UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 10-01
Economic Series (01)
January 2010

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 916249875

PRICE REGULATION IN OLIGOPOLY[1]

Luis C. Corchón

Departamento de Economía,

Universidad Carlos III, Madrid, Spain

Félix Marcos

Departamento de Fundamentos del Análisis Económico.

Universidad Complutense de Madrid, Madrid, Spain

Abstract. *In this paper we consider price regulation in oligopolistic markets when firms are quantity setters. We consider a market for a homogeneous good with a special form of the demand function (ρ-linearity), constant returns to scale and identical firms. Marginal costs can take two values only: low or high. The regulator knows all parameters except marginal costs. Assuming that the regulator is risk neutral, we characterize the optimal policy and show how this policy depends on the basic parameter of demand and costs.*

[1] We thank Carmen Beviá, Ángeles de Frutos and Natalia Fabra for very useful comments. The first author acknowledges financial support from SEJ2005-06167/ECON.

1: INTRODUCTION

One of the main causes of market failure is lack of competition. Thus, a completely informed and benevolent regulator can find an allocation in which all agents are better off than under oligopolistic competition. A great deal of attention has been devoted to the case in which the regulator lacks information to implement efficient allocations and a single firm supplies the whole market (see the survey by Armstrong and Sappington (2005) and the references therein). In contrast, the case of regulation under oligopoly has been relatively neglected: All the papers we are aware of are generalizations of optimal regulatory schemes proposed for the case of monopoly: Shaffer (1989) and Kim and Chang (1993) generalized the work of Loeb and Magat (1979), Schwermer (1994) and Lee (1997) generalized the work of Sappington and Sibley (1988) and López-Cuñat (1995) generalized the work of Baron and Myerson (1982).

In this paper we study the performance of a particular mechanism, price regulation, under oligopolistic competition and compare its welfare properties with those of oligopolistic competition, also referred to in the sequel as the free market. The motivation for our study is that price regulation is (or has been) used to regulate oligopolistic markets like gasoline, natural gas, electric power generation, telecommunications, health care, pharmaceuticals, etc. Again, the initial work in this problem was done in the case of a regulated monopoly, see Littlechild (1986). Price regulation is presented in textbooks as an inefficient mechanism under perfect competition. In this paper we attempt to bridge monopoly and perfect competition by considering the intermediate case, namely, oligopoly. We will see that, in some cases, price regulation is more efficient than the free market.

In order to keep the story as simple as possible we assume a very stylized scenario which is presented in Section 2. The product is homogeneous. The unique consumer has a ρ-linear demand curve, which is a generalization of linear and isoelastic forms, see Anderson and Renault (2003), which is known by the regulator. The industry is composed of a fixed number of identical firms with constant marginal costs, i.e. we assume that costs are perfectly correlated (see Armstrong, Cowan and Vickrey (1994)). The assumption that firms are identical is useful for separating the problem of regulating a market where several firms have inefficient technologies from the problem of pure regulation of oligopoly. The marginal cost can take two values, high and low. The probabilities of occurrence of these two values are known by the risk-neutral regulator who maximizes expected social welfare. The free market is represented by Cournot equilibrium where firms know the true value of the marginal cost.

In Section 3, we show that the optimal policy is either the free market or a regulated price equal to one the two values of the marginal cost (Lemma 1). This is an interesting result from the normative point of view (i.e. how price regulation should be done) because it simplifies the task of maximizing expected social welfare.

In sections 4 and 5 we study how the optimal policy depends on the underlying parameters. Instead of comparing expected social welfare under free market and regulation we compare percentages of welfare losses. This is because the percentage of welfare loss under the free market does not depend on both the marginal costs and the intercept of the demand function so the analysis is simplified greatly (Lemma 2).

We show that the optimal policy depends on four variables: The number of firms, the parameter ρ of the demand function, the parameter β related with the ratio between high and low marginal costs and the parameter q which is the ratio of probabilities of occurrence of high versus low marginal costs. As we expect, the desirability of regulation decreases with the number of competitors, see Remark 1. Also for very small or very high values of q, regulation is the optimal policy because in this case the regulator faces very little uncertainty (Proposition 1). When the high and the low marginal costs are similar, so β is close to one, the optimal policy is to set a regulated price equal to the value of the large marginal cost. This is because this choice yields almost as much social welfare as the low marginal cost and avoids the risk that firms do not produce. Contrarily, when both marginal costs are sufficiently far apart, the optimal policy is to set a regulated price equal to the low value of the marginal cost because this option yields great social welfare. For intermediate values of β the free market may or may not be the optimal policy (Propositions 2 and 3). However β also depends on the intercept of the demand function. When this intercept is close to the high value of the marginal cost, the best policy is a regulated price equal to the low value of the marginal cost since, if costs are high, the loss is small. When the intercept is very large the best policy is a regulated price equal to the high value of the marginal cost in order to secure positive output whatever happens. Finally we study the shape of the optimal policy with respect to ρ. Unfortunately, in this case there is no clear cut result except for values of ρ close to zero or to infinite (Proposition 4). In the first case the optimal policy is to regulate with the regulated price equal to the low level of the marginal cost since demand is almost rigid. In the second case regulation is the optimal policy with the regulated price equal to the high value of the marginal cost since demand is very elastic.

In Section 6 we tackle the case where the regulator, instead of setting a price sets a ceiling for the price, i.e., a price cap. We present a necessary and sufficient condition for the price caps to be equal to the regulated price considered before (Proposition 5). This condition implies that the variability of costs is not very large relatively to ρ and the number of firms. This condition does not look unreasonable in some instances.

