How to Allocate Forward Contracts

The case of electricity markets

María-Ángeles de Frutos and Natalia Fabra
Universidad Carlos III de Madrid*

16 April 2010

Abstract

Several regulatory authorities worldwide have imposed forward contract commitments on electricity producers as a way to mitigate their market power. In this paper we analyze the impact of such commitments on equilibrium outcomes in a model that reflects important institutional and structural features of electricity markets. We show that, when firms are asymmetric, the distribution of contracts amongst firms matters. In the case of a single dominant firm, the regulator can be confident that allocating contracts only to that firm will be pro-competitive. When the asymmetries are less extreme, however, certain contract allocations can yield anti-competitive outcomes by eliminating the more competitive equilibria. Our analysis thus suggests that forward contracts should be allocated so as to (virtually) reduce asymmetries across firms.

Keywords: Forward contracts, discrete supply functions, electricity markets, antitrust remedies, simulations.

JEL Classification Numbers: L13, L94, G13.

*Department of Economics. Universidad Carlos III de Madrid. Calle Madrid 126. 28903 Getafe, Madrid. Spain. Emails: frutos@eco.uc3m.es; nfabra@eco.uc3m.es. We are grateful to Antonio Cabrales, Álvaro Cartea, Jim Bushnell, Alberto de Frutos, Giulio Federico, José Luis Ferreira, Mónica Gandolfi, Ángel Hernando, Xavier Jarque, Kai-Uwe Kuhn, Juan Toro, Achim Wambach, three anonymous referees, and seminar audiences at the UCEI (Berkeley), Universidad de Málaga, Universidad Carlos III de Madrid, IESE (Barcelona), EARIE Conference (Toulouse), ASSET Meeting (Florence), Universidad Complutense (Madrid), the CEPR Conference on Applied IO (Manheim), IFN (Stockholm) and HEREC (Helsinki). Pablo Angulo provided excellent research assistance. We also thank Fundación Ramón Areces for financial support.

1 Introduction

Concerns over the exercise of market power in electricity markets have led several competition and regulatory authorities to impose forward contract commitments on the dominant producers.[1] Such contracts have taken various forms, but they all have one important feature in common: they commit producers to receiving a fixed price for a certain fraction of their output before wholesale market competition takes place. The 'vesting contracts' introduced at privatization in the British electricity market or the 'Competition Transition Costs' for stranded costs recovery in Spain, provide two well-known examples of such forward commitments.[2] More recently, several regulators worldwide have been forcing large electricity producers to auction off 'virtual power plants' (VPPs), which essentially work as forward sales. VPPs have also been used as antitrust remedies in several competition policy cases, including merger control proceedings and abuse of dominance investigations.[3] More generally, several authors have blamed the poor performance of some electricity markets on the lack of sufficient forward contracting, and propose to foster it for these markets to deliver efficient outcomes (Wolak (2007b) and Bushnell *et al.* (2008)).

In this paper we investigate how such forward contract commitments affect firms' strategic behavior and equilibrium outcomes in electricity markets.[4] As compared to the no-contracts case, forward contracts can potentially result in higher prices if sufficiently many forward contracts are awarded to firms with little but yet some market power. However, such anti-competitive effects never arise whenever contracts are awarded to the dominant firm in the market. Therefore, the relevant question is how to allocate forward contracts among firms to avoid their potential anti-competitive effects while enhancing their pro-competitive role. In this respect, the current paper provides a clear policy answer: forward contracts should be awarded in ways that align all firms' interests by (virtually) reducing their asymmetries. This can be achieved not only by mitigating the dominant firms' incentives to increase prices through contract sales, but also by encouraging the medium-sized firms to purchase such contracts in order to counterbalance the market power of the bigger firms.[5]

Our analysis reflects important institutional and structural features of electricity markets. Firstly, firms compete by submitting supply functions with a finite number of steps, as it is the case in all

[1] Market power concerns in electricity markets have also fostered the establishment and promotion of forwards markets, as in the Pennsylvania-New Jersey-Maryland market (PJM) or in the Australian National Electricity Market. However, these are not the subject of this paper to the extent that participation in such markets is typically voluntary and hence endogenous.

[2] See Wolfram (1999) for a description of the first, and Fabra and Toro (2005) and Kühn and Machado (2006) for a description of the second.

[3] For instance, VPPs have been used in the mergers cases EDF/EnBW in 2000 and Nuon/Reliant in 2003, in the alleged price-squeeze case involving EDF/Direct Energy in 2007, or in the abuse of dominance by ENEL in 2006. In Spain and Portugal, VPPs have also been used in an attempt to make the market more competitive. For a description and analysis of VPPs, see Schultz (2007) and Federico and López (2009). Some unconventional forms of forward contracts have been used in other competition policy settings, such as certain voluntary operating restrictions adopted by firms in antitrust lawsuits, see Borenstein (1996).

[4] Such contractual arrangements encompass several types of vertical commitments, including vertical integration. To the extent that they can be considered to be exogenous (Bushnell *et al.* (2008)), our paper also sheds light on their effects.

[5] This is in contrast to the Spanish experience with VPPs, as the medium-sized firms were not allowed to buy such contracts.

electricity markets in practice; secondly, firms own a portfolio of several production technologies, thus giving rise to (weakly) increasing marginal cost functions that might differ across firms; and thirdly, firms are allowed to hold exogenously given forward contracts, which are financially settled once the market closes.[6]

Despite the complexity of the problem, we show that all the equilibria have a simple pattern: all firms but one (referred to as *non-price-setters*) behave as price-takers, i.e., they produce the same as if they bid at marginal costs,[7] while the remaining firm (referred to as the price-setter) sets the price at the level that maximizes its profits over the residual demand (Theorem 1).[8] Therefore, there are as many candidate equilibrium outcomes as firms in the market, all of which differ in the identity of the price-setter. In general, the resulting equilibria cannot be Pareto-ranked as, all else equal, firms prefer to be non-price-setters rather than price-setters.

A central result of the paper is that not all candidate equilibria might be sustainable (Corollary 2). A firm that is willing to set a very high price when everyone else behaves competitively might have incentives to deviate from any candidate equilibria at which the price-setter chooses a very low price: the deviating firm would lose output, but such output loss might be more than compensated by the price increase. This limits the set of firms that can act as price-setters in equilibrium. The equilibrium set is nevertheless non-empty, as no firm wants to deviate from the highest price candidate equilibrium (Corollary 1).

The main results of the paper are contained in Proposition 4, which shows that the impact of forward contracts on equilibrium prices derives from two effects: the change in the price-setter's profit-maximizing price, and the change in the non-price-setters' deviation incentives. On the one hand, the price-setter's profit-maximizing price is lower with contracts given that market prices only affect its uncovered sales. On the other hand, a lower price also makes it more attractive for a non-price-setter to deviate to a higher price. If contracts are symmetrically distributed across symmetric firms, the only relevant effect of contracts is the one on the price-setter's profit-maximizing price. Hence, an increase in contracts up to firms' competitive quantities is unambiguously pro-competitive (Lemma 5). However, this prediction may be reversed when firms are asymmetric, as the effects of contracts on the non-price-setters' deviation incentives, and thus on equilibrium existence, start to play a role. Indeed, a novel result from the paper - namely, that an increase in the contract coverage of the price-setter can lead to higher prices- comes exactly from the impact of contracts on equilibrium existence. The increase in the price-setter's contract coverage, which lowers its profit-maximizing price, may trigger a deviation by some other firm, thus making such equilibrium disappear. This result is therefore related to a shrinking of the set of equilibrium outcomes and is not a standard type of comparative static result.

The above conclusions support the main message of the paper: since contract distribution and

[6] Nyborg and Strebulaev (2004) also study auctions where bidders have exogenously given forward contracts. However, in that paper short-sellers face the risk of being squeezed in the secondary market, thus affecting the auction itself. Short-squeezes are not an issue in our setting as electricity markets are typically very liquid and most contracts are settled by differences with respect to the spot market price.

[7] To be more precise, this holds true as long as no firm has an excessive amount of contracts. Otherwise, the non-price-setters might produce in an inefficient manner. This is analyzed in detail in Section 6.

[8] Using data from the UK electricity market, Crawford et al. (2007) have shown that this pattern of asymmetric bidding is observed in practice.

contract volume are crucial in determining the effects of forward contracts, there is scope for making them pro-competitive. In markets with large asymmetries across firms, only the dominant firm should be forced to hold forward contracts; getting contract volume right is less critical, as contracts in this case would at worst be ineffective. Regulators should be more cautious in the presence of mild asymmetries between large and medium-sized firms, as it is in such cases when the potential anti-competitive effects of contracts are more likely to arise. Still, it is in these contexts when contracts may have a stronger role to play, as encouraging the smaller firms to purchase such contracts may further mitigate the market power of the bigger firms.

In order to illustrate our theoretical results, we have performed a simulation exercise that uses a rich data set of the Spanish electricity market. Assuming that contract volume remains fixed while demand varies over the year, the analysis shows that the pro-competitive effect of contracts dominates over the anti-competitive one. Still, the latter shows up in the simulations at certain hours, depending on contract volume and contract allocation.

There is already a large body of theoretical work on the impact of forward trading on the performance of oligopolistic markets.[9] However, existing papers are not fully applicable to the problem at hand, to the extent that they assume that ex-ante symmetric firms choose their contracts prior to competing either *à la* Cournot (Allaz and Vila (1993) and Bushnell (2007)) or *à la* Bertrand (Mahenc and Salanié (2004)).[10] Instead, forward contract commitments are not endogenously chosen by firms but rather imposed by regulators. Also, costs and capacity asymmetries are pervasive among electricity producers. These two differences explain why and when our predictions differ. In the existing papers, and regardless of whether firms compete *à la* Cournot or *à la* Bertrand, forward sales (purchases) induce firms to compete (less) more fiercely given that spot market prices only affect their net-selling (net-buying) positions. However, once contracts are endogenized, the Cournot model predicts that contracts are pro-competitive because all firms are net-sellers at the subgame perfect equilibrium, whereas the opposite holds true under the Bertrand model. In contrast, our model predicts that exogenously given contracts might have anti-competitive effects even if firms are net-sellers.[11]

As a by-product, our analysis also contributes to the literature on share auctions.[12] In a common value setting, Wilson (1979) shows that there exist equilibria with prices below the common value. Kremer and Nyborg (2004) demonstrate that these kind of equilibria can be eliminated in a discrete setting, similar to the one employed in the current paper, where quantities must be discrete though prices need not. Restricting bidders to submit a finite number of price-quantity pairs implies that there is a positive mass at the margin, so that competition for the margin destroys the underpricing

[9] There is also extensive empirical literature which confirms that contracts affect the performance of spot markets. See Bushnell, Mansur and Saravia (2008), Fabra and Toro (2005), Hortacsu and Puller (2008), Kühn and Machado (2006), Mansur (2007) or Wolak (2000, 2007).

[10] Newbery (1998) and Green (1999) obtain mixed results in models in which firms compete by choosing continuous supply functions. Various papers analyze the dynamic effects of contracts (Ferreira (2003); Green and Le Coq (2006); Liski and Montero (2006)), and tend to conclude that they have anti-competitive effects.

[11] It is simple to show that, in a Cournot model with exogenously given contracts and asymmetric firms, forward contracting is always (weakly) pro-competitive, regardless of contract distribution.

[12] In most of the papers in this literature, bidders submit demand functions (to buy some underlying good) rather than supply functions (to supply electricity), as in the current paper. However, as it is well known, whether one casts the model in terms of demand or supply functions is immaterial because demand and supply games are isomorphic.

equilibria found by Wilson. In the current paper, in contrast, bidders can exploit the fact that (weakly) increasing marginal costs lead to downward sloping residual demand functions, in the same way as bidders can engage in demand reduction in a setting *à la* Wilson (see also Ausubel and Cramton (2002)). In sum, by relaxing the flat common value assumption, our paper recovers the inefficiencies in Wilson in a discrete setting *à la* Kremer and Nyborg.[13]

The paper is structured as follows. In Section 2 we describe the general model, a simple example of which is solved in Section 3. Sections 4 to 6 are devoted to the analysis of the general model, including the characterization of firms' optimal behavior, equilibrium outcomes, equilibrium existence and multiplicity, and the impact of forward contracts. Section 7 contains a simulation exercise, while Section 8 concludes. All proofs are contained in the Appendix.

2 Description of the Model

We consider a model in which $N \geq 2$ firms compete to supply a perfectly divisible good. Market demand, $D(p)$, can be either price-inelastic or downward-sloping, $D'(p) \leq 0$, and its inverse function is denoted $P(q)$.

Firm n's productive capacity K_n, $n = 1, ..., N$, is made up of several units. Each unit has constant marginal costs of production up to its capacity limit. We impose no constraints on the number of units firms have (other than it must be finite), and allow for all types of asymmetries (both in size and cost) among the units owned by a firm, as well as across firms. By stacking firm n's units in increasing cost order, we construct its marginal cost curve, $c_n(q)$, which is a left-continuous non-decreasing step function.[14] We use $C_n(q)$ to denote firm n's cost function, i.e., $C_n(q) = \int_0^q c_n(z)dz$. In line with the literature on electricity auctions, we assume that information on firms' costs is complete because electricity generators share similar production technologies, and are thus well aware of the efficiencies of their plants and the cost of the fuels.

Firms compete by simultaneously submitting a finite number of price-quantity pairs. Prices cannot exceed the 'market-reserve price' p^R (which, for simplicity but without loss of generality, is assumed to exceed the highest marginal cost), and firms cannot produce above their capacities. Note that restricting firms to submit a finite number of price-quantity pairs implies that firms' strategies are left-continuous non-decreasing step functions with a finite number of steps. We assume that both the "height" (prices) and "length" (quantities) of the steps are continuous choice variables.

By ordering firms' price-quantity pairs in increasing price order, we construct their bid functions, i.e., for firm n,

$$b_n = \{(p_{ns}, q_{ns})\}_{s=1}^{\bar{s}}, \ p_{ns} \in \left[0, p^R\right] \text{ with } p_{ns+1} \geq p_{ns}, \ q_{ns+1} \geq q_{ns} \text{ with } q_{n\bar{s}} \leq K_n,$$

[13] To be sure, the reasons why we recover the underpricing equilibria are similar to the ones that explain why the competitive outcome is not sustainable under Bertrand competition with capacity constraints, even though it constitutes the unique equilibrium outcome under pure Bertrand competition. Within the electricity auctions literature, simplified versions of our model also lead to a similar prediction (von der Fehr and Harbord (1993), García-Díaz and Marín (2003), Fabra *et al.* (2006), and Crawford *et al.* (2007)).

[14] As will become clear in Section 4, the possibility that firms have increasing marginal cost functions is a key ingredient of the model, as it implies that firms face step-wise downward sloping residual demand functions when rival firms bid at marginal costs.

where $\bar{s} < \infty$ is the maximum number of admissible steps in a firm's bid function. Consistently with actual rules in electricity markets, we will assume that the number of admissible steps does not constrain firms from bidding each unit at its own marginal cost, i.e., $\bar{s}$ is large enough so as to allow firms to *at least* replicate their marginal cost curves.[15] At each step s in firm n's bid function, p_{ns} specifies the minimum price at which the firm is willing to produce up to quantity q_{ns}. For a given bid profile $b = \{b_n\}_{n=1}^{N}$, we construct the aggregate supply function, denoted $S(q)$, which determines the lowest price at which all firms in the market are willing to produce up to quantity q.

