On The Allocative Effects of Rent Seeking

LUIS C. CORCHÓN
Universidad Carlos III

Abstract

We consider the effects of rent-seeking activities on resource allocation. Before rent-seeking activities take place, there are prior probabilities that an object will be given to one of several agents. The posterior probability depends on prior probabilities and the expenses incurred by all agents. In the case of two agents who equally value the object, prior and posterior probabilities coincide, and thus rent seeking has no effect on resource allocation. If there are two agents with different valuations of the object or we have more than two agents, rent seeking matters and posterior probabilities reflect the valuations of the agents.

1. Introduction

Some years ago, it was found that a firm located in southern Europe engaged in expenditures that may be interpreted as trying to influence the judges of a famous contest for a prize. A friend of mine (a native from the northern country in which the prize is awarded) argued that since all potential recipients have the opportunity to make the same expenditure, such activities do not change the outcome of the process and therefore, prizes given in these circumstances were, after all, fair.

The theory of rent seeking, developed after the seminal paper of Tullock (1980), addresses the question of the characteristics of equilibrium expenses (symmetry, rent dissipation, etc.; see Pérez-Castrillo and Verdier 1992 for a characterization of the equilibrium set). However, to the best of my knowledge, there has been no analysis of the problem of how rent-seeking activities distort the decision of who obtains a prize. For instance, it was found that several cities paid expenses for some members of the

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Department of Economics, Universidad Carlos III, Madrid, C/Madrid 126, Getafe. 28903 Madrid, Spain (lcorchon@eco.uc3m.es).

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Olympic Games Committee. The question is: did these expenses change the odds in favor of one of the contending cities?

In order to investigate this issue, we make several simplifying assumptions. We assume that the number of rent-seeker agents is exogenously given (see Corcoran 1984 for a model where the number of participants is a variable), we assume that information is symmetric, and we disregard all dynamic aspects of the problem (see Cairns 1989). However, our model generalizes the standard rent-seeking model in several aspects: different agents may value the object differently and the probability of obtaining the object is a general function of rent-seeking expenses. We also outline how illegal lobbying can be considered.

With this model at hand we prove that my friend was right in the case in which there are two agents with equal valuations of the object to be allocated (Propositions 1 and 2, part i)). When there are more than two agents or the two agents have different valuations of the object, rent seeking affects the decision on who receives the prize. In the case of two agents with different valuations, the posterior probability that the object is allocated to agent i is larger than the corresponding prior probability if, and only if, the valuation of i is larger than the valuation of j (Proposition 2, part ii). Under an additional assumption on the form in which posteriors are formed, in the case of two or more agents, the ranking of posterior probabilities reflects the ex ante value attached by the agents to the object (Proposition 3). Thus, our results point out that, in certain cases, there is a positive relation between rent seeking and efficiency, a point that should be considered when discussing the social desirability of rent seeking. See Section 4 for further discussion of this point.

The paper ends with an Appendix that discusses the continuity of the function relating rent-seeking expenses and the probability of obtaining the object.

2. The Model with Two Agents

Consider the following situation: an object is going to be allocated to one of two competing agents. Agent i values the object in V_i, $i = 1,2$. When the valuations of both agents are identical we will denote them as V. Before any rent-seeking activity is undertaken, agents have common prior probabilities that the object will end up in the hands of one of them. Let us denote this probability q_i, $i = 1,2$. Prior probabilities are common knowledge. Agents can engage in activities that change these probabilities by means of expenses G_i, $i = 1,2$. The posterior probability that the object will be allocated to agent i is denoted by p_i, $i = 1,2$. This probability depends on prior probabilities and expenses, that is, $p_i = p_i(q, G)$, where q (resp. G) is a vector composed of q_i's (G_i's resp.). For instance,

$$p_i = G_i \cdot q_i \Big/ \left(\sum_{j=1}^{2} G_j \cdot q_j \right), \qquad i,j = 1,2. \tag{1}$$

This corresponds to a much-used specification in the literature on rent-seeking if priors are identical among agents.

The previous model can deal with a variety of situations, but in this paper we want to focus our attention on three specific problems:

(1) The Procurement Problem: The object is a contract to provide a good used by the government (i.e., a fighter aircraft). Rent seekers are the firms supplying the good. A committee takes the decision on the contract, and the members of the committee receive the expenses.

(2) Awarding a Prize: The object is a prize. Rent seekers are the potential recipients. A committee takes the decision on who receives the prize, and the members of the committee receive the expenses.

