Of molecules and men

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Abstract

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Keywords:

1 Introduction

2 A trivial model

Utility of patient/doctor pair (PDP) i from being treated with branded drugs (or molecules) A and B is respectively:

$$\begin{aligned} U_A^i &= \theta_A + a_A + \varepsilon^i - \delta p_A, \\ U_B^i &= \theta_B + a_B - \delta p_B, \end{aligned}$$

where θ_J is drug J's (average) intrisic therapeutic quality; a_J is for the advertisement, kickbacks, scientific-based advocacy on the drug's merits, etc, that increase the value of the drug beyond its intrinsic therapeutic value; ε^i is the patient's responsiveness to drug A or the opposite of its side effects on patient i (in comparison with drug B: this is a relative idiosyncratic shock); δ is the PDP's sensitivity to prices (the MRS with therapeutic value), and p_J is the *net* price paid by the PDP (net of social security insurance subsidies).

We shall assume for simplicity that ε^i is uniformly distributed over $[-e/2; e/2]$. This means that:

$$f(\varepsilon) = 1/e, \ \forall\varepsilon \in [-e/2; e/2]$$

$$F(\varepsilon) = \begin{cases} 0, \forall\varepsilon < -e/2 \\ \frac{\varepsilon+e/2}{e} = \frac{1}{2} + \frac{\varepsilon}{e}, \forall\varepsilon \in [-e/2; e/2] \\ 1, \forall\varepsilon > e/2. \end{cases}$$

`At some point, we'll have to check if we can get closed form or numerical solutions with the normal distribution.`

By assumption θ_A and θ_B are strictly larger than $e/2$, so that drug A is potentially useful to all patients, and drug B can still charge a non-negative price and experience positive demand if $p_A = a_A = 0$ `(find a better motivation. the point will be that, when` $\theta_A = \theta_B$`, equ'm prices will be` $e/2$`).`

Lemma 1 $\theta_J > \frac{3c\delta e-2}{2c\delta}$ *is a sufficient condition for* $q_J = 1$ *when* $J = A, B$ *faces no competition.*

Proof. *See Section 6.* ■

Utility of PDP from being treated with G, the generic equivalent of drug A, is:

$$U_G^i = \theta_A + \gamma^i a_A + \varepsilon^i - \delta p_G,$$

where $\gamma^i \in [0,1]$ is the spillover effect of the advertisement for A onto the generic. When $\gamma^i = 0$, the PDP treats the branded drug A and its bio equivalent generic as different. For instance, the PDP is exclusively sensitive to kickbacks or finds it simpler to remember the name of the brand or the shape and color of the box. In contrast, $\gamma^i = 1$ for PDPs that are exclusively sensitive to the scientific evidence provided about the drugs' genuine qualities: the therapeutic effects are, by assumption, exactly equal between the branded drug and its generic equivalent. So, all the scientific evidence produced regarded drug A carries over to its bio equivalent generic.

Advertisement costs

Convincingly advertising a drug's qualities and managing to convince doctors to accept kickbacks has a strictly increasing and convex cost:

$$C(a_J) = c\frac{a_J^2}{2},$$

where $c > 0$ is a parameter.

3 Duopoly prior to generic entry

When both drugs still benefit from patent protection, each PDP has a choice between branded molecules A and B. We assume a mass 1 of PDPs who clearly need to buy one of the two molecules (θ large enough for their medical condition: see Lemma above). PDPs compare their utility of using each drug and go for the one that produces the highest utility: they go for drug B iff

$$\begin{aligned} \theta_A + a_A + \varepsilon^i - \delta p_A &\leq \theta_B + a_B - \delta p_B, \text{ or} \\ \varepsilon^i &\leq \Delta\theta_B + \Delta a_B - \delta \cdot \Delta p_B, \end{aligned}$$

where $\Delta\theta_B \equiv \theta_B - \theta_A$, $\Delta a_B = a_B - a_A$, etc.

The quantity sold by B is thus:

$$q_B = F_\varepsilon \left[\Delta\theta_B + \Delta a_B - \delta \cdot \Delta p_B\right] = \frac{1}{2} + \frac{\Delta\theta_B + \Delta a_B - \delta \cdot \Delta p_B}{e} \tag{1}$$

and the quantity sold by A is $1 - q_B$.

