Ascending Auctions with Costly Monitoring*

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Abstract

This paper analyzes discriminatory ascending auctions where there are limited opportunities to bid throughout the auction, and where rival bidder activity is not continuously monitored. We characterize the theoretical properties of a Perfect Bayesian Equilibrium in which bids are increasing in valuations. We analyze identification from data on bids and bidding times, and develop a semiparametric estimator for the model. We apply the framework to a financial market where state governments purchase certificates of deposits from banks by an auction mechanism where bids are based on interest rate offers. Our data set covers the onset of the recession in 2008, allowing us to study the effects of the financial crisis on this local market. The estimated monitoring costs that banks incur, together with the social losses induced by not invariably funding the banks with the highest valuations, are indicative of market microstructure conduct and performance in more general limit order market settings.

1 Introduction

This paper is a theoretical analysis and empirical application of discriminatory ascending auctions with costly monitoring. In our framework there is a fixed finite time horizon for conducting the auction, and bidders have limited opportunities to place and update their bids. Bids cannot be withdrawn or reduced, and the next opportunity to revise old bids or place new ones is a random event whose probability distribution is determined by the bidder's monitoring choices. We characterize symmetric pure strategy perfect equilibria, analyze identification for a cross section of auction records on bidding, and develop a semiparametric estimator for such models. The model is applied to a monthly financial market where the treasurers of state governments purchase from local banks savings vehicles, certificates of deposit (CD), for unallocated state funds via a procurement auction mechanism. We demonstrate an identification strategy, and develop an estimator. Our estimates explain how local financial markets reacted to the credit crisis of 2008, focusing on changes in the potential numbers of bidders, the distribution of their valuations, and how that affected their bidding and monitoring behavior. Finally we estimate the welfare costs of using this particular trading mechanism in the CD market, namely the value of resources used to identify the winners of the auction plus the social surplus lost from selecting winners who have lower private values than those who bid too little, or did not bid at all.

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The rules governing the auction and the main features of our data are described in the Section 2. Each month banks bid for state funds via an open cry multiunit online auction. Entry occurs anonymously throughout the auction, and as it progresses nobody knows which bidders have dropped out. Some banks submit bids above the reserve price at the beginning of the half hour auction period and identical units are often sold at different prices. These institutional arrangements and stylized facts cannot be reconciled to a Japanese auction, where the field of bidders is announced before the auction begins, and bidders publically withdraw as the auctioneer increases the price until the supply of units matches the demand by the bidders who are left, who all pay the same price. In an ascending electronic auction where banks can bid at the final instant, standard auction theory predicts that every bank would do that, snipe, to prevent other bidders from revising their own bids in response. While some sniping occurs in our data, it is not unusual for winning bids to be made well before the end of the auction. Bids above the reserve price are sometimes submitted near the beginning of an auction, and they tend to increase with the lowest price necessary to win, further evidence against the equilibrium outcome in sealed bid auctions, where bidders cannot react to each other. Yet our data also exhibit bids that jump in a discrete manner, in contrast to an English auction, where the auctioneer's ask price increases incrementally.

To accommodate the peculiar features of our data, Section 3 develops a theoretical model estimated in the latter parts of the paper. We treat the auction as a dynamic game where players with private valuations have limited opportunities to place new bids, update stale bids by reacting to the current lowest price in the money, and decide how closely to monitor proceedings. The player's monitoring choice determines the rate at which bidding opportunities arrive throughout the auction game; a higher arrival rate is more costly to implement than a lower rate, and it can be revised with each bid update. At his first bidding opportunity, the player knows the lowest price at which he would win a unit if no other bids were placed throughout the auction, the on-the-money bid (ONM). At that time, and all future bidding opportunities, he can pass (or not revise any previous bid) bid ONM, or even higher, that is in-the-money (INM). If he receives another bidding opportunity later in the auction, he observes the updated ONM only if his current outstanding bid is out-of-the-money (OUTM), that is less than ONM. We derive the first order conditions characterize the optimal monitoring rates and interior INM bidding in perfect Bayesian equilibrium where bidding that is increasing in valuations. Bidding is not strictly monotone in valuations, because players with different slightly valuations would bid ONM in otherwise identical situations.

Identification is established in Section 5. We first show that the valuations of players bid INM and therefore satisfy a first order condition in equilibrium are pointwise identified. Thus the highest bid ever observed by a group of players in different auctions that are observed to make the same INM bid in response to the same history up until that point, is a consistent estimate of their common valuation. Second, we note that as soon as a player who previously bid INM falls out OUTM, he will take the first opportunity to bid if at some later point in the auction he is observed to bid. This establishes a simple set of sufficient conditions in which a bidder would bid if he or she could do so, thus identifying the monitoring technology selected as a function of the history of the auction (including the remaining amount of time to bid) and the previous INM bid which is a sufficient statistic for the player's valuation. In estimation we implement a maximum likelihood estimator to recover the distribution of the duration times. Having identified both the valuation and the choice of monitoring rate for INM bids, we now apply the first order conditions for the monitoring choices to identify the marginal cost of increasing monitoring at the the player's equilibrium choice. Using Euler equation methods the cost technology is estimated in this third step. The final step in identification, and estimation, is to recover the underlying distribution of player valuations, by adjusting the truncated distribution of bidders who bid ITM at least once to account for players who do not bid at all, and those who only bid ONM. This exercise in selection correction is accomplished by using the model structure to identify the sets of valuations that lead to players to stay out or bid ONM.

As indicated above, estimation proceeds sequentially by forming sample analogues to their population counterparts exploited in identification. The estimated structural parameters are presented in Section ??. Estimated monitoring costs account for about 5 percent of the estimated value of trading. Our model degenerates to more standard first price sealed bid or Japanese button auctions when monitoring costs are eliminated; which specialization applies hinges on whether or not players always have an opportunity to respond to rival bids. In this important respect, monitoring costs are shown to be a key feature for explaining the dynamic of bidding activity through the auction, the initial burst, a lull in the middle and the most concentrated activity towards the end; the sealed bid auction predicts all bidding takes place at the end of the auction, while in the Japanese auction the time profile of increasing bids is indeterminate. Similarly the existence of monitoring costs also explains why some bids are INM well before the auction ends, a stylized fact that cannot be reconciled with either specialization. Imposing the assumption of no monitoring costs and assuming a Japanese auction format yields a valuation distribution that is first order stochastically dominated by the distribution estimated from our monitoring cost model, which in turn is stochastically dominated by the valuation distribution estimated under the first price sealed bid assumption. Theoretically and empirically, our model combines elements of the other two. Our estimates allow us to estimate the effects of the financial crisis of late 2008. We find that liquidity dries up. Finally we compute the social costs of this auction mechanism. Section 7 concludes, while an appendix contains details about the estimation strategy and the asymptotic properties of the estimators properties.

Our study draws from both the theory of auctions and the econometric issues associated with identifying and estimating auction models. Theoretical models that analyse jump bidding are, amongst others, Hörner and Sahuguet (2007) and Avery (1998). Birulin and Izmalkov (2011) consider English auctions with re-entry. There are a number of theoretical papers that deal with ascending bidding models that are not strategically equivalent to a Japanese auction. For example, Avery (1998) considers jump bidding models. Daniel and Hirshleifer (1998) consider auctions with costly sequential bidding. The latter shows that in the absence of bidding costs an equilibrium can exist where the transaction price is not equal to the second highest valuation. Birulin and Izmalkov (2011) considers English auctions with re-entry. Estimation follows the spirit of indirect estimation strategies such as Hotz and Miller (1993), Guerre, Perrigne, and Vuong (2000) and Manski (1993). The literature on the structural estimation of auctions was initiated by the work of Donald and Paarsch (1993), Elyakime, Laffont, Loisel, and Vuong (1994) and Guerre, Perrigne, and Vuong (2000). Donald and Paarsch (1993) estimate auction models, where the underlying valuation distribution is parameterized and bidding strategies must be computed. Guerre et al. (2000) consider two step indirect non-parametric estimators that can avoid the computation of strategies and the specification of underlying valuation distributions.

The estimation of dynamic auctions is considered in Jofre-Bonet and Pesendorfer (2003) and Groeger (2010). These papers consider repeated first price auctions in dynamic environments and extend Guerre et al. (2000). Jofre-Bonet and Pesendorfer (2003) consider the effect of capacity constraints on bidding. Groeger (2010) studies repeated first price auctions with endogenous participation and synergies in participation. Our paper differs from the aforementioned papers in that on the one hand we consider the auction as a dynamic game, but on the other hand we exclude dynamic linkages between auctions. The presence of multiple units and the open cry format makes this market differ from standard ascending auction models and existing multiunit auction mechanisms in the literature. The literature on the empirical analysis of ascending auctions has mainly made use of the Japanese button auction model to estimate underlying valuations. In this setting the transaction price is the second highest valuation. One exception is Haile and Tamer (2003) who attempt to construct an incomplete model of ascending auctions that allows for a number of different equilibrium outcomes. The authors are able to provide informative bounds on the valuation distribution.

2 Auctioning Certificates of Deposit

A Certificate of Deposit (CD) is similar to a standard savings account. The key difference is that a CD has a specific, fixed term, usually three, six or twelve months. At maturity the money may be withdrawn together with the accrued interest. Our application analyzes CD auctions in Texas held by that state's government. Prior to entering a CD auction a bank must undergo a pre-qualification process. During this process the level of collateral a bank holds is ascertained. Texas limits participation to local banks to ensure that tax money does not leave the state and can be used to inject liquidity into the local economy. Participating banks compete to sell these savings vehicles to the state treasurer through an on line auction. The money sold in these auctions from state funds have no immediate purpose.[1] This section is an overview of our data. We describe the institutional setting and the auction mechanism; we provide several measures of the size of the auctions, and present evidence illustrating the magnitude of the difference between the highest and lowest winning bids. Then we summarize the main features of bids and their timing.

2.1 Auction format

Auctions take place each month, usually during the first week. Before the auction begins, the state government announces a reserve interest rate, which determines the minimal acceptable bid. Bidding takes place over 30 minutes. Banks can submit a succession of increasing bids on up to five separate parcels, at different interest rates for different dollar amounts, The size of each parcel is a multiple of \$100,000, with a minimum is \$100,000, and a maximum is \$7million. The collateral of a bank determines the upper limit on the sum of its parcel sizes. A bid for a parcel can be increased as many times as the bank wants during the 30 minute period, but not decreased, and the size of the parcel cannot be reduced either. The only information available to the bank throughout the auction is whether its most recent bid on each parcel would win if no further bids were tendered by anybody, equivalently whether which of their parcels are out of the money (OUTM) or not. Parcels are ranked by the interest rate on the most recent bid; size does not affect ranking. When the auction ends, an amount up to the total funds are dispersed to the banks offering the highest interest rates. Banks pay the (most recent and highest) interest rate they bid on their winning parcels, and nothing on parcels that lose. In the event of a tie for the lowest winning bid, the earliest order rate at that interest rate is given precedence. Moreover if the parcel size of the lowest winning bid exceeds the amount left for allocation, the bank is obliged to partially fill its order with the remaining funds.[2] Summarizing, this is a discriminatory price, multiunit ascending auction in which banks receive limited information about the status of their own bids throughout the bidding phase.

2.2 Auction Size

Our data set contains 140 CD auctions in Texas for the years 2006 through 2010. The majority of auctions involve six month CDs. There are five auctions in our data set that are for 12 month

[1] There are a number of other states that use the same online auction platform, for example Idaho, Iowa, Louisiana, Massachusetts, New Hampshire, Ohio, Pennsylvania and South Carolina. We focus on Texas sinceit is the most active state.

[2] For example, if there is \$700,000 to be allocated, but the lowest winning bid is for \$1 million at an interest rate of 10 percent, then the bank making that bid must issue a CD for \$700,000 at the 10 percent rate.

CDs. The amount to be auctioned is on average $76 million. The majority of auctions were for $80 million. Eleven auctions were for $50 million. Summary characteristics of the auction are displayed in Table 1. There is a potential pool of 53 banks, and an average of 25 banks enter each auction. With an average order comprising 1.6 parcels per auction, most banks only submit one parcel. Row 7 of the table, "Number of Parcels", shows that banks rarely increase or split parcels. For the most part, each bank bids on a fixed amount of funds at a uniform interest rate, both of which vary by bank. On average, 76 percent of participating banks win an award from a total of $75 million, and their average award size is $4 million.

Table 1: Summary Statistics on Auctions

	Mean	**Standard Deviation**	**Min**	**Max**
Number of Banks who bid	24.500	6.498	9.000	41.000
Number of Bids Per Bank	8.006	13.348	1.000	139.000
Proportion of Bids In The Money (INM)	0.606	0.227	0.000	1.000
Proportion of Bids Out of The Money (OUTM)	0.293	0.216	0.000	0.745
Proportion of Final Bids OUTM upon Submission	0.060	0.052	0.000	0.265
Proportion of Bids On The Money (ONM)	0.201	0.137	0.000	0.553
Proportion of Winning Bids ONM	0.046	0.039	0.000	0.182
Proportion of Bids with Quantity Changes	0.003	0.027	0.000	0.500
Number of Parcels	1.601	1.026	1.000	5.000
Proportion of Banks who win	0.758	0.180	0.414	1.000
Annual Reserve Coupon Rate	2.193	2.107	0.060	5.300
Total Award Amount (millions dollars)	75.769	10.509	50.000	80.000
Award to Winning Bank (millions dollars)	4.125	2.744	0.100	7.000
MLT (dollars)	1803.217	6551.870	0.000	210000.000

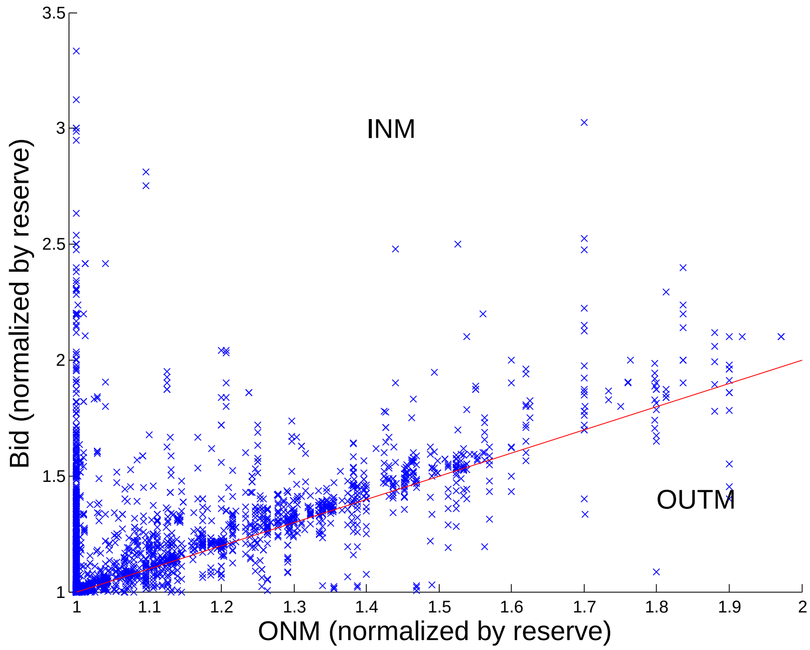
The table shows that on average 61 percent of the bids are in-the-money (INM), in other words so high that at least one previous lower bid would win if no further bids are tendered. Conversely 16 percent of the bids are out-of-the-money (OUTM); nothing is gained from making such a bid aside from knowing that the bank must make a more attractive offer to win any award. This third category of bids, accounting for the remaining 6.1 percent, are on-the-money (ONM), defined as the lowest interest rate amongst those bids that would win if every bank stopped bidding at that point in the auction.