(3) Elections: The object is to win office. Rent seekers are the political parties. Expenses are advertising. Advertising influences the probabilities of winning office.

The previous model is a noncooperative game in which strategies are expenses and the payoff function for agent $i = 1,2$ is expected profit, that is,

$$\Pi_i = p_i(q, G_1, G_2) \cdot V_i - G_i .$$

Since q and V_i are fixed, the payoff of i can be written as $\Pi_i(G_1, G_2)$, $i = 1,2$.

A Nash equilibrium of the above game is a pair (G_1^*, G_2^*) so that G_i^* maximizes $\Pi_i(G_i, G_j^*)$, $i, j = 1,2$. Since our interest is to analyze the effects of rent seeking we will consider only equilibria in which all agents make positive expenses (we note that asymmetric equilibria with some $G_i^* = 0$ may exist, see Pérez-Castrillo and Verdier 1992, p. 342).

$$\text{Let } p_i^* = p_i(q, G_1^*, G_2^*).$$

Computing the Nash equilibrium of this game for the specific $p_i(\ )$ function given in equation (1) above, assuming that $V_1 = V_2$, we find that

$$G_1^* = G_2^* = q_1 q_2 V \quad \text{and} \quad p_i^* = q_i, \quad i = 1,2.$$

This is interesting on two counts. First, the Nash equilibrium is symmetric in spite of the fact that payoff functions are possibly different (since q_1 and q_2 are not necessarily identical). Second, in this case, rent seeking does not distort resource allocation since posterior probabilities equal prior probabilities. In terms of the examples spelled out above, expenses do not influence who gets the contract/prize or which party wins office. In the rest of this paper we will investigate if this is a general result or not, and in the latter case, what the proper generalization is.

Even though the focus of this paper is not on the existence of a Nash equilibrium, we will first show that if $V_1 = V_2$ there is a symmetric Nash equilibrium. In order to do that we need the following assumptions on $p_1(\ )$:

(a) $p_1(\ )$ is continuously differentiable in G_1 and G_2 for strictly positive values of these variables.

(b) $p_1(\ )$ is concave on G_1 and convex on G_2.

(c) $p_1(\ )$ is homogeneous of degree zero on G_1 and G_2.

(d) $\delta p_1(q, y, y)/\delta G_1$ goes to zero when y goes to infinity, and it goes to a number greater than $1/V$ when y goes to zero.

(e) $p_1(q, G_1, G_2) > q_1$ if and only if $G_1 > G_2$.

Assumptions (a) and (d) are merely technical. Assumption (b) implies that the payoff of agent i is concave on G_i, $i = 1,2$, which is a standard condition in game theory. Assumption (c) says that units in which expenses are measured do not affect p_1. This assumption creates a technical difficulty that is discussed in the Appendix. Assumption (e) says that the only way to increase the posterior probability beyond the prior probability is by making larger expenses than those made by the other firm. Assumptions (a) and (e) imply that $p_1(q, z, z) = q_1$ for any $z > 0$, that is, when both agents make equal expenses, prior and posterior probabilities are identical. The class of functions satisfying (a)–(e) is not empty; see, for example, (1) above. A more general class of functions is the one given by

$$P_1 = G_1^a f_1(q_1) \Big/ \left(\sum_{j=1}^{2} G_j^a f_j(q_j) \right), \qquad 0 < a \leq 1, \tag{1a}$$

where the functions $f_j(\ )$ satisfy $q_1 = f_1(q_1)/(\sum_{j=1}^{2} f_j(q_j))$. Another class of functions satisfying (a) to (e) is

$$P_1 = G_1^a f_1(q_1) \Big/ \left(\sum_{j=1}^{2} G_j f_j(q_j) \right)^a, \qquad 0 < a \leq 1, \tag{1b}$$

with the same restriction on the functions $f_j(\ )$.

