Auctioning Off With a Split Mind: Privatization Under Political Constraints

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Abstract

When privatizing, governments have conflicting objectives, like raising revenues and minimizing induced unemployment. We construct two mechanisms that take into account both criteria: a first-score auction in which bidders bid both in terms of price and retained excess labor, and a first-price auction in which bidders bid only over price but they also commit to keep a predetermined by the government number of employees. When bidders differ in their costs of accommodating excess labor, the resulting competition softens, and governments may optimally want to appear strong against labor redundancies. In the first auction this is done by setting a scoring rule that does not correspond to their genuine preferences, and in the second by announcing a smaller labor requirement. Nonetheless, such policies require strong commitment ability.

Keywords: multi-objective auctions, multidimensional private information, privatization.

JEL Classification Numbers: D44, D82, L33.

1 Introduction

One of the most compound transactions of the three year long privatization program was finished today. In the last one and a half year it was established that in significant part of the cases the privatization is not simply buying and selling, but a complex process taking employment, regional and development interests into consideration as well.
Miklós Kamarás, CEO of the Hungarian Privatization and State Holding Company[1]

Most auction theory papers represent the auctioneer as single minded, preoccupied only with the maximization of revenues she is able to raise. In practice though, auction organizers care about more than just the price at which they are able to sell an object. This may happen for example because they may have preferences over the post-auction use of the object they sell. Equivalently, if they are buyers organizing a procurement auction, because they may be facing a group of potential sellers who offer a wide palette of differentiated objects.[2]

We, therefore, usually observe auction rules requiring the bidders to submit multidimensional bids, and specifying that the method determining the winning bid takes into account all different characteristics of the bids.[3] Along these lines, this paper drops the unidimensionality of bids assumption, and analyzes an auctioneer who asks the bidders to bid over two dimensions.

Moreover, when the single-mindedness of the auctioneer's objective is dropped, the unidimensionality of the asymmetric information in the bidders-auctioneer relationship becomes even more problematic. If, for example, the bidders compete to win the contract for a complex project, it is difficult to claim that all relevant private information each of the bidders has about her ability to perform different aspects of the project the auctioneer cares about can be summarized into one sufficient statistic.

The paper is an attempt to study the use of auctions in the privatization of public firms. Maskin (1992) offered compelling arguments explaining the reasons for which transition

[1]Excerpt from a speech on the occasion of the signing ceremony of the privatization of the Dunaferr Rt. steel mill on September 30, 2004. (See corporate website http://www.duraferr.hu/english/08-media/news20040930.htm)

[2]Standard examples in the literature include the auctioning of monopoly franchises, and the procurement for technologically advanced military equipment.

[3]The World Bank, for example, instructs that the procurement rules of the projects it funds should take explicitly into account quality differences among bidders. The European Commission's directives on public procurement specify the use of the criterion of "the most economically advantageous tender", which takes into account the offered price, quality, completion date and other parameters.

economies in Eastern Europe should award the privatized firms via auctions to the bidders that maximize the firms' efficiency. Nonetheless, in practice we observe that governments often ask bidders to submit, in addition to the price they are willing to pay, some aspects of the post-privatization policies of the firm under their management, most notably, but not exclusively, their future employment and investment plans.[4] For reasons that have to do with political considerations, governments, especially those in transition economies, want to reduce the social costs of firing workers during the restructuring face following a privatization.[5]

Even in Germany, which was in a better position than any other transition economy to handle the social costs of restructuring as the privatization of eastern German firms took place after the reunification, the German privatization agency (Treuhandanstalt) was taking into account in its decisions employment guarantees. Actually, Birgit Breuel, the second CEO of Treuhandanstalt, shortly after her appointment stated explicitly: "We have two criteria: first the price we can get and second the amount to be invested and the number of jobs. The latter has begun to play a bigger role for us".[6] Because of adopting these criteria the privatization agency was willing to trade off revenues, or even to accept offering subsidies, in exchange of securing employment. A number of studies have documented and tried to estimate the magnitude of this trade off. E.g., Carlin and Mayer (1994) report that depending on the sector of the privatized firms Treuhandstalt effectively accepted a sale price reduction between 12,000 DM and 50,000 DM per retained position.[7], [8] In a more general study, Chong and López-de-Silanes (2002) report that in a sample of 400 privatizations around the world they found that the use of employment guarantees before privatization reduces the sale price by 16 percent, and that this reduction is robust and statistically significant in the various

[4]For example, according to the Russian Privatization Act, when awarding public assets, authorities should also take into account non-price criteria, such as benefits to localities (Articles 20 and 21).

[5]The issue of the attenuation of the social costs of transition steered a lot of discussion regarding the optimal speed of reforms. (See, for example, Aghion, Blanchard and Carlin (1997).)

[6]As quoted in Lundberg and Donahue (1992).

[7]Nonetheless, there were instances where the buyers received large subsidies to retain employment positions, as in the privatization of Rostock's Shipyards in which the buyer received a subsidy of 710.000 DM per saved job in order to keep 1,400 employees. (See Lichtblau (1993), as referred to in Dyck and Wruck (1998).)

[8]On the other hand, Hau (1998) reports a positive correlation between sale price and preserved employment, but this is more an endogeneity result due to the sectoral composition of the privatized firms, as firms that operate in sectors that are in decline require a much larger labor shading but also attract lower (or even negative) prices as they have a lower prospect of turnaround.

econometric specifications they employ.

We have opted not to challenge the government's objectives, but to discuss their repercussions. How do the government's objectives influence the privatization process and the bidders' behavior? Which auction rules should be preferred given these pre-determined objectives? Finally, we attempt to shed some light on the reasons for which foreign capital has mostly shied away from investing in privatized firms in transition economies.

Consider an auctioneer (government) that wants to sell a firm to a group of prospective buyers. The government wants to maximize the privatization proceeds, but at the same time it also wants to secure the maximum possible employment level in the privatized firm. In principle, an auction should take into account both criteria. Moreover, each prospective buyer has two pieces of private information: A signal about the profitability of the efficiently run firm under her management, and a bidder specific cost of accommodating excess labor. Clearly, depending on the importance given to each criterion, a different outcome will arise, both in terms of what the government will be able to achieve and of the identity of the winner. Essentially, the government can influence the degree of heterogeneity among buyers, and consequently affect the competitive pressure under which they participate in the auction.

We construct two alternative mechanisms: First, a one-stage score-based auction, in which the government requests tenders specifying both the price the bidders are willing to pay, as well as the bidders' future employment plans. Then by employing a scoring rule, it converts the bids into scores, and based on the scores' ranking, it awards the firm. Secondly, a two-stage price-only auction with fixed-labor-requirement, i.e., an auction in which bidders bid only over price but at the same time they commit to secure the positions of a number of employees predetermined by the government.

Both mechanisms are empirically relevant as they have been employed in practice. For example, Treuhandanstalt's method is similar to our score-based auction. Potential buyers were submitting their entire business plan and their bids were evaluated based on all aspects of the plans (See, Priewe (1993)). Although the relative weights given to different attributes of the bids were not made public, as it will be explained later in the paper, it was possible to be deducted that sales prices (or more general, reduction to financial obligations of the group of firms) were receiving twice the weight the employment guarantees were getting.[9] In other

[9]There are other examples of auctions that fit our modeling framework as well. According to the latest EU directives on procurement auctions, when the criterion of the most economically advantageous offer is used the contracting entity should preannounce the relative weight each attribute is receiving in the evaluation, and if that is not possible the attributes should be presented in a descending order of importance.

occasions, like the privatization of Hellenic Shipyards in Greece in 2001, the privatization authority asked for bidders to commit to retain at least 1400 employees for 6 years, to keep operating its main shipyard in Scaramangas for 10 years and to infuse into the firm capital in the form of an increase of equity equal to twice the price the bidders were offering to buy the firm. Only if these conditions were accepted (plus some other technical issues) the bids would have been considered valid, and then evaluated according to the sales price that was offered.[10]

The main difference between these two mechanisms is that the score-based auction allows for more flexibility in the bidders' strategies. Bidders have the ability to differentiate themselves, by bidding more aggressively over the dimension (i.e., price or labor) they have a comparative advantage. On the other hand, in the fixed-labor-requirement auction, all bidders are forced to behave in a uniform fashion with respect to their future employment pledges. Nonetheless, this forced uniformity in their behavior affects them differently. Bidders with low costs of accommodating excess labor, depending on the set labor requirement, gain a comparative advantage compared to the bidders with higher costs.

Under both mechanisms the government can influence the degree of competitiveness in the contest. In the score-based auction, this is done by the choice of relative weights in the employed scoring rule. A large relative weight on the bidders' employment pledges favors the bidders who are efficient in accommodating excess labor and reduces the competitive pressure they feel. Analogously, in the fixed-labor-requirement auction, the number of labor positions required to be retained directly determines the degree of comparative advantage a bidder enjoys. The more stringent is this requirement, the more advantaged become the bidders who are efficient in accommodating excess labor.

