Security Prices and Market Transparency: The Role of Prior Information on Market Quality

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Abstract

This paper analyzes how prior specification affects the implications of pre-trade transpareny. We find that results related to the amount of information that prices reveal are robust to prior specification. In competitive markets, priors have no impact on market quality, whereas the opposite holds under imperfect competition. More importantly, market depth and volatility might be positively related with informative priors.

Key words: Market Microstructure, Transparency, Prior Information, Market Quality

1 Introduction

One of the most surprising phenomena in the microstructure of financial markets is the heterogeneity in pre-trade transparency exhibited by different trading venues.[1] Although one could argue that most of the modern stock trading platforms distribute information on depth, accessible to traders either subscribing directly to the market feed, or purchasing a consolidated feed, it is also true that in the last ten years there has been a tendency to introduce anonymity into stock, bond, and foreign exchange markets.[2] Similarly, we are nowadays envisioning the evolution to dark trading in exchange markets.[3] An investigation on dark trading can be found in Bloomfield et al. (2011). They compare visible markets in which all orders must be displayed, Iceberg (or reserve) markets that allow both displayed and partially displayed orders, and Hidden markets in which orders can be non-displayed.[4]

There is broad agreement that transparency matters; it affects the informativeness of the order flow and hence the process of price discovery, but the key effects of transparency on security markets are complex and contradictory. As pointed out by Eom et al. (2007) "...there is no consensus on whether an increase in pre-trade transparency results in an improvement or deterioration in market quality." These authors study changes in pre-trade transparency in the Korea Exchange. They conclude that market quality is increasing in pre-trade transparency. In the same line Boehmer et al. (2005) find that the introduction of OpenBook by the NYSE leads to a more active management of trading strategies and improvements in terms of liquidity and informational efficiency. These results contrast

[1] Pre-trade transparency refers to the wide dissemination of price quotations and orders before trade takes place.

[2] Anonymity was introduced into the French Stock Exchange in 2001 and into the Italian one in 2004. See B. Rindi (2008) for a comparison of anonymous versus pre-trade transparent regimes.

[3] Dark pools are trading systems where there is no pre-trade transparency of orders in the system. They can be split into two types: systems such as crossing networks in which cross orders are not subject to pre-trade transparency requirements, and trading venues, such as regulated markets and MTFs, that use waivers for avoiding to display orders. By contrast, Lit markets are pre-trade transparent.

[4] Other issues related to pre-trade transparency are the comparison of floor versus automated trading systems, sunshine trading and the logic for trading halts, among others.

with findings derived in Madhavan et al. (2005). This paper shows that an increase in pre-trade transparency in the Toronto Stock Exchange leads to wider spreads, lower depth, and higher volatility. This is consistent with the empirical evidence from the French Stock Exchange where liquidity increased after anonymity was introduced (see Foucault et al. (2007)), the same occurred when brokers identification codes were removed at The Tokyo Stock Exchange (see Comerton-Forde et al. (2005)). Similarly, the experiments by Bloomfield et al. (2011) support the robustness of informational efficiency and liquidity to opacity regimes too.

One of the studies on the effects of pre-trade transparency on market performance is Madhavan (1996). He shows that there exists an inverse relationship between price volatility and market depth; for some parameter configurations an increase in transparency delivers the desirable properties of higher liquidity and lower price volatility, whereas for others it can exacerbate volatility and decrease liquidity.[5] These results are derived under the proviso that rational investors hold improper or non-informative priors about the liquidation value of the risky asset.[6] We here propose to frame Madhavan's analysis in a more canonical way. We assume that rational investors are endowed with proper priors. The assumption of proper priors is more realistic to us, as in financial markets traders negotiate regularly and therefore, ex-ante; they are not completely ignorant about the liquidation value of the assets they trade.

Our modeling strategy allows tackling two issues simultaneously. First, the impact of transparency on metrics of market quality when investors hold proper priors and hence the role played by prior beliefs in determining such outcomes.[7] Second, to derive a set

[5] Madhavan focuses on anonimity versus transparency. The impact of anonymity on liquidity in an electronic limit order book is also investigated in Foucault, Moinas, and Theissen (2007), while Theissen (2003) examines anonymity in a setting where there is a market maker alongside a limit order book.

[6] Bayesians believe only in proper priors as a foundation for statistics. For a discussion on the relationship between proper and improper prior distributions, see O'Hagan (1994).

[7] It is important to note that most theoretical analysis on transparency set models in which agents hold proper priors. This is so in the seminal papers by Admati and Pfleiderer (1991), Biais (1993), Pagano and Roëll (1996), among others. Consequently, the comparison of results should be made under the same modeling conditions.

of market indicators whose sign depend on prior specification, which may turn out to be useful in empirical investigations on transparency. As a by-product, we have established how stringent the assumption of uninformative priors turns to be.

We show that in competitive markets, priors have no impact on market quality, whereas under imperfect competition the implications of market transparency depend on prior specification. More importantly, the inverse relationship between price volatility and market liquidity, obtained in competitive markets or when investors have improper priors, may not hold with imperfect competition and proper priors, i.e., the inverse relationship between market depth and price volatility reported in Madhavan (1996) may not hold if investors have proper priors.[8] If market depth and volatility are negatively related, an increase in transparency either stabilizes prices and increases market liquidity (both of them suitable properties of a financial market) or increases volatility and reduces liquidity (both of them undesirable for a market). Since preferences for both of these market indicators are aligned, one could conclude that transparency is unambiguously a good or a bad property for a market to have. However, if such a (negative) relation does not hold then there are trade-offs among these two market indicators, and a clear ranking between transparent and opaque markets may not exist.

Additionally, no matter prior specification, transparency increases the precision of traders' predictions about the liquidation value. However, the comparison on market liquidity across market structures does also depend on prior specification as with proper priors the opaque market is deeper for a larger parameter specification set.

The remainder of this paper is organized as follows. Section 2 outlines the notation and the hypotheses, which are common for both settings. Section 3 characterizes the unique symmetric linear equilibrium in both market structures. Section 4 examines the

[8] Although we lack of a clear-cut intuition for the need of uninformative priors to deliver Madhavan's results, we believe that the arguments behind them are along the following lines. The use of improper priors is equivalent to the use of no prior information. By contrast, informative priors convey information on what are the relevant values of the parameters when it comes to study the properties that a given model satisfies.

economic implications of transparency and Section 5 provides some concluding comments. Finally, proofs are relegated to the Appendix.

2 The Model

We consider a pure exchange economy where at time 0 a risky asset is traded in an auction market against a risk-less bond, whose return is normalized to zero. The liquidation value of the risky asset in period 1 is represented by the random variable $\widetilde{v}$. The risky asset is traded at the automatic market-clearing price p, so that its return is given by its liquidation value minus the price.

In this economy there are N strategic traders indexed by n who are endowed with assets and are privately informed about the liquidation value of the risky asset. Trader n holds an initial endowment of y_n shares of the riskless asset and an initial endowment of ω_n shares of the risky asset. Traders have CARA preferences over final wealth $\widetilde{W}_{1n}$ of the form $U(\widetilde{W}_n) = -\exp\{-\rho\widetilde{W}_n\}$, where the common coefficient of risk aversion is $\rho > 0$, q_n are the units of the risky asset traded by investor n and W_n is his final wealth given by[9]

$$\widetilde{W}_n = (\widetilde{v} - p)\, q_n + \widetilde{v}\omega_n + y_n.$$

Traders are risk averse and they choose their desired order quantities to hedge a shock to their endowments. But portfolio hedging is not their sole motive for trade, as strategic traders posses private information.[10] Let $\tilde{s}_n = v + \widetilde{\varepsilon}_n$ denote the private signal of the n-th trader where v is the realized value of the risky asset and $\widetilde{\varepsilon}_n$ is a random variable normally distributed.[11] Traders use their information to speculate in the market so that part of trade is information-motivated with transaction volume arising endogenously. In addition to the demands of the speculative traders, there is another component of order flow, denoted by $\tilde{z}$, which represents the imbalances arising from uninformed or noisy

[9] The derivation of this expression is provided in the Appendix.

[10] Before trading takes place, each strategic trader receives a private signal conveying information about the liquidation value of the risky asset.

[11] Throughout the paper, given the random variable $\tilde{x}$, we will denote its realized value by x.

traders whose demands are price inelastic. We assume that $\tilde{z}$ is normally distributed with a mean normalized to zero and a variance equal to σ_z^2.