PROPOSITION 1: *Under (a)–(d) above if $V_1 = V_2$, there is a Nash equilibrium with $G_1^* = G_2^*$. If (e) also holds, $p_i^* = q_i$, $i = 1,2$.*

Proof: Let x^* be a solution of the equation

$$\delta p_1(q, x, x)/\delta G_1 = 1/V.$$

The existence of x^* is guaranteed by (a) and (d). Then,

$$V \cdot \delta p_1(q, x^*, x^*)/\delta G_1 - 1 = 0,$$

which is the first order condition for maximization of $\Pi_1(\ )$ with respect to G_1. Since $\Pi_1(\ )$ is concave on G_1, this implies that x^* maximizes $\Pi_1(G_1, x^*)$ with respect to G_1. Now, from (c)

$$x^* \cdot \delta p_1(q, x^*, x^*)/\delta G_1 + x^* \cdot \delta p_1(q, x^*, x^*)/\delta G_2 = 0,$$

which, since $x^* > 0$, implies that

$$\delta p_1(q, x^*, x^*)/\delta G_1 = -\delta p_1(q, x^*, x^*)/\delta G_2.$$

Thus,

$$-V \cdot \delta p_1(q, x^*, x^*)/\delta G_2 = 1.$$

Since $p_1(\ )$ is convex on G_2, x^* maximizes

$$(1 - p_1(q, x^*, G_2)) \cdot V - G_2 = \Pi_2(G_2, x^*)$$

with respect to G_2. Thus $G_1 = G_2 = x^*$ is a Nash equilibrium, and by (a) and (e) $p_i^* = q_i$, $i = 1,2$. ■

The next proposition provides a converse to Proposition 1 and also deals with the case in which the V_is are not identical.

PROPOSITION 2: *At any interior Nash equilibrium:*

(i) *Under (a) and (c), $G_1^* = G_2^*$ if and only if $V_1 = V_2$. If (e) also holds, $p_i^* = q_i$, $i = 1,2$, if and only if $V_1 = V_2$.*

(ii) *Under (a) and (c), $G_1^* > G_2^*$ if and only if $V_1 > V_2$. If (e) also holds, $p_1^* > q_1$ if and only if $V_1 > V_2$.*

Proof: From the first order conditions of a Nash equilibrium,

$$V_1 \cdot \delta p_1(q, G_1^*, G_2^*)/\delta G_1 - 1 = 0 = -V_2 \cdot \delta p_1(q, G_1^*, G_2^*)/\delta G_2 - 1.$$

From (c) we have that

$$G_1^* \cdot \delta p_1(q, G_1^*, G_2^*)/\delta G_1 + G_2^* \cdot \delta p_1(q, G_1^*, G_2^*)/\delta G_2 = 0.$$

From the first order conditions of a Nash equilibrium it follows that $\delta p_1(q, G_1^*, G_2^*)/\delta G_1 > 0$. Thus, from the last two equations we find that $G_1^*/G_2^* = V_1/V_2$. Thus, $G_1^* >$ (resp. $=$) G_2^* if and only if $V_1 >$ (resp. $=$) V_2. Finally, $p_1^* >$ (resp. $=$) q_1 if and only if $V_1 >$ (resp. $=$) V_2 follows from (e). ■

If $V_1 = V_2$, all firms would be better off if rent seeking were effectively banished since they incur expenditures just to maintain the probability of getting the object. However, if V_1 is much greater than V_2, firm 1 may be better off under rent seeking than under no rent seeking. For instance, in the case in which the functional form (1) holds, the expected profits of firm 1 in the Nash equilibrium are $q_1^2 \cdot V_1^3/(\sum_{j=1}^{2} V_j \cdot q_j)^2$. Since expected profits under no rent seeking are $q_1 \cdot V_1$, it is easily calculated that the former are greater than the latter if and only if $V_1 > V_2 \cdot (1 + (1/q_1)^{1/2})$.

Propositions 1 and 2 can be worked out with similar conclusions in the case in which lobbying is illegal. In this case, if the lobbying activity of agent i is exposed, this agent is fined with a quantity F_i that depends on G_i, that is, $F_i = F(G_i)$. Let r_i be the probability that the lobbying is discovered. This probability is a function of G_i, that is, $r_i = r(G_i)$. Let $L_i = r(G_i) \cdot F(G_i) = L(G_i)$ be the expected loss of i if her lobbying activities are exposed. Expected payoffs of i are

$$\Pi_i(G_i, G_j) \cdot r(G_i) + \Pi_i(G_i, G_j) \cdot (1 - r(G_i)) - L(G_i) = \Pi_i(G_i, G_j) - L(G_i).$$

Propositions 1 and 2 can be proved by adapting Assumptions (a)–(e) and assuming that $L_i(\ )$ is differentiable and convex.