Our results can be used to judge the success (or lack of) of price regulation as done in practice. In particular they stress the importance of having a good estimate of the intercept of the demand function and ρ. Since these magnitudes cannot be discerned by looking at points around equilibrium, price regulation in practice might be more difficult than presented in this paper.

Finally, section 6 gathers our final comments on the limitations of our work and mentions some possible extensions.

2. THE MODEL

There is a representative consumer with a utility function $U = Ax - bx^{\rho+1}/(\rho+1) - px$ where x is aggregate output and p is the market price. We assume $b\rho > 0$ and $\rho > -1$.[2] Maximization of utility yields an inverse demand function $p = A - bx^{\rho}$. Because $b\alpha > 0$, p is decreasing on x. Notice that if $\rho < 0$, $b < 0$ and $A = 0$ we have an isoelastic demand function $p = -bx^{\rho}$ and if $\rho = 1$ demand is linear. There are n identical firms producing a single output denoted by x_i. Thus, $x = \sum_{i=1}^{n} x_i$. Marginal costs are constant and denoted by c. Defining $a = A - c$, profits for firm i can be written as $B_i = (a - bx)x_i$. Assume $ab > 0$, which implies that positive outputs are possible. If firms are quantity setters, it is easy to check that second order conditions of profit maximization hold and that equilibrium is symmetric. Thus, first order conditions of profit maximization yield the Cournot equilibrium outputs and profits, denoted by the superscript c, namely.

$$x^C = \left[\frac{an}{b(n+\rho)}\right]^{\frac{1}{\rho}}; \qquad B_i^C = b\rho n^{\frac{1-\rho}{\rho}}\left[\frac{a}{b(n+\rho)}\right]^{\frac{1+\rho}{\rho}} \quad . \tag{1}$$

Social welfare, denoted by W, is the sum of profits and the utility of the representative consumer, i.e.

$$W = U + \sum B_i = ax - \frac{bx^{\rho+1}}{\rho+1}.$$

Social welfare in the Cournot equilibrium, denoted by W^C, is,

$$W^C = \frac{a^{\frac{1+\rho}{\rho}}\rho n^{\frac{1}{\rho}}(n+\rho+1)}{b^{\frac{1}{\rho}}(n+\rho)^{\frac{1+\rho}{\rho}}(\rho+1)}. \tag{2}$$

[2] ρ must be larger than *-1* because if $\rho < -1$ utility tends to minus infinite if x tends to zero.

In an optimal allocation, social welfare is maximized. Since the second order condition holds and the allocation is symmetric, the first order condition of social welfare maximization yields the optimal allocation, namely

$$x^O = \left(\frac{a}{b}\right)^{\rho}, \quad W^O = \frac{\rho a^{\frac{1+\rho}{\rho}}}{b^{\frac{1}{\alpha}}(1+\rho)}. \tag{2'}$$

Define the percentage of welfare loss, denoted by *PWL*, of an allocation yielding, say, social welfare *W'*, as follows:

$$PWL = \frac{W^O - W'}{W^O}.$$

The percentage of welfare loss under Cournot competition, denoted by PWL^C, is

$$PWL^C = 1 - \frac{n^{\frac{1}{\rho}}(n+\rho+1)}{(n+\rho)^{\frac{1+\rho}{\rho}}} = PWL^C(\rho, n). \tag{3}$$

Plotting equation (3) for given *n*, it is seen that PWL^C is non monotonic in ρ. It is easily seen that $dPWL^C / dn = -(1+\rho)n^{\frac{1-\rho}{\rho}}(n+\rho)^{-2\frac{1+2\rho}{\rho}} < 0$ so PWL^C is decreasing in *n*.

3. REGULATION

We assume that the regulator controls the market price. The regulator is uncertain of the value of marginal cost which can take two values: *y* with probability π and *z* with probability $1-\pi$. We will refer to these values as “states of the world”. Once the regulated price, *p,* is set, firms decide output. If *p* is smaller than the actual value of the marginal cost, firms do not produce and the social welfare is zero. If *p* is larger than or equal to the actual value of marginal cost, they produce an identical quantity each $x_i = x/n$ that satisfies demand $x = ((A-p)/b)^{\frac{1}{\rho}}$. Thus, social welfare in state $c = y, z$ with regulated price *p* is

$$W^R = Ax - \frac{bx^{\rho+1}}{\rho+1} - cx = \left(\frac{A-p}{b}\right)^{\frac{1}{\rho}}\left(\frac{\rho A + p - (1+\rho)c}{1+\rho}\right) \text{ if } x > 0.\ W^R = 0 \text{ otherwise.}$$

We assume that the regulator is risk neutral and maximizes expected social welfare. Firstly we show that the regulator chooses either $p = y$ or $p = z$.

Lemma 1. *A regulator maximizing expected social welfare chooses a regulated price equal to y or to z.*

Proof. Let *U(x)* be the utility function of the consumer and *x(p)* is demand at price *p*. Thus, if $x > 0$ social welfare can be written as $W = U(x(p)) – cx(p)$. Differentiating the previous equation we easily see that if $p > c$ social welfare is decreasing in *p*.
Suppose that $p > y$. Expected social welfare can be increased by decreasing *p* so this price cannot be optimal. Suppose now that $z < p < y$. In this case, as before, a decrease in the regulated price has no effect on social welfare if $c = y$ but it increases social welfare if $c = z$. Therefore, again a price decrease improves expected social welfare. Since the regulator will never choose a price below *z*, the lemma is proved.■

The intuition behind Lemma 1 is simple. The regulator wants prices to be as low as possible. However, if she chooses too low a price she risks that the industry will not produce at all. Thus, if the regulator chooses a price that is larger than *y* (resp. *z*) she bears as much risk of zero output as choosing $p = y$ (resp. *z*) but if costs are *y* (resp. *z*) she is not doing as good as well as she can by setting the price equal to *y* (resp. *z*).

