The Reverse War of Attrition

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Abstract

This paper introduces a new contest model with unobservable actions in which the designer maximizes discounted aggregate effort by choosing a starting time and a deadline. At the deadline, the contestant who exerted most effort wins a prize, which consists of the endowment of the designer and collected interest.

The contest has a unique Nash equilibrium. In the main model, the designer should announce the contest immediately with a relative short deadline to promote intense competition. I study the implications of different types of asymmetries, a different contest success function and a different goal function of the designer.

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1 Introduction

In many (public) architecture, innovation and procurement contests, the designer announces the contest along with the deadline and the winner prize.[1] All contestants can exert effort between the announcement and the deadline, at which the best-performing contestant receives the prize. While there is a lot of recent literature on contests, the deadline—an important choice variable of the designer—receives relatively little attention. Some contest models such as all-pay auctions or Tullock contests do not model the time dimension at all, while other contest models with unobservable actions abstract from discounting and/or assume infinite or endogeneous deadlines.[2]

In this paper, I analyze the problem of an impatient contest designer who maximizes discounted expected total effort by the contestants; effort is interpreted as a proxy for output/innovation. The designer has a fixed monetary endowment available for the contest.[3] He chooses when to announce the contest and for how long to run the contest. At the deadline, the contestant who exerted most effort wins the prize, i.e., the endowment and accumulated interest; ties are broken randomly.

In the first step, I characterize the equilibria in a two-player contest depending on the starting time/deadline combination in Propositions 1-3. Proposition 1 determines combinations for which contestants exert effort during the entire contest in equilibrium with probability 1, whereas Propositions 2 and 3 characterize equilibria for other regions of the parameter space.

Given the characterization for the contest game, I move to the problem of the contest designer in Propositions 4 and 5. There are two mitigating effects: a higher deadline increases expected effort, but due to the discounting, it also reduces the valuation of the designer. The derivation shows that the increase

[1] For a directory of major competitions in landscape architecture, urban planning, design and other applications, see for instance `http://www.competitions.org/` or `http://www.designboom.com/competition/`.

[2] For the former category, see, e.g., Tullock (1980), Hillman and Samet (1987), Hillman and Riley (1989), Baye, Kovenock, and de Vries (1996), Konrad (2002), Siegel (2009a,b) and Alcalde and Dahm (2010); for the latter category, see, e.g., Taylor (1995) and Seel and Strack (2013, 2015).

[3] This assumption is relaxed in Section 3.3.

in expected effort due to a larger deadline is overcompensated by the increase in the waiting time. Thus, the main model yields two strong predictions: (i) it is always optimal to announce a contest immediately and (ii) the deadline of a contest should be relatively short.

The remainder of the paper studies the robustness of the predictions to changes in the main assumptions. By Propositions 7-9, the main predictions extend to n symmetric players and to two players with different marginal effort cost. In the latter case, the deadline helps to level the playing field as in Che and Gale (1998), Kirkegaard (2012) and Siegel (2014).

The main results also extend to a Tullock lottery contest success function. Thus, while leading to qualitative different types of equilibria in a static setting, the two most commonly used contest success functions yield similar predictions when taking the time dimension into account.

Both main predictions—the optimality of announcing a contest immediately and the relatively short deadline—do not extend if the designer maximizes highest discounted effort of one participant. In this case, an intermediate deadline is sometimes optimal for the designer. Intuitively, only one of the contestants needs to increase his effort sufficiently in expectation compared to a short deadline, which might happen with sufficiently high probability. Finally, the main results extend to an impatient contest designer, but a very patient contest designer might prefer not to announce the contest immediately and to have a larger deadline.

Related Literature

The model can be seen as a modified war of attrition with discounting and a deadline.[4] As in a standard war of attrition, the player who stays in longer wins the prize and both players incur effort cost independently of the outcome.

Compared to a war of attrition, however, the present paper reverses the informational assumptions, i.e., no player can observe his rival. While a war

[4] The war of attrition was introduced by Maynard Smith (1974) and has been extensively studied since then; seminal papers include Hendricks, Weiss, and Wilson (1988), Krishna and Morgan (1997) and Bulow and Klemperer (1999).

of attrition captures applications such as animal conflicts (Maynard Smith, 1974, Bishop and Cannings, 1978) or price wars (Fudenberg and Tirole, 1986), this model applies to procurement contests or design competitions, in which research is conducted secretly and the progress of all participants is evaluated at the deadline. For further applications, see also Taylor (1995).

The different informational assumptions lead to a reversed timing structure in equilibrium: players exert effort from a certain time onwards. Hence, the player who starts earliest wins the game, whereas the player who persists longest wins in a war attrition. The resulting payoffs are pay-your-effort rather than incurring the effort cost of the player who resigns earlier. Finally, differing from a war of attrition which has a myriad of asymmetric equilibria, this model has a unique equilibrium. Thus, despite a similar game structure, the different informational assumptions reverse many standard results of a war of attrition.

For a fixed starting time and a fixed deadline, equilibrium behavior is isomorphic to an all-pay auction with exponential bidding cost and a bid cap. The designer's choice of the starting time and deadline provides an additional twist, since it determines the size of the bid cap and it endogenizes cost-prize ratio in the isomorphic all-pay model.

Finally, the paper relates to a relatively novel literature on optimal deadlines started by Damiano, Li, and Wing (2012). They analyze a war of attrition with private information and a common interest part and characterize the welfare maximizing deadline. My focus, however, lies on the maximization of expected total effort and expected maximal effort. Lang, Seel, and Strack (2014) consider a stochastic contest model with discrete jumps and without discounting. They provide a partial ranking of expected total effort.

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, I derive the equilibrium of the contest and the optimal starting time/deadline combination for the contest designer. Section 4 is devoted to different extensions of the main model. The results are discussed in Section 5. Most proofs are relegated to the appendix.

2 The Model

Consider a model with a risk-neutral contest designer and $i = 1, 2$ risk-neutral contestants. The contest designer decides on the time $\underline{T}$ (starting time) at which he announces the contest and on the time T (deadline) at which the contest ends. She has an endowment P, on which she collects interest at the interest rate r until the deadline T. At any point $t \in [\underline{T}, T]$, each contestant decides whether to exert effort $e_t^i = 1$ or not to exert effort $e_t^i = 0$. The effort decisions of each player are unobservable to his rival. Exerting effort induces a flow cost of c, while no effort induces no flow cost. The net present value of total costs at time $t = 0$ is thus $\int_{\underline{T}}^{T} c e_t^i \exp(-rt)dt$.[5]

At the contest deadline T, the designer pays $P \exp(rT)$ to the contestant who exerted most effort; ties are broken randomly. Thus, the net present value of winning the prize is P. The contest designer chooses $\underline{T}$ and T to maximize the expected discounted sum of efforts $\mathbb{E}[\exp(-rT) \sum_{i=1}^{2} \int_{\underline{T}}^{T} e_t^i dt]$. I solve the contest using the Nash equilibrium concept, since no new information about the rival's strategy arrives over time.

