Welfare Gains from Trade under Capacity Constraints *

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Abstract

This paper proposes a trade model where firms are heterogeneous both in productivity and capacity. If a firm cannot freely adjust its production because it has a binding capacity constraint, it will face a trade-off between domestic and export sales, and will raise prices in order to take advantage of access to larger markets. This generates markets with a competitive structure where the impact of trade on welfare is not straightforward. By calibrating the model to illustrate the theoretical predictions, I show that if capacity is scarce, opening up to trade can have a negative impact on welfare. Furthermore, the larger the country's trading partner, the smaller are the distortionary effects induced by capacity constraints.

JEL Codes: F12, L11, L13, R13. Keywords: Capacity Constraints, Firm Heterogeneity, Endogenous Markups, Market Size, Trade Liberalization.

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1 Introduction

It is a well known fact that openness of national economies to international trade has a positive influence on the country's competitiveness, welfare and growth. The reallocation of sales and resources towards more productive firms has been identified as the main source of gains following a trade liberalization — this is the well-known selection effect described by Melitz (2003) and documented by Roberts and Tybout (1997) and Bernard and Bradford Jensen (1999). However, such a reallocation is unlikely to happen if strict regulations prevent resources from moving towards the most productive sectors and to the most efficient firms within sectors. Many recent studies have documented that a trade liberalization may fail to fully benefit the population in the presence of heavily regulated countries, labor market rigidity or financial constraints.¹ In the presence of such domestic restrictions, firms are not fully flexible in their production decisions. While there is an extensive industrial literature on capacity constraints affect firm behavior in an international trade setting.

I propose a trade model with firms that are heterogeneous both in productivity and in capacity. In the wake of a trade liberalization, if a firm is capacity constrained, it will face a trade off between selling at home and selling abroad to take advantage of the larger market, and may end up raising prices. How the rise in price and the substitution of domestic sales for foreign sales induced by capacity constraints interact with the selection effect, and thus the final impact of trade liberalization on the economy, is no longer straightforward. Thus, I calibrate the model to several European Economies in order to illustrate theoretical predictions of the effect of trade on welfare. I show that if capacity is tight, opening up to trade can have a negative impact on welfare. The increased market opportunities tighten the constraints on firms, which cannot freely expand their production, and therefore sell lower quantities of their goods to the markets at a higher price. The surplus extracted by consumers thus decreases and counteracts any gains in variety and efficiency that trade may induce. Finally, the larger the country's trading partner, the smaller these distortionary effects are overall. This paper suggests that capacity constrained firms have strong internal linkages across destinations and thus pass any distortionary behavior onto each of their markets.

In standard models of trade with heterogeneous firms, individual participation in international markets is driven solely by the productivity of the firm and production decisions

¹ See Bolaky and Freund (2004), Behrens et al. (2007), Hasan et al. (2007) and Manova (2013).

²From Levitan and Shubik (1972) and Spence (1977) to Knittel and Lepore (2010).

are completely flexible.³ In the model presented in this paper, the existence of capacity constraints restricts production and pricing decisions, which are no longer completely flexible. Firms interact in a monopolistically competitive market for their differentiated varieties in the spirit of Melitz and Ottaviano (2008). Firms set prices and markups endogenously according to their production capacity and demand for their good. Larger markets exhibit tougher competition, resulting in lower average markups and higher aggregate productivity (selection effect). The model produces an equilibrium identical to that of Melitz and Ottaviano (2008) in the limit, when the probability of firms being capacity constrained approaches zero. Instead, the existence of capacity constraints changes the structure of firms' marginal costs. Firms cannot freely expand production to access markets if they are facing a binding capacity constraint; thus, they no longer have constant marginal costs and consequently face a trade-off between domestic and export sales (substitution effect). Furthemore, since firms producing at capacity are unable to expand production in order to take advantage of access to larger markets, they raise prices even in the presence of tougher competition (composition effect). Hence, the existence of capacity constrained firms may lead to a market wide softening of competition as markets grow.

By incorporating competition channels and quantitative restrictions, the model has implications that differ from standard models of trade with heterogeneous firms. First, costly trade does not completely integrate markets and thus the existence of capacity constraints may magnify or mitigate the competitive effects of opening up to trade. In particular, price competition is softened in markets with high concentrations of capacity constrained firms. Second, while the classic predictions hold (i.e. more productive firms are larger and tend to export), the model can also be used to explain the existence of highly productive firms with relatively low export sales or no presence at all in the export market. In standard models of trade with heterogeneous firms, matching both the number of exporters and the intensity of exporting has been a challenge since, by construction, foreign sales are proportional to domestic sales.⁴ Finally, the model allows us to predict positive and negative correlations between domestic and export sales at the firm level, depending on the status of the firm: firms facing a binding capacity constraint substitute sales across destinations, while unconstrained ones do not.⁵

The impact of trade on welfare is not straightforward due to the new forces induced by capacity constraints. Thus, I calibrate the model and asses the consequences of opening

³See among others, Bernard et al. (2003), Melitz (2003), Helpman et al. (2004), Ghironi and Melitz (2005), Arkolakis (2010), Davies and Eckel (2010) and Forslid et al. (2011).

⁴Due to the use of CES preferences and fixed cost of exporting.

⁵This is consistent with recent empirical studies which have documented a negative correlation between exporting firm's domestic and export sales growth in the presence of financial constraints (See Blum et al. (2010) and Ahn and McQuoid (2013) among others).

up to trade — considering both symmetric and asymmetric trade;⁶ and the implications of a unilateral trade liberalization. In particular, the model is calibrated to match the firm size distribution (mean and variance) of three European countries. I use data from France, Germany and Spain provided by the survey 'European Firms in a Global Economy' (EFIGE).⁷. In the theoretical model, a binding capacity constraint implies that firms produce below their first-best level and remain small. The empirical counterpart matches this fact, and in countries with fewer large firms, such as Spain, firms have a higher probability of being constrained.

Using the calibrated model, I perform several exercises. First The first exercise consists of analyzing under which conditions the presence of capacity constraints does not undermine the positive effects of trade. I demonstrate that markets may be distorted to such an extent that consumer welfare falls with trade. Firms shift the weight of their pricing distortions across markets and substitute sales across destinations, scaling sales back in each one. This reduces the surplus extracted by consumers for each good, which may counteract the variety and efficiency gains. This result is reminiscent of second-best results of Bhagwati and Ramaswami (1963) — with one distortion in place (capacity constraints), reducing a second distortion (such as trade barriers) need not make the country better off. Furthermore, it provides an explanation of why many countries are resistant to trade liberalization. Firms facing binding capacity constraints eliminate the expected gains from trade that come through enhanced price competition. In addition, the simulations highlight the importance of a country's trading partner. In the absence of capacity constraints, any gains from a larger trading partner would be offset by increased competitiveness (a greater number of more productive firms competing in a given market drive down markups). In the presence of capacity constraints, however, a larger market provides a buffer to the negative effects brought about by these restrictions.

The second exercise focuses on trade policy between asymmetric partners. In Melitz and Ottaviano (2008), trade barriers and foreign market characteristics only have direct effects on exporters, and only impact equilibrium indirectly. With capacity constraints, market accessibility directly shapes the implicit marginal costs of all constrained firms. This has a direct structural influence on the equilibrium equations, and thus firms may get more than what they bargained for from trade policy. For example, a unilateral increase in trade costs by the potentially constrained country intensifies the competition in the market, increases the number of varieties and as a result consumer welfare rises. Constrained

 $^{^{6}}$ I consider both trade between two countries facing potentially binding constraints — symmetric trade; and trade between two countries where only the home country faces potentially binding capacity constraints — asymmetric trade.

 $^{^7\}mathrm{The}$ EFIGE survey was conducted during the year 2009 and is representative of the manufacturing sector in each country

firms substitute sales away from the home market and the excess demand is then served by productive foreign firms. In such a case, a protectionist policy clearly improves competition. These results underline how the welfare loss associated with unilateral liberalization is driven by the de-localization of firms, which favors the non-liberalizing trading partner, and is intensified by the presence of capacity constraints in the liberalizing country.

The model proposed remains highly tractable, even when extended to a general framework with multiple asymmetric countries integrated to different extents through asymmetric trade costs. It therefore provides a useful modeling framework that is particularly well suited to the analysis of trade and regional integration policy scenarios in environments with heterogeneous firms, endogenous markups and non-price restrictions in the market.

As in Melitz and Ottaviano (2008), the present paper highlights the role of market size in determining changes in the performance measures of firms. It goes one step further, however, by showing how de-localization effects from trade liberalization are directly affected by the size of the country's trading partner. Moreover, I am able to explain various occurrences in the data that are anomalous to "new" new trade theory models. Particularly, I provide a plausible explanation for the substitution patterns of exporters across destinations. Vannoorenberghe (2012), Blum et al. (2013) and Liu (2014) have also proposed models which aim to explain the substitution of sales across markets. These models feature heterogeneous firms with fixed capital and Cobb-Douglas production, and thus firms can reach any finite level of production at increasing marginal costs. Hence, they cannot accommodate some firms exhibiting a substitution of sales across destinations while others exhibit a complementary relationship in the same economy. More importantly, they cannot generate the small efficient firms or weak price competition in large markets highlighted above which are key to understanding potential losses from trade liberalization.