Lemma 1 implies that the regulated price is either *y* or *z*. These two magnitudes are defined as follows:

$$EW^{Y} = \frac{\rho\pi}{(1+\rho)b^{\frac{1}{\rho}}}(A-y)^{\frac{1+\rho}{\rho}} + (1-\pi)\left(\frac{A-y}{b}\right)^{\frac{1}{\rho}} \frac{\rho(A-z)-(A-y)+(A-z)}{1+\rho} \tag{4}$$

$$EW^{Y} = (1-\pi)\left(\frac{A-z}{b}\right)^{\frac{1}{\rho}} \frac{\rho(A-z)}{1+\rho} = \frac{\rho(1-\pi)}{b^{\frac{1}{\rho}}(1+\rho)}(A-z)^{\frac{1+\rho}{\rho}}. \tag{5}$$

The regulator chooses $p = y$ if $EW^Y > EW^Z$ and $p = z$ otherwise. The regulator is indifferent between both prices if $EW^Y = EW^Z$. Define $a_H = A - z$, $a_L = A – y$ and $\beta = a_H / a_L$. Notice that $\beta > 1$ if $\rho > 0$ and $0 < \beta < 1$ if $\rho < 0$ (the case of $\beta = 1$, namely $y = z$, is trivial and it is not considered). With this notation in hand, $EW^Y = EW^Z$ can be written as $\pi\rho + (1-\pi)(\rho\beta - 1 + \beta) = \rho(1-\pi)\beta^{\frac{1+\rho}{\rho}}$. Defining $q = \pi / 1 - \pi$, the regulator is indifferent between setting the price at *y* or at *z* if and only if

$$q = \frac{\rho\beta\left(\beta^{\frac{1}{\rho}} - 1\right) - \beta + 1}{\rho}. \tag{6}$$

4. COMPARING THE FREE MARKET AND REGULATION

In order to compare the expected welfare under free market (e. g., Cournot competition) and in the social optimum we will find it more convenient to work in terms of percentages of welfare losses. As we will see, this greatly simplifies our analysis.

From equations (4) and (5) we calculate the percentage of the expected welfare losses from price regulation. The optimal expected welfare, denoted by EW^0, is the expected social welfare obtained if for each state of the world, the planner had complete information, i.e. when $c = y$ (resp. $c = z$) the regulated price would be y (resp. z). Thus,

$$EW^0 = \frac{\rho}{b^{\frac{1}{\rho}}(1+\rho)}\left(\pi a_L^{\frac{1+\rho}{\rho}} + (1-\pi)a_H^{\frac{1+\rho}{\rho}}\right). \tag{7}$$

Next, we notice that the percentage of expected welfare losses from Cournot equilibrium equals the percentage of welfare losses obtained in (3) above. The intuition behind this result is that the parameter a, which encapsulates all the uncertainties faced by the regulator, enters multiplicatively the welfare both at Cournot and the optimal allocation (see equations (2) and (2')). Consequently, a cancels out in the formula of the welfare losses percentage which does not depend on the state of the world. We record this result as Lemma 2 below.

Lemma 2. *The percentage of expected welfare loss under Cournot competition equals the percentage of welfare loss.*

The proof is left for the reader. From now on and abusing notation we will denote the percentage of expected welfare loss under Cournot competition by PWL^C. We are now ready to compare free market and price regulation.

1) Consider first that the regulator chooses $p = y$. We will compare the percentage of welfare losses from price regulation when $p = y$, denoted by PWL^Y, with the welfare losses under Cournot competition.

$$PWL^Y = \frac{EW^0 - EW^Y}{EW^0} = \frac{\rho\beta^{\frac{1+\rho}{\rho}} - \rho\beta - \beta + 1}{\rho\left(q + \beta^{\frac{1+\rho}{\rho}}\right)} = PWL^Y(\rho, \beta, q) \tag{8}$$

$PWL^Y(\bullet)$ is increasing in ρ and β if $\rho > 0$ (decreasing if $\rho < 0$) and decreasing in q (see Appendix). Comparing (8) with the welfare losses under Cournot equilibrium, easy calculations show that $PWL^C = PWL^Y$ iff

$$q = \frac{(\rho\beta + \beta - 1)(n+\rho)^{\frac{1+\rho}{\rho}} - \rho\beta^{\frac{1+\rho}{\rho}}(n+\rho+1)n^{\frac{1}{\rho}}}{\rho n^{\frac{1}{\rho}}(n+\rho+1) - \rho(n+\rho)^{\frac{1+\rho}{\rho}}}. \tag{9}$$

2) Now consider that the regulator chooses $p = z$. The expected welfare is

$$EW^Z = (1-\pi)\left(\frac{A-z}{b}\right)^{\frac{1}{\rho}} \frac{\rho(A-z)}{1+\rho} = \frac{(1-\pi)\rho}{b^{\frac{1}{\rho}}(1+\rho)} a_H^{\frac{1+\rho}{\rho}}. \tag{10}$$

The percentage of welfare losses from price regulation when $p = z$, denoted by PWL^Z is

$$PWL^Z = 1 - \frac{(1-\pi)a_H^{\frac{1+\rho}{\rho}}}{\pi a_L^{\frac{1+\rho}{\rho}} + (1-\pi)a_H^{\frac{1+\rho}{\rho}}} = \frac{q}{q+\beta^{\frac{1+\rho}{\rho}}} = PWL^Z(\rho,\beta,q). \tag{11}$$

