

# Trade disclosure and price dispersion <sup>☆</sup>

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## Abstract

This paper studies the implications of trade reporting in a two-stage trade model similar to Journal of Financial Economics 14, 71–100. We find that the degree of market transparency has important effects on market equilibria. In particular, we show that dealers operating in a transparent structure set regret-free prices at each period. In contrast, dealers in an opaque market invest in acquiring information at the beginning of the trading day. Moreover, we show that in equilibrium there is price dispersion in the opaque market, whereas this is not the case if orders are reported. Additionally, we show that trade disclosure increases the informational efficiency of transaction prices and reduces volatility. Finally, concerning the welfare of market participants, we obtain ambiguous results.

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## 1. Introduction

As we enter the 21st century the demand for a global equity market seems to be growing. Investing institutions, investment banks and companies are already increasingly global, and technology is pushing in that direction. There is however a possible problem lurking in the wings: regulation. Already regulatory differences are complicating the existence of a single European equity market as requirements on accounting standards, provisions for disclosure or transparency rules all vary hugely between markets. The need to better understand the relationship between market structure and market quality is greater than ever in a EU seeking to construct a single financial market.

One of the challenges regulators face is agreeing on the desired level of transparency in stock exchange dealings.<sup>1</sup> In the United States, Arthur Levitt, who was chairman of the **Securities and Exchange Commission (SEC)** throughout the Clinton presidency, devoted much time to improving standards of disclosure and transparency in the equity markets. The view of the Commission is straightforward: “The Commission has long believed that transparency the real time, public dissemination of trade and quotation information plays a fundamental role in the fairness and efficiency of the secondary markets...transparency helps to link dispersed markets and improves the price discovery, fairness, competitiveness and attractiveness of US markets.”<sup>2</sup> In the same vein, the SEC also argued “...transparent disclosure of quotes and trades promotes best execution.”<sup>3</sup> In contrast, in the UK, the **Securities and Investment Board (SIB)** has argued that there are important differences between quotation transparency and trade transparency, and that transparency (in the context of prompt publication of large trades) should be restricted if it is necessary to assure adequate liquidity.<sup>4</sup>

The Federation of European Securities Exchanges (FESE) has made it clear that real time reporting is basically state of the art and should be the standard in Europe’s financial markets. It claims that reporting of “standard” trades at the end of the day is not deemed sufficient.<sup>5</sup> Nevertheless, markets with low degrees of transparency seem to be doing quite well. In less than forty years, the Eurobond market has gone from zero to becoming the second largest bond market in the world, and the largest for corporate bonds. And that happened despite the market’s having almost none of the characteristics which are claimed by some to be essential for a

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<sup>1</sup>Real world trading systems exhibit considerable heterogeneity in the degree of transparency they offer. Automated limit order book systems such as the type used by the Toronto Stock Exchange and the Paris Bourse offer high degrees of transparency. Foreign exchange and corporate junk bond markets offer very little transparency, whereas other dealer markets such as Nasdaq or the London Stock Exchange offer moderate degrees of transparency (see **Madhavan, 2000**).

<sup>2</sup>See SEC Market 2000 Study, Chapter IV-1.

<sup>3</sup>See Release No. 34-36310; File No. S7-30-95.

<sup>4</sup>For a further discussion of these issues see **Bloomfield and O’Hara (1999)**.

<sup>5</sup>Many non-exchange traded securities are traded in markets which are virtually opaque or dependent on newsletter-like surveys for price discovery. Similarly, some derivatives are traded over-the-counter in markets where trading information is not readily available as is the case with futures and options exchanges.

market which is fair and efficient for all. Compared to equity markets, its transparency has been limited, and yet investors have received good returns over many years and their confidence in the market has been steadfastly maintained. Is there then a basis for imposing post trade transparency on transactions?

Transparency is generally regarded as central to influencing the liquidity and quality of price formation. Changes in transparency regimes alter the information sets of market participants, change their optimal behavior and hence influence the price formation process. Prices, on the other hand, have an impact on not only the fairness and efficiency of the markets, but also on their attractiveness. The economic literature has shown steady interest in the welfare implications of transparency. It has been a popular belief that open sharing of information is beneficial to market participants. However, real life markets offer mixed evidence.<sup>6</sup> How transparency affects market behavior is a question addressed by several papers, which shows both the importance and the complexity of this issue. Our contribution here is to delve deeper into the effects of post trade transparency in the performance of the market in a model based on [Glosten and Milgrom \(1985\)](#). We think that the study of this issue within a well known framework may help in understanding its implications. More precisely, we model a quote driven market in which the daily trade takes place in two different intervals of time, whereas new information only arrives at the beginning of the day. Using this set up we compare two market structures: a post trade transparent market, in which trades are made public, and a post trade opaque market, in which trades are not disclosed. In both market structures the information contained in customer orders is valuable.<sup>7</sup> To undertake the comparison we first provide an explicit characterization and computation of dealers' equilibrium pricing strategies in the two market structures. This allows us to understand what the driving forces behind dealers' behavior are.

We show that prices in an opaque market result from the interplay between informational and strategic considerations, whereas in a transparent market prices are only informationally driven. Dealers operating in a transparent market set regret free prices at each period making zero expected profits in each of the two trading rounds. By contrast, dealers in an opaque market set prices away from the short run equilibrium. We show that in the opaque market structure price setting dealers invest in acquiring information by setting more attractive prices from investors' viewpoint at the beginning of the trading day. They depart from maximizing current expected profits in order to produce information that will yield

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<sup>6</sup>On one hand, most public B2B exchanges have found great difficulties in signing up suppliers. Furthermore, many firms have switched from public exchanges to private ones, which are less transparent. For example, Cisco, Dell, and Hewlett-Packard have established private exchanges with their suppliers and business partners (see [Zhu, 2004](#)). On the other hand, the market for electricity derivatives, which lacks any transparency, has suffered from the collapse of Enron, a major innovator and trader of electricity derivatives. It may be argued that many of the current problems with electricity derivatives result from problems in the underlying market for electricity itself, which is also quite opaque.

<sup>7</sup>Benn Steil, an analyst at the Council on Foreign Relations, has argued in an interview with the Economist that NYSE specialists' enviably profitability is linked largely to their knowledge of order flow (see [The Economist, May 5th 2001](#)).

future expected profits.<sup>8</sup> More importantly, in equilibrium, dealers try to attract order flow in both directions, i.e., they try to be competitive in both sides of the market and to do so they jointly set their ask and bid.<sup>9</sup> This result departs from most of the theoretical results in the literature on dealer markets where dealers set their asks and bids independently. Nevertheless, our result is consistent with the findings in the experimental works by Bloomfield and O'Hara (1999, 2000) on how dealers behave in opaque and in transparent markets.

This paper provides a series of testable predictions on the impact of transparency on price dynamics. Note that the explicit computation of the equilibrium pricing strategies allows us to examine the impact of market opaqueness on metrics of market quality such as spreads, volatility and price efficiency. Some of the results we obtain are similar to those delivered by Madhavan (1995) or Bloomfield and O'Hara (2000), among others. Nevertheless, we also offer new predictions that might be useful to econometricians with price data. Among the similar results we also find that post trade opaqueness has the following effects. (1) It results in a reversal in the normal intraday pattern of the bid ask spread. (2) It increases price volatility because the differences between the dealers who transact in the two periods are bigger in the opaque market and this is reflected in prices. (3) It reduces transaction price efficiency because less information is impounded in prices. The main new predictions we offer may help to reconcile the theoretical results on trade disclosure with the empirical evidence from opaque markets such as the FX market or the corporate junk bond markets. In particular, (1) We show that transaction prices in an opaque market do not follow a martingale, and consequently, the first order differences in prices may not be uncorrelated. This topic is of relevance since until now in all the information based models this property was satisfied.<sup>10</sup> (2) In the opaque market, spreads increase over time even if there was no order in the past.<sup>11</sup> Similarly, we show that if dealers have asymmetric beliefs about the value of the security, then spreads are history dependent. (3) Finally, we show that prices in opaque markets are more spread out. In particular we show that in equilibrium there is price dispersion in the opaque market whereas this is not the case if trades are reported.<sup>12</sup>

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<sup>8</sup>The fact that market makers experiment with prices is not new. For instance, Leach and Madhavan (1992, 1993) deliver price experimentation when trading is accomplished through a single market maker (i.e., a specialist). By contrast, in the present paper we show that this phenomenon can occur in a market with a competitive group of market makers if the market is opaque.

<sup>9</sup>This result may seem to be in conflict with the empirical evidence reported in Hansch et al. (1998) which suggests that a majority of dealers try to attract order flow primarily in one direction. Note that their empirical findings are consistent with the inventory model of dealership markets. Since in our model dealers are risk neutral, there is no contradiction between their empirical findings and our results.

<sup>10</sup>Note that in many of these models post-trade transparency is either explicitly or implicitly assumed. We find here that this serial uncorrelation property depends on the degree of post-trade transparency of the market.

<sup>11</sup>Peng (2001) provides evidence that the bid-ask spreads increase over time when no orders arrive. This empirical finding is supportive of this prediction.

<sup>12</sup>Empirical research on the corporate junk bond market (an opaque market) shows evidence of price dispersion across dealers. See, for instance, Saunders et al. (2002) where this finding is present for a sample of bond trades conducted by a major asset manager/dealer in the OTC corporate bond market.

Our study builds on a large body of research investigating how transparency affects market behavior. A part of this literature has analyzed how transparency before the trade, pre trade transparency, affects market behavior. Some of these studies have focused on issues related to *visibility of market orders*. So, Madhavan (1996) shows that disclosing information about the composition of order flows can increase price volatility and lower market liquidity. Pagano and Röell (1996) find that trading costs for uninformed traders are generally lower in more transparent markets. Some other studies have focused on the *visibility of market quotes*. In particular, Biais (1993) and Frutos and Manzano (2002) compare centralized and fragmented markets and show that the ability to observe price setters' quotes affects spreads and the welfare of market participants.

Another part of this literature has focused on the delayed reporting of trades. In particular, Madhavan (1995) shows that delayed publication benefits large traders who place multiple trades. Gemmill (1996) analyzes the effects of changing trade reporting requirements on the London Stock Exchange. He concludes that disclosure does not have a relevant effect on liquidity trading. By contrast, Porter and Weaver (1998) show that dealers in the NASDAQ systematically delay trade reporting, which suggests that it is beneficial to them. Recently, experimental studies have been used to test theories concerning market structure. Naik et al. (1999) find that the full and prompt disclosure of first stage trade details may reduce the welfare of the public investor. The closest paper to ours is Röell (1991) which is also based on Glosten and Milgrom. Röell studies trade reporting by comparing a transparent and an opaque market in a two stage dynamic game. The main differences between the two papers are to be found in the models' specification and in the equilibria implications. In Röell, the size of the order flow may convey information. This assumption is crucial, as the active market maker will learn perfectly the identity of the investor via his order size. Therefore, in her set up, an active market maker knows whether an informed trader has traded, and whether he bought or sold. By contrast, in our model only unitary orders are considered, which implies that a market maker never knows the identity of the investor she has traded with. Röell's paper and ours also differ in the tie breaking rule employed whenever dealers set identical prices (identical asks and bids). Here ties are broken by flipping a coin so that a dealer might get to attend a sell but not a buy. In contrast, in Röell's work, nature picks a dealer and she will attend both sides of the market in the event of a tie. Due to this difference, Röell provides a symmetric equilibrium in pure strategies for the opaque market, whereas no symmetric equilibrium exists for the opaque market under our tie breaking rule. With respect to their implications, the two papers differ from each other as well. The main contribution of our paper, not only from a theoretical perspective but also from an empirical viewpoint, is the finding that price dispersion is an equilibrium phenomenon. The empirical research on opaque markets (recall footnote 12) has shown evidence of price dispersion across dealers. This finding is not compatible with an equilibrium price schedule involving symmetric pure strategies in the first period like the one proposed by Röell.