Each firm maximizes its profits:

$$\pi_J = P_J \; q_J - C\left(a_J\right), \; J = A, B,$$

where P_J is the gross price obtained by the firm. If patients are subsidized in proportion $(1-k)$, we have: $p_J = kP_J$, where k is the patient's co-payment factor.

FOCs are:

$$q_J + P_J \frac{\partial q_J}{\partial p_J} k = 0 \tag{2}$$

$$P_J \frac{\partial q_J}{\partial a_J} = C'\left(a_J\right) = c a_J. \tag{3}$$

The FOC (2) yields:

$$P_J = \frac{q_J}{k\left|\frac{\partial q_J}{\partial p_J}\right|} = q_J \frac{e}{k\delta}. \tag{4}$$

Substituting this into (3) then yields:

$$\begin{aligned} ca_J &= q_J \frac{e}{k\delta}\frac{1}{e} \\ a_J &= \frac{q_J}{ck\delta}. \end{aligned} \tag{5}$$

In words:

Proposition 1 *Prior to generic entry, both firms choose a price and an advertisement level that are proportional to their market share. Both price and advertisement are decreasing in the PDP's sensitivity to prices, $k\delta$ and, ceteris paribus, equilibrium prices are increasing in the degree of differentiation, e.*

(`note: shouldn't we drop` k `completely, and instead lump it directly into` δ?)

3.1 Implicit solutions

To ensure that we can use calculus to find the equilibrium, we need that molecule A can be sufficiently superior to B for some PDPs, and conversely for other PDPs. The following condition will prove useful to eliminate uniform dominance by one molecules or corner solutions:

Condition 1 $e > 2/3\left(\frac{1}{\delta c} + |\Delta\theta_B|\right)$. *[Or: $|\Delta\theta_B| < \frac{3c\delta e - 2}{2\delta c}$]*

Proof. *(Proof made on some piece of paper – lost it already – was quite straightforward)* ■

Modified + some NEW

Some open thoughts (by Micael): I took some time to understand this condition. When $\Delta\theta_B = 0$ it seems as if this condition should not matter (numerator and denominator are then equal, and you end up with $a_J = \frac{1}{2c\delta}$ and $P_J = \frac{e}{2\delta}$ no matter what). So, is this condition needed in the symmetric case, or not? It does! This may be due to the fact that, to obtain this condition, we would at some point be dividing 0 by 0, which is indeterminate. So, I believe that this condition is required in all cases. I tried to solve the whole stuff by hand (well: pencil and paper) and this is really long (4 eq'ns, 4 unknowns, long fractions) and I am repeatedly making typos... Not sure it's worth it! So, as I am saying below, I think we should simply impose this condition throughout! As Carmine remarks, this might even be related in some ways to Lemma 1 (beware of the difference: Lemma 1 is about the absolute value of θ).

In this part, we look for implicit solutions, that require fewer functional assumptions, and prove handy to isolate some relationships between variables.

Normalizing k to 1 from now on, we have:

$$\begin{aligned} P_A &= \frac{1-F_\varepsilon}{\delta}e \\ P_B &= \frac{F_\varepsilon}{\delta}e \\ a_A &= \frac{1-F_\varepsilon}{c\delta} \\ a_B &= \frac{F_\varepsilon}{c\delta} \end{aligned} \tag{6}$$

Hence:

$$\begin{aligned} P_B - P_A &= \frac{2F_\varepsilon - 1}{\delta}e \\ a_B - a_A &= \frac{2F_\varepsilon - 1}{c\delta}, \end{aligned}$$

where:

$$\begin{aligned} F_\varepsilon &= \frac{1}{2} + \frac{\Delta\theta_B + \Delta a_B - \delta \cdot \Delta p_B}{e} \\ &= \frac{1}{2} + \frac{\Delta\theta_B + \frac{2F_\varepsilon - 1}{c\delta} - \delta \cdot \frac{2F_\varepsilon - 1}{\delta}e}{e} \\ &= \frac{1}{2} + \frac{\Delta\theta_B}{e} + \frac{2F_\varepsilon - 1}{c\delta e} - (2F_\varepsilon - 1) \\ \left(2 - \frac{2}{c\delta e} + 1\right) F_\varepsilon &= \frac{1}{2} + \frac{\Delta\theta_B}{e} - \frac{1}{c\delta e} + 1 \\ (3c\delta e - 2)\, F_\varepsilon &= \frac{c\delta e}{2} + \Delta\theta_B c\delta - 1 + c\delta e \end{aligned}$$

$$\boxed{F_\varepsilon = \frac{1}{2}\frac{(3c\delta e - 2) + 2\Delta\theta_B c\delta}{3c\delta e - 2}} \tag{7}$$