My paper is also related to several papers in the industrial organization literature that analyze the role of capacity constraints in the presence of international markets. Maggi (1996) discusses the effects of "soft" capacity constraints in a three-country duopoly model with homogeneous firms. His work demonstrates that the distortion of competition from capacity constraints has substantial impact on market outcomes, especially in the light of globalization, even when firms are homogeneous. Krishna and Panagariya (2000) discuss how pre-imposed quantitative restrictions enter differently from price restrictions, and analyze the implications of this difference for the conduct of second-best optimum policies in the pure theory of international trade — custom unions theory, the principle of targeting and immiserizing growth. Their work highlights the importance of considering secondbest solutions when dealing with a perfectly competitive economy with an homogeneous good, although it abstracts from imperfect competition models. My paper adds to the knowledge in this area by extending the analysis to heterogeneous firms and firm level decisions.

The paper is organized as follows. Section 2 describes the model in a closed economy and how capacity constraints interact with production decisions. Section 3 opens the model to international trade and elaborates on the implications of capacity constraints for firm dynamics. Section 4 discusses the data and calibration strategy. Section 5 summarizes the implications of opening up to trade if there are potentially binding capacity constraints in the economy. Finally, Section 6 concludes.

2 Closed Economy

This section analyzes the consequences of firms facing hard capacity constraints in a closed economy, where consumers face a linear demand system with horizontal product differentiation. Such demand, developed by Ottaviano et al. (2002), generates a tractable endogenous distribution of markups across firms that responds to the toughness of competition in a market — to the number and average productivity of competing firms in that market.

2.1 Preferences and Demand

Consider an economy with L consumers, each supplying one unit of labor, with a linear quadratic utility.

$$U^{i} = q_{0}^{i} + \alpha \int_{\omega \in \Omega} q_{\omega}^{i} d\omega - \frac{\gamma}{2} \int_{\omega \in \Omega} \left(q_{\omega}^{i} \right)^{2} d\omega - \frac{\eta}{2} \left(\int_{\omega \in \Omega} q_{\omega}^{i} d\omega \right)^{2}, \tag{1}$$

where q_0^i and q_{ω}^i represent the individual consumption levels of the numeraire good and each variety ω .⁸ The demand parameters α , η and γ are all positive. The first two index the substitution between the differentiated varieties relative to the numeraire, while the latter indexes the degree of product differentiation between the varieties.⁹

The marginal utilities for all goods are bounded, and thus, positive demand for a particular good is not guaranteed. For simplicity, I assume that consumers always have positive demands for the numeraire good $(q_0^c > 0)$. The inverse demand for each variety ω is then given by:

$$p_{\omega} = \alpha - \gamma q_{\omega}^{i} - \eta Q^{i}, \qquad \text{for } q_{\omega}^{i} \ge 0.$$
(2)

⁸The superscript i indicates the individual consumer.

⁹At the limit, when $\gamma = 0$, varieties are perfect substitutes and $Q^i = \int_{\omega \in \Omega} q^i_{\omega} d\omega$.

Let $\Omega^* \subset \Omega$ be the subset of varieties that are consumed $(q^i_{\omega} > 0)$, then invert and aggregate the equation to yield the linear market demand system for these varieties:

$$q_{\omega} \equiv Lq_{\omega}^{i} = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p_{\omega} + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \overline{p}, \qquad \forall \omega \in \Omega^{*},$$
(3)

where N is the measure of varieties consumed in Ω^* and $\overline{p} = (1/N) \int_{\omega \in \Omega^*} p_{\omega} d\omega$ is the average price.

The set Ω^* is the largest subset of Ω that satisfies

$$p_{\omega} \le \frac{\gamma \alpha + \eta N \overline{p}}{\eta N + \gamma} \equiv p_{max},\tag{4}$$

where p_{max} represents the price at which demand for a variety is driven to 0.10

The effects of aggregate market outcomes on individual welfare are characterized by the indirect utility function,

$$U^{i} = I^{i} + \frac{1}{2} \left(\eta + \frac{\gamma}{N} \right)^{-1} (\alpha - \overline{p})^{2} + \frac{1}{2} \frac{N}{\gamma} \sigma_{p}^{2}, \tag{5}$$

where I^i is the consumer's income and $\sigma_p^2 = \frac{1}{N} \int_{\omega \in \Omega^*} (p_\omega - \overline{p})^2 d\omega$ represents the variance of prices. To ensure positive demand levels for the numeraire, I assume that $I^i > \int_{\omega \in \Omega^*} p_\omega q_\omega^i d\omega = \overline{p}Q^i - N\sigma_p^2/\gamma$.

Welfare naturally rises with decreases in average prices \overline{p} . It also rises with increases in the variance of prices σ_p^2 (holding the mean price \overline{p} constant), as consumers then shift their expenditures towards lower priced varieties as well as the numeraire good. Finally, the demand system exhibits "love of variety": holding the price distribution constant (namely holding the mean \overline{p} and variance σ_p^2 of prices constant), welfare rises with increases in product variety N.

2.2 Production and Firm Behaviour

The only factor of production is labor, which is inelastically supplied in a competitive market. The numeraire good is produced under constant returns to scale at unit costs and sold in a competitive market. These assumptions on the numeraire good and the labor market together imply unit wages.

¹⁰Notice that the price-demand equation implies that $p_{max} < \alpha$. In contrast to the case of CES demand, the price elasticity of demand, $\epsilon_{\omega} \equiv \left|\frac{\partial q_{\omega}}{\partial p_{\omega}}\frac{p_{\omega}}{q_{\omega}}\right| = \left(\frac{p_{max}}{p_{\omega}} - 1\right)^{-1}$, is not uniquely determined by the level of product differentiation γ . Given this, lower average prices \overline{p} or a larger number of competing varieties N induce a decrease in the price bound p_{max} and an increase in the price elasticity of demand ϵ_{ω} at a given p_{ω} .

Firms outside of the numeraire market incur product development and production start-up costs. By making the irreversible investment (f_E) to enter the market, firms learn about their marginal costs c and their production capabilities K.¹¹ Marginal costs are drawn from a common and known distribution G(c) that has support $[0, c_M]$. Similarly, plant capacity is randomly drawn from the common and known distribution H(K) over the strictly positive support $[0, K_{max}]$. Production exhibits constant returns to scale technology at marginal cost c unless the firm is constrained by its plant capacity draw K. The marginal cost of production beyond these "hard" capacity constraints is infinite.

This model does not allow for a firm's specific productivity draw to directly impact its capacity draw, but rather endogenizes the distribution of capacity throughout the market cut-off cost level. Therefore, if the constraint is binding or not for a firm will depend on the marginal costs of the firm and the market conditions, but the capacity draws are independent of the productivity draws. This is consistent with capacity being acquired randomly from a source with incomplete information regarding the individual productivity of firms. For example, if there is incomplete information in the lending market, then lenders uncertain of the productivity of a firm may under- or over-invest in its capacity without the firm becoming aware of its allotment until entry.

For simplicity, I assume that firms cannot recover their entry cost, and the capacity and cost draws are permanent over the life of the firm.¹² Capacity defines the quantity a given firm can produce at its constant marginal cost (c), thus the assumption that capacity is permanent over the life of the firm may seem very harsh. The model could quite easily be extended to one in which firms can adjust their capacity by incurring a constant per unit cost r, such that each unit produced beyond K is at a marginal cost of c+r. This feature, while providing a more realistic approach to capacity constraints, does not have a qualitative impact on the predictions of the model but over-complicates the mathematics. Hence, we can think of this model as one in which firms face prohibitively high costs of adjusting their capacity levels.

Since the entry cost is sunk, firms that can cover their marginal cost survive and produce regardless of whether they are constrained by their capacity draw or not. All the other firms exit the industry. The surviving firms compete in a monopolistically competitive market for differentiated goods taking the average price level \bar{p} and number of firms N as given.

Firms maximize their profits using the residual demand function from Equation 3

¹¹I suppress the firm-specific subscript here and in the following to save on notation.

¹²Fixing capacity levels throughout the life of the firm is the extreme case of specifying increasing marginal costs in this model.

subject to their realized capacity constraint K and marginal cost c:

$$\arg\max_{p(c)} \quad \left\{ (p(c) - c) q(c) |_{q(c) = min\{q(p(c)), K\}} \right\}$$

$$[FOC] \quad p(c) = \begin{cases} c + \frac{\gamma}{L} q(c) & for q(c) < K \\ c + \lambda + \frac{\gamma}{L} q(c) & for q(c) = K, \end{cases}$$
(6)

where λ is the shadow value of capacity.

Let c_D reference the cost of the firm which is indifferent between producing and exiting. This firm's price equals its marginal costs and it faces zero demand for its variety. Thus, $p(c_D) = c_D = p_{max}$.