We explicitly characterize the equilibrium bids in both auctions, and we perform comparative statics' exercises with respect to the instruments the government has at her disposal in each mechanism to influence the intensity of competition among bidders. In the score-based auction, the government would like to commit to a scoring rule that undervalues excess labor compared to its true preferences. By doing so, the government succeeds in reducing the comparative advantage of "insiders" (i.e., of those who are efficient in accommodating excess labor). Analogously, in the fixed-labor-requirement auction the government would like to announce a less stringent requirement than it would have, had all bidders been equally efficient in accommodating excess labor. Moreover, in this case, by doing so, the government

[10]See article in Greek newspaper $K\alpha\theta\eta\mu\epsilon\rho\iota\nu\eta$ (Kathimerini), August 10, 2001.

succeeds in attracting more bidders. As a result, governments should be inclined to appear tough against labor redundancies before privatizations.

Nonetheless, such behavior can be productive only if the government has commitment power due to her reputation. Lack of commitment ability does not allow the government to persuade bidders that, in a score-based auction, at the end it will order the bids according to the announced scoring rule. Bidders, anticipating the government reneging from its announced scoring rule, will bid as if they face a scoring rule based on true preferences. This commitment problem appears to be less severe in the second mechanism, as the government announces the fixed-labor-requirement before the auction takes place. Nonetheless, also there it is possible for the government to start renegotiating with the winner to raise the secured employment positions at the cost of making concessions on the price the winner will end up paying.

Due to the bidimensionality of the private signals, the paper departs from the related literature on multidimensional score-based auctions. Both Che (1993) and Branco (1997) assume unidimensional private information.[11] Moreover, we do not consider any correlation between these two private information parameters. This comes in contrast with Cornelli and Li (1997) who study optimal privatization mechanisms in an environment not unlike ours. They also endow their government with conflicting objectives, and allow a potential bidder to hold two correlated private signals. In the presence of correlation between private information parameters, all private information can be essentially treated as one variable.[12] Boone and Goeree (2009) study optimal privatization auctions in a common valuation setting and discuss the repercussions of the presence of an insider with superior information about the firm. There is also an ever growing literature on multidimensional mechanism design involving multiple instrument-multiple private information variable environments (e.g., Rochet and Choné (1998), Asker and Cantillon (2010)).

The following section sets up the model and presents the two mechanisms. Section 3 analyses the score-based auction and characterizes the optimal scoring rule. Section 4 addresses the fixed-labor-requirements auction. Finally, Section 5 concludes.

[11]In Che (1993) the fact that the optimal allocation can be implemented via a standard two-instrument direct revelation mechanism (as in Riordan and Sappington (1987)). Branco's (1997) optimal direct revelation mechanism is a bit more complicated due to a common-cost component he assumes.

[12]Nonetheless, they search for the optimal privatization scheme, while we content ourselves with studying pre-specified mechanisms.

2 The Model

There is a government that organizes an auction to privatize a public firm. There are 2 bidders, indexed by i, interested in acquiring this firm.

2.1 Preferences and Technology

The government preferences are summarized in the following utility function $U_G(p, l)$. Where U_G is increasing in both of its arguments, i.e., the government wants: a) to maximize the price (p) at which it will be able to sell the firm, and b) to secure the maximum possible employment level in the privatized firm (l). It is assumed that G wants to maximize a convex combination of these two variables.

$$U_G = \alpha p + (1 - \alpha)l, \tag{1}$$

where $\alpha \in (0, 1)$ represents the relative weight of price in the utility function. These preferences can be understood as coming out of a political economy model in which the government wants to raise revenues to reduce its budget deficit, but at the same time it wants to appeal to its citizenry for re-election purposes.

Obviously, the bidders want to acquire the firm at low price, and to pledge employment plans that maximize the value of the firm. Each bidder receives a private signal x_i, representing the value of the firm under her management according to the optimal business plan (i.e., if allowed to run the firm unimpeded). The signals are assumed to be independently and identically distributed (i.i.d.) according to a cumulative distribution function F. The associated density function, f, is assumed to be absolutely continuous and strictly positive over its closed and bounded support $[\underline{x}, \overline{x}] \equiv X \subset I\!R_+$.

Moreover, we assume that if the bidder is forced to alter her business plan to accommodate excess labor she has to pay extra costs. Excess labor is defined as the difference between the employed labor force and the labor needed to create the maximal firm value. By normalizing the optimal employment level to 0, we are able to denote l as excess labor. The cost of accommodating excess labor is denoted by θ_i, which also is private information to each bidder. $\theta_i \in [\underline{\theta}, \overline{\theta}] \equiv \Theta \subset I\!R_{++}$, i.i.d. across bidders, distributed according to G, with absolutely continuous and strictly positive over its support density function, g. Given that bidders are risk neutral their utility function is given by the following parametric specification

$$U_i = x_i - \theta_i l_i - p_i. \tag{2}$$

2.2 Description of the Two Alternative Auction Mechanisms

2.2.1 Score-based Multidimensional Auction

We consider a one-stage mechanism. The government announces the rules of the auction, solicits bids, denoted as b_i, which are price-employment plan pairs, (p_i, l_i), and then, according to the announced rules, it chooses the winner. In order to be able to create a complete ordering of the bids, it uses a scoring rule, $S(p, l)$, where $S : \mathbb{R}^2_+ \longrightarrow \mathbb{R}_+$. The scoring rule is assumed to take the following parametric form:

$$S(p, l) = \lambda p + (1 - \lambda)l, \tag{3}$$

where $\lambda \in (0, 1)$ represents the relative weight of price in the scoring rule. It may appear strange that the scoring rule adds dollars and people, which clearly measure up to different scales. To solve this measurability problem, in practice scoring rules do not use absolute values but relative terms that are expressed in comparable scales. Nonetheless, as the analysis of the auction is not affected by this rescaling of the variables we choose for simplicity of the exposition to present the scoring rule in absolute terms.[13]

Notice that λ is a choice variable for the government, which is employed strategically to maximize its utility, i.e., we do not equate λ to α. Moreover, the scoring rule is common knowledge prior to the auction.[14] Nonetheless, the credible announcement of a λ different than α requires a commitment power from the side of the government which may not be realistic. Both polar cases of full and no commitment are discussed.

Given the rules of the auction, the bidding strategies map the bidders' private information into bids, $B_i(x_i, \theta_i)$, i.e., $B_i : X \times \Theta \longrightarrow \mathbb{R}^2_+$.

[13] This change in the variables can be done the following way: First define p^r as the ratio p/p_{max}, where p_{max} refers to the maximum value of the price attribute received in the auction. Notice that p^r can take values between 0 and 1. Then define l^r as the fraction of the redundant workers the bidder pledges to keep. Again, l^r is defined in the zero, one interval. A scoring rule that is defined as $S^r(p^r, l^r) = \lambda^r p^r + (1 - \lambda^r)l^r$ does not have this problem. Notice, though that because, in general, the p's are much larger numbers than the l's, the value of λ will be much smaller than that of an equivalent λ^r.

[14] One should mention that the Treuhandanstalt privatization agency was never reporting the relative weights it was using to evaluate the different attributes of the bids, and in that sense the process was much less transparent then the one we analyze. But as Dyck and Wruck (1998) report, these weights could have been uncovered by checking the performance-related contracts that were used to incentivize the Treuhandanstalt's managers who were in charge of the privatization of groups of firms.

We consider a first-score auction, which is a generalization of the first-price auction in a multidimensional framework. In such an auction the bidder with the highest score wins and is required to materialize the promise she made with her bid; i.e., to pay price p and to commit to excess labor l.[15]

2.2.2 Fixed-Labor-Requirement Unidimensional Auction

We now consider a two-stage mechanism. The government first announces the amount of excess labor prospective buyers must commit to keep, denoted by L. Then the bidders, given L, decide whether to participate in the auction. If they participate, they submit bids b_i, which are now unidimensional, i.e., bidders bid only in terms of price, p_i. Then, the winner is determined according to a standard first-price auction.

Given the fixed-labor-requirement announced by the government, the bidding strategies map the bidders' bidimensional private information into unidimensional bids, $B_i(x_i, \theta_i \mid L)$, i.e., $B_i : X \times \Theta \longrightarrow \mathbb{R}_+$. Note, that, in this case, the strategic variable for the government is L.

3 Solving the Score-based Multidimensional Auction

3.1 Reduction to a Unidimensional Auction

The problem bidder i faces in the score-based auction can be disentangled by looking at two subproblems. First, given her private information, the bidder has to find the score she would like to reach, i.e., solve for the optimal strategy that maps from her private information into scores. And then, she has to find which is the optimal way to reach that score, i.e., the optimal combination of p and l.[16]

Suppose that given the realizations of her private information variables, bidder i's equilibrium strategies prescribe a score $\overline{s}$. Bidder i wants then to find the least costly pair (p, l)

[15] A second-score auction is an auction where the highest bidder is still the winner, but has to pay a price p cum to commit to excess labor l such that the combination of this pair matches the second highest score. For a more detailed analysis of score auctions the reader should refer to Che (1993) and Asker and Cantillon (2008).