The difference between INM and ONM bids is reflected in the normalized interest rate spread, defined as the difference between the highest winning interest rate and highest losing interest rate, all divided by the highest winning rate. The average normalized interest rate spread is 21 percent and its standard deviation is 20 percent. The bottom rows of Table 1 summarise these results in terms of dollar amounts. Another way of summarizing the different terms of trade auction is the money-left-on-the-table (MLT), found by subtracting from interest payments winners promise the amount of interest that would have been paid if the highest OUTM bid had used to a uniform interest rate of auction winners. Table 1 shows the average MLT is $1,803 but has a standard deviation more than three times as large, dictated in part by observations in the upper tail.

2.3 Bidding Patterns

Figures 1 through 5 amplify the importance of the three bidding categories by providing some simple dynamics. Figure 1 plots each bid against the current ONM (that is the lowest bid that would win an award if no more bidding took place). Thus, crosses on the 45 degree line represent ONM bids the higher they are the further the auction progressed. Crosses to the right of the 45 degree line represent INM bids and crosses to the left represent OUTM bids. The dotted lines indicate iso-times at the minutes indicated. By inspection, Figure 1 shows that At the beginning of the auction (when ONM is at the origin), the dispersion of bids vertically arrayed is greater than at any other point during the auction. From the scatter of points arrayed in vertical lines, we can see from Figure 1 that there are many bids in all three categories throughout the auction.

Figure 1: Submitted bid against lowest INM bid



To help explain why we observe all three categories of bidding throughout the auction, we separated ONM bids from INM bids (that are strictly greater than the interest rate necessary to win an award if no further bids are submitted). Figure 2 shows that a preponderance of ONM bids were preceded by slightly lower OUTM bids (those that cannot win an award given the current state of the auction). In other words, if they had not already submitted an INM bid, banks ascertain the current ONM bid by creeping up to it with a sequence of OUTM bids.

Figure 2: ONM versus Previously Submitted Bids

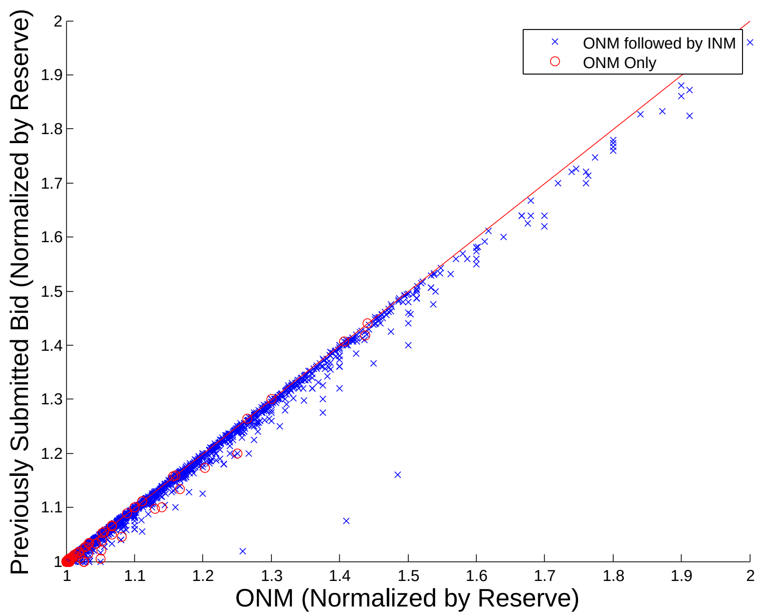
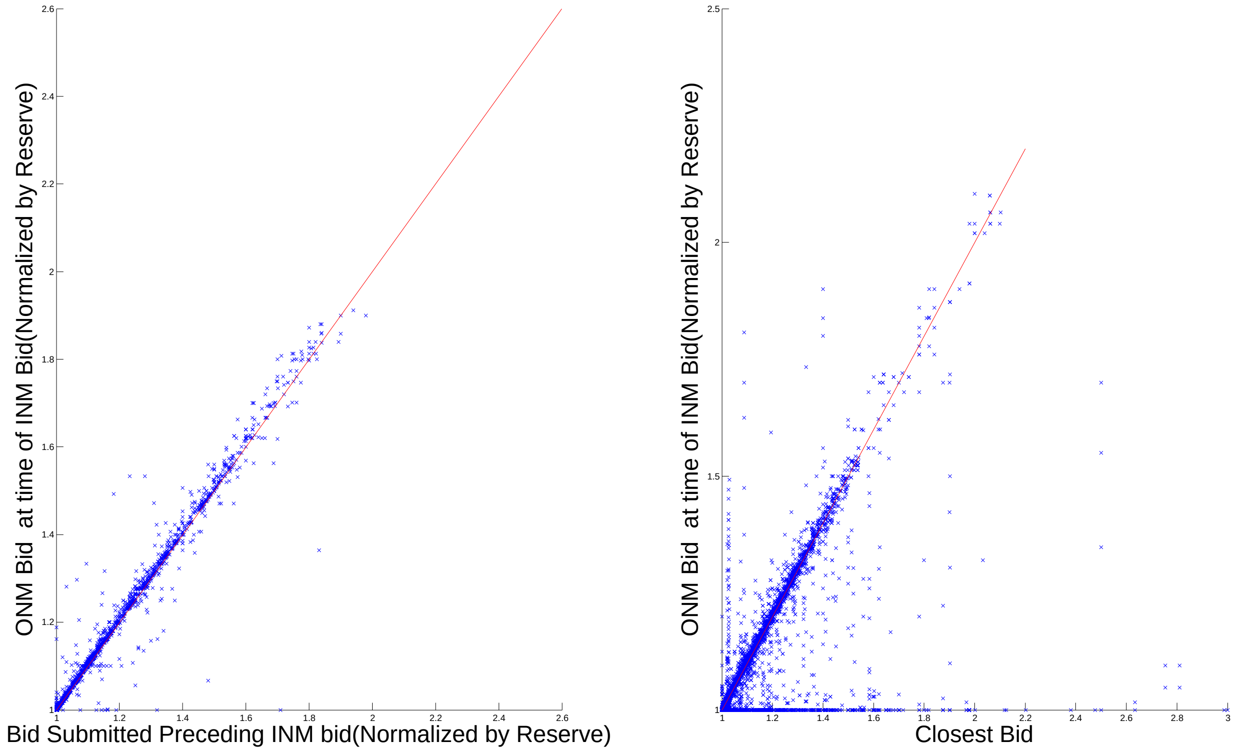


Figure 3: In the Money Bids versus Previously Submitted Bids



Taken together Figures 2 and 5 illustrate a pronounced tendency for bidders who ultimately place a bid that is ONM or INM to first determine where the boundary is by creeping to it with incremental bids, and only then deciding how much to jump into the money, if at all.

Figure 4: In the Money Bids versus Previously Submitted Bids Multi Parcel

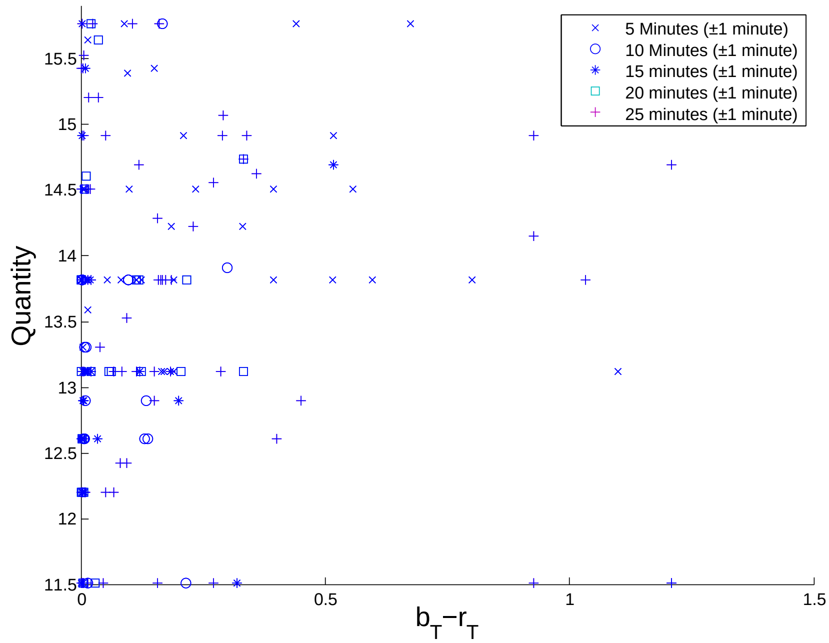
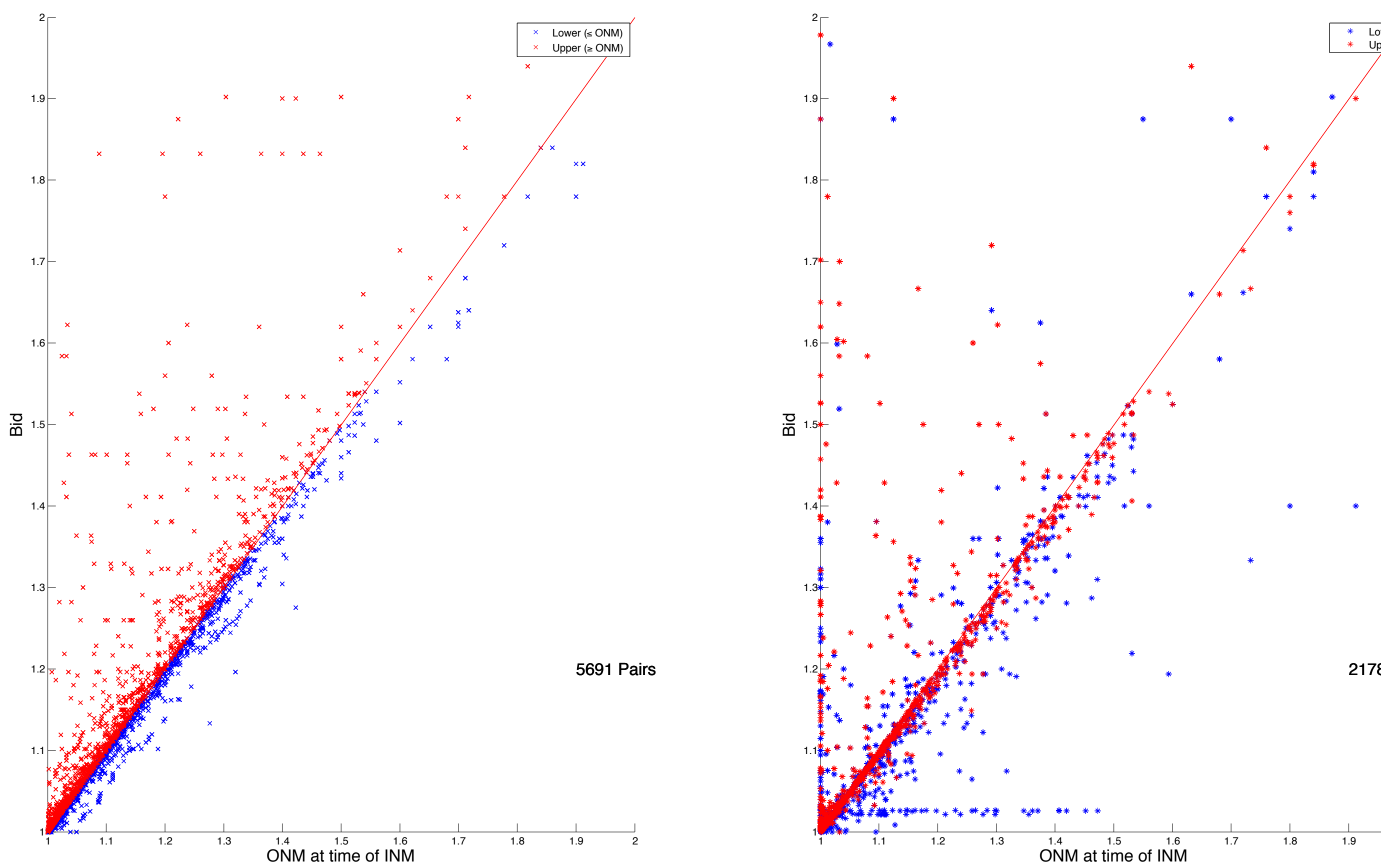


Figure 5: In the Money Bids versus Previously Submitted Bids Multi Parcel

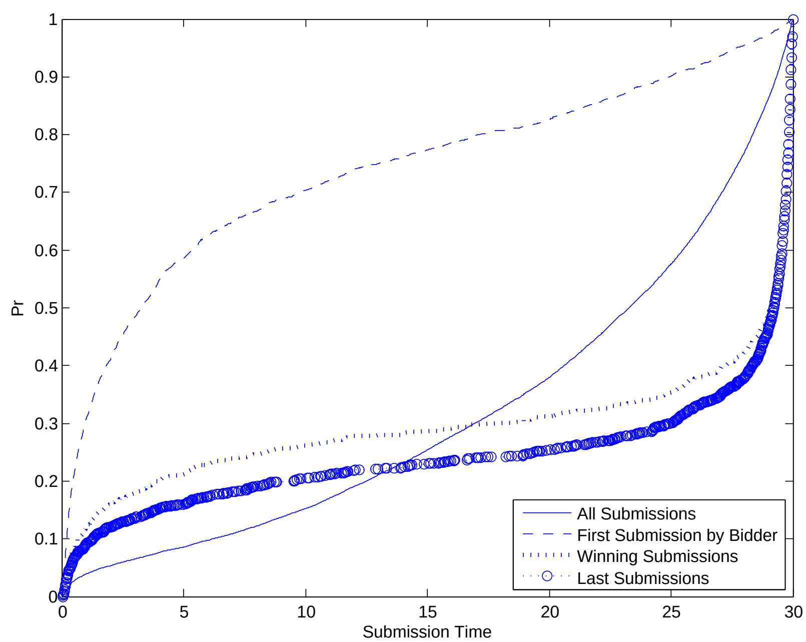


2.4 Submission Times

Data on bidding submission times also provides useful information about what type of model might fit the data. In Figure 6 we provide the empirical distribution of submission times for all bids, all first bids, all last bids, and for all winning bids. (The 30 minute auction period is divided into 3 minute intervals in the figure.) Most first bids are made before the 6 minute mark, and the convex shape of the distribution of all bids betrays less activity in the middle of the auction than at either end. This concentration is accentuated in last bids: more than half of all final bids are submitted in the final two minutes of the auction, and more than 10 percent of them are made in the first two minutes.

Sniping does not predominate bidding behavior of winning banks though. Figure 6 shows that most winning bids have already been submitted by the 25 minute mark. The winning bid distribution climbs to the 20th percentile within 6 minutes of the auction beginning but flattens between 9th and of 24th minute. The fact that a winning bid is frequently submitted in the early stages of the auction provides further evidence that some bidders do not win by incrementally increase their bids as necessary but enter with a jump giving their rivals plenty of time to respond.