3 Equilibrium Analysis

The goal of this section is to compute the optimal starting time/deadline combination $(\underline{T}, T)$ for the contest designer. To do so, I first derive different categories of Nash equilibria in the contest depending on the parameters. Secondly, I determine the expected aggregate effort for each equilibrium category in closed form. This allows me to find the starting time/deadline combination which maximizes aggregate effort in the last step. Superscripts for the players are omitted whenever there is no ambiguity.

3.1 Nash Equilibria in the Contest

If a contestant exerts effort for a fixed amount of time, due to the discounting, it is cheapest if he starts exerting effort as late as possible. This directly yields

[5] Section 4.5 contains a detailed discussion about the implied assumptions on discounting of the designer and the participants.

the following lemma.

Lemma 1 (Delay). *If a player exerts effort on some interval* $(s, \tilde{s}]$*, then the player also exerts effort for almost all* $t \in [\tilde{s}, T]$*.*

Hence, the decision problem of each player reduces to finding a *starting time* $s \geq \underline{T}$ such that the player exerts effort at time t if and only if $s \leq t \leq T$.

The following lemma is a by now standard result in game theory with a continuous state space and holds for all deadlines (the proof is omitted since it directly follows from the arguments in Baye, Kovenock, and de Vries, 1996):

Lemma 2 (No Interior Mass Point). *In equilibrium, no player starts with strictly positive probability at a time* $\underline{T} < s < T$*. At least one player starts before time* T *with probability 1.*

Lemma 3 (Zero Profits). *Assume that* $\exp(-r\underline{T}) - \exp(-rT) > \frac{Pr}{2c}$*. In any Nash equilibrium, both players make zero profits.*

In the following, I distinguish three different categories of deadlines and derive the respective equilibria. Lemmas 2 and 3 are crucial steps in the equilibrium characterization for intermediate and long deadlines.

Proposition 1 (Short Deadlines). *Assume that* $\exp(-r\underline{T}) - \exp(-rT) \leq \frac{Pr}{2c}$*. In the unique Nash equilibrium, both players always exert effort, i.e.,* $s = \underline{T}$*.*

Proposition 2 (Intermediate Deadlines). *Assume that* $\frac{Pr}{c} > \exp(-r\underline{T}) - \exp(-rT) > \frac{Pr}{2c}$*. In the unique Nash equilibrium, each player randomizes his starting time* s *according to the cumulative distribution function*

$$F(s) = \begin{cases} 2\left(1 - \frac{c}{Pr}(\exp(-r\underline{T}) - \exp(-rT))\right) & \textit{for all } 0 \leq s < \tilde{s} \\ 1 - \frac{c}{Pr}(\exp(-rs) - \exp(-rT)) & \textit{for all } s \in [\tilde{s}, T] \\ 1 & \textit{for all } s > T\,, \end{cases}$$

where $\tilde{s} = -\frac{1}{r}\log(2\exp(-r\underline{T}) - \exp(-rT) - \frac{Pr}{c})$*.*

Proposition 3 (Long Deadlines)**.** *Assume that* $\frac{Pr}{c} \leq \exp(-r\underline{T}) - \exp(-rT)$. *In the unique Nash equilibrium, each player randomizes the starting time* s *according to the cumulative distribution function*

$$F(s) = \begin{cases} 0 & \textit{for all } 0 \leq s \leq \hat{s} \\ 1 - \frac{c}{Pr}(\exp(-rs) - \exp(-rT)) & \textit{for all } s \in (\hat{s}, T] \\ 1 & \textit{for all } s > T \ , \end{cases}$$

where $\hat{s} = -\frac{1}{r}\log(\frac{Pr}{c} + \exp(-rT))$.

For short deadlines, both players exert effort during the entire contest, which is reminiscent of the equilibrium in a war of attrition with a short deadline.

The equilibrium for intermediate deadlines is also similar to the symmetric equilibrium of a war of attrition with an intermediate deadline. More precisely, the "early starters" in this game correspond to the players who persist till the end in a war of attrition; see, e.g., Bishop and Cannings (1978) or Hendricks, Weiss, and Wilson (1988). There is an interior interval in which no player starts which corresponds to the interval in which no player resigns in a war of attrition.

As in the case of intermediate deadlines, long deadline also lead players randomize on an interval to make their rivals indifferent in equilibrium. In the case of long deadlines, however, no player exerts effort throughout the entire time interval $[\underline{T}, T]$ with positive probability, since this is too costly. This type of equilibrium does not arise in a war of attrition.

3.2 The Designer's Problem

We can now tackle the designer's problem: which starting time/deadline combination maximizes expected discounted equilibrium effort?

Using the unique Nash equilibrium for any deadline, we can derive a closed-form solution to the discounted total effort $\mathbb{E}(\sum_{i=1}^{2}\int_{\underline{T}}^{T} e_t^i \exp(-rT)dt)$ for all three regions (short, intermediate, long) of the parameter space.

For the case of Proposition 1, discounted total efforts are

$$\sum_{i=1}^{2} \int_{\underline{T}}^{T} e_t^i \exp(-rT) dt = 2(T - \underline{T}) \exp(-rT) \tag{1}$$

A closed-form solution for the aggregate discounted effort in the other regions of the parameter space is presented in the appendix. The direct approach would be to maximize $\underline{T}$ and T on all three regions of the parameter space in order to find local maxima and to compare them across regions. This would, however, result in calculations which are difficult to handle analytically for the latter two cases. Thus, a different approach is called for.

Using an argument from mechanism design, I first show that for any given T, the optimal solution is to choose a $\underline{T}$ such that the resulting deadline is short. In a second step, I derive a closed-form solution for the optimal starting time/deadline combination $(\underline{T}, T)$.

For $\underline{T} = 0$ and a short deadline, the designer's problem from Eq. (1) reduces to maximizing the function $2T \exp(-rT)$. This function attains its maximum at $T^* = \frac{1}{r}$.