Informed traders submit their (net) demand functions to a unique auctioneer, who aggregates all the demand schedules and the noisy demand and calculates the price at which the market clears. At this price, the auctioneer allocates quantities to satisfy traders' demands. In this set-up, we will analyze and contrast two trading mechanisms that differ in their degree of transparency regarding the random imbalance of uninformed liquidity traders. The first mechanism is transparent so that the realization of $\tilde{z}$ is disclosed to all market participants prior to trading, the second one is opaque in that rational traders do not receive any information about the composition of order flow. In what follows, we will use superscripts O and T to refer to the opaque and the transparent markets, respectively.

We end the model assuming that all the random variables $\widetilde{\varepsilon}_1, ..., \widetilde{\varepsilon}_N, \tilde{\omega}_1, ..., \tilde{\omega}_N$ and $\tilde{z}$ are uncorrelated with $\widetilde{v}$, and normally and independently distributed with a mean normalized to zero and a variance equal to σ_ς^2 (its inverse, the precision, will be denoted by τ_ς) for $\varsigma = \varepsilon, \omega, z$.[12] Finally, their joint distribution is assumed common knowledge.

Priors modeling strategy

Regarding the true value of the asset two possible modeling approaches can be followed. The first one is to assume that there is a non-informative prior with uniform density on $\mathcal{R}^1$. Under this approach, the posterior of $\widetilde{v}$ given a signal realization s_n, is normally distributed with mean s_n and variance σ_ε^2. The second one is to assume that the prior of $\widetilde{v}$ is proper,[13] in particular, as it is standard in the canonical models on market microstructure, that $\widetilde{v}$ is

[12] Models assuming a CARA utility function plus a normal distribution of all random variables abound in the market microstructure literature. Moreover, they are the most commonly used even nowadays, as, for instance, in Rindi (2008).

[13] The first approach corresponding to improper priors is the one used in Madhavan (1996). The second one is used in most models of market microstructure (see for instance, Hellwig (1980), Diamond and Verrechia (1981), Glosten and Milgrom (1985), Kyle (1985, 1989), Huddart, Hughes and Levine (2001), among others).

normally distributed with mean $\overline{v}$ and variance σ_v^2. With this specification, the posterior distribution of $\widetilde{v}$, given an observation of the investor n's private signal s_n, is normally distributed (see DeGroot, 1970) with

$$E(\widetilde{v}|s_n) = \frac{\sigma_\varepsilon^2 \overline{v} + \sigma_v^2 s_n}{\sigma_v^2 + \sigma_\varepsilon^2} \qquad \text{and} \qquad var(\widetilde{v}|s_n) = \frac{\sigma_v^2 \sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}.$$

The prior variance σ_v^2 represents the weakness of the prior. Whenever σ_v^2 is large, a relatively large range of values are plausible a priory, representing weak prior information, whereas when σ_v^2 is small, a relatively narrow range of values are plausible a priory, representing strong prior information. Thus, for a high σ_v^2, we have $E(\widetilde{v}|s_n) \simeq s_n$ and $var(\widetilde{v}|s_n) \simeq \sigma_\varepsilon^2$, and for a low σ_v^2 we have $E(\widetilde{v}|s_n) \simeq \overline{v}$ and $var(\widetilde{v}|s_n) \simeq \sigma_v^2$. Note that the posterior $N(s_n, \sigma_\varepsilon^2)$ cannot be obtained as the posterior distribution of any proper prior distribution. Nevertheless, the normality assumption allows to get arbitrarily close to a setup with improper priors, as does Madhavan's (1996), by taking a large variance of $\widetilde{v}$.

In this paper, we will assume that priors are proper. In particular, we will assume that $\widetilde{v}$ is normally distributed with finite mean $\overline{v}$ and variance σ_v^2. By doing so, we are able to determine how the strength of prior information affects the robustness of the results on market metrics provided by Madhavan (1996). Moreover, we will try to recover his results, whenever possible, by taking a large variance of $\widetilde{v}$.[14]

3 The Symmetric Linear Equilibrium

In the market structure M, $M = O, T$, we will denote by $q_n^M(\bullet; I_n^M)$ the net demand schedule of informed trader n, given that he has observed the information set I_n^M. When the market is opaque, I_n^M is a vector which contains trader n's private information, i.e., $I_n^O = (s_n, \omega_n, y_n)$, whereas when the market is transparent, I_n^M contains not only the private signals observed by trader n, but also the public realization of $\tilde{z}$, i.e., $I_n^T = (s_n, \omega_n, y_n, z)$. Thus, $q_n^M = q_n^M(p; I_n^M)$ will represent the net quantity demanded by the n-th informed

[14] The use of limits to recover the results under improper priors is standard. See, for instance, Morris and Shin (2003) in the context of global games.

investor, for particular realizations of both the price of the risky asset $\widetilde{p}$ and the vector which collects his information $\widetilde{I}_n^M$.

In this economy, we will search for a rational expectations equilibrium under imperfect competition (*REE*, for short) in which traders follow identical linear demand functions. A *REE* defines a Bayes-Nash equilibrium in demand functions. In words, it is a price and a set of demand schedules such that the market clears and each trader n, given his information set, submits demand orders which maximize his conditional expected utility taking into account the effect of his trading on prices and taking as given the strategies of other traders. We next define it formally.

Definition 1: A rational expectations equilibrium for the trading mechanism M, $M = M = O, T$, is a price p^M and a vector of strategies $q^M = \left(q_1^M, \cdots, q_N^M\right)$, such that:
i). Excess demand is zero at the equilibrium price

$$\sum_{n=1}^{N} q_n^M(p^M; I_n^M) + z = 0.$$

ii). Each trader n maximizes the expected utility of final period wealth given the strategies of other traders,

$$q_n^M(p^M; I_n^M) \in \arg\max_{\{q_n^M\}} E\left(U\left[\left(\widetilde{v} - p^M\right) q_n^M + \widetilde{v}\omega_n + y_n\right] \middle| p^M, I_n^M\right), \text{ for } n = 1, ..., N.$$

As mentioned above, for the tractability of the analysis, we will focus on symmetric linear rational expectations equilibrium (*SLE,* for short).

Definition 2: A symmetric linear equilibrium is a rational expectations equilibrium in which traders' net demand functions are identical linear functions, that is,

$$q_n^M(p; I_n^M) = \mu^M + \beta^M s_n - \alpha^M \omega_n - \delta_{\{M=T\}} \xi^T z - \gamma^M p, \text{ for } M = O, T.$$

The parameters μ^M, β^M, α^M, ξ^T and γ^M are constants and $\delta_{\{M=T\}}$ is an indicator function that takes value one if $M = T$ and zero if $M = O$.

Following Kyle (1989),[15] in a *SLE* the optimal demand function for trader n is given by

$$q_n^M(p; I_n^M) = \frac{E(\widetilde{v}|p, I_n^M) - p - \rho var(\widetilde{v}|p, I_n^M)\omega_n}{\frac{1}{(N-1)\gamma^M} + \rho var(\widetilde{v}|p, I_n^M)}. \tag{1}$$

This demand maximizes traders' utility whenever the second order condition for a maximum holds, or, equivalently, if inequality below is satisfied

$$\frac{2}{(N-1)\gamma^M} + \rho var(\widetilde{v}|p, I_n^M) > 0. \tag{2}$$

We next compute the optimal demand schedules in the two trading protocols.

3.1 The Opaque Market

When the market is opaque, investor n's *SLE* demand function is given by

$$q_n^O(p; s_n, \omega_n) = \mu^O + \beta^O s_n - \alpha^O \omega_n - \gamma^O p, \text{ for all } n. \tag{3}$$

To study the existence of a *SLE*, we first write all coefficients of speculators' demands as functions of the coefficient α^O. This coefficient is then characterized as a root of a polynomial. If such a root exists, then one might conclude that a *SLE* exists. These facts are formally stated in the following results:

Lemma 1: If there exists a SLE then

$$\mu^O = \frac{2\alpha^O}{\rho\left(N-(N-1)\alpha^O\right)\sigma_v^2}\bar{v}, \tag{4}$$

$$\beta^O = \frac{\alpha^O}{\rho\sigma_\varepsilon^2}, \tag{5}$$

$$\gamma^O = \frac{\alpha^O}{\rho\sigma_\varepsilon^2}\left(1 + \frac{2\sigma_\varepsilon^2}{\left(N-(N-1)\alpha^O\right)\sigma_v^2}\right). \tag{6}$$

Lemma 2: If there exists a SLE, $\alpha^O \in \left(\underline{\alpha}, \frac{N-2}{N-1}\right)$, *where* $\underline{\alpha} = \max\left\{0, \frac{\phi(N-2)-N}{(\phi+1)(N-1)}\right\}$. *In addition, this coefficient is a root of a polynomial of degree three* $Q(\alpha) = a\alpha^3 + b\alpha^2 + c\alpha - d$,

[15] Another derivation of this formula can be found in the proofs of Propositions 1 and 3 in Madhavan (1996), which also follow the proofs in Kyle (1989).

whose coefficients are given by $a = (\phi + 1)(N-1)^2$, $b = (N - \phi(N-2))(N-1)$, $c = (N-1)\varphi$, *and* $d = \varphi(N-2)$, *with* $\phi = \rho^2\sigma_\varepsilon^2\sigma_\omega^2$ *and* $\varphi = \rho^2\sigma_\varepsilon^2\sigma_z^2$.