The stop-out price, p^*, at which all transactions take place, is defined as follows,

$$p^* = \max_{q} \{p = S(q) \,| S(q) \leq P(q)\}.$$

In words, the stop-out price p^* is the point on the aggregate supply function, $S(q)$, at which the market clears. If the demand function $P(q)$ is downward-sloping, it need not always intersect the (possibly) discontinuous aggregate supply function, in which case p^* is the highest price on the aggregate supply function at which there is excess demand. To the contrary, if demand is inelastic, there are potentially many market-clearing prices when the demand function intersects the supply function at the right end of a step. In this case, p^* is the lowest price at which the market clears, given that it must be on the (left-continuous) aggregate supply function.[16]

Firms are called to produce in increasing price order up to p^*. We use q_n^* to denote the quantity allocated to firm n. If there is excess supply at p^*, we assume efficient rationing on-the-margin, i.e., if several units have been bid at p^*, they split residual demand proportionally to the quantities offered at exactly p^*, unless their marginal costs differ, in which case the low cost units are dispatched first.[17] By using efficient rationing, the set of equilibria of our game approximates the set of equilibria of a game in which rationing pro-rata on-the margin is used but where firms choose their bid prices on a finite grid, which is what occurs in real markets. In contrast, assuming rationing pro-rata on-the margin in our set-up would lead to a problem of non-existence of equilibrium similar to the one that arises under a Bertrand game with asymmetric costs.

We label prices and quantities as either competitive or non-competitive. The competitive price, denoted p^c, is the point on the aggregate cost function at which demand and competitive supply intersect. As before, if they do not intersect, we assume that p^c is the highest price on the aggregate

[15] The limit on the number of bids is typically set for each production unit rather than at the firm level. For instance, in the original market design in England and Wales, firms were allowed to submit up to 3 incremental prices per unit; up to 25 price-quantity pairs per unit in Spain; and up to 40 per unit in Texas. In practice, firms use even fewer bidpoints than the ones they are allowed to (Hortacsu and Puller (2008)).

[16] These assumptions are consistent with most auction rules in practice. For instance, in the Spanish electricity market, demand bids cannot determine the stop-out price (see www.omel.es). The fact that the auctioneer chooses the lowest market-clearing price whenever there are multiple market-clearing prices is reasonable to the extent that it is the most favourable one from consumers' point of view. This is also assumed in Kremer and Nyborg (2004) and Kastl (2008)- note however that in these papers the stop-out price is assumed to be the highest market-clearing price as the auctioneer is selling rather than buying the underlying good.

[17] Several papers in the electricity auctions literature also assume efficient rationing on-the-margin (see García-Díaz and Marín (2003) and Fabra *et al.* (2006), among others). Instead, papers in the Treasury auctions literature typically adopt the rationing pro rata on-the-margin rule, which rations the marginal bids at p^* proportionally to the total quantity offered at exactly p^*, regardless of their marginal costs (see Back and Zender (1993) and Kastl (2008), among others).

cost function at which there is excess demand. Formally,

$$p^c = \max_q \{p = C(q) \,|\, C(q) \leq P(q)\}.$$

The resulting competitive quantities are denoted $(q_1^c, ..., q_N^c)$. All other prices and quantities are referred to as non-competitive. Similar labels are used to classify market outcomes.

An important feature of the model is that firms might be subject to forward contracts. We use τ_n to denote firm n's contract price, and $x_n \geq 0$ to denote firm n's contract quantity;[18] both τ_n and x_n are fixed when firms submit their bids. Consequently, when the stop-out price is p^* and firm n's dispatched quantity is q_n^*, firm n's profits are given by

$$\pi_n(p^*, q_n^*) = p^* q_n^* - C_n(q_n^*) + [\tau_n - p^*]\, x_n, \tag{1}$$

where the first two terms give the firm's market profits, and the last term gives the firm's contract profits.[19] To fix ideas, one can think of these contracts as being purely financial, i.e., firm n continues to supply all its quantity q_n^* to the market at p^* and the contract's counterpart, e.g. a big customer, continues to buy all its demand from the market at p^*. The contract requires firm n to pay (receive) the difference between the contract price and the stop-out price times the contract quantity, $[\tau_n - p^*]\, x_n$, whenever positive (negative). Re-writing the above expression as

$$\pi_n(p^*, q_n^*) = p^* [q_n^* - x_n] - C_n(q_n^*) + \tau_n x_n, \tag{2}$$

shows that firms' bidding incentives depend on their net-positions, $[q_n^* - x_n]$, which are positive for the net-sellers, $q_n^* > x_n$, and negative for the net-buyers, $q_n^* < x_n$. The last term, $\tau_n x_n$, is fixed when firms compete in the spot market; as such, it has no effect on bidding incentives (indeed, one could set $\tau_n = 0$ without loss of generality). We will assume that total contract volume never exceeds demand at the competitive price, $\sum_n x_n \leq D(p^c)$, thus ruling out the cases in which $x_n \geq q_n^c$ holds for all firms n (with at least one strict inequality).

Firm n's problem is to choose a finite number of price-quantity pairs that maximize π_n given its rivals' supply functions. We focus on Nash equilibria in pure strategies. All aspects of the model are common knowledge among firms.[20]

Before we proceed, it is convenient to set some terminology and notation. We first define which firms are *marginal*:

[18] We adopt the convention that $x_n > 0$ corresponds to firm n selling contracts (i.e., taking a short-position). We do not allow firms to buy forward contracts since in real markets regulators typically impose sale obligations.

[19] In models of vertical integration, the first two terms would represent the profits of the upstream subsidiary, while the third term would accrue to the downstream subsidiary.

[20] When applied to electricity markets, it could be argued that firms face demand uncertainty (or demand variation) at the bidding stage. However, this issue depends on the duration of bids as compared to the frequency of market clearing: for instance, when firms' submit supply functions that remain valid for a single period of market clearing, there will be little or no relevant variation in demand; however, with bids that remain good for a whole day, demand will vary considerably over the pricing period. Accordingly, our paper applies to the first case, which is typically referred to as the short-lived bids case (as opposed to the second case, referred to as long-lived bids case). The former is in place in most electricity markets in practice (see Fabra et al. (2006) and García-Díaz and Marín (2003)). Last, the contracting stage may be affected by demand variation, as contracts typically remain fixed for longer periods of time. However, this has no effect on the bidding stage as long as the features described above are met.

Definition 1 *For an arbitrary bid function profile resulting in an outcome* $\{p^*; (q_1^*, ..., q_N^*)\}$ *, firm* n *is marginal if its bid function has some step* s *at the stop-out price,* $p_{ns} = p^*$*.*

We use the above definition to also classify firms as either *price-setters* or *non-price-setters*:[21]

Definition 2 *For an arbitrary bid function profile resulting in an outcome* $\{p^*; (q_1^*, ..., q_N^*)\}$ *, firm* n *is a price- setter if it is a marginal firm and if it is at least partly dispatching its marginal step,* $q_n^* \in (q_{ns-1}, q_{ns}]$*. Otherwise, firm* n *is a non-price-setter.*

Finally, both the stop-out price and the dispatched quantities depend on the demand, $D(p)$, and the bid function profile, b. However, in order to simplify notation, we suppress these arguments whenever clear from the context.

3 Illustrative Example

We start by analyzing a simple example to convey the intuitions of the main results of the paper. In particular, we fix $N = 2$ and assume that demand is perfectly inelastic at $D = 3$. There exist four types of units, each with capacity normalized to one, whose marginal costs are 0, 1, 2 or 2.5. Firm 1 owns 4 units, one of each cost type, while firm 2 only has 3 units, not owning the unit with marginal costs 2. More specifically, their marginal cost functions are $c_1 = \{(0,1), (1,2), (2,3), (2.5,4)\}$ and $c_2 = \{(0,1), (1,2), (2.5,3)\}$. Accordingly, firms 1 and 2 will be respectively referred to as the "large firm" and the "small firm". Finally, we assume without loss of generality that the contract price is zero, $\tau_n = 0$.

We first show that in the absence of contracts, the competitive outcome cannot be sustained in equilibrium. Suppose that both firms bid at marginal costs, $b_n = c_n$, $n = 1, 2$, so that the aggregate supply function is $S = \{(0,2), (1,4), (2,5), (2.5,7)\}$. Since the auctioneer has to dispatch three units to satisfy demand, the competitive outcome is $\{p^c = 1; (q_1^c = 1.5, q_2^c = 1.5)\}$, with profits $\pi_n^c = 1$, $n = 1, 2$. If firm 1 deviates to bidding all its units at 2.5, i.e., $b_1' = \{(2.5,4)\}$, the aggregate supply function becomes $S' = \{(0,1), (1,2), (2.5,7)\}$, the stop-out price is raised to $p^* = 2.5$, and firms' dispatched quantities are $q_1^* = 1$ and $q_2^* = 2$ (by the efficient tie-breaking rule, firm 1's first unit is dispatched at capacity, as it has lower marginal costs than any of the other units that tie at the margin). Thus, firm 1 makes a larger profit, $\pi_1' = 2.5 > \pi_1^c$, and firm 2 gains even more, $\pi_2' = 4 > \pi_2^c$. Note that one can also rule out existence of a competitive equilibrium by letting firm 2 deviate from marginal cost bidding. In this case, firm 2 would optimally raise the price to 2, e.g. by bidding at $b_2' = \{(2,2), (2.5,3)\}$, in order to increase its profits to $\pi_2' = 2 > \pi_2^c$; again, the other firm gains even more, $\pi_1' = 3 > \pi_1^c$. Thus, the competitive outcome cannot be sustained in equilibrium, unless firms used weakly-dominated strategies, a possibility ruled out throughout the paper.

The two bid function profiles considered above, $\{b_1', c_2\}$ and $\{c_1, b_2'\}$, are indeed in equilibrium. Under both profiles, one firm is setting the stop-out price at the level that maximizes its profits over its residual demand (which coincides with the marginal cost of its rival's first undispatched unit), while the other firm cannot increase its profits as it is producing the maximum it can without

[21] We have inherited the price-setter and non-price-setter terminology from Crawford et al. (2007).

incurring in losses. The two equilibria are not outcome equivalent, as the first results in a high price, $p^* = 2.5$, while the second results in a lower price, $p^* = 2$. However, none of them can be ruled out by appealing to Pareto dominance, given that each firm is strictly better-off when the rival sets the price.[22] To see this, recall that at the first equilibrium firms' profits are $\pi_1 = 2.5$ and $\pi_2 = 4$, whereas at the second firms' profits are $\pi_1 = 3$ and $\pi_2 = 2$.[23]

Besides the two equilibria described above, there are many other equilibrium bid profiles; in particular, there exist several equilibria with both firms bidding above marginal costs. The reason for this multiplicity is that several bids are outcome irrelevant. However, conditionally on the identity of the firm that sets the price, all equilibria are outcome equivalent to the two equilibria just described. In sum, even though the strategy space is quite large, we need just focus on candidate equilibrium outcomes, of which there are at most as many as firms in the market.

To illustrate the impact of contracts, let us first allocate all contracts to the large firm, $x_1 \in (1, 2] > x_2 = 0$. If firm 2 bids at marginal costs, firm 1's profit-maximizing price now equals $p^* = 1$ rather than $p^* = 2.5$. To see this, note that if firm 1 sets the stop-out price at $p^* = 2.5$, it now becomes a net-buyer with $x_1 > q_1^* = 1$. As such, it prefers to reduce the price to $p^* = 1$ by e.g. bidding at marginal costs. Indeed, since marginal cost bidding allows firm 1 to save the price difference over its net-buying position, its profits increase by $[1 - 2.5]\,[1 - x_1] > 0$. Therefore, the equilibrium at which firm 1 sets the price at $p^* = 2.5$ can no longer be sustained, whereas the equilibrium at which firm 2 sets the price at $p^* = 2$ can still be sustained (firm 2's incentives are unchanged as it has no contracts, while firm 1 does not find it profitable to reduce the price as at this equilibrium it is no longer a net-buyer, $x_1 \leq q_1^* = 2$). Since only the low-price equilibrium outcome survives, allocating contracts to the large firm is pro-competitive.

Alternatively, let us now allocate all contracts to the small firm, $x_2 \in (1, 2] > x_1 = 0$. By the same logic, the equilibrium with firm 2 setting the price at $p^* = 2$ disappears: as a net-buyer, firm 2 would rather bid at marginal costs in order to reduce the price from $p^* = 2$ to $p^* = 1$. However, firm 1 would then respond by setting the price at $p^* = 2.5$, which implies that the only surviving equilibrium outcome is the one with the high price. Hence, forward contracts are anti-competitive in this case. Table 1 summarizes these results.

To sum-up, this example illustrates that the impact of forward contracts on bidding incentives and equilibrium outcomes critically depends on its distribution among firms. Even though contracts reduce firms' incentives to increase prices, equilibrium prices need not be lower as contracts might also jeopardize the existence of the equilibria in which the contracted firm sets the price. Indeed, contracts might lead to (weakly) higher prices whenever they are awarded in sufficiently large quan-

[22] We are grateful to a referee for pointing out that, in some pathological examples, the low-price equilibrium is (weakly) Pareto dominated by the high-price equilibrium. This may occur when the length and height of firms' marginal cost functions equal one and demand is an integer, so that a firm might be indifferent between being a non-price-setter and selling one more unit at a lower price, or being a price-setter and selling one unit less at a higher price. For instance, in this example, if the high marginal cost equals 3 rather than 2.5, the high price equilibrium becomes $p^* = 3$, with $\pi_1 = 3$ and $\pi_2 = 5$ so that the low-price equilibrium is now weakly Pareto dominated. However, this Pareto ranking is not robust to slight perturbations. If this marginal cost equals $3 - \varepsilon$ (for ε small enough) instead of 3, or if demand is $3 - \varepsilon$ instead of 3, then firm 1 would strictly prefer to be a non-price-setter rather than a price-setter and the two equilibria can no longer be Pareto ranked.

[23] This also implies that both equilibria are coalition-proof (see Bernheim, Peleg and Whinston (1987)).

	Equilibrium Prices	
	No contracts	$x_i \in (1,2] > x_j = 0$
Firm i is large	$\{2.5, 2\}$	$\{\varnothing, 2\}$
Firm i is small	$\{2, 2.5\}$	$\{\varnothing, 2.5\}$

Table 1: Equilibrium clearing prices as a function of firms' forward contract positions

Note: the first (second) term in brackets is the price that firm i (firm j) would set in equilibrium when its rival behaves as a price-taker; there is an $\varnothing$ if such an equilibrium does not exist.

tity to the firm that would set lower prices without contracts (in this example, the small firm). Let us note that this effect can only be uncovered once the symmetry assumption is relaxed, as otherwise the equilibrium price would be the same regardless of which firm sets prices, and hence regardless of whether contracts are allocated to one firm, to the other, or to none.

4 Analysis of the Model

In this section, we characterize equilibrium bidding behavior and equilibrium outcomes in the general model. Rather than deriving equilibrium strategies, we instead deduce structural features that any equilibrium must have. As it is common in the analysis of uniform-price auctions, we first refine the equilibrium set by restricting attention to strategies that are not weakly-dominated. In the absence of contracts, bidding below marginal costs is a weakly-dominated strategy (García-Díaz and Marín (2003) and Crawford *et al.* (2007)). However, when firms hold contracts, this is not generally the case, as shown next.