If the maximum for a short deadline and $\underline{T} = 0$ lies within the interior of the parameter space for short deadlines, the local maximum is also the global maximum, since the maximal possible effort (both players always exert effort) is used in the maximization problem. Such an interior equilibrium exists if $\frac{Pr}{2c} \geq 1$ or $\frac{Pr}{2c} < 1$ and

$$1 - \exp(-rT) \leq \frac{Pr}{2c}. \tag{2}$$

Transforming Eq. (2), I get $T \leq -\frac{1}{r} \log(1 - \frac{Pr}{2c})$

Hence, for $\frac{Pr}{2c} \geq 1$, the global maximum is attained at $T^* = \frac{1}{r}$. In the same way, for $\frac{Pr}{2c} < 1$, the unique global maximum is attained at $T^* = \frac{1}{r}$ if T^* is attained for a short deadline, i.e.,

$$\frac{1}{r} \leq -\frac{1}{r} \log(1 - \frac{Pr}{2c}).$$

Solving this equation, I get

$$\frac{Pr}{2c} > 1 - \frac{1}{e},$$

where e the Euler's number. Summing up the two cases $\frac{Pr}{2c} \geq 1$ and $\frac{Pr}{2c} < 1$, I obtain the following lemma:

Lemma 4. *The unique global maximum is attained at an interior solution for a short deadline and* $\underline{T} = 0$ *if* $\frac{Pr}{2c} > 1 - \frac{1}{e}$.

The result is intuitive, since for a high interest rate, the designer wants to get the discovery quickly and is not too concerned about the quality.

Note that there is a qualitative difference in the equilibrium for short and intermediate deadlines, i.e., for the latter, contestants do not spend full effort with probability one. Thus, the effort for intermediate deadlines also increases in T, but at a smaller rate than for short deadlines. The next proposition shows that the increase in waiting time overcompensates the increase in effort for intermediate deadlines for the contest designer. Thus, it is always optimal to pick a short deadline.

Proposition 4 (Optimal Starting Time)**.** *(i) For* $\frac{Pr}{2c} \geq 1$ *and any given* T, $\underline{T} = 0$ *maximizes total expected discounted effort.*
(ii) For $\frac{Pr}{2c} < 1$ *and* $T \leq -\frac{1}{r}\log(1 - \frac{Pr}{2c})$, $\underline{T} = 0$ *maximizes total expected discounted effort.*
(iii) For $\frac{Pr}{2c} < 1$ *and any* $T > -\frac{1}{r}\log(1 - \frac{Pr}{2c})$, $\underline{T} = -\frac{1}{r}\log(\frac{Pr}{2c} + \exp(-rT))$ *maximizes total expected discounted effort.*

Proof. The cases (i) and (ii) are trivial: both players exert effort throughout the entire game (see equilibrium for short deadlines), i.e., effort is maximal.

For Case (iii), note that both players exert effort at any time after $\underline{T}$ in equilibrium. In the following, I show that setting $\underline{T}$ as described in the proposition yields the maximal possible discounted effort of any mechanism with maximal transfer P, deadline T which respects the participation constraints. Thereby, it is clearly also the optimal choice for the contest.

For T such that $\underline{T} > 0$, let us maximize total effort subject to the participation constraint, i.e., the sum of the effort costs should not exceed the prize. By Lemma 1, I only need to find the optimal starting times. Thus, I get

$$\max_{s_1,s_2}(T - s_1) + (T - s_2)$$

subject to

$$\int_{s_1}^{T} c\exp(-rt)dt + \int_{s_2}^{T} c\exp(-rt)dt \le P.$$

The solution to this problem is $s_1 = s_2 = -\frac{1}{r}\log(\exp(-rT)+\frac{Pr}{2c})$, i.e., exactly the starting time $\underline{T}$.

$\square$

Thus, to find the optimal $(T, \underline{T})$-combination for the designer, it remains to find the optimal T. Note that if $Pr \ge 2c$ or $Pr < 2c$ and $T \le -\frac{1}{r}\log(1 - \frac{Pr}{2c})$, the optimization problem is $\max 2T\exp(-rT)$. On the other hand, for $Pr < 2c$ and $T > -\frac{1}{r}\log(1 - \frac{Pr}{2c})$, the optimization problem is $\max 2(T - \underline{T})\exp(-rT)$. Plugging in the optimal $\underline{T}$ from Proposition 4 (iii), I obtain $\max 2(T + \frac{1}{r}\log(\frac{Pr}{2c} + \exp(-rT)))\exp(-rT)$.

Lemma 5. *The function $2(T+\frac{1}{r}\log(\frac{Pr}{2c}+\exp(-rT)))\exp(-rT)$ is decreasing in T for all $T \ge -\frac{1}{r}\log(1 - \frac{Pr}{2c})$.*

Lemma 5 allows us to state the main characterization result for the contest designer:

Proposition 5 (Optimal Starting Time and Deadline)**.** *In equilibrium, the contest designer chooses the starting time $\underline{T} = 0$. If $\frac{Pr}{2c} \ge 1 - \frac{1}{e}$, the optimal deadline is $T = \frac{1}{r}$. If $\frac{Pr}{2c} < 1-\frac{1}{e}$, the optimal deadline is $T = -\frac{1}{r}\log(1-\frac{Pr}{2c})$.*

Proof. By Proposition 4, the optimal starting time is $\underline{T} = 0$ for cases (i) and (ii). By Lemma 5, the payoff is decreasing in T for the parameters considered in case (iii) of Proposition 4. Thus, the optimal T in this case is also chosen such that the optimal $\underline{T} = 0$. Since these cases contain all parameters, I obtain $\underline{T} = 0$.

To prove the second part of the statement, recall that the payoff is increasing for short deadlines until $T = \frac{1}{r}$. Thus, T is either the interior maximum

$T = \frac{1}{r}$ if $\frac{Pr}{2c} > 1 - \frac{1}{e}$ or the corner solution $T = -\frac{1}{r}\log(1 - \frac{Pr}{2c})$ which is the maximal T such that $\underline{T} = 0$.

$\square$

Thus, the model yields a very strong prediction about the optimal contest starting time and deadline: it is always optimal to start the contest without any delay and the deadline of the contest should be short enough such that both contestants choose to exert effort in the entire contest.