Lemma 2 shows that α^O is a function of N and of two payoff unrelated components in the trade: a liquidity component due to traders' endowments of the risky asset as endowments can be thought of as liquidity shocks, $\phi = \rho^2\sigma_\varepsilon^2\sigma_\omega^2$, and a liquidity component due to the presence of noise traders, $\varphi = \rho^2\sigma_\varepsilon^2\sigma_z^2$.[16]

Using lemmas above one can asses the impact of prior specification on the coefficients of a *SLE* for an opaque market.

Corollary 1: Sensitivity of traders' demands to private information (β^O, α^O) *is independent of prior specification, whereas price sensitivity* (γ^O) *is smaller if priors are uninformative.*

The logic behind these results is as follows. An increase in σ_v^2 (a less informative prior) has two opposite effects: on the one hand, it makes the risky asset a riskier prospect so that risk-averse investors will reduce their demand (β^O decreases); but, on the other hand, it increases the quality of the informed traders' information (as σ_v^2 increases) so that demand becomes more sensitive to signals (β^O increases).[17] In a *SLE*, these two effects exactly offset each other so that both β^O and α^O become independent of σ_v^2. Regarding γ^O, it captures the whole impact of the priors due to price informativeness. To see arguments above, we start by rewriting investor n's optimal demand as

$$q_n^O(p; I_n^O) = \beta^O(s_n - p) - \alpha^O\omega_n + \frac{2\alpha^O}{\rho(N-(N-1)\alpha^O)\sigma_v^2}(\bar{v} - p).$$

The optimal demand is made of three terms. The two first ones, $\beta^O(s_n - p)$ and $\alpha^O\omega_n$, are independent of prior specification as they only depend on either the impact of the signal $(s_n - p)$ or on the initial endowment (ω_n) which are both private information. The last term, $\frac{2\alpha^O}{\rho(N-(N-1)\alpha^O)\sigma_v^2}(\bar{v} - p)$, is prior-sensitive. When σ_v^2 is small, it is more likely that the

[16] A similar decomposition is used by Naik et al. (1999) to characterize the *SLE* in a model with interdealer trading.

[17] Note that σ_v^2 is directly related with the quality of the informed traders' private information about $\tilde{v}$, as $1/(1 + \frac{\sigma_\varepsilon^2}{\sigma_v^2})$ is the squared correlation coefficient between $\tilde{v}$ and $\tilde{s}_n$.

realization of $\tilde{v}$ will be close to $\bar{v}$. This makes strategic investors' buying shares if $p < \bar{v}$. As σ_v^2 increases, the probability that the realization of $\tilde{v}$ gets close to $\bar{v}$ diminishes, and the difference $\bar{v} - p$ is less relevant for trading decisions. In the limit, with uninformative priors, this third component fully vanishes.[18]

Decomposition above sheds light on investors' motivations to trade. Part of their trade represents portfolio hedging (the second term $-\alpha^O \omega_n$) whose magnitude is independent of priors' modelling. With improper priors it is a constant proportion of the difference between the private signal s_n and price (the first term in $q_n^O(p; I_n^O)$). When priors are informative or proper it depends as well on the difference between the unconditional expected liquidation value and the price (the last term in $q_n^O(p; I_n^O)$). Intuitively, under improper priors traders ignore the initial public information (the prior) as it is completely uninformative. Consequently their demand orders only take into account their private information. With proper priors, investors' orders take into account both the private information and the initial public information traders have. Moreover, as $\frac{2\alpha^O}{\rho(N-(N-1)\alpha^O)\sigma_v^2} > 0$ -recall that $\alpha^O < 1$- it follows that, at the same price, if $\bar{v} > p$, traders' demands are larger the more precise their priors are.

We end this subsection providing a sufficient condition for the existence and uniqueness of a *SLE*.

Proposition 1: If $N \geq 3$, *then there exists a unique SLE.*

This result shows existence and uniqueness of the *SLE* under general conditions. We only require that $N \geq 3$ as in Kyle (1989).[19] A *SLE* does not always exist because, with infinitely inelastic demand by noise traders, speculators may have "too much" monopoly power if there are not enough of them.

[18] The intuition is not due to the utility function being CARA as it is valid under alternative utility functions.

[19] When $N = 2$ the polynomial $Q(\alpha)$, stated in Lemma 2, simplifies to $Q(\alpha) = (\phi + 1)\alpha^3 + 2\alpha^2 + \varphi\alpha$, which lacks of any real strictly positive roots.

3.2 The Transparent Market

In a transparent market, the symmetric linear demand strategy can be written as

$$q_n^T(p; s_n, \omega_n, z) = \mu^T + \beta^T s_n - \alpha^T \omega_n - \xi^T z - \gamma^T p, \text{ for all } n. \tag{7}$$

The following result explicitly characterizes the unique *SLE* corresponding to the transparent market. In this market structure equilibrium coefficients are independent of σ_z^2 as liquidity demand is now common knowledge.

Proposition 2: i). A SLE in a transparent market exists iff $N < (N-2)\phi$, $\phi = \rho^2\sigma_\varepsilon^2\sigma_\omega^2$. *If it exists, then* $\alpha^T = \frac{\phi(N-2)-N}{(\phi+1)(N-1)}$.

ii). If a SLE exists then

$$\begin{aligned}
\mu^T &= \frac{2\alpha^T}{\rho\left(N-(N-1)\alpha^T\right)\sigma_v^2}\bar{v}, \\
\gamma^T &= \frac{\alpha^T}{\rho\sigma_\varepsilon^2}\left(1+\frac{2\sigma_\varepsilon^2}{\left(N-(N-1)\alpha^T\right)\sigma_v^2}\right), \\
\beta^T &= \frac{\alpha^T}{\rho\sigma_\varepsilon^2} \textit{ and } \xi^T = \frac{1}{N+\phi}.
\end{aligned}$$

Proposition 2 part i) shows that in a transparent market a *SLE* may fail to exist as existence requires the liquidity shock via endowments be large enough. The rationale for this inequality is as follows. Whenever the price is not a good estimator of the private information (because either σ_ω^2 or σ_ε^2 are high), or, alternatively, agents face a high inventory cost from maintaining their initial holdings of the risky asset (the coefficient of risk aversion is high), then they find profitable to participate in the market as ϕ will be large enough. But if traders are well informed and/or there is little endogenous liquidity needs (σ_ω^2, σ_ε^2 and ρ are low so that ϕ is also low), then agents are unwilling to reveal their information to others as they find the readjustment of their portfolio very expensive. They prefer to consume their initial endowments giving *boost* to a market breakdown. When this is the case, a *SLE* fails to exist.[20] Part ii) shows that the coefficient associated with

[20] Note that the condition $N < (N-2)\phi$ can be alternatively written as $\tau_\epsilon < \rho^2(N-2)/\tau_\omega N$ which is the condition for existence of a *SLE* under improper priors.

z is negative, so that strategic traders place orders that partly accommodate the noise demand.

Regarding the effect of prior specification, we derive similar comparative statics results as in the opaque market structure, summarized below.

Corollary 2: Sensitivity of traders' demands to private information and to noise demand (β^T, α^T, ξ^T) is independent of prior specification, whereas price sensitivity (γ^T) is smaller if priors are uninformative.

3.3 Equilibrium Comparison

We end this subsection discussing the impact of transparency on the strategic behavior of investors, as it influences not only existence of equilibrium, but also the price intercept and the slope of traders' demands.

Regarding existence of equilibrium, note that if a *SLE* exists for the transparent market, it exists for the opaque market as well. The condition for existence in the former, $N < (N-2)\,\phi$, must meet $N > 2$ but as N is a natural number then it requires $N \geq 3$. Note that this is the condition for existence in the latter. Thus, trading is more robust in the opaque market than in the transparent market. In other words, transparency may induce a form of market failure.[21] Moreover, it is important to point out that the conditions for existence of SLE are the same as the ones derived by Madhavan (1996). This result is not surprising given that under non-informative priors, the existence conditions are independent of σ_v^2.