Hence:

Modified

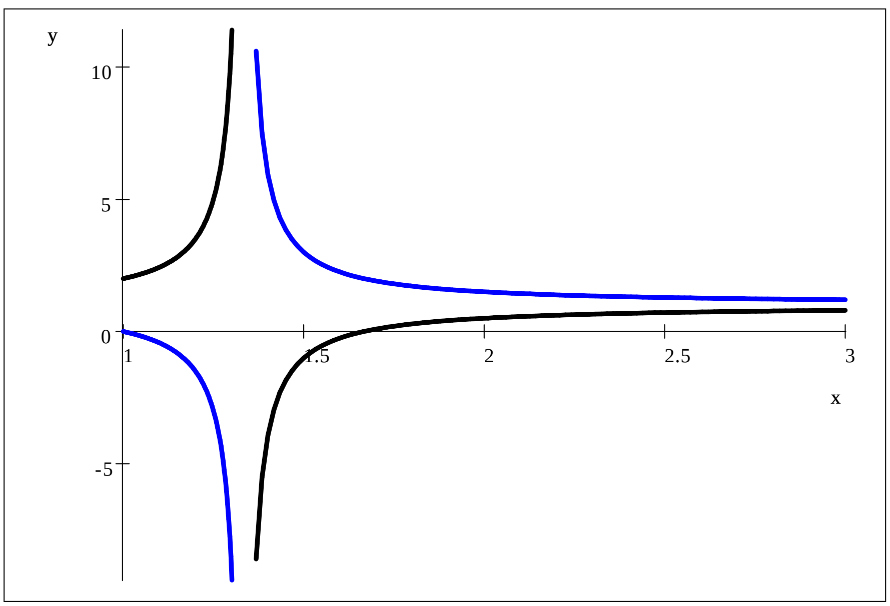
Proposition 2 *When condition 1 holds, the best molecule (J such that $\Delta\theta_J > 0$) sells more, at a higher price, and is advertised more than the worst one.*

3.2 Closed form solutions

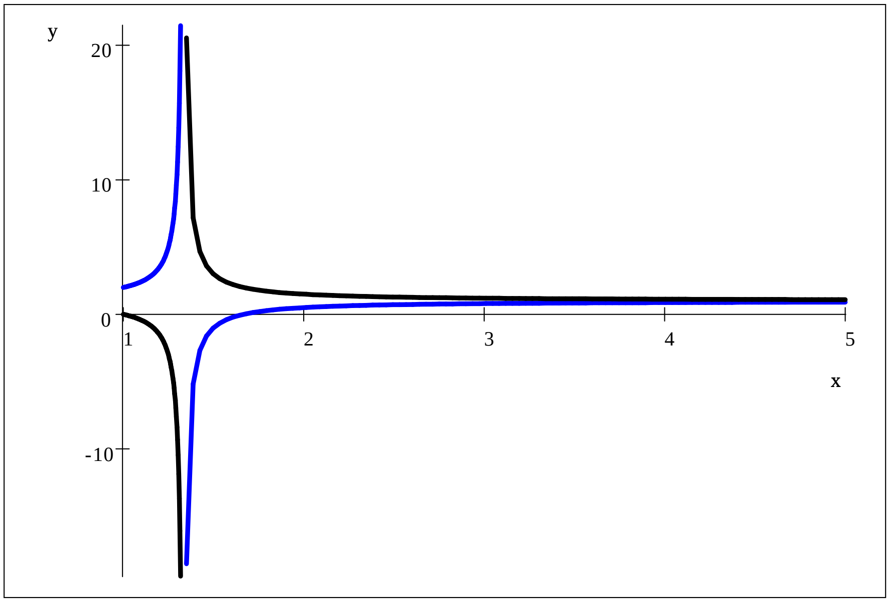
Using (1) to directly substitute for q shows that:

$$
\begin{aligned}
p_A^* &= \frac{e}{2\delta}\frac{2 + c\delta\left(2\Delta\theta_B - 3e\right)}{2 - 3c\delta e} \\
p_B^* &= \frac{e}{2\delta}\frac{2 - c\delta\left(2\Delta\theta_B + 3e\right)}{2 - 3c\delta e} \\
a_A^* &= \frac{1}{2c\delta}\frac{2 + c\delta\left(2\Delta\theta_B - 3e\right)}{2 - 3c\delta e} \\
a_B^* &= \frac{1}{2c\delta}\frac{2 - c\delta\left(2\Delta\theta_B + 3e\right)}{2 - 3c\delta e}
\end{aligned}
$$