3.3 Variable Prize

So far, we have assumed that the endowment of the contest designer is fixed. While this covers applications in which a principal endows the designer with a certain budget, there are other cases in which the designer cannot only choose the starting time and deadline, but also the prize. While the previous analysis only requires the goal function to be some monotone increasing function of the expected discounted effort, the specific form is needed to derive a closed-form solution in this section. I consider the following problem:

$$G(\underline{T}, T, P) = \underset{\underline{T}, T, P}{\text{maximize}} \ \exp(-rT)\mathbb{E}(\sum_{i=1}^{2}\int_0^T e_i^t dt) - P. \tag{3}$$

The following proposition fully characterizes the optimal contest:

Proposition 6. *Suppose the contest designer maximizes Eq.* (3).
(i) If $c \geq 1$, $P = 0$ *is optimal.*
(ii) If $c < 1$, *the optimal solution is given by* $P = (1 - \exp(c-1))\frac{2c}{r}$, $\underline{T} = 0$ *and* $T = \frac{1-c}{r}$. *The resulting profit of the designer is* $\frac{2}{r}(\exp(c-1) - c) > 0$.

Proof. Proposition 5 shows the optimal starting time and deadline for every given prize. Thus, taking the resulting values as given, it remains to find the optimal prize. Plugging $T = -\frac{1}{r}\log(1 - \frac{Pr}{2c})$ and $\underline{T} = 0$ into Eq. (3) yields

$$G(P) = -\frac{2}{r}\log(1 - \frac{Pr}{2c})(1 - \frac{Pr}{2c}) - P.$$

The first-order condition is

$$\frac{dG(P)}{dP} = \frac{1}{c}(1 + \log(1 - \frac{Pr}{2c})) - 1 = 0,$$

which reduces to $P = (1 - \exp(c-1))\frac{2c}{r}$.

Thus, P is positive if and only if $c < 1$ (the fact the P is the maximum follows directly from the second-order condition or a sign test). This establishes Part (i) of the proposition. For Part (ii), we plug the value for P back into the expression for T to obtain $T = \frac{1-c}{r}$. Plugging P and T into the goal function, I get $G = \frac{2}{r}(\exp(c-1) - c) > 0$.

$\square$

Intuitively, the principal does not treat the prize as given anymore, but bears the cost of a prize increase himself. Thus, he has to balance the higher induced effort against the cost of providing a higher prize. If effort costs are too high as in (i), a contest with a positive prize results in negative profits, i.e., $P = 0$ (no contest) is optimal. If a contest yields positive profits, by Proposition 5, the resulting optimum always results in an interior solution for a short deadline, since $T < \frac{1}{r}$.

3.4 Relation to the All-Pay Auction

Suppose players are restricted to choose optimally, i.e., they choose a starting time s and they only exert effort between s and T. The resulting reduced-form game for a fixed deadline and a fixed prize is isomorphic to an all-pay auction with a cost function $c(x) = c\int_0^x \exp(-r(T-t))dt$, where a bid $x = T - s$ corresponds to a starting time s and a bid cap at $\bar{x} = T - \underline{T}$.

Differing from the standard all-pay auction model, however, the choice of a starting time, deadline and prize affects both the cost structure and the maximal feasible bid. These effect are driven by the discounting. Hence, the paper gives a microfoundation for bid caps and—more importantly—it allows us to analyze the effects of discounting which are absent in the standard static setting as well as in previous dynamic models which abstract from discounting.

4 Extensions

In this section, I want to understand how robust the optimal contest starting time and deadline are to different changes in the model.

4.1 More Players

Consider the above model with n symmetric players $i = \{1, 2, \dots, n\}$ and suppose that in case of a tie, all players with the highest effort win with the same probability. Proposition 4 and 5 directly extend to n players by replacing 2 by n. Proposition 2 and 3 do not extend in a straightforward way. In these cases, there is a multiplicity of equilibria in which some players do not exert any effort with probability one.[6] However, using the n-player version of Proposition 4 and 5, we can compare the maximal expected efforts for different number of players. This results in the following proposition.

Proposition 7. *The maximal discounted effort which a contest designer can generate increases in the number of contestants.*

Thus, in principle it is best for the contest designer to invite as many participants as possible with the maximal T which still leads to the short deadline equilibrium. This finding differs from the seminal paper by Che and Gale (2003) who conclude that, in many instances, it is optimal to restrict the contest to two participants. However, it needs to be taken with a grain of salt, since especially for a large number of players, the homogeneity assumption is quite strong. Therefore, I consider different types of heterogeneity in the next sections. For tractability, I henceforth restrict attention to the two players.

4.2 Asymmetric Cost Functions

As in the baseline model, two players compete for the prize as described in Section 2. Differing from that model, however, I now assume that players have different flow costs, without loss of generality, $c_1 < c_2$.

[6] A full characterization of these equilibria does not seem to deliver additional insights and is therefore beyond the scope of this paper.

By the proof of Proposition 1, there is a unique equilibrium in which both players exert effort during the entire contest (see Proposition 1) if this leads to a (weakly) positive payoff for both players. If this condition is satisfied for player 2, then it also satisfied for player 1 who has lower flow costs. Thus, we obtain:

Proposition 8. *Assume that* $\exp(-r\underline{T}) - \exp(-rT) \leq \frac{Pr}{2c_2}$. *In the unique Nash equilibrium, both players always exert effort, i.e.,* $s = \underline{T}$.

For intermediate and long deadlines, I derive the equilibrium distributions in the appendix. Using these distributions, I show that the main result continues to hold in the presence of asymmetric cost functions in the following proposition.

Proposition 9. *In equilibrium, the contest designer chooses the starting time* $\underline{T} = 0$. *If* $\frac{Pr}{2c_2} \geq 1 - e$, *the optimal deadline is* $T = \frac{1}{r}$. *If* $\frac{Pr}{2c_2} < 1 - e$, *the optimal deadline is* $T = -\frac{1}{r}\log(1 - \frac{Pr}{2c_2})$.

Intuitively, for short deadlines, the equilibria are not changed. For intermediate and long deadlines, player 1 randomizes in the same way as before (with $c = c_2$). Player 2, however, uses a lower average effort than before. Since even for the higher effort levels derived in Proposition 2 and 3, there is a short deadline which dominates both intermediate and long deadlines, the main result continues to hold for intermediate and long deadlines with asymmetric cost functions.

4.3 Tullock Lottery Contest Success Function

So far, I have assumed that the player who exerts most effort wins the contest with probability 1. I now extend the analysis to a Tullock lottery contest success function, i.e., each player's probability of winning the contest is proportional to his share in the total effort (with the convention that the probability is $\frac{1}{2}$ if total effort is 0).