PWL^Z is decreasing in β if $\rho > 0$ (increasing if $\rho < 0$), increasing in ρ if $\rho > 0$ (decreasing if $\rho < 0$) and increasing in q. Similar to case 1) above, the value of q equalizing welfare losses under regulation with $p = z$ and Cournot equilibrium, is

$$q = \frac{\beta^{\frac{1+\rho}{\rho}}\left[(n+\rho)^{\frac{1+\rho}{\rho}} - n^{\frac{1}{\rho}}(n+\rho+1)\right]}{n^{\frac{1}{\rho}}(n+\rho+1)}. \tag{12}$$

Notice that when β tends to 1, PWL^Y tends to zero but PWL^Z tends to $q/(q+1)$. The reason for that is the existence of a discontinuity: if the state of the world is y but the regulator set z there is no production so the welfare loss is positive even for $y \simeq z$. However, if the regulator set y the welfare loss tends to zero when y tends to z.

Summing up, our main insight in this section is that, in order to find the optimal policy, a regulator has to look at three equations only, namely (3), (8) and (11).

5. THE OPTIMAL POLICY: COMPARATIVE STATICS

In this section we characterize the values of ρ, β, n and q for which regulation or the free market are the optimal choices. We have two kinds of results: results that do not depend on the sign of α and results that depend on this sign.[3] In the first class we have the characterization of the optimal policy when n and q vary. In the second class we have the characterization of the optimal policy when α and β vary. Firstly, we notice the following remark:

Remark 1. *Given ρ, β, and q there is a n^1 such that if $n \in (1, n^1]$ the optimal policy is regulation (either with $p = z$ or with $p = y$). If $n > n^1$ the optimal policy is free market.*

The remark follows from the fact that, as we remarked after equation (3), the welfare loss under Cournot competition decreases with n and tends to zero when n tends to infinite, and that the expected welfare loss under regulation does not depend on n. This remark is just a re-statement of the well known fact that under identical firms with no economies of scale, the more competition in the market, the better. Clearly, when n^1 is 1, the interval $(1, n^1]$ is empty and free market is always the optimal policy.

Now we turn to the study of the optimal policy when q varies. For given values of n, ρ and β denote by q^1 the value of q in equation (6). Similarly, define q^2 and q^3 as the values of q at equations (9) and (12) respectively.

Proposition 1. *Let ρ, β, and n be given. Suppose that $q^2 \geq q^3$. If $q \in (0, q^3]$, the optimal policy is regulation with $p = z$. If $q \in [q^3, q^2]$, the optimal policy is free market. If $q \in$*

[3] Recall that when $\alpha > 0$ the demand function is concave or convex depending on α being larger or smaller than one, respectively. When $\alpha < 0$ the demand function is convex.

$[q^2, \infty)$ the optimal policy is regulation with $p = y$. Suppose that $q^2 < q^3$. If $q \in (0, q^1]$,

the optimal policy is regulation with $p = z$. If $q \in [q^1, \infty)$ the optimal policy is regulation

with $p = y$.

Proof. Notice the following facts**:**

a) $PWL^Y(\rho, \beta, \bullet)$ is decreasing in q and tends to zero when q tends to infinite.
b) PWL^Z $PWL^Z(\rho, \beta, \bullet)$ is increasing in q, tends to zero when q tends to zero and tends to one when q tends to infinite.
c) PWL^C is larger than zero and does not depend on β.

Notice that a) and b) above imply that q^1 and q^3 exist. Then, the proposition follows from a), b) and c) by noticing that the values of q are defined by the equality between PWL at regulated prices y or z and the Cournot equilibrium. ■

The intuition of Proposition 1 is as follows. When q is small enough, the optimal policy is regulation with $p = z$ because the probability of $c = z$ is very large so the welfare loss of regulation is negligible because the case $c = y$, seldom occurs (PWL^Z is close to zero). When q increases, on the one hand, PWL^Z increases because the probability of $c = y$ increases (in this case there is no production), and on the other hand PWL^Y decreases for the same reason. Thus for the intermediate value theorem there is a value of q (q^1) for which $PWL^Z = PWL^Y$. If for q^1, $PWL^Y < PWL^C$ regulation is always better because PWL^Y is decreasing in q and tends to zero when q tends to infinite. Contrarily, if for q^1, $PWL^Y > PWL^C$, there is an interval $[q^2, q^3]$ where free market is the optimal policy.

We now turn our attention to the shape of the optimal policy when β or ρ vary. In this case we have to consider two cases: when ρ is positive and β lies between one and infinite and when ρ is negative and β lies between zero and one.

Take ρ and q as given. Let β^1 be the value of β solving $PWL^Y(\rho, \beta, q) = PWL^Z(\rho, \beta, q)$, i.e. it solves equation (6). Let β^2 be the value of β solving $PWL^C(\rho, n) = PWL^Y(\rho, \beta, q))$, i.e. it solves equation (9). β^1 and β^2 exist and are unique (see the proof of the next proposition). Let β^3 be the value of β that solves PWL^C $(\rho,\ n) = PWL^Y,\ (\rho,\ \beta,\ q)$ i.e. it solves equation (12). If such a value exists it is unique. If this value does not exists set $\beta^3 = 1$ (the lower bound for β). Now we are prepared to prove our next result.