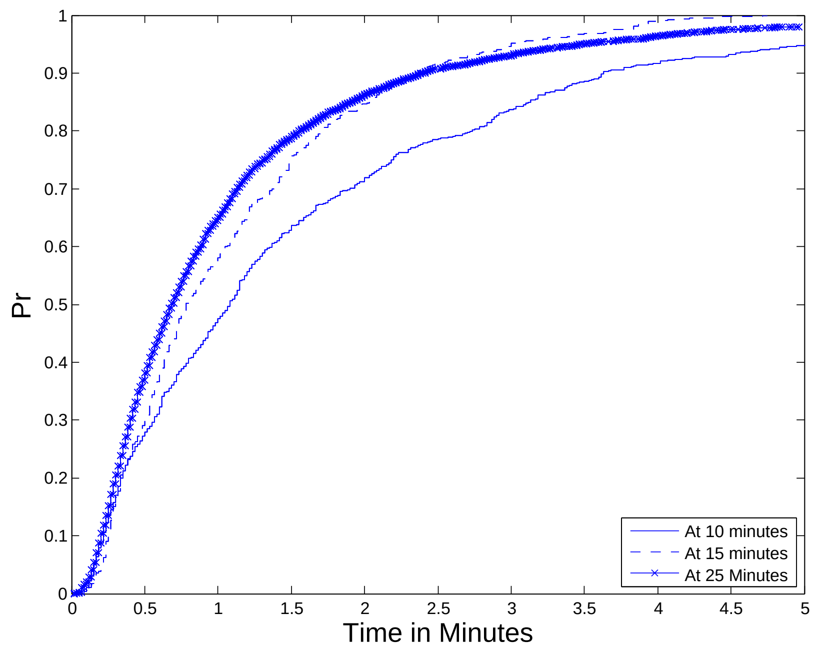
Figure 6: Empirical Distribution of Order Submission Times



How quickly bidders respond to rivals by updating their previous bids reflects how intensively they monitor the auction. However the average duration time between submissions by the same bidder is a biased estimators of the expected length of durations between successive monitoring opportunities, because bidders do not necessarily update their bids at every opportunity. But if a bidder was pushed out of the money, had limited bidding opportunities to revise his previous bid, and a sufficiently high valuation, he might increase his offer at the first available opportunity. That motivates why we display duration times between a bidder being pushed out of the money to when an updated bid restores him into the money relative to all bidders who fall out of the money.

Figure 7 shows four empirical distributions of reaction times. For every auction at 10, 15 and 25 minute mark into the bidding, we select all bidders who fall out of the money at some point within the preceding five minute window. We then form percentiles measuring the proportion of those bidders who re-submit a new bid in the money within a given amount of time up to five minutes.

Figure 7: Empirical Distribution of Reaction Times



This measure of monitoring intensity does not account for bidders who withdraw from the auction because the lowest bids in the money have overtaken their gross value from winning since their previous bids were placed, a factor that shifts all three curves to the left. Nevertheless it is noteworthy that the distribution of reaction times at earlier points in the auction first order stochastically dominate the distribution of reaction times at later points in the auction, suggesting that bidders increase their monitoring as the auction proceeds. Reviewing the data on submission times, it is implausible to argue that the final bid distribution in these auctions matches those we would observe in a discriminatory first price sealed bid auction.

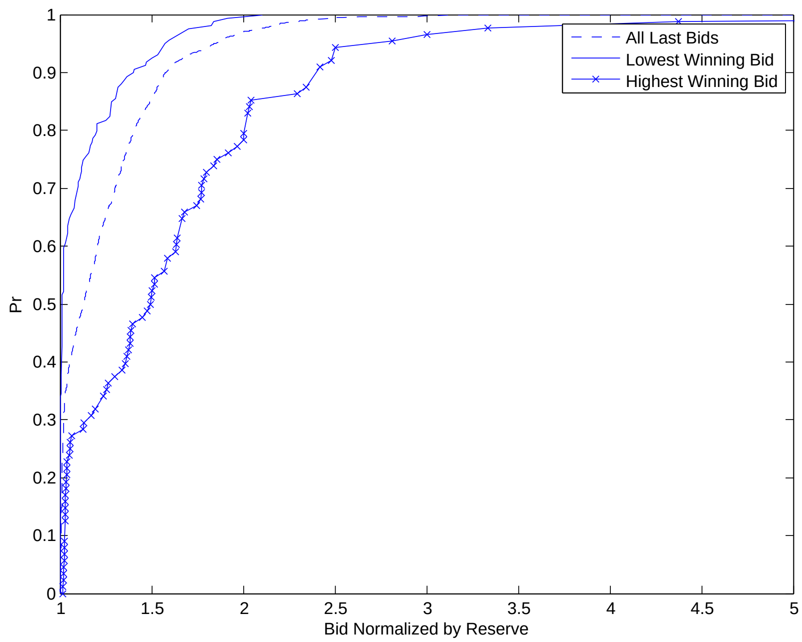
2.5 Summarizing the Data

From our discussion it appears the number of bidders is uncertain until the auction ends. Bidding activity is most intense at the beginning and end of the auction (like a limit order market). Bids often increase by more than the minimal amount required to stay in the money (in contrast to English auctions). Winning bids are sometimes submitted early in the auction, and sniping (common on eBay) is infrequent.

The interest rate spread of winning bids is notable (in contrast to Japanese auctions). Because bidders leave money on the table, these auctions do not seem to belong to the broad class of auctions Haile and Tamer (2003) investigate. In their analysis of ascending auctions Haile and Tamer (2003) bound valuations (and their distribution) on the basis of two behavioral premises. First, players never bid above their (private) valuations. Second, if the current high bid is below his valuation a player will submit another bid. Because players in these auctions can bid essentially any real number that exceeds their previous bid, it seems optimal to only bid ONM. In that case, all winning bids pay the same price the two distributions of highest and lowest winning bids coincide. Furthermore, the distribution of lowest winning bids first order stochastically dominates the distribution of all last (winning and losing) bids.

As can be seen in Figure 8 the distribution of lowest winning bids is first order stochastically dominated by the distribution of the highest winning bids and the distribution of all last (winning and losing) bids. This suggests that at least one of the two Haile and Tamer assumption is violated. One reason why the second assumption might not hold is that opportunities to bid might be limited by competing uses for a trader's time (such as trading other financial securities or professional duties).

Figure 8: Bid Distributions



3 Model

Taken together our observations outlined in Section 2 motivate our model of auctions with bidding frictions.

To rationalize the bidding behavior observed in the data we draw upon the stylized facts described above and model the institution. The data exhibit heterogeneity in parcel size across banks, but that banks rarely increase or split their parcels, so our model assumes the size of the parcel as exogenous but heterogenous across banks. Since most banks bid on only one parcel within an auction, the model assumes that each parcel is uniquely assigned to a bank; that is we ignore the interdependence created by bidding on two or more parcels at once.[3] Since banks with only one parcel can and do creep up to the ONM interest rate with successive OUTM bids, and INM bids are typically preceded by an ONM bid, our model assumes that the bank observes the ONM rate whenever its previous bid is OUTM. Consistent with the institutional arrangments, the bank receives no information when presented with an opportunity to update a bid that is INM, except that its current bid is still INM. Thus data on OUTM bids are treated as a procedure for determining the ONM rate, and play no further role in the theoretical or empirical analysis. Because the institutional mechanism for updating a bid involves typing a few keystrokes on a computer, and banks creep up to the ONM rate not seeming to economize on the number of bidding entries, our model also assumes the act of bidding is not costly. Instead of continuously updating an ONM bid near the end of the auction period, or sniping, more than half the winning banks submit their final bids long before the auction ends: to accommodate these features of the data, the bidders in our model have only imperfect monitoring capabilities. Intuitively there are competing uses for bank time; alternatives to devoting attention to the auction and reacting immediately to changes in the bidding history of that security, or being available to bid at the endpoint of the auction, include monitoring other securities the bank trades in, seeking new clients, or engaging in administrative work. In the model monitoring opportunities are modeled as random events, driven by a stochastic process that is controlled by the bank at a cost.

Prior to the beginning of the auction, a price is set exogenously, players in this dynamic game observe their private values and the size of their parcel. They individually and privately determine at a cost their monitoring intensities, which will stochastically determine the arrival of their first bidding opportunity after the auction begins. If and when a player receives his first bidding opportunity, he observes the current ONM price, submits a bid, and chooses the intensity of the arrival rate for the next bidding opportunity.[4] He can either maintain the same level or increase it at future bidding opportunities, and if he updates his previous bid from OUTM to INM he can reset the monitoring rate too. The only new information a player receives at a bidding opportunity is the current ONM price. The highest bids are fully filled first, and in the event of a tie, the earliest entry at the tied rate is given priority; to fill out the total funds available, partial awards to the lowest winning bidder are typical. If a player wins the auction, he pays his own final bid on each unit awarded.

- *Time* is continuous on the interval $[0, T]$, or alternatively $[0, T)$, a distinction that is immaterial when monitoring costs per unit time are strictly positive, but is critical for determining equilibrium outcomes for the specialization where there are no monitoring costs.

[3] It is straightforward to extend our theoretical model to accommodate banks that bid on one parcel, but notationally cumbersome. In our empirical analysis we conduct our analysis on the full sample and also a restricted sample of those banks bidding on just one parcel to test whether our results are sensitive to this assumption.

[4] If players had the choice of bidding or not at their first opportunity, everyone would do so. This is because their valuations exceed the reservation rate,they might not have another opportunity, and no information is conveyed to rivals by bidding at the reservation rate.

- *Total units awarded:* The maximal total amount awarded to all the winning bidders in aggregate is denoted by Q.

- *Reserve price:* A reservation price denoted by r_0 is announced prior to bidding. At any given time $t \in [0, T]$ during the auction, we denote by r_t the price that the lowest winning bidder would pay if no further bids are received in the auction. We call r_t the ONM bid; bids above r_t are INM, and bids below r_t are OUTM.

- *Number of bidders:* The players in the game are a set of potential bidders denoted $\mathcal{I} \equiv \{1, ..., I\}$.

- *Value and size of parcels:* Player $i \in \mathcal{I}$ draws a single indivisible parcel of exogenous size, denoted by $q_i \in [0, Q]$, along with a private value $v_i \in [r, \infty)$, independently and identically from commonly known joint distribution $F_{q,v}(.)$ prior to the beginning of the auction[5].

- *Action space:* Player $i \in \mathcal{I}$ makes choices at the beginning of the auction, as well as at the random times $\tau_{i1}, \tau_{i2}, \ldots, \tau_{i\rho}$ where ρ_i indicates the total number of bidding opportunities player i had in the auction. We denote his bid update at τ_{is} by b_{is} for $s \in \{1, \ldots, \rho_i\}$ and set $b_{i0} = 0$. At the beginning of the auction the i^{th} player selects a monitoring intensity for his first bidding opportunity, denoted by $\lambda_{i0} \in [0, \infty)$. For the s^{th} monitoring time player i updates his bid $b_{is} \in [b_{is-1}, \infty)$ and revises his monitoring intensity $\lambda_{is} \in [0, \infty)$.

- *Monitoring times:* The distribution of the first monitoring time is Poisson with parameter λ_{i0}. Let r_{is} denote the ONM rate at time τ_{is}. (Thus if $\tau_{is} = t$ then $r_{is} = r_t$.) Also let τ^*_{is} denote the time b_s falls OUTM or T, whichever comes first.[6] We assume players only have a chance to update their bids and reset their monitoring intensities after their current bid falls OUTM; thus if $s < \rho_i$ then $\tau_{i,s+1} > \tau^*_{is}$. Beginning at τ^*_{is} the distribution of the $(s+1)^{th}$ monitoring time is Poisson with parameter λ_{is}, a choice variable. That is given $\tau^*_{is} < T$, the random variable $\tau_{i,s+1}$ has conditional probability density function $\lambda_{is} e^{-\lambda_{is}(\tau_{i,s+1} - \tau_{is})} \Big/ \left[1 - e^{-\lambda_{is}(T - \tau^*_{is})}\right]$.

- *Information:* At his s^{th} bidding opportunity, the information set of player i at time τ_{is} includes his valuation and parcel size (v_i, q_i) , plus past bidding times $\tau_{i,s-k}$, previous bids $b_{i,s-k}$, and the associated ONM rates $r_{i,s-k}$ for $k \in \{1, \ldots, s-1\}$. We consolidate our notation by expressing the information of player i at τ_{is} as $h_{is} \in H_s$ for $s \in \{0, 1, \ldots, \rho_i\}$, where $h_{i0} \equiv (v_i, q_i)$ and:

$$h_{is} \equiv (v_i, q_i, \{\tau_{is'}, r_{is'}\}_{s'=1}^{s})$$

  for $s \in \{1, 3, \ldots, \rho_i\}$. Thus ρ_i, the total number of bidding opportunities player i had in the auction is only revealed at T.

- *Monitoring costs:* The per unit time cost of choosing monitoring intensity λ_{is} at time τ_{is} is defined on Λ by a convex increasing positive real valued mapping $g(\cdot)$ where $g(0) = g'(0) = 0$. Monitoring costs begin at τ^*_{is} and are incurred in the interval $(\tau^*_{is}, \tau_{i,s+1}]$ implying total monitoring costs associated with the bid at τ_{is} are $g(\lambda_{is})(\tau_{i,s+1} - \tau^*_{is})$.

- *Award rules:*Who wins an award, and what proportion of the parcel is filled, depends on all final bids, parcel sizes, and final submission times. We denote this set of variables by $l \in \mathcal{L}$ where

[5] Kastl (2011) and Hortacsu and McAdams (2010) consider divisible good auctions where the number of units to be bid on can be continuously controlled. In our setting we have discrete packages and we abstract from quantity choices.

[6] Formally $\tau^*_{is} \equiv \min\{t : r_{it} > b_{is}\}$ if $r_T > b_{is}$, with $\tau^*_{is} \equiv T$ otherwise.

$l \equiv \{b_{i\rho_i}, q_i, \tau_{\rho_i}\}_{i=1}^{I}$ and let $M_i(l) : \mathcal{L} \rightarrow [0,1]$ denote the award to player i as a proportion of his parcel size q_i. Funds are allocated by price, ties being settled by time priority. Denote this ordering by " $\succ$ ". It is formally defined by two criteria. If $b_i > b_j$, then $i \succ j$. If $b_i = b_j$ and $\tau_i < \tau_j$, then $i \succ j$. The ordering is strict, because bids cannot arrive simultaneously, and if the auction is filled there is exactly one bid b_k for quantity q_k that uniquely satisfies the two inequalities:

$$q_k + \sum_{j \in \mathcal{I}} 1\{b_j \succ b_k\} q_j > Q > \sum_{j \in \mathcal{I}} 1\{b_j \succ b_k\} q_j$$

Everyone ranked above Player k is fully funded, everyone ranked below k recieves nothing, while k receives the residual at price b_k as a partial award:

$$M_i(l) = \begin{cases} 1 \text{ if } \sum_{j \in \mathcal{I}} 1\{b_i \succ b_j\} q_j < Q \\ 0 \text{ if } \sum_{j \in \mathcal{I}} 1\{b_j \succ b_i\} q_j > Q \\ q_k^{-1}\left[Q - \sum_{j \in \mathcal{I}} 1\{b_j \succ b_k\} q_j\right] \text{ if } i = k \end{cases}$$

- *Payoffs:* Player i pays $q_i b_i M_i(b,q)$.The payoff to bidder i is:

$$q_i (v_i - b_i) M_i(l) - \sum_{s=1}^{\rho_i} g(\lambda_{i\tau_s}) \,|q_i, v_i \tag{1}$$

Total auction revenue is:

$$R \equiv \sum_{i \in \mathcal{I}} q_i b_i M_i(l)$$

In the special case where the submissions do not fill the total amount being auctioned, $\sum_{i=1}^{I} q_i \leq Q$, then every submission is fully funded, $M_i(l) = 1$ for all $i \in \mathcal{I}$, and total auction revenue reduces to the sum of all final bids weighted by their quantities, $\sum_{i=1}^{I} q_i b_i$.