The article is organized as follows. In the next section we present a sequential trade model. Section 3 characterizes the equilibrium in the transparent market. Section 4

derives dealers' pricing strategies in an opaque market. Section 5 compares some market indicators corresponding to both market structures. Section 6 discusses the robustness of the results. Concluding comments are presented in Section 7. Proofs are included in the Appendix.

## 2. The model

In this section we describe the basic structure of our sequential trade model, which is similar to [Glosten and Milgrom \(1985\)](#) or [Easley and O'Hara \(1987\)](#). We consider an economy with a single risky asset, whose liquidation value is denoted by  $v$ . The risky asset can take on two possible values, 0 and 2, both equally likely. Potential buyers and potential sellers trade the risky security with market makers or dealers, who are responsible for supplying liquidity by simultaneously setting prices at which they will buy and sell the asset. We will assume that there are only two dealers, dealer  $D$  and dealer  $D'$ , who are both risk neutral.

Liquidity is demanded by two possible types of investors: informed traders and uninformed traders.<sup>13</sup> Informed traders know the liquidation value of the risky asset perfectly. If an informed trader observes the high liquidation value, then she will buy the stock if the smallest ask price is below 2; if she has observed the low liquidation value, then she will sell it if the highest bid price is above 0. Uninformed traders do not know the liquidation value and they are hence equally likely to be potential buyers or potential sellers. They differ in their trading motivations. These may reflect their liquidity needs, their price sensitivity, or individual specific trading rules. These factors influence the willingness of an uninformed trader to transact. We will here assume that with probability  $1 - p$  they decide not to trade.<sup>14</sup>

Trades occur throughout the trading day. We divide the day into two intervals of time,  $t = 0$  and  $t = 1$ . At each time interval, first market makers select ask and bid prices at which they are willing to sell or to buy one unit of the asset. Then, a trader is selected according to a probabilistic arrival process described below, and she decides her order size; i.e., whether to buy one unit at the smallest ask price ( $q_t + 1$ ), to sell one unit at the highest bid price ( $q_t - 1$ ), or not to trade at all ( $q_t = 0$ ).

The probabilistic structure of a trading day is depicted in the tree diagram in [Fig. 1](#). The first node of the tree corresponds to nature selecting whether information will be good or bad. This node is only reached at the beginning of the trading day, meaning that new information occurs only between trading days. In the second node, an investor is selected. With half probability she is an informed trader, and with the complementary probability she is an uninformed trader. The third node corresponds

<sup>13</sup>It is well known that in order to avoid a no-trade equilibrium, at least some traders must transact for non-speculative reasons such as liquidity needs (see [Milgrom and Stokey, 1982](#)).

<sup>14</sup>[Easley and O'Hara \(1992\)](#) propose a similar specification. [Leach and Madhavan \(1993\)](#) likewise allow the possibility of no trade by an uninformed investor but in their model the probability with which no trade occurs is determined endogenously, whereas here it is assumed to be an exogenous variable.

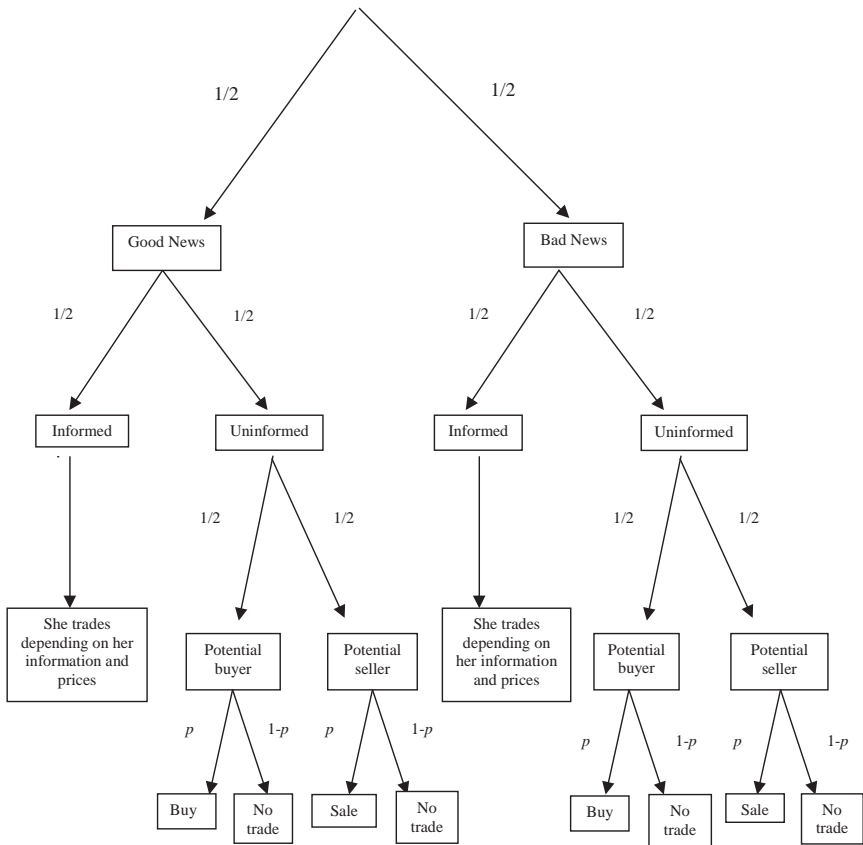


Fig. 1. The probabilistic structure of trade. At the beginning of the trading day, nature chooses the type of information. Then, in the second node, an investor (informed or uninformed) is selected. The third node corresponds to the trading decision each trader will make if he has an opportunity to trade. Finally,  $p$  is the probability that an uninformed trader in the market will choose to trade.

to the trading decision each trader will make if given the opportunity to trade. Whether an informed trader buys or sells depends upon the relationship between the value of the risky asset she has observed and the prices. An uninformed trader is equally likely to be a potential buyer or a potential seller. Moreover, she will trade with probability  $p$ , with  $1 > p > 0$ , and will not trade with the remaining probability  $1 - p$ . Note that when  $p$  approaches zero, an order to trade can only come from an informed trader. When  $p$  approaches one, uninformed traders always choose to trade. Throughout the paper we further assume that  $p = 1/2$  which facilitates exposition; however, as we go along, we will explain how the results extend easily to

any  $p \in [0, 1]$ . At the end of this second interval of time the liquidation value of the risky asset is made public and agents consume.

Since the problem we are addressing involves a multi dealer dynamic pricing game of incomplete information, the equilibrium concept we use is that of perfect Bayesian equilibrium. We search for dealers' pricing strategies with the typical property found in rational expectations models of incorporating the information the trade itself reveals.

Finally, the liquidity value of the risky asset and the investor's arrival process are assumed independent random variables. The joint distribution of all these random variables will be common knowledge.

### 3. The post-trade transparent market

In a transparent market, at the end of the first round, all the trading information related to  $t = 0$  is publicly disclosed. Both dealers will hold the same information and will hence quote the same bid and ask prices. Further, competition combined with risk neutrality dictates that any rents earned on trades would be bid away. Consequently, prices will equal reservation quotes so that the expected profit on any trade will be zero. The result is due to the price competition among symmetric risk neutral dealers.

When computing the optimal pricing strategies, we consider that the informed trader's strategy is to sell when  $v = 0$  and to buy if  $v = 2$ . We will then show the optimality of this behavior. The following proposition explicitly characterizes the Bayes Nash equilibrium in a market with post trade transparency.<sup>15</sup>

**Proposition 1.** *There exists a unique Bayes Nash equilibrium in the post trade transparent market, where the equilibrium price quotation function at  $t = 0$ ,  $P_0(q_0)$  satisfies*

$$P_0(q_0) = E(v|q_0) = 1 + \frac{2}{3} q_0,$$

and the equilibrium price quotation function at  $t = 1$ ,  $P_1(q_1; q_0)$  satisfies

$$P_1(q_1; q_0) = E(v|(q_0, q_1)) \begin{cases} 1 + \frac{12}{13} q_1 & \text{if } q_1 = q_0 \neq 0, \\ 1 & \text{if } q_1 \neq q_0 \neq 0, \\ 1 + \frac{2}{3} q_1 & \text{if } q_0 = 0. \end{cases}$$

The logic behind these expressions is clear. Consider, for instance, a potential sequence of buy orders. Dealers know that this potential sequence of trades could be generated by (1) the independent arrival of informed traders in periods 0 and 1, (2) the arrival of an informed trader in period 0 followed by a liquidity trader in period 1, (3) the arrival of a liquidity trader in period 0 followed by an informed trader in

<sup>15</sup>The derivation of the equilibrium in our transparent market is similar to that of Roëll (1991) and Madhavan (1995). However, the equilibria are not identical because of the different models we consider.



period 1, or (4) the independent arrival of liquidity traders in both periods. In cases (1) (3) above, there is an investor who is informed, and her order reveals that the value of the risky asset is the high one, whereas in case (4) the potential sequence of trades is not informative about the liquidation value. Since  $1/13$  is the conditional probability of case (4) given the potential sequence of trades, it follows that  $E(v|q_0 = 1, q_1 = 1) = 25/13$ . Now, Bertrand competition combined with risk neutrality implies that  $P_1(1; 1) = 25/13$ . The expressions for the other prices are obtained similarly.

When the first round ends without any trading activity, dealers do not revise their quotes as they do not observe any new relevant information about  $v$ . Otherwise, following a trade, they set new prices since the type of trade has signal value about  $v$ . In particular, when there is trade continuation, a sell (buy) in the first period decreases (increases) the bid (ask) price in the second period. By contrast, when there is trade reversal, dealers set their ask and their bid in the second period equal to the ex ante expected value of the security. In either case, bids and asks are not symmetric around previous transaction prices. Note that the midquote following a sell (buy) is higher (lower) than the previous transaction prices.

Finally, equilibrium prices lie in the interval  $(0, 2)$ . This ensures that the informed trader's strategy to sell (buy) the asset if she has observed the low (high) value is optimal in both periods.

#### 4. The post-trade opaque market

In an opaque market, the trading information related to  $t = 0$  is not made public, which prevents free riding from non trading dealers. A dealer may hence now choose to invest in producing information by pricing more aggressively in the first round so as to use his private knowledge from trade to extract rents in the second round. By doing so, dealers depart from maximizing expected profits in each period to maximize the sum of their profits.<sup>16</sup>

An important feature of our modeling strategy is that the amount of information dealers can acquire in the opaque market depends on whether they choose to be *competitive* in both sides of the market or just in one side. A dealer who is competitive in both sides of the market is perfectly informed about the occurrence and sign of the order at the first round. A dealer who is competitive in one side becomes perfectly informed if he attended the order.<sup>17</sup> Otherwise, he is unable to distinguish between the event of no order arrival and the event in which the order was attended by his competitor. We will say that a dealer specializes if he chooses to

<sup>16</sup>Any equilibrium strategy in the opaque market must yield zero overall expected profits as there is price competition between risk-neutral dealers who are ex-ante identical. However, contrary to the transparent market, equilibrium strategies do not necessarily yield zero expected profits in each period.

<sup>17</sup>Throughout this section when we write "informed" or "uninformed" referring to a dealer, we are specifying his knowledge, or lack of it, about the order type in the first round, and not the knowledge about the liquidation value of the security.

be competitive only in one side of the market. We will say that a dealer invests in perfect learning if he chooses to be competitive in both sides of the market.

In order to obtain the optimal pricing strategies we solve the model by backward induction under the provision that an informed trader sells when  $v = 0$  and buys if  $v = 2$ . That is, given the information sets the dealers bring into the second round, we solve for the dealers' optimal pricing strategies at  $t = 1$ . Given these optimal strategies at  $t = 1$ , we then solve for the optimal strategies at  $t = 0$ .

