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ISOLATING CHANGES IN NET RESIDENTIAL SEGREGATION FROM THE EFFECT OF DEMOGRAPHIC FACTORS IN THE U.S., 1989-2005

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Abstract

This paper investigates residential segregation trends net of changes in the racial and the neighborhood marginal population distributions. It follows two alternative strategies. First, it uses indices of two types. Indices of the first type emphasize an evenness segregation concept and are only invariant to changes in the marginal distribution by race, while those of the second type emphasize a representativeness segregation concept and are only invariant to changes in the marginal distribution by neighborhood. Second, it uses the mutual information, or the *M* index that is not invariant to changes in either of the marginal distributions but admits two decompositions. Each of the decompositions isolates a term which (a) is invariant to changes in the marginal distribution of one of the two variables and the entropy, or diversity, of the other, and (b) reflects changes in either an evenness or a representativeness segregation notion. According to the *M* index, net residential segregation in both an evenness and a representative sense considerably decreases for the U.S. public school student population in urban areas in 1989-2005. Because of their failure to control for changes in the spatial entropy, invariant indices of the first type register a smaller decline in the evenness sense, while because of their failure to control for changes in the racial entropy invariant indices of the second type register an increase in residential segregation in the representativeness sense. Within the evenness perspective, all racial groups experiment a reduction in net segregation which is greatest for Hispanics.

Keywords: Multigroup Segregation; Multilevel Segregation; Residential Segregation; Mutual Information; Entropy Indices; Invariance Properties; Econometric Models.

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I. INTRODUCTION

During the past few decades the U.S. has become increasingly racially and ethnically diverse because minorities have grown much more rapidly than the white population thanks to their greater fertility and/or immigration rates. As a consequence, whites represent less than 50% of the population in many metropolitan areas, and are no longer the majority group in some others. Moreover, the racial composition in different areas presents wide variations over the country as a whole. For example, just as blacks in the past, Asians or Hispanics today are relatively concentrated in some parts of the country and almost absent in many others.[3] These changes have taken place along a demographic wave that has consistently moved the country's center of population west- and southwards, with California and Texas currently the most populous states. This paper is concerned with the measurement of residential segregation trends under these circumstances.

Multigroup residential segregation usually refers to the tendency of more than two racial groups to have different distributions over locational units within a larger area, say over neighborhoods within a country. This view of residential segregation coincides with the one referred to as "evenness" by Massey and Denton (1988) in the context of occupational segregation by gender. As such, the notion of evenness is compatible with different views about the role that the marginal population distribution by race should play in the measurement of residential segregation and its changes. Some people would argue that a race's contribution to residential segregation should depend on its size, so that if the most segregated races grow faster over time, then the country's segregation should increase. Others would think that, as long as all the racial distributions over neighborhoods remain constant, the measurement of residential segregation should not change. This property is known as composition invariance, scale

[3] The terms *white*, *black*, *Asian* and *Native American* are used throughout the paper to refer to non-Hispanic members of these racial groups. Asians include Native Hawaiian and Pacific Islanders; Native Americans include American Indians and Alaska Natives (Innuit or Aleut). The term *Hispanic* is an ethnic rather than a racial category since Hispanic persons may belong to any race. The term racial group is used throughout to refer to each of these five racial/ethnic categories.

invariance, or invariance 1, and was originally discussed in the context of intertemporal (or international) comparisons of occupational segregation by gender.[4] An index of segregation that satisfies this property will be said to be invariant 1 or ***I1*** for short.

Residential segregation admits an alternative interpretation which focuses on the extent neighborhoods become black ghettos, barrios, and Chinatowns, that is, the extent to which neighborhoods have racial compositions which are different from the population as a whole. This notion of segregation, which is referred to as "representativeness" by Frankel and Volij (2008a), is the multigroup extension of the "isolation" dimension distinguished in Massey and Denton (1988) in the two-group case. There are different views about how the marginal population distribution by neighborhoods should affect the measurement of residential segregation viewed as representativeness. Some people would argue that a neighborhood's contribution to residential segregation should depend on its size, so that if the most segregated neighborhoods grow faster over time, then the country's segregation should increase. Others would think that as long as the neighborhoods' racial mix remains constant, residential segregation should not change. This property is known as invariance 2 or, in the context of occupational segregation, occupational invariance (see Watts, 1998, and Blackburn *et al.*, 1993, 1998). An index of segregation that satisfies this property will be said to be invariant 2, abbreviated as ***I2***.

This paper searches for intertemporal comparisons in residential segregation in the U.S. using two strategies. In the first place, in agreement with those who insist that changes in segregation should be invariant to changes in the population marginal distributions, residential segregation is measured with several invariant segregation indices. In practice, the literature has emphasized ***I1*** indices. In particular, the two indices used in this paper are the general Atkinson index, referred to as the A index, which is the only ***I1*** index that satisfies four additional properties, and a multigroup version of the Dissimilarity

[4] See, *inter alia*, James and Taeuber (1985), Watts (1992, 1998), Charles (1992, 1998), Charles and Grusky (1995, 2004), Grusky and Charles (1998), and Hutchens (1991, 2001, 2004).

index, or *D* index, which is a multigroup segregation version of the original dissimilarity index due to Duncan and Duncan, 1955 (see Frankel and Volij, 2008b, for a discussion and characterization results). We do not know of any ***I2*** multigroup segregation index or any index that simultaneously satisfies both invariant properties in the multigroup case.[5] However, as pointed out in Mora and Ruiz-Castillo (2008a), by just changing the roles of racial groups and neighborhoods, any ***I1*** index becomes an ***I2*** one. Thus, the reciprocal versions of the *A* and the *D* indices, which will be referred to as *A** and *D**, are the ***I2*** indices used in this paper.

In the second place, one may use a segregation index that is neither ***I1*** nor ***I2*** but whose intertemporal changes admit a decomposition that isolates a term that captures segregation changes net of the impact of pure demographic factors, or net segregation changes.[6] This is the case of the mutual information, or *M* index, which was originally proposed by Theil and Finizza (1971) and whose underlying ordering has been recently characterized by Frankel and Volij (2008a).[7] The *M* index simultaneously captures the notions of evenness and representativeness, and it is equal to the demographically weighted average of local segregation indices for each race or each neighborhood. Therefore, changes in residential segregation using this index result from changes in the way races are distributed over neighborhoods and changes in the population marginal distribution over neighborhoods, and/or changes in the neighborhoods' racial composition and changes in the population marginal distribution over races. Hence, the *M* index is neither ***I1*** nor ***I2***. However, Mora and Ruiz-Castillo (2008a) show that changes in the *M* index can be decomposed in two complementary ways: the first decomposition isolates an ***I1*** term, maintaining constant the marginal distribution over

[5] In the two-group case, Charles (1992) and Charles and Grusky (1995) discuss a segregation index that is both ***I1*** and ***I2***.
[6] In the context of occupational segregation by gender, many authors have defended this strategy before (see, *inter alia*, Blau and Hendricks, 1979, Jonung, 1984, Beller, 1985, and Watts, 1992, 1998).
[7] Very few segregation indexes have been similarly characterized. In the two-group case, Chakravarty and Silber (1992) characterize an index of absolute segregation, while Chakravarty and Silber (2007) axiomatically derive a class of numerical indexes of relative segregation that parallel the multidimensional Atkinson inequality indices. Two members of that class are monotonically related to the square root index, independently characterized by Hutchens (2004), and the *M* index. In the multigroup case, Alonso-Villar and Del Río (2008) characterize local segregation indexes in the mutual information context, while Frankel and Volij (2008b) provide an ordinal characterization of the general and the symmetric Atkinson indexes.