- *Strategy space:* A strategy for player i, denoted by σ_i, is an initial monitoring choice, denoted by $\lambda_{i0}^{\sigma} : H_0 \rightarrow [0, \infty)$, plus at each monitoring opportunity $s \in \{1, 2, \ldots, \rho_i\}$, a sequence of bids, denoted by $b_{is}^{\sigma} : H_s \rightarrow [b_{is-1}, \infty)$ where $b_{i0} \equiv r$, as well as monitoring rate adjustments, denoted by $\lambda_{is}^{\sigma} : H_s \rightarrow [0, \infty)$.

4 Equilibrium

The existence of a symmetric Perfect Bayesian Equilibrium with monotone weakly increasing bidding strategies can be established by extending the analyses of Athey (2001) and Reny (2011). This section describes the main features of solving the optimization problem in equilibrium. To reduce notational clutter we now drop i subscripts, for example setting $\tau_{is} \equiv \tau_s$ and $M_s(l) \equiv M(l)$. At his s^{th} opportunity to bid, the value function for the player is defined as:

$$V_s(h_s) = \max_{\{b_r, \lambda_r\}_{r=s}^{\rho}} E_s\left[(v - b_\rho) q M(l) - \sum_{r=s}^{\rho} g(\lambda_r)(\tau_{r+1} - \tau_r)\right] \tag{2}$$

where $T \equiv \rho + 1$, and the expectation $E_s[\cdot]$ is taken conditional on the information set of player i, that is $h_s \equiv h_{is}$, over the final state of the auction, l. Similarly the program solved at the beginning of the auction to determine the initial monitoring rate is given by:

$$V_0(h_0) = \max_{\lambda_0}\left[\left(1 - e^{-\lambda_0 T}\right) V_1(h_1) - \left(1 - T\lambda_0 e^{\lambda_0 T} - e^{\lambda_0 T}\right) \lambda_0^{-1} g(\lambda_0)\right]$$

The value function $V_s(h_s)$ has a recursive formulation. If $\tau_s^* = T$, the player receives a payout of $(v - b_s) qM(l)$, because b_s is a winning bid. If $\tau_s^* < T$ and the player does not increase his bid again, b_s becomes a losing bid; with probability $e^{-\lambda_s (T - \tau_s^*)}$ the player does not receive another opportunity to bid and pays $g(\lambda_s)(T - \tau_s^*)$. Otherwise the player pays $g(\lambda_s)(\tau_{s+1} - \tau_s^*)$ and has another opportunity to bid and reset the monitoring rate at $\tau_{s+1} \in (\tau_s^*, T)$. We rewrite (2) recursively as:

$$V_s(h_s) = \max_{(b,\lambda_s)} \{ E_s[I\{b \geq r_T\}(v-b) M(l) | b] q - h(\lambda_s | h_s, b) g(\lambda_s) + E_s[I\{s < \rho\} V_{s+1}(h_{s+1}) | b] \}$$

where we show in the lemma below that:

$$h(\lambda_s | h_s, b_s) = E_s \left[I\{b < r_T\} \left\{ \lambda_s^{-1} + e^{-\lambda_s (T - \tau_s^*)} (T - \tau_s^*) + \frac{T - \tau_s^*}{e^{\lambda_s (T - \tau_s^*)} - 1} \right\} | b_s \right] \tag{3}$$

is the expected amount of time the bank pays at rate $g(\lambda_s)$ for monitoring. Note that whether and when the player will have another chance to bid depends on the monitoring rate through the expression:

$$E_s[I\{s < \rho\} V_{s+1}(h_{s+1}) | b] = E_s \left[I\{b < r_T\} \int_{\tau_s^*}^{T} \frac{\lambda_s e^{-\lambda_s (\tau_{s+1} - \tau_s)}}{1 - e^{-\lambda_s (T - \tau_s^*)}} V_{s+1}(h_{s+1}) d\tau_{s+1} | b \right]$$

If the reservation price at τ_s exceeds his valuation, the player stops monitoring the auction, effectively dropping out of the auction. If not, he increases his bid in return for the possibility of winning the auction. In this respect, bidding more closely resembles the Japanese auction rather than the English, since players with valuations higher than the ONM, when ONM is higher than their current bid are compelled to immediately update regardless of what other bidders do, as in an English auction where players can wait until a later point in the auction to reveal themselves. If his valuation is high enough, he solves a first order condition and bid INM, as in a first price sealed bid auction, and otherwise bids the ONM rate r_{is}, as in a Japanese or English auction. However the equations determining the optimal behavior not only reflect the uncertainty about future bidding opportunities, but also the effect of bidding on the future ONM rival bids, and hence the feedback into future monitoring.

The necessary first order condition for an interior solution is:

$$\begin{aligned} & (v - b_s) q \frac{\partial}{\partial b_s} E_s[I\{b \geq r_T\} M(l) | b_s] - E_s[I\{b \geq r_T\} M(l) | b_s] q \\ = \quad & g(\lambda_s) \frac{\partial}{\partial b_s} h(\lambda_s | h_s, b_s) - \frac{\partial}{\partial b_s} E_s[I\{\rho > s\} V_{s+1}(h_{s+1}) | b_s] \end{aligned} \tag{4}$$

There are two differences between a sealed bid auction equilibrium and this ascending auction. In a discriminatory multiunit auction the first order condition equates the expression on the left side to zero. Here there are two additional effects that are captured by the expressions on the right side of the equation. They stem from the fact that increasing the bid raises the threshold for the OUTM rate, serving to postpone the next bidding opportunity, but also encourages a more aggressive response from rivals, thus accelerating the increasing reservation rate. The first expression on the right side is the effect of bidding on the expected length of time the monitoring cost is $g(\lambda_s)$; the second expression is the effect of bidding on the timing and hence the value of the state variables for the continuation value $V_{s+1}(h_{s+1})$.

When a player receives a bidding opportunity and has a valuation higher than ONM, he will bid ONM if his valuation is sufficiently close. Consider the expected value of bidding at each of the existing prices in the auction book and amongst this set of bids let b^* denote the price which maximizes expected revenue for a player with valuation $\widehat{v}$. Let $\widehat{b}$ denote the bid at which his first order condition is solved and suppose he is indifferent between bidding and at the price point b^*. Since the player is indifferent between bidding either price:

$$
\begin{aligned}
& E\left[M\left(h_T, b_\tau\right)|h_j, b^*\right]\widehat{v} - E\left[b_\tau M\left(h_T, b_\tau\right) + \sum_{k=j}^{\tau} g(\lambda_k)\,|h_j, b^*\right] \\
= \quad & E\left[M\left(h_T, b_\tau\right)\Big|h_j, \widehat{b}\right]\widehat{v} - E\left[b_\tau M\left(h_T, b_\tau\right) + \sum_{k=j}^{\tau} g(\lambda_k)\,\Big|h_j, \widehat{b}\right]
\end{aligned}
$$

Making $\widehat{v}$ the subject of the equation yields:

$$
\widehat{v} = \frac{E\left[b_\tau M\left(h_T, b_\tau\right) + \sum_{k=j}^{\tau} g(\lambda_k)\,|h_j, b^*\right] - E\left[b_\tau M\left(h_T, b_\tau\right) + \sum_{k=j}^{\tau} g(\lambda_k)\,\Big|h_j, \widehat{b}\right]}{E\left[M\left(h_T, b_\tau\right)|h_j, b^*\right] - E\left[M\left(h_T, b_\tau\right)\Big|h_j, \widehat{b}\right]} \tag{5}
$$

It is straightforward to show that if a player with valuation $\widehat{v}$ is indifferent between bidding $\widehat{b}$ and b^*, then all players with valuations lying between $\widehat{v}$ and v^* would bid b^*, where v^* is the valuation of a player who would solve his first order condition by selecting b^*. Since the price bid does not change with valuations in this interval, it follows that the remaining players in the game cannot fully deduce the valuations of the players who bid on price points. By way of contrast the valuations of players who bid at isolated points are fully revealed because they uniquely satisfying a first order conditions.

First order conditions also characterize his monitoring expenditure; spending more raises the likelihood of having an opportunity to update the current bid. At that point he will also optimally adjust or stop his monitoring of the auction book. Denote $g'(\lambda) \equiv \partial g(\lambda)/\partial\lambda$. By assumption $g'(\infty) = \infty$. Consequently the optimal choice of λ is found by solving its associated first order condition given by:

$$
\begin{aligned}
& h\left(\lambda_s\,|h_s, b_s\right) g'\left(\lambda_s\right) + h'\left(\lambda_s\,|h_s, b_s\right) g(\lambda_s) \\
= \quad & E_s\left[I\left\{\rho > s\right\}\left\{\lambda_s^{-1} - \left(\tau_s^* - \tau_s\right) + \frac{\left(T - \tau_s^*\right)}{e^{\lambda_s\left(T-\tau_s^*\right)} - 1}\right\} V_{s+1}\left(h_{s+1}\right)|b_s\right]
\end{aligned} \tag{6}
$$

where:

$$
h'\left(\lambda_s\,|h_s, b_s\right) = -E_s\left[I\left\{b < r_T\right\}\left\{\begin{array}{l} \lambda_s^{-2} + e^{-\lambda_s\left(T-\tau_s^*\right)}\left(T - \tau_s^*\right)^2 \\ +e^{\lambda_s\left(T-\tau_s^*\right)}\left(T - \tau_s^*\right)^2\left(e^{\lambda_s\left(T-\tau_s^*\right)} - 1\right)^{-2} \end{array}\right\}|b_s\right] < 0 \tag{7}
$$

is the marginal reduction in expected monitoring time paid at rate $g(\lambda_s)$. Intuitively, the left side of (6) is the marginal cost of increasing the monitoring rate, the difference between the product of $h\left(\lambda_s\,|h_s, b_s\right)$, the amount of time spent monitoring at λ_s and the increased cost per unit time $g'(\lambda)$, and the expected marginal reduction in the amount of amount of spent $h'\left(\lambda_s\,|h_s, b_s\right)$ multiplied by the unit rate $g(\lambda_s)$. On the right side sum is the effect on the value function $V_{s+1}\left(h_{s+1}\right)$ through the change in the state variables h_{s+1} induced by a higher probability and earlier opportunity to bid. The first order condition for the initial monitoring intensity λ_0 can be found by specializing the equation above.

Lemma 1 below summarizes our discussion, and its proof we provide conditions for finding the threshold valuation that separates boundary condition bidders from those solving a first order condition, as a mapping of the ONM bid, the time remaining in the auction, and the quantity bid. It also shows that optimal response in equilibrium does not depend on previous monitoring opportunities, only the amount of time left in the auction.

Lemma 1 *A perfect Bayesian equilibrium exists where bids are monotone increasing in valuations. There exists a mapping* $\overline{v} : [0, 30] \times R^{+} \times R^{+} \rightarrow [r_0, \infty)$ *denoted by* $\overline{v}(\tau, r, q)$ *and satisfying the inequality* $r < \overline{v}(\tau, r, q)$ *such that, for all* s*, if* $r \leq v < \overline{v}(\tau_s, r_s, q)$ *then the player bids* r*, if* $\overline{v}(\tau_s, r_s, q) \leq v$ *then the player solves* (23)*, and if* $v < r$ *then the player drops out of the bidding. When given the opportunity* λ_s *solves* (6) *.*

There are two special cases that emerge from setting monitoring costs to zero. If the bidding interval was $[0, T]$ as specified in our framework, then there would be no incentive to post a bid before T and the auction would degenerate to a sealed bid first price auction. We would expect to see (practically) all the final and winning bids placed in the very last instants of the auction, in stark contrast to the results of Figure 4. Alternatively suppose the bidding interval was $[0, T)$. In our model this small change does not have any implications for the equilibrium bidding or monitoring. However if there are no monitoring costs there are huge implications. Following the natural extension of Haile and Tamer, bidding times are indeterminate, while there is jump bidding, it is never above the reservation price, as shown in Figure 1, and a uniform price is paid by all winning bidders at the end of the auction, so there is no money left on the table, as Table 1 shows. To summarize the absence of impediments to bidding lead to very different equilibrium outcomes that depend on a mathematical technicality that evaporates when monitoring costs are included, but neither specialization can rationalize key features of the data that from the model at least seem perfectly plausible. We conclude that this data should be modeled with an ascending auction mechanism where there are monitoring costs, which is our justification for the identification and estimation analysis that follows.

5 Identification and Estimation

The primitives of the model are the reservation price, r, the number of players I, the number of units for sale, Q, the joint probability density function for the valuations of the players and their demand quantities, denoted by $f(v, q)$, plus the monitoring cost rate function, $g(\lambda)$. Given values for these primitives, an equilibrium for the model generates a joint probability distribution for the sequence of bids, monitoring rates and reservation prices, along with the identities of the bidders (the participating players). The data set we analyze comprises records of many auctions in which the initial reservation price r is recorded, along with the number of units for sale, Q,as well as the bid sequence and the identities of the bidders, which allows us to construct the reservation prices. We do not observe their monitoring choice sequences or I, the number of players. However it is straightforward to show that with strictly positive probability, all the players bid. Consequently I is identified, and is consistently estimated by the maximum number of players observed bidding in one of the sampled auctions. This only leaves $f(v, q)$ and $g(\lambda)$ to identify. We follow a standard assumption in the literature on structural estimation that the data for all the auctions are generated by same equilibrium, that they have the same monitoring cost structure, and all valuations are drawn from the same probability distribution.

We successively analyze, in four steps, the valuations of players who bid, their monitoring rates, the probability distribution of valuations for all players whether they bid or not, and finally the monitoring cost function. At each step we establish identification, and then show how the sample utilized in estimation. Our estimators, which the Appendix describes in detail, are for the most part constructed from sample analogs to the conditions and nonlinear regression functions that establish identification.

5.1 Identifying and estimating the valuations of players who bid

The valuations of players who quit the auction by bidding OUTM, and those of players who make interior bidding choices INM are point identified. The valuations of players who are observed to only bid ONM are not point identified, but their bids do provide information about the probability distribution of valuations. and we show those valuations are bounded. Reweighting the distribution formed from valuations of players who bid OUTM and/or INM at some point during the auction with information about the quantiles of valuations for those bidding ONM, we obtain a consistent estimator of the probability distribution of players who bid.

A player whose final bid is OUTM at the time he bids fully reveals his valuation. His behavior is essentially identical to a player in a Japanese auction who quits before the auction is over. In both settings as the price rises players stay in the auction until their valuations are reached, and then immediately exit. In symbols:

$$v_i = b_{i\rho} \text{ if } b_{i\rho} < r_{i\rho} \tag{8}$$

.

Assuming interior bids are strictly monotone in valuations, it follows that for a given history of identical interior bids (across different auctions) were induced by the same valuation. Therefore the highest bid ever observed amongst this subset of players lower bounds their common valuation. Since the lowest winning bid may be arbitrarily close to any valuation, we can show that the highest winning bid observed amongst the subset converges from below to the common valuation. Let $b_{is} \equiv b\left(h_{is}\right)$ denote an equilibrium interior bid by player i at t when his partial auction history is h_{is}, meaning $b_{is} > r_{is}$. We let $h_{is'} \succeq (h_{is}, b_{is})$ mean that for any time $s \in (t, T]$, the partial history h_s succeeds h_t or is a continuation of h_t. Then:

$$v_i = \sup_{h_s \succeq (h_t, b_{it})} \left\{ b\left(h_s\right) | h_t, b_{it} \right\} \text{ if } b_{i\rho} \geq r_{i\rho} \text{ and } b_{is} > r_{is} \text{ for some } s \in \{1, 2, \ldots, \rho\} \tag{9}$$

Intuitively, given the opportunity, a player bids up to his valuation in an independent private value ascending auction and quits if r_{it}, the ONM bid, rises above it. So to prove this result we need to show that with positive probability there are histories $h_s \succeq (h_t, b_{it})$ such that $b\left(h_{is}\right)$ is arbitrarily close to v_i, say because with positive probability there are sufficiently many players with higher valuations that would bid if they could.