Lemma 1 *For firm n, it is weakly-dominated (i) to bid below marginal costs for quantities above its contract cover, $q_n > x_n$, as well as (ii) to bid above marginal costs for quantities not exceeding its contract cover, $q_n < x_n$.*

In words, weak-dominance arguments eliminate *below marginal cost bidding* only for quantities above the firm's contract cover, $q_n > x_n$, i.e., such that the firm is a net-seller.[24] At lower quantities, the firm is a net-buyer, and as such it would like to exercise monopsony power by bidding some units below marginal cost. Consistently with this, weak-dominance arguments also eliminate *above marginal cost bidding* for quantities below the firm's contract cover, $q_n < x_n$. We cannot rule out either below or above marginal cost bidding for $q_n = x_n$ because bid functions are step functions. With continuous bid functions instead, bidding $q_n = x_n$ at marginal costs would be a dominant strategy.

In what follows, we will first fix the identity of the price-setter in order to characterize the non-price-setters' optimal bidding behavior.

[24] Kastl (2008) shows that in discrete multi-unit uniform-price auctions, a rational bidder (without contracts) may submit a bid above its marginal valuation (in the current paper, a bid below marginal costs). This occurs only when the number of admissible steps in the bid functions is lower than the number of units, as it implies that bidders have to bundle bids for several units together. However, this does not arise in our paper given that, consistently with practice, bidders can at least submit as many bids as units they own.

Lemma 2 *At any Nash Equilibrium in which firm i is a price-setter, all other firms j, $j \neq i$, are fully dispatching all their units with marginal costs* strictly below *the equilibrium price p^*.*

The intuition underlying Lemma 2 above is simple. Given that firm i is dispatching some output at p^*, it cannot be the case that some other firm j, $j \neq i$, has some unit with marginal costs strictly *below* p^* that has not been called to produce. If it instead bid such an undispatched unit slightly below p^*, firm j would earn a positive profit margin over its increased production, with only (if any) a slight reduction in the price. Key to this result is the fact that firms submit a finite number of price-quantity pairs, which implies that there is a positive output mass at the margin. Hence, when firm j reduces its bid, the quantity gain always outweighs the price reduction as the latter can be made arbitrarily small.

Lemma 3 *At any Nash Equilibrium in which firm i is a price-setter, firm j, $j \neq i$, is not dispatching any unit with marginal costs* strictly above *the equilibrium price p^* if either one of the following two conditions holds:*

1) firm j is a net-seller or has a balanced position, i.e., $q_j^* \geq x_j$, or

2) there is at least one marginal firm that is not fully dispatching its marginal step, i.e., $p^* = p_{ks}$ and $q_k^* < q_{ks}$, $k \neq j$.

By elimination of weakly-dominated strategies, net-sellers cannot sell their marginal output below marginal costs. Similarly, firms with a balanced position do not find it profitable to bid below marginal costs in equilibrium given that by bidding at marginal costs they could save the difference between their marginal costs and the equilibrium price times their reduced output. Hence, $q_j^* \geq x_j$ is sufficient to guarantee that firm j does not dispatch any unit with marginal costs below p^*.

The same result does not apply in general to an equilibrium in which firm j is a net-buyer, unless some other firm k has bid some step at p^* which has not been fully dispatched (i.e., $p^* = p_{ks}$ and $q_k^* < q_{ks}$). When this is the case, firm j can avoid producing at a loss by bidding some of its output slightly above p^*, with no effect on the price. However, if all the marginal firms are fully dispatching their marginal steps, firm j may be unable to reduce its production so as to avoid productive losses unless it raises the price high enough. As firm j is a net-buyer, the price increase - which may no longer be infinitesimal - may reduce the firm's profits (firm j reduces productive losses but buys its negative net position at a higher price). There is hence no guarantee that in an equilibrium in which firm i is a price-setter, the other firms produce in an efficient manner unless they are all net-sellers. Note that this result only arises with contracts as, otherwise, all firms would trivially be net-sellers.

The next Proposition combines the two lemmas above to provide conditions under which at any equilibria the non-price-setters behave *as if* they were price-takers. This does not imply that they do not act strategically; to the contrary, the non-price-setters are the ones that benefit the most from the (potential) exercise of market power, while the price-setters must bear the cost.

Proposition 1 *At any Nash Equilibrium in which firm i is a price-setter, firm j, $j \neq i$, produces the same "as if" it were bidding at marginal costs if either one of the two conditions in the statement of Lemma 3 hold.*

The conditions under which Proposition 1 holds relate to equilibrium features, which are endogenous, such as the identity of the price-setter or the non-price-setters' equilibrium net-positions. Nevertheless, one can guarantee that at any equilibrium Proposition 1 always holds if all firms are net-sellers at the competitive outcome. This condition relates to the primitives of the game, which are no longer endogenous.

To see why this is the case, note that weak-dominance arguments imply that if $x_n < q_n^c$ holds for all firms, they must all bid their competitive quantities at or above marginal costs. Therefore, the equilibrium price p^* cannot be lower than p^c. This implies that those firms that bid at marginal costs must be producing more than at the competitive outcome, and are thus net-sellers; while those firms that bid above marginal costs must also be net-sellers by elimination of weakly-dominated strategies. Therefore, since condition 1) of Lemma 3 is satisfied, Proposition 1 applies.

In contrast, Proposition 1 does not generally hold if some firms are net-buyers at the competitive outcome. First, since $p^* \leq p^c$ cannot be ruled out, even the firms that are bidding at marginal costs may produce below their competitive quantities, and hence remain/become net-buyers. Moreover, even if $p^* > p^c$, and some firms expand production above their competitive quantities, such an increase in quantity might not be enough so as to exceed their contract positions.

For these reasons, it will be useful to analyze these two cases separately, which we respectively refer to as the *regular* cases (in which $x_n < q_n^c$ for *all* firms) and the *irregular* cases (in which $x_n < q_n^c$ holds for some but *not all* firms). Arguably, the *regular* cases are the empirically most relevant ones (in practice, regulators never force firms to holding contracts above their competitive quantities), but for completeness we will also cover the *irregular* cases in Section 6.

5 Regular Cases

We start the analysis of the regular cases by identifying conditions under which the competitive outcome constitutes the unique equilibrium outcome of the game. On the one hand, we provide primitives of the game which are sufficient for all firms to behave competitively. On the other hand, we derive properties of the equilibrium bid profiles that sustain the competitive outcome.

Proposition 2 *Let $x_n < q_n^c$ hold for all firms.*

(i) If for any n, all firms but firm n can jointly serve total competitive demand $D(p^c)$ without losses, then the competitive outcome, $p^* = p^c$ and $q_n^* = q_n^c$ for all n, is the unique equilibrium outcome.

(ii) Whenever there is supply rationing at p^c, any Nash equilibrium results in the competitive outcome if and only if there is more than one price-setter.

If every possible combination of $(N-1)$ firms can jointly serve total demand at the competitive price, the residual demand faced by the Nth firm would fall down to zero if it deviated optimally from the competitive equilibrium. Hence, all firms have no option but to behave competitively. If this condition did not hold, the Nth firm would have the possibility of manipulating the price up a notch. If such a firm is marginal at the competitive outcome, it will certainly bid above marginal costs in order to make profits out of its marginal output. However, if it is not marginal, meaning that it is making strictly positive profits out of all its dispatched units, it might not find it profitable to deviate if the losses from reducing output exceed the gains due to the price increase. Hence, while the condition in part (i) of Proposition 2 above is sufficient for the competitive outcome to emerge, it is nevertheless not necessary.

But for knife-edge cases with no supply rationing at p^c, the coexistence of multiple price-setters is both necessary and sufficient for the competitive outcome to be sustainable. If there was only one price-setter, its bid would determine the stop-out price, and the firm would have incentives to engage in *supply reduction*.[25] The upshot of this is that with multiple price-setters, the equilibrium must be competitive as any of them would otherwise gain by slightly undercutting the price in order to achieve a positive increase in output. It follows that there cannot occur (payoff-relevant) ties at the margin among dispatched units, unless the equilibrium is competitive. In contrast, ties at the equilibrium price with only one firm dispatching output at the margin will be (almost always) the rule. This will be clearly the case with inelastic demand, as the price-setter will optimally drive the price up to the next step in its rivals' bid functions.[26]

The reason why the statement of part (ii) does not include certain knife-edge cases is simple. If there was no supply rationing at p^c, there could exist equilibria with $p^* > p^c$ with ties at the margin, as long as firms still produce their competitive quantities. Since all firms are rationed at p^c, they would also be rationed at prices slightly above p^c, and would hence have no incentives to fight for market share at the margin. One simple example in which this is the case has $D = 2$, $c_1 = c_2 = \{(0,1)\}$ and $c_3 = \{(c,1)\}$. In equilibrium, $p^* = c > 0 = p^c$ while $q_n^* = q_n^c = 1$ for $n = 1, 2$, and $q_3^* = q_3^c = 0$. Both firms 1 and 2 could be price-setters if they bid at $b_1 = b_2 = \{(c,1)\}$, but the same outcome would also arise with just one of them bidding at c while the other bids below. In this sense, if there is no supply rationing at p^c, ties at the margin among dispatched units can occur in equilibrium, but such ties are payoff irrelevant.

An important consequence of Proposition 2 is that *at any non-competitive Nash equilibrium, there is a unique price-setter.* This fact allows us to proceed by fixing the identity of the price-setter and treating all other firms as non-price-setters. This approach is appropriate even for competitive equilibria, as there would be more than one price-setter (Proposition 2) but they would also behave as non-price-setters in equilibrium (Proposition 1).

5.1 The non-price-setters' and the price-setter's optimal behavior

In order to derive the necessary and sufficient conditions for equilibrium bidding, we will first characterize firms' optimal bidding behavior conditional on their identities.

By Proposition 1, we already know that the non-price-setters behave as price-takers, i.e., they have to decide how much to produce at a given stop-out price, p^*. Formally,

$$q_j^{NPS}(p^*) \in \arg\max_{q_j^*} \pi_j^{NPS}\left(p^*; q_j^*\right),$$

[25] Arguments here are identical to those in Ausubel and Cramton (2002) to get optimal *demand reduction* in uniform-price auctions.

[26] With a rationing pro rata on-the-margin rule, such a tie at the margin would not arise. Nevertheless, the outcome would (almost perfectly) approximate the equilibrium outcome under the efficient tie-breaking rule. Note that if the tie-breaking rule did not allocate the marginal output to the low cost firm first, such a firm would avoid the tie by bidding slightly below its rivals' first non-accepted bid.

where

$$\pi_j^{NPS}\left(p^*;q_j^*\right)=p^*\left[q_j^*-x_j\right]-C_j\left(q_j^*\right)+\tau_j x_j.$$

Note that, in order to produce $q_j^{NPS}(p^*)$, the non-price-setters can bid at marginal costs or use any other outcome equivalent strategy. However, their choice of bidding strategies is not irrelevant, as these determine the shape of the residual demand faced by the price-setter and hence its optimal bidding behavior. For this reason, we will not assume that the non-price-setters bid at marginal costs, unless we make it explicit. It follows that we can readily compute the price-setter's production *in equilibrium*, but not outside the equilibrium. In particular, at any candidate equilibrium with $p^*=p^c$, the price-setter produces q_i^c, while if $p^*>p^c$, given that the market must clear, the price-setter produces

$$q_i^{PS}(p^*)=D(p^*)-\sum_{j\neq i}q_j^{NPS}(p^*).$$

However, the above equation might not be satisfied at prices other than the equilibrium price, given that the non-price-setters need not be bidding at marginal costs outside the equilibrium, and given that the market need not always clear.

The nature of the price-setter's problem clearly differs from that of the non-price-setters'. Instead of choosing how much to produce at a given price, the price-setter behaves as if it were to choose the stop-out price that maximizes its profits over its residual demand, i.e., total demand minus the quantity that the non-price-setters are willing to supply at each price. Formally,

$$p_i^{PS}(b_{-i})\in\arg\max_{p^*}\pi_i^{PS}(p^*;b_{-i}), \tag{3}$$

where

$$\pi_i^{PS}(p^*;b_{-i})=p\left[q_i^*(p^*;b_{-i})-x_i\right]-C\left(q_i^*(p^*;b_{-i})\right)+\tau_i x_i,$$

and,

$$q_i^*(p^*;b_{-i})=\max\left\{0,D(p^*)-\sum_{j\neq i}q_j^*(p^*;b_{-j})\right\}.$$

Since both the cost function and the residual demand faced by the price-setter are step-functions, its profit function may fail to be differentiable, so that the price-setter's profit-maximizing price might not be obtained as the solution to a first order condition. Therefore, in order to understand the price-setter's bidding incentives, consider the change in firm i's profits when it raises the stop-out price from p^* to some $p'>p^*$,

$$\pi_i^{PS}\left(p';b_{-i}\right)-\pi_i^{PS}(p^*;b_{-i})=\left[p'-p^*\right]\left[q_i^*\left(p';b_{-i}\right)-x_i\right]-\int_{q_i^*(p';b_{-i})}^{q_i^*(p^*;b_{-i})}\left[p^*-c_i(z)\right]dz. \tag{4}$$

As in any standard monopoly problem, a price increase implies greater revenues through the firm's net-position - the first term in (4), - but it also implies a profit loss due to the output reduction - the second term in (4). Accordingly, the price-setter's incentives to raise the price are stronger the bigger its net-position is, the less elastic its residual demand is, and the smaller the price-cost margin on its lost production is. It then follows that firm i's profit-maximizing price given its rivals' strategies, $p_i^{PS}(b_{-i})$, is non-increasing in its contract cover, x_i. This mimics the standard result that smaller firms (here, firms with smaller net-positions) have weaker incentives to raise prices.

We conclude this subsection by comparing the price-setter's and non-price-setters' profits. To simplify notation, we will write $\pi_i^{PS}(p^*)$ and $\pi_j^{NPS}(p^*)$, for $j \neq i$, to respectively denote the price-setter's and non-price-setters' profits when the former sets the stop-out price at p^* and all the latter produce $q_j^{NPS}(p^*)$.

Lemma 4 *Let $x_n < q_n^c$ hold for all firms. For any stop-out price p^*, (i) the non-price-setters' profits $\pi_j^{NPS}(p^*)$ are increasing in p^*, and (ii) they weakly exceed those they would get as a price-setter at the same stop-out price, $\pi_i^{PS}(p^*) \leq \pi_i^{NPS}(p^*)$.*

Since under the regular cases all firms are net-sellers in equilibrium, any price increase makes the non-price-setters strictly better-off. Because of this, we will assume that whenever the price-setter is indifferent between multiple prices, it always chooses the highest one as it is the Pareto dominant one. By definition, the price-setter is indifferent between these prices, but the non-price-setters are strictly better off when the highest maximizer is chosen (as shown in Lemma 4 above). Finally, the profits that a firm earns as a non-price-setter are bounded below by the profits it could obtain as a price-setter: both the non-price-setters and the price-setter are paid the same price, but the price-setter sells (weakly) less and thus gives up a positive profit margin on its reduced production.

5.2 Equilibrium characterization

We have already characterized firms' optimal behavior conditional on their identities, but the equilibrium characterization also requires an assessment of whether the price-setter prefers to become a non-price-setter and *vice-versa*. An equilibrium outcome is a collection of quantities produced by the non-price-setters and a price chosen by the price-setter such that no firm wants to deviate, either by changing its quantity or price choice, or by changing its identity. The following Theorem provides necessary and sufficient conditions for equilibrium bidding in the regular cases.

Theorem 1 *Let $x_n < q_n^c$ hold for all firms. A strategy profile b constitutes a Nash equilibrium in which firm i is the price-setter if and only if the following three conditions hold:*

1) $p^* = p_i^{PS}(b_{-i}) \geq p^c$ and $q_j^* = q_j^{NPS}(p^*)$ for all $j \neq i$.

