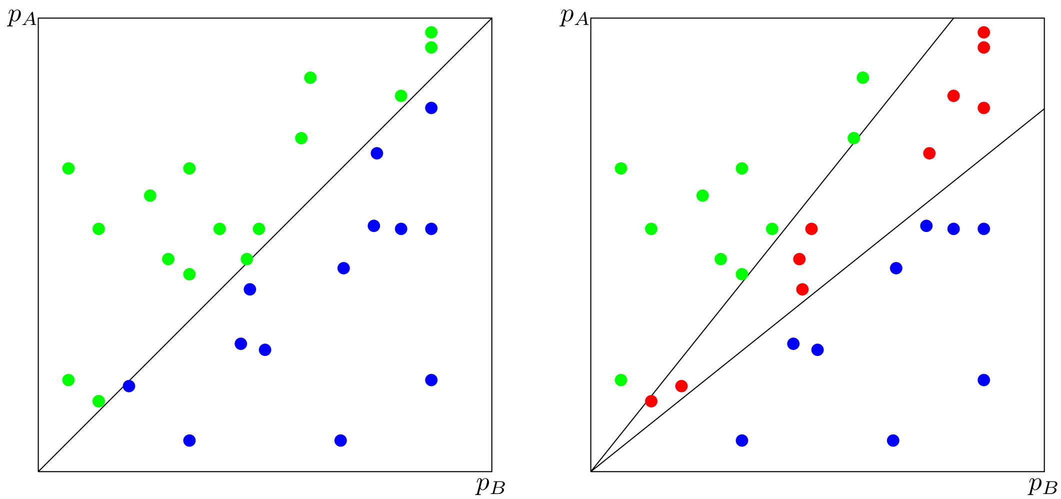
$$\hat{Q}_i = \frac{\sum_{y_i} \sum_{k=1}^{N} \mathbf{1}\{y_{i,k} = y_i\}}{N} \tag{16}$$

Secondly define the estimator $\hat{R}_{i,\epsilon}$ as follows:

$$\hat{R}_{i,\epsilon} = \frac{\sum_{y_i} \sum_{k=1}^{N} \mathbf{1}\{y^+_{i,k,\epsilon} - y^-_{i,k,\epsilon} = y_i\}}{2N\epsilon} \tag{17}$$

[4] A similar estimation procedure could be carried out for other demand functions not corresponding to procurement auctions.

Figure 2: Illustration: Estimation



Note that $Q_i\Big(x_i^*(1\pm\epsilon)|x_i^*\Big)$ denotes the expected quantity that firm i produces, assuming all other bidders play according to the strategy profile $b_j(c_j) = c_j x_j^*$ and firm i bids according to the strategy profile $b_j(c_j) = c_j x_j^*(1\pm\epsilon)$. Hence $\hat{R}_{i,\epsilon}$ is a consistent estimator for $\frac{}{\epsilon}\Bigg(Q_i\Big(x_i^*(1-\epsilon)|x_i^*\Big) - Q_i\Big(x_i^*(1+\epsilon)|x_i^*\Big)\Bigg) \approx -x_i^* q_i(x_i^*|x_{-i}^*)$. Hence as $\epsilon \to 0$, then $\hat{R}_{i,\epsilon}$ becomes a consistent estimator of $-x_i^* q_i(x_i^*|x_{-i}^*)$. This motivates us to suggest the following estimator $\hat{x}_i$ for the mark-up x_i:

$$\hat{x}_i = \frac{\hat{R}_{i,\epsilon}}{\hat{R}_{i,\epsilon} - \hat{Q}_i} \tag{18}$$

The appropriate bandwidth ϵ depends on the dispersion of prices observed and the number of observations N. A larger ϵ will tend to increase bias, whereas a smaller ϵ will tend to increase the variance of the estimate. Figure 2 illustrates how for a given ϵ, the parameters $\hat{R}_i$ and $\hat{Q}_i$ can be found in a given sample with two firms.