racial groups *and* the entropy or diversity of the population neighborhood distribution, while the second decomposition isolates an ***I2*** term, maintaining constant the marginal distribution over neighborhoods *and* the entropy of the population marginal distribution over races. The invariant term in the first decomposition reflects changes in the groups' conditional distributions over neighborhoods and, therefore, can be interpreted as changes in evenness. The invariant term in the second decomposition reflects changes in the neighborhoods' racial mix and, therefore, can be interpreted as changes in representativeness. This paper contains the first empirical application of this methodology.[8]

We use data about the student population enrolled in public schools in Core Based Statistical Areas (CBSAs) –urban clusters of 10,000 or more inhabitants, referred to in the sequel as *cities*– during the 1989-90 and 2005-06 academic years. We aggregate the data at school districts that, although they may better correspond to municipalities in many cases, will be referred to as *neighborhoods*. To understand the interest of the empirical application, imagine a situation where school district authorities all over the country are able to implement a policy that reproduces in all schools the racial mix of the district to which they belong. In this scenario, the public student population would still experience the residential segregation studied in this paper, arising from the location decisions about neighborhoods and cities taken by their parents or caretakers among the adult population. Using the same data, Mora and Ruiz-Castillo (2008b) report changes in residential segregation within the student population using the *M* index. They find that residential segregation declines during the period for all minorities, but since their joint population share increases by almost 13% and the segregation level of all of them is rather high, all minorities taken together contribute to an increase in the *M* index. At the same time, the segregation index for whites increases, whereas their population share declines; however, since their demographic

[8] Frankel and Volij (2008b) compare the *M* index (as well as other normalized and un-normalized segregation indices which are neither ***I1*** nor ***I2***) with the *A* and D indices. These authors, however, do not consider ***I2*** indices or, above all, decompositions of segregation changes of not invariant indices –like the *M* index– that isolate invariant terms. On the other hand, although Frankel and Volij (2008b) use the same data set than us, they study U.S. school rather than residential segregation during a slightly different time period.

weight remains high whites also contribute positively to an increase in the *M* index.[9] Even if one is willing to accept that race and neighborhood sizes should affect the measurement of segregation in a moment in time, the above results provide a good example of the need for disentangling changes in evenness and representativeness from changes due merely to variations in the population marginal distributions over racial groups or spatial units.

This paper adopts a multilevel approach where overall residential segregation is due to the fact that people of different races cluster, not only in different neighborhoods within a city, but also in different cities.[10] This poses no problem for the *M* index because it can be expressed as the sum of two terms capturing between- and within-cities residential segregation. Frankel and Volij (2008b) show that the *A* index also satisfies this property. Although the *D* index is not decomposable in that sense, a measure of overall and between-cities segregation can still be legitimately obtained with this index by computing them separately for neighborhoods and cities, respectively. For comparison purposes with the *M* and the *A* indices, the difference between these two terms can be interpreted as a measure of within-cities segregation. An identical operation can be performed for the *A** and the *D** indices.

The main results of the paper can be summarized as follows. Firstly, racial groups become more evenly distributed over neighborhoods. Correspondingly, the ***I1*** indices record a reduction in residential segregation in the evenness sense. However, since they do not control for changes in the spatial entropy this reduction is smaller than that indicated by the evenness term in the first decomposition of the *M* index. Secondly, neighborhoods' racial compositions become closer to each other leading to a reduction in the representativeness term of the second decomposition of the *M* index. However, because they do not control for changes in the racial entropy, ***I2*** indices record an increase in residential segregation in

[9] Analogous, although less dramatic results are found along the spatial dimension: when the spatial units are cities rather than neighborhoods and cities are grouped into 12 regions, the *M* index capturing between-cities residential segregation declines in 7 regions representing almost 60% of the population, but the increase in this index in the remaining 5 regions lead to an increase in the segregation for the country as a whole.

[10] For other instances of the multilevel approach to residential and school segregation, see Massey and Hajnal (1995), Riardon *et al.* (2000), Fisher *et al.* (2004), and Mora and Ruiz-Castillo (2008b, 2008c).

the representativeness sense. Thirdly, within the evenness perspective provided by the first decomposition of the M index, it is found that both between- and within-cities net residential segregation decreases for all minorities. The reduction in net between-cities segregation has been greatest for Hispanics, while the reduction in net within-cities segregation is similar for all minorities. Although of a smaller order of magnitude, whites also experience a reduction in net overall residential segregation due to a corresponding reduction in within-cities segregation.

The rest of the paper is organized in three Sections. Section II presents some notation, the independence properties and the indices to be used. Section III is devoted to the empirical results, while Section IV contains some concluding comments.

II. NOTATION, INVARIANCE PROPERTIES, AND SEGREGATION INDICES

II.1. Notation

Assume a country partitioned into N geographical units or neighborhoods, indexed by $n = 1,\ldots, N$. Each individual living in a neighborhood belongs to only one of five racial groups, indexed by g = *whites*, *blacks*, *Hispanics*, *Asians*, and *NA* (Native Americans). The data available can be organized into the following 5 x N matrix:

$$X = \left\{T_g^n\right\}_{g,n} = \begin{bmatrix} T_{whites}^1 & \cdots & T_{whites}^N \\ \vdots & \ddots & \vdots \\ T_{NA}^1 & \cdots & T_{NA}^N \end{bmatrix} \tag{1}$$

where T_g^n is the number of individuals of racial group g living in neighborhood n.

The information contained in the joint absolute frequencies of racial groups and neighborhoods, X, is usually summarized by means of numerical indices of segregation, $S(X)$. Let $T = \sum_{n=1}^{N} \sum_{g} T_g^n$ be the total population, and denote by $P_{gn} = \left\{ \frac{T_g^n}{T} \right\}_{g,n=1}^{N}$ the joint distribution of racial groups and neighborhoods. In the following, the discussion will be restricted to indices that capture a *relative* view of segregation in which all that matters is the joint distribution, i.e. indices which admit a representation as a function of P_{gn}:[11]

$$S(X) = S_I\left(P_{gn}\right).$$

In order to introduce the segregation notions of evenness and representativeness formally, note that any bivariate joint distribution P_{gn} admits two alternative factorizations into a conditional and a marginal:

$$p_{gn} = p_{n|g}\,p_g = p_{g|n}\,p_n, \tag{2}$$

where

$p_{n|g}$: proportion of individuals in group g who are located in neighborhood n,

$p_{g|n}$: proportion of individuals in neighborhood n who belong to group g,

p_g : proportion of individuals from racial group g in the population, and

p_n : proportion of individuals from neighborhood n in the population.