2) $\pi_i^{PS}(p^*) \geq \pi_i^{NPS}(p)$ for all $p < p^*$ such that $q_i^{NPS}(p) + \sum_{j \neq i} q_j(p; b) = D(p)$.

3) $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}\left(p_j^{PS}\right)$ for all $j \neq i$ such that $p_j^{PS}(b_{-j}) > p^*$.

In equilibrium, the price-setter chooses the price that maximizes its profits over the residual demand, $p^* = p_i^{PS}(b_{-i})$. By weak-dominance arguments, it must be either equal or above the competitive price. All the other firms must behave as price-takers given p^*, and hence produce the same as if they were bidding at marginal costs (Proposition 1).

By condition 1) of Theorem 1 above, all firms are already optimizing conditionally on their identities. Thus, the only relevant deviations are those by which firms reverse their identities. Since the price-setter might consider becoming a non-price-setter in order to sell more at a lower stop-out price,[27] condition 2) is needed to rule out such deviations. In turn, since all the non-price-setters are net-sellers in equilibrium, those with profit-maximizing prices no larger than p^* never find it

[27] Note that at the resulting price, there must be market clearing, as otherwise the deviant could bid so as to sell the same quantity at the price that clears the market.

optimal to deviate: not only they would sell their production at a (weakly) lower price, but they would also sell less. Hence, the only relevant deviations are those by the remaining non-price-setters, but condition 3) rules them out.

5.3 Equilibrium existence and multiplicity

For a given price-setter, there exist multiple bid function profiles that constitute an equilibrium (all those satisfying Theorem 1). This derives from the fact that firms only care about one point in their bid functions, the one corresponding to the stop-out price. Furthermore, the multiplicity of equilibrium bid functions may pave the way for multiplicity of equilibrium outcomes to emerge. Fortunately, this is not the case, as stated next.[28]

Proposition 3 *(i) There exists an equilibrium in which firm i is the price-setter if and only if the equilibrium in which firm i sets the price at* $p^* = p_i^{PS}(c_{-i})$ *while all the other firms bid at marginal costs exists. (ii) Furthermore, if there also exist other equilibria in which firm i is the price-setter, the one above is the Pareto dominant one.*

Proposition 3 implies that, conditionally on the identity of the price-setter, there is no loss of generality (as far as equilibrium outcomes are concerned) in restricting attention to equilibria in which the non-price-setters bid at marginal costs and the price-setter maximizes its profits over the resulting residual demand. This claim is supported by two important facts: if such an equilibrium does not exist, there does not exist any other equilibrium in which the same firm acts a the price-setter; while if it exists, it is either the unique equilibrium or the Pareto dominant one.

The next result, which is a corollary of Theorem 1, guarantees equilibrium existence. In particular, the candidate equilibrium with the highest price always exists.

Corollary 1 *The equilibrium with* $p^* = \max_i p_i^{PS}(c_{-i})$ *always exists.*

So far, we have shown that, conditionally on the identity of the price-setter, equilibrium multiplicity is irrelevant for equilibrium outcomes. However, the multiplicity of equilibria that differ in the identity of the price-setter might potentially result in different equilibrium prices. This multiplicity was highlighted in the illustrative example provided in Section 3 but it holds more generally. Furthermore, existence of a candidate equilibrium implies that all other candidate equilibria with higher equilibrium prices also exist. To understand this, note that the profits that a firm achieves as a price-setter are given, but the profits it makes as a non-price-setter are increasing in the equilibrium price (Lemma 4). Hence, if none of the firms has incentives to deviate from a candidate equilibrium, it must also be the case that none of them wants to deviate from a candidate equilibrium with a higher price. For similar reasons, if a candidate equilibrium does not exist, there does not exist any other candidate equilibrium with a lower price. These results are stated in the last corollary of Theorem 1.

[28] In contrast, multiplicity of equilibrium outcomes is pervasive in auctions with continuous bid functions (seeWilson (1979), Klemperer and Meyer (1989) and Back and Zender (1993), among others). See also the analysis of the irregular cases in Section 6, which also give rise to multiple equilibria.

Corollary 2 *If the equilibrium with* $p^* = p_i^{PS}(c_{-i})$ *exists, the equilibria with* $p^* = p_n^{PS}(c_{-n}) \geq p_i^{PS}(c_{-i})$ *also exist. Alternatively, if it does not exist, the equilibria with* $p^* = p_n^{PS}(c_{-n}) \leq p_i^{PS}(c_{-i})$ *do not exist either,* $n = 1, ..., N$.

Combining the two corollaries above, it follows that under the same primitives of the game, a competitive equilibrium cannot coexist with a non-competitive equilibrium.

5.4 The impact of forward contracts

We are now ready to analyze the impact of forward contract commitments on equilibrium outcomes. To do so, we will focus on those cases under which the equilibrium is non-competitive in the absence of contracts. The reason is two-fold: first, these are indeed the cases under which forward contract commitments are used by regulators in practice; and second, if the equilibrium was competitive even without contracts, introducing them would trivially make no difference. Since our aim is to perform comparative statics with respect to changes in contracts, in what follows, with some abuse of notation, we will write $p_i^{PS}(x_i)$ to denote the profit-maximizing price of firm i when its rivals bid at marginal costs and its contract obligation is x_i. We will also index firms according to their profit-maximizing prices at the no-contracts case, i.e., $p_1^{PS}(0) \geq p_2^{PS}(0) \geq ... \geq p_N^{PS}(0)$.[29]

Suppose first that firms are symmetric in all respects. The next lemma characterizes the impact on prices and productive efficiency of increasing total contracts when they are either symmetrically or asymmetrically distributed among firms.

Lemma 5 *In a symmetric oligopoly,*

(i) If forward contracts are equally distributed among firms, i.e., $x_1 = ... = x_N = x < q^c$, *equilibrium prices are non-increasing in* x *and productive efficiency is non-decreasing in* x.

(ii) If forward contracts are not equally distributed among firms, the highest equilibrium price is (weakly) higher and its associated productive efficiency is (weakly) lower than under the equal contract distribution.

Since firms are fully symmetric, there exist N price-equivalent equilibrium outcomes that only differ in the identity of the price-setter. As firms' contract cover is increased, the equilibrium price is reduced and productive efficiency is improved.[30] Furthermore, any departure from the symmetric contract distribution would weaken the positive effect of contracts as firms' ex-ante symmetry, which induces more competitive outcomes, would no longer be preserved. For given contracts, similar results also arise in Allaz and Vila (1993)'s and Bushnell (2007)'s Cournot models, as well as in Newbery's (1998) Supply Function Equilibrium model.

We next allow for all types of asymmetries among firms, and perform comparative statics with respect to contract volume up to firms' competitive quantities, depending on the distribution of contracts across firms.

[29] With inelastic demand and flat cost functions, the profit-maximizing price of all firms would be the same, as it would be equal to the price cap. Hence, contracts would have an effect on equilibrium existence (as shown in Proposition 4 below), but this effect would not be reflected in equilibrium prices.

[30] In contrast, if total contract volume was further increased (taking us away from the regular cases), firms would start exercising monopsony power, leading to prices below the competitive price. Furthermore, since the price-setter would produce more than at the competitive outcome, productive inefficiencies might emerge.

Proposition 4 *Consider an asymmetric oligopoly, such that at the no-contracts case equilibrium prices are* $\left\{p_1^{PS}(0), ..., p_i^{PS}(0)\right\}$ *for* $1 \leq i \leq N$*, while prices* $p_n^{PS}(0)$ *for* $i < n \leq N$ *cannot be sustained because firm 1 would deviate.*

(i) If forward contracts are awarded to firm 1 *only, prices are (weakly) lower than at the no-contracts case. Furthermore, a (weak) non-monotonic relationship between contract volume and equilibrium prices may arise.*

(ii) If forward contracts are awarded to firm i *only, there exists* $x_i' \in (0, q_i^c)$*, such that any contract allocation* $x_i < x_i'$ *leads to (weakly) lower prices than at the no-contracts case, whereas any contract allocation* $x_i \geq x_i'$ *leads to (weakly) higher prices. Hence, there is a (weak) non-monotonic relationship between contract volume and equilibrium prices.*

(iii) If forward contracts are awarded to firms $n > i$ *only, they have no effect on equilibrium outcomes.*

At the no-contracts case, firm 1 and firm i set the highest and lowest equilibrium prices respectively, while firms $n > i$ behave as price-takers at any equilibrium. Accordingly, we say that firm 1 and firm i have 'high' and 'low' market power respectively, while firms $n > i$ have 'no' market power at all. The impact of forward contracts on equilibrium prices depends on how contracts are awarded among these firms.

To understand the results in Proposition 4 above, it is important to first recall that as a firm's contracts go up, its profit-maximizing price (weakly) goes down. In turn, given that a low price makes it relatively more attractive for an uncontracted non-price-setter to become the price-setter, the equilibrium in which the contracted firm sets the price might *disappear*. By the opposite logic, the contracted firm now finds it more appealing to be the non-price-setter, so that if no other firm has incentives to deviate, there can now *appear* new equilibria involving lower prices. These effects are illustrated in Figures 1 to 4.

If all contracts are awarded to the firm with 'high' market power, as in part (i), contracts (weakly) reduce prices with respect to the no-contracts case. This holds true regardless of whether the equilibrium in which the contracted firms sets the price disappears (Figure 1), and regardless of whether new equilibria arise (Figure 3), given that in any case the remaining equilibria result in lower prices.

The above conclusion may be reversed when all contracts are awarded to the firm with 'low' market power, as in part (ii). In this case, it is still true that contracts (weakly) reduce prices when the firm with 'low' market power sets the price. However, prices might go up when such equilibrium disappears (for $x_i \geq x_i'$). Given that the equilibrium price will then be set by firms with higher profit-maximizing prices, contracts in this case may result in (weakly) higher prices as compared to the no-contracts case (Figure 2).[31] Last, if contracts are awarded to firms with 'no market power', as in part (iii), contracts simply have no effect as such firms behave as price-takers with or without contracts (Figure 4).

[31] The cases in which firms $1 < n < i$ hold contracts are similar to case (ii) in the Proposition. There exists $x_n' \in (0, q_n^c)$ such that allocating contracts $x_n < x_n'$ to firm n leads to (weakly) lower prices because the equilibrium at which firm n sets the price still exists. However, contracts $x_n \geq x_n'$ eliminate such equilibrium, with some of the remaining equilibria leading to either higher or lower prices. Hence, the effects of contracts in these cases depend on equilibrium selection, but the anti-competitive effects cannot be ruled out.

To conclude, when contracts are allocated to firms with market power, an increase in contract volume does not always lead to price reductions. Indeed, an increase in contract volume may lead to (weakly) higher prices whenever such an increase in contracts destroys the equilibrium in which the contracted firm sets the price (Figures 1 to 3).

[INSERT FIGURES 1-4 AROUND HERE]

Interestingly, note that the classification of firms as having either 'high', 'low' or 'no' market power, and hence the effect of forward contracts on prices, depends on several factors, including firms' cost functions, firms' sizes and, in general, the degree of firms' asymmetries. Large asymmetries, such that only one firm has market power at the no-contracts case, lead to a clear-cut policy conclusion: the dominant firm should be forced to hold contracts; getting contract volume wrong in this case is not very costly, as contracts would in any case be effective. To the contrary, mild asymmetries among firms (particularly so, between the firms with 'high' and 'low' market power) might give rise to the anti-competitive effects identified in Proposition 4. It is in these cases when the regulator should be most cautious when deciding on contract volume and its distribution among firms. However, it is also in these cases when contracts can potentially play a more crucial role, as encouraging firms with 'medium' and 'low' market power to purchase such contracts may counterbalance the market power of the dominant firms.

6 Irregular Cases

In this section, we assess whether the results we have derived so far are robust to some firms holding fewer contracts than their competitive quantities while others hold more, i.e., $x_n < q_n^c$ for some firms and $x_n \geq q_n^c$ for others.

Similar results to those found under the regular cases regarding equilibrium characterization, equilibrium existence and multiplicity, also arise under the irregular cases as long as *in equilibrium* all firms are either net-sellers or have a balanced position, i.e., as long as $x_i \leq q_i^*$ holds for all firms. The intuition is simple: if all firms' net-positions at the candidate equilibrium have the same sign (or no sign at all), they face no conflict of interests among them. In particular, none of the non-price-setters wants to deviate in order to reduce the price and, given that Proposition 1 applies, none of them wants to change its production as they are all producing the same as if they were bidding at marginal costs.

In contrast, the properties of the equilibria in which net-sellers and net-buyers coexist might drastically differ from those derived under the regular cases, as the coexistence of net-buyers and net-sellers gives rise to a conflict of interests among them. Several implications follow from this.

First, as already argued, there is no guarantee that the non-price-setters produce efficiently, given that Proposition 1 need not hold for the net-buyers. To see this, consider for instance a duopoly facing inelastic demand, $D = 6$, with costs $c_1 = \{(0,1),(1,5),(c,6)\}$, $c_2 = \{(0,1),(c,6)\}$, where $c > 1$, and contracts $x_2 = 6 > q_2^c = 1$ and $x_1 = 0 < q_1^c = 5$. The bid profile $b_1 = \{(z,1),(p^R,6)\}$ and $b_2 = \{(0,5),(z,6)\}$, where $1 < z < c$, constitutes a Nash equilibrium at which firm 1 is the price-setter and firm 2, despite being a non-price-setter, produces with units whose marginal costs exceed p^* (in particular, $q_2^* = 5$ whereas $c_2(5) = c > z = p^*$), thus failing to satisfy Proposition 1.

Second, the conditions that guarantee existence of competitive equilibria are more stringent in the irregular than in the regular cases (Proposition 2). On the one hand, the fact that all firms but one can exhaust competitive demand does not guarantee that the equilibrium will be competitive, as the net-buyers face incentives to deviate by reducing the price rather than by increasing it. On the other hand, neither the existence of several price-setters implies that the equilibrium must be competitive, nor the existence of a unique price-setter implies that the equilibrium must be non-competitive. To see this, consider again the market structure described above. The strategies $b_1' = \{(z, 5), (p^R, 6)\}$ and $b_2' = \{(z, 6)\}$ generate a Nash equilibrium in which both firms are price-setters and still $p^* = z > 1 = p^c$. This equilibrium hinges on a property which holds more generally under the irregular cases, namely, that it is not possible to rule out payoff-relevant ties at the margin. Whereas the net-buyers weakly prefer to tie so as to avoid productive losses, the net-sellers might be capacity-constrained to benefit from the increased demand they would face if they were to break the tie by reducing their bids.

Third, the necessary and sufficient conditions for equilibrium bidding must now take into account not only that deviations up and deviations down might be profitable, but also that the candidate stop-out price can be above, equal or below the competitive price. At an equilibrium with $p^* \geq p^c$, condition 3) in Theorem 1 has to be strengthened so that $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ holds, not only for all firms $j \neq i$ with $p_j^{PS}(b_{-j}) > p^*$ (as in the regular cases), but also for all firms $j \neq i$ which satisfy $x_i + x_j > q_i^* + q_j^*$. In words, the modified condition 3) requires that deviations by non-price-setters with profit-maximizing prices below p^* be ruled out, but only when the deviant remains a net-buyer after deviating to a lower price. To see this, note that the quantity sold by the deviant after a price reduction would at least be equal to $q_i^* - x_i + q_j^*$,[32] so that a firm with $x_j \leq q_i^* - x_i + q_j^*$ would become a net-seller and hence would no longer benefit from lowering the price. Similar arguments explain why an equilibrium with $p^* \leq p^c$ requires $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ to hold for all $j \neq i$ with $p_j^{PS}(b_{-j}) < p^*$, but also for all $j \neq i$ satisfying $x_i + x_j < q_i^* + q_j^*$.