Recall that, due to the discounting, the optimal decision of a player reduces to finding a starting time $s_i \geq \underline{T}$ such that $e_i^t = 1$ if and only if $t \geq s_i$.

The optimization problem of player i is thus:

$$\max_{s_i} P\frac{T-s_i}{2T-s_i-s_j} - \int_{s_i}^{T} c\exp(-rt)dt$$

The first derivative of this function is

$$\frac{d\Pi_i}{ds_i} = P\frac{s_j-T}{(2T-s_i-s_j)^2} + c\exp(-rs_i).$$

Note that as s_i decreases, the increase in winning probability becomes smaller and the marginal cost increases. Thus, there is a unique solution to the above equation. If $\underline{T}=0$, always effort by both players is the Nash equilibrium if nobody profits by increasing his starting time[7]:

$$\frac{d\Pi_i}{ds_i}(s_1=0, s_2=0) = P\frac{-T}{(2T)^2} + c \leq 0,$$

i.e., if $\frac{P}{cT} \geq 4$.

If always effort is not the Nash equilibrium, I obtain the symmetric Nash equilibrium by setting

$$\frac{d\Pi_i}{ds_i}(s_1=s_2=s) = P\frac{s-T}{(2T-2s)^2} + c\exp(-rs) = 0$$

which yields

$$\exp(-rs)(T-s) = \frac{P}{4c}.$$

Thus,

$$s = \frac{rT - W(\frac{P\exp(rT)r}{4c})}{r},$$

where W denotes the Lambert W Function.

A few remarks are in order. First, differing from the all-pay contest success function, for a Tullock lottery contest success function, there always exists a symmetric pure-strategy equilibrium in which both contestants exert effort after a certain starting time. Thus, a restriction on $\underline{T}$ (weakly) reduces

[7] Existence and uniqueness of all equilibria in this section follows from Theorem 1 in Szidarovszky and Okuguchi (1997).

effort. Hence, without loss of generality, I henceforth set $\underline{T} = 0$. Second, I only need to compare the effort on two intervals:

For $T \leq \frac{P}{4c}$, the effort of each contestant is T, while for $T > \frac{P}{4c}$, the effort of each contestant is $T - s = \frac{W(\frac{P\exp(rT)r}{4c})}{r}$.

For the first interval, I obtain $T^* = \frac{1}{r}$ as before. For the second interval, one has to maximize $\frac{W(\frac{P\exp(rT)r}{4c})}{r}\exp(-rT)$. The first derivative of this function is

$$-\frac{r\exp(-rT)W(\frac{P\exp(rT)r}{4c})^2}{W(\frac{P\exp(rT)r}{4c}+1)} < 0,$$

since $W(x) > 0$ for all $x > 0$. Thus, it is never optimal to choose $T > \frac{P}{4c}$. Summing up, I obtain the following proposition:

Proposition 10. *The optimal starting time is always* $\underline{T} = 0$. *The optimal deadline is* $T = \frac{1}{r}$ *if* $\frac{1}{r} < \frac{P}{4c}$ *and* $T = \frac{P}{4c}$ *otherwise.*

While the exact parameters differ slightly, the optimal equilibrium for a Tullock contest success function is qualitatively the same as for the all-pay contest success function. More precisely, the optimal contest length is either in the interior of the parameter space for short deadlines or at the upper end of that parameter space.

4.4 Different Goal Functions of the Contest Designer

The related literature focuses on two different goal functions of the contest designer, maximizing expected total effort as analyzed in the main part and maximizing the expected maximum effort; see, for example, Taylor (1995), Moldovanu and Sela (2001), or Seel and Wasser (2014). Thus, I now consider a contest designer who maximizes $\exp(-rT)\{E(\max\{\int_{\underline{T}}^{T} e_1^t dt, \int_{\underline{T}}^{T} e_2^t dt\})\}$. By the next proposition, short deadlines need not be optimal in this case:

Proposition 11. *Choosing* $\underline{T} = 0$ *and* T *such that the resulting deadline is short is not necessarily maximizing the discounted expected maximum effort* $\exp(-rT)\{E(\max\{\int_{\underline{T}}^{T} e_1^t dt, \int_{\underline{T}}^{T} e_2^t dt\})\}$.

The proof in the appendix constructs a counterexample. While for the parameters $\frac{Pr}{2c} \geq 1$, the proof of the main result directly extends, there are other parameters for which the equilibrium construction in Proposition 2 yields a higher expected maximum effort. In the counterexample, there is a high probability that at least one of the contestants spends the maximal possible effort.

4.5 Different Discount Factors

Up to now, I have assumed that the discount rate of the contest designer from receiving the output triggered by the efforts satisfies $\delta = \frac{1}{1+r}$.

If the contest designer is less patient, i.e., $\delta < \frac{1}{1+r}$, the qualitative results of the paper continue to hold. Intuitively, a short deadline becomes even more attractive, since the contest designer urgently needs the output. On the other hand, if the contest designer is more patient, i.e., $\delta > \frac{1}{1+r}$, the results might break down. Keeping the duration of the contest constant, a later starting time means lower costs for the same prize (both discounted back to $T = 0$). In turn, this implies that contestants exert effort for a longer time period in equilibrium. In the main model, the computation shows the this effect is more than offset by the fact that the designer has to wait longer for the output. But clearly, if the patience of the designer is sufficiently high, the effect of lowering costs for the contestants becomes dominant. Hence, choosing $\underline{T} = 0$ with T such that the resulting deadline is short no longer ensures the equilibrium which generates the highest discounted effort.

For the contestants, I have also assumed that their effort costs shrink at the discount rate. There are several reasons why (perceived) effort costs are decreasing over time such as technological progress, a lower opportunity cost in the future since other projects have upcoming deadlines which require effort now, or behaviorally, simple procrastination. The main results continue to hold as long as effort costs shrink, but at most at the speed of the discount rate. If effort costs shrink faster than the discount rate (e.g., due to fast technological progress), a larger deadline might be optimal.

5 Conclusion

The main model has yielded two strong predictions: (i) A contest designer designer chooses to announce the contest immediately and (ii) a relatively short deadline is optimal. Both predictions are not obvious, since both a longer deadline and a later starting time (for a given contest length) lead to higher effort. However, this effect is dominated by the impatience of the contest designer. The findings are robust to changes in the number of players and asymmetric marginal cost functions in the two-player case. Moreover, they also extend if the contest designer is more patient.