[16] It should be noted that the second subproblem is a decision theoretic one, that is solved independently of the auction environment the bidder is participating in. This is the case because the bidder's place in the auction, and hence her probability of winning, depends only on the score and not how this score was reached.

that ensures to her $\overline{s}$. This problem can be represented by the following program: $\max_{p,l} U_i$ subject to $S(p,l) = \overline{s}$. Its solution is presented in the following Lemma.

Lemma 1 *For all possible scores, $\overline{s}$, a bidder may want to reach, her optimal bidding strategy satisfies the following bang-bang property*

$$\begin{array}{llll} \forall\, \theta_i < \tilde{\theta}, & p^* = 0 \text{ and } l^* > 0 & s.t. & S(0, l^*) = \overline{s} \\ \forall\, \theta_i > \tilde{\theta}, & l^* = 0 \text{ and } p^* > 0 & s.t. & S(p^*, 0) = \overline{s}, \end{array}$$

where $\tilde{\theta} \equiv \frac{1-\lambda}{\lambda}$.

Proof: Since the scoring rule is linear the constraint of the program is $\overline{s} = \lambda p + (1-\lambda)l$. This equation defines a linear isoscore curve with slope $\frac{dp}{dl} = -\frac{1-\lambda}{\lambda}$. Given that U_i is also linear, the program has a corner solution. Depending on the slope of the linear indifference curve ($\frac{dp_i}{dl_i} = -\theta_i$), the bidder bids over one variable and sets the other one to zero. Given that λ is invariant on s, the slopes of the isoscore curves are constant for all $\overline{s}$, and hence the cut off $\tilde{\theta}$ is the same for all $\overline{s}$.[17] Q.E.D.

Clearly, bidding behavior is lop-sided. Bidders bid a positive amount only over the dimension they have a comparative advantage. The ratio $\frac{1-\lambda}{\lambda}$ expresses the exchange rate the government uses to translate excess labor into money. Bidders, whose cost of pledging excess labor (i.e., the bidder's exchange rate) is lower than the government's exchange rate, prefer to reach their desired score by pledging a lot of excess labor and pay nothing. On the other hand, bidders, whose cost of pledging excess labor is high, bid the opposite way. This allows us to separate the bidders into two categories: We name bidders who bid in terms of excess labor (i.e., those whose $\theta_i \leq \tilde{\theta}$) as type I bidders, and bidders that bid in terms of price (the complementary set) as type II bidders.

The bang-bang property of the solution depends crucially on the linearity of both the score function and the bidders' payoffs.[18] Nonetheless, the risk neutrality of the bidders, which is responsible for the linearity of the bidders' payoffs, is a standard in the auction

[17] It should be noted though that the homotheticity of the scoring rule, i.e., the fact that λ does not depend on the level of the score, is crucial for the result. Had not this been the case, the slope of the isoscore curve would have been changing with the score, and hence $\tilde{\theta}$ would have been a function of $\overline{s}$. In such a case, it would have been possible that depending on the targeting score a bidder would some times bid over prices and some other over excess labor.

[18] The same property is derived by Ewerhart and Fieseler (2005), in a specific competitive procurement environment employing unit-price contracts.

literature assumption. As for the linearity of the scoring rule, we appeal to reality, since the score is often a weighted sum of the various attributes of the bid.[19]

No doubt, this property of the bids seems extreme. It resembles though the properties of observed bids in privatizations. It is usually the case that workers, in worker-management buyouts, attempt to buy a firm at negligible price stressing the beneficial aspects of such "acquisition" in terms of local employment. On the other hand, foreign investors rarely pledge voluntarily employment plans. When they end up doing that, it is usually as a concession after negotiations.[20]

Remark. It is interesting to do comparative statics' with respect to the government's choice of the scoring rule. Obviously, $\frac{d\tilde{\theta}}{d\lambda} < 0$. This means that when the government is more interested in raising funds, a larger proportion of bidders bid over prices. Hence, the government may affect the nature of the bids and the competitiveness of the environment the bidders are facing.

One should notice though that it's not necessary for both types to be present in an auction. Remember that $\theta \in [\underline{\theta}, \overline{\theta}] \subset I\!R_{++}$, while $\tilde{\theta}$ can take values from 0 to $+\infty$, depending on whether λ is 1 or 0, accordingly. If $\tilde{\theta} \leq \underline{\theta}$, which occurs whenever $\lambda \in [1/1 + \underline{\theta}, 1]$, there are only type II bidders who bid in terms of price. On the other hand, if $\tilde{\theta} \geq \overline{\theta}$, which occurs whenever $\lambda \in [0, 1/1 + \overline{\theta}]$, there are only type I bidders who bid in terms of labor.

Therefore, depending on the value $\tilde{\theta}$ takes, auction participants may either all bid in terms of one dimension (in terms of labor if they are all type I bidders or in terms of price if they are all type II bidders) or some may bid in terms of labor and others in terms of price (i.e., both type of bidder coexist). It is in this last case that we truly have a multidimensional auction, in the sense that in the first two cases we can treat bids as unidimensional.

Thus, when $\tilde{\theta} \leq \underline{\theta}$, only type II bidders participate and bid in terms of price. Since their values of θ do not play any role, the auction can be analysed as a standard symmetric unidimensional auction where bidders' private information is also unidimensional. When $\tilde{\theta} \geq \overline{\theta}$, only type I bidders participate and bid in terms of labor. Nonetheless, now their values of θ are relevant and the auction is a unidimensional auction but one in which bidders' private information is bidimensional.

[19] According to the World Bank's toolkit or practitioner's guide to privatization "... a weighted average of the technical and financial bids of all retained bidders is calculated to determine the winning bid." (Welch and Fremont (1998))

[20] E.g., see Bornstein (2000).

The analysis becomes more complicated when $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$, and hence both types of bidders coexist and some submit bids in terms of price and other in terms of labor. Nonetheless, even in this case, we do not need to analyze separately the behavior of both types to characterize the equilibrium of the auction. Type II bidders' behavior can be suppressed. This is accomplished by focusing on a truncated auction in which all type II bidders are replaced by $\tilde{\theta}$-type bidders who bid in terms of excess labor. The following Proposition shows that such truncated auction is equivalent to the original one.

Proposition 1 *(i) The original auction is equivalent to a truncated auction in which (a) all bidders with $\theta_i \geq \tilde{\theta}$ are replaced with $\tilde{\theta}$-type bidders who bid in terms of excess labor, and (b) the winner of the auction has the option to choose a new price-excess labor combination with which she will reach the score she has to match.*
(ii) Moreover, any equilibrium strategy for the bidders with $\theta_i < \tilde{\theta}$ in the original auction is also an equilibrium strategy in the truncated auction.

Proof: (i) First, consider any strategy $p_{\theta_i}(x)$, a bidder with $\theta_i \geq \tilde{\theta}$ may follow in the original auction. By following such strategy, she gets a score $s_{\theta_i}(x) = \lambda p_{\theta_i}(x)$. Note that it is equally costly for all $\theta_i \geq \tilde{\theta}$ bidders to reach the same score. This implies that without loss of generality we can restrict attention to strategies such that $p_{\theta_i}(x) = p_{\tilde{\theta}}(x),\ \forall\ \theta_i \geq \tilde{\theta}$. Moreover, a bidder of such type instead of following strategy $p_{\tilde{\theta}}(x)$, she can follow strategy $l_{\tilde{\theta}}(x)$, such that $(1-\lambda)l_{\tilde{\theta}}(x) = \lambda p_{\tilde{\theta}}(x)$ and still reach the same score, i.e., her probability of winning is not affected by this change in strategies. The fact that the truncated auction allows the winner to resubmit a new price-excess labor combination sufficient to reach the score she has to match (a score that is the same as in the original auction) implies that all bidders with $\theta_i \geq \tilde{\theta}$ can make this change without any additional cost, and hence they are not made worse off in the truncated auction. Given that the new strategies were available in the original auction the winner is not made better off either.

(ii) Now consider any bidder with $\theta_i < \tilde{\theta}$. Suppose that this bidder's strategy in the original auction is $l_{\theta_i}(x)$. By following such strategy she reaches a score of $s_{\theta_i}(x) = (1 - \lambda)l_{\theta_i}(x)$. The probability of winning the auction depends exclusively on whether the attained score is the highest among the scores of all other bidders. By the previous argument, the change in the auction does not affect the score all bidders with $\theta_i > \tilde{\theta}$ end up reaching. This implies that the bidder under consideration faces an isomorphic game, and hence does not need to contemplate a change in her strategy. Since this is true for all bidders with $\theta_i < \tilde{\theta}$, all possible outcomes attained in the original auction are also feasible in the truncated auction. Q.E.D.

This result greatly simplifies the analysis of the auction. It allows us to approach the auction as a unidimensional auction with bidimensional private information. In a sense, the truncated auction is like the auction where there are only type I bidders, with the difference that now there is an artificially created mass point in the right end of the distribution of θ's.

3.2 Characterization of Equilibrium Bids

We now characterize the equilibrium of the auction described in Proposition 1, when x and θ are uniformly distributed.[21] Moreover, we assume w.l.o.g. that x lies in the interval $[0, 1]$.