Let $P_g = \left\{p_g\right\}_g$ be the marginal distribution by racial groups and $P_{n|g} = \left\{p_{n|g}\right\}_{n=1}^{N}$ be the conditional distribution by neighborhoods of individuals in group g. Since $P_{n|g}$ summarizes all the relevant information regarding how different racial groups are located across neighborhoods, any index of segregation based on the notion of evenness must be a function of $P_{n|g}$. From equation (2) we have:

$$S_I\left(P_{gn}\right) = S_E\left(P_{n|g}, P_g\right). \tag{3}$$

Therefore, any relative index of segregation admits an evenness representation that, without imposing

[11] This property, satisfied by most segregation indices, is referred to as Size Invariance in James and Taeuber (1985) and as Weak Scale Invariance in Frankel and Volij (2008a). For a study that focuses on translation invariant segregation indices that represent an absolute view of segregation, see Chakravarty and Silber (1992).

further assumptions, will not be invariant to changes in the marginal distribution by racial groups. Alternatively, denote by $P_{g|n} = \left\{ p_{g|n} \right\}_g$ the conditional distribution by race of individuals in neighborhood n and by $P_n = \left\{ p_n \right\}_{n=1}^N$ the marginal distribution by neighborhood size. Since $P_{g|n}$ summarizes all the relevant information regarding how neighborhoods differ in their racial compositions, any index of segregation based on the notion of representativeness must be a function of $P_{g|n}$. From equation (2) we have:

$$S_I\left(P_{gn}\right) = S_R\left(P_{g|n}, P_n\right). \tag{4}$$

Therefore, any relative index of segregation admits a representativeness representation that, without imposing further assumptions, will not be invariant to changes in the marginal distribution by neighborhoods, P_n.

As can be seen in expression (1), where the rows are racial groups and the columns are neighborhoods, evenness and representativeness are dual concepts: deviations from evenness (representativeness) correspond to differences in the row (column) percentages. The following observation indicates how close these two views are to each other.

Remark 1. If a segregation index S that captures the notion of evenness when applied to the 5 x N array $X = \left\{T_g^n\right\}$ is applied to the N x 5 array $X' = \left\{T_n^g\right\}$ where the role of neighborhoods and racial groups are reversed, then what will be called the reciprocal index S^* applied to X' captures equally well the notion of representativeness (and vice versa).

In general, $S(X)$ and $S^*(X')$ will provide a different segregation value for the same data. When this is not the case, the segregation index under consideration is said to be transpose invariant.

II. 2. Invariance Properties

In the literature on occupational segregation by gender it has long been noted that both the overall gender composition of employment and the distribution of the population across occupations typically change over time and/or space. Similar phenomena are present in other segregation contexts. As indicated in the Introduction, in the past few decades the U.S. has become increasingly racially diverse and the mean population center of the country has moved consistently west- and southwards. Equations (3) and (4) highlight that those changes in the marginal distributions may result in changes in any index of segregation $S_I(P_{gn})$, independently of the notion of evenness and representativeness that it may capture. However, many people would argue that, as long as the distributions of racial groups over neighborhoods remain constant, the measurement of segregation viewed as evenness should not change. Similarly, as long as the neighborhoods' racial mix remains constant, segregation viewed as representativeness should not change.

The following two axioms have been proposed to capture these ideas. To motivate the first one, consider a situation in which only the size of one or more racial groups varies, so that the marginal distribution P_g changes, but the allocation of racial groups across neighborhoods, $P_{n|g}$, remains constant. Under these circumstances, it has been argued that the segregation index should not change.

Invariance 1 (*I1*): (*Composition Invariance* in James and Taeuber, 1985, and Watts, 1998; *Homogeneity* in Hutchens, 1991; *Scale Independence* in Frankel and Volij, 2008b). If $X = \{T_g^n\}$ and $\tilde{X} = \{\tilde{T}_g^n\}$ are two matrices such that $\tilde{T}_g^n = \lambda_g T_g^n$ for all *n* and *g* with $\lambda_g > 0$ for each *g*, then $S(X) = S(\tilde{X})$.

<u>Remark 2.</u> The only relevant magnitudes in the domain of an ***I1*** index are the conditional neighborhood distributions by racial group, $P_{n|g}$; that is, if $S(X)$ satisfies ***I1***, then $S_E(P_{n|g}, \overline{P}_g) = S_E(P_{n|g}, \overline{\overline{P}}_g)$ for any two marginal distributions $\overline{P}_g$ and $\overline{\overline{P}}_g$.

For the next property, consider situations in which the school size distribution P_n changes, while the racial mix within each neighborhood $P_{g|n}$ remains constant. It has also been argued that under these conditions segregation should not change.

Invariance 2 (*I2*): (*Occupational Invariance* in Watts 1998, and Blackburn *et al.* 1993, 1995). If $X = \left\{T_g^n\right\}$ and $\tilde{X} = \left\{\tilde{T}_g^n\right\}$ are two matrices such that $\tilde{T}_g^n = \lambda^n T_g^n$ for all *n* and *g* with $\lambda^n > 0$ for each *n*, then $S(X) = S(\tilde{X})$.

Remark 3. The only relevant magnitudes in the domain of ***I2*** indices are the neighborhoods' racial composition, $P_{g|n}$; that is, if *S*(*X*) satisfies ***I2***, then $S_R(P_{g|n}, \overline{P}_n) = S_R(P_{g|n}, \overline{\overline{P}}_n)$ for any two marginal distributions $\overline{P}_n$ and $\overline{\overline{P}}_n$.

Remark 4. In view of Remark 1, the reciprocal *S**(*X*) of any ***I1*** index *S*(*X*) becomes an ***I2*** index (and vice versa).

II. 3. Invariant Segregation Indices

The Atkinson segregation indices, which are based on the Atkinson inequality indices, were first introduced by James and Taeuber (1985) for *G* = 2. Given any scalar $\delta \in (0,1)$, the Atkinson index in this case equals:

$$A_\delta(X) = 1 - \left[\sum_{n=1}^{N} \left(p_{n|g=1}\right)^{\delta} \left(p_{n|g=2}\right)^{1-\delta}\right]^{1/(1-\delta)}.$$

As Frankel and Volij (2008b) argue, this index is difficult to generalize to more than two groups since the outer exponent, $1/(1-\delta)$, is the reciprocal of the weight on a particular group. Instead, they propose an increasing transformation of the original index that represents the same ordering. Let $\pi = (\pi_1, \pi_2, ..., \pi_G)$ be a vector of *G* nonnegative weights that sum up to one. The general Atkinson index with weights π, $A_\pi(X)$, is defined by

$$A_\pi(X) = 1 - \sum_{n=1}^{N} \prod_g \left(p_{n|g}\right)^{\pi_g}.$$

The symmetric Atkinson index, which is obtained when all weights π_g are equal, is ***I1***. More generally, whenever the weights π are invariant to the marginal distribution of groups, P_g, then $A_\pi(X)$ will be ***I1***.

The reciprocal version of $A_\pi(X)$ is defined by

$$A^*_{\pi^*}(X) = 1 - \sum_g \prod_{n=1}^{N} \left(p_{g|n}\right)^{\pi^*_n},$$

where $\pi^* = (\pi^*_1, \pi^*_2, ..., \pi^*_N)$ are N nonnegative weights that sum up to one. It follows from Remark 4 that if the neighborhoods' weights π^* are invariant to the marginal distribution of neighborhoods, P_n, then $A^*_{\pi^*}(X)$ is ***I2***.

A general multigroup version of the Index of Dissimilarity –first proposed by Duncan and Duncan (1955) in the two-group context– can be defined as

$$D_\pi(X) = \frac{1}{2(G-1)} \sum_{n=1}^{N} \sum_g \left| p_{n|g} - \sum_{g'} \pi_{g'} p_{n|g'} \right|.$$

Frankel and Volij (2008b) define the Unweighted Dissimilarity Index by giving the same weight to each ethnic group, $\pi_g = 1/G$ for all g. Whenever the weights π are invariant to P_g, as in the case of the Unweighted Dissimilarity, $D_\pi(X)$ is ***I1***. It follows from Remark 4 that the reciprocal version of the index, i.e.

$$D^*_{\pi^*}(X) = \frac{1}{2(N-1)} \sum_g \sum_{n=1}^{N} \left| p_{g|n} - \sum_{n'=1}^{N} \pi^*_{n'} p_{g|n'} \right|,$$

is ***I2*** if the neighborhoods' weights $\pi^* = (\pi^*_1, \pi^*_2, ..., \pi^*_N)$ are invariant to the marginal distribution of neighborhoods, P_n.