Fourth, multiplicity of equilibrium outcomes pervades the irregular cases. As in the regular cases, some of this multiplicity can be avoided by appealing to Pareto dominance arguments. In example above, both b and b' are Nash equilibria in which firm 1 is a price-setter. But profile b (which generated a Nash equilibrium in which Proposition 1 failed to hold) is Pareto dominated by profile b', which satisfies Proposition 1. The net-buyer (firm 2) is trivially better-off under b' than under b as both generate the same stop-out price while firm 2's productive losses are lower. In turn, since this implies that the net-seller (firm 1) produces more, firm 1 is also better-off under b' as compared to b. Nevertheless, there is still some remaining multiplicity among equilibria which cannot be Pareto ranked (as we illustrate below by means of an example).

Finally, there is one last feature that also distinguishes the irregular from the regular cases. Namely, in contrast to Proposition 3, equilibria in which the non-price-setters bid at marginal costs may fail to exist under the irregular cases, given that marginal cost bidding exacerbates firms' conflict of interests. As compared to the case in which the non-price-setters bid above marginal costs, marginal cost bidding by the non-price-setters makes it easier for an over-contracted firm to

[32] At prices below p^*, firm i would at most sell x_i. Hence, in order to become a price-setter at a lower price, firm j would have to at least produce the quantity it was producing before the deviation, q_j^*, plus the output that firm i is no longer producing, which is at least equal to $q_i^* - x_i$.

set a low price: it reduces the amount of output that it has to bid below marginal costs in order to drive the stop-out price below the competitive one, rendering such a deviation more attractive. Since firms with fewer contracts may find it profitable to set a much higher price, an equilibrium in which the non-price-setters bid at marginal costs may thus fail to exist.[33]

To illustrate some of the features discussed above, we conclude this section by extending the illustrative example presented in Section 3 (two symmetric firms with costs $c_n = \{(0,1), (1,2), (2,3)\}$, $n = 1, 2$, facing inelastic demand, $D = 3$). Since $q_n^c = 1.5$, let $x_1 = 0 < x_2 \in (2,3]$ to allow for a (relevant) irregular case to emerge.[34] Focusing on Pareto undominated Nash equilibria, the following bid profile generates a continuum of equilibria parametrized by the stop-out price, which is determined by the first (second) step in firm 1's (firm 2's) bid function. More precisely, we claim that the following bid profile constitutes a Nash equilibrium,

$$b_1 = \left\{(z,1), \left(p^R, 3\right)\right\} \text{ and } b_2 = \{(0,2), (z, x_2), (2,3)\},$$

where $z \in [2(3 - x_2), 2]$. Since $D = 3$, the resulting outcome is $q_1^* = 1 > x_1$, $q_2^* = 2 < x_2$ and $p^* = z \in [2(3 - x_2), 2]$; firm 1 is net-seller and price-setter, and firm 2 is net-buyer and non-price-setter (note that firm 2 is marginal as it is bidding one step at z, but it is not dispatching it). Equilibrium profits are

$$\pi_1^{PS}(b) = z \text{ and } \pi_2^{NPS}(b) = z[2 - x_2] - 1.$$

To check that firm 1 is best-responding to b_2, note that for any strategy b_1' resulting in $p^* \in [0, z]$, it profits are $p^* \leq \pi_1^{PS}(b) = z$; alternatively, if b_1' results in $p^* > z$, firm 1's profits are bounded below by $2[3 - x_2] \leq \pi_1^{PS}(b) = z$. Consider firm 2 now. For any strategy b_2' resulting in $p^* < z$, firm 2's profits are $p^*[3 - x_2] - 3$; since firm 2 would become a net-seller, $3 - x_2 \geq 0$, it does not gain by lowering the price, i.e., $p^*[3 - x_2] - 3 \leq z[3 - x_2] - 3 \leq \pi_2^{NPS}(b) = z[2 - x_2] - 1$, with strict inequality if $z < 2$. If b_2' results in $p^* = z$, firm 2's production and profits are identical to those under b_2. Finally, if b_2' results in $p^* > z$, the losses on its net-position increase, thus rendering it unprofitable.

The equilibria constructed above yield a continuum of prices $p^* = z \in [2(3 - x_2), 2]$. They are thus not outcome equivalent despite the fact that firm 1 is the price-setter under all of them. They arise because firm 2, which is a net-buyer, strategically bids at z so as to 'cap' the price set by firm 1, by offering some of its output below marginal costs. These equilibria might result in prices below, at or above the competitive price, and might also involve productive inefficiencies by the non-price-setter (i.e., there is a second source of productive inefficiency on top of the price-setter withholding profitable production). Such multiplicity of equilibrium outcomes, which is reminiscent of Wilson's results, only arises in the presence of forward contracts.

Interestingly enough, the solution is continuous in the amount of contracts held by firm 2, which enlarge the set of equilibrium prices: when $x_2 \to 2$, $p^* = 2$ is the unique equilibrium price (which

[33] Since there is complete information, Equilibrium existence can nevertheless be guaranteed .by appealing to Reny's better reply security (see Corollary 5.2 to Theorem 3.1 in Reny (1999)): due to the efficient tie-breaking rule, bidders' payoffs are secure and their sum is upper semi-continuous, so that an equilibrium always exists. Note further that existence of a pure-strategy equilibrium follows from Corollary 14 to Theorem 6 in Jackson and Swinkels (2005) as the multi-unit uniform-price auction analyzed here satisfies assumption 8' in their paper (see page 122).

[34] Even though $x_2 \in [1.5, 2]$ would also belong to the irregular cases, in Section 3 we already showed that at the unique equilibrium outcome, both firms' net-positions have the same sign and hence face no conflict of interests.

coincides with the one reported in Section 3 for $x_2 \leq 2$), while when $x_2 = 3$, any price $p^* \in [0, 2]$ can be sustained in equilibrium. Last, one cannot derive a general Pareto ranking among the multiple equilibria: the price-setter is strictly better-off at the equilibrium with the highest price (whose outcome is the same as when the non-price-setter bids at marginal costs); however, that is precisely the worst equilibrium from the non-price-setter's point of view.

To conclude, whereas under the regular cases any potential multiplicity of equilibrium outcomes must derive from the coexistence of equilibria in which different firms act as price-setters, the irregular cases lead to a continuum of equilibria even when conditioning on the identity of the price-setter. Some of these equilibria might involve allocative inefficiencies due to either monopoly or monopsony power, but they might also lead to productive inefficiencies that cannot be ruled by Pareto dominance. From a policy point of view, the analysis of the irregular cases again points out at a similar conclusion as the one above, but for a different reason: *more is not always better* since an increase in contract volume widens up the range of inefficiently low price equilibria. Furthermore, an increase in some firms' forward contracts commitments above their competitive output implies that the regulator loses all control as to which equilibria will be played, therefore running the risk that a particularly welfare detrimental equilibrium will be chosen.

7 Simulating the Impact of Forward Contracts

We next apply the theoretical model to simulate equilibrium bidding behavior and market outcomes in the Spanish electricity market during 2005. The aim is to illustrate with real data the strategic effects of contracts that we have described in Section 5. Appendix B contains details on the Spanish electricity market as well as on the procedures we have followed to compute firms' marginal costs.

We have considered alternative scenarios regarding total contract volume and its distribution across firms. In particular, focusing on the equilibria in which only the two main firms (Endesa and Iberdrola) are price-setters, we have computed both the competitive as well as the equilibrium market outcomes under the no-contracts case and the cases in which either Endesa (END) or Iberdrola (IB) hold contracts, ranging from 1 to 8 GWs.[35,36]

Table 2 reports the markups that result from comparing the simulated equilibrium price to the price that would arise in a competitive market (as suggested in Borenstein *et al.* (2002)).[37] Markups are computed at four demand levels (expressed in percentiles), under the no-contracts case and under the cases in which Endesa has contracted either 2 or 5 GWs, and Iberdrola has contracted either 6 or 8 GWs (results for all other cases are qualitatively similar). By comparing the markups across

[35] Since the simulations are conducted on an hourly basis over a year, there are at least 8,760 and at most (if both firms act as price-setters) 17,520 equilibrium market outcomes under each of the 17 cases considered, plus the 8,760 competitive outcomes (these are the same regardless of whether firms hold contracts or not)- adding up to over 300,000 simulated market outcomes in total. Simulations have been produced by ENERGEIA, a simulation software developed by the authors.

[36] To have an idea of what this range of contract cover meant for firms over their total capacity, let us note that Endesa's and Iberdrola's total capacity in 2005 was almost 11 GWs and 8.5 GWs, respectively. Table 4 in Appendix B provides information on firms' total capacity and technology mix.

[37] We have chosen to report these markups rather than prices for clarity. Nevertheless, note that both markups and prices illustrate identical results to the extent that the competitive price is the same regardless of which firm sets the price and regardless of the level of contracting.

	No Contracts		END 2 GWs		END 5 GWs		IB 6 GWs		IB 8 GWs	
Price-setter	IB	END	IB	END	IB	END	IB	END	IB	END
Peak load	50.0%	50.0%	50.0%	50.0%	50.0%	*–	50.0%	50.0%	*–	50.0%
75%	11.2%	15.0%	11.2%	*11.6%	11.2%	*–	*–	15.0%	*–	15.0%
50%	–	15.9%	*5.2%	*10.7%	*5.2%	*–	–	15.9%	–	15.9%
25%	23.4%	23.6%	23.4%	23.6%	23.4%	*–	23.4%	23.6%	23.4%	23.6%

Table 2: The impact of forward contracts on markups $\frac{p^{*}-p^{c}}{p^{*}}$ in the Spanish electricity market during 2005

Note on Table 2: The table reports the simulated mark-ups $\frac{p^{*}-p^{c}}{p^{*}}$ for four demand levels (the year's peak load, and the 75%, 50% and 25% demand percentiles). The results are divided in columns, depending on the identity of the price-setter. A table entry is left empty if, for the associated demand level and contract volumes, there is not an equilibrium in which such a firm behaves as price-setter. An asterisk denotes that the equilibrium has changed with respect to the no-contracts case.

firms at the no-contracts case (first two columns in Table 2), we can readily verify that Endesa's profit-maximizing price exceeds that of Iberdrola's for all demand levels considered, except for peak load, at which both profit-maximizing prices coincide.

Let us first consider the effects of contracts when awarded to the firm with the high profit-maximizing price, Endesa. First, contracts may reduce Endesa's profit-maximizing price as a price-setter; this is, for instance, the case when Endesa contracts 2 GWs and demand is at its 75% or 50% percentile. Second, contracts may give rise to a new equilibrium in which Iberdrola sets a lower price; this is the case when Endesa contracts either 2 or 5 GWs and demand is at its 50% percentile. Last, contracts may eliminate certain equilibria at which Endesa sets the price; this is the case when Endesa contracts 5 GWs for all demand levels. Therefore, contracts by Endesa have (weakly) pro-competitive effects.

However, such a conclusion is reversed when contracts are awarded to the firm with the low profit-maximizing price, Iberdrola. More specifically, contracts by Iberdrola have (weakly) anti-competitive effects when they destroy the low-price equilibrium outcomes. This is the case when Iberdrola contracts either 6 or 8 GWs and demand is at its 75% percentile.

The effects of contracts reported so far vary with the demand level. For example, whereas at very high or very low demand levels contracts barely have any effect on equilibrium outcomes, their effect for intermediate demand levels can go in either direction depending on contract volume and contract allocation. In real markets, since demand changes over time while contract volumes remain fixed, the overall effect of contracts will depend on the relative occurrence of periods in which contracts are either pro-competitive or anti-competitive. Therefore, with illustrative purposes, we have assessed the effect that contracts would have had on the Spanish electricity prices during 2005 by computing total payments to producers over the year.

Table 3 reports the change in total payments when contracts are introduced. Given that there may be multiplicity of equilibrium outcomes depending on which firm sets the price, we have reported

Contracts by ENDESA (GWs)	Min	Max
	Δ Payments	
1	-84	-107
2	-143	-194
3	-377	-410
4	-457	-577
5	-439	-608
6	-456	-632
7	-548	-639
8	-709	-654

Contracts by IBERDROLA (GWs)	Min	Max
	Δ Payments	
1	-24	-78
2	-54	-161
3	-88	-222
4	-117	-280
5	-181	-379
6	-200	-434
7	-169	-437
8	-171	-437

Table 3: The impact of forward contracts on total payments to producers (Million €) for the Spanish electricity market during 2005

Note on Table 3: Total payments to producers under the competitive outcome are 9,599 M€; the minimum value under the no-contracts case is 11,422 M€, while the maximum is 11,728 M€. The table reports how these figures change when forward contracts are introduced. Given that there might be multiplicity of equilibrium outcomes, the Min and the Max columns report the minimum and maximum change in total payments.

the minimum and the maximum change in payments. Under all contract cases, total payments to generators go down, thereby indicating that the pro-competitive effects of contracts seem to dominate over the anti-competitive ones. However, the anti-competitive effects can also be inferred from these figures as they account for the non-monotonic relationship between payments to producers and total contract volume. For instance, such non-monotonicity arises when Iberdrola's contracts are increased above 6 GWs, when savings are reduced from 200 M€ to either 169 M€ or 171 M€.

8 Conclusions

In this paper we have analyzed the impact of forward contract commitments (or more generally, the impact of exogenous vertical commitments) on the performance of spot markets in a model that tries to capture the essential institutional and structural features of electricity markets. Instead of assuming either Cournot or Bertrand competition, we have tried to model the actual market rules that govern most electricity markets in practice. In particular, we have assumed that firms compete by submitting discrete supply functions. Furthermore, we have put no restrictions on the market demand function - which could be either downward-sloping or price-inelastic,- or the firms' cost functions - which could result in either constant or step-wise increasing marginal costs, and could be symmetric or asymmetric across firms. Thus, the model is flexible enough so as to make it comparable with other more stylized models, at the same time as it allows for all degrees of complexity. Indeed, we have used it to simulate real electricity market outcomes in order to provide an order of magnitude for the model predictions.

We find that forward contracts play a key role in shaping equilibrium market outcomes. To start with, forward contracts determine the set of weakly-dominated strategies, thus potentially ruling out equilibria that would exist without contracts, or giving rise to new equilibria that only exist with contracts. Indeed, we show that the effects of contracts on equilibrium existence are crucial. If contracts are awarded to firms with strong incentives to exercise market power (i.e., typically the large firms and/or those facing relatively inefficient rivals), forward contracts may destroy the equilibria at which such firms set prices. Since the surviving equilibria involve lower prices, forward contracts are unambiguously pro-competitive. However, the contrary occurs if contracts are awarded to firms with weak but yet some market power. In particular, contracts might destroy the low price equilibria, and hence have anti-competitive effects. Allocating contracts to firms with no market power has no effects on equilibrium existence, and it is thus irrelevant as far as market outcomes are concerned. The effects of contracts on equilibrium existence also suggest that *more is not always better*. That is, if an increase in contract volume destroys the equilibrium at which the contracted firm sets the price, more contracts might lead to higher prices.