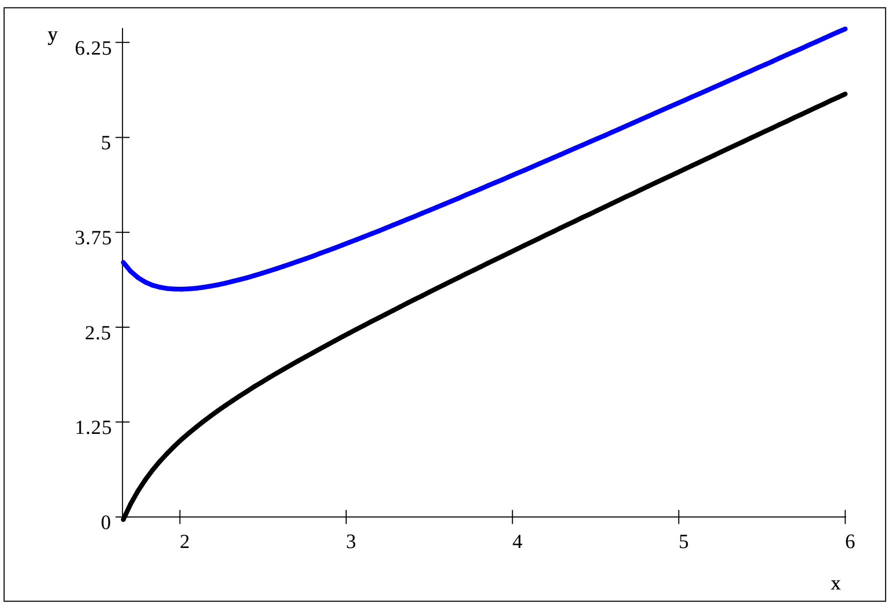
Let $c = 1, \delta = 1/2, \Delta\theta_B = -1/2$: the following plot shows (a_A^* in blue, a_B^* in black) that e (on the hz axis) must be at least as large as $5/3$ to have both advertisement levels positive.



Now, the same with $\Delta\theta_B = 1/2$:



and the price levels for the same parameters:



It emerges that:

Modified

Proposition 3 *When condition 1 holds, the equilibrium is interior (i.e.* p_J^**,*$a_J^* > 0$ *for* $J = A, B$*). Moreover, when the degree of differentiation, e, increases: the advertisement of the best/worst molecule decreases/increases; the price of the worst molecule increases; and both the advertisement and price gaps decrease.*

NEW

Remark: this means that me HAVE TO IMPOSE CONDITION 1 if we want to use comparative statics and exploit calculus (FOCs) to characterize the equilibrium in a simple way. Otherwise, we have to assume that one of the variables is set to zero, e.g. zero advertising by one firm, check the best responses of the other firm, and the price of the firm with zero advertisement, and then check back if indeed zero advertisement is part of the best response for the former firm... That looks ugly!!!

Proof. To show the results concerning a_J^*, note that:

$$a_A^* = \frac{1}{2c\delta} \frac{2 + c\delta(2\Delta\theta_B - 3e)}{2 - 3c\delta e} = \frac{1}{2c\delta} - \frac{\Delta\theta_B}{3c\delta e - 2}$$

$$\frac{\partial}{\partial e} = \frac{3c\delta}{(3c\delta e - 2)^2} \Delta\theta_B,$$

which is negative iff $\Delta\theta_B < 0$, i.e. if A has the best drug.

Concerning the price: let $\Delta\theta_B < 0$, so that B is the worst molecule, and let us look at the evolution of the price of B. From (7):

$$\frac{\partial F_\varepsilon}{\partial e} = -3c\delta \frac{\Delta\theta_B c\delta}{(3c\delta e - 2)^2} = \frac{-3c\delta}{3c\delta e - 2}(F_\varepsilon - 1/2).$$

From (6):

$$\begin{aligned} \frac{\partial P_B}{\partial e} &= \frac{F_\varepsilon}{\delta} + \frac{e}{\delta}\frac{\partial F_\varepsilon}{\partial e} \\ &= \frac{F_\varepsilon}{\delta} - \frac{3ec}{3c\delta e - 2}(F_\varepsilon - 1/2) \\ &= \frac{3c\delta e - 2 - 3c\delta e}{\delta(3c\delta e - 2)} F_\varepsilon + \frac{3ec/2}{3c\delta e - 2} \\ &= \frac{3c\delta e/2 - 2F_\varepsilon}{\delta(3c\delta e - 2)}, \end{aligned}$$

which is strictly positive if $F_\varepsilon < 1/2$, and by (7) and Proposition 2, the latter happens iff $\Delta\theta_B < 0$.