II. 4. The Mutual Information Index of Segregation

In information theory, the expression

$$M_g = \sum_{n=1}^{N} p_{n|g} \log\left(\frac{p_{n|g}}{p_n}\right),$$

is known as the expected information of the message that transforms the set of proportions P_n to the set of proportions in group g, $P_{n|g}$.[12] Since M_g measures the extent to which the conditional distribution $P_{n|g}$ differs from the marginal distribution P_n, it can be seen as a local index of segregation for racial group g when segregation is interpreted as a deviation from evenness. The mutual information index M is the weighted average of the local segregation indices for racial groups, with weights equal to the proportion of individuals from each group:

$$M = \sum_g p_g M_g . \tag{2}$$

Thus if the groups that are more segregated grow faster over time, the M index will register an increase in segregation.

Alternatively, the expression

$$M^n = \sum_g p_{g|n} \log\left(\frac{p_{g|n}}{p_g}\right) \tag{3}$$

is the expected information of the message that transforms P_g to $P_{g|n}$. Since it measures the extent to which the racial composition in neighborhood n differs from the one for the population as a whole, M^n can be seen as a local index of segregation in neighborhood n when segregation is interpreted as a deviation from representativeness. As Frankel and Volij (2008a) and Mora and Ruiz-Castillo (2005) show, the weighted average of information expectations defined in (3) with weights equal to the proportion of individuals in each neighborhood coincide with the overall index of segregation as defined in equation (2), that is:

$$M = \sum_{n=1}^{N} p_n M^n . \tag{4}$$

Thus if the neighborhoods that are more segregated grow faster over time, the M index will register an increase in segregation.[13]

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[12] In principle, the logarithm could be computed in any base. In the empirical application we will use the natural logarithm.

[13] The M index can also be motivated using the entropy concept, as in as Theil and Finizza (1971) Theil (1972), Reardon and

Equations (2) and (4) indicate that the M index is transpose invariant, it captures the notions of evenness and representativeness in a symmetric fashion, and it is sensitive to variations in the marginal distributions by racial groups and neighborhoods, i.e. it is neither ***I1*** nor ***I2***.

II.5 Decompositions of Pairwise Comparisons of the *M* Index[14]

Differences in the index of segregation between any two situations may result from differences in the marginal distributions, P_g and P_n, as well as from differences in the conditional distributions, $P_{n|g}$ and $P_{g|n}$. As indicated in the Introduction, there are reasons to argue that pairwise comparisons of segregation should be made independent from the effect of differences in the marginal distributions. Such comparisons can be accomplished in at least two ways. First, if the index is invariant, then the comparison of the index will also be invariant. Second, if the index is not invariant, then observed differences may be decomposed so that one of the terms in the decomposition reflects changes in segregation which are due only to changes in one of the conditional distributions. This is the strategy applied for the M index in the rest of this Section.

For the sake of concreteness, assume that there is data on a country in two periods, $X(t),\ t = \{t^1, t^2\}$ and that we are concerned with the intertemporal comparison of segregation, $\Delta M = M\left(X(t^2)\right) - M\left(X(t^1)\right)$. Mora and Ruiz-Castillo (2008a) show that in pairwise comparisons the M index admits two decompositions into three terms. Given the set $\pi = (\pi_1, \pi_2, ..., \pi_G)$ of G nonnegative weights that sum up to one, the first decomposition is the following:

$$\Delta M = \Delta P_{n|g}(\pi) + \Delta E^n + \Delta P_g(\pi), \tag{5}$$

where

$$\Delta P_{n|g}(\pi) = \sum_{g=1}^{G} \pi_g \sum_{n=1}^{N} \left\{ p_{n|g}(t^2) \log\left(p_{n|g}(t^2)\right) - p_{n|g}(t^1) \log\left(p_{n|g}(t^1)\right) \right\},$$

$$\Delta E^n = E\left(P_n(t^2)\right) - E\left(P_n(t^1)\right),$$

Firebaugh (2002), and Frankel and Volij (2008a), as well as a statistical measure of association, as in Zoloth (1974, 1976), Flückiger and Silber (1999), and Mora and Ruiz-Castillo (2008c).

[14] This Section draws substantially from Mora and Ruiz-Castillo (2008a).

$$\Delta P_g(\pi) = \sum_{t=t^1,t^2} (-1)^{1(t=t^1)} \left\{ \sum_{g=1}^{G} \left(p_g(t) - \pi_g \right) \sum_{n=1}^{N} p_{n|g}(t) \log\left(p_{n|g}(t) \right) \right\},$$

$1(\cdot)$ is the indicator function, and $E(P) = \sum_i p_i \log\left(1/p_i\right)$ is the entropy of the probability distribution $P = \{p_i\}$.[15] The three terms in equation (5) can be interpreted as follows. First, ΔE^n captures segregation changes due to intertemporal changes in the entropy of neighborhoods. Second, $\Delta P_g(\pi)$ captures segregation changes due to differences between the marginal distribution by groups and the groups' weights, i.e. differences between $P_g(t)$ and π. Third, $\Delta P_{n|g}(\pi)$ gives the difference in segregation that arises from changes in $P_{n|g}(t)$ when both the overall entropy of neighborhoods and the marginal distribution $P_g(t)$ remain constant, and the latter equals π. Notice that $\Delta P_{n|g}(\pi)$ is ***I1*** in the sense that it equals zero if $T_g^n(t^2) = \lambda_g T_g^n(t^1)$.

Given the set $\pi^* = (\pi_1^*, \pi_2^*, ..., \pi_N^*)$ of N non-negative weights that sum up to one, the second decomposition is the following:

$$\Delta M = \Delta P_{g|n}(\pi^*) + \Delta E_g + \Delta P_n(\pi^*), \tag{6}$$

where

$$\Delta P_{g|n}(\pi^*) = \sum_{n=1}^{N} \pi_n \sum_{g=1}^{G} \left\{ p_{g|n}(t^2) \log\left(p_{g|n}(t^2) \right) - p_{g|n}(t^1) \log\left(p_{g|n}(t^1) \right) \right\},$$
$$\Delta E_g = E\left(P_g(t^2)\right) - E\left(P_g(t^1)\right),$$
$$\Delta P_n(\pi^*) = \sum_{t=t^1,t^2} (-1)^{1(t=t^1)} \left\{ \sum_{n=1}^{N} \left(p_n(t) - \pi_n^* \right) \sum_{g=1}^{G} p_{g|n}(t) \log\left(p_{g|n}(t) \right) \right\}.$$

The three terms in equation (9) can be interpreted as follows. First, ΔE_g captures segregation changes due to changes in the entropy of racial groups. Second, $\Delta P_n(\pi^*)$ measures segregation changes due to differences between $P_n(t)$ and π^*. Third, $\Delta P_{g|n}(\pi^*)$ gives the difference in segregation that arises from

[15] Entropy measures the diversity exhibited by the distribution in the sense that it equals to zero if and only if all individuals are members of a single group, and it is maximized if and only if individuals are evenly distributed among all the groups.

changes in $P_{g|n}(t)$ when both the overall entropy of racial groups and $P_n(t)$ remain constant, and the latter equals π^*. Note that $\Delta P_{g|n}(\pi^*)$ is ***I2*** in the sense that it equals zero if $T_g^n(t^2) = \lambda_n T_g^n(t^1)$.