3. The Model with an Arbitrary Number of Agents

In the case of more than two agents, Propositions 1 and 2 no longer hold. For instance, consider the generalization of the functional form given by (1) above to the case of n agents,

$$p_i = G_i \cdot q_i \Big/ \left(\sum_{j=1}^{n} G_j \cdot q_j \right), \quad i, j = 1, \ldots, n. \tag{1'}$$

We find that the Nash equilibrium of the game with n agents with identical valuations is[1]

$$p_i^* = 1 - (n-1) \Big/ q_i \left(\sum_{j=1}^{n} 1/q_j \right).$$

In general, p_i^* and q_i will be different (they are equal if priors are identical). For instance, if $q = (0'375, 0'375, 0'25)$ we find that $p^* = (0'43, 0'43, 0'14)$. However, under (1′), it is easily seen that $p_i^* >$ (resp. $=$) p_j^* if and only if $q_i >$ (resp. $=$) q_j. In other words, rent-seeking expenses do not change the ranking of probabilities. We will show that a generalization of this property holds when agents have different valuations and probability functions are of the following form:

$$p_i = p(G_i \cdot q_i, \Psi(G_1 \cdot q_1, G_2 \cdot q_2, \ldots, G_n \cdot q_n)), \; i = 1, \ldots, n. \tag{2}$$

Let $x \equiv G_i \cdot q_i$ and $z \equiv \Psi(G_1 \cdot q_1, G_2 \cdot q_2, \ldots, G_n \cdot q_n)$. Then,

$$\partial p_i / \partial G_i = q_i (\partial p / \partial x + \partial p / \partial z \cdot \partial \Psi / \partial x).$$

Let us write the right hand side of the above equation as

$$q_i \cdot F(G_i \cdot q_i, \Psi(G_1 \cdot q_1, G_2 \cdot q_2, \ldots, G_n \cdot q_n)) \equiv q_i \cdot F(G_i \cdot q_i, z).$$

[1]We assume that $q_i \geq (n-2)/\sum_{j \neq i} 1/q_j$, $i = 1, \ldots, n$. This assumption holds if a priori probabilities do not differ too much from each other.

Now, we assume the following:

(f) $F(\ )$ is decreasing on G_i, given z, and $p(\ )$ is increasing on G_i, given z.

An example of a functional form like (2) satisfying (f) is the following (which generalizes (1′) above):

$$p_i = f(G_i \cdot q_i) \Big/ \sum_{j=1}^{n} f(G_j \cdot q_j), \quad i, j = 1, \ldots, n,$$

where $f(\ )$ is strictly increasing, differentiable, and concave with non-negative values. Taking $z \equiv \sum_{j=1}^{n} f(G_j \cdot q_j)$, it is clear that $p_i = f(G_i \cdot q_i)/z$, and thus $p_i(\ )$ is increasing on G_i, given z. Also,

$$\partial p_i/\partial G_i = V_i\, q_i \cdot f'(G_i \cdot q_i) \cdot \left(\sum_{j \neq i} f(G_j \cdot q_j) \Big/ \left(\sum_{j=1}^{n} f(G_j \cdot q_j \right) \right)^2$$

$$= V_i\, q_i \cdot f'(G_i \cdot q_i) \cdot (z - f(G_i \cdot q_i)/(z)^2$$

(derivatives are denoted by primes). Thus, under the above conditions on $f(\ )$, $F(\ )$ is decreasing on G_i, given z.

PROPOSITION 3: *Suppose that* (2) *and* (*f*) *hold. Then, for any i and k with* $G_i^* > 0$ *and* $G_k^* > 0$,

(i) $p_i^* = p_k^*$ *if and only if* $V_i \cdot q_i = V_k \cdot q_k$.
(ii) $p_i^* > p_k^*$ *if and only if* $V_i \cdot q_i > V_k \cdot q_k$.

Proof: Consider the first order condition of payoff maximization for i:

$$V_i \cdot \partial p_i / \partial G_i - 1 = 0.$$

Or equivalently,

$$V_i \cdot q_i \cdot F(G_i^* \cdot q_i\,, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n{}^* \cdot q_n)) - 1 = 0.$$

Also, for agent k,

$$V_k \cdot q_k \cdot F(G_k^* \cdot q_k, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n^* \cdot q_n)), - 1 = 0.$$

From the last two equations, it follows that

$$V_i \cdot q_i \cdot F(G_i^* \cdot q_i, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n^* \cdot q_n))$$

$$V_k \cdot q_k \cdot F(G_k^* \cdot q_k, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n^* \cdot q_n)).$$

Clearly, if $V_i \cdot q_i = V_k \cdot q_k, G_i^* \cdot q_i = G_k^* \cdot q_k$, and thus $p_i^* = p_k^*$. Conversely, if $p_i^* = p_k^*$, (2) implies that $G_i^* \cdot q_i = G_k^* \cdot q_k$ and the above equations imply that $V_i \cdot q_i = V_k \cdot q_k$.