As for the shape of the demand functions, transparency has two main effects. An increment in the price of the risky asset makes agents more optimistic about its liquidation value, which leads to a smaller reduction in the individual demands as compared to the opaque market. Second, demands become less sensitive to traders' liquidity shocks and

[21] This form of market failure refers to the non-existence of the type of equilibrium we focus on, the SLE. For the comparison to be meaningful, we will focus on parameter values that satisfy $N < (N-2)\,\phi$.

private signals. The rationale is that in the transparent market, there is a higher need for camouflage which makes demands less sensitive to private information.
Next corollary summarizes these results.

Corollary 3: Traders' demands are less sensitive to both private information and price in a transparent market as

$$\alpha^T < \alpha^O,\ \beta^T < \beta^O \text{ and } \gamma^T < \gamma^O.$$

In sum, transparency reduces endogenous liquidity trading creating less risk sharing ($\alpha^T < \alpha^O$), and making orders less responsive to private information about the liquidation value ($\beta^T < \beta^O$). The magnitude of these effects is independent of traders' priors. By contrast, transparency reduces demands' price-responsiveness $\left(\gamma^T < \gamma^O\right)$, with $\gamma^O - \gamma^T$ being larger the preciser priors are.

4 Transparency and Market Quality

In this section we examine the economic implications of pre-trade transparency. Specifically, we analyze the differences between the opaque and the transparent market in terms of some measures of market quality.

4.1 Informational efficiency

The market microstructure literature provides several sensible measures of the informational efficiency of a market.[22] One is the precision of the information held by informed traders, measured by $var^{-1}(\widetilde{v}|p^M, I_n^M)$. Other is the informational content of the equilibrium price, which captures the information revealed by prices to uninformed traders, measured by $var^{-1}(\widetilde{v}|p^M)$.

Proposition 3: At the time the trade is made and z is realized, pre-trade transparency

[22] See Kyle (1989) for a thorough discussion of these measures.

unambiguously increases the precision of the information held by informed traders, i.e.,

$$var^{-1}\left(\widetilde{v}|p^T, I_n^T\right) > var^{-1}(\widetilde{v}|p^O, I_n^O).$$

In the transparent market, the information held by agents is preciser than in the opaque market. Next, we show that prices by themselves may not contribute to this greater precision as in equilibrium $var^{-1}(\widetilde{v}|p^O)$ migth be larger than $var^{-1}\left(\widetilde{v}|p^T\right)$.

Proposition 4: i). Prices are more informative in the transparent market iff the following inequality holds

$$\alpha^O < \frac{\alpha^T}{1 - N\xi^T} = \frac{(N+\phi)\left(\phi\left(N-2\right) - N\right)}{\phi\left(\phi+1\right)\left(N-1\right)}.$$

ii). If $\phi\left(2 + N^2 - 4N\right) - N^2 > 0$ holds, or if σ_z^2 is low enough then prices are always more informative in the transparent market.[23]

Notice that the expressions of the market clearing price are given by

$$\tilde{p}^O = \frac{1}{N\gamma^O}\left(N\mu^O + \beta^O \Sigma_{j=1}^N \tilde{s}_j - \alpha^O \Sigma_{j=1}^N \tilde{\omega}_j + \tilde{z}\right), \text{ and} \tag{8}$$

$$\tilde{p}^T = \frac{1}{N\gamma^T}\left(N\mu^T + \beta^T \Sigma_{j=1}^N \tilde{s}_j - \alpha^T \Sigma_{j=1}^N \tilde{\omega}_j + \left(1 - N\xi^T\right)\tilde{z}\right). \tag{9}$$

Hence,

$$\begin{aligned} var^{-1}\left(\widetilde{v}|p^O\right) &= var^{-1}\left(\widetilde{v}|\frac{1}{\rho\sigma_\varepsilon^2}\Sigma_{j=1}^N s_j - \Sigma_{j=1}^N s_j\omega_j + \frac{z}{\alpha^O}\right) \\ var^{-1}(\widetilde{v}|p^T) &= var^{-1}\left(\widetilde{v}|\frac{1}{\rho\sigma_\varepsilon^2}\Sigma_{j=1}^N s_j - \Sigma_{j=1}^N \omega_j + \frac{\left(1 - N\xi^T\right)z}{\alpha^T}\right). \end{aligned}$$

As $\widetilde{v}$ and $\tilde{z}$ are uncorrelated, prices are more informative in the transparent market if $\alpha^O < \alpha^T/1 - N\xi^T$. Transparency has two opposite effects on the informativeness of prices: on the one hand, it reduces endogenous liquidity trading by making pricess less informative ($\alpha^O > \alpha^T$). On the other hand, it facilitates that noise trading be accommodated ($\xi^T > 0$) leading prices to be more informative. The dominant effect is in general ambiguous, but there are, at least, two instances at which transparency enhances price informativeness.

[23] A particular case in which prices are more informative in the transparent market is when N converges to infinity. A result consistent with those reported in Admati and Pfleiderer (1991).

If σ_z^2 is small enough then α^O approaches α^T making the first effect insignificant. As the second effect is independent of σ_z^2, transparency increases the informational content of prices. Similarly, if ϕ is large enough then transparency increases the informativeness of prices. The first effect is not very significant as $\alpha^O - \alpha^T$ is small when ϕ is large. The second effect, despite being small as well, becomes the dominant one. As conditions stated in the previous two propositions are independent of σ_v^2, the next result trivially follows:

Corollary 4: Results related to informational efficiency are robust to prior specification.

4.2 Market Liquidity

In order to measure the impact of transparency on market liquidity, we now compare the market depth in the two market structures. Following Kyle (1985), the market depth is defined as the quantity of noise trading required to induce the price of the risky asset to boost by one unit. Formally, market depth is given by

$$\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} = N\gamma^O \text{ and } \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1} = \frac{N\gamma^T}{1 - N\xi^T}. \tag{10}$$

From Equation (1) and the market clearing condition, it follows that

$$\begin{aligned} p &= \frac{\sum_{n=1}^N E(\tilde{v}|h_n^M, s_n) + \left(\frac{1}{(N-1)\gamma^M}\right) z + \left(z - \sum_{j=1}^N \omega_n\right) \rho var(\tilde{v}|h_n^M, s_n)}{N}, \text{ where} \\ h_n^M &= \beta^M \sum_{j \neq n} s_j - \alpha^M \sum_{j \neq n} \omega_j + \delta_{\{M=O\}} z. \end{aligned}$$

Thus noise trading affects market price through three channels captured by the three terms in equation above: an adverse-selection effect, a strategic effect and a risk-bearing effect.

The <u>*adverse-selection effect*</u> is captured by the first term via h_n^M. An increase in z increases h_n^O without affecting h_n^T (in the transparent market this effect is absent as noise demand is displayed). Speculator n assumes that an increment might be due to his competitors receiving favorable signals about the payoff of the risky asset. Each speculator therefore adjusts his forecast upwards which generates a price boost.

The <u>*strategic effect*</u>, the second term $z/(N-1)N\gamma^M$, measures the competitiveness of the

market or its size via N, as well as the price sensitivity of strategic traders' demands. As they are less price-sensitive in the transparent market, this second effect is stronger in the transparent market.[24]

Finally, there is a *risk-bearing effect*. The market-clearing price must accommodate for inducing risk-averse speculators to trade. As speculators are better informed in the transparent market, this third effect is more important in the opaque market.

When N goes to infinity the strategic-behavior effect vanishes. The equilibrium price is unambiguoisly more sensitive to changes in the noise demand in the opaque market, i.e., the transparent market is deeper. This result may no longer hold under imperfect competition.

Two sufficient conditions for obtaining a larger market depth in a transparent market are given below.

Proposition 5: If

$$\frac{\sigma_\varepsilon^2(1+\phi)}{N\sigma_v^2} + \left(1 - \phi\frac{(N^2-4N+2)}{N^2}\right) < 0,$$

or, alternatively, if σ_z^2 is sufficiently small, then the transparent market is deeper.