To understand the differences between these decompositions and what can be accomplished with ***I1*** and ***I2*** indices, consider a situation in which both conditional and marginal distributions are changing. Thus, net segregation in both an evenness and a representativeness sense is changing, but it is desirable to isolate such changes from the effect attributable to demographic changes in the two marginal distributions. The first term in equation (5) captures the change in net segregation in an evenness sense, holding constant the effects attributable to the change in the marginal distribution by race and to the change at least in the entropy of the other variable. Instead, ***I1*** indices will mix up the change in evenness with the effect attributable to the change in the marginal distribution by neighborhood. Similarly, the first term in equation (6) captures the change in net segregation in a representativeness sense, holding constant the effects attributable to the change in the marginal distribution by neighborhood and to the change at least in the entropy of the other variable. Instead, ***I2*** indices will mix up the change in representativeness with the effect attributable to the change in the marginal distribution by race.

Decompositions (5) and (6) can only be computed after specific values for π and π^* are chosen. Naïve candidates are $P_g(t^1)$ and $P_g(t^2)$ for π and $P_n(t^1)$ and $P_n(t^2)$ for π^*. However, as Karmel and MacLachlan (1988) argue, the interpretability of the invariant decompositions obtained using these weights is compromised by the fact that the conditional and the marginal distributions are constrained by equation (2). Following the approach advocated in Deutsch *et al.* (2006), Mora and Ruiz-Castillo (2008a) show that the average of the two decompositions obtained using $P_g(t^1)$ and $P_g(t^2)$ as π and the average of the two decompositions obtained using $P_n(t^1)$ and $P_n(t^2)$ as π^* are free of this criticism.

III. EMPIRICAL RESULTS

III.1. Data and Demographic Trends

We use the Common Core of Data (CCD) compiled by the National Center for Educational Statistics (NCES).[16] This dataset contains school enrolment records by racial groups from all public schools in the United States. Results are reported for the academic years 1989-90 (the first year for which complete enrolment data is available) and 2005-06. As defined by the U.S. Office of Management and Budget (OMB), the term Core Based Statistical Area (CBSA) is a collective term for both metropolitan and micropolitan statistical areas. A metropolitan area contains a core urban area of 50,000 or more population, and a micro area contains an urban core of at least 10,000 (but less than 50,000) population. Schools districts are retrospectively assigned CBSA codes based on 2005 ZIP codes so that comparisons over time can be made without changes in city boundary definitions affecting the results. As indicated in the Introduction, cities and neighborhoods are identified with CBSAs and school districts within them, respectively, while the student population enrolled in public schools in such cities and neighborhoods will be simply referred to in the sequel as the population.

Since the decomposition methods for changes in residential segregation according to the M index are valid for the homogeneous case in which the two situations under comparison share a common structure, a sample has been constructed with a common set of 834 cities and 5,429 neighborhoods.[17] Basic statistics about this population, including its racial and spatial entropy as a measure of diversity, are offered in Table 1.

Table 1 around here

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[16] Other versions of this data set have been used in studies of residential and school segregation by Reardon *et al.* (2000), Frankel and Volij (2008a, 2008b), and Mora and Ruiz-Castillo (2008b, 2008c).

[17] The main difference between the common sample used in this paper and the one in Ruiz-Castillo (2008b) is twofold: the latter includes 869 cities and 5,834 neighborhoods in 1989, 1.3% fewer whites, and 1% and 0.3% more blacks and Hispanics, respectively; on the other hand, in 2005 there are 999 cities and 7,707 neighborhoods, 0.8% and 0.7% fewer whites and Hispanics and 1.4% more blacks. However, as will be seen below, residential segregation changes according to the M index in the two papers are of the same order of magnitude.

As can be seen in Panel A, minorities (namely, Native Americans, blacks, Asians, and Hispanics) already represent 34.8% of the 24.8 million people of the total population in 1989; furthermore, since all of them grow rather rapidly while whites actually decrease during this period, they represent as much as 48.1% of the 25.5 million people of the total population in 2005. Among minorities, the black, Native American and Asian populations increase by 10%, 30% and 32%, respectively, while Hispanics increase by 72%. As a result, there is a considerable change in the marginal distribution by race: in 2005 whites represent 13.3 percentage points less than in 1989, and Hispanics, blacks, and Asians 10.1, 1.7 and 1.3 points more, respectively. Not surprisingly, the racial diversity of the population, measured by its entropy, considerably increases by 17.6% during this period (see Panel C in Table 1).

The spatial distribution of the population across neighborhoods and cities is markedly skewed to the right, as shown by the large difference between the average and the median number of individuals per neighborhood and per city in both years (see Panel B in Table 1). At the neighborhood level, average and median sizes slightly increase during 1989-2005. Changes in the marginal distribution by neighborhoods are hard to summarize, but the entropy – a measure of diversity or concentration – remains essentially stable (see Panel C). At the city level, the mean slightly increases, while the median and the entropy slightly decrease.

III. 2. Changes in Segregation According to the *M* Index

What is the impact on residential segregation of such rather important demographic changes? In order to understand the interplay between changes in segregation and demographic changes when segregation is measured according to the *M* index, recall that, from the evenness point of view, overall segregation in 1989 is $M = \sum_g p_g M_g$, where $\left(p_g M_g\right)$ is the contribution to this magnitude by racial group *g*. Therefore, the change in overall segregation, ΔM, can be expressed as

$$\Delta M = \sum_g \Delta\left(p_g M_g\right) .$$

On the other hand, overall segregation can be decomposed as the sum of a between- and a within-cities segregation term, $M = BC + WC$. Therefore,

$$\Delta M = \Delta BC + \Delta WC;$$

that is, changes in overall segregation result from changes in between- and within-cities segregation, ΔBC and ΔWC. Similarly, for each g we have that $M_g = BC_g + WC_g$, so that $BC = \sum_g p_g BC_g$ and $WC = \sum_g p_g WC_g$, where $\left(p_g BC_g\right)$ and $\left(p_g WC_g\right)$ are the contributions to BC and WC by racial group g, respectively. Therefore, we have

$$\Delta BC = \sum_g \Delta\left(p_g BC_g\right),$$

$$\Delta WC = \sum_g \Delta\left(p_g WC_g\right).$$

The information about total changes (ΔM, ΔBC, ΔWC), as well as the details by racial group (ΔM_g, ΔBC_g, ΔWC_g) is in Panel A in Table 2. Panel B reports the changes in the contributions to each of the indices by each racial group, $\left(\Delta\left(p_g M_g\right), \Delta\left(p_g BC_g\right), \Delta\left(p_g WC_g\right)\right)$.

Table 2 around here

Regarding changes in racial groups' segregation indices, ΔM_g, the most remarkable feature is perhaps the increase in whites' index of overall segregation and the decrease in that index for all minorities. But behind this general pattern, the following three points should be emphasized. First, in 2005 blacks, Hispanics, and Native Americans have simultaneously less between- and within-cities segregation than in 1989. Second, between-cities segregation strongly declines for Asians, but their within-cities segregation term is the only one increasing among the minorities. Third, whites register an important increase of both between- and within-cities segregation.