Proposition 2. *Suppose that $\rho > 0$, n and q are given. Suppose that $\beta^3 > \beta^2$. If $\beta \in (1, \beta^2]$ the optimal policy is regulation with $p = y$. If $\beta \in [\beta^2, \beta^3]$, the optimal policy is free market. If $\beta \in [\beta^3, \infty)$, the optimal policy is regulation with $p = z$. Suppose that $\beta^2 > \beta^3$. If $\beta \in (1, \beta^1]$, the optimal policy is regulation with $p = y$. If $\beta \in [\beta^1, \infty)$, the optimal policy is regulation with $p = z$.*

Proof. Notice the following facts**:**

d) PWL^Y tends to zero if β tends to one. $PWL^Y(\rho,\ \bullet,\ q)$ is increasing on β and tends to one if β tends to infinite.
e) PWL^Z tends to $q/(q+1)$ if β tends to one. $PWL^Z(\rho,\ \bullet,\ q)$ is decreasing on β and tends to zero if β tends to infinite.
f) PWL^C does not depend on β.

Notice that a) and b) above imply that β^1 and β^2 exist. The proposition then follows from a), b) and c) by noticing that the values of β are defined by the equalities between *PWL* at regulated prices *y* or *z* and the Cournot equilibrium. ■

The intuition of Proposition 2 is as follows. When β is close to *1*, regulation is better than free market because the regulator faces very little uncertainty since *y* and *z* are almost identical so the expected welfare loss of regulation tends to zero. In this case the regulator sets $p = y$, because the welfare loss of setting *y* when costs are *z* is very small compared with the loss of setting z when the cost is y (in the later firms will not produce). When β is large enough, regulation is better with $p = z$ because when y tends to A, the surplus of setting a regulated price of y tends to zero so the welfare loss when $p = z$ and $c = y$ is very small. Thus for the intermediate value theorem there is a value of β (β^1) for which $PWL^Z = PWL^Y$. If for β^1 $PWL^Y < PWL^C$, regulation is always better because PWL^Z is decreasing in β and tends to zero when β tends to infinite. Contrarily if for β^1 $PWL^Z > PWL^C$, there is an interval $[\beta^2, \beta^3]$ where free market is the optimal policy.

Proposition 3. *Suppose that $\rho \in (-1, 0)$, n and q are given. Suppose that $\beta^2 > \beta^3$. If $\beta \in (0, \beta^3]$, the optimal policy is regulation with $p = z$. If $\beta \in [\beta^3, \beta^2]$, the optimal policy is free market. If $\beta \in [\beta^2, 1)$, the optimal policy is regulation with $p = y$. Suppose that $\beta^3 > \beta^2$. If $\beta \in (0, \beta^1]$, the optimal policy is regulation with $p = z$. If $\beta \in [\beta^1, 1)$, the optimal policy is regulation with $p = y$.*

Proof. Notice the following facts

a) PWL^Y tends to zero if β tends to one. $PWL^Y(\rho, \bullet, q)$ is decreasing on β and tends to one if β tends to zero.
b) PWL^Z tends to $q/(q+1)$ if β tends to one. $PWL^Z(\rho, \bullet, q)$ is increasing on β and tends to zero if β tends to zero.
c) PWL^C does not depend on β.

The proposition follows from a), b) and c) by noticing that the values of β are defined by the equalities between *PWL* at regulated prices *y* or *z* and the Cournot equilibrium. ■

The intuition of Proposition 3 is as follows. When β is small enough regulation is better with $p = z$, because the surplus when $c = z$ is very large and the welfare loss when $p = y$ is relatively negligible. When β is large enough regulation with $p = y$ is the optimal policy because *y* is close to *z* and the welfare loss when $c = z$ is very small. On the contrary, if $p = z$, the welfare loss of $c = y$ can be very large because there is no production. When β increases, PWL^Z increases and PWL^Y decreases so there is a value of β (β^I) for which $PWL^Z = PWL^Y$. If for β^I $PWL^Y < PWL^C$, regulation is always better. Contrarily, if for β^I $PWL^Y > PWL^C$, there is room for free market.

Propositions 2 and 3 say two things: Firstly, when *y* and *z* are almost identical -so β is close to *1*- regulation with price equals *y* (i.e. the safe option because it secures a positive output, no matter what) is the optimal choice. Secondly, when *y* and *z* are very different –so β is very large when ρ is positive and β is close to zero when ρ is negative- regulation with price equals *z* (i.e. the risky option) is the optimal choice given the large social welfare that is created in this case. However, the relationship between β and the marginal costs is not straightforward because β also depends on *A*. Thus fix *y* and *z* such that *z/y* is low. Firstly consider the case in which ρ is positive and *A* is positive but close to *y*. Thus, β is large and the optimal choice is a regulated price of *z* because the social welfare obtained when marginal costs are *y* is small. Secondly, consider the case in which ρ is negative and *A* is zero. Because *z/y* is low, β is low and the optimal choice is a regulated price of *y* because the social welfare that would be lost if price were *z* and the state of the world were *y* would be large. Thus the same values of the marginal costs are associated with very different policies.

Propositions 1, 2 and 3 imply that when β or *q* are small or large enough, regulation is always the optimal policy. This is very intuitive because in these cases the uncertainty faced by the regulator disappears. Notice that, when $\rho < 0$ a price equal to *y* is optimal for values of β close to 1 (which is the largest value of β) and that when $\rho > 0$ this price is also optimal when β is close to 1 (which is the smallest value of β).

Finally we consider variations of ρ. This case is more complicated than the previous ones because PWL^C depends on ρ. Consequently, our conclusions are not as sharp as in previous results.

Proposition 4. *Suppose that β, n and q are given and that $\rho > 0$. For ρ close enough to 0 the optimal policy is regulation with $p = z$. When ρ tends to infinite the optimal policy is regulation with $p = y$.*

Proof. Notice the followings facts.

a) PWL^C is positive when $\rho \to 0$. $PWL^C(\bullet, n)$ is quasi-concave with a maximum and decreasing when α is large enough. When $\rho = 1$, $PWL^C > 0$ and when $\rho \to \infty$, $PWL^C \to 0$.[4]

b) $PWL^Z(\bullet, \beta, q)$ is increasing with ρ. $PWL^Z \to 0$ if $\rho \to 0$.

c) $PWL^Y(\bullet, \beta, q)$ is decreasing in ρ, $PWL^Y \to 1$ if $\rho \to 0$.