Another main finding is the similarity of the equilibrium structure for the optimal contest with an all-pay contest success function and the optimal contest with a Tullock lottery contest success function. This is in sharp contrast to the standard models, in which all-pay contests induce mixed equilibria and Tullock lottery contests induce pure equilibria. Thus, making the timing structure explicit entails a closer connection between the models.

In accordance with previous literature, a contest designer who is only interested in the effort of one contestant might lead to different results type of effort-maximizing equilibrium.

More generally, reversing the main informational assumptions of a war of attrition changes payoffs and optimal behavior. While a war of attrition typically has a multiplicity of equilibria, the present model yields a unique prediction. For some, yet not all deadlines, the construction is reminiscent of the symmetric equilibrium of a war of attrition. Moreover, the unique equilibrium allows for comparative statics in the discounted effort due to changes in the starting time and deadline.

6 Appendix

Proof of Lemma 3. There exists no equilibrium in which both players start with probability 1 at $s = \underline{T}$, since this would lead to negative profits.

By Lemma 2, the supremum of the starting times contained in the randomization of one player loses with probability 1. By continuity, this player, without loss of generality player 2, gets a payoff of zero.

By standard arguments, both players randomize with a positive density on the same intervals (otherwise, one player could increase his starting time on that interval and obtain the same winning probability at a lower cost).

Case 1: Exactly one player starts with positive probability at $s = \underline{T}$.

In this case, there exists an $\epsilon > 0$ such that the rival does not start in an interval $(0, \epsilon)$ with positive probability, since starting at 0 would increase the expected payoff. Hence, the first player has an incentive to start at $\frac{\epsilon}{2}$ instead of $s = 0$, since both starting guarantee him to win with probability 1, but the latter one induces a higher cost. This contradicts the equilibrium assumption.

Case 2: Both players start with positive probability $m \in (0, 1)$ at $s = 0$.

This entails $F_1(0) > F_2(0)$, since player 1 has a higher equilibrium profit. By the argument in Case 1, there exists an $\epsilon > 0$ such that no player starts with positive probability in $(0, \epsilon)$. Since both players randomize both positive density on the same intervals, I can define the infimum of starting times above 0 which are contained in the randomization of both players by $\tilde{s}$. Both players are indifferent between starting at 0 and at $\tilde{s}$, which leads to a contradiction, since the gain of player 2 from starting at 0 instead of $\tilde{s}$ is higher, because $F_1(0) > F_2(0)$.

Case 3: No player starts with positive probability at $s = \underline{T}$.

Player 1 receives positive profits by the lowest starting time contained in the support of his randomization. Thus, player 2 also receives positive profits by using the lowest starting time in the randomization of player 1, since it guarantees him to win with probability 1 incurring the same cost as player 1. This contradicts the equilibrium assumption. □

Proof of Proposition 1. **Existence:** If both players exert effort during the entire game, both win the prize with probability $\frac{1}{2}$. For any (pure strategy) deviation, a player wins the prize with probability 0, since the rival exerts more effort in this case. Thus, the best possible deviation is to exert no effort at all, which leads to a payoff of 0. The equilibrium payoff is thus greater or equal to the payoff from the best deviation if

$$\frac{P}{2} - \int_{\underline{T}}^{T} c\exp(-rt)dt \geq 0\,.$$

Solving this equation, I get

$$\exp(-r\underline{T}) - \exp(-rT) \leq \frac{Pr}{2c}\,.$$

Uniqueness: There exists no equilibrium in which a player starts at a time $\underline{T} < s < T$ with positive probability by Lemma 2. Towards a contradiction, consider an equilibrium in which at least one player does not start at $s = \underline{T}$ with probability 1. Then, by Lemma 2, the lowest bid of one player loses with probability 1, i.e., at least one player, say player i, makes zero profits .

I distinguish two cases:

1. Player $j \neq i$ starts at time $t = \underline{T}$ with probability 1. As I have argued in the existence part, it is not optimal for player i to use a strategy which starts with positive probability in $(\underline{T}, T)$ against the strategy of player j. Thus, the remaining candidates for an equilibrium strategy of player i place mass $m \in [0, 1)$ at $s = \underline{T}$ (full effort) and $1 - m$ at $s = T$ (no effort). Note that no effort leads to strictly lower payoff for player i if $\exp(-r\underline{T}) - \exp(-rT) < \frac{Pr}{2c}$. Thus, in this case, $m = 1$ (full effort) is a profitable deviation.

It remains to rule out a different equilibrium in the boundary case $\exp(-r\underline{T}) - \exp(-rT) = \frac{Pr}{2c}$ and $m < 1$. Compare the payoff of player j from $s = \underline{T}$ to $s = T - \epsilon$:

$$\Pi^j(s = T - \epsilon) - \Pi^j(s = \underline{T}) = (1 - m - \epsilon) - (1 - \frac{m}{2} - \frac{1}{2}) = \frac{1 - m}{2} - \epsilon\,.$$

Thus, for any $m \in [0, 1)$, there exists an $\epsilon > 0$ (e.g., $\epsilon = \frac{1-m}{4}$), such that starting at $s = T - \epsilon$ is strictly better for player j than starting at $s = \underline{T}$. This contradicts the initial assumption that player j starts at $\underline{T}$ with probability 1.

2. Player $j \neq i$ starts at time $t = \underline{T}$ with a probability less than 1. Recall that player i makes zero profits for his supposed equilibrium strategy. Consider the deviation which starts at time $t = \underline{T}$ for player i. This strategy guarantees a winning probability above $\frac{1}{2}$, since player j does not start at time $t = \underline{T}$ with probability 1. Hence,

$$\Pi^i(s = \underline{T}) > \frac{P}{2} - \int_{\underline{T}}^{T} c\exp(-rt)dt \geq 0\,,$$

i.e., player i has a profitable deviation. $\square$

Proof of Proposition 2. If a player does not start at $s = \underline{T}$ with positive probability, the other player can start at $s = \underline{T}$ and make positive profits which violates Lemma 3. Hence, both players have to start at $s = \underline{T}$ with positive probability. This entails zero profits if

$$\int_{\underline{T}}^{T} c\exp(-rt)dt = P(\frac{F(\underline{T})}{2} + 1 - F(\underline{T})) = P(1 - \frac{F(\underline{T})}{2})\,.$$