The intuition behind Proposition 5 runs as follows. When σ_z^2 is small enough, α^O is close to α^T, making $\gamma^O - \gamma^T$ very small. As ξ^T is independent of σ_z^2, (10) allows us to conclude that in this case transparency increases market depth. Regarding the sufficient condition

$$\frac{\sigma_\varepsilon^2(1+\phi)}{N\sigma_v^2} + \left(1 - \phi\frac{(N^2-4N+2)}{N^2}\right) < 0$$

its intuition is less clear. When N converges to infinity it simplifies to $1-\phi < 0$.[25] Thus,

[24] To clarify the role of the strategic-behavior effect, note that under perfect competition the optimal investors' demands are given by

$$q_n^M(p; I_n^M) = \frac{E(\widetilde{v}|p, I_n^M) - p - \rho var(\widetilde{v}|p, I_n^M)\omega_n}{\rho var(\widetilde{v}|p, I_n^M)}$$

(see, for instance, Diamond and Verecchia (1981)). Using these demands' expressions and the market clearing condition, it follows that the equilibrium price has only the terms associated with the adverse-selection effect and the risk-bearing effect.

[25] This inequality coincides with the condition that guarantees the existence of the SLE in competitive markets under the transparent regime.

transparency increases market liquidity if the market is sufficiently competitive. With uninformed priors, it becomes

$$N^2 < \phi(N^2 - 4N + 2).$$

With informative priors the sufficient condition is more stringent making the opaque market deeper for a larger set of parameter specifications. Because of this, when investors have proper priors there are more circumstances in which transparency damages market depth.

Next corollary summarizes arguments above.

Corollary 5: The comparison of the market liquidity between the two market structures is ambiguous and depends on the specification of priors.

Next figure illustrates that as N increases the parameter configurations in which the transparent market is more liquid is higher (recall that $DL < 0$ implies that the transparent market is deeper). In the limit, when N converges to infinity, the transparent market is more liquid for all σ_z^2. The black line corresponds to $N = 10$, the red one to $N = 20$ and the green one to $N = 30$.

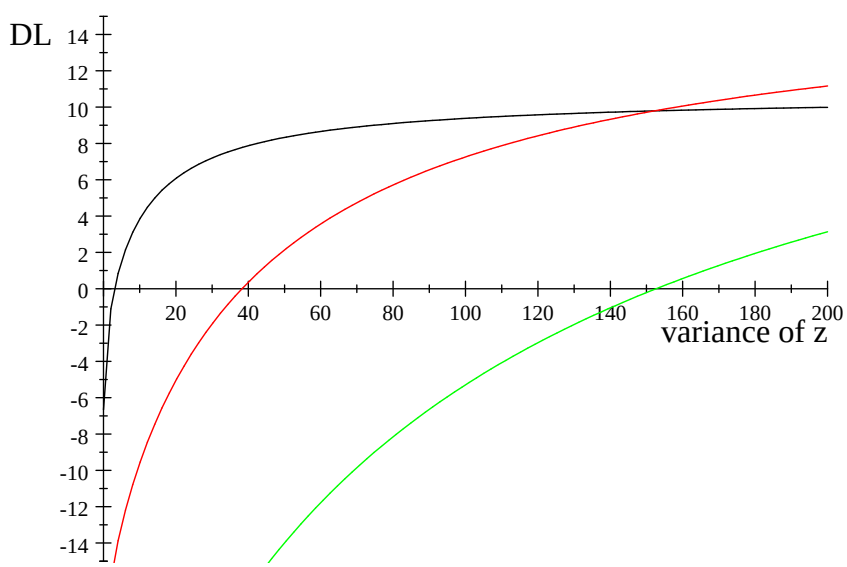

Figure 1. Differences in market liquidity as a function of σ_z^2.

The ambiguity of the effect of changes in pre-trade transparency on market liquidity stated in Corollary 5 is consistent with the empirical evidence. Madhavan et al. (2005) find that an increase in pre-trade transparency in the Toronto Stock Exchange led to lower depth. This result contrasts with the empirical investigation of pre-trade transparency at the NYSE conducted by Boehmer et al. (2005).

4.3 Volatility

The volatility of equilibrium prices is measured by $var(\widetilde{v} - \widetilde{p}^M)$. From Expressions (8) and (9), it follows that

$$\begin{aligned}
\widetilde{v} - \widetilde{p}^O &= \widetilde{v} - \frac{\mu^O}{\gamma^O} - \frac{\beta^O}{\gamma^O}\frac{\sum_{j=1}^{N}\tilde{s}_j}{N} + \frac{\alpha^O}{\gamma^O}\frac{\sum_{j=1}^{N}\tilde{\omega}_j}{N} - \frac{1}{N\gamma^O}\tilde{z} \text{ and} \\
\widetilde{v} - \widetilde{p}^T &= \widetilde{v} - \frac{\mu^T}{\gamma^T} - \frac{\beta^T}{\gamma^T}\frac{\sum_{j=1}^{N}\tilde{s}_j}{N} + \frac{\alpha^T}{\gamma^T}\frac{\sum_{j=1}^{N}\tilde{\omega}_j}{N} - \frac{1 - N\xi^T}{N\gamma^T}\tilde{z}.
\end{aligned}$$

Straightforward computations yield

$$var(\widetilde{v} - \widetilde{p}^M) = \left(\frac{\partial \widetilde{p}^M}{\partial \tilde{z}}\right)^2 \sigma_z^2 + g(\alpha^M, r), \text{ with} \tag{11}$$

$$g(\alpha^M, r) = \left(1 - \frac{\beta^M}{\gamma^M}\right)^2 \sigma_v^2 + \left(\frac{\beta^M}{N\gamma^M}\right)^2 N\sigma_\varepsilon^2 + \left(\frac{\alpha^M}{N\gamma^M}\right)^2 N\sigma_\omega^2, \tag{12}$$

where r stands for the quotient $\sigma_\varepsilon^2/\sigma_v^2$.

Substituting the values of the equilibrium coefficients, $g(\alpha^M, r)$ simplifies to

$$g(\alpha^M, r) = \frac{\sigma_\varepsilon^2\left(4Nr + (\phi+1)\left(N - \alpha^M(N-1)\right)^2\right)}{N\left(N - \alpha^M(N-1) + 2r\right)^2}.$$

To analyze the difference in price volatility,

$$DV = var(\widetilde{v} - \widetilde{p}^O) - var(\widetilde{v} - \widetilde{p}^T),$$

we decompose it into two terms, DV_1 and DV_2, with

$$\begin{aligned}
DV_1 &= \left[\left(\frac{\partial \widetilde{p}^O}{\partial \tilde{z}}\right)^2 - \left(\frac{\partial \widetilde{p}^T}{\partial \tilde{z}}\right)^2\right]\sigma_z^2 \text{ and} \\
DV_2 &= g(\alpha^O, r) - g(\alpha^T, r).
\end{aligned}$$

The first term DV_1 shows that the difference in price volatility partly stems from the difference in market liquidity DL. However, the presence of a second term, DV_2, indicates that there are other factors affecting the difference in price volatility. Whenever DV_2 vanishes or it is small enough, then

$$\begin{aligned} sign[DV] &= sign\left[\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^2 - \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^2\right] \\ &= -sign\left[\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} - \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1}\right] = -sign\left[DL\right]. \end{aligned}$$

An inverse relationship between market depth and price volatility emerges. The mechanism with greater market price volatility provides the lower market depth.[26]

A set-up in which an inverse relationship emerges is in large markets. Note that

$$\lim_{N\to\infty} var(\widetilde{v} - \widetilde{p}^O) = \lim_{N\to\infty} var(\widetilde{v} - \widetilde{p}^T) = 0.$$

Consequently,

$$\lim_{N\to\infty} \left[var(\widetilde{v} - \widetilde{p}^O) - var(\widetilde{v} - \widetilde{p}^T)\right] = 0.$$

Moreover, after some algebra, it is easy to show that DV_1 is of the order of $\frac{1}{N^2}$, but appealing to the Mean Value Theorem, $\lim_{N\to\infty} N^2 DV_2 = 0$. Thus, DV_2 converges to zero quicklier than DV_1 and the result follows.

The analytical derivation of sufficient conditions on the primitives that guarantee a direct relationship between price volatility and market depth is not easy. However, there are some particular cases where we can ensure that this type of relationship is feasible. For instance, if

$$\frac{\sigma_\varepsilon^2(1+\phi)}{N\sigma_v^2} + \left(1 - \phi\frac{(N^2 - 4N + 2)}{N^2}\right) > 0$$

holds, then there exists a unique value of σ_z^2, say $\hat{\sigma}_z^2$, such that both markets are equally liquid (see the proof of Proposition 5), i.e., $DV_1 = 0$ and, therefore, $DV = DV_2$. Thus

[26] With uninformative priors

$$\beta^O/\gamma^O = \beta^T/\gamma^T = 1 \text{ and } \alpha^O/\gamma^O = \alpha^T/\gamma^T = \rho\sigma_\varepsilon^2,$$

so that $g(\alpha^O, r) = g(\alpha^T, r)$ (see Equation (12)) and $DV_2 = 0$ follows. Consequently, $sign\left[DV\right] = -sign\left[DL\right]$.

around $\hat{\sigma}_z^2$ we find parameter configurations where price volatility and market liquidity are lined up. For completeness, we next include some numerical examples illustrating this point.