6 General Bertrand competition

Now consider the case of a general demand function, this allows for discriminatory multi-unit procurement auctions or Bertrand competition with differentiated products. Depending on the prices submitted, each firm faces demand $y_i = D_i(p_i, p_{-i})$ where $y_i \in \{0, 1, 2, ..., K\}$. Again profits are given by $\pi_i(y_i, p_i, c_i) = y_i(p_i - c_i)$, now allowing for $y_i \in \{1, \ldots, k\}$.

Similarly the definition of beliefs is modified to allow for different levels of production $y_i \in \{0, 1, 2, ..., K\}$. Each firm i has a belief $\Omega_i : C_i \times P_i \times Y_i \mapsto [0, 1]$. The term $\Omega_i(y_i|p_i, c_i)$ captures the probability firm i assigns to producing a quantity exactly equal to y_i, given that his marginal cost is c_i and his price is p_i. The optimality condition, taking expectations across all possible levels of production becomes:

Condition 4 (General Optimality). *For all c_i*

$$b_i(c_i) = \min \left\{ \arg\max_{p_i} \left\{ \sum_{y_i=0}^{K_i} \Omega_i(y_i|p_i, c_i)\pi_i(c_i, p_i, y_i) \right\} \right\}$$

Furthermore, define $\phi_i(y_i|p_i, c_i, b_{-i})$ to be the probability that firm i produces a quantity y_i given (i) firm i submits a price p_i, (ii) firm i has a marginal cost of c_i and (iii) opponents play according to strategies b_{-i}. Hence:

$$\phi_i(y_i|p_i, c_i, b_{-i}) := \int_{\mathbb{R}_+^{n-1}} f(c_{-i}|c_i)\mathbf{1}_{\left\{y_i = D_i\left(p_i, b_{-i}(c_{-i})\right)\right\}} dc_{-i} \tag{19}$$

Given ϕ, the equilibrium condition for belief formation under Bayesian updating is the same as in the special case of a single unit procurement auction, allowing for different levels of production $y_i \in \{1, \ldots, k\}$.

Condition 5 (Bayesian updating (General)). *Suppose firms are playing according to strategies $(b_1, ..., b_n)$ and beliefs $(\Omega_1, ..., \Omega_n)$ are formed according to Bayesian updating. Then for all $i \in I$, $y_i \in \{1, \ldots, K\}$, $p_i \in P_i$ and $c_i \in C_i$, beliefs are given as follows:*

$$\Omega_i(y_i|p_i, c_i) = \phi_i(y_i|p_i, c_i, b_{-i})$$

Similarly the condition for belief formation under coarse updating remains unchanged, except for allowing different levels of y_i.

Condition 6 (Coarse updating (General)). *Suppose firms are playing according to strategies* $(b_1, ..., b_n)$ *and beliefs* $(\Omega_1, ..., \Omega_n)$ *are formed according to mark-up updating. Then beliefs are given as follows:*

$$\Omega_i(y_i|x_i c_i, c_i) = \int_0^\infty \phi_i\Big(y_i\Big|x_i\tilde{c}_i, \tilde{c}_i, b_{-i}\Big) f_i(\tilde{c}_i) d\tilde{c}_i$$

Note that the definitions of equilibrium under Bayesian updating and under mark-up updating remain unchanged. Definitions 1 and 2 also apply in the case of Bertrand competition with a general demand function.