To understand the changes in the contributions to residential segregation at the country level by racial group, it is necessary to take into account variations in the population marginal distribution by race (see column 3 in Table 1). As can be seen in the last column of Panel A in Table 2, the change in overall residential segregation for the country, ΔM, is equal to 4.2, or 11.6% of the level reached in 1989. In the mutual information framework, this means that a randomly selected individual's neighborhood in 2005 conveys more information about its race than in 1989. To see how this is possible, note that

$$\Delta M = \sum_{g} p_g^* \Delta M_g + \Delta p_g M_g, \tag{7}$$

where p_g^* is the marginal distribution in 2005. To begin with, consider the trends for whites. Whites' contribution to segregation increases by 3.0 index points –which accounts for 71.4% of the increase in overall segregation for the country as a whole– because the increase in their segregation index in the first term of equation (7) more than compensates the negative effect of its population share decrease in the second term of that equation. On the contrary, changes in the contributions to overall segregation in the case of Hispanics, Asians, and Native Americans are positive because their population shares increase. For blacks, in contrast, changes in the contribution to overall segregation are negative because the change in the contribution to within-cities segregation turns out to be negative in spite of the fact that their population share also increases.

Although useful for understanding the role of changes in the local indices M_g in segregation changes for the country as a whole, decompositions like (7) fail to properly isolate changes in residential segregation from changes in the marginal distributions. Sub-sections III.3 and III.4 will present two strategies to achieve this end.

III.3. Changes in Residential Segregation According to Invariant Indices

A potential drawback of measuring segregation using the symmetric Atkinson index and the

Unweighted Dissimilarity index, both of which employ equal weights for all racial groups, is that their values might be unduly sensitive to the inclusion of a relatively small group. Indeed, when Native Americans, who account for less than one percentage point of the total population in the U.S., are included in the sample, the value of the symmetric Atkinson index increases by 12.5% in 1989 and by 11.3% in 2005. The corresponding values for the Unweighted Dissimilarity index are 3.6% and 2.3%.[18] An alternative strategy is to compute the general Atkinson and Dissimilarity indices, $A_{\pi}(X)$ and $D_{\pi}(X)$, using as weights π the demographic importance of each of the racial groups in a given reference year. A similar strategy can be implemented for the reciprocal indices using as weights the demographic importance of the neighborhoods in a reference year. For comparability with the *M* index results, we employ $P_g(1989)$ and $P_g(2005)$ as weights for computation of *A* and *D* and $P_n(1989)$ and $P_n(2005)$ as weights for *A** and *D**. As with *M*, we only report the averages of the resulting indices, which will be denoted by $\overline{A}, \overline{A}^*, \overline{D}$, and $\overline{D}^*$.[19] Frankel and Volij (2008b) show that the *A* index is decomposable into a between-cities and a within-cities term as the mutual information *M* index. As pointed out in the Introduction, although the *D* index is not decomposable in that sense, the difference between a measure of overall segregation and a measure of between-cities segregation can be interpreted, for comparison purposes, as a measure of within-cities segregation. An identical operation can be performed for the *A** and the *D** indices. Panel A in Table 3 contains the changes in $\overline{A}, \overline{A}^*, \overline{D}$, and $\overline{D}^*$, and their between- and within-cities decompositions.

Table 3 around here

The message is clear: the sign of the change in residential segregation during 1989-2005 depends on the notion of segregation advocated. On the one hand, taking into account that according to the *M* index residential segregation for all minorities goes down by a considerable amount, one would expect any ***I1***

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[18] Similarly, reciprocal indices that place the same weight to each neighborhood may be very sensitive to the exclusion of small neighborhoods. For example, if the smallest neighborhoods which account for one percentage point of the student population are excluded from the sample, then the symmetric Atkinson index increases by 15.3% in 1989 and by 6.9% in 2005. The corresponding figures for the Unweighted Dissimilarity are 3% and 2.4%.

[19] Detailed results are available upon request.

index to register a decrease in residential segregation from an evenness perspective. Indeed, both $\overline{A}$ and $\overline{D}$ report an overall residential segregation reduction by 16.6% and 4.9%, respectively.[20] It is observed that most of this decrease is attributable to a reduction in between-cities segregation. On the other hand, according to $\overline{A}^*$ and $\overline{D}^*$ overall residential segregation from a representativeness perspective increases by 24.9% and 26.9%, respectively. This is the result of increases in both between- and within-cities segregation. However, according to the reciprocal Atkinson $\overline{A}^*$ index both terms are equally responsible for the overall increase, while according to the reciprocal Dissimilarity $\overline{D}^*$ index about 2/3 of the overall increase is attributable to the increase in the between-cities component.

Finally, the pattern of change by racial group is also quite different (see Panel B in Table 3). According to $\overline{A}^*$, due to a reduction in their contribution to between-cities residential segregation, the more important minorities –blacks, Hispanics, and Asians– contribute negatively to the overall change; consequently, the increase in overall segregation is accounted for by a positive and strong white contribution. However, according to $\overline{D}^*$ not only whites, but also the remaining groups, contribute positively to the general increase in residential segregation.

III. 4. Changes in Net Segregation According to the *M* Index

In the rest of this Section, the results for decompositions (5) and (6) introduced in Section II.5 are presented. In particular, we report the average of the two decompositions obtained using $P_g(1989)$ and $P_g(2005)$ as π and the average of the two decompositions obtained using $P_n(1989)$ and $P_n(2005)$ as $\pi*$.[21] Results are shown in Panel A of Table 4.[22] Panel B contains the changes in the conditional

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[20] These results are compatible with the changes in school segregation reported in Frankel and Volij (2008b) using ***I1*** indices. The reduction in school segregation according to the Atkinson index using as weights the marginal distribution by race in 1987-88 and 2005-06 are 23.1% and 29.1%. Using the Unweighted Dissimilarity index, this reduction is 6.3%.

[21] Detailed results are available upon request.

[22] As was observed in Section III.2, the change in between- and within-cities segregation in this paper's common sample is equal to 2.5 and 1.7. These values do not differ greatly from 1.0 and 2.5, which are the corresponding values reported by Mora and Ruiz-Castillo (2008b) for the entire unbalanced sample; therefore, the change in overall segregation in both

distributions for each g within the evenness perspective

$$\Delta M_g = \sum_{n=1}^{N} \left\{ p_{n|g}(t^2) \log\left(p_{n|g}(t^2) \right) - p_{n|g}(t^1) \log\left(p_{n|g}(t^1) \right) \right\}. \quad (8)$$

Table 4 around here

This information merits the following four comments. Firstly, the more remarkable result is perhaps the dramatic decrease in net overall segregation according to both the evenness and the representativeness views: 22.9 index points, or 63.2% of the overall residential segregation reached in 1989 according to evenness, and 13.9 points or 38.4% according to representativeness. Within the mutual information framework, both views complement each other by emphasizing a different diagnostic. In the first place, there is a reduction in net residential segregation according to an evenness perspective, i.e. $\Delta \overline{P}_{n|g} < 0,$ indicating that racial groups become more evenly distributed over neighborhoods. This means that when we randomly choose an individual in 2005 we learn from her race less about the neighborhood she lives in than in 1989. Both in absolute and relative terms, the more dramatic change is the reduction in within-cities net segregation, that is, when the selection of an individual of a given race takes place once a certain city has been chosen. In the second place, the second decomposition shows a reduction in net residential segregation in a representativeness perspective, i.e. $\Delta \overline{P}_{g|n} < 0,$ indicating that neighborhoods racial compositions are closer to each other. Thus, when we choose any individual in a given neighborhood we learn less in 2005 about her race than in 1989. This negative change in net segregation actually takes place at the city level: once a city has been chosen, if we select an individual in a given neighborhood, we learn slightly more about her race in 2005 than in 1989. In other words, within-cities net segregation slightly increases during the period.