Furthermore, among the active firms some will face a binding constraint on production. For each firm with cost draw c, there exists a restrictive capacity level $K^*(c)$, such that firms drawing $K < K^*(c)$ face a binding capacity constraint, while firms drawing $K \ge K^*(c)$ are unconstrained. Each firm's specific productivity, combined with market characteristics, endogenously determines its cut-off capacity level $K^*(c)$, which is characterized by:

$$K^*(c) = \frac{L}{\gamma} \left(p_{max} - p(c) \right) \Rightarrow K^*(c) = \frac{L}{2\gamma} (c_D - c).$$

Thus, the random draws of capacity imply a distribution of implicit marginal costs, which is characterized by the restrictive capacity level $K^*(c)$.

Finally, define the implicit marginal costs as the sum of the actual marginal costs and the shadow value of capacity, $\kappa = c + \lambda$. These implicit marginal costs are endogenously determined by the firm's marginal cost (c), the firm's capacity (K) and the cut-off cost level of the market (c_D):

$$\kappa(c, K, c_D) = \begin{cases} c & \text{if } K \ge K^*(c) \\ c_D - \frac{2\gamma}{L} K & \text{if } K < K^*(c) \end{cases}$$
(7)

I can then characterize firm behavior solely with the implicit marginal costs κ and the characteristics of firm c_D . Combining a firm's optimal behavior and residual demand yields the optimal price, markup, quantity and profit equations:

$$p(\kappa) = \frac{1}{2}(c_D + \kappa) \tag{8}$$

$$\mu(\kappa) = \frac{1}{2}(c_D + \kappa) - c \tag{9}$$

$$q(\kappa) = \frac{L}{\gamma} \left(c_D - \frac{1}{2} (c_D + \kappa) \right)$$

$$= \frac{L}{2\gamma}(c_D - \kappa) \tag{10}$$

$$\pi(\kappa) = p(\kappa)q(\kappa) - q(\kappa)c$$

$$= \frac{1}{2}(c_D + \kappa)\frac{L}{2\gamma}(c_D - \kappa) - \frac{L}{2\gamma}(c_D - \kappa)c$$

$$= \frac{L}{4\gamma}(c_D - \kappa)(c_D + \kappa - 2c).$$
(11)

Lower cost firms set lower prices and earn higher revenues and profits than firms with higher costs. All else being equal, firms facing a binding constraint charge higher prices and set higher markups than their unconstrained counterparts. That is, prices and markups are monotonically increasing in a firm's implicit marginal costs. However, profits are strictly decreasing in implicit marginal costs. Despite the higher markups, constrained firms make lower profits as the quantity restriction outweights any pricing gains.

In the absence of binding capacity constraints, the model is identical to the set-up of Melitz and Ottaviano (2008), and the more profitable firms are the more efficient ones. However, by introducing capacity constraints into their set-up, I match more realistic scenarios where highly efficient producers remain small and less profitable due to binding capacity constraints. For example, this is consistent with the results of Clementi and Hopenhayn (2006), who conclude that efficient firms may produce at less than optimal scale due to an inability to acquire their optimal level of capital if there is contracting uncertainty in the lending market.

2.3 Free Entry Equilibrium

Prior to entry, the expected firm profit is $\int_0^{c_D} \int_0^{K_{max}} \pi(\kappa) dH(\kappa) dG(c) - f_E$. If this profit were negative, no firms would enter the industry. As long as some firms produce, the expected profit is driven to 0 by the unrestricted entry of new firms. This, along with Equation 7, yields the free entry condition:

$$\overline{\Pi} \equiv \int_0^{c_D} \left[1 - H(K^*(c)) \right] \pi(c) dG(c) + \int_0^{c_D} \int_0^{K^*(c)} \pi(\kappa) dH(K^*(c)) dG(c) = f_E.$$
(12)

 $K^*(c)$ and $\pi(\kappa)$ are determined by the distribution of implicit marginal costs implied by the distribution of capacity. Since κ is monotonic in K, a monotonic transformation can be applied and the free entry condition can be rewritten as:

$$\int_0^{c_D} \left[1 - H\left(\frac{L}{2\gamma}(c_D - c)\right) \right] \pi(c) dG(c) + \frac{L}{2\gamma} \int_0^{c_D} \int_c^{c_D} \pi(\kappa) h(\frac{L}{2\gamma}(c_D - \kappa)) d\kappa dG(c) = f_E,$$

which determines the cost cut-off c_D . In turn, this cut-off determines the number of

surviving firms.

$$c_D = p(c_D) = p_{max} = \frac{\gamma \alpha + \eta N \overline{p}}{\eta N + \gamma},$$

with $\overline{p} = \frac{1}{M} \int_{\omega \in \Omega^*} p_{\omega} d\omega = \frac{1}{M} \int_{\omega \in \Omega^*} \frac{1}{2} (c_D + \kappa) d\omega = \frac{1}{2} (c_D + \overline{\kappa})$, where $\overline{\kappa}$ is the weighted average of the marginal costs faced by constrained and unconstrained producers, that is $\overline{\kappa} = \frac{1}{G(c_D)} \left(\int_0^{c_D} \left[1 - H \left(\frac{L}{2\gamma} (c_D - c) \right) \right] c dG(c) + \frac{L}{\gamma} \int_0^{c_D} \int_c^{c_D} \kappa (\frac{L}{2\gamma} (c_D - \kappa)) d\kappa dG(c) \right)$. The number of varieties in the economy¹³ is then:

$$N = \frac{\gamma(\alpha - c_D)}{\eta(c_D - \overline{p})} = \frac{2\gamma(\alpha - c_D)}{\eta(c_D - \overline{\kappa})}.$$

Given a production technology referenced by G(c), the average productivity will be higher (lower $\bar{\kappa}$) when the sunk costs are lower, when varieties are closer substitutes (lower γ), and in bigger markets (more consumers L). In all these cases, the firm exit rates are also higher and the pre-entry probability of survival $G(c_D)$ is lower. The demand parameters α and η that index the overall level of demand for the differentiated varieties (relative to the numeraire) do not affect the selection of firms and industry productivity — they only affect the equilibrium number of firms.

Competition is "tougher" in larger markets (large L), as more firms compete and average prices $\overline{p} = (c_D + \overline{\kappa})/2$ are lower. A firm with cost c responds to this tougher competition by setting a lower markup relative to the markup it would set in a smaller market.

Competition is "weakened" in markets where capacity is restricted (low K_{max}) as the constrained firms raise the average price level, and thus there are more varieties sold in the economy to satisfy demand. I refer to this effect as the *composition effect*, since its force relies on the mix of constrained and unconstrained firms in the economy.

This equilibrium exists and is unique regardless of the distribution of K and c with the proper parametrization of the support of capacity.¹⁴

2.4 Parametrization of technology

In order to illustrate more easily the results discussed in the previous section, I use a specific parametrization and give a closed-form solution.

Assume that each firm draws its capacity K from a uniform distribution over the pos-

¹³The number of entrants is directly related to the number of varieties in the economy, and is given by $N_E = \frac{N}{G(c)}$.

¹⁴It can be shown that $\overline{\Pi}$ is strictly increasing in c_D with an application of the Leibniz rule under relatively unrestrictive assumptions regarding the support of capacity, and since $\lim_{c_D\to\infty}\overline{\Pi} = \infty$ and $\lim_{c_D\to c}\overline{\Pi} = 0$, a unique solution exists. See the Appendix for proof.

itive support $[0, K_{max}]$ and that productivity draws 1/c follow a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k \ge 1$. This implies a distribution of cost draws c given by:

$$G(c) = \left(\frac{c}{c_M}\right)^k, \ c \in [0, \ c_M]$$

Given this parametrization, the cut-off cost level c_D that solves Equation 12 is:

$$c_D^{k+2}\left(1 - \frac{L}{2\gamma(k+3)}\frac{c_D}{K_{max}}\right) = \left[\frac{2(k+1)(k+2)\gamma(c_M)^k f_E}{L}\right].$$

For convenience, I define a measure of the ease of capacity acquisition relative to the degree of market competition, $\delta = \frac{K_{max}}{c_D}$. A relatively large δ implies a large capacity acquisition process (high K_{max}) or a lower capacity premium due to high competition (low c_D). Furthermore, since $K_{max} > \frac{L}{2\gamma}$, positive expected profits are guaranteed.¹⁵ Finally, I also define a technology index, $\phi = 2(k+1)(k+2)(c_M)^k f_E$, which combines the effects of a better distribution of cost draws (lower c_M) and lower entry costs (f_E).

The solution of the free entry condition can then be expressed as:

$$c_D = \left[\left(\frac{1}{1 - \frac{L}{2\gamma\delta(k+3)}} \right) \frac{\gamma\phi}{L} \right]^{\frac{1}{k+2}}.$$
(13)

Although it is not a closed-form solution, it allows us to more easily interpret the effects that the existence of constrained firms has in the economy. As the ease of acquiring capacity increases ($\delta \rightarrow \infty$), the probability of being constrained tends to zero. Consequently expected profits and the model in its entirety approach that of Melitz and Ottaviano (2008).