Other possible proof: let $\Delta\theta_B > 0$ and look at p_A^*:

$$p_A^* = \frac{e}{2\delta} \frac{2 + c\delta(2\Delta\theta_B - 3e)}{2 - 3c\delta e} = \frac{e}{2\delta}\left(1 - \frac{2c\delta}{3c\delta e - 2}\Delta\theta_B\right)$$

$$\begin{aligned}
\frac{\partial p_A^*}{\partial e} &= \underbrace{\frac{1}{2\delta}\left(1-\frac{2c\delta}{3c\delta e-2}\,\Delta\theta_B\right)}_{>0}+\frac{3c^2\delta e}{\left(3c\delta e-2\right)^2}\,\Delta\theta_B, \\
&= \frac{1}{2\delta}-\frac{2c\delta}{3c\delta e-2}\,\frac{\Delta\theta_B}{2\delta}+\frac{3c^2\delta e}{\left(3c\delta e-2\right)^2}\,\Delta\theta_B \\
&= \frac{1}{2\delta}+\frac{3c^2\delta e-c\left(3c\delta e-2\right)}{\left(3c\delta e-2\right)^2}\,\Delta\theta_B \\
&= \frac{1}{2\delta}+\frac{2c}{\left(3c\delta e-2\right)^2}\,\Delta\theta_B
\end{aligned}$$

■

4 Competition post generic entry

Here, we are looking at the "long-run", i.e. when A has been wiped out of the market.

By assumption, many generic producers compete against one another. Since their drugs are perfect substitutes, we assume that the price is equal to marginal cost: $P_G = 0$. The first order condition on a_G is similar to (3), and implies $a_G^* = 0$. Thus, we are in a situation in which B competes against a molecule that is cheap but no longer advertised.

Therefore, B's market share is:

$$q_B = F_\varepsilon\left[\Delta\theta_B + a_B - \delta\cdot P_B\right] = \frac{1}{2}+\frac{\Delta\theta_B + a_B - \delta\cdot P_B}{e}, \qquad (8)$$

and (4) and (5) are still valid. It thus follows that:

$$\begin{aligned}
P_B &= \left(\frac{1}{2}+\frac{\Delta\theta_B + a_B - \delta\cdot P_B}{e}\right)\frac{e}{\delta} \\
P_B &= \frac{e}{4\delta}+\frac{\Delta\theta_B + a_B}{2\delta}
\end{aligned}$$

This implies:

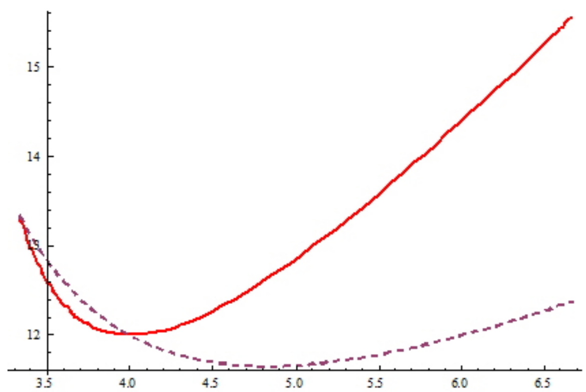
$$\begin{aligned}
q_B &= F_\varepsilon\left[\Delta\theta_B + a_B - \delta\cdot P_B\right] = \frac{1}{2}+\frac{\Delta\theta_B + a_B}{e}-\left(\frac{1}{4}+\frac{\Delta\theta_B + a_B}{2e}\right) \\
&= \frac{1}{4}+\frac{\Delta\theta_B + a_B}{2e}
\end{aligned}$$