Secondly, these results differ markedly from those obtained with the ***I2*** indices, $\overline{A}^*$ and $\overline{D}^*$,

samples is equal to 4.2 versus 3.5 index points. Given these differences, we feel comfortable that the following results obtained in this paper are equally applicable to the segregation changes observed in the former one.

according to which when a representativeness perspective is adopted residential segregation increases during the period 1989-2005. Moreover, from an evenness perspective the reduction in residential segregation according to the ***I1*** indices $\overline{A}$ and $\overline{D}$ is of a smaller order of magnitude than the reduction according to the term $\Delta\overline{P}_{n|g}$ in the first decomposition of ΔM. How can these discrepancies be explained? The key fact is that invariant indices only neutralize changes in one of the marginals. On one hand, ***I2*** indices convey changes in residential segregation according to a representativeness view independently of changes in the marginal distribution over neighborhoods, ΔP_n. But in so doing they mix up the effect of changes in the conditional distribution that focus on the racial composition by neighborhood, $\Delta P_{g|n}$, with the impact of changes in the marginal distribution by race, ΔP_g. As can be seen in the third term of the first decomposition, the impact of the latter on residential segregation is positive and strong (particularly for between-cities segregation). By holding constant the change in racial entropy, the second decomposition of the *M* index partially controls for this effect, allowing the reduction in residential segregation in a representativeness view to reveal itself through the term $\Delta\overline{P}_{g|n}$ at the between-cities level. In the absence of such a control, the ***I2*** indices register an increase in residential segregation in the representative sense (particularly at the between-cities segregation level, as expected). On the other hand, ***I1*** indices convey changes in residential segregation according to an evenness view independently of changes in the marginal distribution over races, ΔP_g. But in doing so they mix up the effect of changes in the conditional distribution that focuses on the spatial distribution by race, , $\Delta P_{n|g}$, with the impact of changes in the marginal distribution by neighborhood, ΔP_n. As can be seen in the third term of the second decomposition, the impact of the latter on residential segregation is slightly positive (as a result of a small positive effect in within-cities segregation, almost offset by a negative effect in between-cities segregation). By holding constant the change in spatial entropy, the first decomposition of the *M* index partially controls for this effect, allowing the reduction in residential segregation in an evenness view to reveal itself through the term $\Delta\overline{P}_{n|g}$, particularly at the within-cities level. In the absence of such a control, the ***I1*** indices register an increase in residential segregation in the evenness sense of a smaller order of magnitude (particularly at the within-cities segregation level, as expected).

Thirdly, it could be argued that, knowing in advance the sign and strength of the racial and spatial entropy changes, either ***I1*** or ***I2*** indices could then be selected accordingly. Even in this case, the analysis of residential segregation changes would be restricted to either an evenness or a representativeness view. An obvious alternative is to use the two decompositions of the *M* index that provide in all cases an integrated version of both views.

Finally, for policy purposes it is important to know about changes in net segregation by race group at different spatial levels, a question that becomes possible in the evenness view (see Panel B in Table 4). Firstly, as expected from the detailed geographical analysis in Mora and Ruiz-Castillo (2008b), the reallocation of Hispanics and, to a lower extent, the remaining minorities over the U.S. cities during 1989-2005 causes strong decreases in between-cities net segregation for all these groups. This substantially confirms the view offered in Table 2 about changes in between-cities segregation indices by racial group, ΔBC_g. Secondly, the previous literature on residential segregation based on pairwise comparisons of within-cities segregation (see *inter alia* Mora and Ruiz-Castillo, 2008b, for extensive references) tends to emphasize the decrease of residential segregation between whites and blacks. In a different vein, Table 2 informs about an increase in within-cities segregation indices for both whites and Asians. Here in Table 4, all groups exhibit a decrease in net within-cities segregation, with blacks, but also Asians and Hispanics showing the strongest decreases. Thirdly, as a consequence, even whites exhibit a reduction in net overall segregation. Nevertheless, the reduction in net overall segregation in an evenness sense is seen to be basically a minority phenomenon with Hispanics clearly at the top.

IV. CONCLUSIONS

Whether the measurement of residential segregation according to an evenness or a representativeness view should depend on race and/or neighborhood sizes is a debatable question. In either case, most people would agree that it is convenient to isolate changes in some notion of net residential segregation from pure demographic changes in the marginal distributions over races and/or neighborhoods –an issue that, with the exception of Frankel and Volij (2008b), has only been discussed in the context of occupational segregation in the two-group case. These authors only compare non-invariant and ***I1*** indices to study school segregation for a very similar sample of the U.S. student population enrolled in urban public schools to the one used in this paper (see footnote 7). In the context of residential segregation among five distinct racial groups during 1989-2005, this paper is the first one to include also ***I2*** indices and, above all, the two decompositions of changes in residential segregation according to the *M* index originally presented in Mora and Ruiz-Castillo (2008a).

The main findings are the following. Firstly, two ***I1*** indices invariant only to changes in the marginal distribution over races –the general Atkinson and a Dissimilarity index– have registered a decrease in residential segregation in an evenness sense. Secondly, their reciprocal indices, invariant only to the marginal distribution over neighborhoods, have registered an increase in residential segregation in a representativeness sense. Thirdly, residential segregation changes have been measured using the *M* index that simultaneously captures both evenness and representativeness segregation notions but it is neither ***I1*** nor ***I2***. During 1989-2005, the interplay between residential segregation changes and changes in the population marginal distributions has given rise to an increase in residential segregation according to the *M* index. However, such changes are decomposable in two ways. In the first decomposition, changes in the conditional distribution by race – net of changes in the marginal distribution by race and in the spatial entropy – have registered a decrease in residential segregation in an evenness sense greater than the one recorded by ***I1*** indices. This decomposition permits changes in net segregation by group and geographical level to be studied. In particular, it has been found that the reduction in net between-cities segregation has been greatest for Hispanics, while the reduction in net within-cities segregation is of a similar order of magnitude for all minorities. Finally, in the second decomposition of the *M* index, changes in the conditional distribution by neighborhood – net of changes in the marginal distribution by neighborhood and in the racial entropy – have registered a decrease in residential segregation in a representative sense, contrary to the increase recorded by ***I2*** indices.

These results illustrate that in practical situations simultaneous changes in both marginal distributions cloud the measurement of net residential segregation. More fundamentally, they exemplify that ***I1*** or ***I2*** indices may provide a misleading view as they only control for the changes in one of the two marginals. Consequently, changes in segregation according to an evenness (representativeness) notion via ***I1*** (or ***I2***) indices include the effects of changes in the marginal distribution of neighborhoods (races). To our knowledge, the two complementary decompositions of the *M* index used in this paper constitute the more satisfactory available alternative. They isolate changes in residential segregation from changes in the marginal distribution of one of the variables and the entropy changes in the other, integrating in a coherent way the evenness and representativeness notions that are simultaneously captured by the *M* index.