Figures 2 and 3 expose a particular case in which

$$\frac{\sigma_\varepsilon^2(1+\phi)}{N\sigma_v^2} + \left(1 - \phi\frac{(N^2-4N+2)}{N^2}\right) > 0.$$

Concretely, $N = 10$, $\phi = 2$, $\sigma_\varepsilon^2 = 10$, $\sigma_v^2 = \frac{1}{2}$ and $\rho = 1$. In this case the value of σ_z^2 such that both markets are equally liquid, $\hat{\sigma}_z^2$, is 3.052.In Figure 2 DL is displayed in terms of σ_z^2. When $\sigma_z^2 < 3.052$ the transparent market is more liquid, whereas when $\sigma_z^2 > 3.052$ the opaque is deeper. Figure 3 represents DV as a function of σ_z^2. One can observe how for low values of σ_z^2 ($\sigma_z^2 < 4.030$) the price volatility is higher in the opaque market, whereas for $\sigma_z^2 > 4.030$ the opposite holds true. Therefore when $\sigma_z^2 \in (3.052\,, 4.030)$, the opaque market is both more liquid and volatile.

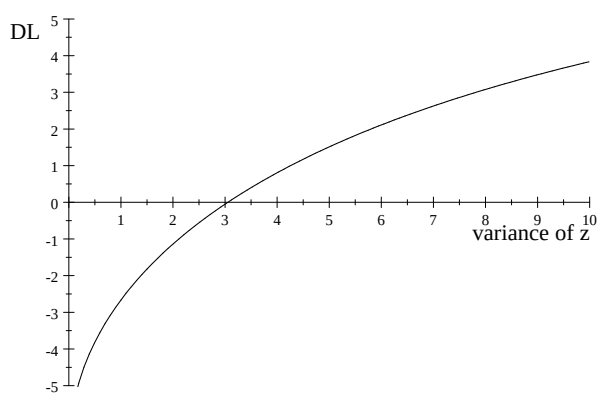

Figure 2. DL in terms of σ_z^2.

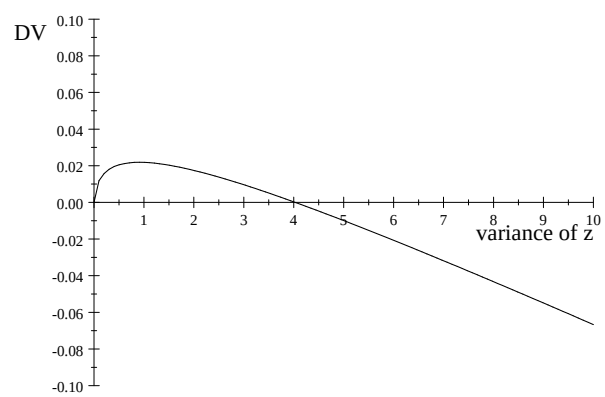

Figure 3. DV in terms of σ_z^2.

Next figures, Figure 4 and Figure 5, display the differences in price volatility in terms of σ_z^2 for different values of N and σ_v^2, respectively. In Figure 4, the black line corresponds to $N = 10$, the red one to $N = 20$ and the green one to $N = 30$ (as N increments the set of parameter configurations in which the transparent market is less volatile becomes higher). In the limit, when N converges to infinity, the transparent market is more liquid

and less volatile for all σ_z^2. In Figure 5 the black line corresponds to $\sigma_v^2 = \frac{1}{2}$, the red one to $\sigma_v^2 = 1$ and the green one to $\sigma_v^2 = 5$. We get that an increase in σ_v^2 leads to a higher set of parameter values in which the transparent market is less volatile

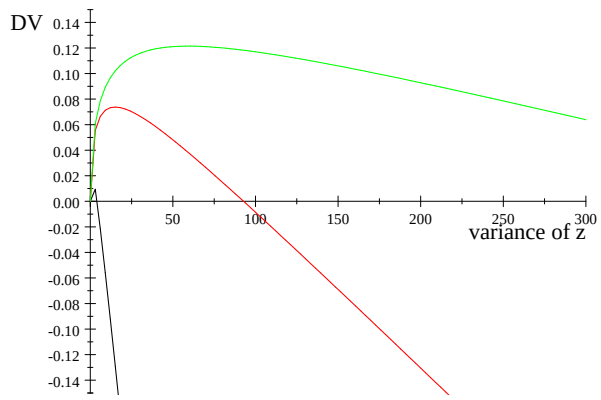

Figure 4. DV for different values of N.

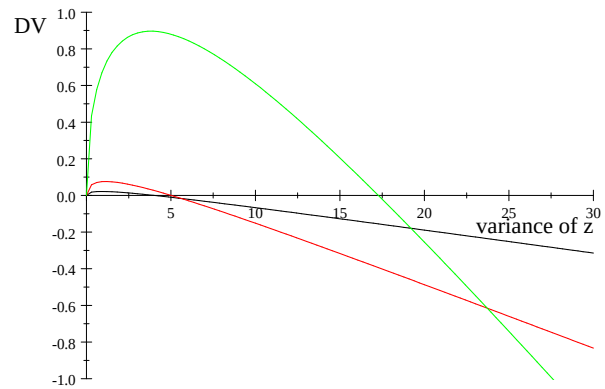

Figure 5. DV for different values of σ_v^2.

Finally, all these results are summarized as follows:

Corollary 6: Under improper priors there is an inverse relationship between market depth and price volatility. This result may no longer hold if priors are proper unless N is large enough.

4.4 Costs of liquidity traders

The expected trading costs of liquidity traders are given by

$$\begin{aligned} C^O &= E\left(\tilde{z}\left(\tilde{p}^O - \tilde{v}\right)\right) = \frac{1}{N\gamma^O}\sigma_z^2 \text{ and} \\ C^T &= E\left(\tilde{z}\left(\tilde{p}^T - \tilde{v}\right)\right) = \frac{1 - N\xi^T}{N\gamma^T}\sigma_z^2. \end{aligned}$$

Note that $C^O > C^T$ if and only if $\frac{1}{N\gamma^O} > \frac{1-N\xi^T}{N\gamma^T}$ or, equivalently, if $\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} < \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1}$. This inequality and Corollary 5 show that the comparison of expected trading costs does also depend on the specification of priors.

When the number of speculators goes to infinity, the expected trading costs of liquidity traders are strictly lower when pre-announcement takes place than when it does not, which is consistent with Proposition 1 in Admati and Pfleiderer (1991). Nevertheless, it might not hold in a model with informative priors and imperfect competition.

5 Conclusions

We have examined the role of traders' priors (proper versus improper) on the implications of market transparency. In competitive markets we have shown that priors have no impact on market quality, whereas under imperfect competition the implications of market transparency depend on prior specification. We show that the inverse relationship between price volatility and market liquidity obtained in Madhavan (1996), assuming improper priors, may not hold with proper priors. The practical implication is that a change in transparency that lowers price volatility does not always reduce the execution costs of liquidity traders.

Our work might be relevant for researchers and regulators alike. From our analysis it can be argued that transparency is beneficial for active securities, independently of the public knowledge traders initially hold on their liquidation value. For inactive securities, one market structure might turn out to be "superior" in markets with a low number of analysts or in those in which traders have little knowledge on the securities they are trading (for instance international traders who often lack of any expertise in the financial markets in which they operate). Otherwise, a clear ranking between transparent and opaque markets might not exist.