Substituting q_B into (5) then yields:

$$\begin{aligned} c\delta a_B &= \frac{1}{4} + \frac{\Delta\theta_B + a_B}{2e} \\ \frac{2c\delta e - 1}{e} a_B &= \frac{e + 2\Delta\theta_B}{2e} \\ a_B^{**} &= \frac{e/2 + \Delta\theta_B}{2c\delta e - 1}. \end{aligned}$$

$$\begin{aligned} P_B^{**} &= \frac{e}{4\delta} + \frac{\Delta\theta_B + \frac{e/2+\Delta\theta_B}{2c\delta e-1}}{2\delta} \\ &= \frac{e}{4\delta} + \frac{\Delta\theta_B\left(2c\delta e - 1 + 1\right) + e/2}{2\delta\left(2c\delta e - 1\right)} = \frac{e}{4\delta} + \frac{4\Delta\theta_B c\delta e + e}{4\delta\left(2c\delta e - 1\right)} \\ &= \frac{4\Delta\theta_B c\delta e + e + \left(2c\delta e^2 - e\right)}{4\delta\left(2c\delta e - 1\right)} = \frac{2\Delta\theta_B ce + ce^2}{2\left(2c\delta e - 1\right)} = \frac{2\Delta\theta_B + e}{2\left(2c\delta e - 1\right)} ce \end{aligned}$$

Comparing P_B^* (in red) and P_B^{**} (in dashed): $\theta_A = 5$; $\theta_B = 6$; $\delta = 1/4$; $c = 1$ and e on the hz axis. Parameters chosen to ensure interior solutions everywhere.



`[Remark: the optimal prices are U-shaped in` e`!]`

Proposition 4 *$q_B^{**} > q_B^*$, $P_B^{**} > P_B^*$ and $a_B^{**} > a_B^*$ iff $c\delta e < 1$.*

Proof. *First, we prove this for the symmetric case $\Delta\theta_B = 0$:*

$$\begin{aligned} q_B^{**} &= \frac{1}{4} + \frac{a_B}{2e} = \frac{1}{4} + \frac{\frac{e/2}{2c\delta e-1}}{2e} = \frac{1}{4} + \frac{\frac{1}{2c\delta e-1}}{4} \\ &= \frac{1}{4}\frac{2c\delta e - 1 + 1}{2c\delta e - 1} = \frac{1}{2}\frac{c\delta e}{2c\delta e - 1} \end{aligned}$$

$$q_B^* = \frac{1}{2}$$

Thus $q_B^{**} > q_B^*$ *iff:*

$$\begin{aligned} \frac{c\delta e}{2c\delta e - 1} &> 1 \\ c\delta e &> 2c\delta e - 1 \\ 1 &> c\delta e \end{aligned}$$

Since:

$$\begin{aligned} P_B &= \frac{e}{\delta} F_\varepsilon \\ a_B &= \frac{F_\varepsilon}{c\delta}, \end{aligned} \tag{9}$$

$1 > c\delta e$ *also implies:* $P_B^{**} > P_B^*$ *and* $a_B^{**} > a_B^*$. ■

Proof. *Now, for the general case:*

$$\begin{aligned} q_B^{**} &= \frac{1}{4} + \frac{\Delta\theta_B + \frac{e/2 + \Delta\theta_B}{2c\delta e - 1}}{2e} = \frac{1}{4} + \frac{\Delta\theta_B (2c\delta e - 1) + e/2 + \Delta\theta_B}{2e (2c\delta e - 1)} \\ &= \frac{1}{4} + \frac{4c\delta e \ \Delta\theta_B + e}{4e (2c\delta e - 1)} = \frac{2c\delta e^2 - e + 4c\delta e \ \Delta\theta_B + e}{4e (2c\delta e - 1)} \\ &= \frac{c\delta e + 2c\delta \ \Delta\theta_B}{2 (2c\delta e - 1)} = \frac{c\delta e/2}{2c\delta e - 1} + \frac{c\delta \ \Delta\theta_B}{2c\delta e - 1} \end{aligned}$$

$$q_B^* = \frac{1}{2} + \frac{c\delta \ \Delta\theta_B}{3c\delta e - 2}$$

$$\begin{aligned} q_B^{**} &= \frac{c\delta e/2}{2c\delta e - 1} + \frac{c\delta \ \Delta\theta_B}{2c\delta e - 1} > \frac{1}{2} + \frac{c\delta \ \Delta\theta_B}{3c\delta e - 2} = q_B^* \\ \frac{c\delta e/2 - c\delta e + 1/2}{2c\delta e - 1} &> \frac{c\delta \ \Delta\theta_B}{3c\delta e - 2} - \frac{c\delta \ \Delta\theta_B}{2c\delta e - 1} \\ \frac{1}{2} \frac{1 - c\delta e}{2c\delta e - 1} &> \frac{2c\delta e - 1 - (3c\delta e - 2)}{(3c\delta e - 2)(2c\delta e - 1)} c\delta \ \Delta\theta_B \\ \frac{1}{2} \frac{1 - c\delta e}{2c\delta e - 1} &> \frac{1 - c\delta e}{(3c\delta e - 2)(2c\delta e - 1)} c\delta \ \Delta\theta_B \end{aligned}$$