From (a), (b) and (c) the proposition follows noticing that it can be shown that when $\rho \to \infty$, $\lim PWL^Y/PWL^C = 0$.■

The intuition behind Proposition 4 is that when ρ is close to zero, the demand function is very elastic, so the welfare gains in the state of the world $c = z$ are very large with respect to the welfare gains when the state of the world is $c = y$. Hence the optimality of setting the regulated price at z. When ρ is close to infinite, the demand function is almost rigid so setting the regulated price at z risks a lot of welfare gains if the state of the world is y. Moreover, in this case (even when the state of the world is z) the price under free market tends to A and welfare losses are larger with free market than with regulation with $p = y$.

Notice that Proposition 4 does not characterize the optimal policy for intermediate values of ρ. However, something can be said about this case: For instance if q is small enough, Proposition 1 implies that free market is never optimal. When n is large enough, it can be shown that the slope of $PWL^C(\bullet, \beta, q)$ is almost zero so there is a single value of ρ for which PWL^C equals PWL^Y and PWL^Z. Thus, the intervals for which the optimal policy is regulation with $p = y$, free market or regulation with $p = z$ follow each other in this order.

Finally, very little can be said when ρ is negative. It can be easily shown that $PWL^Z(\beta, \bullet, q) \to 0$ *if* $\rho \to 0$ but the limits of PWL^Y and PWL^C are not zero, so regulation with $p = z$ is the optimal policy. The reason is that demand is very elastic. This is different from the result in Proposition 4 above because there β was larger than one and in this case β is between zero and one. When ρ tends to -1, the welfare loss of price regulation with $p = y$ and free market tend to zero but the welfare loss of price regulation with $p = z$ tends to $q/(q + 1)$. Thus the latter cannot be optimal. Depending on the parameters, the optimal policy could be regulation with $p = y$ or free market. For instance, when $\beta = 0.4$, $q = 0.1$ and $n = 10$, free market is optimal. The reason is that marginal costs are far apart, the

[4] See Anderson-Renault (2003), p. 262.

probability of occurrence of y is small and competition is high. When $\beta = 0.9$, $q = 10$ and $n = 2$, regulation is optimal. The reason is that marginal costs are close, the probability of occurrence of y is large and competition is between two firms only.

6. PRICE CAPS

In many practical applications, the regulator, instead of setting a price, sets a price cap, i.e. a ceiling for the price.[5] In our case this means that, once the regulated price has been set, say to c, the regulator would allow for any increase in output starting from $p^{-1}(c)/n$. We will see that under an additional (and not unreasonable) condition, price caps coincide with regulated prices.

Proposition 5. *The price arising from a price cap coincides with the optimal regulated price for all states of the world iff the following inequalities hold*

$$\text{If } \rho > 0,\ \beta \leq (\rho + n)/n.$$
$$\text{If } \rho < 0,\ \beta \geq (\rho + n)/n.$$

Proof. Firstly suppose that the regulated price is z. If the state of the world is y, production is zero and no firm has incentives to increase production because costs are never recovered for any positive output. If the state of the world is z and a firm increases output it will face losses.

Secondly, suppose that the regulated price is y. If the state of the world is y, again, if a firm increases output it will face losses. So we are left with the case in which the state is z. Let us write the price in the Cournot equilibrium when marginal costs are $c = y, z$ as $p^C(c)$. Easy calculations show that

$$p^C(c) = \frac{A\rho + cn}{n + \rho}. \tag{13}$$

Notice that $p^C(\bullet)$ is increasing in c. Thus if $p^C(z) > y$ the price cap is always binding. But if $p^C(z) < y$ and the regulator sets a price of y, firms might be interested in increasing the output until they reach the Cournot equilibrium.

Let $x(y)$ be the output that results from setting $p = y$, i.e. $x(y) = p^{-1}(y)$. If starting from a price y, a single firm infinitesimally increases the output, the resultant change in profits will be along the inverse demand function and it amounts to

$$y - z + \frac{dp(x(y))}{dx}\frac{x(y)}{n}. \tag{14}$$

[5] An analysis of the investment incentives under price caps and other market mechanisms is done by Fabra, Fehr and Frutos (2009).

Taking into account the first order condition of profit maximization in a Cournot equilibrium when the state of the world is z, equation (14) can be written as

$$y+\frac{dp(x(y))}{dx}\frac{x(y)}{n}-p^{C}(z)-\frac{dp(x^{C})}{dx}\frac{x^{C}}{n}. \tag{15}$$

Consider the following function: $\Psi(x) \equiv dp(x)/dx\ (x/n) + p(x)$. It is easily seen that

$$\frac{d\Psi(x)}{dx}=\frac{dp(x)}{dx}\frac{x}{n}(\rho+1)+\frac{dp(x)}{dx}<0. \tag{16}$$

Combining equations (15) and (16), we see that an infinitesimal increase in the output of a single firm increases the profits for this firm so the situation where the price equals the regulated value is no longer an equilibrium. Since profit functions are concave, these infinitesimal variations are all we have to consider.