6 Appendix

Derivation of Equations (1) and (2). Conditional on his information set, agent n chooses y_n shares of the riskless asset and Q_n shares of the risky asset so as to

$$\max_{Q_n, y_n} E(-e^{-\rho(vQ_n + y_n)} | p, I_n^M)$$
$$s.t.\ pQ_n + y_n = p\omega_n + y_{n,0},$$

or, equivalently, as $y_n = p(\omega_n - Q_n) + y_{n,0}$, to

$$\begin{aligned} & \max_{Q_n} E(-e^{-\rho W_n} | p, I_n^M) \\ s.t.\ W_n &= vQ_n + p(\omega_n - Q_n) + y_{n,0}. \end{aligned}$$

Let q_n denote the units of the risky asset traded by investor n. Thus, $q_n = Q_n - \omega_n$. The previous optimization problem is equivalent to

$$\begin{aligned} & \max_{q_n} E(-e^{-\rho W_n} | p, I_n^M) \\ s.t.\ W_n &= (v - p) q_n + v\omega_n + y_{n,0}. \end{aligned} \tag{13}$$

Suppose that investors, other than n, use identical linear net demands given by

$$q_j^M(p; I_j^M) = \mu^M + \beta^M s_j - \alpha^M \omega_j - \delta_{\{M=T\}} \xi^T z - \gamma^M p, \text{ for all } j \neq n.$$

Then, from the market clearing condition, it follows that investor n faces a linear residual supply curve given by

$$p = p_n^M + \lambda^M q_n, \tag{14}$$

in which

$$p_n^M = \frac{(N-1)\mu^M + \beta^M \sum_{j \neq n} s_j - \alpha^M \sum_{j \neq n} \omega_j - (N-1)\delta_{\{M=T\}} \xi^T z + z}{(N-1)\gamma^M} \text{ and } \lambda^M = \frac{1}{(N-1)\gamma^M}.$$

Hence, investor n has to choose the quantity she trades so as to maximize her expected utility conditional on $\left(p_n^M, I_n^M\right)$. Furthermore, as $W_n | p_n^M, I_n^M$ is normally distributed it follows that

$$E(-e^{-\rho W_n} | p_n^M, I_n^M) = -e^{-\rho\left(E\left(W_n | p_n^M, I_n^M\right) - \frac{\rho}{2} var\left(W_n | p_n^M, I_n^M\right)\right)}.$$

Her optimization problem simplifies to

$$\max_{q_n} E\left(W_n|p_n^M, I_n^M\right) - \frac{\rho}{2} var\left(W_n|p_n^M, I_n^M\right),$$

with, recall (13),

$$\begin{aligned} E\left(W_n|p_n^M, I_n^M\right) &= \left(E\left(v|p_n^M, I_n^M\right) - p_n^M - \lambda^M q_n\right) q_n + E\left(v|p_n^M, I_n^M\right)\omega_n + y_{n,0}, \text{ and} \\ var\left(W_n|p_n^M, I_n^M\right) &= var\left(v|p_n^M, I_n^M\right)\left(q_n + \omega_n\right)^2. \end{aligned}$$

Substituting the two expressions above into the previous optimization problem and maximizing with respect to q_n, the first and the second order condition require respectively, that the equality and inequality stated below hold:

$$E\left(v|p_n^M, I_n^M\right) - p_n^M - 2\lambda^M q_n - \rho var\left(v|p_n^M, I_n^M\right)\left(q_n + \omega_n\right) = 0,$$
$$-2\lambda^M - \rho var\left(p_n^M, I_n^M\right) < 0.$$

From (14) and the first equality, the value of q_n is deduced, with

$$q_n = \frac{E\left(v|p_n^M, I_n^M\right) - p - \rho var\left(v|p_n^M, I_n^M\right)\omega_n}{\lambda^M + \rho var\left(v|p_n^M, I_n^M\right)}.$$

Finally, taking into account the expression of the residual supply curve, given in (14), the two (in)-equalities are equivalent to Equations (1) and (2) in the main text of the paper, as

$$q_n^M(p; I_n^M) = \frac{E(\widetilde{v}|p, I_n^M) - p - \rho var(\widetilde{v}|p, I_n^M)\omega_n}{\frac{1}{(N-1)\gamma^M} + \rho var(\widetilde{v}|p, I_n^M)},$$
$$\frac{2}{(N-1)\gamma^M} + \rho var(\widetilde{v}|p, I_n^M) > 0.$$

Proof of Lemma 1: From the market clearing condition, it follows that the vector $\left(p, I_n^O\right)$ is informatively equivalent to the vector $\left(h_n^O, s_n\right)$, $h_n^O = \beta^O \sum_{j\neq n} s_j - \alpha^O \sum_{j\neq n} \omega_j + z$, so that $E(\widetilde{v}|p, I_n^O) = E(\widetilde{v}|h_n^O, s_n)$, and $var(\widetilde{v}|p, I_n^O) = var(\widetilde{v}|h_n^O, s_n) = a_2\sigma_\varepsilon^2$ with

$$E(\widetilde{v}|p, I_n^O) = \bar{v} + a_1\left(N\left(\gamma^O p - \mu^O\right) - \beta^O s_n + \alpha^O\omega_n - \beta^O(N-1)\bar{v}\right) + a_2\left(s_n - \bar{v}\right),$$

where

$$
\begin{aligned}
a_1 &= \frac{\sigma_\varepsilon^2 \sigma_v^2 \beta^O (N-1)}{D^O}, \\
a_2 &= \frac{\sigma_v^2 \left[\left(\beta^O\right)^2 (N-1)\sigma_\varepsilon^2 + \left(\alpha^O\right)^2 (N-1)\sigma_\omega^2 + \sigma_z^2 \right]}{D^O}, \text{ and} \\
D^O &= \left(\beta^O\right)^2 (N-1)\, \sigma_\varepsilon^2 \left(N\sigma_v^2 + \sigma_\varepsilon^2\right) + \left[\left(\alpha^O\right)^2 (N-1)\sigma_\omega^2 + \sigma_z^2 \right] \left(\sigma_v^2 + \sigma_\varepsilon^2\right).
\end{aligned}
$$

Plugging the expression for $E(\widetilde{v}|p, I_n^O)$ obtained above into (1), and equating coefficients according to (3), it follows that:

$$
\mu^O = \frac{\bar{v}(1-a_2) - a_1 \left(\beta^O (N-1)\bar{v} + N\mu^O\right)}{\frac{1}{(N-1)\gamma^O} + \rho var(\widetilde{v}|p, I_n^O)}, \tag{15}
$$

$$
\beta^O = \frac{a_2 - a_1 \beta^O}{\frac{1}{(N-1)\gamma^O} + \rho var(\widetilde{v}|p, I_n^O)}, \tag{16}
$$

$$
\alpha^O = \frac{-a_1 \alpha^O + \rho var(\widetilde{v}|p, I_n^O)}{\frac{1}{(N-1)\gamma^O} + \rho var(\widetilde{v}|p, I_n^O)}, \text{ and} \tag{17}
$$

$$
\gamma^O = \frac{1 - a_1 N \gamma^O}{\frac{1}{(N-1)\gamma^O} + \rho var(\widetilde{v}|p, I_n^O)}. \tag{18}
$$

From system above we obtain all coefficients as functions of α^O. To do so, we first derive two auxiliary equations. The first one is obtained after computing $\left(1-\alpha^O\right)/\gamma^O$, using (17) and (18), and performing simple manipulations, which gives

$$
a_1 \gamma^O \left(N - (N-1)\,\alpha^O\right) = \frac{N-2}{N-1} - \alpha^O. \tag{19}
$$

The second one follows from rearranging (18), and combining it with (19), resulting in

$$
\rho var(\widetilde{v}|p, I_n^O) = \frac{2\alpha^O}{\gamma^O \left(N - (N-1)\,\alpha^O\right)}. \tag{20}
$$

For getting β^O we first divide (16) and (17) and, then, using the fact that $var(\widetilde{v}|p, I_n^O) = a_2 \sigma_\varepsilon^2$ in the resulting equation, we obtain the expression given in (5).