It is, now, helpful to introduce a new random variable, the ratio between the two pieces of private information each bidder has: $w = x/\theta$, which lies in the interval $[0, \frac{1}{\underline{\theta}}]$. This ratio is a sufficient statistic of the bidders' private information, and it allows us to construct a complete ordering with respect to the desirability each bidder has about the object being auctioned off, based on her private signals; a notion equivalent to the bidder's pseudotype in Asker and Cantillon (2008) parlance.

Before proceeding one caveat has to be pointed out: We provide the equilibrium bids for $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$. Clearly, as we have discussed earlier $\tilde{\theta}$ may lie outside that interval. In this case the equilibrium bids would be the same if we replace $\tilde{\theta}$ with $\theta_t \equiv \min\{\max\{\tilde{\theta}, \underline{\theta}\}, \overline{\theta}\}$. Nonetheless, to avoid extra notational complexity we keep on employing $\tilde{\theta}$. The analysis of the two cases where $\tilde{\theta}$ lies outside θ's support is discussed afterwards.

Proposition 2 *The equilibrium bids in a first price auction are increasing and continuous functions of the ratio w. Further, the equilibrium is symmetric with bids given by*

$$l^*(w) = \begin{cases} \frac{w}{2} & \text{if } w < 1/\tilde{\theta}, \\ \frac{w\left[\ln w - \frac{1}{2}w^2\underline{\theta}^2 + C\right]}{2\overline{\theta}w - 1 - w^2\underline{\theta}^2} & \text{if } w \geq 1/\tilde{\theta}, \end{cases}$$

where $C = \overline{\theta}/\tilde{\theta} - \frac{1}{2} + \ln \tilde{\theta}$.

Proof: Assume players follow strategies that are increasing functions of w. An (x, θ)-type bidder will choose the ratio that maximizes her expected payment. Thus the problem faced by this bidder is to choose a type z to maximize her expected profits:

$$(w - l(z))M_{\tilde{\theta}}(z),$$

[21] The characterization of the equilibrium can be delivered for any parametrization of the distribution functions. Uniformity is only assumed for simplicity.

where $M_{\tilde{\theta}}(w)$ is the cumulative distribution function of w, so that $M_{\tilde{\theta}}(z)$ represents type z's probability of winning the auction.

To compute the *cdf* of w one has first to notice that it is the quotient of two uniform random variables x and θ with the additional complication that θ has a mass point at $\tilde{\theta}$ (recall that all types $\theta > \tilde{\theta}$ have been replaced by $\tilde{\theta}$). Thus for any $w \leq 1/\tilde{\theta}$ we have that $M_{\tilde{\theta}}(w)$ is equal to

$$\frac{1}{\overline{\theta} - \underline{\theta}} \left[\int_{\tilde{\theta}}^{\overline{\theta}} \int_{0}^{w\tilde{\theta}} dx d\theta + \int_{\underline{\theta}}^{\tilde{\theta}} \int_{0}^{w\theta} dx d\theta \right].$$

For values of w in $\left[1/\tilde{\theta}, 1/\underline{\theta}\right]$ there is a $\hat{\theta}$ such that $w = 1/\hat{\theta}$ so that such a value of w can only be obtained if $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$. The *cdf* of the quotient for values of w belonging to this interval becomes

$$\frac{1}{\overline{\theta} - \underline{\theta}} \left[\int_{\hat{\theta}}^{\overline{\theta}} \int_{0}^{1} dx d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \int_{0}^{w\theta} dx d\theta \right].$$

After computing the integrals in the expressions above we get that

$$M_{\tilde{\theta}}(z) = \begin{cases} \frac{1}{\overline{\theta} - \underline{\theta}} z \left(\left(\overline{\theta} - \tilde{\theta} \right) \tilde{\theta} + \frac{\tilde{\theta}^2 - \underline{\theta}^2}{2} \right), & \text{if } 0 \leq z < 1/\tilde{\theta}, \\ \frac{1}{\overline{\theta} - \underline{\theta}} \left[\left(\overline{\theta} - \widehat{\theta}(z) \right) + z \left(\frac{\widehat{\theta}^2(z) - \underline{\theta}^2}{2} \right) \right], & \text{if } 1/\tilde{\theta} \leq z \leq 1/\underline{\theta}. \end{cases}$$

Since, in a symmetric equilibrium, $z = w$ and $\widehat{\theta}(z) = 1/w$, the first order condition of the maximization problem yields $l(w) = w/2$, whenever $w < 1/\widetilde{\theta}$. For $w \geq 1/\widetilde{\theta}$, the optimal bid is the solution to the following differential equation,

$$\left(\overline{\theta} - 1/w + w \left(\frac{1 - w^2 \underline{\theta}^2}{2w^2} \right) \right) l'(w) - (w - l(w)) \left(\frac{1 - w^2 \underline{\theta}^2}{2w^2} \right) = 0.$$

Integrating by parts the first term in the expression above, and simplifying we get the bids. Notice that the constant of integration (C) is set to ensure the continuity of the bidding functions. Finally, straightforward computations show that the bid functions are increasing in w, as it was first assumed in order to get the equilibrium bids. Q.E.D.

We now turn to the extreme cases where there are only bidders of one type. If $\tilde{\theta} < \underline{\theta}$ then all bidders bid in terms of price. This means that their θs do not play any role and the auction turns into the standard private values auction with uniformly distributed unidimensional private information, where the optimal bids are $p^*(x) = x/2$, $\forall x$. If on the other hand, $\tilde{\theta} > \overline{\theta}$ then all bidders bid in terms of labor. Equilibrium bids are then the same as in Proposition 2, after substituting $\tilde{\theta}$ with $\overline{\theta}$.

3.3 Expected Score

Given that we have analyzed the behavior of the bidders we can now describe the outcome of the auction for the government. In a conventional unidimensional auction this means to compute the expected revenues to the auctioneer. The equivalent result in this framework is to compute the expected score of the auction.

Taking into account the result in Lemma 1, if the winner is of type I, i.e. she has $\theta \leq \tilde{\theta}$, we know that she bids in terms of labor and her score can be directly computed by multiplying her bid $l^*(x, \theta)$ by the relative weight labor has in the scoring rule, $(1 - \lambda)$. On the other hand, if the winner is of type II, i.e. she has $\theta > \tilde{\theta}$, she bids in terms of price and then to compute her bid we just have to remember from Proposition 1 that $p^*(x) = \frac{1-\lambda}{\lambda} l^*(x, \tilde{\theta})$. To get her score we just have to multiply her bid by λ, which is the relative weight of price in the scoring rule. But now it is easy to observe that the score of this type II winner is given by $\lambda p^*(x) = \lambda \frac{1-\lambda}{\lambda} l^*(x, \tilde{\theta})$, which is equal to $(1 - \lambda) l^*(x, \tilde{\theta})$.

Therefore, to compute the expected score of the auction we do not really have to track down the identity of the winner (whether they are type I or type II bidders). It is sufficient to compute the expected pledge of excess labor received in the truncated auction (denoted by $El(\tilde{\theta})$) and multiply it with its relative weight, $ES(\tilde{\theta}) = (1 - \lambda) El(\tilde{\theta})$.

We now turn to compute the expected pledge of excess labor. To do so, first notice that for any equilibrium bid, there exists a type with $\theta = \underline{\theta}$ that makes the same bid. Clearly, a $(0, \underline{\theta})$-type bidder would submit the smallest of the equilibrium bids, while a $(1, \underline{\theta})$-type the largest of the equilibrium bids. Thus, we can compute the expected excess labor just by computing the expected bid that a $(x, \underline{\theta})$-type bidder will make in equilibrium, and then by integrating over all possible values such a $\underline{\theta}$-bidder may have.[22]

Since the expected excess labor is the bid times the probability of winning, we first compute the probability that a randomly selected bidder bids lower than a $(x, \underline{\theta})$-type bidder does. Straightforward calculations yield:

$$M_{\tilde{\theta}}(x/\underline{\theta}) = \begin{cases} \frac{x}{2\underline{\theta}(\overline{\theta}-\underline{\theta})} \left[2\overline{\theta}\tilde{\theta} - \underline{\theta}^2 - \tilde{\theta}^2 \right] & \text{if } x < \underline{\theta}/\tilde{\theta}, \\ \\ \frac{1}{2x(\overline{\theta}-\underline{\theta})} \left[2\overline{\theta}x - x^2\underline{\theta} - \underline{\theta} \right] & \text{if } x \geq \underline{\theta}/\tilde{\theta}. \end{cases}$$

[22] See Che and Gale (1998), where a similar procedure is used to produce expected revenues.