In order to find equilibria in this case, define the quantity firm i expects to produce as $\overline{\Omega}_i(p_i; c_i)$. This quantity is calculated by summing over the beliefs of firm i across all quantities y_i:

Definition 5 (Expected quantity).

$$\overline{\Omega}_i(p_i; c_i) = \sum_{y_i=0}^{K} y_i \Omega_i(y_i|p_i, c_i)$$

The maximization problem of firm i can now be written as follows:

$$\max_{p_i} \Big\{\overline{\Omega}_i(p_i; c_i)(p_i - c_i)\Big\} \tag{20}$$

Taking first order conditions and setting them equal to zero leads to the following proposition:

Proposition 6.1. *Suppose beliefs* (Ω_i) *and strategy profiles* $(b_i)_{i\in I}$ *form an equilibrium under Bayesian updating. Then for all* $i \in I$ *and* $c_i \in C_i$ *the following condition is satisfied:*

$$b_i(c_i) = c_i + \frac{\overline{\Omega}(b_i(c_i); c_i)}{-\overline{\omega}_i(b_i(c_i); c_i)}$$

Now consider coarse updating with general Bertrand competition. We define $\overline{Q}_i(x_i|x_{-i})$ to be the quantity firm i expects to sell, when it sets a mark-up x_i and its competitors play according to constant mark-up strategies x_{-i}, when different levels of production are possible.

Definition 6. *Suppose firms are playing according to constant mark-up strategies, where* $b_i(c_i) = x_i c_i$ *for all* i. *Then* $\overline{Q}_i(x_i|x_{-i})$ *is defined to be:*

$$\overline{Q}_i(x_i|x_{-i}) = \sum_{y_i=1}^{K} \int_{\mathbb{R}_+^n} f(\mathbf{c}) y_i \mathbf{1}\Big\{y_i = D_i\Big(p_i, b_{-i}(c_{-i})\Big)\Big\} d\mathbf{c}$$

In this case the firm's maximization problem becomes:

$$\max_{x_i} \Big\{\overline{Q}_i(x_i|x_{-i})(x_i^* - 1)c_i\Big\} \tag{21}$$

Moreover the subjective profit that the firm expects to earn becomes:

$$\Pi^S(x_i, x_{-i}|c_i) = \overline{Q}_i(x_i|x_{-i})(x_i^* - 1)c_i \tag{22}$$

We can now extend the result presented for procurement auctions above:

Proposition 6.2. *Suppose assumption 1 and assumption 2 hold. Then there exists a coarse equilibrium of the form* $b_i(c_i) = x_i^* c_i$ *for all* $i \in I$. *Moreover* $(x_1^*, ..., x_n^*)$ *satisfy the following system of equations:*

$$x_i^* = 1 + \frac{\overline{Q}_i(x_i^*|x_{-i}^*)}{-\overline{q}_i(x_i^*|x_{-i}^*)} \text{ for all } i \in I$$

Therefore the equilibria under general Bertrand competition with a constant cost per unit, have the same form as those in the special case of single unit procurement auctions.

7 Conclusion

In this paper we have used the psychological theory of cue competition to suggest a new model for price competition that is consistent with a number of facts observed in practice. First it is consistent with survey evidence, which suggests firms use mark-up pricing bidding and bid a constant proportion of their marginal cost. Secondly it is consistent with experimental evidence, which suggests firms never bid close to marginal cost. Thirdly the theory of coarse updating tends to lead to less cost dispersion for a fixed price distribution than the standard model. This is desirable, since one criticism of estimates based on the standard model is that cost dispersion seems too high. Despite the large number of experiments conducted on price competition and the survey and anecdotal evidence for mark-up pricing, - as far as we are aware- there are no experiments that directly test for the use of mark-up pricing. Such experiments may shed light on the importance of mark-up updating and provide additional insight into how players choose bidding strategies.

A further advantage of the model proposed here is that preliminary simulations have shown that it is easier to estimate than the standard model. This means that it can be estimated with less data, and hence can be applied in a wider range of situations. In particular, a procurement auction with more than two (asymmetric) firms is relatively easy to estimate using the procedure outlined above, while this is difficult to estimate using the standard model as discussed in Campo & Vuong (2003). Quantifying this advantage (ie how much less data is needed), running further simulations and developing an application are all areas for future work.