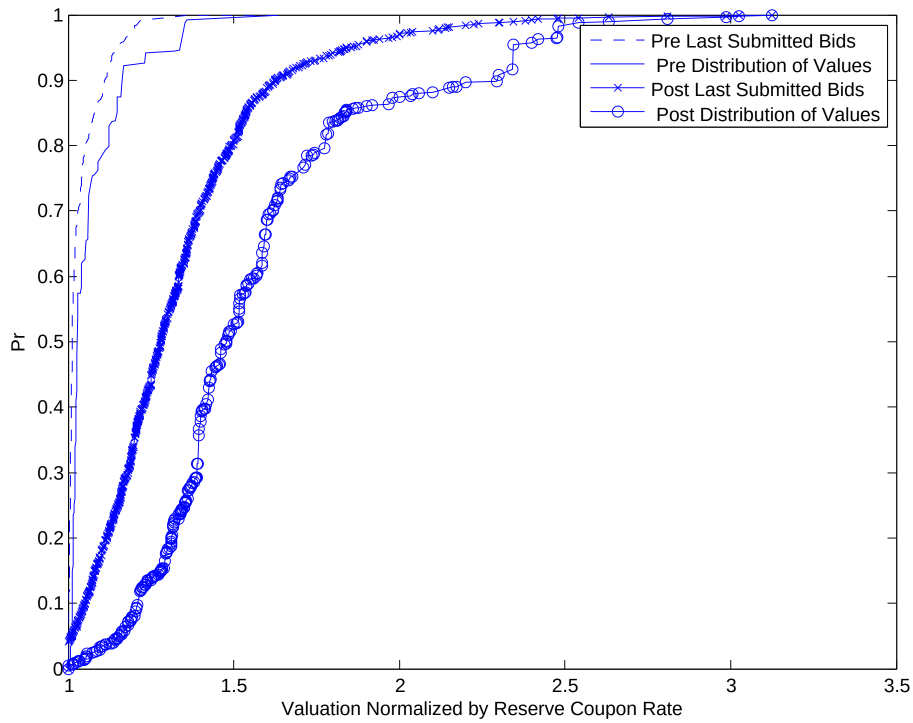
If players are observed to only bid ONM, then their point estimates are not point identified. Denoting by $\widehat{v}_i$ the valuation of a player whose last or final ONM bid is observed $r_{i\rho}$ clearly $\widehat{v}_i \geq r_{it}$ otherwise player i would have bid less. But depending on the slope of the marginal benefit from raising a bid at the ONM amount, players with valuations only slightly higher than r_{it}, say $\widehat{v}_i$, might optimally bid r_{it}. In that case, monotonicity implies every player with a valuation between r_{it} and $\widehat{v}$ would bid r_{it} in those circumstances too. These remarks also lead to a minimal bound on valuations that are associated with bidding ONM. Valuations and final bids.

$$v_i \in [b_{i\rho},] \text{ where } \widehat{v}_i \text{ is the the valution of the "relevant lowest FOC bid"} \tag{10}$$

How close are the bounds we would obtain from not putting any structure on the institutional detail surrounding the bidding. Figure 10 compares the distribution of final bids with the distribution of valuations estimated from our model for interior bids for data prior to 2008 and post 2008. These distributions are based on the sample of interior bids.

By construction valuations obtained from interior bids are at least as high as their final bids and typically higher, the valuation intervals obtained from ONM bids are lower bounded by the bids, while final bids OUTM equal their valuations. For these reasons the estimated probability distribution of valuations belonging of players who bid first order stochastically dominates the

Figure 9: Valuation Distributions Using Close Bids, normalized by reserve coupon rate



distribution of final bids. As a fraction of the final bid, the estimated mean valuation is XX higher and the margin observed is YY.

5.2 Effect of auction format on estimated distribution of valuations

Thus far the estimation of valuations has not exploited the assumptions in the model about the monitoring technology. Consequently the bidding strategies underpinning identification and estimation are optimal for any nonanticipating times that stochastically determine the player's sequence of bidding opportunites. We now compare these estimates with those obtained from two alternative auction formats. Both alternatives assume, contrary to the assumptions of our model, that at the time players submit their final bid they know there will be no further opportunities to bid. In a standard first price sealed bid (FPSB) auction there is a common reservation price and every player believes all final bids are simultaneous. In a variation on the FPSB auction, each bidder knows ONM when he submits his final bid, and recognizes the others may have opportunities to submit their final bids after him. How are the estimates of valuations affected by these two different assumptions about auction format?

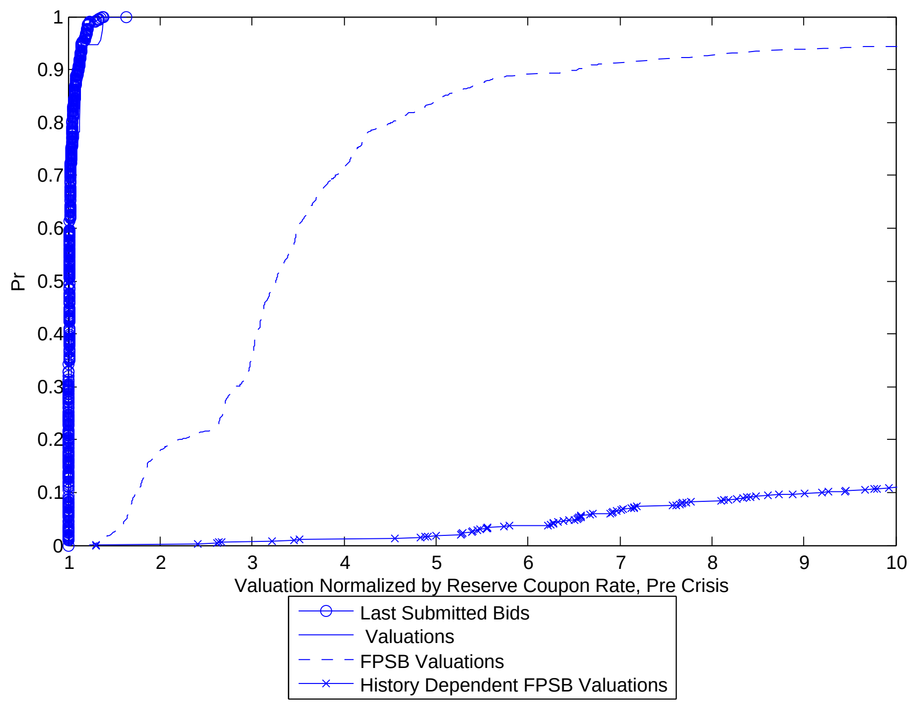
5.2.1 Valuations implied by a first price sealed bid auction

Perhaps the simplest FPSB auction that might be broadly consistent with our institutional setting is when players do not take account of the ONM price when submitting their final bid the end of the auction, and treat all previous bids as uninformative, similar to Bajari and Hortacsu (2003). Dropping the last two terms in (23) (because in this specialization bidders do not pay any monitoring costs and know when they have no further bidding opportunities), and similarly dropping the subscipt on the expectations operator (since bidders don't use any information about the partial history of the auction when they make their final bid), the first order condition for bidders becomes:

$$(v - b_\rho)\, q \frac{\partial}{\partial b_\rho} E\left[I\left\{b_\rho \geq r_T\right\} M\left(l\right)|b_\rho\right] = E\left[I\left\{b_\rho \geq r_T\right\} M\left(l\right)|b_\rho\right] q \qquad (11)$$

and the resulting estimator is a straightforward application of the approach in Guerre et al. (2000).

Figure 10: Valuations compared to FPSB estimates, normalized by reserve coupon rate



The distribution function estimated this way first order stochastically dominates the estimated distribution from our specification in quite dramatic fashion. Valuations are implausibly estimated orders of magnitude higher than the bids even though there are on average, more than twenty bidders in each auction.

While this is an implausible result under the assumption of FPSB, it is easy to explain if our original specification generates the data and the econometric specification is misspecified as FPSB. The most obvious source of differences arise from the treatment of bids that are OUTM when submitted. In the ascending auctions specification, the valuations equate with such bids, but in the FBSB format the valuations are strictly higher than these OUTM bids. Similarly bids ONM are matched to lower valuations in the ascending than the FPSB format, since in the former valuations are are weighte Finally, in equilibrium the derivative of the probability of winning with respect to the current bid averaged across partial histories h_ρ, that is $\partial E\left[I\left\{b_\rho \geq r_T\right\} M\left(l\right)|b_\rho\right] / \partial b_\rho$ is lower in a model if future bidding opportunities are uncertain than if players know they have reached their opportunity to bid. Since there may be an opportunity to bid later on, lowballing can be corrected at some future time before the auction ends if the ONM amount increases in the interim, and also discourages rival bidders from raising the ONM price. The low derivative is complemented by at least one additional term that separates the bid from its valuation in the first order condition;

thus not all the difference between the quotient of the winning probability and its derivative can be attributed to the difference between the bid and the valuation. **(What is the size of the difference when we substitute our estimates into the FOC? How far off zero are they?)**

5.2.2 Valuations implied by a sequential auction

Our data show that, amongst winning bids, the later the player bids, the closer his bid is to the the lowest winning bid, a finding at odds with the predictions of the FPSB auction format described above. Taken together with such implausibly high valuations, it might seem reasonable to modify the FPSB auction format to capture the possibility that bidders account for previous bids through their effects on ONM, which can be deduced at the time players submit their (final) bids, as discussed in Section 2. This model is similar in spirit to Daniel and Hirshleifer (1998). Suppose the timing sequence when bidders make their final bids is exogenously determined. The first order condition pertaining to this specialization is found by dropping the terms on the right side of (23); recognizing that the bidding process in this specialization markedly differs from the process in the original specification the subscript terms on the expectations operator are nevertheless left intact. Thus the essential difference in the econometric specification between this specialization and the FPSB amounts to the conditioning sets that affect the probability of winning and its derivative. Valuations estimated by OUTM bidders are identical for the original and this specialization, thus removing one of the distinctions of the FPSB. However the slope of the bid function is more similar to FPSB than the original.

The empirical distribution of valuations under the assumption that the auction is FPSB first order stochastically dominates the distribution of valuations generated by the ascending auction with monitoring, and if we are to take it seriously, suggests that competitive forces play a very small role in bidding. For example, valuations are a multiple of bids. Our description of the data provide several reasons why the FPSB model is misspecified. In estimation we can factor out the components that generate the misspecification using the first order condition for bidding (23) .

5.3 Monitoring choices of interior bids

Second we infer the monitoring rates from bidders whose bids fall OUTM. When reviewing a player's history within the data, we do not observe monitoring opportunities that the player decides not to bid on. The identification strategy is isolate those phases in the history when we are certain the player would bid if she could. Suppose the bid of player i at τ_{is} falls OUTM at some later point during the auction denoted by $\widehat{\tau}_{is}$, and she subsequently raises her bid at $\tau_{i,s+1} > \widehat{\tau}_{is} > \tau_{is}$. The latter bid shows her valuation can justify INM or ONM bids at $\widehat{\tau}_{is}$. Thus by Lemma 1, in equilibrium she bids at her first opportunity after falling OUTM. Extending this insight, once a bid falls OUTM, the player takes her first opportunity to rebid so long as her valuation is INM, because choosing the same monitoring rate and updating his bid to INM or ONM dominates passing. The per time cost is the same, as are future bidding opportunities, but she gains from bidding if no further bidding opportunities arrive and her revised bid remains ITM.

If the ONM price exceeds the i^{th} player's valuation v_i when the auction ends at T, we define T_i as the random time when the ONM price crosses her valuation, and otherwise we set $T_i = T$. It follows from this discussion that the timing of the next bidding opportunity is governed by an exponential hazard with rate $\lambda(h_{is})$ between $\widehat{\tau}_{is}$ and T_i if $\tau_{is} = \tau_{i\rho}$ and s is the last time i bids, and between $\widehat{\tau}_{is}$ and $\tau_{i,s+1}$ if the player updates her bid. Identification of the parameter $\lambda(h_{is})$ for all interior bids at τ_{is} follows directly by obtaining a closed form solution from the score for this mixed exponential density.

Lemma 2 *For all h_{is} satisfying $b_{i,s-1} < r_{is}$ and $\overline{v}(\tau_s, r_s, q) < v$, the optimal monitoring rate solves:*

$$\lambda\left(h_{is}\right)=\frac{\Pr\left\{\tau_{is}<\tau_{i\rho}\left|h_{is}\right.\right\}}{\Pr\left\{\tau_{is}<\tau_{i\rho}\left|h_{is}\right.\right\}E\left[\tau_{i,s+1}-\widehat{\tau}_{is}\left|\tau_{is}<\tau_{i\rho},h_{is}\right.\right]+\Pr\left\{\tau_{is}=\tau_{i\rho}\left|h_{is}\right.\right\}E\left[T_{i}-\widehat{\tau}_{is}\left|\tau_{is}=\tau_{i\rho},h_{is}\right.\right]}$$

It can be

5.4 Distribution of valuations for actual and potential bidders

From the first two steps we infer the initial monitoring rates that determine whether a player ever bids or not, and hence recover from the distribution of the selected players who bid the exante unconditional distribution of valuations for all players. All players would bid in the auction if they were confronted with the coupon reservation price of r_0. The valuations of those bidding at the reservation price are not uniquely identified since $\overline{v}\left(\tau,r_0,q\right)>r_0$. However monotonicity in the first order condition guarantees the truncated distribution of bidders solving an interior solution is identified. From the timing of first bids for those players with identical valuations we identify the initial choice of the monitoring rate as a function of their valuation, denoted by $\lambda_0\left(v\right)$. Since the selection rule and the truncation distribution of valuations for first bidders is identified, and the truncated distribution covers the support of the underlying distribution of valuations down to $\overline{v}\left(\tau,r_0,q\right)$, it follows that the latter is identified too.

Lemma 3 *For all h_{is} satisfying $b_{i,s-1}<r_{is}$ and $\overline{v}\left(\tau_s,r_s,q\right)<v$, the optimal monitoring rate solves: or taking limits we have:*

$$\Pr\left\{\overline{v}\left(\tau,r_0,q\right)<v\leq v_k\right\}=\frac{E\left[I\left\{v\leq v_k\right\}/\lambda_0\left(v\right)\right]}{E\left[1/\lambda_0\left(v\right)\right]}$$

where the expectation is taken over the truncated distribution.

Figure 11 depicts our estimates of the parent distribution from which the participating players are drawn through their first bidding opportunity. The most noticeable features are that bids are made on projects that, from a distributional perspective, first order dominate the untruncated valuations.