From a policy perspective, our analysis thus implies that forward contracts should be awarded in ways that align all firms' interests by (virtually) reducing their asymmetries. Paradoxical though it may seem, it is as important to mitigate the large firms' incentives to increase prices as it is to enhance those of smaller competitors. This could be achieved by encouraging the medium to small firms in the industry to act as counterparts of the forward contract commitments imposed on the dominant producers. Similarly, restricting certain firms from entering into these contracts can be misplaced.[38] Regarding contract volume, forcing firms to hold too few or too many forward contracts might be at best ineffective. Since the optimal contract volume ultimately depends on firms' cost structures and demand, it should be determined on a case-by-case basis.

Extending these conclusions to the analysis of vertical integration would imply that vertical mergers involving large upstream competitors would be pro-competitive, while vertical mergers involving smaller upstream firms might be anti-competitive, particularly so when the merging party is a large downstream firm.

We have focused on exogenously given contracts since we believe, in line with Bushnell *et al.* (2008), that many "*vertical arrangements [in electricity markets] are better understood and can reasonably be considered to be exogenous.*" Still, a further step of the analysis would be to allow for more general types of contracts by investigating the incentives to sign new contracts and hence their endogenous distribution across firms. The current paper provides the needed first step to perform such an analysis.

To conclude, even though our analysis has been inspired by the workings of electricity markets, we believe that its implications have broader applicability. Since the most relevant features of our model are not unique to electricity markets, similar analyses could be applied to other contexts. Indeed, there are several other markets in which forward contracts and auctions coexist (e.g. Treasury markets, gas markets, etc.), or markets which are organized in ways that make auction theory useful for understanding firms' strategic behavior (Klemperer (2003)).

[38] For instance, EDP and Unión Fenosa are not allowed to participate in the Spanish VPPs. Similarly, Electrabel and Essent were not initially allowed to participate in the VPPs in the Netherlands (Essent objected to being excluded from the auction and the court finally allowed it to participate).

References

[1] Allaz, B. and J.-L. Vila (1993) "Cournot Competition, Forwards Markets and Efficiency," *Journal of Economic Theory* 59, 1-16.

[2] Ausubel, L. M. and P. Cramton (2002) "Demand Reduction and Inefficiency in Multi-Unit Auctions," University of Maryland, Working Paper 96-07.

[3] Back, K. and J. Zender (1993) "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment," *Review of Financial Studies* 6 (4), 733-764.

[4] Bernheim, B.D., B. Peleg, and M.D. Whinston (1987) "Coalition-Proof Nash Equilibria I. Concepts," *Journal of Economic Theory* 42 (1), 1-12.

[5] Borenstein, S. (1996) "Settling for Coupons: Discount Contracts as Compensation and Punishment in Antitrust Lawsuits," *Journal of Law and Economics* 39 (2), 379-404.

[6] Bushnell, J. (2003) "A Mixed Complementarity Model of Hydro-Thermal Electricity Competition in the Western U.S.," *Operations Research* 51 (1), 81-93.

[7] Bushnell, J. (2007) "Oligopoly Equilibria in Electricity Contract Markets," *Journal of Regulatory Economics* 32 (3), 225-245.

[8] Bushnell, J., E. Mansur and C. Saravia (2008) "Vertical Arrangements, Market Structure and Competition: An analysis of Restructured U.S. Electricity Markets," *American Economic Review*, 98 (1), 237-266.

[9] Crampes, C. and N. Fabra (2005) "The Spanish Electricity Market: Plus Ça Change...," *Energy Journal* 26, 117-144.

[10] Crawford, G. S., J. Crespo and H. V. Tauchen (2007) "Bidding Asymmetries in Multi-Unit Auctions: Implications of Bid Function Equilibria in the British Spot Market for Electricity," *International Journal of Industrial Organization*, 25 (6), 1233-1268.

[11] Fabra, N., N.-H. von der Fehr and D. Harbord (2006) "Designing Electricity Auctions," *Rand Journal of Economics*, 37 (1), 23-46.

[12] Fabra, N. and J. Toro (2005) "Price Wars and Collusion in the Spanish Electricity Market," *International Journal of Industrial Organization* 23 (3-4), 155-181.

[13] Federico, G. and A. López (2009) "Divesting Power," Mimeo, IESE Business School.

[14] von der Fehr, N.-H. M. and D. Harbord (1993) "Spot Market Competition in the UK Electricity Industry," *Economic Journal* 103, 531-546.

[15] Ferreira, J. L. (2003) "Strategic Interaction between Futures and Spot Markets," *Journal of Economic Theory* 108, 141-151.

[16] García-Díaz, A. and P. Marín (2003) "Strategic Bidding in Electricity Pools with Short-Lived Bids: An Application to the Spanish Market," *International Journal of Industrial Organization* 21, 201-222.

[17] Green R. J. (1999) "The Electricity Contract Market in England and Wales," *Journal of Industrial Economics* 47 (1), 107-124.

[18] Green R. J. and C. Le Coq (2006) "The Length of the Contract and Collusion," CSEM Working Paper 154, University of California, Berkeley.

[19] Green, R. J. and D. M. Newbery (1992) "Competition in the British Electricity Spot Market," *Journal of Political Economy* 100 (5), 929-53.

[20] Hortacsu, A. and S. Puller (2008) "Understanding Strategic Bidding in Multi-Unit Auctions: A Case Study of the Texas Electricity Spot Market," *RAND Journal of Economics*, 39 (1), 86-114.

[21] Jackson, M. and J. Swinkels (2005) "Existence of Equilibrium in Single and Double Private Value Auctions," *Econometrica* 73 (1), 93-139.

[22] Kastl, J. (2008) "On the Properties of Equilibria in Private Value Divisible Good Auctions with Constrained Bidding," Mimeo, Stanford University.

[23] Klemperer, P. (2003) "Alfred Marshall Lecture: Using and Abusing Economic Theory," *Journal of the European Economic Association* 1 (2-3), 272-300.

[24] Klemperer, P. D. and M. A. Meyer (1989) "Supply Function Equilibria in Oligopoly Under Uncertainty," *Econometrica* 57 (6), 1243-77.

[25] Kremer, I. and K. Nyborg (2004) "Underpricing and Market Power in Uniform-Price Auctions," *Review of Financial Studies* 17, 849-877.

[26] Kühn, K.-U. and M. Machado (2006) "Bilateral Market Power and Vertical Integration in the Spanish Electricity Spot Market," CEPR Discussion Paper N. 4590.

[27] Liski M. and J.-P. Montero (2006) "Forward Trading and Collusion in Oligopoly," *Journal of Economic Theory* 131, 212-230.

[28] Mahenc, P. and F. Salanié (2004) "Softening Competition through Forward Trading," *Journal of Economic Theory* 116, 282-293.

[29] Mansur, E. T. (2007). "Upstream Competition and Vertical Integration in Electricity Markets," *Journal of Law and Economics* 50 (1), 125-156.

[30] Newbery, D. M. (1998) "Competition, Contracts, and Entry in the Electricity Spot Market," *RAND Journal of Economics* 29 (4), 726-749.

[31] Nyborg, K. G. and Strebulaev (2004) "Multiple Unit Auctions and Short Squeezes," *The Review of Financial Studies* 17 (2), 545-580.

[32] Reny, P. J. (1999) "On the Existence of Pure and Mixed Strategy Nash Equilibria in Discontinuous Games," *Econometrica* 67 (5), 1029-1056.

[33] Schultz, C. (2007) "Virtual Capacity and Competition," Mimeo, University of Copenhagen.

[34] Wilson, R. (1979) "Auctions of Shares," *Quarterly Journal of Economics* 93 (4), 675-689.

[35] Wolak, F. A. (2000) "An Empirical Analysis of the Impact of Hedge Contracts on Bidding Behavior in a Competitive Electricity Market," *International Economic Journal* 14 (2), 1-39.

[36] Wolak, F. A. (2007a) "Quantifying the Supply-Side Benefits from Forward Contracting in Wholesale Electricity Markets," *Journal of Applied Econometrics* 22, 1179-1209.

[37] Wolak, F. A. (2007b) "Regulating Competition in Wholesale Electricity Supply," mimeo.

[38] Wolfram, C. D. (1999) "Measuring Duopoly Power in the British Electricity Spot Market," *American Economic Review* 89 (4), 805-826.

Appendix

Notation

The following pieces of notation are used throughout the Appendix. We will denote by $\underline{q}_n(p)$ the maximum quantity that firm n can produce at marginal costs *strictly below* p, and by $\bar{q}_n(p)$ the maximum quantity that firm n can produce at marginal costs *not exceeding* p. Formally,

$$\begin{aligned}\underline{q}_n(p) &= \max\{q : q \in [0, K_n] \text{ } and \text{ } c_n(q) < p\}, \text{ and} \\ \bar{q}_n(p) &= \max\{q : q \in [0, K_i] \text{ } and \text{ } c_n(q) \leq p\}.\end{aligned}$$

Since the marginal cost curve, $c_n(q)$, is a left-continuous non-decreasing step function, by treating all production units with equal marginal costs as the same unit we can write it as a finite number of cost-quantity pairs, $c_n = \left\{(c_{ns}, q_{ns})_{s=1}^{\tilde{s}_n}\right\}$ with $c_{ns+1} > c_{ns}$, $q_{ns+1} > q_{ns}$ and $q_{n\tilde{s}_n} = K_n$. Since $\underline{q}_n(p)$ and $\bar{q}_n(p)$ are on the corners of firm n's marginal cost function, the following properties trivially follow:

(i) $\underline{q}_n(p)$ *and* $\bar{q}_n(p)$ *are non-decreasing in* p.
(ii) $\underline{q}_n(p) = \bar{q}_n(p) = q_{ns}$ *for all* $p \in (c_{ns}, c_{ns+1}]$.
(iii) $\underline{q}_n(p) = q_{ns-1} < \bar{q}_n(p) = q_{ns}$ *for* $p = c_{ns}$.
(iv) $\underline{q}_n(p') \geq \bar{q}_n(p)$ *for all* $p' > p$.

Since marginal cost functions are non-decreasing, the maximum quantities that a firm can produce either below or at marginal costs are non-decreasing in p, (i). If p does not intersect the firm's marginal cost function (or equivalently, if it falls on a vertical segment), then $\underline{q}_n(p)$ and $\bar{q}_n(p)$ are equal, (ii). Otherwise, $\underline{q}_n(p) < \bar{q}_n(p)$, with p reflecting the marginal costs at which the firm produces the quantities in $(q_{ns-1}, q_{ns}]$, (iii). Last, when comparing $\underline{q}_n(p')$ and $\bar{q}_n(p)$ at different prices $p' > p$, note that $\underline{q}_n(p')$ exceeds $\bar{q}_n(p)$ whenever there is a step in firm n's marginal cost function in between p' and p, and they are both equal otherwise, (iv). These results are illustrated in Figure 5.

[INSERT FIGURE 5 AROUND HERE]

A Proofs

Proof of Lemma 1. (i) Let us fix the bid functions submitted by firms other than n at b_{-n}, and assume firm n follows the strategy $\hat{b}_n$, under which there is at least one step to the right of x_n at prices below marginal costs. For the sake of exposition, but without any loss of generality, let us assume that this happens at only one step $\tilde{s}$ such that $x_n \leq q_{n\tilde{s}-1}$. Next, we show that $\hat{b}_n$ is weakly-dominated by the strategy b'_n, which replicates $\hat{b}_n$ except for quantities in $(q_{n\tilde{s}-1}, q_{n\tilde{s}}]$ as these are now offered at marginal costs. Let $\hat{b}$ (respectively, b') denote the bid profile $\left(\hat{b}_n, b_{-n}\right)$ (respectively, (b'_n, b_{-n})). We first note that the equilibrium price under $\hat{b}$ is no larger than under b' as $\hat{b}_n \leq b'_n$ while b_{-n} is the same under both profiles. Consequently, $\hat{p} \leq p'$.[39] Trivially, if either $\hat{p} = p' \leq \hat{p}_{n\tilde{s}-1}$ or $\hat{p} = p' \geq \hat{p}_{n\tilde{s}+1}$ hold, the two strategies give firm n the same profits. We focus next in the remaining possibilities. If demand is price-inelastic, we must have $\hat{p} < p'$ and $\hat{q}_n = q'_n > x_n$; if demand is downward-sloping, we have $\hat{p} \leq p'$ and $\hat{q}_n > q'_n > x_n$. Profits at the two profiles are given by

$$\begin{aligned} \pi_n\left(b'\right) &= \left[q'_n - x_n\right] p' - C_n\left(q'_n\right) + \tau_n x_n \text{ and} \\ \pi_n\left(\hat{b}\right) &= \left[\hat{q}_n - x_n\right] \hat{p} - C_n\left(\hat{q}_n\right) + \tau_n x_n. \end{aligned}$$

Thus,

$$\pi_n\left(b'\right) - \pi_n\left(\hat{b}\right) = \left[q'_n - x_n\right]\left[p' - \hat{p}\right] - \int_{q'_n}^{\hat{q}_n} \left[\hat{p} - c_n(z)\right] dz > 0,$$

as either $\hat{q}_n = q'_n$ so that the integral is zero while $\left[q'_n - x_n\right]\left[p' - \hat{p}\right] > 0$; or $q'_n < \hat{q}_n$ so that $\int_{q'_n}^{\hat{q}_n} \left[\hat{p} - c_n(z)\right] dz < 0$ as $\hat{p} < c_n(q)$ for all $q > x_n$ and hence for $q \in (q'_n, \hat{q}_n]$ while $\left[q'_n - x_n\right]\left[p' - \hat{p}\right] \geq 0$. Since profits under b' are no smaller than under $\hat{b}$, the statement follows.

(ii) Let us fix the bid functions submitted by firms other than n at b_{-n}, and assume that firm n follows the strategy $\hat{b}_n$, under which there is at least one step to the left of x_n at prices above marginal costs. Compare the profits made by firm n under $\hat{b}_n$ and b'_n, with the latter constructed as above, i.e., it replicates $\hat{b}_n$ except for quantities in $(q_{n\tilde{s}-1}, q_{n\tilde{s}}]$, with $q_{n\tilde{s}} \leq x_n$, as these are now offered at marginal costs. The equilibrium price under $\hat{b}$ is no smaller than under b' as $\hat{b}_n \geq b'_n$ while b_{-n} is the same under both profiles. Consequently, $\hat{p} \geq p'$. If $\hat{p} = p' \leq \hat{p}_{n\tilde{s}-1}$ or $\hat{p} = p' \geq \hat{p}_{n\tilde{s}+1}$ holds, the two strategies give firm n the same profits. Since $\hat{p} = p' = \hat{p}_{n\tilde{s}}$ cannot hold, the only remaining possibility is $\hat{p} = \hat{p}_{n\tilde{s}} > p'$ so that $x_n \geq q'_n > \hat{q}_n$ if demand is downward-sloping and $x_n \geq q'_n = \hat{q}_n$ if demand is price-inelastic. Thus, unless demand is inelastic and it happens to be such that $x_n = q'_n = \hat{q}_n$ so that profits under the two strategies are identical, the difference in profits at the two profiles is

$$\pi_n\left(b'\right) - \pi_n\left(\hat{b}\right) = \left[x_n - \hat{q}_n\right]\left[\hat{p} - p'\right] + \int_{\hat{q}_n}^{q'_n} \left[p' - c_n(z)\right] dz > 0,$$

as either the integral is strictly positive or the second term, $\left[x_n - \hat{q}_n\right]\left[\hat{p} - p'\right]$, is strictly positive. Since profits under b' are no smaller than under $\hat{b}$, the statement follows. ■

[39] There is a knife-edge case with downward-sloping demand in which $\hat{p} > p'$. This only occurs if demand cuts aggregate supply generated by $\hat{b}$ at $\hat{p}_n$ but does not intersect aggregate supply generated by b', so that the price falls down to the price at the previous step. In this case, however, the strategy $\hat{b}_n$ is weakly dominated by one that moves the price for quantities in $(x_n, \hat{q}_{n\tilde{s}-1})$ above marginal costs so that there is market clearing and hence a new price that exceeds $\hat{p}$, as claimed.