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Table 1. Descriptive Statistics for the U.S. Urban Student Population Enrolled in Public Schools

PANEL A: Race Frequencies

	1989 (1)		2005 (2)		Change (3) = (2) – (1)		Change in %: (4) = (3)/(1)x100	
	millions	In %	millions	In %	millions	In %	millions	In %
Minorities	8.6	34.8	12.2	48.1	3.6	13.3	42.1	38.1
Native Americans	0.2	0.7	0.2	0.9	0.1	0.2	33.8	30.0
Asians	1.0	4.2	1.4	5.5	0.4	1.3	36.1	32.3
Blacks	4.0	16.1	4.5	17.8	0.5	1.7	13.7	10.5
Hispanics	3.4	13.8	6.1	23.9	2.7	10.0	77.3	72.4
Whites	16.1	65.2	13.2	51.9	- 2.9	- 13.3	- 18.1	- 20.3
All	**24.8**	**100.0**	**25.5**	**100.0**	**0.7**	**0.0**	**2.9**	**0.0**

PANEL B: Number of Students per Organizational Unit

	Per Neighborhood				Per City			
	1989	2005	Change	% Change	1989	2005	Change	% Change
Average	4,560	4,691	130.7	2.9	29,683	30,534	850.6	2.9
Median	1,967	2,112	145.0	7.4	6,132	5,823	-308.5	-5.0

PANEL C: Population Diversity Measured by its Entropy

	1989 (1) (% max)	2005 (2) (% max)	Change (3) = (2) - (1)	Change in % (4) = (3)/(1) x 100
Racial Entropy				
Racial Groups (E_g)	101.3 (62.9)	119.1 (74.0)	17.8	17.6
Spatial Entropy				
Neighborhoods (E^n)	760.2 (88.4)	764.6 (88.9)	4.5	0.6
Cities (E^{Cities})	524.2 (77.9)	517.4 (76.9)	-6.8	-1.3

Notes: **% max** in Panel C reports the value of the entropy as a percentage of its maximum value. The data source for this and the following tables is the Common Core of Data (CCD) compiled by the National Center for Educational Statistics (NCES). See footnote 2 in the text for race categories definitions. Cities and neighborhoods refer to Core Based Statistical Areas and school districts, respectively. The population refers to the urban student population within the common set of 834 cities and 5,429 neighborhoods, accounting for 93.2 and 70.6% of those enrolled in public schools in 1989 and 2005, respectively.

Table 2. Changes in Segregation According to the Mutual Information Index. 1989-2005

Panel A: Changes in Segregation Indices

		Details by Racial Group					All	
		Whites	**Blacks**	**Hispanics**	**Asians**	**Native Americans**		
Overall	$\Delta M_g =$	8.9	-8.5	-32.6	-11.0	-20.3	$\Delta M =$	4.2
Between-cities	$\Delta BC_g =$	4.5	-1.2	-26.8	-15.2	-9.0	$\Delta BC =$	2.5
Within-cities	$\Delta WC_g =$	4.4	-7.3	-5.8	4.2	-11.3	$\Delta WC =$	1.7

Panel B: Changes in Contributions by Racial Group

		Whites	**Blacks**	**Hispanics**	**Asians**	**Native Americans**
Overall	$\Delta\left(p_g M_g\right)=$	3.0	-0.3	0.9	0.5	0.1
Between-cities	$\Delta\left(p_g BC_g\right)=$	1.7	0.3	0.3	0.0	0.2
Within-cities	$\Delta\left(p_g WC_g\right)=$	1.3	-0.7	0.6	0.5	0.0

Notes: See Section II.4 in the main text for the definition of the indices M_g and M. See Section III.2 in the main text for the definitions of BC_g, WC_g, BC, and WC, and the definitions of the racial contributions, $p_g M_g, p_g BC_g, p_g WC_g$.

Table 3. Changes in Segregation for Invariant Indices. 1989-2005

Panel A: Changes in Segregation Indices

	I1 Indices (Evenness)		*I2* Indices (Representativeness)	
	Change	**% Change**	**Change**	**% Change**
General Atkinson	$\Delta\overline{A}$	$\Delta\overline{A}/\overline{A}x100$	$\Delta\overline{A}^*$	$\Delta\overline{A}^*/\overline{A}^*x100$
Overall	-7.6	-16.6	8.4	24.9
Between-cities	-5.6	-21.2	3.8	17.7
Within-cities	-1.9	-10.2	4.6	37.2
Dissimilarity	$\Delta\overline{D}$	$\Delta\overline{D}/\overline{D}x100$	$\Delta\overline{D}^*$	$\Delta\overline{D}^*/\overline{D}^*x100$
Overall	-2.9	-4.9	7.8	26.9
Between-cities	-2.4	-5.2	5.9	21.4
Within-cities	-0.4	-3.8	2.0	119.2

Panel B: Racial Contributions to Changes in *I2* Indices (Representativeness)

	Whites	**Blacks**	**Hispanics**	**Asians**	**Native Americans**
General Atkinson					
Overall	18.2	-2.3	-6.8	-0.7	0.0
Between-cities	15.1	-1.9	-8.3	-1.1	-0.1
Within-cities	3.1	-0.4	1.5	0.4	0.1
Dissimilarity					
Overall	2.9	1.0	3.3	0.5	0.2
Between-cities	1.7	0.8	2.8	0.4	0.2
Within-cities	1.2	0.2	0.5	0.1	-0.1

Notes: See Section II.2 in the main text for the definition of the indices. For each index, **Change** is the change in the year-specific averages across the resulting values obtained when using as weights the sample marginal distributions in both 1989 and 2005. The column **Racial contributions** reports the racial contributions to each ***I2*** index, i.e. $1/G-\prod_{n=1}^{N}\left(p_{g|n}\right)^{\pi_n^*}$ for the Reciprocal Atkinson and $\frac{1}{2(N-1)}\sum_{n=1}^{N}\left|p_{g|n}-\sum_{n'=1}^{N}\pi_{n'}^{*}p_{g|n'}\right|$ for the Reciprocal Dissimilarity. The Within-cities Dissimilarity is defined as the difference between the Overall and the Between-cities indices.

Table 4. Net Segregation Changes from the Mutual Information Index. 1989-2005

PANEL A: Decompositions of Gross Changes

	Change		***I1* Decomposition (Evenness)**			***I2* Decomposition (Representativeness)**		
	ΔM	**(in %)**	$\Delta \overline{P}_{n\|g}$	ΔE^{n}	$\Delta \overline{P}_{g}$	$\Delta \overline{P}_{g\|n}$	ΔE_{g}	$\Delta \overline{P}_{n}$
Overall	4.2	11.6	-22.9	4.5	22.6	-13.8	17.8	0.2
Between-cities	2.5	11.7	-7.7	-6.8	17.0	-14.5	17.8	-0.8
Within-cities	1.7	11.5	-15.2	11.3	5.6	0.7	0.0	1.0

PANEL B: Changes in Evenness by Racial Group

	Whites	**Blacks**	**Hispanics**	**Asians**	**Native Americans**
Overall	-8.5	-34.8	-53.0	-37.0	-29.9
Between-cities	0.4	-7.8	-31.5	-11.3	-13.7
Within-cities	-8.9	-27.1	-21.5	-25.8	-16.1

Notes: See equations (5) and (6) in the main text for the definitions of the two decompositions. $\Delta \overline{P}_{n|g}$ denotes the average of the terms $\Delta P_{n|g}(\pi)$ in decomposition (5) when taking as π weights $P_g(1989)$ and $P_g(2005)$, while $\Delta \overline{P}_{g}$ denotes the corresponding average of the terms $\Delta P_g(\pi)$. $\Delta \overline{P}_{g|n}$ denotes the average of the terms $\Delta P_{g|n}(\pi^*)$ in decomposition (6) when taking as π^* weights $P_n(1989)$ and $P_n(2005)$, while $\Delta \overline{P}_{n}$ denotes the corresponding average of the terms $\Delta P_n(\pi)$. See equation (8) in the main text for the definition of the changes in evenness by racial group.