In order to finish the proof, notice that $p^{C}(z) < y$ arises iff $a_L(\rho + n) < a_H n$. If $\rho > 0$ this inequality is equivalent to $\beta > (\rho + n)/n$. If $\rho < 0$, this inequality is equivalent to $\beta < (\rho + n)/n$. ■

Proposition 5 says that if costs do not display too much variability, firms will not take the opportunity to increase the output when the regulated price is y and the state of the world is z. This is because costs are not sufficiently far apart to justify an increase in output. Proposition 5 suggests that this case arises under not unreasonable assumptions. For instance assume that there are four firms in the market. If demand is linear, β should be smaller than $(n+1)/n$. This implies that the variability in costs must be less than *25%*. When demand is isoelastic, with, say, elasticity of *1.5*, ρ equals *.66*. This implies that the variability of costs must be less than *16.66%*. Both numbers do not seem far fetched even though further research on this issue is necessary.

7. EXTENSIONS AND FINAL COMMENTS

In this paper we have studied, in a very simple framework, the optimal regulatory policy in an oligopolistic market where marginal costs are unknown to the regulator. There are many aspects of paramount practical importance that are left out of our study because they would require a fresh modelling, i.e. quality of the product, repeated interaction among firms and the regulator, different objectives for the regulator (i.e. pro-consumer or pro-firms) or variability of the demand.[6] Other aspects could be incorporated or, at least, discussed in our framework like the three points considered below.

1: Many marginal costs.

[6] For the latter case see Earle, Schmedders and Tatur (2009).

In this case, it is easy to see that our Lemma 1 can be generalized to: “The regulator should never set a price different from a possible value of the marginal cost”. The proof is just identical, for any price between two values of the marginal cost, apply the proof in the main text. Of course the formulae showing welfare losses of regulated prices are less transparent but the general principle applies: an expected social welfare maximizing regulator only considers as an optimal choice either marginal costs or the free market price to be an optimal choice. This may be called the generalized “price equals marginal costs” principle. See more on this in Point 2 below.

2. Non-constant average costs.

Some of the markets in which price regulation has been used are characterized by large fixed costs (electricity, hospitals, etc.). Our paper can be interpreted as a model where fixed costs are known to the regulator and they are financed by non-distortionary taxation. The consideration of Ramsey pricing (i.e. price equals average costs) would require different calculations, even though the principles established in this paper still apply. Next, let us consider increasing marginal costs. Let $\rho = b = 1$, $n = 1$ and costs are $cx^2/2$ with $c = y, z$. We will assume that once the price has been set by the regulator, firms are free of supplying whatever quantity they find more convenient.[7] This implies that the monopolist supplies at price equals marginal costs unless this quantity exceeds the one that can be sold in the market at the regulated price. Consider regulation at price equals marginal costs if the true state is y. If the regulator decreases this price slightly we have two effects. On the one hand if the state is y the output and social welfare decrease (following the “price equals marginal cost” rule). On the other hand if the state is z output and social welfare increase following the expansion of demand. By differentiating expected welfare and setting this to zero, we get that the optimal price is

$$p = \frac{A\left[\dfrac{\pi}{y} + 1 - \pi\right]}{\dfrac{1+y}{y}\pi + (1-\pi)(1+z)}. \tag{??}$$

Notice that when $\pi = 0$ (resp. 1), the regulated price is $A/(1+z)$ (resp. $A/(1+y)$), which results from equalizing price to the marginal cost under state z (resp. y). The general principle would be that besides price equals marginal costs there are other prices that might be optimal for the regulator because they are a good compromise between different states of the world.

3: Other forms of demand.

[7] Under constant returns to scale, if price equals marginal cost, the firm is indifferent among any output as long as it is sold in the market. Therefore, the implicit assumption in our previous analysis is that the regulator can choose from any output from those in which the firm is indifferent. When marginal costs are increasing we have to be specific about the behaviour of firms.

There are cases in which the percentage of welfare losses under free market depends on the state of the world. In this case, the comparative static results will not be as clean as those case presented in this paper. Assume that the utility function of the representative consumer is cubic, written as $Ax - bx^2 - d\,x^3 - px$. Thus, $p = A - 2bx - 3dx^2$ (see McHardy, (2000)). As in the main text set $a \equiv A - c$. The optimal and the free market output are respectively

$$x^O = \frac{-b + \sqrt{b^2 + 3ad}}{3d} \quad \text{and} \quad x^C = \frac{-b(1+\frac{1}{n}) + \sqrt{b^2(1+\frac{1}{n})^2 + 3ad + 6\frac{a}{n}}}{3d + \frac{6}{n}}. \qquad (??)$$

Setting $b = d = n = 1$ by simplicity, we see that if a tends to zero PWL^C tends to 0.111 and if a tends to infinite, PWL^C tends to 0.116. Thus, in this case PWL^C depends on a.

REFERENCES

Anderson, S. and R. Renault (2003). "Efficiency and Surplus Bounds in Cournot Competition". *Journal of Economic Theory*, 113, 254-264.

Armstrong, M., M., S. Cowan, J. Vickers (1994). *Regulatory Reform: Economic Analysis and British Experience*. MIT Press, Cambridge, MA.

Armstrong, M. and D. Sappington (2005). "Recent Developments in the Theory of Regulation". In *Handbook of Industrial Organization*, Vol. III, M. Armstrong and R. Porter (eds.).

Baron, D. and R. Myerson (1982). "Regulating a Monopolist with Unknown Costs". *Econometrica*, 50, 4, 911—930.

Earle, R., K. Schmedders and T. Tatur (2007). "On Price Caps under Uncertainty". *Review of Economic Studies,* 74, 1, 93-111.

Fabra, N., N.-H. von der Fehr and M. A. de Frutos (2009). "Market Design and Investment Incentives". Working Paper, Carlos III University, November.