A final advantage of the model proposed here is that when costs are highly correlated beliefs under coarse updating closely resemble beliefs under Bayesian updating. This means that firms have little incentive to deviate from a coarse equilibrium. This case is particularly relevant, because firms costs are often highly correlated.

There are at two distinct ways to extend the model of coarse updating proposed here. The first extension is to consider a wider range of utility functions and not just to model Bertrand competition. We believe this is a promising area for future research, since it allows to model cognitively constrained agents in a tractable way.

The second extension involves considering different ways of categorising information. By assuming that firms focus on the mark-up variable we implicitly define a mapping from $\phi : C_i \times P_i \mapsto X_i$: more precisely choosing $\phi(c_i, p_i) = p_i/c_i$ determines which categories are evaluated together. Choosing different ways of categorising the information would lead to different results. One obvious choice would be for firms to focus on the price variable and choose $\phi(c_i, p_i) = p_i$. The reason this paper focuses on the mark-up variable is that (i) it is widely discussed in the business literature and (ii) beliefs are almost correct when costs are highly correlated. However investigating other ways of categorising information may also be more relevant in other situations.

8 Appendix

8.1 Proof of proposition 3.4

Consider a mark-up equilibrium $b_i(c_i)$. From the optimality condition, it follows that:

$$\frac{b_i(c_i)}{c_i} = \min\left\{\arg\max\left\{\sum_{y_i=1}^{N} y_i \Omega_i(y_i|x_i c_i, c_i)(x_i - 1)\right\}\right\}$$

Since $\Omega_i(y_i|x_i c_i, c_i) = \Omega_i(y_i|x_i)$, the right hand side does not depend on c_i. Hence the left hand side does not depend on c_i either. Therefore for some x_i^*, it follows that $\frac{b_i(c_i)}{c_i} = x_i^*$ and hence $b_i(c_i) = x_i^* c_i$. It follows that all mark-up equilibria are in linear strategies.

8.2 Proof of proposition 3.5

Consider expected profits, namely $\Pi_i(x_i, x_{-i}|c_i) = c_i(x_i - 1)Q_i(x_i|x_{-i})$. Define $BR_i(x_{-i})$ as follows:

$$BR_i(x_{-i}) = \min\left\{\arg\max_{x_i \in [1,\infty)} \left\{(x_i - 1)Q_i(x_i|x_{-i})\right\}\right\}$$

Now define a sequence inductively as follows. First define $x_i^{(0)} = 1$. Secondly define $x_{-i}^k = \left(x_1^{(k+1)}, ...x_{i-1}^{(k+1)}, x_{i+1}^{(k)}, ..., x_n^{(k)}\right)$ and finally define $x_i^{(k+1)} = BR_i(x_i^{(k)})$. Define also that $\overline{x} = \max\{\overline{x}_i\}$. We argue that (i) $x_i^{(k+1)} \in [1, \overline{x}]$, (ii) $x_i^{(k)}$ is weakly increasing, (iii) $x_i^{(k)} \to x_i^*$ and (iv) $x^* = (x_1^*, ..., x_n^*)$ is an equilibrium.

In order to prove (i) $BR_i(x_{-i}) \geq 1$ from the definition of $BR_i(x_{-i})$. Moreover suppose $x_j \leq \overline{x}$ for all $j \neq i$. Then by assumption 2 $BR_i(x_{-i}) \leq \overline{x}$, since for any $\hat{x}_i > \overline{x} \geq \overline{x}_i$ it follows that $\Pi_i(x_i, x_{-i}|c_i) \leq \Pi_i(\overline{x}, x_{-i}|c_i)$.

In order to prove (ii) note from above that $x_i^{(k)} \geq 1$. The result then follows by induction using assumption 1: if $x'_{-i} \geq x_{-i}$, then $BR_i(x'_{-i}) \geq BR_i(x_{-i})$

Moreover in order to prove (iii) note that $x_i^{(k)}$ is a weakly increasing sequence which is bounded above. Hence it converges to x_i^*.