The zero cut-off profit condition and Equation 13 characterize the competitive equilibrium.¹⁶ The average price \overline{p} , the number of firms in equilibrium N and the welfare U are then given by:

$$\overline{p} = \left(\frac{2k+1}{2(k+1)} + \frac{L}{4\gamma\delta(k+1)(k+2)}\right)c_D,$$

$$N = \frac{2\gamma(k+1)(\alpha - c_D)}{\eta\left(c_D - c_D\frac{L}{2\gamma\delta(k+2)}\right)},$$

$$U = 1 + \frac{1}{2\eta}\left(\alpha - c_D\right)\left(\alpha - \frac{\frac{k+1}{k+2} - \frac{L(k+1)}{2\gamma\delta(k+2)(k+3)}}{1 - \frac{L}{2\gamma\delta}\frac{1}{(k+2)}}c_D\right)$$

¹⁵Notice that otherwise $K^*(c) > K_{max}$, which is infeasible.

¹⁶Note that the variance can be written as a function of the cost draws. In particular, $\sigma_p = \frac{1}{4}\sigma_{\kappa}$.

The traditional result, when production is completely flexible, is that bigger markets induce tougher selection (lower cutoff c_D), leading to higher average productivity, lower average prices and a higher number of varieties in the economy. Therefore, in a bigger market, welfare is enhanced through this *selection effect*.

The presence of constrained firms dampens these pro-competitive effects through the *composition effect*. Capacity constrained firms sell at inflated prices since they respond to the tightness of their constraint and not to the higher degree of competition. Constrained firms create an excess demand in the bigger market, which allows more firms to exist in it (N rises), and has an ambiguous effect on the average prices in the economy: \bar{p} may rise if the composition effect overcomes the selection effect. Therefore, the *composition effect* ambiguously impacts welfare — weaker competition in the market due to the presence of constrained firms' but more varieties on the market. Ultimately, fluctuations in welfare are dominated by changes in the degree of competition c_D .

Proposition 1

Firms serving larger markets face stronger selection, $\frac{\partial c_D}{\partial L} < 0$. In such a market:

- The number of varieties grows, $\frac{\partial N}{\partial L} > 0$.
- The effect on the average price level is ambiguous, $\frac{\partial \bar{p}}{\partial L} \leq 0$.
- Consumer welfare rises, $\frac{\partial U}{\partial L} > 0$.

Proof. See the Appendix

Figure 1a and Figure 1b provide a numerical example of a parameterized equilibrium as market size varies with respect to an economy without constrained firms. In Figure 1a, I show the evolution of the average price level and the competition level as market size increases (\bar{p} and c_D), and compare it with the flexible production situation which is equivalent to the Melitz and Ottaviano (2008) model (\bar{p}^{MO} and c_D^{MO}). The presence of constrained firms in the economy decreases the level of competitiveness in the economy and raises average prices. Figure 1b depicts how the number of firms and welfare move with market size, both when there are no constrained firms in the economy with constrained firms, despite the number of varieties consumed being higher. The difference increases as the market size grows larger, and the number of constrained firms in the economy increases, thus confirming the excess demand due to constrained firms and the Free Entry condition kicking in by increasing the number of firms in the market.

Parameter values: $\alpha = 15$, $\eta = 10$, $\gamma = 2$, k = 2, $f_E = 1$, $c_M = 10, K_{max} = 350$.



Figure 1: Model Parametrization in Autarky

3 Open Economy

The previous section highlighted the pro- and anti-competitive effects at play when the market increases its size. This closed-economy model could be immediately applied to a set of open economies that are integrated through trade. In this case, the transition from autarky to free trade is very similar to an increase in market size but not equivalent. First, because goods are not freely traded, markets are connected in ways that preclude the analysis of each market in isolation. And second, because capacity constraints significantly impact the structure of competition in each market. Therefore, firms may forego sales in one market to serve another and pass along their distortionary behaviour.

3.1 **Production and Exporting Decisions**

Suppose there are two countries, home (H) and foreign (F). The two markets are segmented, although firms can produce in one market and sell in the other at a cost. In particular, firms exporting to country l (l = H, F), incur an additional per unit iceberg transport cost of $\tau^h > 1 \forall l \neq h$, which captures the barriers to trade between the two countries.

The countries only differ in two dimensions: market size L^l and barriers to imports τ^l . The consumers in both countries share the same inverse demand functions given by Equation 2. Let p_{max}^l denote the price threshold for positive demand in market l. Then

Equation 4 implies

$$p^{l} = \frac{\gamma \alpha + \eta N^{l} \overline{p}^{l}}{\eta N^{l} + \gamma}, \qquad l = H, F,$$
(14)

where N^l is the total number of firms selling in country l (sum of domestic firms and foreign exporters) and \overline{p}^l is the average price (across both local and exporting firms) in country l. Let $p_D^l(c)$ and $q_D^l(c)$ represent the domestic levels of the profit maximizing price and quantity sold for a firm producing in country l with cost c. Such a firm may also decide to produce some output $q_X^l(c)$ that it exports at a delivered price $p_X^l(c)$. Consequently, the utilization of capacity for firms producing in country l is $q_D^l(c) + \tau^h q_X^l(c)$, and the profit maximization problem of a firm located in country l is:

$$\begin{split} & \underset{p_{D}^{l}(c), p_{X}^{l}(c)}{\text{arg max}} & \left\{ \begin{pmatrix} p_{D}^{l}(c) - c \end{pmatrix} q_{D}^{l}(c) + \begin{pmatrix} p_{X}^{l}(c) - \tau^{h}c \end{pmatrix} q_{X}^{l}(c)|_{q_{D}^{l}(c) + \tau^{h}q_{X}^{l}(c) \leq K} \\ & \\ p_{D}^{l}(c) = c + \frac{\gamma}{L^{l}}q_{D}^{l}(c) & \text{if } q_{D}^{l}(c) + \tau^{h}q_{X}^{l}(c) < K \\ & \\ p_{X}^{l}(c) = \tau^{h}c + \frac{\gamma}{L^{h}}q_{X}^{l}(c) & \text{if } q_{D}^{l}(c) + \tau^{h}q_{X}^{l}(c) < K \\ & \\ p_{D}^{l}(c) = c + \lambda + \frac{\gamma}{L^{l}}q_{D}^{l}(c) & \text{if } q_{D}^{l}(c) + \tau^{h}q_{X}^{l}(c) = K, \\ & \\ p_{X}^{l}(c) = \tau^{h}(c + \lambda) + \frac{\gamma}{L^{h}}K & \text{if } q_{D}^{l}(c) + \tau^{h}q_{X}^{l}(c) = K, \end{split}$$

Define the implicit marginal costs for firms located in country l for a firm drawing marginal cost c and capacity K as $\kappa^l = c + \lambda$, which equals

$$\kappa^{l}(c, c_{D}^{l}, c_{X}^{l}, K) = \begin{cases} c & \text{if } K \geq K^{l^{*}}(c) \\ \widetilde{c}^{l} - \frac{2\gamma K}{\widetilde{L}^{l}} & \text{if } K < K^{l^{*}}(c), \end{cases}$$

where $\tilde{L}^{l} = L^{l} + (\tau^{h})^{2}L^{h}$, and $\tilde{c}^{l} = \frac{L^{l}c_{D}^{l} + (\tau^{h})^{2}L^{h}c_{X}^{l}}{\tilde{L}^{l}}$, which can be interpreted as the global competition faced by firms in l. The domestic and export cost cut-offs, c_{D}^{l} and c_{X}^{l} , are the upper bound costs for firms selling in their domestic market and in the foreign market $h \neq l$ respectively. These cut-offs must then satisfy:

$$\begin{split} c_D^l &= \sup\{c : \pi_D^l(c) \ge 0\} = p_{max}^l, \\ c_X^l &= \sup\{c : \pi_X^l(c) \ge 0\} = \frac{p_{max}^h}{\tau^h}. \end{split}$$

Finally, $K^{l^*}(c)$ is the capacity level at which a firm with marginal costs c is indifferent between being constrained or not, and does not change its pricing decisions. It is endogenously determined by the cost cut-offs in each country $(c_D^l \text{ and } c_X^l)$ and the marginal cost of the firm (c):

$$K^{l^{*}}(c) = q_{D}^{l} + \tau^{h} q_{X}^{l} = \frac{L^{l}}{\gamma} \left(p_{D}^{l}(c) - c \right) + \tau^{h} \frac{L^{h}}{\gamma} \left(p_{X}^{l}(c) - \tau^{h} c \right)$$

$$\rightarrow K^{l^*}(c) = \frac{\widetilde{L}^l}{2\gamma} \left(\widetilde{c}^l - c \right)$$