Suppose now that $V_i \cdot q_i > V_k \cdot q_k$. Thus

$$F(G_i^* \cdot q_i, \Psi(G_1^* \cdot q_1, G_2{}^* \cdot q_2, \ldots, G_n^* \cdot q_n))$$

$$< F(G_k^* \cdot q_k, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n^* \cdot q_n)),$$

and $G_i^* \cdot q_i > G_k^* \cdot q_k$ follows from $F(\ )$ decreasing on G_i^*. Thus, $p_i^* > p_k^*$.

Finally, suppose that $p_i^* > p_k^*$. Then, (2) implies that $G_i^* \cdot q_i > G_k^* \cdot q_k$. Plugging this inequality into the first order conditions of payoff maximization,

$$V_i \cdot q_i \cdot F(G_i^* \cdot q_i, \Psi(G_1^* \cdot q_1, G_2{}^* \cdot q_2, \ldots, G_n^* \cdot q_n))$$

$$= V_k \cdot q_k \cdot F(G_k^* \cdot q_k, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n^* \cdot q_n))$$

$$< V_k \cdot q_k \cdot F(G_i^* \cdot q_i, \Psi(G_1^* \cdot q_1, G_2^* \cdot q_2, \ldots, G_n^* \cdot q_n)),$$

which implies that $V_i \cdot q_i > V_k \cdot q_k$. ■

Notice that Proposition 3 also holds in the case of two agents.

4. Discussion of Results

In this paper we have identified a case in which rent-seeking activities do not matter in terms of resource allocation: two agents with identical valuation of the object. When there are more agents, or two agents value the object differently, rent seeking makes a difference. In the first case, posterior probabilities reflect a priori expected valuations, that is, $p_i^* \geq p_k^*$ if and only if $V_i q_i \geq V_k q_k$. In the second case, the increase in posterior probabilities with respect to priors reflect valuations, that is, $V_1 > V_2$ if and only if $p_1^* > q_1$. In both cases, rent-seeking activities increase the chance that the agents who value the object most (in real or in expected terms) get the object. The interpretation of this fact in the three examples spelled out above is, however, very different. In the procurement problem we may say that rent seeking is socially beneficial because it makes the allocation of the object more efficient.[2] In the case of the prize, rent seeking makes it more likely that candidates with high valuations (not necessarily the best) win the prize. Finally, in the case of political competition, advertisement favors parties with very high valuation for office, that is, either corrupt or very patriotic political parties.[3]

[2] The notion of efficiency (ex ante or ex post) used here depends on whether we refer to Proposition 2 (ex post) or to Proposition 3 (ex ante).

[3] In this discussion we have neglected the negative effects (on, say, public morality) of the transfers of money between candidates and committee members.

Appendix: Discussion of the Function $p_i(\ )$.

The assumption that the function $p_i(\ )$ is homogeneous of degree zero creates a difficulty: if $p_i(\ )$ is continuous at zero expenses by both firms, $p_i(\ )$ must be constant on G_1/G_2. Indeed, fix any G_1/G_2. Take a sequence of G_1 and G_2 where both tend to zero but the ratio remains constant. By homogeneity of degree zero the value of the function $p_i(\ )$ remains constant in the sequence and by continuity this value, say, a, must be the value of the function in the limit. Consider now other G_1/G_2 and take a sequence with the same properties as the one before. By the same argument, the value of the function in the sequence must be the value in the limit, and this must be a. Thus the function is constant on G_1/G_2. Since this is absurd, we must admit that the function $p_i(\ )$ is either discontinuous or undefined on (0, 0). The latter occurs in the case where $p_i = G_i \cdot q_i/(\sum_j G_j \cdot q_j)$, $i, j = 1,2$ (equation (1) in the text). It is clear that the function above can be redefined to take an arbitrary value, say, q_i, when $G_1 = G_2 = 0$.

The fact that the probability function $p_i(\ )$ is discontinuous at (0, 0) says that a little rent seeking does indeed make a difference when compared with the situation where there is no rent seeking–an intriguing fact.

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