#### 4.1. Optimal quotes at $t = 1$

In the second period there are two relevant continuation paths to consider depending on dealers' decision to be competitive in both sides of the market or just in one side:

- (1) The continuation path that follows dealers' specialization at  $t = 0$ , i.e., a dealer setting the best ask and his competitor setting the best bid.
- (2) The continuation path that follows investment in perfect learning at  $t = 0$ , i.e., a dealer setting both the best ask and the best bid.<sup>18</sup>

The key difference between these two paths is the amount of informational asymmetry among dealers. In the second path, we have the competition between a dealer with complete information and a dealer completely ignorant about the order type of the first period. By contrast, in the first path, a non trading dealer knows at least that the order was not the one for which he was competitive. Thus, in the second path one dealer follows his priors and the other revises his beliefs, whereas in the first one both dealers revise their beliefs but each one incorporates different information.<sup>19</sup>

At the second period of trade, dealers are not concerned about the learning effect of their actions, as there is no other period at which they may profit from the acquired information. It is hence optimal for them to treat each side of the market (buys and sells) independently. This independence and the symmetry of the model allows us to concentrate, without loss of generality, on buy orders so that we will here only develop the ask price.

##### 4.1.1. Specialization

Specialization gives rise to a game of incomplete information in which each dealer may have two types; i.e., there are four potential players. If dealer  $D$  specialized in buy orders and dealer  $D'$  in sell orders, then these potential players are dealer  $D$  who observed a buy order, dealer  $D$  who did not trade and hence does not know whether

<sup>18</sup>The continuation path that follows after dealers set equal prices in the first period can be analyzed by using the results for specialization and/or perfect learning. We will further elaborate on this point as we discuss these paths in more detail.

<sup>19</sup>The information set of a dealer at  $t = 1$  reflects all the values of  $q_0$  to which he assigns positive probability of their having happened.

$q_0 = 0$  or  $q_0 = 1$ , dealer  $D'$  who observed a sell order and, finally, dealer  $D'$  who did not trade and hence does not know whether  $q_0 = 0$  or  $q_0 = 1$ . We will refer to them as  $D(1)$ ,  $D(0, 1)$ ,  $D'(1)$  and  $D'(1, 0)$ , respectively. Note that the two types of each dealer correspond to the two possible information sets each dealer may have at  $t = 1$  when there is specialization. Furthermore, the realization of  $q_0$  will determine which types are actually present at  $t = 1$ .

To analyze specialization, we start by deriving the reservation ask prices.

**Lemma 2.** *If at  $t = 0$  dealer  $D$  set the best ask and dealer  $D'$  set the best bid, then the reservation selling quotes at  $t = 1$  are the following:*

$$A_{r,1}^{D(1)} = \frac{25}{13}, \quad A_{r,1}^{D'(1,0)} = 1 + \frac{2}{3}\eta + \frac{12}{13}(1 - \eta), \quad A_{r,1}^{D(0,-1)} = 1 + \frac{2}{3}\theta \quad \text{and} \\ A_{r,1}^{D'(-1)} = 1,$$

where

$$\theta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0, 1\}, q_1 = 1) = \frac{6}{11}$$

and

$$\eta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0, 1\}, q_1 = 1) = \frac{6}{19}.$$

In the opaque market reservation quotes differ across dealers. Those with pessimistic information about the value of the security can use this information to undercut the price of their competitors in order to make extra profits. This undercutting generates a situation similar to the Edgeworth cycle that results in the non existence of a pure strategy equilibrium.<sup>20</sup>

**Proposition 3.** *Under specialization there is no equilibrium continuation in which dealers use pure price strategies.*

Nevertheless there exists an equilibrium in which dealers randomize as shown in the next proposition.

**Proposition 4.** *If at  $t = 0$  dealer  $D$  set the best ask and dealer  $D'$  set the best bid, then the ask quotation at  $t = 1$  is set according to the following mixed strategies equilibrium:*

$D(1)$  who knows that the past order was a buy sets his reservation selling price,  $\frac{25}{13}$ .  
 $D'(1)$  who knows that the past order was a sell randomizes in the interval  $[S, Z]$

<sup>20</sup> $D(1)$  will set an ask price equal to his reservation ask price,  $25/13$ . If dealer  $D'(1, 0)$  were to match this price, then the best response by  $D(0, 1)$  is to slightly undercut this price. Given this strategy,  $D'(1, 0)$  finds it profitable to undercut so as to win no matter his opponent's type. This undercutting fosters a new undercutting by  $D(0, 1)$ , and prices will reach  $A_{r,1}^{D'(1,0)}$ . At this point, dealer  $D'(1, 0)$  is better off raising his price back to  $25/13$  and price undercutting begins anew. Dealers' best replies generate a cycle with no end. The intuition is similar to that in [Dennert \(1993\)](#) where no equilibrium in pure price strategies exists. There is a discontinuity of the payoffs such that even a slight change of prices produces a discontinuous shift in the expected market maker's clientele.

according to the cumulative distribution function

$$F_{D'(-1)}(A) = \frac{11}{5} \left( \frac{A - S}{A - 1} \right).$$

$D'(1, 0)$  with information set  $\{1, 0\}$  plays a mixed strategy with support  $[Z, \frac{25}{13}]$ . He assigns probability according to the distribution  $F_{D'(1,0)}$  which has a mass point at  $25/13$ , where

$$F_{D'(1,0)}(A) = \frac{3(A - Z)}{3A - 5}.$$

$D(0, -1)$  with information set  $\{0, -1\}$  randomizes in the interval  $[S, \frac{25}{13})$  according to the cumulative distribution function  $F_{D(0,-1)}$  which has a kink at  $Z$ . Furthermore,

$$F_{D(0,-1)}(A) = \begin{cases} \frac{A - S}{A - 1} & \text{if } S \leq A \leq Z, \\ \frac{19A - 35}{6A - 10} & \text{if } Z \leq A < \frac{25}{13}. \end{cases}$$

Finally, the values of  $S$  and  $Z$  are  $S = \frac{6Z+5}{11}$  and  $Z = \frac{335}{179}$ .

The properties of the mixed strategies equilibrium deserve some comments. In this equilibrium a dealer informed about a previous purchase never wins, and he sets an ask price equal to his reservation ask price. The two possible types of dealer  $D'$  randomize over linked pairs of prices.<sup>21</sup> In particular,  $D'(-1)$  randomizes in the interval  $[S, Z]$ , whereas  $D'(1, 0)$  randomizes in  $[Z, \frac{25}{13})$ . Dealer  $D'(1, 0)$  gets zero expected profit as he may share the market with dealer  $D(1)$ . Both types of dealer  $D'$  may have  $D(0, -1)$  as their opponent. Because of this,  $D(0, -1)$  randomizes over the union of the asks set by his two potential opponents, i.e., over the interval  $[S, \frac{25}{13})$ . Furthermore, he derives a positive expected profit from any of the ask prices he sets. The equilibrium strategies can be depicted by means of a box, shown in Fig. 2. In the  $x$  axis we arrange dealers depending on their willingness to transact. In the  $y$  axis, we plot prices. In this box, when analyzing the ask side, we consider the southwest corner as the origin. In contrast, we use the northwest corner as the origin when we study the bid side.

Both dealers' expected profits at  $t = 1$  coincide. We can hence focus, without loss of generality, on dealer  $D$ . First note that he can only benefit from his private information in case of reversal. If  $q_0 = 1$ , then he only makes profits if a sell order comes. Consequently, his expected profits are  $\frac{5}{24}(S - 1)$ , where  $\frac{5}{24}$  is the probability of reversal from a buy to a sell. If  $q_0 \neq 1$ , then he will only profit if  $q_1 = 1$ . The expected profits he will make equal  $\frac{11}{40}(S - \frac{15}{11})$ , where  $\frac{11}{40}$  is the probability that

<sup>21</sup>Randomization over linked pair of prices is also found in the dynamic auction analyzed by Frutos and Rosenthal (1998).

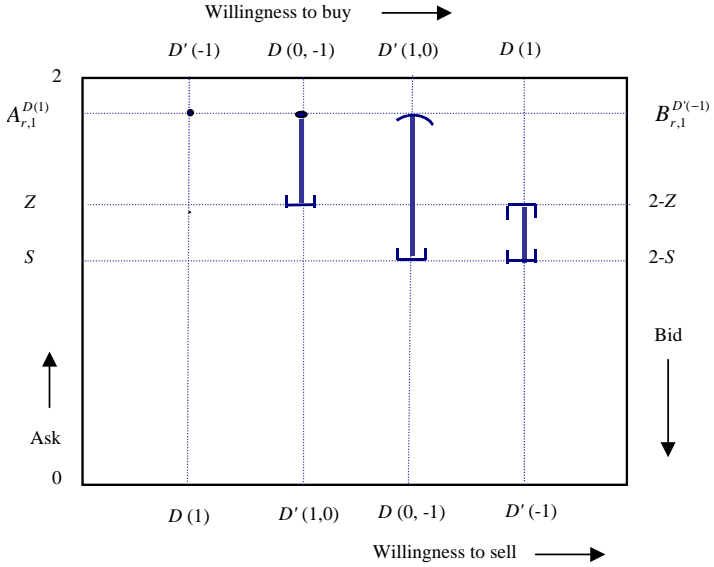


Fig. 2. Range of prices in the mixed strategies equilibrium under specialization. This figure shows the range of prices in the mixed strategy equilibrium assuming that, at  $t = 0$ , dealer  $D$  specialized on buy orders and dealer  $D'$  on sell orders. In this continuation path there are four potential players:  $D(1)$ ,  $D'(1,0)$ ,  $D(0,-1)$  and  $D'(-1)$ .  $D(1)$  represents dealer  $D$  who observed a buy order,  $D(0,-1)$  denotes dealer  $D$  who does not know whether  $q_0 = 0$  or  $1$ . The types of dealer  $D'$  are analogously defined. In the x-axis we arrange dealers depending on their willingness to transact. In the y-axis, we plot prices.

$D(0,-1)$  assigns to a future buy. Adding up, using Bayes' rule, the overall expected profits from trading at  $t = 1$  are  $\frac{1}{16}(4S - 5) = \frac{1775}{31504}$ .

#### 4.1.2. Investment in perfect learning

When a dealer is competitive in both sides of the market, at the end of the period he can perfectly infer the order size. Note that if he did not trade in the first period, then he correctly infers that  $q_0 = 0$ . Thus, under perfect learning by dealer  $D$  the potential players (or types of players) at  $t = 1$  are: dealer  $D$  who observed a buy order, dealer  $D$  who knows that  $q_0 = 0$ , dealer  $D$  who observed a sell order, and dealer  $D'$  who is completely uninformed. We will refer to them as  $D(1)$ ,  $D(0)$ ,  $D(-1)$  and  $U$ , respectively. The following lemma derives their reservation selling prices.

**Lemma 5.** *If at  $t = 0$  dealer  $D$  was competitive in both sides of the market, then the reservation selling prices at  $t = 1$  are the following:*

$$A_{r,1}^{D(1)} = \frac{25}{13}, \quad A_{r,1}^{D(-1)} = 1 \quad \text{and} \quad A_{r,1}^{D(0)} = A_{r,1}^U = \frac{5}{3}.$$

As in the previous path, dealers' information determines their willingness to sell. Obviously, the informed dealer has the same reservation quotes as if he were in a

transparent market, whereas the uninformed dealer holds the same reservation quotes in both periods. If there were trading activity at  $t = 0$ , then the informed dealer has a double advantage over his competitor. On the one hand, he has a more accurate estimate of the liquidation value of the risky asset. On the other hand, he knows precisely what information his competitor possesses. This double advantage will have an impact not only on profits but also on the pricing strategies of both dealers.