Kim, J.-C. and K.-B. Chang (1993). "An optimal tax/subsidy for output and pollution control under asymmetric information in oligopoly markets". *Journal of Regulatory Economics*, 5, 2, 183-197.

Lee, S.-H. (1997). "A Note on Regulating Oligopolistic Industries: A Hierarchical Model". *Journal of Regulatory Economics*, 12, 1, 91-97.

Littlechild, S. (1986). "Economic Regulation of Privatized Water Authorities". Report for the Department of Environment, HMSO, London.

Loeb, M. and W. Magat (1979). "A Decentralized Method for Utility Regulation," *Journal of Law and Economics*, 22, 2, 399-404.

López-Cuñat, J. (1995). "Regulating Oligopolies" *Revista Española de Economía*, Special Issue on Regulation, 61-91.

McHardy, J.P. (2000). Miscalculations of monopoly and oligopoly welfare losses with linear demand. Hull Economic Research Papers, November.

Sappington, D. and D. Sibley (1988): "Regulating Without Cost Information: The Incremental Surplus Subsidy Scheme," *International Economic Review*, 29, 2, 297-306.

Schwermer, S. (1994). "Regulating oligopolistic industries: A generalized incentive scheme". *Journal of Regulatory Economics*, 6, 1, 97-108.

Shaffer, S. (1989). "A first-best regulatory tax for oligopoly". *Journal of Regulatory Economics*, 1, 4, 373-389.

APPENDIX

1) $PWL^Y(\bullet, \rho, q)$ is increasing in β if $\rho > 0$ and decreasing if $-1 < \rho < 0$.

$$\frac{\partial PWL^{Y}(\beta,\rho,\mathrm{q})}{\partial \beta}=\frac{q\rho\left[(1+\rho)\beta^{\frac{1}{\rho}}-\rho-1\right]+\beta^{\frac{1+\rho}{\rho}}(1+\rho)-\beta^{\frac{1}{\rho}}(1+\rho)}{\rho^{2}\left(q+\beta^{\frac{1+\rho}{\rho}}\right)}=$$

$$=\frac{(1+\rho)\left[q\rho\left(\beta^{\frac{1}{\rho}}-1\right)+\beta^{\frac{1}{\rho}}(\beta-1)\right]}{\rho^{2}\left(q+\beta^{\frac{1+\rho}{\rho}}\right)}\Bigg|_{\substack{>0\,if\ \rho>0\\ <0\,if\ \rho<0}}$$

Notice that if ρ > 0, β > 1; if ρ < 0, 0 < β <1, and $\beta^{\frac{1}{\rho}}>1$ always.

2) *PWL*^{Y} *(β, ●, q)* is increasing in *ρ* if *ρ > 0* and decreasing if *-1 < ρ < 0.*

$$\frac{\partial PWL^{Y}(\beta,\rho,q)}{\partial \rho}=\frac{\beta^{\frac{1+\rho}{\rho}}\frac{\rho-1-\rho}{\rho^{2}}LN\beta\left(q+\beta^{\frac{1+\rho}{\rho}}-\beta^{\frac{1+\rho}{\rho}}\right)}{\left(q+\beta^{\frac{1+\rho}{\rho}}\right)^{2}}+\frac{\beta\beta^{\frac{1+\rho}{\rho}}\frac{\rho-1-\rho}{\rho^{2}}LN\beta}{\left(q+\beta^{\frac{1+\rho}{\rho}}\right)^{2}}+$$

$$+\frac{(\beta-1)\left(q+\beta^{\frac{1+\rho}{\rho}}+\rho\beta^{\frac{1+\rho}{\rho}}\frac{\rho-1-\rho}{\rho^{2}}LN\beta\right)}{\rho^{2}\left(q+\beta^{\frac{1+\rho}{\rho}}\right)^{2}}=$$

$$=\frac{-q\beta^{\frac{1+\rho}{\rho}}LN\beta-\beta\beta^{\frac{1+\rho}{\rho}}LN\beta+(\beta-1)\left(q+\beta^{\frac{1+\rho}{\rho}}\left(1-\frac{LN\beta}{\rho}\right)\right)}{\rho^{2}\left(q+\beta^{\frac{1+\rho}{\rho}}\right)^{2}}=$$

$$=-\frac{q\left(\beta^{\frac{1+\rho}{\rho}}LN\beta+1-\beta\right)+\beta^{\frac{1+\rho}{\rho}}\left(\beta\,LN\beta-\beta+1+\frac{(\beta-1)LN\beta}{\rho}\right)}{\rho^{2}\left(q+\beta^{\frac{1+\rho}{\rho}}\right)^{2}}\Bigg|_{\substack{<0\forall\rho>0\\ >0\forall\rho\in(-1,0)}}$$

If *ρ > 0 (resp.< 0)*, this expression is negative: both parenthesis in the numerator are positive (resp. negative), because they are increasing in β and tend to zero if β→1.

$$\frac{\partial}{\partial\beta}\left(\beta^{\frac{1+\rho}{\rho}}LN\beta+1-\beta\right)=\frac{1+\rho}{\rho}\beta^{\frac{1}{\rho}}LN\beta+\beta^{\frac{1}{\rho}}-1>0$$

$$\lim_{\beta\to 1}\left(\beta^{\frac{1+\rho}{\rho}}LN\beta+1-\beta\right)=0$$

$$\frac{\partial}{\partial\beta}\left(\beta LN\beta-\beta+1+\frac{(\beta-1)LN\beta}{\rho}\right)=LN\beta+\frac{LN\beta+\frac{\beta-1}{\beta}}{\rho}>0$$

$$\lim_{\beta\to 1}\left(\beta LN\beta-\beta+1+\frac{(\beta-1)LN\beta}{\rho}\right)=0\,.$$