To derive the expression of γ^O, we first compute $var(\widetilde{v}|p, I_n^O)$. Substituting the values of a_1 and a_2 into (16), and using (20) in the resulting equation, we get

$$
\beta^O = \frac{\sigma_v^2 \gamma^O \left[(\alpha^O)^2 (N-1)\sigma_\omega^2 + \sigma_z^2\right]}{\left(\frac{1}{(N-1)} + \frac{2\alpha^O}{(N-(N-1)\alpha^O)}\right) D^O}. \tag{21}
$$

Multiplying both sides of the previous equality by $\beta^O\sigma_\varepsilon^2$ and taking into account the expression of a_1

$$\left(\beta^O\right)^2\sigma_\varepsilon^2=\frac{\gamma^O a_1\left[(\alpha^O)^2(N-1)\sigma_\omega^2+\sigma_z^2\right]}{\left(\frac{1}{(N-1)}+\frac{2\alpha^O}{(N-(N-1)\alpha^O)}\right)(N-1)},$$

and from (19), it follows that

$$\left(\beta^O\right)^2\sigma_\varepsilon^2=\frac{\left(\frac{N-2}{N-1}-\alpha^O\right)\left[(\alpha^O)^2(N-1)\sigma_\omega^2+\sigma_z^2\right]}{N+\alpha^O(N-1)}. \tag{22}$$

Using the previous equality in the expression of $var(\widetilde{v}|p,I_n^O)$, we get $var(\widetilde{v}|p,I_n^O)=\frac{2\sigma_\varepsilon^2\sigma_v^2}{(N-(N-1)\alpha^O)\sigma_v^2+2\sigma_\varepsilon^2}$. Substituting this expression in (20) and operating, the expression for γ^O given in (6) is derived. Finally, μ^O as given in (4), is obtained by dividing (15) by (18), which gives

$$\frac{\mu^O}{\gamma^O}=\frac{\bar{v}(1-a_2)-a_1\left(\beta^O(N-1)\bar{v}+N\mu^O\right)}{1-a_1N\gamma^O},$$

which combined with (20) and $var(\widetilde{v}|p,I_n^O)=\sigma_v^2(1-a_1\beta^O(N-1)-a_2)$ give (4). ■

Proof of Lemma 2: From (21) and (20) we obtain

$$\beta^O=\frac{\sigma_v^2\left((\alpha^O)^2(N-1)\sigma_\omega^2+\sigma_z^2\right)}{\left(\frac{1}{(N-1)\gamma^O}+\rho var(\widetilde{v}|p,I_n^O)\right)D^O}.$$

Since (2) implies $\beta^O>0$ it follows that $\alpha^O>0$ because of (5). The inequality $\alpha^O<(N-2)/(N-1)$ follows from (22). Combining this results with (6) we have that $\gamma^O>0$. To derive α^O we first note (18) and (19) imply $\left(N-2-\alpha^O(N-1)\right)\rho var(\widetilde{v}|p,I_n^O)=2(N-1)a_1\alpha^O$. Since $var(\widetilde{v}|p,I_n^O)=a_2\sigma_\varepsilon^2$, plugging the values of a_1 and a_2 into the previous expressions, straightforward computations give

$$\rho\frac{\left(N-2-(N-1)\alpha^O\right)\left[\left(\alpha^O\right)^2(N-1)\sigma_\omega^2+\sigma_z^2\right]}{N+\alpha^O(N-1)}=\beta^O(N-1)\alpha^O.$$

Since $\beta^O=\alpha^O/\rho\sigma_\varepsilon^2$ (see (5)), by substituting β^O in equation above, we obtain an equation whose unique unknown is α^O which is a solution of a polynomial of degree three

$$Q(\alpha)=a\alpha^3+b\alpha^2+c\alpha-d.$$

Finally, notice that $Q(\alpha)=0$ can be rewritten as

$$(N-1)(1+\phi)\alpha^2\left(\frac{\phi(N-2)-N}{(\phi+1)(N-1)}-\alpha\right)+\varphi\left(\frac{N-2}{N-1}-\alpha\right)=0,$$

which implies that $\alpha^O > \frac{\phi(N-2)-N}{(\phi+1)(N-1)}$. ■

Proof of Proposition 1: Lemmas 1 and 2 imply that to study the existence of a *SLE* we only need to analyze the roots of $Q(\alpha)$ belonging to $\mathcal{I} = \left(\underline{\alpha}, \frac{N-2}{N-1}\right)$, where $\underline{\alpha} = \max\left\{0, \frac{\phi(N-2)-N}{(\phi+1)(N-1)}\right\}$. Evaluating this polynomial in the extremes of the interval $\mathcal{I}$,[27] we have $Q\left(\underline{\alpha}\right) < 0$ and $Q\left(\frac{N-2}{N-1}\right) > 0$, which guarantees existence of a zero in $\mathcal{I}$. Finally, uniqueness follows from the fact that $Q(\alpha)$ is strictly increasing in $\mathcal{I}$. ■

Proof of Proposition 2: It is omitted since it is similar to the previous ones. ■

Proof of Proposition 3: In equilibrium, we have $var(\widetilde{v}|p, I_n^M) = \frac{2\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2(N-(N-1)\alpha^M)+2\sigma_\varepsilon^2}$. Thus, $var^{-1}(\widetilde{v}|p, I_n^M) = \frac{\sigma_v^2\left(N-(N-1)\alpha^M\right)+2\sigma_\varepsilon^2}{2\sigma_v^2\sigma_\varepsilon^2} > 0$. As $\alpha^T < \alpha^O$, it follows that $var^{-1}(\widetilde{v}|p, I_n^T) > var^{-1}(\widetilde{v}|p, I_n^O)$. ■

Proof of Proposition 4: Using (3) and (7), the market clearing condition imply (8) and (9). As all random variables are normally distributed we have that

$$var^{-1}\left(\widetilde{v}|p^M\right) = \left(\sigma_v^2 - \frac{cov^2(\tilde{v}, \tilde{p}^M)}{var(\tilde{p}^M)}\right)^{-1}, \ M = O, T.$$

From Equations (8) and (9), we have

$$var^{-1}\left(\widetilde{v}|p^O\right) = \frac{1}{\sigma_v^2\sigma_\varepsilon^2}\left(\sigma_\varepsilon^2 + \frac{N\sigma_v^2}{\phi + 1 + \frac{1}{N\left(\alpha^O\right)^2}\varphi}\right)$$

and

$$var^{-1}(\widetilde{v}|p^T) = \frac{1}{\sigma_v^2\sigma_\varepsilon^2}\left(\sigma_\varepsilon^2 + \frac{N\sigma_v^2}{\phi + 1 + \frac{\left(1 - N\xi^T\right)^2}{N\left(\alpha^T\right)^2}\varphi}\right).$$

Notice that $var^{-1}\left(\widetilde{v}|p^O\right) < var^{-1}(\widetilde{v}|p^T)$ is equivalent to $1/\alpha^O > \left(1 - N\xi^T\right)/\alpha^T$. Substituting α^T and ξ^T by their values, this inequality simplifies to

$$\alpha^O < \frac{\alpha^T}{1 - N\xi^T} = \frac{(N+\phi)\left(\phi\left(N-2\right) - N\right)}{\phi\left(\phi+1\right)\left(N-1\right)}.$$

[27] Rearranging (18), we have $\frac{N-2}{N-1} = \gamma^O\left(\rho var(\widetilde{v}|p, s_n, y_n) + a_1 N\right)$, which implies that $N > 2$ must hold for a SLE to exist as both $\gamma^O > 0$ and $a_1 > 0$.

It is easy to see that $\frac{\alpha^T}{1-N\xi^T} > \frac{N-2}{N-1}$ iff $\phi\left((N-2)^2-2\right)-N^2>0$. As $\alpha^O < \frac{N-2}{N-1}$, then $\alpha^O < \frac{\alpha^T}{1-N\xi^T}$ holds in this case, and consequently, prices are more informative in the transparent market. Finally, this result is also satisfied provided that α^O is low enough, which is equivalent to saying that σ_z^2 is low enough. Just note that $\frac{\alpha^T}{1-N\xi^T} > \alpha^T$ and σ_z^2 low enough guarantees that α^O gets close to α^T. ■

Proof of Proposition 5: From (3) and (7), the market clearing condition implies that

$$\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} = N\gamma^O \text{ and } \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1} = \frac{N\gamma^T}{-N\xi^T+1} = \frac{N\gamma^T\left(N+\phi\right)}{\phi}$$

Substituting the equilibrium values we have that the difference in liquidity DL among the two markets is given by

$$DL = \frac{N}{\rho\sigma_\varepsilon^2}\left(\alpha^O\left(1+\frac{2r}{N-\left(N-1\right)\alpha^O}\right)-\frac{\left(N+r\left(1+\phi\right)+\phi\right)\left(\left(N-2\right)\phi-N\right)}{\left(\phi+1\right)\left(N-1\right)\phi}\right).$$

where $r=(\sigma_\varepsilon^2/\sigma_v^2)$. DL is strictly increasing in α^O. At $\alpha^O=\alpha^T$, it attains a negative value. At $\alpha^O=\left(N-2\right)/\left(N-1\right)$ it is also negative if $\frac{\sigma_\varepsilon^2(1+\phi)}{N\sigma_v^2}+\left(1-\phi\frac{\left(2-4N+N^2\right)}{N^2}\right)<0$, so that $DL<0$, and the transparent market is deeper. Otherwise, the result follows when σ_z^2 is low enough as then α^O is close enough to α^T. ■

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