Proof of Lemma 2. Since we must show $q_j^* \geq \underline{q}_j(p^*)$, assume by contradiction that there is an equilibrium at which some firm j, $j \neq i$, for which $q_j^* < \underline{q}_j(p^*)$ holds. As there is a finite number of bid-points, there exists $\hat{\epsilon} > 0$ such that there are no price-bids in the interval $(p^* - \hat{\epsilon}, p^*)$. Pick an $\epsilon < \hat{\epsilon}$ and consider the deviation by firm j of moving the original bid for quantities in $\left(q_j^*, \underline{q}_j(p^*)\right]$ down to $p^* - \epsilon$. Let the resulting bid profile, denoted by $b' = \left\{b_1, .., b_j', .., b_N\right\}$, generate a stop-out price p', and a quantity q_j' for firm j. Since $q_j' > q_j^*$ (the deviant's quantity replaces part of firm i's marginal output), the difference in firm j's profits is given by:

$$\pi_j\left(b'\right) - \pi_j(b) = \left[p' - p^*\right]\left[q_j' - x_j\right] + \int_{q_j^*}^{q_j'} \left[p^* - c_j(z)\right] dz.$$

There are two cases to consider: (a) If $p' = p^* - \epsilon$, then the difference in profits is positive: the second term of the above equation is positive by definition of $\underline{q}_j(p^*)$ as $p^* > c_j(q)$ for any positive $q < q_j' \leq \underline{q}_j(p^*)$, whereas the first term can be made arbitrarily small by taking ϵ small enough. (b) If $p' = p^*$, the deviant now sells the additional output $\underline{q}_j(p^*) - q_j^*$ at a price above its marginal costs and thus gets more profits. Since the deviation is profitable, we reach a contradiction which proves the result. ■

Proof of Lemma 3. To show $q_j^* \leq \bar{q}_j(p^*)$ for all $j \neq i$, assume for the sake of contradiction that there is some firm j for which $q_j^* > \bar{q}_j(p^*)$ holds in equilibrium. By weak-dominance arguments, it must be the case that $q_j^* \leq x_j$ as it could not otherwise bid below marginal costs to dispatch $q_j^* > \bar{q}_j(p^*)$. It hence follows that $q_j^* > x_j$ suffices for $q_j^* \leq \bar{q}_j(p^*)$ to hold. To show that $q_j^* = x_j$ or $q_k^* < q_{ks}$ for some marginal firm k, $k \neq j$, are also sufficient conditions to ensure $q_j^* \leq \bar{q}_j(p^*)$, let $\bar{q}_j(p^*) < q_j^* \leq x_j$. Consider the deviation by firm j of moving the original bid(s) for quantities in $\left(A, q_j^*\right]$ above p^* (e.g. firm j could bid such quantities at marginal costs). The difference in firm j's profits is given by:

$$\pi_j\left(b'\right) - \pi_j(b) = \left[p' - p^*\right]\left[q_j' - x_j\right] + \int_{q_j'}^{q_j^*} \left[c_j(z) - p^*\right] dz. \tag{5}$$

If $q_{ks} < q_k^*$ let $A = q_j^* - \left[q_{ks} - q_k^*\right]$. At the resulting bid function profile b' the stop-out price remains p^* as j's dispatched output under b is replaced by firm k, so that $q_j' < q_j^*$. Consequently, the first term in equation (5) is zero while the second is strictly positive (the deviant now reduces its output and therefore its losses), so that $\pi_j(b') > \pi_j(b)$. Since the deviation is profitable, we have reached a contradiction as desired.

If $q_{ks} = q_k^*$ for all marginal firms k, $k \neq j$, but $q_j^* = x_j$ let $A = q_j^* - \bar{q}_j(p^*)$. Now, firm j's deviation implies a price increase, $p' > p^*$, thus implying that the first term in equation (5) may be negative. However, as we can rewrite equation (5) as the sum of two integrals, recalling that $q_j^* = x_j$, it follows that

$$\pi_j\left(b'\right) - \pi_j(b) = \int_{x_j}^{q_j'} \left[p' - p^*\right] dz + \int_{q_j'}^{x_j} \left[c_j(z) - p^*\right] dz = \int_{q_j'}^{x_j} \left[c_j(z) - p'\right] dz > 0,$$

since $c_j(z) > p'$ for all $z \in \left(\bar{q}_j(p^*), q_j^*\right]$ and hence for $z \in \left[q_j', x_j\right]$. The deviation is again profitable, reaching a contradiction. ■

Proof of Proposition 1. We must show that $q_j^* \in \left[\underline{q}_j(p^*), \bar{q}_j(p^*)\right]$ for any non-price-setter firm j. By appealing to Lemma 2 it follows that $q_j^* \geq \underline{q}_j(p^*)$ holds. Similarly, if $q_j^* > x_j$ or if $q_k^* < q_{ks}$ for some marginal firm k, $k \neq j$, then $q_j^* \leq \bar{q}_j(p^*)$ follows from Lemma 3. Thus $q_j^* \in \left[\underline{q}_j(p^*), \bar{q}_j(p^*)\right]$ as claimed. ■

Proof of Proposition 2. (i) We first show that if $D(p^c) \leq \min_i \sum_{j \neq i} \bar{q}_j(p^c)$ holds, then at any Nash equilibrium $p^* = p^c$. Assume, by contradiction, that $p^* > p^c$. Since p^* must be set by at least one firm, assume that firm i is a price-setter. By Lemma 2, $\sum_{j \neq i} q_j^* \geq \sum_{j \neq i} \underline{q}_j(p^*)$, so that $q_i^* = D(p^*) - \sum_{j \neq i} q_j^* \leq D(p^*) - \sum_{j \neq i} \underline{q}_j(p^*)$. Furthermore, since $\sum_{j \neq i} \underline{q}_j(p^*) \geq \sum_{j \neq i} \bar{q}_j(p^c)$, then $q_i^* \leq D(p^c) - \sum_{j \neq i} \bar{q}_j(p^c) \leq 0$, contradicting that firm i is a price-setter. Since $p^* = p^c$, it follows that $q_n^* = q_n^c$ for all n, by Proposition 1.

(ii) [*Only if*] Assume, by contradiction, that there is a unique price-setter, while $p^* = p^c$ and $q_n^* = q_n^c$ for all n hold. Note that by definition of p^c, it must be the case that $p^c \geq c_n(q_n^c)$. For all the firms that are not marginal such a condition is satisfied with strict inequality, $p^c > c_n(q_n^c)$, so that $q_n^c = \bar{q}_n(p^c)$. Moreover, there must be at least one marginal firm for which such a condition is satisfied with equality, $p^c = c_n(q_n^c)$, so that $q_n^c < \bar{q}_n(p^c)$ as we have ruled out the cases under which there is no supply rationing at the competitive outcome. Two possibilities can emerge.

1. Firm i is the unique marginal firm. Hence, $p^c = c_i(q_i^c)$. Firm i can profitably deviate by bidding its marginal output up to $p^c + \epsilon$, for ϵ small enough so that there are no other bids in $(p^c, p^c + \epsilon)$. Such a deviation is trivially profitable, $p^c = c_i(q_i^c)$ implies that it was making no profit out of the marginal output.

2. Both firms i and j are marginal. We cannot have $p^c = c_j\left(q_j^c\right) = c_i(q_i^c)$, as both firms would be partly dispatching their marginal steps, which contradicts the fact that there is only one price-setter. Hence, let $c_j(q_j^c) < c_i(q_i^c)$. If $p^c = c_j(q_j^c) < c_i(q_i^c)$, firm i would be selling q_i^c at a price below marginal costs, which is ruled out by weak-dominance. Alternatively, if $c_j(q_j^c) < p^c = c_i(q_i^c)$, both firms must be dispatching their marginal steps as demand would not otherwise be covered, $D(p^c) = \sum_{j \neq i} \bar{q}_j(p^c) + q_i^c$. The contradiction proves our claim.

[*If*] If there were more than one price-setter while $p^* > p^c$, then $q_n^* \geq \underline{q}_n(p^*)$ for all n must hold by Lemma 2. Since $\underline{q}_n(p^*) \geq \bar{q}_n(p^c) \geq q_n^c$, then $q_n^* \geq q_n^c$ for all n. If for at least one of them $q_n^* > q_n^c$ holds, then $D(p^*) \geq \sum_n q_n^* > \sum_n q_n^c = D(p^c)$, an impossibility. Consequently, $q_n^* = q_n^c$ for all n. However, $q_n^* = q_n^c$ and $p^* > p^c$ can only hold simultaneously when $q_n^* = \bar{q}_n(p^*) = \bar{q}_n(p^c) = q_n^c$ for all n, so that firms are not rationed at the competitive outcome. A possibility which has been nevertheless ruled out. ■

Proof of Lemma 4. To show (i), recall that $q_j^* \geq \underline{q}_j(p^*)$ holds for all $j \neq i$ by Proposition 1. Since $p^* \geq p^c$, then $\underline{q}_j(p^*) \geq \underline{q}_j(p^c)$ as $\underline{q}_j$ in a non-decreasing function of p. Consequently, $\pi_j^{NPS}(p^*)$ is an increasing function of p^* as $q_j^{NPS}(p^*) \geq q_j^c > x_j$.

(ii) Since $p^* \geq p^c$, then $q_j^{NPS}(p^*) \geq q_j^c \geq q_j^{PS}(p^*)$, with strict inequality if $p^* > p^c$. Thus,

$$\pi_j^{NPS}(p^*) - \pi_j^{PS}(p^*) = \int_{q_j^{PS}(p^*)}^{q_j^{NPS}(p^*)} [p^* - c_j(z)]\, dz \geq 0,$$

as $p^* \geq c_j(q_j^{NPS}(p^*))$. ■

Proof of Theorem 1. *[Only if]* Suppose that there exists a pure-strategy equilibrium b in which firm i sets the price at $p^* = p_i^{PS}(b_{-i}) \geq p^c$ and firms' payoffs are $\pi_i^{PS}(p^*)$ and $\pi_j^{NPS}(p^*)$, $i, j = 1, ...N$, $j \neq i$. If this is the case then Condition 1 follows from Proposition 1 and optimal behavior by the price-setter, and Conditions 2 and 3 follow trivially from the definition of Nash equilibrium.

[If] To show that no firm profits by deviating from strategies that satisfy Conditions 1 to 3, consider first the non-price-setters j, $j \neq i$. By Condition 1, they do not want to change their quantity given p^*. Thus, the only relevant deviations are those that allow the firm to become the price-setter at a price above p^*. Deviating to a price equal to or lower than p^* is not profitable as by Lemma 4, $\pi_j^{NPS}(p)$ is increasing in p and $\pi_j^{NPS}(p) \geq \pi_j^{PS}(p)$. Hence, those firms j with $p_j^{PS}(b_{-j}) \leq p^*$ will trivially not deviate. Those firms j with $p_j^{PS}(b_{-j}) > p^*$ will not deviate as $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^{PS})$ holds by Condition 3. Last, by Condition 1 the price-setter is already maximizing its profits over its residual demand, so that any deviation by firm i must imply becoming a non-price-setter at a lower price, $p < p^*$, while increasing its production to $q_i^{NPS}(p)$. Such deviation is not profitable by Condition 2. ■

Proof of Proposition 3.

We first prove part (ii). Let $\hat{b}$ and b' be two equilibrium bid profiles such that that under $\hat{b}$ all firms j bid at marginal costs (i.e., $\hat{b}_{-i} = c_{-i}$) while firm i sets the price at $p^* = \hat{p}$, whereas under b' at least one firm j, $j \neq i$ does not bid at marginal costs while firm i sets the price at $p^* = p'$. Trivially, if $\hat{p} = p'$ both equilibria are outcome-equivalent as prices are the same and firms' $j \neq i$ quantities must also coincide since they must satisfy Proposition 1. If $\hat{p} > p'$ then any non-price-setter prefers $\hat{b}$ to b' as shown in Lemma 4. This is also the case for the price-setter: if $p' = p^c$, the price-setter prefers $\hat{p}$ to p' by revealed preference, as it could have chosen to also bid at marginal costs to set the price at p^c, but it chose to set $\hat{p} > p^c$ instead**;** if $p' > p^c$, then

$$\pi_i^{PS}\left(\hat{p}; \hat{b}_{-i}\right) \geq \pi_i^{PS}\left(p'; \hat{b}_{-i}\right) = \pi_i^{PS}\left(p'; b'_{-i}\right),$$

where the first inequality follows from the fact that $\hat{p}$ is an equilibrium under $\hat{b}$ which requires that $\hat{p} \in \arg\max_p \pi_i^{PS}\left(p; \hat{b}_{-i}\right)$, and the second equality from the fact that p' is an equilibrium under b' so that Proposition 1 holds and hence $q_j^*\left(p'; b'_{-i}\right) = q_j^*\left(p'; \hat{b}_{-i}\right) = q_j^*\left(p'; c_{-i}\right)$ for all $j \neq i$ so that $q_i^*\left(p'; b'_{-i}\right) = q_i^*\left(p'; \hat{b}_{-i}\right)$. Since all firms are better-off at $\hat{b} = (\hat{b}_i, c_{-i})$, it is the Pareto-dominant one, as claimed.

If $\hat{p} < p'$, we show next that p' is also an equilibrium under $\hat{b}$. For the sake of contradiction assume it is not so that one of the three conditions in Theorem 1 must fail to hold. Since firms $j \neq i$ bid at marginal costs, they are trivially producing optimally conditionally on being non-price-setters; furthermore, given that the non-price-setters do not want to become the price-setter under $\hat{p}$, $\hat{p} < p'$, the same must hold true under p' so that Condition 3 of Theorem 1 is satisfied. As p' is an equilibrium under b' then for any $p \leq p'$ such that $\sum_n q_n(p; b'_{-i}) = D(p)$,

$$\pi_i^{PS}\left(p'; b'_{-i}\right) = \pi_i^{PS}\left(p'; \hat{b}_{-i}\right) \geq \pi_i^{NPS}\left(p; b'_{-i}\right) = \pi_i^{NPS}\left(p; \hat{b}_{-i}\right),$$

where the first equality and second inequality follow from the fact that p' is an equilibrium under b' (so that Conditions 1 and 2 in Theorem 1 hold) and the last equality from the fact that the non-price-setters' profits are independent of their rivals' strategies. Hence, since this implies that Condition 2 of Theorem 1 does also hold, it must then be the case that $p' \notin \arg\max \pi_i^{PS}\left(\hat{b}_{-i}\right)$, so that

$$\pi_i^{PS}\left(\hat{p};\hat{b}_{-i}\right) > \pi_i^{PS}\left(p';\hat{b}_{-i}\right) = \pi_i^{PS}\left(p';b'_{-i}\right) \geq \pi_i^{PS}\left(\hat{p};b'_{-i}\right).$$

where the last inequality from the fact that $p' \in \arg\max \pi_i^{PS}\left(b'_{-i}\right)$. Thus, $\pi_i^{PS}\left(\hat{p};\hat{b}_{-i}\right) > \pi_i^{PS}\left(\hat{p};b'_{-i}\right)$ or equivalently,

$$\int_{q_i\left(\hat{p};b'_{-i}\right)}^{q_i\left(\hat{p};\hat{b}_{-i}\right)} \left[\hat{p} - c_i(z)\right] dz > 0.$$

However, integral above cannot be positive. If $q_i\left(\hat{p};b'_{-i}\right) = q_i\left(\hat{p};\hat{b}_{-i}\right)$, integral above is zero. If $q_i\left(\hat{p};b'_{-i}\right) > q_i\left(\hat{p};\hat{b}_{-i}\right)$, because some bidder j bids units above marginal costs, then $\hat{p} \geq c_i(q_i\left(\hat{p};b'_{-i}\right))$ implies that the integral is negative. (c) Finally, if $q_i\left(\hat{p};b'_{-i}\right) < q_i\left(\hat{p};\hat{b}_{-i}\right)$ then some bidder j bids units below marginal costs at b'_{-i}. Since such units are dispatched under $\hat{b}_{-i}$ bidding them below marginal costs is ruled out by weak-dominance so that $q_i\left(\hat{p};b'_{-i}\right) < q_i\left(\hat{p};\hat{b}_{-i}\right)$ cannot hold in equilibrium. Since integral above cannot be positive we ran into a contradiction proving that $p' > \hat{p}$ must also be an equilibrium when firms j bid at marginal costs as it satisfies the three conditions in Theorem 1. Last, by the same arguments as above, the equilibrium with p' Pareto dominates the equilibrium with $\hat{p}$.