Price competition among asymmetrically informed dealers results in the non-existence of a pure strategy equilibrium.<sup>22</sup> The mixed strategies equilibrium is given in the next proposition.

**Proposition 6.** *Under perfect learning, the equilibrium ask quotation at  $t = 1$  is set according to the following mixed strategies:*

*$D(1)$  who knows that there was a buy order sets his reservation selling price,  $\frac{25}{13}$ .*

*$D(0)$  who knows that there was no trade randomizes by setting prices in the interval  $[\frac{35}{19}, \frac{25}{13}]$ . He assigns probability according to the distribution  $G_{D(0)}$ , where*

$$G_{D(0)}(A) = \frac{19A - 35}{6A - 10}.$$

*$D(-1)$  who knows that there was a sell order randomizes in the interval  $[\frac{5}{3}, \frac{35}{19}]$ . He assigns probability according to the distribution  $G_{D(-1)}$ , where*

$$G_{D(-1)}(A) = \frac{8}{5} \left( \frac{3A - 5}{A - 1} \right).$$

*Finally, the uninformed dealer randomizes in the interval  $[\frac{5}{3}, \frac{25}{13}]$  according to the cumulative distribution function  $G_U$ , which has a mass point at  $\frac{25}{13}$ , where*

$$G_U(A) = \begin{cases} \frac{3A - 5}{3(A - 1)} & \text{if } \frac{5}{3} \leq A \leq \frac{35}{19}, \\ \frac{36A - 65}{12(3A - 5)} & \text{if } \frac{35}{19} \leq A < \frac{25}{13}. \end{cases}$$

---

<sup>22</sup>In equilibrium, the dealer with the smallest reservation ask,  $D(-1)$ , will always undercut the price set by his competitor. The uninformed dealer accounts for this fact, and hence he optimally sets a price equal to the reservation ask of dealer  $D(1,0)$ . This behavior makes both  $D(-1)$  and  $D(0)$  slightly undercut the price set by dealer  $U$ . But then, the uninformed dealer only wins when meeting dealer  $D(1)$ . He accounts for this fact, and hence he optimally increases his price up to the reservation ask of dealer  $D(1)$ , making both  $D(-1)$  and  $D(0)$  further increase the quotes they offer, setting an ask slightly below the ask set by dealer  $U$ . But then the uninformed dealer can decrease his quote, beating any of his opponents while making profits. As the uninformed dealer resets the price, the informed immediately follows and price undercutting begins anew. This will activate behavior leading to a cycle with no end, which results in the non-existence of a pure strategy equilibrium.

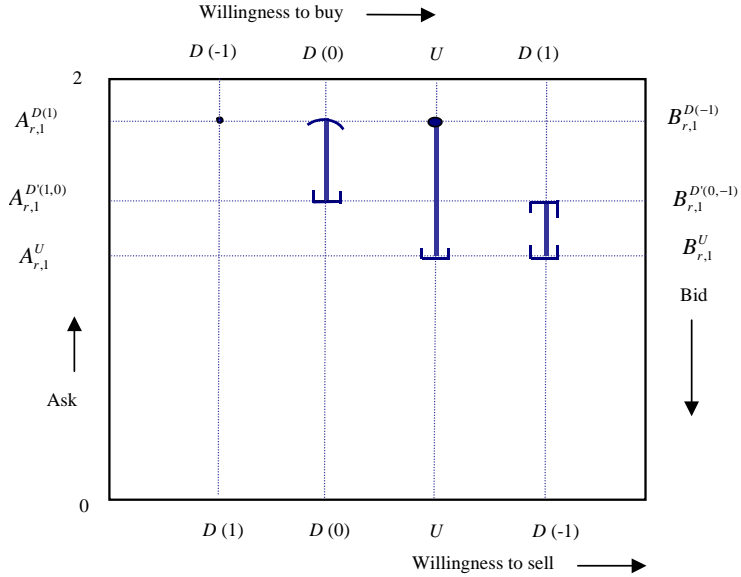


Fig. 3. Range of prices in the mixed strategies equilibrium under perfect learning. This figure shows the range of prices in the mixed strategy equilibrium assuming that, at  $t = 0$ , dealer  $D$  was competitive in both sides of the market. The potential players in this continuation path are:  $D(1)$ ,  $D(0)$ ,  $D(-1)$  and  $U$ .  $D(j)$  represents dealer  $D$  who observed  $q_0 = j$  and  $U$  denotes dealer  $D'$  who is completely uninformed about the order type in the first round. In the  $x$ -axis of this box we arrange dealers depending on their willingness to transact (note that  $D(0)$  and  $U$  have the same willingness to transact, we here plot them separately to facilitate the comprehension of this figure). In the  $y$ -axis, we plot prices.

In equilibrium, the potential types of the informed dealer  $D$  randomize over linked intervals of prices. Type  $D(-1)$  randomizes over the interval  $[\frac{5}{3}, \frac{35}{19}]$ , type  $D(0)$  randomizes over  $[\frac{25}{19}, \frac{35}{13}]$  and the type with the most positive information about the true value,  $D(1)$ , sets his reservation selling price,  $\frac{25}{13}$ . The uninformed dealer randomizes in the convex hull of the interval of prices used by the potential types of his competitor, i.e., in the interval  $[\frac{5}{3}, \frac{25}{13}]$ . He makes zero expected profits with any of the pure strategies he uses.

The equilibrium strategies of this continuation path are shown in the Fig. 3.

As in the previous continuation path, the informed dealer only expects positive profits in case of reversals. So, dealer  $D(-1)$  only expects positive profits if there is a buy order in which case he makes profits of  $\frac{2}{3}$ . Consequently, his expected profits are  $\frac{5}{24} \times \frac{2}{3}$ , where  $\frac{5}{24}$  is the probability of a reversal from a sell to a buy. By symmetry, these are also the expected profits of dealer  $D(1)$ . Finally, dealer  $D(0)$  is indifferent between  $q_1 = 1$  and  $q_1 = -1$  since his expected profits in the two possible events are equal. In either case, he plays a mixed strategy with support  $[\frac{35}{19}, \frac{25}{13}]$ , which yields

expected profits of  $5/48$ . Direct computations yield expected profits from trading at  $t = 1$  for the informed dealer equal to  $25/192$ .

#### 4.2. Optimal quotes at $t = 0$

Given the optimal responses corresponding to  $t = 1$ , we now calculate the Perfect Bayesian equilibrium in the opaque market. It is important to point out that opaqueness in the first round may generate equilibrium prices that depart from the independence property. In most microstructure models [see, for instance, [Glosten and Milgrom \(1985\)](#), [Easley and O'Hara \(1987\)](#), [Dennert \(1993\)](#), and [Leach and Madhavan \(1993\)](#)], each market maker faces two independent bidding problems: one for the ask and other for the bid side of the market. In our model, at  $t = 0$ , each dealer realizes that when setting his ask price, he needs to consider the relative position of his bid with respect to his competitor's bid, to deduce his expected profits, and vice versa.

Using the equilibrium continuation derived before, the overall expected profits accruing to dealer  $D$  if he sets prices  $1 + a_0^D$  and  $1 - b_0^D$  while his competitor sets prices  $1 + a_0^{D'}$  and  $1 - b_0^{D'}$ , are as follows:

- If he is competitive in both sides of the market ( $a_0^D < a_0^{D'}$  and  $b_0^D < b_0^{D'}$ ),<sup>23</sup> then

$$E[\Pi^D] = \frac{1}{8}(3a_0^D + 3b_0^D - \frac{71}{24});$$

- If he is only competitive in the ask side ( $a_0^D < a_0^{D'}$  and  $b_0^D > b_0^{D'}$ ), then

$$E[\Pi^D] = \frac{1}{16}(6a_0^D + 4S - 9);$$

- If he is only competitive in the bid side ( $a_0^D > a_0^{D'}$  and  $b_0^D < b_0^{D'}$ ), then

$$E[\Pi^D] = \frac{1}{16}(6b_0^D + 4S - 9);$$

- If he is not competitive in any side of the market ( $a_0^D > a_0^{D'}$  and  $b_0^D > b_0^{D'}$ ), then  $E[\Pi^D] = 0$ .

To better understand the expected profits above, consider first the case in which a dealer is competitive in both sides of the market and, further, chooses the same selling and buying fees. Let  $\Delta_0^{TS}$  be the fee that makes null the expected profits attending both sells and buys. Similarly, consider the case in which a dealer is competitive in just one side of the market. Let  $\Delta_0^{OS}$  denote the fee that makes null the expected profits attending either a buy or a sell. Using this notation we can rewrite

<sup>23</sup>It is easy to see that perfect learning dominates any continuation path in which there are equal prices either in one side of the market or in both sides. We will hence consider here that dealers set different prices. This restriction is satisfied in equilibrium as we will later show.



dealer  $D$ 's expected profits as follows:

Event	Expected profits
$a_0^D < a_0^{D'}$ and $b_0^D < b_0^{D'}$	$\frac{3}{8}(a_0^D + b_0^D - 2\Delta_0^{TS})$
$a_0^D < a_0^{D'}$ and $b_0^D > b_0^{D'}$	$\frac{3}{8}(a_0^D - \Delta_0^{OS})$
$a_0^D > a_0^{D'}$ and $b_0^D < b_0^{D'}$	$\frac{3}{8}(b_0^D - \Delta_0^{OS})$
$a_0^D > a_0^{D'}$ and $b_0^D > b_0^{D'}$	0

Notice that for any given pair of prices set by dealer  $D'$ , dealer  $D$  prefers to attend both sides of the market instead of specializing on the bid side if and only if  $a_0^D \geq 2\Delta_0^{TS} - \Delta_0^{OS}$ . Similarly, he prefers to attend both sides instead of specializing on the ask side if and only if  $b_0^D \geq 2\Delta_0^{TS} - \Delta_0^{OS}$ . The next proposition shows that in an opaque market one dealer (let us say, without loss of generality, dealer  $D$ ) will be competitive in both sides of the market and will hence gain perfect information about the order flow. Consequently, in equilibrium there will be investment in perfect learning. The next proposition summarizes the strategic behavior of dealers in an opaque market.

**Proposition 7.** *The following set of strategies constitutes a Perfect Bayesian equilibrium for the opaque market:*

At  $t = 0$ :

- Dealer  $D$  sets the following ask and bid:  $1 + \Delta_0^{TS}$  and  $1 - \Delta_0^{TS}$ , and
- Dealer  $D'$  randomizes by setting prices  $(A, B)$  such that

$$\{(B, A) \in [1 - \Delta_0^{OS}, 1 - \Delta_0^{TS}] \times [1 + \Delta_0^{TS}, 1 + \Delta_0^{OS}]: A - B = \Delta_0^{OS} + \Delta_0^{TS}\}, \quad (1)$$

where  $\Delta_0^{TS} = \frac{71}{144}$  and  $\Delta_0^{OS} = \frac{6101}{11814}$ .

He chooses among the prices that satisfy (1) according to a uniform cumulative distribution function.

At  $t = 1$ , dealers follow the equilibrium strategies described in Proposition 6.<sup>24</sup>

Note that, in equilibrium, for any price set by dealer  $D'$  his opponent is better off by attending both sides of the market as it is satisfied that  $a_0^D - \Delta_0^{TS} \geq 2\Delta_0^{TS} - \Delta_0^{OS}$ . Our prediction is supported by laboratory experiments conducted by Bloomfield and

<sup>24</sup>It is important to point out that the proposed strategies in either Roëll (1991) or Madhavan (1995) do not constitute an equilibrium in our framework. In Madhavan both dealers set the same prices at  $t = 0$ . At  $t = 1$ , the uninformed dealer quotes a price equal to the expected value of the security given that a continuation occurs, whereas the informed dealer (marginally) improves these quotes if and only if there is a reversal. Note that they play a pure strategies equilibrium while we have shown that in (our) opaque market there is no continuation equilibrium in pure strategies. In Roëll, dealers play a symmetric equilibrium in pure strategies at  $t = 0$ . We have here shown that there is no equilibrium which involves equal prices at  $t = 0$ .