If $1 - c\delta e < 0$:

$$\frac{3c\delta e - 2}{2c\delta} < \Delta\theta_B,$$

which violates condition 1. So, for an initially interior equilibrium, $1 - c\delta e < 0$ *implies that* $q_B^{**} < q_B^*$. *Conversely,* $1 - c\delta e > 0$ *implies that* $q_B^{**} > q_B^*$. *Again, these effects extend to prices and advertisement.* `[Beware: for some parameter values,` $P_B^* > 0$ `but` $P_B^{**} < 0$ `-- corner solution]` ■

5 Side case: if A only competes with G

In this case, each PDP buys A iff:

$$\begin{aligned}\theta_A + a_A + \varepsilon^i - \delta p_A &> \theta_A + \gamma^i a_A + \varepsilon^i - \delta p_G \\ \left(1-\gamma^i\right) a_A &> \delta\ \Delta p_A.\end{aligned}$$

That is, PDPs with $\gamma^i = 1$ have no generic aversion and buy the cheapest product. For those with $\gamma^i = 0$, it is as if advertisement for A has no spillover effect on the demand for G. It follows that:

$$q_A = F_\gamma \left(1 - \frac{\delta\ \Delta p_A}{a_A}\right)$$

Normalizing p_G to 0 and taking FOCs:

$$\begin{aligned}P_A^{*G} &= \frac{q_A^3}{\left(f_\gamma \delta\right)^2} \\ a_A^{*G} &= \frac{q_A^2}{c\delta f_\gamma}\end{aligned}$$

We can compare this with the case in which there is A vs B and $\Delta\theta_B = 0$:

$$a_A^{*B} = \frac{1}{2c\delta}.$$

The ratio between the two is thus:

$$\frac{a_A^{*G}}{a_A^{*B}} = \frac{2q_A^2}{f_\gamma},$$

so advertisement can go up or down, depending on f_γ. Not very interesting I believe!

6 Monopoly

Following Maria-Angeles's idea, we need to identify the parameters such that, if A or B was alone, it would cover the whole market. For A this means that a_A^* and p_A^* are such that:

$$\theta_A + a_A^* - \frac{e}{2} = \delta p_A^* \tag{10}$$

Otherwise,

$$q_A = \frac{1}{2} + \frac{\theta_A + a_A - \delta p_A}{e}.$$

Normalizing k to 1, profits are:

$$p_A\ q_A - C(a_A) = \frac{\theta_A + a_A^* - \frac{e}{2}}{\delta} - ca_A^2/2,$$

if (10) holds. In this case:

$$\frac{\partial \pi_A}{\partial a_A} = \frac{1}{\delta} - ca_A = 0 \Leftrightarrow a_A = \frac{1}{c\delta}.$$

In that point, one must have:

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_A} &= \underbrace{\frac{1}{2} + \frac{\theta_A + \frac{1}{c\delta} - \delta p_A}{e}}_{q_A \equiv 1} - p_A \underbrace{\frac{\delta}{e}}_{\partial q_A/\partial p_A} < 0. \\ 1 &< p_A \frac{\delta}{e} \Leftrightarrow p_A > e/\delta \end{aligned}$$

This must be negative in $\theta_A + a_A - \delta p_A = e/2$:

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_A} &= 1 - p_A \frac{\delta}{e} < 0 \\ e/\delta &< p_A = \frac{\theta_A + \frac{1}{c\delta} - \frac{e}{2}}{\delta} \\ \frac{3e}{2} &< \theta_A + \frac{1}{c\delta} \\ \theta_A &> \frac{3e}{2} - \frac{1}{c\delta} \end{aligned}$$

$$\boxed{\theta_A > \frac{3c\delta e - 2}{2c\delta}}$$

Logically, this is also a sufficient condition for B since the constraint must be more binding for $i = -e/2$ when considering buying A than when considering buying B.