5.5 Monitoring costs

We also utilize our estimates of monitoring intensities and valuations to obtain estimates of the cost function for monitoring from the Euler equations characterizing the first order condotions of the interior solutions. The monitoring cost function, $g\left(\lambda\right)$, is identified from the first order condition for monitoring (6). We exploit the fact that the expectations error, that is the difference between the conditional expectation and its outcome, is orthogonal to the elements in the information set, which can therefore be used as instruments in estimation. It follows directly from the definition of the expected monitoring time at level λ, given by (3), its derivative (7) and (6) that:

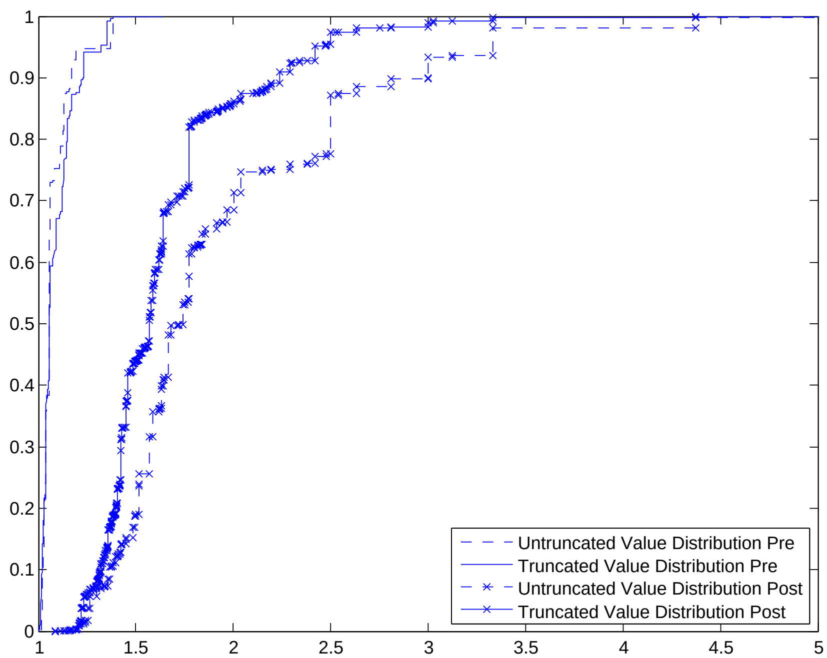
$$E_s\left[y-x_1g'\left(\lambda_s\right)+x_2g(\lambda_s)+\sum_{r=s+1}^{\rho}x_{3r}g(\lambda_r^e)\left|b_s\right.\right]=0 \tag{12}$$

where:

$$y\equiv I\left\{\rho>s\right\}\left\{\lambda_s^{-1}-\left(\tau_s^*-\tau_s\right)+\frac{\left(T-\tau_s^*\right)}{e^{\lambda_s\left(T-\tau_s^*\right)}-1}\right\}\left(v-b_\rho^e\right)qM\left(l^e\right)$$

$$x_1\equiv I\left\{b<r_T\right\}\left\{\lambda_s^{-1}+e^{-\lambda_s\left(T-\tau_s^*\right)}\left(T-\tau_s^*\right)+\frac{T-\tau_s^*}{e^{\lambda_s\left(T-\tau_s^*\right)}-1}\right\}$$

Figure 11: Untruncated versus Truncated Valuation Distribution Pre and Post



$$x_2 \equiv I\{b < r_T\} \left\{ \begin{array}{l} \lambda_s^{-2} + e^{-\lambda_s (T - \tau_s^*)} (T - \tau_s^*)^2 \\ + e^{\lambda_s (T - \tau_s^*)} (T - \tau_s^*)^2 \left(e^{\lambda_s (T - \tau_s^*)} - 1 \right)^{-2} \end{array} \right\}$$

$$x_{3r} \equiv I\{\rho > s\} \left\{ \lambda_s^{-1} - (\tau_s^* - \tau_s) + \frac{(T - \tau_s^*)}{e^{\lambda_s (T - \tau_s^*)} - 1} \right\} (\tau_{r+1} - \tau_r)$$

Setting:

$$g(\lambda) = \sum_{k=0}^{K} \theta_k \lambda^k$$

and substituting this parameterization for $g(\lambda)$ into (12) we group terms in θ_k to obtain:

$$E_s \left\{ y - \sum_{k=0}^{K} \left[x_1 k \lambda_s^{k-1} + x_2 \lambda_s^k + \sum_{r=s+1}^{\rho} x_{3r} (\lambda_r^e)^k \right] \theta_k \, | b_s \right\} = 0$$

In this way we establish sufficient conditions under which $g(\lambda)$ is identified within the class of analytic functions, are that $E[zz']$ is invertible, where the K dimensional random variable $z \equiv (z_1, \ldots, z_K)$ are instruments for this linear projection defined by its components:

$$z_k \equiv E_s \left[x_1 k \lambda_s^{k-1} + x_2 \lambda_s^k + \sum_{r=s+1}^{\rho} x_{3r} (\lambda_r^e)^k \, | b_s \right]$$

Substituting out our estimates for v and λ we can estimate θ in the above directly. We now report our estimates of the monitoring costs.

We first present estimation of the valuations. We sometimes find a number of negative interest rates which we find implausible. We drop these estimates from the discussion. Then we present our estimates of the costs of monitoring, found by computing the expected benefits of increasing monitoring at any given point in the auction.

Table 2: Monitoring Cost Estimates

		Cost Function Parameters (std. error)	
		Poly of degree 2	Poly of degree 3
Pre 2008	θ_0	0.02 (0.00)	48.42 (1.25)
	θ_1 coeff. on λ	8.98 (0.42)	3.74 (0.47)
	θ_2 coeff. on $\lambda.^2$	0.00 (0.00)	0.03 (0.00)
	θ_3 coeff. on $\lambda.^3$		-0.00 (0.00)
Post 2008	θ_0	-0.17 (0.01)	52.37 (0.93)
	θ_1 coeff. on λ	79.76 (0.53)	134.58 (0.86)
	θ_2 coeff. on $\lambda.^2$	-0.00 (0.00)	0.04 (0.00)
	θ_3 coeff. on $\lambda.^3$		-0.00 (0.00)

Table 3: Monitoring Costs over Realized Profits $\frac{\sum_{t=1}^{\tau} g(\lambda_t)}{(v-b)M(b,h_T)}$

		Mean	**Standard Deviation**	**Min**	**Max**
Pre 2008	Poly of Degree 2	0.03	0.19	0.00	4.87
	Poly of Degree 3	0.02	0.13	0.00	3.27
Post 2008	Poly of Degree 2	0.14	1.07	0.00	36.28
	Poly of Degree 3	0.23	1.83	0.00	62.38

6 Efficiency and Welfare

An important feature of this auction is the monitoring costs bidders pay to stay informed. It is reflected in the raw data by the spread of winning bids, and the times at which winning bids are submitted. In the equilibrium of our model the highest valuation players might not bid, and even if they do, might not win the auction. Our structural estimates of monitoring costs are statistically significant and quantitatively important. This section develops measures of the potential gains from trading in this auction market and reports our estimates of how efficiently players in the auction extract them.

6.1 Measures of Conduct and Performance

We use five measures of welfare to assess the auction format. Gross social surplus, the total value of funding the best projects in the absence of monitoring costs, is a natural benchmark to begin the comparisons. Then we derive the first best solution for a social planning problem, in which a team of project managers with the same objective as the planner choose monitoring rates for their respective projects, and if their projects become eligible for funding, they truthfully reveal the value of their projects, so that the planner can fund those that maximize the aggregate value of the program. Compared to the gross social surplus, the first best solution accounts for the monitoring costs of making the market.

Then we compute the second best solution that arises when there is moral hazard, because the social planner cannot observe monitoring effort and hidden information, because the planner cannot observe the value of each project. The welfare difference between the two planning problems is, loosely speaking, the cost of price discovery. We compare these three hypothetical scenarios with the current auction format to account for what happens to the gross total surplus.

Finally we compare the existing format with a discriminatory price sealed bid auction for eligible projects. Modifying the existing format, this can be simply accomplished by hiding from players whether their bids are in the money or not until after the thirty minute bidding period has elapsed. The existing format encourages low valuation project managers to take an option by initially placing low bids and effectively withdrawing from the auction if bids from higher valuation projects displace them, but benefiting by winning if competition from rivals is lacking; a sealed bid auction may reduce overall monitoring costs, by concentrating all monitoring to achieve eligibility in the first (and only) bid, but lacks this dynamic feature.

6.1.1 Gross social surplus

The (expected) gross social surplus is found by selecting the K most valuable projects from the pool of I projects whose valuations are drawn from the parent density function $f(v)$ with cumulative distribution function $F(v)$. Denoting by $m_{j,I}$ the mean value of the j^{th} least valuable project in a pool of size I, it follows that the gross social surplus is:

$$\sum_{j=I-K+1}^{I} m_{j,J} = \sum_{j=I-K+1}^{I} \frac{I!}{j!\,(I-j)!} \int vF^{j-1}(v)\,[1-F(v)]^{I-j} f(v)\,dv \tag{13}$$

6.1.2 First best solution ot the planning problem

There are two factors that explain the difference between the gross social surplus and the welfare obtained from the first best planning solution. First, the social planner incurs monitoring costs that are a function of the valuations, paying $\lambda^{o}(v)$ per unit of time while the project remains ineligible. Thus $g[\lambda(v)]$ is the expected monitoring cost of a project with value v, and:

$$I \int g[\lambda(v)]\, f_{\lambda}(v)\,dv$$

is the total monitoring cost, the independently and identically distributed probability density function for the projects is denoted by $f_{\lambda}(v)$, the subscript λ indicating the effect of monitoring on the projects tagged for possible funding. This density relates to $f_{\lambda}(v)$, the density from which flagged projects are drawn through a monitoring function $\lambda(v)$ via the mapping:

$$f_{\lambda}(v) = f(v)\,\lambda(v)\,/E[\lambda(v)] \tag{14}$$

In the sorting phase the planner allocates the fixed amount of funds to the projects of highest value. Suppose there are I projects of which J are identified by the planner, and a maximum of K can be funded. We denote by $\lambda(v)$ a monitoring function the planner might adopt, interpreting it as the monitoring intensity undertaken by the project manager to seek to contact the planner as instructed.

Second, only J eligible projects, numbering K, can be funded from the total pool of I projects. If $J \leq K$ then all the identified projects are funded, and the gross benefits are Jm_{λ}, where m_{λ} is the mean value for eligible projects when the monitoring technology $\lambda(v)$ is adopted. Alternatively $J > K$ and the planner selects the K most valuable projects for funding. Denoting by $m_{\lambda j:J}$ the mean value of the j^{th} least valuable project, it follows that the gross expected benefits in this case are:

$$\sum_{j=J-K+1}^{J} m_{\lambda j:J} = \sum_{j=1}^{J} m_{\lambda j:J} - \sum_{j=1}^{J-K} m_{\lambda j:J} = Jm_{\lambda} - \sum_{j=1}^{J-K} m_{\lambda j:J}$$

where the mean of this distribution is then:

$$m_\lambda \equiv \int v f_\lambda(v)\, dv \tag{15}$$

and the mean of its j^{th} order statistic (out of J) is:

$$m_{\lambda j:J} = \frac{J!}{j!\,(J-j)!}\int v F_\lambda^{j-1}(v)\left[1 - F_\lambda(v)\right]^{J-j} f_\lambda(v)\, dv \tag{16}$$

Denoting by $p_{\lambda J}$ the probability that J projects are identified to the social planner for potential funding, it now follows that the gross expected benefits to the social planner from selecting $\lambda(v)$ as a monitoring function are:

$$Jm_\lambda - \sum_{J=K+1}^{I}\sum_{j=1}^{J-K} p_{\lambda J} m_{\lambda j:J}$$

where:

$$p_{\lambda J} = \frac{I!}{J!\,(I-J)!}\left[\int \exp\left(-\lambda(v)\,T\right) f(v)\, dv\right]^J \left[1 - \int \exp\left(-\lambda(v)\,T\right) f(v)\, dv\right]^{I-J} \tag{17}$$

To solve the social planner's problem we must relate the set of potential projects to the set of identified projects through the monitoring technology. As before the unconditional probability density function characterizing valuations is denoted by $f(v)$.where the expectation is taken with respect to $f(v)$.

6.1.3 Second best solution to social planning problem

From a social perspective this is a generalized moral hazard problem. A total of I players manage one project each of heterogenous size q, exerting effort λ to make them eligible for funding by, and then in a second stage making an offer through messages b to be selected for funding. The social planner seeks to maximize the expected value from allocating total available funds Q to a subset of eligible projects net of expected monitoring costs incurred by all players seeking eligibility. Players personally and privately bear both the cost of monitoring their own projects and also the direct benefits if funded, so neither the costs nor the benefits are observed by the social planner. The planner only observes the field of eligible projects, recruited from an unobserved pool, the size of each eligible project, q, the time the project became eligible for funding, τ, and the messages, b, the player managing the eligible project sends about its value v. The dual challenge is to design a transfer function $c(q, \tau, b)$ that motivates players to individually monitor their projects to ensure optimal participation in the final allocation mechanism through their individual choices of λ, a moral hazard problem, and to induce those players whose projects are identified as candidates for funding, to reveal enough information about their own project valued at v to promote efficient allocation of funding, a problem in hidden information.

Consider a social planner who can fully commit to a transfer function that might conceivably punish players who report too early in the recruitment process for appearing to search too intensively relative to the social optimum. Following Myerson, all Bayesian equilibrium outcomes of any game induced by the mechanisms available to the social planner can be implemented by a direct mechanism in which player is asked to reveal his type, and finds it optimal to be respond truthfully. In a direct mechanism the space of signals is just the valuation space, and in equilibrium the best response is to set $b = v$ regardless of τ, the time the project is identified as a candidate for funding. Noting that the same valuation leads to the same choice of monitoring but multiple values for τ, it follows from the first order condition the player solves when revealing his valuation that the cross partial derivative of $c(q,\tau,b)$ with respect to its second two arguments is zero, or more generally that given q, the mapping $c(q,\tau,b)$ is additive in b and τ, taking the form:

$$c(q,\tau,b) = c_1(q,\tau) + c_2(q,b)$$

The additivity in transfer functions breaks any strategic linkage between time spent monitoring and bidding. It implies that this direct mechanism is renegotiation proof, and therefore can be implemented sequentally by the social planner; no commitment is called for at the design stage. Since project size is exogenous and the cost function associated with bidding does not depend on τ,the solution to the allocative direct mechanismit is for each funded project to pay a uniform price per unit of the highest losing bid. This simple result is a well known transparent extension of the ubiquitous second price auction. Thus:

$$c_2(q,b) =$$

To solve for the (second best) optimal transfer cost from monitoring we write down the expected value to the player is:

$$\left(1 - e^{-\lambda T}\right) E\left[M(l)\,vq - c_2(q,b)\right] - \int_0^T \lambda e^{-\lambda\tau} c_1(q,\tau)\, d\tau - g(\lambda)\, h(\lambda)$$

The first order condition for this problem is:

$$e^{\lambda T} E\left[M(l)\,vq - c_2(q,b)\right] + \int_0^T (\lambda\tau - 1)\, e^{-\lambda\tau} c_1(q,\tau)\, d\tau = g'(\lambda)\, h(\lambda) + g(\lambda)\, h'(\lambda)$$

The objective of the social planner is the same in the first best problem as in the interim problem, and the second stage of selecting the funded projects from the eligible set is also identical. The basic difference is that in the first best solution the planner chooses monitoring as a function of valuation and quantity, whereas in the second best solution, players choose monitoring as a function of their valuation and quantity, and the only influence the planner can have in their choice is through a tax on the speed at which they enter their orders. If all eligible projects were invariably funded, the bid price would be the reservation rate, all three solutions would coincide, so the optimal congestion transfer tax would be zero. Only when the number of eligible funds exceeds the the available funds do the solutions diverge. In the first best solution, the planner economizes on the eligible number of low valuation projects by monitoring them with low intensity. Compared to that solution, we see that in second best problem this case the transfer function inducing monitoring serves two roles pulling in opposite directions. As in the first best solution, players might be encouraged to monitor so that the social planner can select from a broader range of projects and thus increase the value of the selected bundle. However since $c_1(q,\tau)$ does not depend on v, encouraging all submissions induces monitoring effort by low quality project managers too and contributes to a cost akin to congestion when there are more candidate projects than than can be funded.