O'Hara (1999, 2000). These authors call this behavior *capturing early order flow*. Consequently, the second round of trade in the opaque market will be characterized by the interplay between a perfectly informed dealer and a completely uninformed dealer. The asymmetry between the dealers gives rise to an equilibrium in the second round in mixed strategies and, consequently, to price dispersion.<sup>25</sup> Moreover, Proposition 7 shows that, in equilibrium, the uninformed dealer may attend the order even in a reversal, contrary to results in Madhavan (1995) and Wu and Zhang (2002).

The next corollary characterizes the corresponding equilibrium price quotation functions.

**Corollary 8.** *There exists a Perfect Bayesian equilibrium in the post trade opaque market, where the equilibrium price quotation function at  $t = 0$ ,  $P_0(q_0)$ , satisfies*

$$P_0(q_0) = 1 + \Delta_0^{TS} q_0$$

and the equilibrium price quotation at  $t = 1$ ,  $P_1(q_1; q_0)$ , is given by

$$P_1(q_1; q_0) = 1 + \min\{a_1^{D(q_0 \times q_1)}, a_1^U\} \times q_1.$$

Both dealers make overall expected profits equal to zero. Nevertheless, the dealer that chooses to be competitive in both sides of the market makes negative expected profits in the initial period and positive expected profits in the final period. Just note that

$$P_0(q_0) = 1 + \Delta_0^{TS} q_0 = 1 + \frac{2}{3} q_0 + (\Delta_0^{TS} - \frac{2}{3}) q_0 = E[v|q_0] + \pi q_0,$$

where  $\pi > 0$ . Notice that  $\pi$  reflects the order flow payment that a dealer assumes in order to gain monopoly power over information by capturing order flow that need not be disclosed. Thus, in equilibrium, there is price experimentation in the post trade opaque market.<sup>26</sup>

Price experimentation is also derived by other models of market making such as Leach and Madhavan (1992, 1993). There price experimentation occurs when trading is accomplished through a single market maker (i.e., a specialist) who may experiment with prices to induce more informative order flow. We show here that investment in producing information is also present in markets with a competitive group of market makers if trade disclosure is not mandatory.

<sup>25</sup>In the dealer market for corporate bonds (a low transparent market), Gehr and Martell (1992), using lower-frequency corporate bond quote data, find that bid-ask spreads between dealers often do not intersect, a finding that is consistent with our equilibrium characterization.

<sup>26</sup>There are several mechanisms by which new information is incorporated into security prices. Among them are price experimentation and price signaling. Note that both facilitate price discovery but they operate very differently. In the former, dealers have the ability to expedite price discovery through binding quotes. In the latter, dealers can use nonbinding price quotes to indicate to others information they hold. Evidence of price signaling is found for the period before the opening of the Nasdaq market. There, dealers enter non-binding quotes that can be considered as signals to indicate to others the equilibrium opening prices conditional on the overnight information (see Cao et al., 2000).

## 5. Comparison across market structures

This section is devoted to the comparison between the two market structures. To this end, the superscript  $T(O)$  in a variable means that it corresponds to the post trade transparent (opaque) market. We will first describe some characteristics of the sequences of prices generated in the two markets to examine the main differences in price dynamics between them. We will then study the implications of structure for metrics of market quality such as spreads, volatility and price efficiency. Finally, we study the impact of transparency on the welfare of market participants.

### 5.1. Price dynamics

Equilibrium price quotations in the two market structures are given by

$$P_0^T(q_0) = 1 + \frac{2}{3}q_0 \quad \text{and} \quad P_1^T(q_1; q_0) = \begin{cases} 1 + \frac{12}{13}q_1 & \text{for } q_1 = q_0 \neq 0, \\ 1 & \text{for } q_1 \neq q_0 \neq 0, \\ 1 + \frac{2}{3}q_1 & \text{for } q_0 = 0, \end{cases}$$

$$P_0^O(q_0) = 1 + \Delta_0^{TS}q_0 \quad \text{and} \quad P_1^O(q_1; q_0) = 1 + \min\left\{a_1^{D(q_0 \times q_1)}, a_1^U\right\} \times q_1.$$

Prices in the first round are more attractive in the opaque market as the competition between dealers to secure an informational advantage makes them set higher bids and lower asks. The price difference between the two structures ( $\Delta_0^{TS} = \frac{2}{3}$ ) is negative and equal to  $-\frac{25}{144}$ .

Consider now the prices at the second round of trade. If there is no trading activity in the first period, then market makers only change their quotes in the opaque market. In the transparent market, dealers do not observe new relevant information and consequently, they do not revise their quotes.<sup>27</sup> By contrast, in the opaque market dealers set different prices across periods even when no transaction in the first period occurs. Recall that the most competitive dealer sets prices at  $t = 0$  that yield negative current expected profits. Consequently, if  $q_0 = 0$ , then he must unwind his position to avoid losses in period 1. This dynamic strategy results in smaller bids and higher asks in the opaque market when there is no trade in the first round.

If there is trading activity at both rounds, prices in the second round are more attractive for investors in the opaque market in the event of a continuation whereas the opposite holds in reversals. In a continuation, a sequence of buys (sells) generates an increasing (decreasing) sequence of prices in either market structure. In either period, the opaque market provides a smaller ask and a larger bid (in expected terms). In a reversal from a buy to a sell, the bid is smaller in the opaque market.

<sup>27</sup>Easley and O'Hara (1992) find that the spreads will decrease over time when no orders arrive in a post-trade transparent market. Their result is due to the fact that the absence of trades may imply that no news arrives, i.e., no informational event takes place, and therefore the likelihood of informed trading decreases. By contrast, in our model, it is assumed that there is always new information.

Another important difference between the two market structures is that in the opaque market quotes corresponding to the second round of trade are uncertain (as dealers play mixed strategies) despite the fact that there is no new shock in the fundamentals. Also note that under opaqueness the probability with which the two dealers set the same quotes in either round is zero, whereas it is one in the transparent market.

### 5.1.1. Spreads

Consider now the impact of post trade opaqueness on spreads.

**Corollary 9.** *The intraday patterns in bid ask spreads are opposite under both market structures. While in the transparent mechanism spreads decrease from period 0 to period 1, in the opaque market structure, they increase.*

In the transparent market, at  $t = 0$  dealers are unwilling to improve competitive prices because their competitors will free ride on the information they have gained from observing the trading history. Furthermore, the expected bid ask spread narrows over the day because the adverse selection faced by the market makers is mitigated as they learn from order flow. In the opaque market, by contrast, a completely different temporal pattern emerges. In the first period, the most competitive dealer quotes a tight spread to acquire private information. This creates a double winner's curse problem in his competitor: i) with respect to the informed investor, and ii) with respect to the informed dealer. This relatively severe adverse selection problem widens his spreads and, by extension, the spreads of the informed dealer who needs to cash in his investment in information acquisition.

### 5.1.2. Volatility

Suppose now that trading activity has occurred at  $t = 0$ . The possible absolute price changes in the two structures are summarized in the next table:

	Transparent	Opaque (expected values)
Continuation	0.256	0.382
Reversal	0.667	1.235

Price volatility is due to differences between the prices set by the dealers who accommodate the order in the two periods. In the transparent market, the only difference between them is the information about  $v$ . By contrast, in the opaque market there are strategic considerations that influence price quotations. To understand this point, consider a continuation in buy orders. In the opaque market, the dealers that will attend the buys are dealer  $D$  in the first period, and the uninformed dealer  $D'$  in the second one. Both dealers have the same information about  $v$ . However, they differ in the strategies they follow (dynamic versus static) and in the degree of uncertainty about the competitor's type (while at  $t = 0$  there is no such uncertainty, at  $t = 1$  dealer  $D'$  who is uninformed does not know his competitor's type). The strategic differences in the opaque market have a larger

impact than the informational differences in the transparent market resulting in a larger price change in the opaque market.

Consider now a reversal from a buy to a sell. In the transparent market it induces a large price change, in comparison to that of a continuation, as beliefs revert. In the opaque market, this informational difference reinforces the strategic difference, resulting in an absolute price change which exceeds that of a continuation. Consequently, in either case,  $|P_1^T - P_0^T| < |E(P_1^O) - P_0^O|$  holds. As the probabilities of continuations and reversals are equal in both market structures, we obtain the following result:

**Corollary 10.** *Price volatility measured by the expected absolute change in price is higher in the opaque market, i.e.,  $(E(|P_1^T - P_0^T|)) < (E(|E(P_1^O) - P_0^O|))$ .*

### 5.1.3. Efficiency

In what follows we use the concepts of efficiency proposed by Roberts (1967). We will say that prices are strong form efficient if they reflect all private information, semi strong form efficient if they reflect all publicly available information, and weak form efficient if they reflect the information in their own past values. Note that whenever some traders have superior information, prices will not exhibit strong form efficiency. This is the case in our setup. In the standard sequential trade model proposed by Glosten and Milgrom (1985), the sequence of transaction prices follows a martingale with respect to the sequence of trades. This property implies that prices are semi strong form efficient in the sense that they reflect all the information available to the market makers. Since these authors model a post trade transparent market, it is not surprising that our transaction prices corresponding to the transparent market hold the same property. However, in the post trade opaque market, transaction prices are not a martingale with respect to past prices and so they are not weak form efficient. To understand this result, notice that at  $t = 0$  when setting an ask price, dealers bear in mind that this price corresponds to a purchase. In the opaque market this is not made public, and consequently, prices at  $t = 1$  do not necessarily incorporate this information. All these results are summarized in the following corollary.

**Corollary 11.** *In equilibrium, transaction prices are semi strong form efficient in the transparent market and they are not weak form efficient in the opaque market.*

## 5.2. Welfare

Now we turn our attention to the effect of transparency on the welfare of market participants.

**Corollary 12.** (a) *Investors who transact in the first period prefer the opaque market.*

(b) *If there is no trading activity in the first period, then investors who transact at  $t = 1$  prefer the transparent market. Otherwise, those investors whose order type coincides (differs) with the previous one are better (worse) off in the opaque market. Moreover, in expected terms, investors who transact in the second period prefer the transparent market.*

(c) *As a group, informed traders are better off in the opaque market, whereas liquidity traders prefer the transparent market.*

This result tells us that the lack of transparency benefits investors who transact at  $t = 0$  at the expense of traders who arrive later. The rationale is as follows. Opaqueness increases competition among dealers to attract valuable order flow, leading to better prices for all period 0 investors. On the other hand, concerning the later period, in case of a continuation all period 1 investors prefer the opaque market.<sup>28</sup> An informed trader prefers the opaque market because it provides her more camouflage. An uninformed trader prefers the opaque market because she will transact with the uninformed dealer, whereas in the transparent market she will transact with an informed dealer. By contrast, in case of no trading in the first round or of a reversal any investor prefers the transparent market. Notice that, in this case, opaqueness reduces price competition because the informed dealer can undercut his competitor's quote while still earning positive profits.