The expected excess labor from an $(x, \underline{\theta})$-type bidder becomes

$$P(x) = \begin{cases} \frac{x^2}{4\underline{\theta}^2(\overline{\theta}-\underline{\theta})} \left[2\overline{\theta}\tilde{\theta} - \underline{\theta}^2 - \tilde{\theta}^2\right] & \text{if } x < \underline{\theta}/\tilde{\theta}, \\ \frac{1}{2(\overline{\theta}-\underline{\theta})} \left[\ln(x/\underline{\theta}) - \frac{1}{2}x^2 + \overline{\theta}/\tilde{\theta} - 1/2 + \ln\tilde{\theta}\right] & \text{if } x \geq \underline{\theta}/\tilde{\theta}. \end{cases}$$

Hence, the expected excess labor to the government from the auction can be expressed as

$$\begin{aligned} El(\tilde{\theta}) &= \frac{1}{\overline{\theta}-\underline{\theta}} \left\{ \int_0^{\underline{\theta}/\tilde{\theta}} \frac{x^2}{2\underline{\theta}^2} \left[2\overline{\theta}\tilde{\theta} - \underline{\theta}^2 - \tilde{\theta}^2\right] dM_{\tilde{\theta}}(x/\underline{\theta}) \right. \\ &\quad \left. + \int_{\underline{\theta}/\tilde{\theta}}^1 \left[\ln(x/\underline{\theta}) - \frac{1}{2}x^2 + \overline{\theta}/\tilde{\theta} - 1/2 + \ln\tilde{\theta}\right] dM_{\tilde{\theta}}(x/\underline{\theta}) \right\} \\ &= \frac{\overline{\theta}\underline{\theta}^2 - 2\widetilde{\theta}\left(3\overline{\theta}\underline{\theta} - \overline{\theta}^2 + \underline{\theta}^2\right) + 6\underline{\theta}\widetilde{\theta}^2 \ln\left(\underline{\theta}/\widetilde{\theta}\right) + \widetilde{\theta}^2\left(2\underline{\theta} + 2\widetilde{\theta} + \overline{\theta}\right)}{6\left(\overline{\theta}-\underline{\theta}\right)^2 \widetilde{\theta}^2}. \end{aligned}$$

By differentiating $El(\widetilde{\theta})$ with respect to $\widetilde{\theta}$ we get that

$$\frac{\partial El(\widetilde{\theta})}{\partial\widetilde{\theta}} = -\frac{\left(\overline{\theta}-\widetilde{\theta}\right)\left[\left(\widetilde{\theta}-\underline{\theta}\right)^2 + \widetilde{\theta}\left(\overline{\theta}-\underline{\theta}\right)\right]}{3\widetilde{\theta}^3\left(\overline{\theta}-\underline{\theta}\right)^2} < 0$$

I.e., expected excess labor is strictly decreasing in $\tilde{\theta}$, and hence it gets its greatest value at $\tilde{\theta} = \underline{\theta}$. [23] The reason behind this result is twofold: first, a lower $\tilde{\theta}$ makes all the mass point bidders more efficient and thus willing bid more - remember they bid $x/2\widetilde{\theta}$, and second, by making the bidders more homogeneous it increases the competitive pressure.[24]

Turning now to the expected score it can be easily found that

$$\begin{aligned} ES(\tilde{\theta}) &= (1-\lambda)El(\tilde{\theta}) \\ &= \frac{\widetilde{\theta}}{1+\widetilde{\theta}} \frac{\overline{\theta}\underline{\theta}^2 - 2\widetilde{\theta}\left(3\overline{\theta}\underline{\theta} - \overline{\theta}^2 + \underline{\theta}^2\right) + 6\underline{\theta}\widetilde{\theta}^2 \ln\left(\underline{\theta}/\widetilde{\theta}\right) + \widetilde{\theta}^2\left(2\underline{\theta} + 2\widetilde{\theta} + \overline{\theta}\right)}{6\left(\overline{\theta}-\underline{\theta}\right)^2 \widetilde{\theta}^2}. \end{aligned}$$

For the same reasons, the expected score is also a strictly decreasing function of $\tilde{\theta}$

$$\frac{\partial ES(\widetilde{\theta})}{\partial\widetilde{\theta}} = -\frac{\left(\overline{\theta}-\widetilde{\theta}\right)\left(\widetilde{\theta}-\underline{\theta}\right)^2}{2\widetilde{\theta}^3\left(\overline{\theta}-\underline{\theta}\right)^2} < 0$$

The above discussion is summarized in the following proposition

[23] A point that deserves being mentioned is that expected labor depends on $\overline{\theta}$ although this type has been effectively replaced by a $\tilde{\theta}$-type. The reason is that the distance between $\overline{\theta}$ and $\tilde{\theta}$ determines the mass of bidders of $\tilde{\theta}$ type.

[24] One should understand though that the first effect is an artefact of the truncated auction we study, when we turn to the full auction it disappears.

Proposition 3 *The truncated auction attains its maximal expected score at* $\widetilde{\theta} = \underline{\theta}$*, or alternatively at* $\lambda = \frac{1}{1+\underline{\theta}}$.

3.4 Expected Utility of the Government

At this point, it should be reminded that the government does not care about the score per se, but about its utility $U_G(p,l) = \alpha p + (1-\alpha)l$. Now when computing the expected utility of the government, given the equilibrium behavior in the truncated auction, one has to take into account that type I bidders will bid in terms of labor the way it was derived in the previous section, while type II bidders will use the opportunity they have to retranslate their "labor bids" into "price bids". Hence, the expected utility of the government can be written as

$$EU_G(\alpha,\tilde{\theta}) = (1-\alpha)E\left[l^{*\text{Type }I}\right] + \alpha E\left[p\right] = (1-\alpha)E\left[l^{*\text{Type }I}\right] + \alpha\tilde{\theta}E\left[l^{*\text{Type }II}\right],$$

where

$$\begin{aligned} E\left[l^{*\text{Type }I}\right] &= \frac{\left(\tilde{\theta}^3 - 6\overline{\theta}\underline{\theta}\tilde{\theta} + 2\overline{\theta}\underline{\theta}^2 + 4\overline{\theta}\tilde{\theta}^2 + 2\underline{\theta}\tilde{\theta}^2 - 3\underline{\theta}^2\tilde{\theta} - 6\underline{\theta}\tilde{\theta}^2\ln(\tilde{\theta}/\underline{\theta})\right)}{6\tilde{\theta}^2\left(\overline{\theta}-\underline{\theta}\right)^2} \quad \text{and} \\ E\left[l^{*\text{Type }II}\right] &= \frac{\left(\overline{\theta}-\tilde{\theta}\right)\left(2\overline{\theta}\tilde{\theta} - \underline{\theta}^2 - \tilde{\theta}^2\right)}{6\tilde{\theta}^2\left(\overline{\theta}-\underline{\theta}\right)^2}. \end{aligned}$$

If the government does not consider misrepresenting its preferences - or alternatively, if it doesn't have commitment power - λ will be set equal to α, or alternatively $\tilde{\theta} = \widetilde{\theta}^{\alpha} \equiv \frac{1-\alpha}{\alpha}$. In this case the expected utility will be equal to the expected score, i.e. $EU_G(\alpha,\widetilde{\theta}^{\alpha}) = (1-\alpha)El(\widetilde{\theta}^{\alpha})$. This is the only case when the government does not care about the identity of the winner.

As soon as the government misrepresents its true preferences it introduces a wedge between expected score and expected utility. This wedge creates a real cost for the government each time a type II bidder wins the auction because then that bidder, by using the exchange rate implied by the scoring rule (i.e., $\widetilde{\theta}(\lambda) = \frac{1-\lambda}{\lambda}$), will convert her bid into price at a discount compared to the government's preferred conversion rate (i.e., $\widetilde{\theta}^{\alpha} = \frac{1-\alpha}{\alpha}$).

Effectively, the wedge between $\widetilde{\theta}^{\alpha}$ and $\widetilde{\theta}(\lambda)$ creates a metric of the degree to which the government misrepresents her preferences. The larger the misrepresentation the larger the discount it offers to a type II winner, and the larger the probability that a winner will be type

II. These two effects, that are detrimental to the government's goal, counter the beneficial effect of enhancing the competition among bidders, presented in Proposition 3. As it will be shown in the next Proposition an interior solution is derived in the optimal misrepresentation of the government's preferences.

Proposition 4 *The optimal scoring rule undervalues excess labor relative to the auctioneer's true preferences.*

Proof: We first show that EU_G is strictly decreasing at $\widetilde{\theta}^\alpha$ so that it is optimal to misrepresent the true preferences. To see this note that

$$\left.\frac{\partial EU_G(\alpha,\tilde{\theta})}{\partial\tilde{\theta}}\right|_{\tilde{\theta}=\frac{1-\alpha}{\alpha}} = -\left(\frac{3\left(\overline{\theta}\alpha-(1-\alpha)\right)\left(\underline{\theta}\alpha-(1-\alpha)\right)^2}{6(\overline{\theta}-\underline{\theta})^2\,(1-\alpha)^2}\right) < 0.$$

In order to set where the optimal $\tilde{\theta}$ lies, we check next the properties of EU_G. Its second order derivative is a convex function which is trivially negative for any $\theta \in \left[\underline{\theta}, \sqrt[3]{\overline{\theta}\underline{\theta}^2}\right]$ as

$$\frac{\partial^2 EU_G(\alpha,\tilde{\theta})}{\partial\tilde{\theta}^2} = -\frac{(1-\alpha)3\underline{\theta}(2\overline{\theta}-\tilde{\theta})(\tilde{\theta}-\underline{\theta})+\alpha\tilde{\theta}(\overline{\theta}\underline{\theta}^2-\tilde{\theta}^3)}{3(\overline{\theta}-\underline{\theta})^2\tilde{\theta}^4}.$$

Consequently, either EU_G is a strictly concave in $\tilde{\theta}$ on all its domain (a sufficient condition for this is $\underline{\theta} \geq 0.767\overline{\theta}$) or it is first concave and then convex with a unique inflection point at some $\theta > \sqrt[3]{\overline{\theta}\underline{\theta}^2}$. Since it exhibits a unique maximum it follows that the government will set an optimal $\tilde{\theta}$ larger than $\widetilde{\theta}^\alpha$. Q.E.D.