We summmarize the results of this subsection in the following lemma.

Lemma 4 *Full committment optimal interim efficient social planning problem can be implemented by a direct mechanism. It decomposes into two transfer functions. The second is a standard auction, and yields the optimal allocation amongst candidate players. The first is a tax or subsidy on the time a project becomes a candidate for funding.*

6.1.4 Comparison with a discriminatory price sealed bid auction

In equilibrium players disperse their monitoring efforts over the entire thirty minute auction period, and funds are not automatically awarded to the highest valuation eligible projects. Within our framework, the monitoring effort of players could be concentrated into their first bidding opportunity, and the loss from not funding the highest valuation projects could be eliminated by conducting a discriminatory price sealed bid auction. These changes could be simply accomplished by not revealing to players, until after the full thirty minute period has elapsed, whether their bids are in the money or not. In this case individual monitoring would be a function of (v, q) alone rather than successive values of r_s as well, and the highest value projects would be selected from the set of eligible ones. To compute the value of conducting a discriminatory price sealed bid auction, we evaluate the social planner's objective function when players solve their monitoring strategies in the Bayesian Nash game for $c_1(q, \tau) = 0$.

In the equilibrium of both formats there is congestion relative to the first best solution, in the sense that all players pay monitoring costs but only some projeects are funded. Providing some information to players about rival bids throughout the auction and allowing them to update disperses monitoring costs and might reduce aggregate monitoring costs and perhaps encourage players to monitor more closely when it is more socially valuable to do so. This might offset the loss from not ranking projects in equilibrium strictly according to value, as happens in a sealed bid private auction. Thus theory does not answer the question about which mechanism is more efficient; this is an empirical matter.

6.2 Results

The sum of realized profits for auction l is:

$$\sum_{i}^{I^l} [v_i^l - b_i^l] M(b_i^l) q_i^l$$

We then compute what the potential profits could have been with knowledge on individual valuations. We first determine which bidders should be in the money, rather than those who were found to be INM by the mechanism. Let this updated INM indicator be denoted by M^*. Then the sum of potential profits is given by:

$$\sum_{i}^{I^l} [v_i^l - b_i^l] M^*(b_i^l) q_i^l$$

For simplicity we assume that

$$\lambda(v) = \sum_{p=1}^{P} \gamma_p \lambda^p \tag{18}$$

and that the social planner picks values of γ_p to maximize the expected social surplus. We first present results on the quadratic information cost structure and a cubic $\lambda(v)$ function. The γ parameters are shown in Table 5. These parameters can be used to generate the truncated valuation distribution and can be compared to the selection the current auction mechanism induces. These are shown in Figure 12 and Figure 13

The player's monitoring choice determines the rate at which opportunities to bid arrive throughout the auction game; a higher arrival rate is more costly to implement than a lower rate, and it can be revised with each bid update. In this way we capture alternative uses of the player's time, such as monitoring other securities the player trades, seeking new clients, or conducting administrative duties for his or her employer. The players' equilibrium choices of monitoring endogenize the intensity of bidding activity throughout the auction. At the beginning of the game, bidding is relatively frequent: until a player has made his first bid, the cost of losing an opportunity to bid is zero from the auction, and we show that everyone who has has an opportunity to make an initial bid will take it; anticipating the value of the initial bid players choose a relatively frequent monitoring. Having placed an initial bid, a player might not increase it at every opportunity he has, depending on whether it is still in the money. For this reason players might economize on monitoring costs after making their first bid. The rush of bids at the end, sniping, is a well understood response by bidders trying to prevent being pushed out of the money from rival bids that arrive later but still in time for the auction close. These collective choices are exacerbated by rational herd behavior. Since players have rational beliefs about their rivals, they reduce their monitoring during the the middle phase of the auction because fewer rival bids arise, compounding the reduced frequency of bidding, but in the last minutes of the auction, anticipating that others will snipe, they increase their monitoring as a defensive strategy, which in turn increases bidding activity.

The results are summarized in Table 4.

Table 4: Private and Social Surplus

		Mean	**Standard Deviation**	**Min**	**Max**
Pre 2008	Fraction of OTM bidders who should be ITM	0.78	0.28	0.00	1.00
	$\sum_{i=1}^{I^l} v_i^l M(b_i^l)$	3879765.57	593490.85	2429404.00	4323249.00
	$\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)$	3902577.93	588256.22	2435750.00	4323249.00
	$\frac{\sum_{i=1}^{I^l} v_i^l M(b_i^l)}{\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)}$	0.99	0.01	0.93	1.00
	$\sum_{i=1}^{I^l} b_i^l M(b_i^l)$	3766933.11	578832.43	2391591.00	4269099.00
	$\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)$	3821871.43	571952.24	2381500.00	4244000.00
	$\frac{\sum_{i=1}^{I^l} b_i^l M(b_i^l)}{\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)}$	0.99	0.03	0.90	1.05
Post 2008	Fraction of OTM bidders who should be ITM	0.72	0.40	0.00	1.00
	$\sum_{i=1}^{I^l} v_i^l M(b_i^l)$	921950.19	912430.50	81315.00	3389840.00
	$\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)$	944056.91	928547.94	81315.00	3414240.00
	$\frac{\sum_{i=1}^{I^l} v_i^l M(b_i^l)}{\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)}$	0.98	0.03	0.88	1.00
	$\sum_{i=1}^{I^l} b_i^l M(b_i^l)$	760673.91	844874.28	57999.00	3346376.00
	$\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)$	838595.40	893196.08	37200.00	3360000.00
	$\frac{\sum_{i=1}^{I^l} b_i^l M(b_i^l)}{\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)}$	0.95	0.20	0.52	1.56

Summaries on auction outcomes across all simulations output can be found in Table **??**. To compute summary statistics we first average outcomes over all simulations and then across auctions.

Table 5: Social Planner Solution

	Pre 2008	**Post 2008**
γ_1	-76.0917	0.1060
γ_2	990.3128	-8.1834
γ_3	15821.6614	105.4699

Figure 12: Pre 2008: Comparison of Truncated Distributions from Social Planning Problem (Quadratic Monitoring Costs)

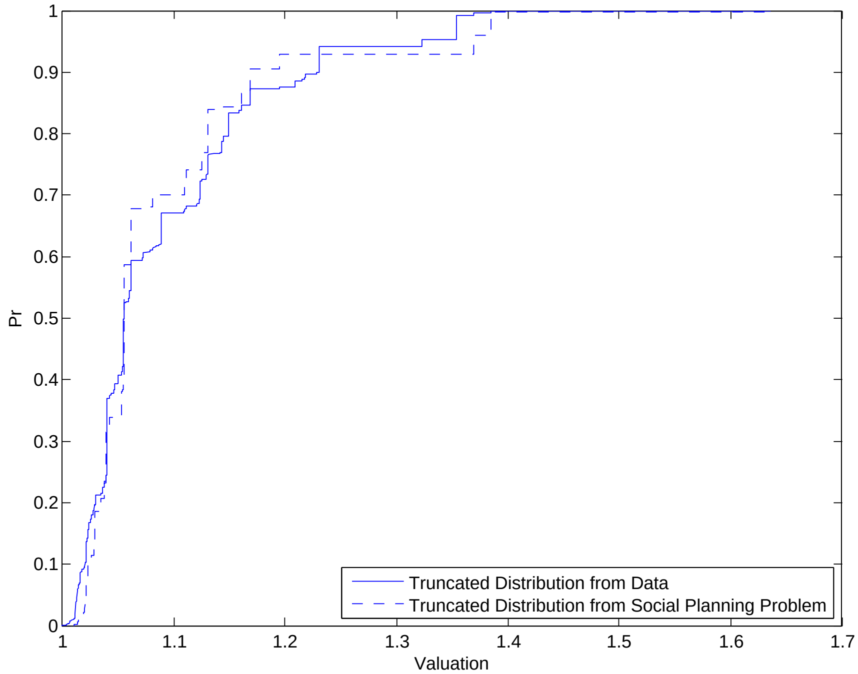


Figure 13: Post 2008: Comparison of Truncated Distributions from Social Planning Problem (Quadratic Monitoring Costs)

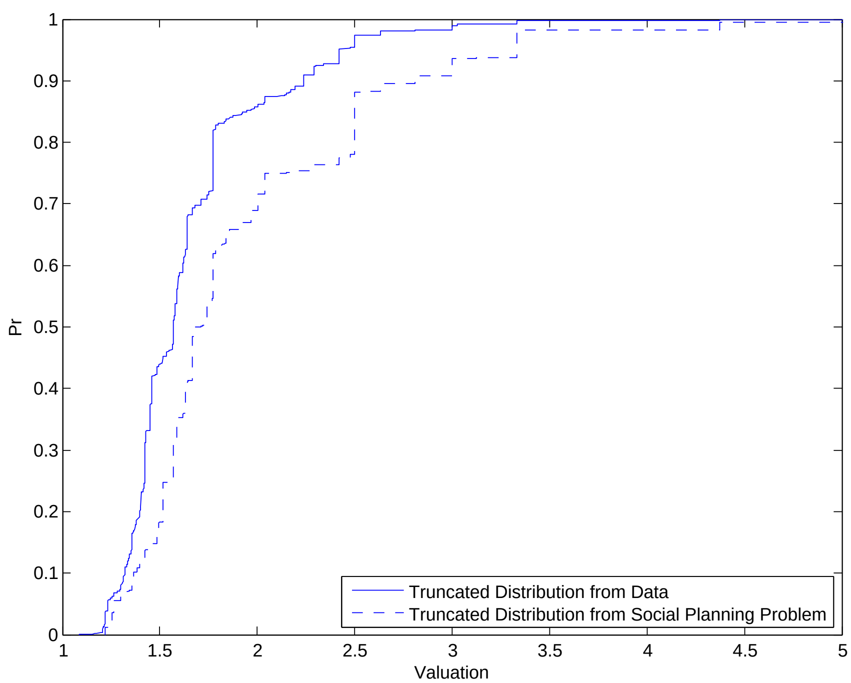


Table 6: Summary Statistics on Social Planner Simulation

		Mean	**Standard Deviation**	**Min**	**Max**
Pre 2008	Number of Bidders	47.8559	11.7195	18.1236	69.0202
	Avg. Monitoring Cost Across Bidders	160.5146	76.6329	23.8342	327.7212
	$\sum_{i=1}^{I} v_i M_{sp}(v_i)$	6955082.1417	1316351.9872	3764680.7801	9887093.0367
	$\frac{\sum_{i=1}^{I} v_i M^*(v_i)}{\sum_{i=1}^{I} v_i M_{sp}(v_i)}$	0.5909	0.1582	0.2933	0.9730
Post 2008	Number of Bidders	18.1071	28.9460	0.0094	71.7114
	Avg. Monitoring Cost Across Bidders	0.1234	0.2612	0.0000	1.1789
	$\sum_{i=1}^{I} v_i M_{sp}(v_i)$	1882073.5362	3027077.1545	96.2925	9246678.9770
	$\frac{\sum_{i=1}^{I} v_i M^*(v_i)}{\sum_{i=1}^{I} v_i M_{sp}(v_i)}$	0.3774	0.0936	0.1675	0.5012

7 Conclusion

This paper constructs and estimates a multiunit ascending auction with costly monitoring. Our modeling approach, estimation strategy and empirical findings establish that there is a role of monitoring costs. Paradoxically the value of a trader's time should be reflected by his inattention to detail. We would expect large successful traders to lose more money in any one market than their less successful rivals. Our estimation approach avoids computation of equilibrium strategies and could potentially be adapted to other environments with even more complex strategy spaces. We then apply the model to CD auctions held by State Treasurers to place unallocated government funds in savings vehicles. Our data allows us to trace the effects of the 2008 recession. We find that the distribution of interest rate valuations changes substantially after the crisis. In particular, the variance of valuations increases substantially. This indicates that bidders were affected very differently from each other by the recession.This paper contributes to understanding dynamic auction mechanisms that do not conform to the Japanese auction idealization.

Finally our model and approach sheds some light on daily trading in limit order markets. Viewed as an electronic limit order market, bidders compete by submitting limit buy orders before a seller enters at a preordained moment and places market orders at . Both types of trading mechanisms The general features of the data, in which order placement activity concentrated at the open and close of trading. Both exhibit isolated limit orders as well as price points, prices where several orders are inverted by placement time. As in our auctions activity is greatest at the beginning and end of the trading period. Our model explains why when there is mutual verification of the monitoring technology, traders tend to place orders at the beginning of the period to deter entry and at the end to prevent responses. At the trading level our model explains why such clumping occurs.

8 Appendix A: Proofs

Lemma 5 *A perfect Bayesian equilibirum exists where bids are monotone increasing in valuations. There exists a mapping* $\overline{v} : [0, 30] \times R^{+} \times R^{+} \to [r_0, \infty)$ *denoted by* $\overline{v}(\tau, r, q)$ *and satisfying the inequality* $r < \overline{v}(\tau, r, q)$ *such that, for all* s*, if* $r \leq v < \overline{v}(\tau_s, r_s, q)$ *then the player bids* r*, if* $\overline{v}(\tau_s, r_s, q) \leq v$ *then the player solves* (23)*, and if* $v < r$ *then the player drops out of the bidding. When given the opportunity* λ_s *solves* (6)*.*

Proof of Lemma 1. Existence from Athey and Reny. ■

Lemma 6 *For all* h_{is} *satisfying* $b_{i,s-1} < r_{is}$ *and* $\overline{v}(\tau_s, r_s, q) < v$*, the optimal monitoring rate solves:*

$$\lambda(h_{is}) = \frac{\Pr\{\tau_{is} < \tau_{i\rho} | h_{is}\}}{\Pr\{\tau_{is} < \tau_{i\rho} | h_{is}\} E[\tau_{i,s+1} - \widehat{\tau}_{is} | \tau_{is} < \tau_{i\rho}, h_{is}] + \Pr\{\tau_{is} = \tau_{i\rho} | h_{is}\} E[T_i - \widehat{\tau}_{is} | \tau_{is} = \tau_{i\rho}, h_{is}]}$$

Proof of Lemma 2. The mixed probability distribution for $\lambda(h_{is})$ is given by the formula: ■

Lemma 7 *For all* h_{is} *satisfying* $b_{i,s-1} < r_{is}$ *and* $\overline{v}(\tau_s, r_s, q) < v$*, the optimal monitoring rate solves: or taking limits we have:*

$$\Pr\{\overline{v}(\tau, r_0, q) < v \leq v_k\} = \frac{E[I\{v \leq v_k\} / \lambda_0(v)]}{E[1 / \lambda_0(v)]}$$

where the expectation is taken over the truncated distribution.