Our comparison between the two market structures generates some stylized facts that can be contrasted with the experience in actual markets. We have shown that liquidity traders prefer greater disclosure, whereas traders with private information prefer opaque trading systems. These results are consistent with the evidence on after hours trading where trade reporting transparency declines as most trades are not reported on the day they occur.<sup>29</sup> The analysis in [Barclay and Hendershott \(2004\)](#) on trading after hours shows that trades may be more informed after hours (their data base shows that the average price impact of a trade is higher after hours than during the trading day). They suggest that uninformed traders have no incentive to move their trades outside of the normal trading day. This evidence is consistent with our welfare comparison. In a related paper, [Barclay and Hendershott \(2003\)](#) find that stock prices after hours are less efficient than prices during the day. After the close there are large bid ask spreads, thin trading, and little new information. The stylized facts generated by our model are in accord with this evidence. Similarly, the empirical investigation of the Italian Treasury bond primary and secondary markets, reported in [Drudi and Massa \(2005\)](#), shows that informed dealers may refrain from trading in the more transparent market in order to exploit their informational advantage in the less transparent one, or they may use the more transparent market in order to manipulate prices.

## 6. Robustness

The model was based on a number of simplifying assumptions, and at this point we are interested in assessing the impact of relaxing them in our conclusions. In this section some alternative formulations are examined.

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<sup>28</sup>This result is consistent with [Madhavan \(1995\)](#) who shows that large traders are better off in an opaque market because this structure allows them to break their orders over time without attracting too much attention of the market, and hence, reducing the corresponding price impact.

<sup>29</sup>On the NASDAQ, data on after-hours trading is integrated into the statistical record next day with a 24 hour cut-off.

### 6.1. Uninformed investors trade

Throughout the paper we have assumed that the probability with which an uninformed investor chooses to trade is  $p = 1/2$ . We now provide the arguments that show the robustness of our results to changes in  $p$ . We concentrate in the opaque market as in the transparent market results are trivially robust to changes in  $p$ . To examine this issue we first consider the two extreme cases ( $p \rightarrow 0$  and 1).

As  $p$  approaches zero all ask prices (under either specialization or perfect learning) converge to 2 with probability one. The intuition is clear: as  $p$  approaches zero, there will be common knowledge that any buy order comes from an informed investor. Consequently, in equilibrium, dealers perfectly anticipate that a buy order implies  $v = 2$ . In this extreme case, dealers will be indifferent between attending both sides or only one side of the market. Furthermore, the two market structures are equivalent.

Consider now that  $p = 1$ . Under specialization, the opaque market becomes informationally equivalent to the transparent market. Note that if each dealer was competitive in one side of the market, a dealer who did not trade can correctly infer that the order type was the one for which he was not competitive. The equilibrium strategies at  $t = 1$  will then converge to the equilibrium strategies in the transparent market, which results in null expected profits in either period. Matters are very different under perfect learning, where the uninformed dealer randomizes by setting ask prices belonging to the interval  $[\frac{3}{5}, \frac{9}{5}]$ , dealer  $D(-1)$  randomizes in the interval  $[\frac{3}{5}, \frac{9}{5})$  and  $D(1)$  sets an ask price equal to  $9/5$ . Expected profits for the informed dealer are hence strictly positive at  $t = 1$ . Perfect learning is hence optimal. In equilibrium, dealer  $D$  will set a selling (buying) price equal to  $1 + 5/16$  ( $1 - 5/16$ ) in the first period, and dealer  $D'$  will randomize uniformly by setting  $(A, B)$  such that  $(B, A) \in [1 - 1/2, 1 - 5/16] \times [1 + 5/16, 1 + 1/2]$  with  $A - B = 13/16$ . At the second stage they will play the strategies described above. The equilibrium is hence qualitatively identical to the one for  $p = 1/2$ . Nevertheless, there will be more price dispersion across dealers when  $p = 1$  than with  $p = 1/2$ .

For intermediate values of  $p$ , results qualitatively similar to the ones derived here are obtained.<sup>30</sup> The rationale behind the robustness with respect to changes in  $p$  is as follows. Expected profits under specialization are concave with a maximum at  $p < 1$ . For small values of  $p$ , an increase in the probability of uninformed trade results in an increase in expected profits since adverse selection decreases. In contrast, when  $p$  is large enough and dealers specialize, their private information is more likely to be inferred and they may hence not earn much in informational rents. In contrast, under perfect learning expected profits are strictly increasing in  $p$ , and hence reach a maximum at  $p = 1$ . To understand this result notice that there are two opposite effects. On the one hand, the signal they gain from the order type is less informative as it is less likely that it will come from an informed investor. Consequently, their informational rents decrease as their private information is less valuable. But, on the other hand, an increase in  $p$  reduces the probability of no trading, and hence, the

<sup>30</sup>In the working paper version of this paper (Frutos and Manzano, 2003) the interested reader can find the model explicitly solved for any  $p \in [0, 1]$ .

types  $D(1)$  and  $D(-1)$ , which are the types of dealers with a double informational advantage with respect to their competitors, are more likely. The second effect dominates making expected profits under perfect learning strictly increasing in  $p$ . The preference for experimentation is hence robust to changes in  $p$ . Further, in equilibrium, dealers cannot tie in the first period as this would give rise to profitable deviations. Consequently, equilibrium price strategies must be like the ones described in the paper for any positive  $p$ .

The comparison between the two market structures in terms of metrics of market quality is also robust to changes in the specification of  $p$ . Furthermore, expected spreads are decreasing in  $p$  in both market structures. The difference between expected spreads (in absolute value) is also decreasing in  $p$  in the transparent market structure. By contrast, in the opaque mechanism, it is increasing in  $p$  unless  $p$  is large enough. The rationale is as follows. At larger values of  $p$ , expected spreads are smaller as the adverse selection problem faced by market makers is mitigated when the probability that an uninformed trader transacts increases. By the same token, an increase in  $p$  leads to a less informative order flow, and therefore, market makers have more similar information across periods of trade. In the transparent market, this is all that counts when setting prices; consequently, the difference between expected spreads must decrease with  $p$ . In the opaque market, by contrast, as  $p$  increases the winner's curse with respect to the informed investor likewise decreases but the one with respect to the informed dealer increases. When  $p$  is small the second effect dominates, whereas the opposite holds if  $p$  is large enough.

## 6.2. Information arrival

We have here assumed that information only arrives at the beginning of the trading day. This assumption is crucial to the model because dealers know when to experiment and when to profit from their information. In a more complex framework with information arriving at random times, unbeknownst to the dealers, it is natural to wonder if the results obtained here will still hold.

To examine the role of random information arrival, we now consider a variation of our original model. More precisely, we now assume that at the beginning of the trading day, nature has three moves in deciding first whether there will be new information in the trading day, second if there is, in which period it is generated and then, what information it will be. Let  $\gamma$  denote the probability that new information is generated during the trading day. In this case, let  $\delta_0$  represent the probability that this information is generated at the beginning of the trading day (obviously, it is generated in the second period with the complementary probability). Note that in this model, all investors are uninformed with probability  $1 - \gamma$ . With probability  $\gamma\delta_0$  the two types of investors (informed and uninformed) can be present in the market at either period. With probability  $\gamma(1 - \delta_0)$  only uninformed investors can trade in the first period, whereas in the second period any type of investor may trade.

The resolution of this extension shows that experimentation is a robust phenomenon. Whenever  $\gamma\delta_0$  is not zero there is an equilibrium in the opaque market in which there is experimentation: one dealer finds it profitable to attend both



sides of the market if there is a non null probability of exploiting the acquired information in subsequent trading. In order to further clarify the impact of  $\gamma$  and  $\delta_0$  on the equilibrium strategies we here now provide, without proof, the equilibrium strategies in the opaque market for  $p = 1$ .<sup>31</sup>

**Proposition 13.** *When the market is opaque and  $\gamma\delta_0 \neq 0$ , the following set of strategies constitutes a Perfect Bayesian equilibrium:*

At  $t = 0$ :

- Dealer  $D$  sets the following ask and bid:  $1 + \delta_0\gamma((4 + \gamma)/16)$  and  $1 - \delta_0\gamma((4 + \gamma)/16)$ , and
- Dealer  $D'$  randomizes by setting prices  $(A, B)$  such that

$$\left\{ (B, A) \in \left[ 1 - \frac{\gamma\delta_0}{2}, 1 + \frac{\delta_0\gamma(4 + \gamma)}{16} \right] \times \left[ 1 + \frac{\delta_0\gamma(4 + \gamma)}{16}, 1 + \frac{\gamma\delta_0}{2} \right] : A = B - \delta_0\gamma\left(\frac{12 + \gamma}{16}\right) \right\}. \quad (2)$$

He chooses among the prices that satisfy (2) according to a uniform distribution probability.

At  $t = 1$ , dealers set the following equilibrium strategies:

- $D(1)$  sets his reservation selling price,  $\frac{4 + \gamma(3\delta_0 + 2)}{\gamma\delta_0 + 4}$ .
- $D(-1)$  randomizes in the interval  $\left[ \frac{2 + \gamma}{2}, \frac{4 + \gamma(3\delta_0 + 2)}{\gamma\delta_0 + 4} \right]$ . He assigns probability according to the distribution  $G_{D(-1)}$ , where

$$G_{D(-1)}(A) = \frac{4(2A - \gamma - 2)}{A(4 - \gamma\delta_0) + 3\gamma\delta_0 - 2\gamma - 4}.$$

- The uninformed dealer, dealer  $D'$ , randomizes in the interval  $\left[ \frac{2 + \gamma}{2}, \frac{4 + \gamma(3\delta_0 + 2)}{\gamma\delta_0 + 4} \right]$  according to the cumulative distribution function  $G_U$ , which has a mass point at  $\frac{4 + \gamma(3\delta_0 + 2)}{\gamma\delta_0 + 4}$ , where

$$G_U(A) = \frac{(2A - \gamma - 2)(4 - \gamma\delta_0)}{2(A(4 - \gamma\delta_0) + 3\gamma\delta_0 - 2\gamma - 4)}.$$

Note that when  $\gamma\delta_0 \rightarrow 1$ , the strategies above converge to the equilibrium strategies described in the previous subsection. In particular, in the second period of

<sup>31</sup>The qualitative features of the equilibria in this extension are similar to the ones in the original model. The equilibria can easily be obtained by applying the techniques developed here. Nevertheless both the notation and the computations become more cumbersome as more parameters are now in place ( $\gamma$  and  $\delta_0$ ). The authors can send the equilibria explicit computations upon request.

trade the uninformed dealer randomizes by setting ask prices belonging to the interval  $[\frac{2}{5}, \frac{3}{5}]$ , dealer  $D(-1)$  randomizes in the interval  $[\frac{2}{5}, \frac{3}{5}]$  and  $D(1)$  sets an ask price equal to  $9/5$ . In the first period, dealer  $D$  will set a selling (buying) price equal to  $1 + 5/16$  ( $1 - 5/16$ ), and dealer  $D'$  will randomize uniformly by setting  $(A, B)$  such that  $(B, A) \in [1 - 1/2, 1 - 5/16] \times [1 + 5/16, 1 + 1/2]$  with  $A - B = 13/16$ .

On the other hand, when  $\gamma\delta_0 \rightarrow 0$ , dealers do not experiment in the opaque market structure. Notice that if  $\gamma = 0$ , in each period dealers set prices equal to the unconditional expected value of the asset, i.e., 1, and if  $\delta_0 = 0$ , dealers set prices equal to the unconditional expected value in the first period and set prices equal to expectation of the value conditional on the order flow in the second period.

The analysis of this extension stresses the importance (and robustness) of strategic information acquisition. Dealers will take actions to elicit information if they can profit from this information later on. This is also the logic behind the results in [Leach and Madhavan \(1992\)](#). They show that dealers invest in the production of information by setting prices to induce statistically more informative order flow. Under certain conditions, they can recoup the cost of this investment by making better pricing decisions in the future with more precise information. They derive price experimentation in a model where order quantity is variable and there is a monopolistic market maker. In contrast, we obtain a similar result in a Glosten and Milgrom type model where the order size is fixed and trading is sequential.