Thus,

$$F(\underline{T}) = 2(1 - \frac{c}{rP}(\exp(-r\underline{T}) - \exp(-rT))).$$

Since a player who starts at $s > \underline{T}$ can only win against players who start above $\underline{T}$, no player starts for $s \in (\underline{T}, \tilde{s})$, since

$$P(1 - F(\underline{T})) - \int_{s}^{T} c\exp(-rt)dt \leq P(1 - F(s)) - \int_{s}^{T} c\exp(-rt)dt < 0.$$

For $s \in [\tilde{s}, T]$, the zero profit condition implies

$$P(1 - F(s)) - \int_{s}^{T} c\exp(-rt)dt = 0\,.$$

Rearranging, I obtain

$$F(s) = 1 - \frac{c}{rP}(\exp(-rs) - \exp(-rT))\,. \tag{4}$$

$\square$

Proof of Proposition 3. By Lemma 3, both players make zero profits. Moreover, they randomize with positive density on the same intervals. This uniquely determines the equilibrium distributions by Eq. (4). No player has an incentive to start at $s < \hat{s}$, since the costs exceed the prize in this case. $\square$

Equilibrium Efforts for Intermediate and Long Deadlines

To obtain the values for intermediate deadlines, I derive expected efforts for the case considered in Proposition 2:

$$\mathbb{E}(\sum_{i=1}^{2}\int_{\underline{T}}^{T} e_t^i \exp(-rT)dt) =$$

$$2\exp(-rT)(2(1-\frac{c}{rP}(\exp(-r\underline{T})-\exp(-rT)))(T-\underline{T})+\int_{\tilde{s}}^{T} f(t)(T-t)dt)$$

Note that

$$\int_{\tilde{s}}^{T} f(t)(T-t)dt = \frac{c}{p}\int_{\tilde{s}}^{T} \exp(-rt)(T-t)dt$$

Integration by parts yields:

$$\frac{c}{Pr}[(2-\exp(-rT)-\frac{Pr}{c})(T+\frac{1}{r}\log(2-\exp(-rT)-\frac{Pr}{c})-\frac{1}{r})+\frac{1}{r}\exp(-rT)]$$

Thus, the expected discounted sum of efforts which the designer collects for intermediate deadlines is given by

$$\begin{array}{rcl}\Pi(T) & = & \sum_{i=1}^{2}\int_{0}^{T} e_t^i \exp(-rT)dt = \\ & 2 & \exp(-rT)(2(1-\frac{c}{rP}(\exp(-r\underline{T})-\exp(-rT)))(T-\underline{T}) \\ & + & \frac{c}{Pr}[(2-\exp(-rT)-\frac{Pr}{c})(T+\frac{1}{r}\log(2-\exp(-rT)-\frac{Pr}{c})-\frac{1}{r}) \\ & + & \frac{1}{r}\exp(-rT)]).\end{array}$$

It remains to consider the discounted sum of efforts for the equilibrium for long deadlines which I derived in Proposition 3:

$$\begin{aligned}\Pi(T) &= \textstyle\sum_{i=1}^{2}\int_{\hat{s}}^{T} e_t^i \exp(-rT)dt \\ &= 2\exp(-rT)(T-\tfrac{1}{r} \\ &+ \tfrac{c}{Pr}(T\exp(-rT)+\tfrac{1}{r}\log(\tfrac{Pr}{c}+\exp(-rT))(\tfrac{Pr}{c}+\exp(-rT))))\end{aligned}$$

Proof of Lemma 5. We have to show that the function

$$\Pi(T) = 2(T + \frac{1}{r}\log(\frac{Pr}{2c} + \exp(-rT)))\exp(-rT)$$

is decreasing in T for all $T \geq -\frac{1}{r}\log(1-\frac{Pr}{2c})$.

The first derivative of this function (dropping the 2 which is irrelevant for the sign of the derivatives) is given by

$$\frac{d\Pi(T)}{dT} = \exp(-rT)(1 - \frac{\exp(-rT)}{\frac{Pr}{2c}+\exp(-rT)} - rT - \log(\frac{Pr}{2c}+\exp(-rT))).$$

This derivative is negative if and only if the second term $g(T) = 1 - \frac{\exp(-rT)}{\frac{Pr}{2c}+\exp(-rT)} - rT - \log(\frac{Pr}{2c}+\exp(-rT))$ is negative. Since it is not straightforward to see this from the equation, I proceed in two steps: First, I show that $g(T)$ is negative at the minimal value of T. In the second step, $g(T)$ is shown to be negative for all T above the minimal value.

Step 1: At $T = -\frac{1}{r}\log(1-\frac{Pr}{2c})$, the equation reduces to

$$(1-\frac{Pr}{2c})(\frac{Pr}{2c} + \log(1-\frac{Pr}{2c})) < 0.$$

Step 2: The derivative of $g(T)$ is given by

$$\frac{dg(T)}{dT} = r[-1 + \exp(-rT)(\frac{\frac{Pr}{2c}}{(\frac{Pr}{2c}+\exp(-rT))^2} + \frac{1}{\frac{Pr}{2c}+\exp(-rT)})].$$

Rearranging, we obtain

$$\frac{dg(T)}{dT} = r[-1 + \frac{\exp(-rT)\frac{Pr}{c} + \exp(-2rT)}{\exp(-rT)\frac{Pr}{c} + \exp(-2rT) + (\frac{Pr}{2c})^2}] < 0.$$

Also note that

$$\frac{d^2\Pi}{dT^2} = \exp(-rT)(\frac{dg(T)}{dT} - rg(T)).$$

Towards a contradiction, suppose that $g(T) \geq 0$ for some $T > -\frac{1}{r}\log(1-\frac{Pr}{2c})$. Then, since $g(T)$ is continuous and $g(-\frac{1}{r}\log(1-\frac{Pr}{2c})) < 0$, $g(T) = 0$ for some T and $\Pi(T)$ is increasing in a neighborhood of T.

However, in this case $\frac{d^2\Pi}{dT^2} < 0$, since $\frac{dg(T)}{dT} < 0$ and $-rg(T) \approx 0$. Thus, $\Pi(T)$ is decreasing in the neighborhood of T which yields a contradiction. Hence, $g(T) < 0$ and $\frac{d\Pi}{dT} < 0$ for all $T \geq -\frac{1}{r}\log(1-\frac{Pr}{2c})$.

Summing up, the first derivative is negative for all $T \geq -\frac{1}{r}\log(1-\frac{Pr}{2c})$, i.e., the payoff decreases in the deadline.