Proof of Lemma 3. More specifically, let T_i denote the random time that player i has his first opportunity to bid when his initial monitoring rate is $\lambda_0(v_i)$. Then:

$$\ln T_i = -\ln \lambda_0(v_i) + \eta_i$$

where η_i has a Type 1 extreme value distribution. Suppose v_i is drawn from a finite set $\{v_1, \ldots, v_I\}$ Suppose momentarily that the support of parent population of the truncated population, ranked from the lowest to the highest, is $\{v_1, \ldots, v_I\}$ with probabilities $\{q_1, \ldots, q_I\}$, and that in the truncated population the probabilities are $\{p_1, \ldots, p_I\}$. Then:

$$p_i = \frac{q_i \lambda_0(v_i)}{\sum_{j=1}^{I} q_j \lambda_0(v_j)}$$

That is:

$$\frac{p_i}{\lambda_0(v_i)} = \frac{q_i}{\sum_{j=1}^{I} q_j \lambda_0(v_j)}$$

But:

$$\sum\nolimits_{i=1}^{I} \frac{p_i}{\lambda_0(v_i)} = \sum\nolimits_{i=1}^{I} \frac{q_i}{\sum_{j=1}^{I} q_j \lambda_0(v_j)} = \frac{1}{\sum_{j=1}^{I} q_j \lambda_0(v_j)}$$

implying:

$$q_i = \left(\frac{p_i}{\lambda_0(v_i)}\right) \Big/ \sum\nolimits_{i=1}^{I} \frac{p_i}{\lambda_0(v_i)}$$

and more generally for any v_k:

$$\Pr\{v \leq v_k\} = \left\{\sum\nolimits_{i=1}^{k} \left(\frac{p_i}{\lambda_0(v_i)}\right) \Big/ \sum\nolimits_{i=1}^{I} \frac{p_i}{\lambda_0(v_i)}\right\}$$

or taking limits we have:

$$\Pr\{v \leq v_k\} = \frac{E\left[I\{v \leq v_k\} / \lambda_0(v)\right]}{E\left[1 / \lambda_0(v)\right]}$$

where the expectation is taken over the truncated distribution. ■

The monitoring cost function, $g(\lambda)$, is identified from the first order condition for monitoring (6). We exploit two features of this equation, Since the optimal choices of λ_r at $r \equiv \tau^{(m)}$ and λ_s at $s \equiv \tau^{(m+1)}$ satisfy the monitoring FOC we can first difference to obtain:

$$\begin{aligned} & \frac{g'(\lambda_s)}{(T-s)} e^{\lambda_s (T-s)} - \frac{g'(\lambda_r)}{(T-r)} e^{\lambda_r (T-r)} \\ = & \left\{ \begin{array}{c} E_{r1}[bM(l)] - b_t E_{r0}[M(l_t)] \\ -E_{s1}[bM(l)] + b_s E_{s0}[M(l_s)] \end{array} \right\} \\ & -v \left\{ \begin{array}{c} E_{r1}[M(l)] - E_{r0}[M(l_t)] \\ -E_{s1}[M(l)] + E_{s0}[M(l_t)] \end{array} \right\} \\ & E_{r1}\left[\sum\nolimits_{k=m+1}^{\tau} g(\lambda^{(k)})\right] - E_{s1}\left[\sum\nolimits_{k=m+2}^{\tau} g(\lambda^{(k)})\right] \end{aligned}$$

But:

$$E_{r1}\left[\sum\nolimits_{k=m+1}^{\tau} g(\lambda^{(k)})\right] - E_{s1}\left[\sum\nolimits_{k=m+2}^{\tau} g(\lambda^{(k)})\right] = g(\lambda_s) + \epsilon_s$$

where ϵ_s is orthogonal to (h_r, v, λ_s). Consolodating we have:

$$v y_{1s} - y_{0s} = g(\lambda_s) - g'(\lambda_s) A_{2s} + g'(\lambda_r) A_{1r} + \epsilon_s \tag{19}$$

where:

$$\begin{aligned}
y_{0s} &= E_{r1}\left[bM\left(l\right)\right]-b_{t}E_{r0}\left[M\left(l_{t}\right)\right] \\
&\quad -E_{s1}\left[bM\left(l\right)\right]+b_{s}E_{s0}\left[M\left(l_{s}\right)\right] \\
y_{1s} &= E_{r1}\left[M\left(l\right)\right]-E_{r0}\left[M\left(l_{t}\right)\right] \\
&\quad -E_{s1}\left[M\left(l\right)\right]+E_{s0}\left[M\left(l_{s}\right)\right] \\
A_{1r} &= \frac{e^{\lambda_{r}(T-r)}}{(T-r)} \\
A_{2s} &= \frac{e^{\lambda_{s}(T-s)}}{(T-s)}
\end{aligned}$$

In this way we establish that $g(\lambda)$ is identified within the class of analytic functions.

More generally $g(\lambda)$ can be modelled as depending on the amount of time left in the auction. For example suppose that monitoring costs are paid per unit time until the next monitoring opportunity, or the auction ends, whichever comes first. We write:

$$\begin{aligned}
g\left(\lambda,s\right) &\equiv h\left(\lambda\right)\int_{0}^{T-s} t\lambda\exp\left(-\lambda t\right)dt \\
&= h\left(\lambda\right)\left\{-\left[t\exp\left(-\lambda t\right)\right]_{0}^{T-s}+\int_{0}^{T-s}\exp\left(-\lambda t\right)dt\right\} \\
&= h\left(\lambda\right)\left\{\frac{1}{\lambda}-\left(T-s+\frac{1}{\lambda}\right)\exp\left[-\lambda\left(s-T\right)\right]\right\}=h(\lambda)A_{3}
\end{aligned}$$

Given that we now have estimated λ and valuations, we can now use:

$$vy_{1s}-y_{0s}=g(\lambda_{s})-g'(\lambda_{s})A_{2s}+g'(\lambda_{r})A_{1r}+\epsilon_{s}$$

To estimate the parameters of the monitoring cost function as a simple regression instrumenting for λ_s using previous λs. We then apply a two stage least squares estimator. To be more precise we assume that:

$$h(\lambda)=h(\lambda;\theta)=\sum_{p=1}^{P}\theta_{p}\lambda^{p}$$

It follows then that: We can then re-write:

$$vy_{1s}-y_{0s}=\sum_{p=1}^{P}\theta_{p}\left[\lambda_{s}^{p}\left(A_{3s}-A_{4s}A_{2s}-p\lambda_{s}^{-1}A_{3s}A_{2s}\right)+p\lambda_{r}^{p-1}A_{1r}A_{4r}+\lambda_{r}^{p}A_{1r}A_{3r}\right]+\epsilon_{s} \tag{20}$$

where A_4 is the derivative of A_3.

Substituting out our estimates for v and λ we can estimate θ in the above directly.

9 Appendix B: Estimation

In this appendix we describe in detail the estimators and their asymptotic properties. Our estimators are based on the conditions and nonlinear regression functions that establish identification. We now turn to the four separate steps comprising estimation, which may be loosely described as successively estimating the valuations of players who bid, their monitoring rates, the parent distribution of valuations for all the players in the game whether they bid or not, and the cost function for monitoring bids.

9.1 Valuations

First we estimate the valuations of players who quit the auction by bidding OUTM, and the valuations of players who make interior bidding choices INM. This only leaves players who are observed to always bid ONM, and we show those valuations are bounded. Reweighting the distribution formed from valuations of players who bid OUTM and/or INM at some point during the auction with information about the quantiles of valuations for those bidding ONM, we obtain a consistent estimator of the probability distribution of players who bid.

The nonparametric estimator for $E(M(b_{\tau_j}, h_{j\tau_j})|h_{js}, b_{j\tau_j})$ is the Nadaraya-Watson estimator. See Pagan and Ullah (1999) for a discussion of the properties of these estimators. Similar, estimators have been used by Hotz et al. (1994) for optimal choice probabilities. The estimator is defined as:

$$\widehat{M}(x) = \frac{\sum_{l=1}^{L} \sum_{j=1}^{N_1} K_X \left((x - x_{jt}^l)/\xi_x \right) M(h_T, b_{jt}^l)}{\sum_{l=1}^{L=1} \sum_{j=1}^{N_1} K_X \left((x - x_{jt}^l)/\xi_x \right)}$$

where $x = (b, h, R, q)$ and K_X is a multi dimensional kernel and ξ is the bandwidth.We use a product kernel for the conditioning variables. We estimate $E[b_{j\tau} M(h_T, b_{j\tau})|h_{\tau_j}, b_{\tau_j}]$ by:

$$\widehat{Mb}(x) = \frac{\sum_{l=1}^{L} \sum_{j=1}^{N_1} K_X \left((x - x_{tj}^l)/\xi_x \right) b_{jt}^l M(h_T, b_{jt}^l)}{\sum_{l=1}^{L=1} \sum_{j=1}^{N_1} K_X \left((x - x_{jt}^l)/\xi_x \right)}$$

As noted before we require the derivatives of the above objects. To estimate these derivatives we simply differentiate the above estimators with respect to the current bid (which is a conditioning variable in the conditional expectation). This approach is described in Härdle (1992) and Pagan and Ullah (1999). Details can be found in the appendix. Bandwidths are selected using a rule of thumb as discussed in Bowman and Azzalini (1997).

9.2 Estimating the parent distribution of valuations

The last two steps use the estimates of the first but are otherwise separate from each other. The initial monitoring rates that determine whether a player ever bids or not is the selection mechanism for bidding. Since we have inferred from the second step, and can associate each first bid with a valuation, or at least a band of valuations, we can recover from the distribution of the selected players who bid the probability distribution of valuations for all players.

9.3 Estimating monitoring costs

Here we describe how our point estimates of monitoring intensities and valuations are used in conjunction with the Euler equations that characterize the first order conditions for bidding and monitoring to estimate the monitoring cost function.

First order condition for bidding The bidding FOC is:

$$\begin{aligned} & (v - b) q \frac{\partial}{\partial b} E_s \left[I \left\{ b \geq r_T \right\} M \left(l \right) | b \right] - E_s \left[I \left\{ b \geq r_T \right\} M \left(l \right) | b \right] q \\ = \quad & g(\lambda_s) \frac{\partial}{\partial b} h \left(\lambda_s \left| h_s, b \right. \right) - \frac{\partial}{\partial b} E_s \left[I \left\{ \rho > s \right\} V_{s+1} \left(h_{s+1} \right) | b \right] \end{aligned}$$

Now consider:

$$
\begin{aligned}
& E_s\left[I\left\{\rho > s\right\}V_{s+1}\left(h_{s+1}\right)|b\right] \\
= \quad & E_s\left[I\left\{b < r_T\right\}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)d\tau_{s+1}\,|b\right] \\
= \quad & \Pr\left\{b < r_T\,|h_s, b\right\}\int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)f_{\tau_s^*|b<r_T,h_s}\left(\tau_s^*\,|b\right)d\tau_{s+1}d\tau_s^* \\
\equiv \quad & \left[1-F_{r_T|h_s}\left(b\right)\right]\int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)f_{\tau_s^*|b<r_T,h_s}\left(\tau_s^*\,|b\right)d\tau_{s+1}d\tau_s^*
\end{aligned}
$$

where $F_{r_T|h_s}\left(b\right)$ is the probability distribution function for r_T conditional on h_s and $f_{\tau_s^*|b<r_T,h_s}\left(\tau_s^*\,|b\right)$ is the probability density function for τ_s^* conditional on the event that $r_{\tau_s} < b < r_T$. Differentiating with respect to b yields:

$$
\begin{aligned}
& \frac{\partial}{\partial b}\left[\left[1-F_{r_T|h_s}\left(b\right)\right]\int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)d\tau_{s+1}d\tau_s^*\right] \\
= \quad & -f_{r_T|h_s}\left(b\right)\int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)d\tau_{s+1}d\tau_s^* \\
& +\Pr\left\{b < r_T\,|h_s, b\right\}\int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)\frac{\partial f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)}{\partial b}d\tau_{s+1}d\tau_s^* \\
& +\Pr\left\{b < r_T\,|h_s, b\right\}\int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}\frac{\partial V_{s+1}\left(h_{s+1}\right)}{\partial b}f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)d\tau_{s+1}d\tau_s^*
\end{aligned}
$$

where $f_{r_T|h_s}\left(b\right)$ is the probability density or derivative for $F_{r_T|h_s}\left(b\right)$ evaluated at b. Now consider the other two expressions. First:

$$
\begin{aligned}
& \int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)\frac{\partial f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)}{\partial b}d\tau_{s+1}d\tau_s^* \\
= \quad & \int_{\tau_s}^{T}\int_{\tau_s^*}^{T}\left[\frac{\lambda_s e^{-\lambda_s(\tau_{s+1}-\tau_s)}}{1-e^{-\lambda_s(T-\tau_s^*)}}V_{s+1}\left(h_{s+1}\right)\frac{\partial f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)/\partial b}{f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)}d\tau_{s+1}\right]f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)d\tau_s^* \\
= \quad & E_s\left[I\left\{\rho > s\right\}V_{s+1}\left(h_{s+1}\right)\frac{\partial f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)/\partial b}{f_{\tau_s^*|bh}\left(\tau_s^*\,|b\right)}\,|b\right]
\end{aligned}
$$

Also define:

$$W_T = I\{b_\rho \geq r_T\}(v-b_\rho)M(l)q$$

and let $f_{W_T|h_{s+1},b}\left(W\,|h_{s+1},b\right)$ denote the density function for W_T condtional on $\left(h_{s+1},b\right)$. Then:

$$
\begin{aligned}
V_{s+1}(h_{s+1}) \quad &= \quad E_{s+1}\left[W_T - \sum_{s'=s+1}^{\rho}h(\lambda_{s'}|h_{s'},b_{s'})g(\lambda_{s'})\right] \\
&= \quad \int_0^{(v-b)q}Wf_{W_T|h_{s+1},b}\left(W\,|h_{s+1},b\right)dW - E_{s+1}\left[\sum_{s'=s+1}^{\rho}h(\lambda_{s'}|h_{s'},b_{s'})g(\lambda_{s'})\right]
\end{aligned}
$$

But:

$$
\begin{aligned}
\frac{\partial}{\partial b} E_{s+1}[W_T | h_{s+1}] &= \frac{\partial}{\partial b} \int_0^{(v-b)q} W f_{W_T|h_{s+1},b}\left(W \left| h_{s+1}, b\right.\right) dW \\
&= \int_0^{(v-b)q} \left[W \frac{\partial f_{W_T|h_{s+1},b}\left(W \left| h_{s+1}, b\right.\right)}{\partial b} \right] dW - (v-b)\, q f_{W_T|h_{s+1},b}\left[(v-b)\, q \left| h_{s+1}, b\right.\right] \\
&= E_{s+1}[W_T \frac{\partial f_{W_T|h_{s+1},b}\left(W_T \left| h_{s+1}, b\right.\right)/\partial b}{f_{W_T|h_{s+1},b}\left(W_T \left| h_{s+1}, b\right.\right)} | h_{s+1}] - (v-b)\, q f_{W_T|h_{s+1},b}\left[(v-b)\, q \left| h_{s+1}, b\right.\right]
\end{aligned}
$$

where we should recognize that $f_{W_T|h_{s+1},b}\left[(v-b)\, q \left| h_{s+1}, b\right.\right]$ may have mass (which adds a couple of terms). Similarly for all $s' \in \{s+1, \ldots, \rho\}$ we have:

$$
\begin{aligned}
& \frac{\partial}{\partial b} E_s\left[h(\lambda_{s'} | h_{s'}, b_{s'}) g(\lambda_{s'}) \left| b \right.\right] \\
= \quad & E_s\left[\left\{ h'(\lambda_{s'} | h_{s'}, b_{s'}) g(\lambda_{s'}) + h(\lambda_{s'} | h_{s'}, b_{s'}) g'(\lambda_{s'}) \right\} \frac{\partial f_{\lambda_{s'}|bh}\left(\lambda_{s'} \left| b\right.\right)/\partial b}{f_{\lambda_{s'}|bh}\left(\lambda_{s'} \left| b\right.\right)} \right]
\end{aligned}
$$

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