This result explains why many governments, when proposing privatization plans, attempt to show that they are tough against labor redundancies, especially when they are keen to attract foreign investors. Nonetheless, when the government's reputation is poor, a commitment to a $\lambda \neq \alpha$ is not credible.

In the preceded analysis it was implicitly assumed that the government cares sufficiently about both objectives, i.e. $\alpha \in \left(\frac{1}{1+\overline{\theta}}, \frac{1}{1+\underline{\theta}}\right)$. If the converse of this condition holds, then $\widetilde{\theta}^\alpha$ lies outside of the interval $(\underline{\theta}, \overline{\theta})$. This means that in these extreme cases, there will either be only type I or type II bidders participating in the auction, and therefore all bidders will either only bid in terms of labor or in terms of price. Hence, if the government mainly care about excess labor, i.e. $\widetilde{\theta}^\alpha \geq \overline{\theta}$, there are only type I bidders who bid in terms of labor and the attained government's expected utility becomes

$$EU_G(\alpha,\tilde{\theta} \geq \overline{\theta}) = (1-\alpha)\frac{5\overline{\theta}^2-\underline{\theta}^2-4\underline{\theta}\overline{\theta}-6\overline{\theta}\underline{\theta}\ln(\overline{\theta}/\underline{\theta})}{6\overline{\theta}\left(\overline{\theta}-\underline{\theta}\right)^2}.$$

On the other hand, if $\widetilde{\theta}^{\alpha} \leq \underline{\theta}$, i.e. the government cares mainly about the amount of revenues it can raise, there are only type II bidders who all bid in terms of price and the attained government's expected utility becomes

$$EU_G(\alpha, \tilde{\theta} \leq \underline{\theta}) = \frac{1}{3}\alpha.$$

4 Solving the Fixed-Labor-Requirement Auction

In practice, it is often the case that governments instead of allowing interested prospective buyers to choose the amount of labor redundancies they will keep, to require from them to commit to a fixed number of labor positions they will retain, let say L. In this case, the auction becomes a unidimensional one, in which bidders bid in terms of price over a firm that has to be managed under the restriction imposed by the announced fixed-labor-requirement.

One can try to draw parallels between the problem bidder i faces in this auction to the one in the score-based auction. The problem being faced under the current specification can be understood as identical to the first subproblem presented in the score-based auction analysis ($B_i^1 : X \times \Theta \longrightarrow I\!R_+$), with the extra restriction that the solution to the second subproblem is no longer unconstraint but has to take into account that by definition the following condition must hold, $l_i = L$.

Nonetheless, one has to remember that unless $L = 0$, the fixed-labor-requirement affects differently different bidders. Depending on each bidder's θ_i, the firm's value under her management is $v_i = x_i - \theta_i L$. Hence, one can reinterpret the auction as one in which bidders receive only one signal v_i, i.e., a linear transformation of the two initial signals x_i and θ_i, which is drawn from a distribution $Z = F \times G$.

Notice, that depending on the magnitude of L the relative weights of x and θ in the determination of v_i change. As a result, when L takes larger values, bidders with low θs become more advantaged while bidders with high θs more disadvantaged. Moreover, it becomes rather apparent that for some combination of xs and θs bidders may find the firm unattractive (defined as $v_i < 0$), and prospective buyers may not be interested in bidding for the firm. The larger is the fixed-labor-requirement, the larger is the probability a bidder may decide not to bid.

4.1 Characterization of Equilibrium Bids

To characterize explicitly the equilibrium of the auction we make the same assumptions as in the analysis of the score-based auction, namely that x and θ are uniformly distributed, and moreover that x lies in the interval $[0,1]$. Clearly, v attains its smallest value when $x=0$ and $\theta=\overline{\theta}$, and its largest value when $x=1$ and $\theta=\underline{\theta}$. I.e., v takes values in the interval $[-\overline{\theta}L, 1-\underline{\theta}L]$.

Notice that the distribution function of v is a linear transformation of two uniforms. Nonetheless, in order to be computed, one has to remember that some values of v are not as likely to appear as others because they cannot be produced with the same mass of combinations of xs and θs. For example, $v=-\overline{\theta}L$ can occur only if $x=0$ and $\theta=\overline{\theta}$, and with no other combination. As larger values of v are considered these can be produced by more combinations of xs and θs. This continues until a particular value of v is reached, which can be produced with the maximum possible combinations of xs and θs. If x and θ have the same range this occurs only at one point, if the ranges are different there is a whole interval of vs that have the same probability mass. As even larger values of v are being considered, after one threshold, fewer combinations of xs and θs are producing these values of v. This trend continues until $v=1-\underline{\theta}L$ which occurs only if $x=1$ and $\theta=\underline{\theta}$. Actually, the probability density function of v is symmetric in its range. The cumulative distribution function of v is then

$$Z(v)=\begin{cases} \frac{1}{\overline{\theta}-\underline{\theta}}\left(v\overline{\theta}+\frac{v^2}{2L}+\frac{\overline{\theta}^2L}{2}\right) & \text{if } -\overline{\theta}L\leq v\leq-\underline{\theta}L, \\ v+\frac{(\overline{\theta}+\underline{\theta})L}{2} & \text{if } -\underline{\theta}L\leq v\leq 1-\overline{\theta}L, \\ 1-\frac{1}{2(\overline{\theta}-\underline{\theta})L}\left(1-v-\underline{\theta}L\right)^2 & \text{if } 1-\overline{\theta}L\leq v\leq 1-\underline{\theta}L. \end{cases}$$

One can readily check that this is a well behaved, continuous and strictly increasing cumulative distribution function. Do notice that for constructing the above cdf we have assumed that $L\in\left[0,1/\overline{\theta}\right]$. For larger values of L, i.e., for $L\in\left(1/\overline{\theta},1/\underline{\theta}\right]$, a positive v can only be generated for some θ so that for each $\tilde{L}\in\left(1/\overline{\theta},1/\underline{\theta}\right]$ there is a $\tilde{\theta}$ such that $v>0$ only if $\theta\in\left[\underline{\theta},\tilde{\theta}\right]$. To avoid this, in what follows we restrict the attention to labor requirements in $\left[0,1/\overline{\theta}\right]$.[25]

The probability that a prospective buyer may not be interested in participating in the auction, given the fixed-labor-requirement announced by the government, can be easily com-

[25] Allowing for larger values of L will make the notation more cumbersome without adding any economic content to the problem.

puted, and it is equal to $Z(v=0) = \frac{(\bar{\theta}+\underline{\theta})L}{2}$. Setting $\frac{(\bar{\theta}+\underline{\theta})}{2} \equiv A$ then $Z(v=0) = AL$. The complementary probability represents the mass of bidders who will participate in the auction. **Note that the labor requirement is not a pre-qualification requierement, i.e., bidders participate in the auction without knowing if there are other bidders. Otherwise, a bidder who knows that his opponent is not partcipitating would bid a price of zero. Consequently, it is in the interest of the auctioneer to avoid running the auction as a two stage game with the first stage being a pre-qualification stage. The equilibrium bids for the participating bidders are presented in the next proposition.**

Proposition 5 *The equilibrium bids in a first-price auction are increasing and continuous functions of v. Further, the equilibrium is symmetric with bids given by*

$$p^*(v) = \begin{cases} \dfrac{v^2}{2(v+AL)} & \text{if} \quad 0 \le v \le 1-\overline{\theta}L, \\ \\ \dfrac{1}{3}\left(\dfrac{3v^2(1-\underline{\theta}L) - 2v^3 - (1-\overline{\theta}L)^3}{2(\overline{\theta}-\underline{\theta})L - (1-v-\underline{\theta}L)^2}\right) & \text{if} \quad 1-\overline{\theta}L \le v \le 1-\underline{\theta}L. \end{cases}$$