## 7. Conclusions

Using a two stage model of trading in a dealer market, we have analyzed here the effects of post trade transparency on the performance of two market structures that differ in the amount of information that is publicly available at the end of the first stage. The main distinctive characteristic of our model is that the lack of transparency forces market makers to consider if they prefer to specialize on one side of the market or to jointly determine their asks and bids to become competitive in both sides of the market.

We can summarize our results as follows. In the opaque market dealers may be able to offer better prices because they can profit in subsequent trading from the private information they infer from the current trades. The upshot of this is that investors that transact in the first period are better off in the opaque market, but this comes at the expense of traders who transact in the second period. Moreover, it is shown that if there is no trading activity in the first period, then market makers only change their quotes in the opaque market. Additionally, we find that the lack of transparency provokes several price distortions: it exacerbates price volatility, it creates price dispersion, and it also reduces price efficiency.

Our analysis has a number of implications for regulatory policy. We find that transparent markets are more informationally efficient. However, regulators must not forget that the increase of informational efficiency may be purchased at the expense of higher spreads because market makers have less incentive to pay to capture the information that a trade with an informed trader will bring. As our

analysis demonstrates, traders with private information prefer systems with less transparency. However, investors with no particular information advantage prefer greater transparency. Therefore, no single transparency regime will be seen as optimal by everybody. Additionally, the lack of post trade transparency can induce fragmentation as traders seek off market venues for their trades. A major feature of securities markets in recent years has been the development of new trading systems and the sharp increase in competition. The arrival of competing trading venues with different transparency arrangements poses the question of the extent to which these trading venues respond to the current regulation requirements imposed on exchanges.<sup>32</sup>

## Appendix A

Since throughout the paper we must determine how the market makers' beliefs evolve over the trading day, we first state the probabilities of the events  $v = 2$  and  $v = 0$  conditional on a sequence of trades. Using Bayes' rule, it follows that

$$\Pr(v = 2 | (q_0, q_1)) \begin{cases} \frac{1}{2} + \frac{6}{13} q_0 & \text{if } (q_0, q_1) \in \{(1, 1), (1, 0)\}, \\ \frac{5}{6} & \text{if } (q_0, q_1) \in \{(1, 0), (0, 1)\}, \\ \frac{1}{6} & \text{if } (q_0, q_1) \in \{(0, 1), (0, 0)\}, \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

Finally, the conditional probability of the event  $v = 0$  is derived as follows:

$$\Pr(v = 0 | (q_0, q_1)) = 1 - \Pr(v = 2 | (q_0, q_1)), \text{ for any } (q_0, q_1).$$

**Proof of Proposition 1.** In the post trade transparent market, the transaction price must equal the expected value of the asset conditional upon the trading history and the incoming order. Hence,  $P_1(q_1; q_0) = E(v | (q_0, q_1))$ . Using the previous conditional probabilities of  $v$ , we obtain the desired expressions.

Similarly,

$$\begin{aligned} P_0(1) &= E(v | q_0 = 1) = \Pr(q_1 = 1 | q_0 = 1)E(v | q_0 = 1, q_1 = 1) \\ &\quad + \Pr(q_1 = 0 | q_0 = 1)E(v | q_0 = 1, q_1 = 0) \\ &\quad + \Pr(q_1 = 1 | q_0 = 0)E(v | q_0 = 0, q_1 = 1). \end{aligned} \tag{3}$$

Using Bayes' rule and the equilibrium quotation functions at  $t = 1$ , the expression for  $P_0(1)$  is obtained. Similar computations provide the expression for  $P_0(0)$ .  $\square$

**Proof of Lemma 2.** If at  $t = 0$  dealer  $D$  set the best ask and he executed the order, then at  $t = 1$  he has the same information as in the transparent market.

<sup>32</sup>In most jurisdictions, regulators have focused transparency requirements exclusively on exchanges. In jurisdictions such as Switzerland, the UK and the US (as long as the trading volume is small) public dissemination of exchange-traded securities that takes place on off-exchange market systems is not required. For more information on these issues see "Transparency and Market Fragmentation," report from the Technical Committee of the IOSC, November 2001.

Consequently, applying Proposition 1, his reservation ask price is derived. If he did not execute the order at  $t = 0$ , he deduces that either  $q_0 = 0$  or  $q_0 = 1$ . Therefore,  $A_{r,1}^{D(0,-1)} = E(v|q_0 \in \{0, 1\}, q_1 = 1) = \theta E(v|q_0 = 0, q_1 = 1) + (1 - \theta)E(v|q_0 = 1, q_1 = 1)$ , where

$$\theta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0, 1\}, q_1 = 1).$$

Using Bayes' rule, we derive the value of  $\theta$ . Finally, using Proposition 1, the expression for this reservation ask price is obtained. Consider now the reservation quotes of the dealer  $D'$  who set the best bid at  $t = 0$ . If he executed the order, then  $A_{r,1}^{D'(-1)} = 1$  follows trivially from Proposition 1. If he did not trade at  $t = 0$ , he deduces that either  $q_0 = 0$  or  $q_0 = 1$ . Therefore,  $A_{r,1}^{D'(1,0)} = E(v|q_0 \in \{0, 1\}, q_1 = 1) = \eta E(v|q_0 = 0, q_1 = 1) + (1 - \eta)E(v|q_0 = 1, q_1 = 1)$ , where

$$\eta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0, 1\}, q_1 = 1).$$

Appealing to Proposition 1, the value of  $A_{r,1}^{D'(1,0)}$  stated in this lemma is derived.  $\square$

**Proof of Proposition 4.** If there was a sell order in  $t = 0$ , then dealer  $D'(1)$  knows that his opponent's information set is  $\{0, 1\}$ . We claim that he randomizes in the interval  $[S, Z]$ . By setting an ask price equal to  $S$ , he wins with probability one achieving expected profits from trading  $\Pi_1^{D'(-1)}(S) = S - 1$ . By playing  $Z$ , he wins with probability  $1 - F_{D(0,-1)}(Z)$  and he loses with the complementary probability. Consequently,  $\Pi_1^{D'(-1)}(Z) = (1 - F_{D(0,-1)}(Z))(Z - 1)$ . Since in a mixed strategy equilibrium expected profits from playing any selling price in the support must be equal,  $\Pi_1^{D'(-1)}(S) = \Pi_1^{D'(-1)}(Z)$  yields

$$F_{D(0,-1)}(Z) = \frac{Z-S}{Z-1}. \quad (4)$$

Consider now dealer  $D$  with information set  $\{0, 1\}$ . This player did not trade in the first round and, hence, he does not know the identity of his opponent. He believes his opponent is  $D'(1, 0)$  with probability  $6/11$ . We claim that he randomizes in the interval  $[S, \frac{25}{13}]$ , where  $S < Z < \frac{25}{13}$ . By playing  $S$ , he wins with probability one, and his expected profits from trading are given by

$$\Pi_1^{D(0,-1)}(S) = S - E(v|q_0 \in \{0, 1\}, q_1 = 1) = S - \frac{15}{11}. \quad (5)$$

By playing  $Z$ , with probability one he wins if his opponent is  $D'(1, 0)$ , whereas with probability one he loses if his opponent is  $D'(1)$ . Since in the first case he deduces that  $q_0 = 0$ , we have

$$\Pi_1^{D(0,-1)}(Z) = \frac{6}{11}(Z - E(v|q_0 = 0, q_1 = 1)) = \frac{6}{11}(Z - \frac{5}{3}). \quad (6)$$

Finally, when setting  $A \rightarrow \frac{25}{13}$ , he wins with probability  $1 - F_{D'(1,0)}(\frac{25}{13})$  if his opponent is  $D'(1, 0)$ , and he always loses if his opponent is  $D'(1)$ . Expected profits converge to

$$\frac{6}{11}(1 - F_{D'(1,0)}(\frac{25}{13}))(\frac{25}{13} - E(v|q_0 = 0, q_1 = 1)) = \frac{6}{11}(1 - F_{D'(1,0)}(\frac{25}{13}))\frac{10}{39}. \quad (7)$$

In a mixed strategy equilibrium, (5) = (6) and (5) = (7). Therefore,

$$S = \frac{15}{11} = \frac{6}{11}(Z - \frac{5}{3}), \quad (8)$$

and

$$S = \frac{15}{11} - \frac{6}{11} \left(1 - F_{D'(1,0)}\left(\frac{25}{13}\right)\right) \frac{10}{39}. \quad (9)$$

Consider now that  $D'$  did not trade in  $t = 0$ . He believes his opponent is  $D(0, -1)$  with probability  $\eta$ , where we recall that  $\eta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0, 1\}, q_1 = 1) = 6/19$ . We claim that he randomizes in the interval  $[Z, \frac{25}{13}]$ . By playing  $Z$ , he wins with probability  $1 - F_{D(0,-1)}(Z)$  if his opponent is  $D(0, -1)$  and with probability one if his opponent is  $D(1)$ . The expected profits from playing this strategy are

$$\eta(1 - F_{D(0,-1)}(Z))(Z - E(v|q_0 = 0, q_1 = 1)) + (1 - \eta)(Z - E(v|q_0 = 1, q_1 = 1)).$$

Finally, when playing  $\frac{25}{13}$  he loses with probability one if his opponent is  $D(0, -1)$  and he obtains zero expected profits if his opponent is  $D(1)$ . Note that he wins but he deduces that  $q_0 = 1$  and  $\frac{25}{13} = E(v|q_0 = 1, q_1 = 1)$ . Thus,  $\Pi_1^{D'(1,0)}(\frac{25}{13}) = 0$ . Equalizing the expected profits from playing these two strategies, we get

$$\frac{6}{19}(1 - F_{D(0,-1)}(Z))(Z - \frac{5}{3}) + \frac{13}{19}(Z - \frac{25}{13}) = 0. \quad (10)$$

Using (4), Eqs. (8) and (10) define a system of two equations and two unknowns whose solution is  $Z = \frac{335}{179}$  and  $S = \frac{2905}{1969}$ . Finally, by substituting the value of  $S$  into (9), we obtain that  $F_{D'(1,0)}(\frac{25}{13}) = \frac{36}{179}$ .

Once the limits of the supports are derived, we can now proceed to compute the distribution functions.

In order to get  $F_{D'(1,0)}$  we first compute the expected profits of dealer  $D(0, -1)$  when choosing  $A > Z$ , which are  $\frac{6}{11}(1 - F_{D'(1,0)}(A))(A - \frac{5}{3})$ . Differentiating with respect to  $A$  and equating the derivative to zero, it follows that  $(A - \frac{5}{3})f_{D'(1,0)}(A) = 1 - F_{D'(1,0)}(A)$ . This f.o.c. generates a differential equation whose boundary condition is  $F_{D'(1,0)}(Z) = 0$ . The solution to the differential equation yields  $F_{D'(1,0)}(A) = 3(A - Z)/(3A - 5)$ .

Similarly, to obtain  $F_{D'(-1)}$  we differentiate the expected profits of dealer  $D(0, -1)$  with respect to  $A$ . This allows us to obtain a differential equation with boundary condition  $F_{D'(-1)}(S) = 0$ . The solution to the differential equation yields  $F_{D'(-1)}(A) = \frac{11}{5}((A - S)/(A - 1))$ .