Firm behavior can be characterized solely by the implicit marginal costs κ and the characteristics of firm c_D^l and firm c_X^l . Combining optimal firm behavior and residual demand yields the optimal price, markup, quantity and profit equations:

$$\begin{split} p_{D}^{l}(\kappa) &= \frac{1}{2}(c_{D}^{l} + \kappa^{l}), & p_{X}^{l}(\kappa) = \frac{1}{2}\tau^{h}(c_{X}^{l} + \kappa^{l}) \\ q_{D}^{l}(\kappa) &= \frac{L^{l}}{2\gamma}(c_{D}^{l} - \kappa^{l}), & q_{X}^{l}(\kappa) = \frac{L^{h}}{2\gamma}\tau^{h}(c_{X}^{l} - \kappa^{l}) \\ \pi_{D}^{l}(\kappa) &= \frac{L^{l}}{4\gamma}(c_{D}^{l} - \kappa^{l})(c_{D}^{l} + \kappa^{l} - 2c), & \pi_{X}^{l}(\kappa) = \frac{L^{h}}{4\gamma}(\tau^{h})^{2}(c_{X}^{l} - \kappa^{l})(c_{X}^{l} + \kappa^{l} - 2c) \end{split}$$

Notice that a constrained firm's optimal decision in a particular market will respond to fluctuations in the competitive environment of alternative markets. In particular the implicit marginal costs of a constrained firm increase when a particular market becomes more appealing. For example, assume there is a shock in the foreign market and it is now more appealing to the home market firms (c_X^l increases). Then, a constrained firm will increase the price it charges for its variety and decrease the quantity it sells domestically in favor of the foreign market. ¹⁷

3.2 Free Entry Equilibrium

Entry is unrestricted in both countries. Firms choose a production location prior to paying the sunk entry cost. Free entry of domestic firms in country l implies zero expected profits in equilibrium, hence

$$\overline{\Pi}^{l} \equiv \int_{0}^{c_{D}^{l}} \left[1 - H(K^{l^{*}}(c)) \right] \pi_{D}^{l}(c) dG(c) + \int_{0}^{c_{X}^{l}} \left[1 - H(K^{l^{*}}(c)) \right] \pi_{X}^{l}(c) dG(c)$$
(15)

$$+ \int_{0}^{c_{D}^{\prime}} \int_{0}^{K^{\prime}(c)} \pi_{D}^{l}(\kappa^{l}) dH(K^{*}(c)) dG(c) + \int_{0}^{c_{X}^{\prime}} \int_{0}^{K^{\prime}(c)} \pi_{X}^{l}(\kappa^{l}) dH(K^{*}(c)) dG(c) = f_{E}$$

Given that κ^l is monotonic in K, I can apply a monotonic transformation and the free entry condition can be written equivalently as:

$$f_E = \int_0^{c_D^l} \left[1 - H\left(\frac{\widetilde{L}^l}{2\gamma}(\widetilde{c_D^l} - c)\right) \right] \pi_D^l(c) dG(c) + \int_0^{c_X^l} \left[1 - H\left(\frac{\widetilde{L}^l}{2\gamma}(\widetilde{c_D^l} - c)\right) \right] \pi_X^l(c) dG(c)$$

¹⁷If c_X^l increases and a firm is constrained, then $\frac{\partial \kappa^l}{\partial c_X^l} = \frac{(\tau^h)^2 L^h}{\tilde{L}^l} > 0$. Therefore, $\frac{\partial q_D^l}{\partial c_X^l} = -\frac{L^l}{2\gamma} \frac{\partial \kappa^l}{\partial c_X^l} < 0$, and $\frac{\partial q_X^l}{\partial c_X^l} = \frac{L^h}{2\gamma} (1 - \frac{\partial \kappa^l}{\partial c_X^l}) > 0$.

$$+ \quad \frac{\widetilde{L^l}}{2\gamma} \int_0^{c_D^l} \int_c^{c_D^l} \pi_D^l(\kappa) h(\frac{\widetilde{L}}{2\gamma} (\widetilde{c_D^l} - \kappa)) d\kappa \ dG(c) + \frac{\widetilde{L^l}}{2\gamma} \int_0^{c_X^l} \int_c^{c_D^l} \pi_X^l(\kappa) h(\frac{\widetilde{L}}{2\gamma} (\widetilde{c_D^l} - \kappa)) d\kappa \ dG(c).$$

The domestic and export cost cut-offs in country l must satisfy $c_D^l = p_{max}^l$ and $c_X^l = \frac{c_D^h}{\tau^h}$. This relationship shows the added efficiency required for firms to break into the export market and is used to simplify the system of expected profits in each country. The expected profit equation is simply interpreted as the expected weighted average of constrained and unconstrained domestic and export profits. Furthermore, the solution to the equilibrium condition exists and is unique regardless of the distribution of K and c with the proper parametrization of the support of capacity.

3.3 Equilibrium Insights

Markets are not completely integrated because trade is costly, and therefore opening up to trade is not just an increase in the market size with a subsequent increase in the competitiveness level of the country. Depending on cross country differences in size and trade openness, the existence of capacity constraints will magnify or mitigate the competitive effects byproduct of the larger market.

Our model's predictions for the effects of bilateral trade liberalization are akin to those emphasized in Melitz and Ottaviano (2008). In particular, trade will have pro-competitive effects in the market, forcing the least productive firms to exit and reallocate market shares towards more productive exporting firms, resulting in an economy with lower average markups and higher aggregate productivity. This is the well-known *selection effect*.

However, the existence of capacity constraints introduces two more effects into the mix when opening up to trade. First, firms facing a binding capacity constraint cannot freely expand production to access markets, and face a trade-off between domestic and export sales (*substitution effect*). Second, in order to take advantage of access to larger markets, constrained firmsraise prices even in the presence of tougher competition. Hence, the existence of capacity constrained firms may lead to a market wide softening of competition as markets grow (*composition effect*).

The selection effect will be more dominant in the economy as the probability of firms facing a binding capacity constraint approaches zero, in which case the model produces an identical equilibrium to Melitz and Ottaviano (2008). However, if firms are capacity constrained, the *substitution effect* and *composition effect* may mitigate or magnify it. The price level falls as competition rises with the introduction of a number of productive foreign producers serving the market. If a high proportion of these foreign firms serving the market are capacity constrained, the market may on average contain a greater fraction of producers selling at high prices than in autarky. In this case, the *composition effect* will dominate, and thus competition will be softened in the home market. However, if a low proportion of these foreign firms serving the market are capacity constrained, then the foreign market becomes more attractive to domestic firms that are capacity constrained, which substitute domestic sales for foreign sales. In this case, the *substitution effect* works along side the *selection effect*, and competition will be strengthened.

In conclusion, the impact of trade on varieties, prices, and competition cannot be deduced in general as they depend greatly on the rigidity of capacity in each market.

3.3.1 Equilibrium under Symmetric Trade

Suppose that there are two identical countries l and h engaging in costly but symmetric trade, such that $L = L^h = L^l$, and $\tau = \tau^l = \tau^h \ge 1$. Then, the effective market size is $\tilde{L} = (1 + \tau^2) L$. Assume that firms draw their potentially binding capacity K from a uniform distribution over the positive support $[0, K_{max}]$ and that productivity draws 1/c follow a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k \ge 1$, regardless of their location.

The cost-cutoffs in each country are identical since the model is fully symmetric, thus $c_D^l = c_D^h = c_D$. The solution to the free entry equation in each country is then given by

$$c_D^{k+2}\left(1+\tau^{-k}-\frac{\widetilde{L}}{2\gamma}\left(1+\tau^{-k-1}\right)\left(\frac{1+\tau}{1+\tau^2}-\frac{k+2}{k+3}\right)\frac{c_D}{K_{max}}\right) = \left[\frac{2(k+1)(k+2)\gamma(c_M)^k f_E}{L}\right].$$

For convenience, just as in the autarky case, I define a measure of the ease of capacity acquisition relative to the degree of market competition, $\delta = \frac{K_{max}}{c_D}$, which helps us to more easily interpret the effects that the existence of constrained firms has in the economy. A relatively large δ implies a large capacity acquisition process (high K_{max}) or a lower capacity premium due to high competition (low c_D). Furthermore, since $K_{max} > \frac{\tilde{L}}{2\gamma}$,¹⁸ expected profits are guaranteed as $\delta \geq \frac{L}{2\gamma} > \frac{L}{2\gamma(k+3)}$. Finally, I also define a technology index, $\phi = 2(k+1)(k+2)(c_M)^k f_E$, and measure the freeness of trade with $\rho \equiv \tau^{-k}$.