Finally to prove (iv) note that $(x_i - 1)Q_i(x_i|x_{-i})$ is continuous. Hence:

$$(x_i^{(k)} - 1)Q_i(x_i^{(k)}|x_{-i}^{(k)}) \geq (\hat{x}_i - 1)Q_i(x_i^{(k)}|x_{-i}^{(k)})$$

Taking the limit as $k \to \infty$ yields:

$$(x_i^* - 1)Q_i(x_i^*|x_{-i}^*) \geq (\hat{x}_i - 1)Q_i(x_i^*|x_{-i}^*)$$

Moreover by assumption 1 whenever $\hat{x}_i < x_i^*$:

$$(x_i^* - 1)Q_i(x_i^*|x_{-i}^*) > (\hat{x}_i - 1)Q_i(x_i^*|x_{-i}^*)$$

Hence $x_i^* = \min\Big\{\arg\max_{x_i}\Big\{(x_i - 1)Q_i(x_i|x_{-i}^*)\Big\}\Big\}$ and this proves the result.

8.3 Proof of proposition 3.6

From proposition 4.1, it follows that:

$$x_i^* = \frac{\int_0^\infty p_i g(p_i, p_i) dp_i}{\int_0^\infty p_i g(p_i, p_i) dp_i - \int_0^\infty \int_{p_i}^\infty g(p_i, p_j) dp_j dp_i}$$

By symmetry $x_i^* = x_j^* = x^*$. Using the definition of $G(p_i, p_j)$ and symmetry, it follows that $g(p_i, p_j) = \frac{1}{x^2} f(p_i/x, p_j/x)$. Hence:

$$x^* = \frac{\int_0^\infty p_i f(p_i/x, p_i/x) dp_i}{\int_0^\infty p_i f(p_i/x, p_i/x) dp_i - \int_0^\infty \int_{p_i}^\infty f(p_i/x, p_j/x) dp_j dp_i}$$

By symmetry $\int_0^\infty \int_{p_i}^\infty f(p_i/x, p_j/x) dp_j dp_i = 1/2$. Substituting $c_i = p_i/x$ leads to:

$$x^* = \frac{2\int_0^\infty c_i f(c_i, c_i) dc_i}{2\int_0^\infty c_i f(c_i, c_i) dc_i - 1}$$

This proves the result.

8.4 Proof of proposition 3.7

It is enough to show that the beliefs formed using coarse updating are arbitrarily close to those formed using Bayesian updating, given players are playing according to linear strategies $b_i(c_i) = x_i^* c_i$. This will ensure that beliefs formed by Bayesian updating are very close to those formed through coarse updating and ensures that players have no incentive to deviate from a coarse equilibrium. If $\Omega_i(y|x_i)$ are the beliefs formed through coarse updating when the opponent plays a strategy $b_i(c_i) = x_j c_i$, then:

$$\min_{c_i \in C_i} \Big\{ \phi_i(1|x_i, x_j, c_i) \Big\} \leq \Omega(y|x_i) \leq \max_{c_i \in C_i} \Big\{ \phi_i(1|x_i, x_j, c_i) \Big\}$$

Hence it is enough to show that for all c_i and c_i' and all strategies x_i and x_j for any $\epsilon > 0$ there exist parameters δ or τ such that:

$$\phi_i(1|x_i, x_j, c_i) - \phi_i(1|x_i, x_j, c_i') \leq \epsilon$$

Using the fact that $F(c_j|\theta) = H(c_j/\theta)$ and $f(c_j|\theta) = \frac{1}{\theta} h(c_j/\theta)$:

$$\begin{aligned}
\phi_i(1|x_i, x_j, c_i) &= \int_{\mathbb{R}_+} g(\theta|c_i) f(c_j|\theta) \mathbf{1}\{x_i c_i \leq x_j c_j\} dc_j d\theta \\
&= \left[\int_{\mathbb{R}_+} g(\tilde{\theta}) f(c_i|\tilde{\theta}) d\tilde{\theta} \right]^{-1} \int_{\mathbb{R}_+} f(c_i|\theta) g(\theta) F\Big(\frac{x_i c_i}{x_j} \Big| \theta \Big) dc_j d\theta \\
&= \left[\int_{\mathbb{R}_+} \frac{g(\tilde{\theta})}{\tilde{\theta}} h(c_i/\tilde{\theta}) d\tilde{\theta} \right]^{-1} \int_{\mathbb{R}_+} \frac{g(\theta)}{\theta} h(c_i/\theta) H\Big(\frac{x_i c_i}{x_j \theta} \Big) d\theta
\end{aligned}$$

Substituting $t = \frac{c_i}{\theta}$ and $s = \frac{c_i}{\tilde{\theta}}$ we reach:

$$\phi_i(1|x_i, x_j, c_i) = \left[\int_{\mathbb{R}_+} \frac{g(c_i/s)}{s} h(s) ds \right]^{-1} \int_{\mathbb{R}_+} \frac{g(c_i/t)}{t} h(t) H\Big(\frac{x_i t}{x_j} \Big) dt$$

Let $\overline{m} = (1 + \tau \underline{z})$ and $\underline{m} = (1 + \tau \overline{z})$. Recall that $(1 + \tau \underline{z})\theta \leq c_i \leq (1 + \tau \overline{z})\theta$ and hence $\underline{m} = (1 + \tau \overline{z}) \leq t \leq (1 + \tau \underline{z}) = \overline{m}$ Hence the integral can be rewritten as follows:

$$\phi_i(1|x_i, x_j, c_i) \quad = \quad \left[\int_{\underline{m}}^{\overline{m}} \frac{g(c_i/s)}{s} h(s) ds\right]^{-1} \int_{\underline{m}}^{\overline{m}} \frac{g(c_i/t)}{t} h(t) H\Big(\frac{x_i t}{x_j}\Big) dt$$

8.5 Proof of proposition 3.8

For the first case suppose $g(\theta)$ has the specific form given in the text. Then $\frac{g(c_i/t)}{t} = \frac{\delta}{2c_i}\Big(\frac{c_i}{t}\Big)^{\delta}$ when $c_i < t$ and $\frac{g(c_i/t)}{t} = \frac{\delta}{2c_i}\Big(\frac{c_i}{t}\Big)^{-\delta}$ when $c_i > t$. We now claim that for any $t \leq s$ and for any c_i:

$$\Big(t\Big/s\Big)^{\delta} \leq \frac{g(c_i/t)}{t}\Big/\frac{g(c_i/s)}{s} \leq \Big(s\Big/t\Big)^{\delta}$$

There are three cases. First assume $t \leq c_i$ and $s \leq c_i$. Then $\frac{g(c_i/t)}{t} = \frac{\delta}{2c_i}\Big(\frac{c_i}{t}\Big)^{-\delta}$ and $\frac{g(c_i/s)}{s} = \frac{\delta}{2c_i}\Big(\frac{c_i}{s}\Big)^{-\delta}$. Dividing $\frac{g(c_i/t)}{\delta t}\Big/\frac{g(c_i/s)}{\delta s}$ leads to the inequality.

Secondly assume $t \geq c_i$ and $s \geq c_i$. Then $\frac{g(c_i/t)}{t} = \frac{\delta}{2c_i}\Big(\frac{c_i}{t}\Big)^{\delta}$ and $\frac{g(c_i/s)}{s} = \frac{\delta}{2c_i}\Big(\frac{c_i}{s}\Big)^{\delta}$. Again dividing $\frac{g(c_i/t)}{\delta t}\Big/\frac{g(c_i/s)}{\delta s}$ leads to the inequality.