Finally, to derive  $F_{D(0,-1)}$  we first study the problem faced by  $D'( -1)$ . His expected profits from setting any  $A < Z$  become  $(1 - F_{D(0,-1)}(A))(A - 1)$ . Differentiating with respect to  $A$ , and using the boundary condition  $F_{D(0,-1)}(S) = 0$ , the c.d.f.  $F_{D(0,-1)}(A) = (A - S)/(A - 1)$  is derived. Regarding the final part of  $F_{D(0,-1)}$ , i.e., for  $A > Z$ , we consider the maximization problem faced by  $D'(1,0)$ . His expected profits from trading will be

$$\frac{6}{19}(1 - F_{D(0,-1)}(A))(A - \frac{5}{3}) + \frac{13}{19}(A - \frac{25}{13}).$$

Differentiating with respect to  $A$ , using  $F_{D(0,-1)}(Z) = \frac{5}{11}$  as boundary condition, we derive  $F_{D(0,-1)}(A) = (19A - 35)/(6A - 10)$ , with  $F_{D(0,-1)}(\frac{25}{13}) = 1$  as claimed.  $\square$

**Proof of Lemma 5.** It is omitted since it is similar to the proof of Lemma 2.  $\square$

**Proof of Proposition 6.** We first note that the uninformed dealer makes zero expected profits from trading at  $t = 1$  when playing the pure strategies:  $A_{r,1}^U$ ,  $A_{r,1}^{D(1,0)}$  and  $A_{r,1}^{D(1)}$ .

The probabilities that  $U$  assigns to meet  $D(1)$ ,  $D(0)$  and  $D(-1)$ , respectively, are by Bayes' rule,  $\Pr(q_0 = 1|q_1 = 1) = \frac{13}{24}$ ,  $\Pr(q_0 = 0|q_1 = 1) = \frac{1}{4}$  and  $\Pr(q_0 = 1|q_1 = 1) = \frac{5}{24}$ . If dealer  $U$  sets  $A$  such that  $\frac{5}{3} < A < \frac{35}{19}$ , then his expected profits from trading are:

$$\frac{13}{24}(A - \frac{25}{13}) + \frac{1}{4}(A - \frac{5}{3}) + \frac{5}{24}(1 - G_{D(-1)}(A))(A - 1).$$

Since these expected profits must be zero, it follows that  $G_{D(-1)}(A) = 8(3A - 5)/(5(A - 1))$ . Similarly, if  $U$  plays  $A$ , with  $\frac{35}{19} < A < \frac{25}{13}$ , then his expected profits from trading are:

$$\frac{13}{24}(A - \frac{25}{13}) + \frac{1}{4}(1 - G_{D(0)}(A))(A - \frac{5}{3}).$$

Since these expected profits must also be zero, we have  $G_{D(0)}(A) = (19A - 35)/(6A - 10)$ .

Consider now player  $D(-1)$ . His expected profits from trading at  $t = 1$  when setting his opponent reservation ask are  $\frac{5}{3}$ . Similarly, by quoting  $\frac{35}{19}$ , his expected profits from trading are  $(1 - G_U(\frac{35}{19}))\frac{16}{19}$ . Since these expected profits must be equal, we must have  $G_U(\frac{35}{19}) = \frac{5}{24}$ .

Regarding  $D(0)$ , his expected profits from quoting  $\frac{35}{19}$  are  $(1 - G_U(\frac{35}{19}))\frac{10}{37}$ . By setting a price  $A \rightarrow \frac{25}{13}$ , his expected profits from trading would converge to  $(1 - G_U(\frac{25}{13}))\frac{10}{39}$ . Equating these two values and operating, we obtain  $G_U(\frac{25}{13}) = \frac{11}{24}$ .

To end this proof, we now proceed to derive the c.d.f.  $G_U$ . We first compute the expected profits of dealer  $D(-1)$  when playing a selling price  $A > \frac{5}{3}$ . Differentiating these expected profits with respect to  $A$ , the f.o.c. generates a differential equation whose boundary condition is  $G_U(\frac{35}{19}) = \frac{5}{24}$ . The solution to the differential equation yields  $G_U(A) = (3A - 5)/(3(A - 1))$ , for all  $\frac{5}{3} \leq A \leq \frac{35}{19}$ .

Similarly, to obtain the last part of  $G_U$  we differentiate dealer  $D(0)$ 's expected profits when  $A > \frac{35}{19}$  to obtain a differential equation whose boundary condition is now  $G_U(\frac{35}{19}) = \frac{5}{24}$ . The solution to the differential equation yields:  $G_U(A) = (36A - 65)/(12(3A - 5))$ , for all  $\frac{35}{19} \leq A < \frac{25}{13}$ .  $\square$

**Proof of Proposition 7.** By following the purported equilibrium strategies  $D$  gets zero expected profits. To study the profitability of potential deviations we first note that we can restrict our attention to deviations that involve setting prices  $(A, B)$  such that  $(B, A) \in [1 - \Delta_0^{OS}, 1 - \Delta_0^{TS}] \times [1 + \Delta_0^{TS}, 1 + \Delta_0^{OS}]$  as any strategy that implies setting a price outside these ranges is weakly dominated. Note that any  $B < 1 - \Delta_0^{OS}$  is payoff equivalent to the strategy  $B = 1 - \Delta_0^{OS}$ . Similarly,  $B > 1 - \Delta_0^{TS}$  yields strictly smaller payoffs than the strategy  $B = 1 - \Delta_0^{TS}$ , for any  $A$  and any  $(A_0^{D'}, B_0^{D'})$ .

Given the equilibrium behavior of dealer  $D'$ , dealer  $D$  when quoting prices  $(A, B)$  gets expected profits given by

$$\begin{aligned} & \frac{3}{8} \{ \Pr(A < A_0^{D'}, B > B_0^{D'}) [A - B - 2\Delta_0^{TS}] \\ & + \Pr(A < A_0^{D'}, B < B_0^{D'}) [A - 1 - \Delta_0^{OS}] \\ & + \Pr(A > A_0^{D'}, B > B_0^{D'}) [1 - \Delta_0^{OS} - B] \}. \end{aligned}$$

Since  $B_0^{D'} > A_0^{D'}$  ( $\Delta_0^{OS} + \Delta_0^{TS}$ ) expected profits above simplify to:

$$\begin{aligned} & \frac{3}{8} \{ \Pr(A < A_0^{D'} < B + \Delta_0^{OS} + \Delta_0^{TS}) [A - B - 2\Delta_0^{TS}] \\ & + \Pr(A_0^{D'} > \max\{A, B + \Delta_0^{OS} + \Delta_0^{TS}\}) [A - 1 - \Delta_0^{OS}] \\ & + \Pr(A_0^{D'} < \min\{A, B + \Delta_0^{OS} + \Delta_0^{TS}\}) [1 - \Delta_0^{OS} - B] \}. \end{aligned}$$

To compute these expected profits we cluster all possible deviations into two groups:

1. Deviations that involve price strategies  $(A, B)$  such that  $A - B \leq \Delta_0^{OS} + \Delta_0^{TS}$ , and
2. Deviations that involve price strategies  $(A, B)$  such that  $A - B > \Delta_0^{OS} + \Delta_0^{TS}$ .

If (1) holds, then expected profits simplify to

$$\begin{aligned} & \frac{3}{8(\Delta_0^{OS} - \Delta_0^{TS})} \{ (B + \Delta_0^{OS} + \Delta_0^{TS} - A) [A - B - 2\Delta_0^{TS}] \\ & + (1 - \Delta_0^{TS} - B) (A - 1 - \Delta_0^{OS}) + (A - 1 - \Delta_0^{TS}) (1 - \Delta_0^{OS} - B) \}. \end{aligned}$$

Differentiating the expression above with respect to  $A$  and  $B$  yields

$$\frac{\partial E(\Pi T)}{\partial A} = \frac{3}{4(\Delta_0^{OS} - \Delta_0^{TS})} (1 + \Delta_0^{TS} - A) < 0, \quad \text{for all } A > 1 + \Delta_0^{TS}$$

and

$$\frac{\partial E(\Pi T)}{\partial B} = \frac{3}{4(\Delta_0^{OS} - \Delta_0^{TS})} (1 - \Delta_0^{TS} - B) > 0, \quad \text{for all } B < 1 - \Delta_0^{TS}.$$

Expected profits are maximized at the purported equilibrium strategy. Hence, no deviation such as the one proposed above is profitable for dealer  $D$ .

If (2) holds, then the probability of attending both sides of the market is zero. Consequently, expected profits simplify to

$$\frac{3}{8(\Delta_0^{OS} - \Delta_0^{TS})} \{ (A - 1 - \Delta_0^{OS})^2 + (1 - \Delta_0^{OS} - B)^2 \},$$

which is always negative. Consequently, these deviations are not profitable either.

Consider now dealer  $D'$ . By following the purported equilibrium strategies he gets zero expected profits and he does not attend any side of the market. To attend it, he must set  $A_0^{D'} < 1 + \Delta_0^{TS}$  and/or  $B_0^{D'} > 1 - \Delta_0^{TS}$ . If he improves only one price, for instance the ask price, then he obtains negative expected profits since  $A_0^{D'} < 1 + \Delta_0^{OS}$ . Otherwise, he also obtains negative expected profits because of  $A_0^{D'} - B_0^{D'} < 2\Delta_0^{TS}$ .  $\square$

**Proof of Corollary 9.** Let  $S_t^T$  ( $S_t^O$ ) denote the expected spread at  $t$  in the post trade transparent (opaque) market, with  $t = 0, 1$ . On the one hand, Proposition 1 implies that  $S_0^T = \frac{4}{3}$  and  $S_1^T = \frac{40}{39}$ . Hence,  $S_0^T - S_1^T > 0$ . On the other hand, from Propositions 7 and 8, we have  $S_0^O = \frac{71}{72}$  and  $S_1^O = 1.636$ . Note that  $S_0^O - S_1^O < 0$ , as claimed.  $\square$

**Proof of Corollary 11.** Consider the transparent market. The martingale property of transaction prices with respect to public information directly follows from the law of iterated expectations. The martingale property dictates that prices are semi strong form efficient.

Consider now the opaque market. Recall that prices are given by

$$P_0^O(q_0) = 1 + \Delta_0^{TS} \times q_0,$$

and

$$P_1^O(q_1, q_0) = 1 + \min\{a_1^{D(q_0 \times q_1)}, a_1^U\} \times q_1.$$

Since there is one to one mapping between  $P_0^O$  and  $q_0$ , it follows that

$$E(P_1^O | P_0^O) = E(P_1^O | q_0).^{33}$$

Suppose now that  $q_0 = 1$ . Notice that

$$A_1^O(1) = 1 + a_1^U \quad \text{and} \quad B_1^O(1) = 1 - \min\{a_1^{D(-1)}, a_1^U\}.$$

Then,

$$E(P_1^O | q_0 = 1) = 1 + \frac{13}{18} E(a_1^U) - \frac{5}{18} E\left(\min\{a_1^{D(-1)}, a_1^U\}\right), \quad (11)$$

where

$$\frac{13}{18} = \Pr(q_1 = 1 | q_0 = 1, q_1 \neq 0). \quad (12)$$

Using Proposition 6, it follows that  $E(a_1^U) = 0.875$  and  $E(\min\{a_1^{D(-1)}, a_1^U\}) = 0.741$ . Plugging expressions above into (11), using (12) and operating, it follows that

$$E(P_1^O | q_0 = 1) = 1 + 0.426.$$

As  $0.426 < \Delta_0^{TS} = 0.493$ , we have that  $E(P_1^O | P_0^O) < P_0^O$  if  $q_0 = 1$ . Analogously, we obtain that  $E(P_1^O | P_0^O) > P_0^O$ , when  $q_0 = -1$ . Consequently,  $E(P_1^O | P_0^O) \neq P_0^O$ .  $\square$

**Proof of Corollary 12.** This proof immediately follows from direct computations.  $\square$

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<sup>33</sup>It is important to notice that transaction prices arise when traders choose to negotiate. Therefore, when computing these expectations we assume that there is trading activity in both periods, that is,  $q_t \neq 0$ ,  $t = 0, 1$ .



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