The solution to the free entry condition can then be expressed as:

$$c_D = \left[\left(\frac{1}{1 + \rho - \frac{\tilde{L}}{2\gamma\delta} \left(1 + \frac{\rho}{\tau}\right) \left(\frac{1+\tau}{1+\tau^2} - \frac{k+2}{k+3}\right)} \right) \frac{\gamma\phi}{L} \right]^{\frac{1}{k+2}}.$$
 (16)

The zero cut-off profit condition and the previous equation characterize the competitive equilibrium. The average price \overline{p} and the number of firms in equilibrium N is then

¹⁸Notice that otherwise $K^* > K_{max}$, which is infeasible.

given by:

$$\overline{p} = \left[\frac{2k+1}{2(k+1)} + \frac{\widetilde{L}}{4\gamma\delta} \left(\frac{1+\frac{\rho}{\tau}}{1+\rho}\right) \left[\left(\frac{1+\tau}{1+\tau^2}\right)^2 + \frac{k}{k+2} - \left(\frac{1+\tau}{1+\tau^2}\right) \frac{2k}{k+1} \right] \right] c_D,$$

$$N = \frac{2\gamma}{\eta} \frac{(\alpha - c_D)}{c_D} \frac{k+1}{\left(1 - (k+1)\frac{\widetilde{L}}{2\gamma\delta} \left(\frac{1+\frac{\rho}{\tau}}{1+\rho}\right) \left[\left(\frac{1+\tau}{1+\tau^2}\right)^2 + \frac{k}{k+2} - \left(\frac{1+\tau}{1+\tau^2}\right) \frac{2k}{k+1} \right] \right)}.$$

Let $\Xi = \left[\left(\frac{1+\tau}{1+\tau^2}\right)^2 + \frac{k}{k+2} - \left(\frac{1+\tau}{1+\tau^2}\right) \frac{2k}{k+1} \right]$ summarize the substitution effect of constrained firms on prices and N. The presence of constrained firms in the economy raises average prices with respect to an economy where firms face no constraints (*composition effect*), and raises the number of varieties in the economy. This effect is reinforced in the presence of trade, since trade introduces more constrained firms to each market through the substitution effect. Constrained firms must choose how much to allocate to each destination, and the degree to which average prizes are boosted depends on the tendency of constrained firms to serve foreign markets. Notice that, since countries are symmetric, there will be no disproportionate trade substitution and the composition effect will be affected the same way in both countries.

The cut-off level and the capacity acquisition parameter completely summarize the distribution of prices and the number of varieties in the economy, and therefore I can uniquely identify consumer welfare from Equation 5 :

$$U = 1 + \frac{\left(\alpha - c_D\right)}{2\eta} \left(\alpha - \frac{\frac{k+1}{k+2} - \frac{\widetilde{L}}{2\gamma\delta} \left(\frac{1+\frac{\rho}{\tau}}{1+\rho}\right) \left[k\Xi - \frac{k-1}{(k+1)(k+2)(k+3)} + \frac{\widetilde{L}}{2\gamma\delta} \left(\frac{1+\frac{\rho}{\tau}}{1+\rho}\right) \left(\frac{k+1}{2}\Xi - \frac{1}{k+2}\right)\right]}{1 - (k+1)\frac{\widetilde{L}}{2\gamma\delta} \left(\frac{1+\frac{\rho}{\tau}}{1+\rho}\right)\Xi} c_D\right).$$

The degree prices and varieties are elevated through the effect of composition is governed by the substitution effect. Therefore, comparisons between trade and autarky levels of competition, prices, varieties, and consumer welfare in the market depend on how transport costs transform the selection effect and the effect of composition.

3.3.2 Equilibrium under Asymmetric Trade

Suppose that two heterogeneous countries H and F begin trading. The producers in country H face potentially binding capacity constraints drawn from the uniform distribution $H(K) \sim U[0, K_{max}]$. To simplify, I assume that the producers located in country F do not face binding capacity draws, which is equivalent to firms drawing capacity from a uniform distribution with the support $\left[\frac{\tilde{L}^F}{2\gamma}\tilde{c}_D^F,\infty\right]$. Productivity draws 1/c follow a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k \geq 1$, regardless

of their location.

Profits in F are separable by destination since firms are not capacity constrained, and thus the free entry condition is:

$$L^{F}(c_{D}^{F})^{k+2} + L^{H}\rho^{H}(c_{D}^{H})^{k+2} = \gamma\phi,$$

where $\rho^H \equiv (\tau^H)^{-k}$ is a measure of the freeness of trade to country H and $\phi = 2(k + 1)(k+2)(c_M)^k \gamma f_E$ is the technology index.

By defining a measure of the ease of capacity acquisition relative to domestic and foreign competition as $\delta^H = \frac{K_{max}}{c_D^H}$ and $\delta^F = \frac{K_{max}}{c_X^H}$, I can rewrite the free entry equation in country H as:

$$c_{D}^{H} = \left[\frac{\gamma\phi}{L^{H}} \frac{1 - \rho^{F} + \frac{\rho^{F}\tilde{L}^{H}}{2\gamma\delta^{F}\delta^{H}} \left[\frac{[L^{H}\delta^{F} + (\tau^{F})^{2}L^{F}\delta^{H}]}{\tilde{L}^{H}} - \delta^{H}\frac{k+2}{k+3}\right]}{1 - \rho^{F}\rho^{H} - \frac{\tilde{L}^{H}}{2\gamma\delta^{F}\delta^{H}} \left[(1 - \rho^{F}\rho^{H})\left(\frac{[L^{H}\delta^{F} + (\tau^{F})^{2}L^{F}\delta^{H}]}{\tilde{L}^{H}} - \delta^{H}\frac{k+2}{k+3}\right) - (\delta^{F} - \delta^{H})\frac{k+2}{k+3}\right]}\right]^{\frac{1}{k+2}}$$
(17)

The effects of capacity are governed by two terms: $\xi_1 = \left[\frac{[L^H \delta^F + (\tau^F)^2 L^F \delta^H]}{\tilde{L}^H} - \delta^H \frac{k+2}{k+3}\right]$ and $\xi_2 = \left[\left(1 - \rho^F \rho^H\right)\xi_1 - (\delta^F - \delta^H)\frac{k+2}{k+3}\right]$. A lower ξ_1 represents a more desirable market abroad, which mitigates the losses from facing a binding capacity draw. At the same time, a more desirable market abroad induces constrained firms to substitute sales away from H, intensifying competition on H. This composition effect is captured by ξ_2 .

Capacity effects depend heavily in the potential market size. In Melitz and Ottaviano (2008), the increased opportunities from interacting with a large trading partner were completely offset by the increased competitiveness induced by a greater number of firms abroad. In this model, the offsetting effects of trading partner size are eliminated. A potentially large export market has an ambiguous effect on the cutoff. Increased export market opportunities attract sales from capacity constrained firms, the domestic market sales of which are replaced by foreign goods produced by highly productive firms. This potentially magnifies competition in the domestic market.

Given the solution for the cut-off in country H, I can characterize the equilibrium. The average price \overline{p} , the number of firms in equilibrium N and the consumer welfare U are then given by:

$$\overline{p^H} = \left[\frac{2k+1}{2(k+1)} + \frac{\widetilde{L}}{4\gamma\delta^F\delta^H(1+\rho^H)}\left(\frac{k+1}{k+2}\delta^F - \frac{k}{k+1}\frac{\left[L^H\delta^F + (\tau^F)^2L^F\delta^H\right]}{\widetilde{L}^H}\right)\right]c_D^H,$$

$$N = \frac{2\gamma}{\eta} \frac{(\alpha - c_D^H)}{c_D^H} \frac{(k+1)(1+\rho^H)}{\left(1 - \rho^H - \frac{\tilde{L}^H}{2\gamma\delta^F\delta^H} \left(\frac{(k+1)^2}{k+2}\delta^F - k\frac{[L^H\delta^F + (\tau^F)^2L^F\delta^H]}{\tilde{L}^H}\right)\right)},$$

$$U^{H} = 1 + \frac{\left(\alpha - c_{D}^{H}\right)}{2\eta} \left(\alpha - \frac{\frac{k+1}{k+2} - \frac{\tilde{L}^{H}}{2\gamma\delta^{F}\delta^{H}} \left(\frac{k^{2}}{k+1}\right) \left[\frac{\tau^{F}L^{F}(\delta^{F} - \delta^{H})}{(1+\rho^{H})\tilde{L}^{H}} + \frac{\tilde{L}^{H}}{4\gamma\delta^{F}\delta^{H}(1+\rho^{H})^{2}} \left(\frac{1}{k} \frac{\left[L^{H}\delta^{F} + (\tau^{F})^{2}L^{F}\delta^{H}\right]}{\tilde{L}^{H}} - \frac{(k+3)(k-1)}{k^{2}(k+2)}\delta^{F}\right)\right]}{1 - \frac{\tilde{L}^{H}}{2\gamma\delta^{F}\delta^{H}(1+\rho^{H})} \left[\frac{(k+1)^{2}}{k+2}\delta^{F} - k\frac{\left[L^{H}\delta^{F} + (\tau^{F})^{2}L^{F}\delta^{H}\right]}{\tilde{L}^{H}}\right]} c_{D}^{H}\right)$$

As in Autarky, the effect of composition impacts welfare ambiguously . While constrained firms soften competition by charging higher prices, which allows more varieties to be present in equilibrium, they also substitute away from H to F, and the presence of highly efficient foreign firms in H is magnified. The net effect of the *selection effect*, the *composition effect* and the *substitution effect* is hard to determine since all of the channels are intertwined. The following section sheds light on how autarky, symmetric trade and asymmetric trade compare by presenting various numerical simulations of the model and examining its response to changing market conditions.