□

Proof of Proposition 7. For a given number of players n and knowing that $T^* = -\frac{1}{r}\log(1-\frac{Pr}{nc})$, the discounted sum of efforts is

$$\Pi(n) = nT^*\exp(-rT^*) = n(-\frac{1}{r}\log(1-\frac{Pr}{nc}))(1-\frac{Pr}{nc}) = \log(1-\frac{Pr}{nc})(\frac{P}{c}-\frac{n}{r}).$$

Taking the derivative of the above expression with respect to the number of players and simplifying, I obtain

$$\frac{d\Pi}{dn} = -\frac{P}{nc} - \frac{1}{r}\log(1-\frac{Pr}{nc}).$$

This expression is larger than zero if and only if $-\frac{Pr}{nc} - \log(1-\frac{Pr}{nc}) > 0$, which holds for all considered parameters, i.e., $\frac{Pr}{nc} \in (0,1)$. Thus, the discounted sum of efforts is increasing in n.

□

Proof of Proposition 9. The proof is split up into two steps. Step 1 derives the starting time distribution for intermediate and long deadlines as two lemmas. The second step uses the lemmas and the findings for symmetric cost functions to establish the result.

Step 1: I first derive the equilibrium distributions for intermediate and long deadlines. In both cases, I omit the uniqueness part in the proof, since it follows the same lines as in the case of symmetric cost functions: first, establish that the payoffs of both players are fixed in equilibrium. Then find the unique distributions which yield these payoffs for any starting time in the support and lower payoffs for other starting times.

Lemma 6. *Assume that* $\frac{Pr}{c_2} > \exp(-r\underline{T}) - \exp(-rT) > \frac{Pr}{2c_2}$*. In the unique Nash equilibrium, players randomize their starting times according to the cumulative distribution functions*

$$F_1(s) = \begin{cases} 2(1 - \frac{c_2}{Pr}(\exp(-r\underline{T}) - \exp(-rT))) & \textit{for all } 0 \leq s < \tilde{s} \\ 1 - \frac{c_2}{Pr}(\exp(-rs) - \exp(-rT)) & \textit{for all } s \in [\tilde{s}, T] \\ 1 & \textit{for all } s > T \\ \end{cases}$$

and

$$F_2(s) = \begin{cases} 2(\frac{c_1}{c_2} - \frac{c_1}{Pr}(\exp(-r\underline{T}) - \exp(-rT))) & \textit{for all } 0 \leq s < \tilde{s} \\ \frac{c_1}{c_2} - \frac{c_1}{Pr}(\exp(-rs) - \exp(-rT)) & \textit{for all } s \in (\tilde{s}, T) \\ 1 & \textit{for all } s \geq T\,, \\ \end{cases}$$

where $\tilde{s} = -\frac{1}{r}\log(2\exp(-r\underline{T}) - \exp(-rT) - \frac{Pr}{c_2})$*.*

Proof. Note that $\Pi_1(s) = P(1 - \frac{c_1}{c_2})$ for all $s \in [\tilde{s}, T)$ and smaller otherwise and that $\Pi_2(s) = 0$ for all $s \in [\tilde{s}, T]$ and smaller otherwise. Since each bid contained in the randomization of each player is an optimal strategy against the rival's distribution, the strategy profile is a Nash equilibrium. □

Lemma 7. *Assume that* $\frac{Pr}{c_2} \leq \exp(-r\underline{T}) - \exp(-rT)$*. In the unique Nash equilibrium, players randomize their starting times according to the cumula-*

tive distribution functions

$$F_1(s) = \begin{cases} 0 & \textit{for all } 0 \leq s \leq \hat{s} \\ 1 - \frac{c_2}{Pr}(\exp(-rs) - \exp(-rT)) & \textit{for all } s \in (\hat{s}, T] \\ 1 & \textit{for all } s > T \end{cases}$$

and

$$F_2(s) = \begin{cases} 0 & \textit{for all } 0 \leq s \leq \hat{s} \\ \frac{c_1}{c_2} - \frac{c_1}{Pr}(\exp(-rs) - \exp(-rT)) & \textit{for all } s \in (\hat{s}, T) \\ 1 & \textit{for all } s \geq T \,, \end{cases}$$

where $\hat{s} = -\frac{1}{r}\log(\frac{Pr}{c_2} + \exp(-rT))$.

Proof. Note that $\Pi_1(s) = P(1 - \frac{c_1}{c_2})$ for all $s \in [\tilde{s}, T)$ and smaller otherwise and that $\Pi_2(s) = 0$ for all $s \in [\tilde{s}, T]$ and smaller otherwise. Since each bid contained in the randomization of each player is an optimal strategy against the rival's distribution, the strategy profile is a Nash equilibrium. □

Step 2: In Lemma 6 and 7, the distribution of player 1 stochastically dominates the distribution of player 2, i.e., we can bound the expected effort of player 2 by expected effort of player 1. For all deadlines, player 1 uses the same equilibrium distributions as in the symmetric setting (with $c_2 = c$) and for short deadlines, both players use the same equilibrium distributions as in the symmetric setting. Thus, Proposition 5 remains valid. □

Proof of Proposition 11. Let $r = 0.01$, $P = 2$, $C = 1$ and $\underline{T} = 0$. Then, for short deadlines, the maximum is obtained at the corner solution $T = -100\log(0.99)$. At this value

$$\max \exp(-rT)\{E(\max\{\int_{\underline{T}}^{T} e_1^t dt, \int_{\underline{T}}^{T} e_2^t dt\})\} = T\exp(-rT) \approx 0.995.$$

Suppose that $\tilde{T} = -100\log(0.989)$ instead. Plugging into Proposition 2, the probability that at least one player exerts the maximal effort is $F(0)^2 +$

$2F(0)(1-F(0)) = 0.99$. In the following, we ignore the 0.01-probability case in which no player exerts the maximal effort; including it would increase expected maximal effort even further. Thus, for deadline $\tilde{T} = -100\log(0.989)$, we obtain:

$$\max \exp(-r\tilde{T})E(\max\{\int_{\underline{T}}^{\tilde{T}} e_1^t dt, \int_{\underline{T}}^{\tilde{T}} e_2^t dt\}) > 0.99\exp(-r\tilde{T})\tilde{T} > 1.08 > 0.995.$$

Thus, the expected discounted maximal effort for an intermediate deadline is higher than for the maximal short deadline.

□

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