Proof: Assume bidders follow strategies that are increasing in v. Let bidder i with v_i chooses optimally $\hat{v}_i$ by maximizing her expected profits

$$\Pr(v_j \le 0)[v_i - p(\hat{v}_i)] + \Pr(v_j > 0)\Pr(v_j \le \hat{v}_i | v_j > 0)[v_i - p(\hat{v}_i)],$$

where $\Pr(v_j \le 0) = Z(0) = AL$, and, using Bayes' rule,[26]

$$\Pr(v_j > 0)\Pr(v_j \le \hat{v}_i | v_j > 0) = \Pr(\hat{v}_i \ge v_j > 0) =$$

$$Z(\hat{v}_i) - Z(0) = \begin{cases} \hat{v}_i & \text{if } 0 \le \hat{v}_i \le 1-\overline{\theta}L, \\ \\ 1 - \frac{(1-\hat{v}_i-\underline{\theta}L)^2}{2(\overline{\theta}-\underline{\theta})L} - AL & \text{if } 1-\overline{\theta}L \le \hat{v}_i \le 1-\underline{\theta}L. \end{cases}$$

i) If $\hat{v}_i \le 1-\overline{\theta}L$, expected profits become:

$$AL[v_i - p(\hat{v}_i)] + [v_i - p(\hat{v}_i)]\hat{v}_i.$$

Taking the first order condition, solving the ensuing differential equation and employing the boundary condition $p(0) = 0$, we get

$$p^*(v_i) = \frac{v_i^2}{2(v_i + AL)}.$$

[26]Note that by setting $\Pr(v_j \le 0) = AL$ we are implicitly assuming that L is no larger than $1/\overline{\theta}$. If this were not the case, i.e., for $L \in \left(1/\overline{\theta}, 1/\underline{\theta}\right)$, we have that $\Pr(v_j \le 0) = 1 - \frac{(1-\underline{\theta}L)^2}{2(\overline{\theta}-\underline{\theta})L}$.

ii) If $\hat{v}_i \geq 1 - \overline{\theta}L$, expected profits become:

$$AL[v_i - p(\hat{v}_i)] + \left(1 - \frac{(1 - \hat{v}_i - \underline{\theta}L)^2}{2(\overline{\theta} - \underline{\theta})L} - AL\right)[v_i - p(\hat{v}_i)] = [v_i - p(\hat{v}_i)]\left(1 - \frac{(1 - \hat{v}_i - \underline{\theta}L)^2}{2(\overline{\theta} - \underline{\theta})L}\right).$$

Differentiating with respect to $\hat{v}_i$, the optimal choice is $\hat{v}_i = v_i$ if

$$-p'(v_i)\left(2(\overline{\theta} - \underline{\theta})L - (1 - v_i - \underline{\theta}L)^2\right) + (v_i - p(v_i))\left(2\left(1 - v_i - \underline{\theta}L\right)\right) = 0,$$

Solving the differential equation we get

$$p^*(v_i) = \frac{C - v_i^2(2v_i - 3(1 - \underline{\theta}L))}{3(2(\overline{\theta} - \underline{\theta})L - (1 - v_i - \underline{\theta}L)^2)},$$

where C is the constant of integration, and to ensure continuity between the two cases is set equal to $(\overline{\theta}L - 1)^3$. Combining the results in cases (i) and (ii) we get the expression in the proposition.

To conclude the proof we first have to show that bids are increasing in v. In case (i), $p'(v) = v(v+2AL)/2(v+AL)^2 > 0$. In case (ii), $p'(v) = (v - p(v))\,(Z'(v))\,/\,(Z(v) + AL) > 0$, given that $v > p(v)$ and $Z'(v) > 0$. Finally, we have to show that no bidder with a v that belongs to case (i) want to bid according to the bid of case (ii) and the other way round, an exercise that is straightforward but tedious and is omitted. Q.E.D.

4.2 Expected Government Utility

To compute the expected utility of the government, one has to remember that a fraction of the prospective buyers may not find it profitable to participate. With probability $Z(0)$ a prospective buyer essentially bids $\{\phi\}$, which is translated as if both $l_i = 0$ and $p_i = 0$.

The expected excess labor the government will be able to secure can be easily found by remembering that with probability $(1 - Z(0)^2)$ at least one prospective buyer will be interested in participating in the auction, and hence the government will secure L positions. Therefore, the expected excess labor is equal to $El = (1 - Z(0)^2)L$.

Analogously, the government's expected revenues can be computed by taking into account that with certain probability the auction will not attract any bids:

$$Ep(v) = Z(0)^2 0 + 2Z(0)(1 - Z(0))E(p^*) + (1 - Z(0))^2 E(\max\{p^*(v_1), p^*(v_2)\}).$$

By substituting for the optimal bidding strategies, and after a series of rather tedious but straightforward computations it can be shown that

$$Ep(L) = \frac{1}{3} - AL + A^2L^2 - \frac{2}{5}A^3L^3 + \frac{1}{60}(\overline{\theta}^3 + 3\overline{\theta}\underline{\theta}^2)L^3.$$

Note that $Ep(L)$ is strictly decreasing in L for any $L \in [0, 1/\overline{\theta}]$.

At this point, the government's expected utility is readily available

$$\begin{aligned} EU_G &= (1-\alpha)El(L) + \alpha Ep(L) \\ &= (1-\alpha)(1-A^2L^2)L + \alpha\left(\frac{1}{3} - AL + A^2L^2 - \frac{2}{5}A^3L^3 + \frac{1}{60}(\overline{\theta}^3 + 3\overline{\theta}\underline{\theta}^2)L^3\right). \end{aligned}$$

We are now in position to maximize government's expected utility with respect to the announced fixed-labor-requirement. Differentiating EU_G with respect to L we get

$$\begin{aligned} \frac{\partial EU_G}{\partial L} &= \left[1 - \alpha\left(1 + \frac{(\overline{\theta}+\underline{\theta})}{2}\right)\right] + \alpha\frac{(\overline{\theta}+\underline{\theta})^2}{2}L \\ &\quad - \left[(1-\alpha)\frac{3(\overline{\theta}+\underline{\theta})^2}{4} + \frac{\alpha}{20}[3(\overline{\theta}+\underline{\theta})^3 - \overline{\theta}^3 - 3\overline{\theta}\underline{\theta}^2]\right]L^2. \end{aligned}$$

It should be stressed that, depending on α, EU_G is not always a concave function, and hence the FOC is not always sufficient for maximization. Nonetheless, one can reach certain results, which are summarized in the following proposition.

Proposition 6 *The optimal fixed-labor requirement, L^*, is non increasing in α. Moreover, for each range of marginal costs of accommodating excess labor, $\Delta = \overline{\theta} - \underline{\theta}$, there exists $\hat{\alpha}(\Delta) \in (0, 1)$ such that $L^* = 0$ if and only if $\alpha \geq \hat{\alpha}(\Delta)$. For $\alpha = 0$, i.e., when the government cares only about excess labor, then $L^* = \frac{2}{\sqrt{3}(\overline{\theta}+\underline{\theta})}$ with $Ep(L^*) > 0$ whereas when $\alpha = 1$, i.e., when the government cares only about revenues, then $L^* = 0$.*

Proof: Assume first $\alpha = 0$, so that $EU_G = El(L) = (1-A^2L^2)L$. Since $El(L)$ is a strictly concave function, from the FOC it follows that $L^* = 2/(\sqrt{3}(\overline{\theta}+\underline{\theta}))$. Note that $L^* \leq 1/\overline{\theta}$ if

$\overline{\theta} \leq 6.46\underline{\theta}$. Furthermore,

$$\begin{aligned}
Ep(L^*) &= \frac{1}{60}\left[2\left(1-\frac{2}{\sqrt{3}}\frac{\overline{\theta}}{\overline{\theta}+\underline{\theta}}\right)^3 + 3\left(1-\frac{2}{\sqrt{3}}\frac{\underline{\theta}}{\overline{\theta}+\underline{\theta}}\right)^3 + 9\left(1-\frac{2}{\sqrt{3}}\frac{\overline{\theta}}{\overline{\theta}+\underline{\theta}}\right)^2\right. \\
&\qquad \left.\left(1-\frac{2}{\sqrt{3}}\frac{\underline{\theta}}{\overline{\theta}+\underline{\theta}}\right) + 6\left(1-\frac{2}{\sqrt{3}}\frac{\overline{\theta}}{\overline{\theta}+\underline{\theta}}\right)\left(1-\frac{2}{\sqrt{3}}\frac{\underline{\theta}}{\overline{\theta}+\underline{\theta}}\right)^2\right] \\
&= \frac{1}{60}\left[3\left(2-\frac{2}{\sqrt{3}}\right)^3 - \left(1-\frac{2}{\sqrt{3}}\frac{\overline{\theta}}{\overline{\theta}+\underline{\theta}}\right)^3 - 3\left(1-\frac{2}{\sqrt{3}}\frac{\overline{\theta}}{\overline{\theta}+\underline{\theta}}\right)\left(1-\frac{2}{\sqrt{3}}\frac{\underline{\theta}}{\overline{\theta}+\underline{\theta}}\right)^2\right] \\
&> \frac{1}{60}\left[24\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)^3 - \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)^3 - 3\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)^3\right] > 0,
\end{aligned}$$

where the first inequality in the last line follows from the fact that the last two terms in the penultimate line take their largest value when $\frac{\overline{\theta}}{\overline{\theta}+\underline{\theta}} \to \frac{\underline{\theta}}{\overline{\theta}+\underline{\theta}} \to \frac{1}{2}$.