4 Quantitative Analysis

The model presented has provided insights that traditional trade models have not been able to replicate, such as the trade off for firms between domestic and export sales. However, it is not intuitive to understand the quasi-closed forms of the equilibrium, and the interaction between all the forces at play when the economy opens up to trade. Thus, I calibrate the model in order to be able to answer some fundamental question regarding trade policy, such as: does the market size matter for welfare gains from trade?; and is there a plausible reason behind a protections policy?.

I calibrate the model using European Data for France, Germany and Spain. The model predicts that some firms remain small despite being very productive, thus, I exploit the firm size distribution of every country to match this fact and provide a plausible framework where we can study trade policy.

I start by presenting how the benchmark model is calibrated in order to fit the real world data, and the robustness of the calibration. Then, I perform three different comparative statistics exercises. First, opening up to trade with an identical country (symmetric trade) under different levels of capacity. Second, opening up to trade with a country where firms face no constraints (asymmetric trade), while there are different levels of capacity in the home country. I also explore here the importance of the market size of the trading partner. Finally, I investigate a unilateral trade liberalization in an asymmetric trade setting.

4.1 Benchmark Calibration

To asses the impact of capacity constraints on the welfare of the economy, I calibrate the model to match the firm size distribution (mean and variance) of three European countries. In particular, I use data from France, Germany, and Spain provided by the survey European Firms in a Global Economy (EFIGE), which was conducted during the year 2009 and is representative of the manufacturing sector in each country.

The benchmark values are chosen for the set of relevant parameters to match the features of the European economy. Following closely Sakane (2011) and Rodríguez-López (2011), I set the technology of the final goods parameters at $\alpha = 10$, $\gamma = 1/2$ and $\eta = 1$. As in Alessandria and Choi (2007) and Obstfeld and Rogoff (2001), I set the steady-state value of iceberg transport cost τ^l equal to 1.4, and the steady-state value of the entry f_E to 1 as in Ghironi and Melitz (2005). The maximum value of the cost distribution c_M is set to 10, without loss of generality.

Countries differ in their market size and their firm size distribution. In particular, the scaling parameter of the Pareto distribution k and the support of the Uniform distribution K_{max} are calibrated to fit the size distribution of the economy by exploiting the mean and dispersion moments. In the model, a binding capacity constraint implies that firms are unable to produce their first best and remain small. This fact is matched by the calibration, and in countries with fewer large firms in the economy, such as Spain, firms have a larger probability of being constrained.

Table 4 lists the calibrated parameters while Figure 2 shows the match of the distribution of firms for each of the countries. The dashed lines represent the model while the solid lines represent the data. The calibration does a very good job of matching Spain and France, while it does a poorer job with Germany.

Description	Value
Strength of product differentiation coefficient	$\alpha = 10$
Product differentiation index	$\gamma = 1/2$
Variety of substitutability	$\eta = 1$
Iceberg transport costs	$\tau = 1.4$
Sunk entry costs parameter	$f_E = 1$
Upper bound of marginal costs	$c_M = 10$
Characterizing parameter of $G(c)$	
France	k=4.8
Germany	k=4.5
Spain	k=4.9
Characterizing parameter of $H(K)$	
France	$K_{max} = 1776$
Germany	$K_{max} = 1169$
Spain	$K_{max} = 1908$
Labor endowment	$L^{Europe} = 1000$
France	L=170
Germany	L = 196
Spain	L=115

 Table 1: Benchmark Parameter Values



Figure 2: Model and Data Firm Size Distribution

4.2 Opening up to Trade with Capacity Constraints

In this section, I analyze the conditions under which the presence of capacity constraints does not undermine the positive effects of trade. I consider both symmetric trade — trade between two countries facing potentially binding constraints; and asymmetric trade — trade between two countries where only the home country faces potentially binding capacity constraints.

4.2.1 Symmetric Trade

Trade intensifies competition by forcing the least productive firms to exit. The presence of capacity constrained firms in the economy raises average prices with respect to an ideal economy where firms face no constraints, which pushes the number of varieties above the autarky levels. Since countries are symmetric, there will be no disproportionate trade substitution and the firm composition will affect both countries equally. Only when capacity is scarce will the negative effects from composition dominate the pro-competitive effects of selection. The following proposition summarizes these effects.

Proposition 2

Symmetric trade intensifies competition,¹⁹ $c_D^S < c_D^A$, ambiguously impacts average prices, $\overline{p}^S \leq \overline{p}^A$, and leads to an increase in the number of firms serving each market $N^S > N^A$.

Proof. See the Appendix

Corollary 1

When capacity is abundant in both countries (large K_{max}), consumer welfare rises under symmetric trade ($U^S > U^A$). Otherwise, consumer welfare can fall.

Figure 3 provides a numerical example of a parametrized symmetric equilibrium for France as capacity becomes more abundant (increasing K_{max}).²⁰ In these graphs I compare symmetric trade not only to autarky, but also to the ideal situation of no binding constraints in the economy, i. e. to the Melitz-Ottaviano world.

Figure 3 (a) shows the evolution of the competition level. Trade intensifies competition. However, the presence of constrained firms in the economy weakens the procompetitive effects of trade. As the probability of being capacity constrained decreases $(K_{max} \text{ increases})$, the level of competition converges $c_D^S \to c_D^{S^{M-O}}$.

Figure 3 (b) depicts the number of varieties consumed in the economy. Opening up to trade induces the least productive firms to leave the economy (*selection effect*), and at the

¹⁹Let S denote the solutions for symmetric trade and A the solutions in autarky.

²⁰Similar numerical simulations can be found for Germany and Spain in the Appendix.

same time the capacity constraint of the average firm tightens. If capacity is scarce (low K_{max}), the number of constrained firms after trade increases and they saturate the market (*composition effect*). These firms split their production between the two countries to maximize profits (*substitution effect*), but are unable to satisfy demand, which attracts a high number of firms to enter the economy and the number of varieties rises. Furthermore, the higher the number of firms that face a binding capacity constraint, the more average prices are pushed upwards, rising even beyond average prices in autarky, as can be seen in Figure 3 (c).

Finally, the evolution of welfare is depicted in Figure 3 (d). As stated in Corollary 1, when capacity is scarce, welfare in the open economy falls below autarky levels. Notice that welfare in the open economy is below autarky levels, not only when $\overline{p}^S > \overline{p}^A$ but also, when average prices have fallen, the number of varieties has increased and the competition levels have risen.



Figure 3: Opening up to Symmetric Trade (France)

What is the logic behind this result? Let $\tau = 1$, then every firm productive enough to be a domestic producer also exports. If a firm facing a binding capacity constraint perfectly splits its production in order to maximize profits, the substitution effect does not play any role in determining welfare after trade.²¹ The positive effects from selection will counteract the negative effects from the firm composition only if capacity is abundant enough. Otherwise, the implicit marginal costs of the average firm increase after trade as the capacity constraint tightens too much. Thus the sales of the average firm decrease ($\downarrow q(\bar{\kappa})$), and the difference between the maximum price and the average price is reduced ($\downarrow (p_{max}^S - \bar{p}^S)$), as can be seen in Figure 4. Hence, the surplus extracted by consumers from each variety, which depends critically on these two variables, decreases and counteracts the variety and efficiency gains.



Figure 4: Opening up to Symmetric Trade II (France)

4.2.2 Asymmetric Trade

For simplicity, I assume that asymmetric trade involves a home country H, where firms face potentially binding capacity constraints, with a foreign country F, where firms do not

 $^{^{21}}$ Since trade is symmetric and the firms from the trading partner behave identically.

face any. Trade intensifies competition by forcing the least productive firms to exit. The presence of capacity constrained firms in the domestic economy raises average prices with respect to an ideal economy where firms face no constraints, which pushes the number of varieties above the autarky levels.

Welfare will decrease in H after trade, as it did in the symmetric trade case, only if access to capacity is extremely restricted, since the exporters in F are wholly comprised of efficient unconstrained firms. By opening up to trade, constrained firms in country H substitute sales and pass along their distortionary pricing behavior to F. The stronger the substitution effect, the smaller the composition effect in H and the more competition effects are magnified.

Equilibrium, and more importantly transition, depends on the relative characteristics of each market, and especially the size of the trading partner. The size of the trading partner is particularly relevant to understand the diffusion of the negative effects introduced by capacity constraints. The simulations in Figures 5 to 8 are based on France. Panel (a) assumes that its trading with a country of exactly the same size, panel (b) with a larger country like Germany, and panel (c) with a smaller country like Spain.

On the one hand, the larger the partner country the more likely it is that opening up to trade is welfare enhancing despite the presence of constraints. On the other hand, the utility of the home country rises well above that of the foreign country if H is relatively larger than F. In such a position, the larger market of H attracts highly productive firms from F, which increases the competition level in the home country and pushes down average prices, thus increasing welfare.