Third assume $t \leq c_i \leq s$. Then using case (i) it follows that:

$$\Big(t\Big/c_i\Big)^{\delta} \leq \frac{g(c_i/t)}{t}\Big/\frac{g(c_i/c_i)}{c_i} \leq \Big(c_i\Big/t\Big)^{\delta}$$

Similarly using case (ii) it follows that:

$$\Big(c_i\Big/s\Big)^{\delta} \leq \frac{g(c_i/c_i)}{c_i}\Big/\frac{g(c_i/s)}{s} \leq \Big(s\Big/c_i\Big)^{\delta}$$

Multiplying these inequalities together we reach case (iii):

$$\Big(t\Big/s\Big)^{\delta} \leq \frac{g(c_i/t)}{t}\Big/\frac{g(c_i/s)}{s} \leq \Big(s\Big/t\Big)^{\delta}$$

Recall that t and s in $[\underline{m}, \overline{m}]$ and hence for any c_i:

$$\Big(\underline{m}\Big/\overline{m}\Big)^{\delta} \leq \frac{g(c_i/t)}{t}\Big/\frac{g(c_i/s)}{s} \leq \Big(\overline{m}\Big/\underline{m}\Big)^{\delta}$$

As $\delta \to 0$ this term goes to 1. Hence as $\delta \to 0$:

$$\begin{aligned}
\phi_i(1|x_i, x_j, c_i) &= \left[\int_{\underline{m}}^{\overline{m}} \frac{g(c_i/s)}{s} h(s) ds\right]^{-1} \int_{\underline{m}}^{\overline{m}} \frac{g(c_i/t)}{t} h(t) H\Big(\frac{x_i t}{x_j}\Big) dt \\
&= \frac{\int_{\underline{m}}^{\overline{m}} h(t) H\Big(\frac{x_i t}{x_j}\Big) dt}{\int_{\underline{m}}^{\overline{m}} \left[\frac{g(c_i/s)}{s} \Big/ \frac{g(c_i/t)}{t}\right] h(s) ds}
\end{aligned}$$

For $\delta \leq \frac{\epsilon}{4\log(\overline{m}) - \log(\underline{m})} = \delta^*(\epsilon)$ the term in square brackets is between $(1-\epsilon/2)$ and $(1+\epsilon/2)$ for all c_i. Hence whenever $\delta < \delta^*(\epsilon)$, then $\phi_i(1|x_i, x_j, c_i) - \phi_i(1|x_i, x_j, c_i') \leq \epsilon$ for all c_i and c_i'. This proves the result.

8.5.1 Case with small τ

Again we aim to prove that $\frac{g(c_i/t)}{t} \Big/ \frac{g(c_i/s)}{s}$ is close to 1 in the interval $[\underline{m}, \overline{m}]$ whenever τ is sufficiently small. First note that as $\tau \to 0$, then $\underline{m} \to 1$ and $\overline{m} \to 1$. Hence it is sufficient to prove that for any $\delta \leq \delta^*(\epsilon)$:

$$1 - \epsilon \leq \frac{g\Big(c_i/(t+\delta)\Big)}{t+\delta} \Big/ \frac{g(c_i/t)}{t} \leq 1 + \epsilon$$

To prove define $\phi(t) = \frac{g(c_i/t)}{t}$. It is enough to show that $\phi'(t)/\phi(t)$ is uniformly bounded. Computing this expression leads to:

$$\begin{aligned}
\Big|\frac{\phi'(t)}{\phi(t)}\Big| &= \Big|\frac{c_i g'(c_i/t)t - g(c_i/t)}{t^2} \Big/ \frac{g(c_i/t)}{t}\Big| \\
&= \frac{1}{t}\Big|\frac{c_i/t g'(c_i/t)}{g(c_i/t)} - 1\Big| \\
&= \frac{1}{t}\Big|\frac{\theta g'(\theta) t}{g(\theta)} - 1\Big|
\end{aligned}$$

Since $t \in [\underline{m}, \overline{m}]$ it follows that $\left|\frac{\phi'(t)}{\phi(t)}\right| < K$ whenever $\frac{\theta g'(\theta)}{g(\theta)}$ is uniformly bounded. From here the result follows.