Consider next $\alpha = 1$, so that $EU_G = Ep(L)$. Given that $Ep(L)$ is strictly decreasing in L, it follows that $L^* = 0$, and $Ep(L^*) = 1/3$.

Focus finally on $\alpha \in (0,1)$. The expected utility of the government is a linear combination between an strictly concave function ($El(L)$) and a strictly convex one ($Ep(L)$). Further inspection of $EU_G(L)$ shows that there is $\tilde{L}$ such that $EU_G(\tilde{L}) = \tilde{L}\left(1-\tilde{L}^2\left(\frac{(\underline{\theta}+\overline{\theta})}{2}\right)^2\right)$ for all α with EU_G being increasing in α for $L \in \left[0, \tilde{L}\right]$, and decreasing for $L \in \left[\tilde{L}, 1/\overline{\theta}\right]$.

For small values of α the concave part dominates and so EU_G exhibits a global interior maximum at L_M^*, $L_M^* > \tilde{L}$, which is given by[27]

$$L_M^* = \frac{\left(20\alpha A^2 + \sqrt{5\left(60\left(1-\alpha\right)^2\left(2A\right)^2 - 2\alpha\left(1-\alpha\right)\left(9\underline{\theta}^3 + 27\overline{\theta}^2\underline{\theta} + 11\overline{\theta}\left(3\underline{\theta}^2 + \overline{\theta}^2\right)\right)\right) + \alpha^2 2A\left(\overline{\theta}-\underline{\theta}\right)^3}\right)}{15(1-\alpha)\left(2A\right)^2 + 3\underline{\theta}^3\alpha + 9\underline{\theta}\overline{\theta}^2\alpha + 6\underline{\theta}^2\overline{\theta}\alpha + 2\overline{\theta}^3\alpha}$$

Implicit differentiation shows that the sign of $dL_M^*/d\alpha$ equals the sign of $dEU_G/d\alpha$ which is negative as $L_M^* > \tilde{L}$.

For large values of α the convex part dominates and the global maximum becomes $L_m^* = 0$.

Consequently, there is $\hat{\alpha}$ such that $EU_G(\hat{\alpha}, 0) = EU_G(\hat{\alpha}, L_M^*)$ so that the optimal labor requirement is L_M^* if $\alpha \leq \hat{\alpha}$ and it is 0 if $\alpha \geq \hat{\alpha}$. It hence follows that the optimal fixed-labor requirement, L^*, is non increasing in α.

Q.E.D.

[27] Note that L_M^* is well defined only if the number in the square root sign is positive, which requires α not being too large. More precisely, it requires $\alpha \leq \bar{\alpha} \leq \frac{3}{2\underline{\theta}+3}$ where $3/(2\underline{\theta}+3)$ corresponds to the value of $\bar{\alpha}$ when $\overline{\theta} = \underline{\theta}$. For $\overline{\theta} = 2$, $\underline{\theta} = 1$ we have $\bar{\alpha} \approx 0.5$.

The intuition behind these results is the following: When the government cares only about revenues, it prefers to set the labor-requirement to zero, as any positive targeting of excess labor would result to lower prices because it reduces the profitability of the firm to all bidders, and moreover, because it introduces heterogeneity among bidders and as a result dampens the competitive pressure for the advantaged bidders.[28] On the other hand, when the government cares only about excess labor, the announced labor-requirement is positive. Nonetheless, the government does not want to turn the auction into a beauty contest, i.e., it still solicits bids with positive prices. Essentially, the government does not have any interest in raising revenues per se, as it values them not. Nonetheless, it collects positive bids as a side-product of its desire to increase the probability of attracting prospective buyers, and hence securing employee positions. Moreover, the government understands that, when it sets the labor-requirement, it effectively gives advantage to the bidders with low cost of accommodating excess labor. The larger is the dispersion of the bidders in this aspect, expressed in terms of the difference $(\overline{\theta} - \underline{\theta})$, the more aware is the government of the deleterious effects of its employment targeting on competitive pressure, and it optimally chooses a less ambitious requirement. Clearly, when the government's objective is more balanced and it cares for both revenues and excess labor, it becomes even more cautious of the negative effects of an aggressive labor-requirement.

5 Discussion and Conclusions

When privatizing, governments in transition economies may have conflicting objectives. Raising revenues but also minimizing induced unemployment are often cited. We construct two mechanisms that take into account both criteria: a first-score auction in which bidders bid both in terms of price and retained excess labor, and a first-price auction in which bidders bid only over price but they also commit to keep a predetermined by the government number of employees. We find that if "insiders" (locals) enjoy a comparative advantage in accommodating labor redundancies compared to "outsiders" (foreign investors), the resulting competition for the privatized firms softens. Taking this effect into account, governments may optimally want to appear strong against labor redundancies, and to set a scoring rule that does not correspond to their genuine preferences in order to reduce the comparative advantage "insiders" enjoy, or announce a smaller labor requirement compared to the one

[28]Notice that as in this case the auction converges to a standard first-price auction, expected revenues converge to the ones reached there (1/3 in our example).

they would have, had all bidders locals and foreigners, have the same cost of accommodating excess labor. Nonetheless, such policy requires strong commitment ability. In the absence of such commitment, foreign capital may eventually shy away from investing in privatization programs, as it has been argued that it has been the case in many transition economies.

Bibliography

Aghion, P., O. Blanchard and W. Carlin, (1997) "The Economics of Enterprise Restructuring in Central and Eastern Europe", in J. Roemer (ed.), *Property Relations, Incentives and Welfare*, McMillan Press, pp. 217-318.

Asker, J. and E. Cantillon, (2010) "Procurement when Price and Quality Matter", *Rand Journal of Economics* 41: 1-34.

Asker, J. and E. Cantillon, (2008) "Properties of scoring auctions", *Rand Journal of Economics* 39: 69-85.

Boone, J. and J. Goeree, (2009) "Optimal Privatization using Qualifying Auctions", *Economic Journal* 119: 277-297.

Bornstein, M., (2000) "Post-Privatization Enterprise Restructuring", *Mimeo.*

Branco, F., (1997) "The Design of Multidimensional Auctions", *Rand Journal of Economics* 28: 63-81.

Carlin, W. and C. Mayer, (1994) "The Treuhand Anstalt: Privatization by State and Market", in O. Blanchard, K. Froot and J. Sachs(eds.), *The Transition in Eastern Europe, Vol. 2: Restructuring*, University of Chicago Press: Chicago, IL.

Che, Y.-K., (1993) "Design Competition through Multidimensional Auctions", *Rand Journal of Economics* 24: 668-680.

Che, Y.-K. and I. Gale, (1998) "Standard Auctions with Financially Constrained Bidders", *Review of Economic Studies* 65: 1-21.

Chong, A. and F. López-de-Silanes, (2002) "Privatization and Labor Force Restructuring around the World", *World Bank Research Paper 2884*.

Cornelli, F. and D. Li, (1997) "Large Shareholders, Private Benefits of Control, and Optimal Schemes of Privatization", *Rand Journal of Economics* 28: 585-604.

Dyck, A. and K. Wruck, (1998) "Organization Structure, Contract Design and Government Ownership: A Clinical Analysis of German Privatization", *Journal of Corporate Finance* 4: 265-299.

Ewerhart, C. and K. Fieseler, (2004) "Procurement Auctions and Unit-price Contracts", *Rand Journal of Economics* 34: 569-581.

Hau, H., (1998) "Privatization under Political Interference: Evidence from Eastern Germany", *European Economic Review* 42: 1177-1201.

Jehiel, P., B. Moldovanu and E. Stacchetti, (1999) "Multidimensional Mechanism Design for Auction with Externalities", *Journal of Economic Theory* 85: 258-294.

Lichtblau, K., (1993) "Privatisierung und Sanierungsarbeit der Treuhandanstalt", *Institute der Deutchen Wirtschaft* Ḱoln.

López-de-Silanes, F., (1997) "Determinants of Privatization Prices", *Quarterly Journal of Economics* 102: 965-1025.

Lundberg, K. and J. Donahue, (1992) "The Treuhandanstalt: Taking the Nation Private", *Kennedy School of Government Case C16-92-1120* Harvard University, MA.

Maskin, E., (1992) "Auctions and Privatization", in H. Siebert (ed.), *Privatization: Symposium in Honor of Herbert Giersch*, Tubingen: Mohr (Siebeck).

Priewe, J., (1993) "Privatization of the Industrial Sector: The Function and the Activities of the Treuhandanstalt", *Cambridge Journal of Economics* 17: 333-348.

Riordan, M. and D. Sappington, (1987) "Awarding Monopoly Franchises", *American Economic Review* 77: 375-387.

Rochet, J.-C. and P. Choné, (1997) "Ironing, Sweeping and Multidimensional Screening", *Econometrica* 66: 783-826.

Welch and Fremont, (1998) *The Case by Case Approach to Privatization: Techniques and Examples*, World Bank.