These results highlight the importance of a country's trading partner. Notice that in the absence of capacity constraints, any gains from a larger trading partner are offset by its increased *competitiveness* (a greater number of more productive firms are competing in that market, driving down markups), whereas in the presence of capacity constraints a larger market provides a buffer to the negative effects brought on by these restrictions.



Figure 5: Competitiveness Level



Figure 6: Number of Varieties



Figure 7: Average Prices



Figure 8: Utility

4.3 Unilateral Trade Liberatlization

In this section, I explore the effects of a unilateral liberalization by country H in which firms face potentially binding capacity constraints, that is, a decrease in τ_H (an increase in ρ_H), holding τ_F (ρ_F) constant. Given the cut-off condition from Equation 17, this leads to an increase in the cost cut-off c_D^H , which means less competition in the liberalizing country, whereas the cut-off c_D^F in the country's trading partner decreases, indicating an increase in competition there. The liberalizing country thus experiences a welfare loss while its trading partner experiences a welfare gain.

In Melitz and Ottaviano (2008), trade barriers and market characteristics abroad have direct effects only on exporters, and impact equilibrium indirectly through the cost cutoffs. With capacity constraints, market accessibility directly shapes the implicit marginal costs of all constrained firms. This has a direct structural influence on the equilibrium equations, and thus firms may get more than what they bargained for from trade policy. In particular, the gains in the non-liberalizing country are intensified.

These results are driven by the change in firm location induced by entry in the long run. The number of entrants in H, the liberalizing country, decreases, while the number of entrants in F increases. Decreasing τ_H , and thus increasing the prevalence of import competition (ρ_H), monotonically increases the cost cut-off. This home-market effect results in less firms serving H and hence a lower degree of competition. Figure 9 (a) illustrates that c_D^H increases in response to decreases in τ_H . As trade becomes relatively less costly in F, constrained firms shift sales to the domestic market in response to the accelerated entry in the foreign markets, as seen in Figure 9 (b). Constrained firms substituting sales away from F leave excess demand that is then served by its productive firms.



Figure 9: Unilateral Liberalization of Country H

These results clearly underline how the welfare loss associated with unilateral liberalization is driven by the shift in the pattern of entry, favoring the non-liberalizing trading partner. The delocalization of firms is intensified by the presence of capacity constraints in the liberalizing country.

5 Conclusions

This paper has proposed a trade model with firms heterogeneous both in productivity and capacity. By introducing capacity constraints, it produces a tractable framework that can explain various occurrences in the data which are anomalous to new trade theory, which previous models are unable to replicate, such as a trade-off between domestic and export sales.

In general, larger markets exhibit tougher competition, resulting in lower average markups and higher aggregate productivity (*selection effect*). However, firms facing a binding capacity constraint are unable to expand production. Thus, in order to take advantage of access to larger markets, they raise prices even in the presence of tougher competition. These firms then face a trade-off between domestic and export sales, which they distribute in order to maximize their profits (*substitution effect*). By doing so, they pass their distortionary behavior on to each of their markets through prices (*composition effect*), inducing a market wide softening of competition as markets grow. Price competition is then softened in markets with high concentrations of capacity constrained firms.

The impact of trade on welfare is not straightforward, as these forces interact with each other. Therefore, I have calibrated the model to illustrate its theoretical predictions with respect to trade and welfare. I have demonstrated that markets may be distorted to such an extent that consumer welfare falls with trade because firms shift the weight of their pricing distortions across markets and substitute sales across destinations (scaling sales back in each market). This reduces the surplus extracted by consumers, which in turn counteracts the variety and efficiency gains. In addition, the simulations highlight the importance of a country's trading partner. If firms could freely adjust their production, any gains from a larger trading partner would be offset by the increased competitiveness in the market. A larger market provides a buffer against the distortionary effects that binding capacity constraints induce in the market.

I have abstracted from a number of potentially relevant features that go beyond the scope of this paper. First, I have assumed that capacity constraints are exogenous to the firm, ignoring the nature of such constraints and the possible correlation they may have upfront with firm productivity. Second, I have focused exclusively on a steady state environment, thus ignoring the transition dynamics. Nevertheless, the model is highly tractable, and thus provides a useful modeling framework within which to analyze trade and market integration policy scenarios. Although not examined in depth, the model identifies a channel through which firms link their decisions across markets. Such linkages have the potential to yield insights into our understanding of how exchange rates or shocks to supply or demand are passed through by firms. Studying the source of these linkages, holds potential to understand how firm behavior shapes fundamentals such as trade elasticities and inflation.

References

- Ahn, J. and A. McQuoid (2013, January). Capacity Constrained Exporters: Micro Evidence and Macro Implications. Working Papers 1301, Florida International University, Department of Economics.
- Alessandria, G. and H. Choi (2007, 02). Do Sunk Costs of Exporting Matter for Net Export Dynamics? The Quarterly Journal of Economics 122(1), 289–336.
- Arkolakis, C. (2010). Market Penetration Costs and the New Consumers Margin in International Trade. Journal of Political Economy 118(6), 1151 – 1199.
- Behrens, K., C. Gaigne, G. I. Ottaviano, and J.-F. Thisse (2007, July). Countries, regions and trade: On the welfare impacts of economic integration. *European Economic Review* 51(5), 1277–1301.
- Bernard, A. B. and J. Bradford Jensen (1999, February). Exceptional exporter performance: cause, effect, or both? *Journal of International Economics* 47(1), 1–25.
- Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003, September). Plants and Productivity in International Trade. *American Economic Review* 93(4), 1268–1290.
- Blum, B. S., S. Claro, and I. Horstmann (2010, May). Facts and Figures on Intermediated Trade. American Economic Review 100(2), 419–23.
- Blum, B. S., S. Claro, and I. J. Horstmann (2013). Occasional and perennial exporters. Journal of International Economics 90(1), 65–74.
- Clementi, G. L. and H. A. Hopenhayn (2006, 02). A Theory of Financing Constraints and Firm Dynamics. *The Quarterly Journal of Economics* 121(1), 229–265.
- Davies, R. B. and C. Eckel (2010, February). Tax Competition for Heterogeneous Firms with Endogenous Entry. *American Economic Journal: Economic Policy* 2(1), 77–102.
- Forslid, R., T. Okubo, and K.-H. Ulltveit-Moe (2011, September). International trade, CO2 emissions and heterogeneous firms. CEPR Discussion Papers 8583, C.E.P.R. Discussion Papers.
- Ghironi, F. and M. J. Melitz (2005, August). International Trade and Macroeconomic Dynamics with Heterogeneous Firms. The Quarterly Journal of Economics 120(3), 865–915.

- Hasan, R., D. Mitra, and K. Ramaswamy (2007, August). Trade Reforms, Labor Regulations, and Labor-Demand Elasticities: Empirical Evidence from India. *The Review of Economics and Statistics* 89(3), 466–481.
- Helpman, E., M. J. Melitz, and S. R. Yeaple (2004, March). Export Versus FDI with Heterogeneous Firms. American Economic Review 94(1), 300–316.
- Knittel, C. R. and J. J. Lepore (2010, March). Tacit collusion in the presence of cyclical demand and endogenous capacity levels. *International Journal of Industrial Organiza*tion 28(2), 131–144.
- Krishna, P. and A. Panagariya (2000, December). A unification of second best results in international trade. *Journal of International Economics* 52(2), 235–257.
- Levitan, R. and M. Shubik (1972, February). Price Duopoly and Capacity Constraints. International Economic Review 13(1), 111–22.
- Liu, Y. (2014). Capital Adjustment Costs: Implications for Domestic and Export Sales Dynamics. Mimeo.
- Maggi, G. (1996, March). Strategic Trade Policies with Endogenous Mode of Competition. American Economic Review 86(1), 237–58.
- Manova, K. (2013). Credit constraints, heterogeneous firms, and international trade. *Review of Economic Studies* 80(2), 711–744.
- Melitz, M. J. (2003, November). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71(6), 1695–1725.
- Melitz, M. J. and G. I. P. Ottaviano (2008). Market Size, Trade, and Productivity. *Review* of *Economic Studies* 75(1), 295–316.
- Obstfeld, M. and K. Rogoff (2001, October). The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? In NBER Macroeconomics Annual 2000, Volume 15, NBER Chapters, pp. 339–412. National Bureau of Economic Research, Inc.
- Ottaviano, G., T. Tabuchi, and J.-F. Thisse (2002, May). Agglomeration and Trade Revisited. *International Economic Review* 43(2), 409–436.
- Roberts, M. J. and J. R. Tybout (1997, September). The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs. *American Economic Review* 87(4), 545–64.

- Rodríguez-López, J. A. (2011). Prices and Exchange Rates: A Theory of Disconnect. *Review of Economic Studies* 78(3), 1135–1177.
- Sakane, M. (2011). Product variety, firm entry, and terms of trade dynamics. Working paper, Tokyo Center for Economic Research.
- Spence, A. M. (1977). Entry, Capacity, Investment and Oligopolistic Pricing. *Bell Journal* of *Economics* 8(2), 534–544.
- Vannoorenberghe, G. (2012). Firm-level volatility and exports. Journal of International Economics 86(1), 57–67.