8.6 Proof of proposition 4.1

First note that:

$$Q_i(x_i^*|x_j^*) = \int_{\mathbb{R}_+^2} f(c_i, c_j)\mathbf{1}\{x_i^* c_i < x_j^* c_j\} dc_i dc_j \tag{23}$$

If firms are playing according to coarse equilibrium strategies then $c_i = p_i/x_i^*$. Note - from the definition of $G(p_i, p_j)$ - that $f(c_i, c_j) = x_i^* x_j^* g(c_i x_i^*, c_j x_j^*)$. Substituting (p_i, p_j) for (c_i, c_j) leads to:

$$\begin{aligned} Q_i(x_i^*|x_j^*) &= \int_{\mathbb{R}_+^{n-1}} g(p_i, p_j)\mathbf{1}\{p_i < p_j\} dp_i dp_j \\ &= \int_0^\infty \int_{p_i}^\infty g(p_i, p_j) dp_j dp_i && (24) \\ & && (25) \end{aligned}$$

Secondly note that:

$$\begin{aligned} -x_i^* q_i(x_i^*|x_{-i}^*) &= x_i \int_0^\infty \frac{c_i}{x_j} f\Big(c_i, \frac{c_i x_i}{x_j}\Big) dc_i \\ &= \int_0^\infty c_i x_i x_j f\Big(c_i x_j, c_i x_i\Big) dc_i \\ &= \int_0^\infty c_i x_i^2 x_j^2 g\Big(c_i x_j x_i, c_i x_i x_j\Big) dc_i && (26) \\ & && (27) \end{aligned}$$

Substituting p_i for $c_i x_i x_j$ leads to:

$$-x_i^* q_i(x_i^*|x_{-i}^*) = \int_0^\infty p_i g(p_i, p_i) dp_i \tag{28}$$

Finally recall that by equation 14:

$$x_i^* = \frac{-x_i^* q_i(x_i^*|x_{-i}^*)}{-x_i^* q_i(x_i^*|x_{-i}^*) - Q_i(x_i^*|x_{-i}^*)} \tag{29}$$

Applying equations 24 and 28 proves the result.

8.7 Proof of proposition 4.2

By proposition 3.2 under Bayesian updating:

$$b(c) = c + \frac{\Omega_i(1|b(c);c)}{-\omega_i(1|b(c);c)}$$

Note that $b(\underline{c}) = \underline{p}$ and $\Omega_i(1|\underline{p};\underline{c}) = 1$. Moreover the chance of winning when submitting a bid p with marginal cost c is $\Omega_i(1|p,c) = G\Big(p|b(c)\Big)$. Differentiating with respect to p and evaluating at $p = \underline{p}$ and $c = \underline{c}$ leads to $\omega_i(1|p,c) = g(\underline{p}|\underline{p})$. Hence:

$$\underline{c}_2 = \underline{p} - \frac{1}{g(\underline{p}|\underline{p})}$$

See Athey & Haile (2007) for a similar formula for standard auctions. On the other hand under coarse updating:

$$\underline{c}_1 = \underline{p}\Big[\frac{\int_0^\infty p_i g(p_i,p_i)dp_i}{\int_0^\infty p_i g(p_i,p_i)dp_i - \int_0^\infty \int_{p_i}^\infty g(p_i,p_j)dp_j dp_i}\Big]$$

Simple algebraic manipulation shows that $\underline{c}_2 < \underline{c}_1$ if and only if

$$2\int_{\mathbb{R}_+} p_i g(p_i,p_i)dp_i > \underline{p}g(\underline{p},\underline{p}) + 1$$

This proves the proposition.

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