We now prove (i). The *[If]* part is trivial, so we omit it. *[Only If]* For the sake of contradiction, suppose that the equilibrium in which firm i is the price-setter at $p^* = p_i^{PS}\left(c_{-i}\right)$ while all other firms bid at marginal costs does not exist. This must be because Condition 3 fails to hold, given that Conditions 1 and 2 trivially hold. If there is multiple profit-maximizing prices any other price $p^* \in \arg\max \pi_i^{PS}\left(c_{-i}\right) < p_i^{PS}\left(c_{-i}\right)$ would also violate Condition 3, given that $p_i^{PS}\left(c_{-i}\right)$ is assumed to be the largest one. To show that there does not exist any other equilibrium in which firm i is the price-setter, argue by contradiction and suppose that some other bid profile b' constitutes an equilibrium. If $p' < p_i^{PS}\left(c_{-i}\right)$ then Condition 3 will again fail to hold contradicting that it constitutes an equilibrium, whereas if $p' > p_i^{PS}\left(c_{-i}\right)$ then p' must also be sustainable when the non-price-setters bid at marginal costs as shown in (ii). The contradiction proves the claim. ■

Proof of Corollary 1. For ease of exposition, let us index firms by their profit-maximizing prices when rivals bid at marginal costs, i.e., $p_n^{PS}\left(c_{-n}\right) \geq p_{n+1}^{PS}\left(c_{-n+1}\right)$. We prove this result by constructing a bid profile that always constitutes an equilibrium, in which firm 1 (i.e., the firm with the highest profit-maximizing price) is the price-setter. Assume that firms $j \neq 1$ bid at marginal costs while firm 1 uses strategy b_1 constructed as follows: any $q_1 < x_1$ is bid in at marginal cost, whereas quantities $q_1 \geq x_1$ are split into two sets, quantities $q_1 \in (x_1, \tilde{q}_1)$ are all bid in at $p_1^{PS}\left(c_{-1}\right)$ where $\tilde{q}_1 = \inf\left\{q : c_1(q_1) \geq p_1^{PS}\left(c_{-1}\right)\right\} - x_1$; and quantities $q_1 \in (\tilde{q}_1, K_1)$ are bid in at marginal costs. Trivially, the equilibrium price under the proposed profile equals $p_1^{PS}\left(c_{-1}\right)$. Note that a price $p^* > p_1^{PS}\left(c_{-1}\right)$ is impossible since at any such price p^* all firms are bidding at marginal costs; similarly, if $p^* < p_1^{PS}\left(c_{-1}\right)$ firm 1 dispatches no more than x_1, implying that firm 1's profits would be less than τx_1, but $\pi_1^{PS}\left(p_1^{PS}\right) \geq \pi_1^{PS}(p^c) \geq \tau x_1$ given that $x_1 < q_1^c$. Now, since $p^* = p_1^{PS}\left(c_{-1}\right)$ and $q_j^* = q_j^{NPS}\left(p^*\right)$ for all $j \neq 1$, Condition 1 of Theorem 1 holds. Condition 2 also holds since if firm 1 deviates to become a non-price-setter, the price would go down to p^c, and

$\pi_1^{NPS}(p^c) = \pi_1^{PS}(p^c) \leq \pi_1^{PS}\left(p_1^{PS}\right)$. Finally, Condition 3 follows from the definition of $p_1^{PS}(c_{-1})$ as $p_1^{PS}(c_{-1}) \geq p_j^{PS}(c_{-j})$ for all $j \neq 1$. ■

Proof of Corollary 2. Assume that the equilibrium in which firm i is the price-setter exists so that $\pi_j^{NPS}\left(p_i^{PS}\right) \geq \pi_j^{PS}\left(p_j^{PS}\right)$ for all $j \neq i$. Since by Lemma 4, the non-price-setters' profits are increasing in p, it follows that $\pi_j^{NPS}\left(p_{i-n}^{PS}\right) \geq \pi_j^{NPS}\left(p_i^{PS}\right)$ for any $n = 1, .., i-1$. Consequently, $\pi_j^{NPS}\left(p_{i-n}^{PS}\right) \geq \pi_j^{NPS}\left(p_i^{PS}\right) \geq \pi_j^{PS}\left(p_j^{PS}\right)$ implies that the equilibria in which firms $\{1, ..., i-1\}$ are the price-setters also exist. The remaining part of the proof follows the reversed arguments. ■

Proof of Lemma 5. Part (i) follows from the fact that $p_n^{PS}(x)$ is equal for all n, it constitutes the unique equilibrium price, and it is (weakly) decreasing in x. As x approaches q^c, productive efficiency is (weakly) greater because firms' market shares converge and marginal cost functions are (weakly) increasing.

To prove part (ii), suppose that contracts are not identically distributed among firms, $x_1 < x_2 \leq ... \leq x_n$, so that $x_1 < \frac{\sum_n x_n}{N} = x$. Since all firms are symmetric in all other respects, $p_1^{PS}(x_1) \geq p_j^{PS}(x_j)$ for all $j \neq 1$; furthermore, existence of an equilibrium with $p_1^{PS}(x_1)$ is guaranteed by Corollary 1. When contracts are identically distributed, the highest equilibrium price is $p_1^{PS}(x)$. Since for $x_1 < x$, $p_1^{PS}(x_1) \geq p_1^{PS}(x)$, the highest equilibrium price is (weakly) higher if contracts are not identically distributed. It follows that productive efficiency at the highest price equilibrium is (weakly) lower under any asymmetric contract distribution. ■

Proof of Proposition 4. Let us first introduce the following piece of notation. For $x_n \geq x_j = 0$, let x'_n be the smallest amount of contracts held by firm n for which the equilibrium in which firm n sets the price does *not* exist. Formally,

$$x'_n \equiv \min\left\{x_n : x_n \in [0, q_n^c] \text{ and } \pi_j^{NPS}\left(p_n^{PS}(x_n)\right) < \pi_j^{PS}\left(p_j^{PS}(0)\right) \text{ for some } j\right\}.$$

(i) By construction, $p_1^{PS}(0)$ is the highest candidate equilibrium price at the no-contracts case, and Corollary 1 guarantees that it is an equilibrium price. Since $p_1^{PS}(x_1)$ is weakly decreasing in x_1, the highest equilibrium price when $x_1 > 0$ is $\max\left\{p_1^{PS}(x_1), p_2^{PS}(0)\right\} \leq p_1^{PS}(0)$. Hence, the highest equilibrium price is (weakly) higher at the no-contracts case. To show that the lowest equilibrium price is also (weakly) higher at the no-contracts case, let $p_i^{PS}(0)$ be the lowest equilibrium price when $x_1 = 0$. By Corollary 2, any price $p_n^{PS}(0) \geq p_i^{PS}(0)$, for $1 < n \leq i$, must also be an equilibrium price when $x_1 = 0$ as well as when $x_1 > 0$. The incentives of all firms other than firm 1 do not depend on x_1, whereas firm 1's incentives to deviate from an equilibrium in which firm n sets the price are decreasing in x_1; hence, if no firm deviates from $p_n^{PS}(0)$ when $x_1 = 0$, no firm will deviate either when $x_1 > 0$. It thus follows that the lowest equilibrium price when $x_1 > 0$ can not be larger than $p_i^{PS}(0)$. Therefore, since the set of equilibrium prices is the same when $x_1 = 0$ or $x_1 > 0$, except (possibly) for the highest price, which is higher when $x_1 = 0$, and the lowest(s) price(s) which is (possibly) lower when $x_1 > 0$, contracts by firm 1 only (weakly) reduce prices.

Let us now show that there can exist a non-monotonic relationship between contracts awarded to firm 1 and equilibrium prices. Since $p_1^{PS}(0)$ is an equilibrium price when $x_1 = 0$, while when $x_1 = q_1^c$ it is not (since $p_1^{PS}(q_1^c) \leq p^c < p_2^{PS}(0)$, firm 2 would trivially deviate from such a low price), there exists $x'_1 \in (0, q_1^c]$ such that the equilibrium with $p_1^{PS}(x_1)$ does not exist for all $x_1 \in [x'_1, q_1^c]$. Let $p_j^{PS}(0)$ be the lowest equilibrium price when $x_1 = x'_1$. By Corollary 2, it must be the case that

$p_1^{PS}(x_1') < p_j^{PS}(0)$ as otherwise $p_1^{PS}(x_1')$ would also be an equilibrium. Thus, if $p_1^{PS}(x_1)$ is close enough to $p_1^{PS}(x_1')$ for x_1 slightly below x_1', then equilibrium prices go up as x_1 approaches x_1'. Note that such non-monotonicity need not always arise, e.g., if for x_1 slightly below x_1', $p_1^{PS}(x_1) \geq p_j^{PS}(0)$.

(ii) Let us allocate all contracts to firm i. Since $p_i^{PS}(0)$ is the lowest equilibrium price at the no-contracts case, by Corollary 2, equilibrium prices are $\left\{p_1^{PS}(0), ..., p_{i-1}^{PS}(0), p_i^{PS}(0)\right\}$. Existence of $x_i' \in (0, q_i^c]$ is guaranteed by monotonicity, since $p_i^{PS}(0)$ is an equilibrium price while $p_i^{PS}(q_i^c) \leq p^c < p_1^{PS}(0)$ is not (firm 1 would trivially deviate from such a low price). Now, as $p_i^{PS}(x_i)$ is non-increasing in x_i, allocating contracts $x_i \in (0, x_i')$ to firm i leads to (weakly) lower prices as compared to the no-contracts case, as equilibrium prices are $\left\{p_1^{PS}(0), ..., p_{i-1}^{PS}(0), p_i^{PS}(x_i)\right\}$ and $p_i^{PS}(0) \geq p_i^{PS}(x_i)$. However, allocating contracts $x_i \in [x_i', q_i^c]$ yields to (weakly) higher prices, as equilibrium prices are $\left\{p_1^{PS}(0), ..., p_{i-1}^{PS}(0)\right\}$ and $p_i^{PS}(0) \leq p_{i-1}^{PS}(0)$. Note that allocating contracts x_i to firm i does not give rise to new equilibria in which firms $n > i$ set prices, as at least firm 1 would deviate from such equilibria. It follows that there exists a (weak) non-monotonic relationship between contract volume and equilibrium prices when contracts are awarded to firm i.

Similar arguments would imply that if all contracts are awarded to some firm $1 < n < i$, there exists $x_n' \in (0, q_n^c)$ such that equilibrium prices are $\left\{p_1^{PS}(0), ..., p_n^{PS}(x_n), ..., p_i^{PS}(0)\right\}$ for $x_n < x_n'$ and $\left\{p_1^{PS}(0), ..., p_{n-1}^{PS}(0), p_{n+1}^{PS}(0), ..., p_i^{PS}(0)\right\}$ for $x_n \geq x_n'$. Contracts $x_n < x_n'$ thus lead to (weakly) lower prices, but the effect of contracts $x_n \geq x_n'$ depend on which equilibrium is chosen, given that $p_i^{PS}(0) \leq p_n^{PS}(0) \leq p_1^{PS}(0)$.

(iii) Since for $n > i$, $p_n^{PS}(0)$ is not an equilibrium price at the no-contracts case, it follows that $x_n' = 0$. Consequently, for any $x_n \geq 0$, $p_n^{PS}(x_n) \leq p_n^{PS}(0)$, so that by Corollary 2, $p_n^{PS}(x_n)$ cannot be sustained in equilibrium. Hence, prices remain the same as in the no-contracts case. ■

B Details on Simulations

The Spanish electricity market is organized similarly to many other wholesale electricity markets around the world. In particular, most transactions take place through an organized exchange, that operates on an hourly basis according to the rules described in Section 2 (see Crampes and Fabra (2005) for more details). The market structure is highly concentrated, with the two largest firms - Endesa and Iberdrola - controlling almost 60% of total thermal capacity, more than 80% of total hydro capacity, and approximately 40% of total renewables. Even though the shares of these technologies on total production vary across years, in 2005 hydro and renewables contributed to cover 8% and 11% of total demand, respectively. Table 4 summarizes the market structure of the main generators in the Spanish electricity market.

In order to conduct the simulations, we have first computed firms' marginal cost curves following similar techniques as in previous papers (Fabra and Toro (2005)).[40] In particular, we have estimated each thermal unit's marginal production costs on a daily basis, taking into account the type of fuel it burns, the cost of the fuel, the plant's heat rate (i.e., the efficiency rate at which each plant converts the heat content of the fuel into output), the short-run variable cost of operating and maintaining it, and the costs of its CO2 emissions. In addition, for coal plants, we have added an estimate of

[40] The data used in the simulations have been obtained from various sources, including The National Energy Commission (CNE), Red Eléctrica de España (REE), OMEL and UNESA.

Firm/ Technology	Nuclear	Coal	CCGT	Oil-Gas	Total	Shares
Endesa	3,511	5,511	1,170	1,779	10,918	33%
Iberdrola	3,222	1,225	3,704	3,050	8,456	26%
Unión Fenosa	702	1,946	1,559	747	4,954	15%
Gas Natural	–	–	2,729	–	2,729	8%
Hidrocantábrico	155	1,549	390	–	2,094	6%
Others	–	909	2,144	731	3,784	11%

Table 4: Installed Thermal Capacity (MWs) by Firm and Technology in the Spanish Electricity Market, 2005 (Source: REE)

the costs of transporting coal from the nearest harbor where it is delivered to the plant where it is consumed. Lastly, each unit's generation capacity has been reduced by its estimated outage rate. By aggregating the capacities of each firm's thermal units in increasing marginal cost order, we have obtained estimates of firms' thermal marginal cost curves for each day of the year.

Furthermore, we have assumed that the marginal costs of producing electricity with hydro and renewables equal zero. The production coming from such sources has therefore been added to the left of each firm's thermal marginal costs curve in order to construct their overall marginal costs curves. We have chosen not to use actual data on hydro production, as it is already the result of firms' strategic decisions. Instead, poundage hydro generation has been set to peak-shave demand on a monthly basis, taking into account maximum hydro flows.[41] Both, run-of-river hydro as well as renewables' production, have been uniformly spread across time. Hydro stocks, run-of-river hydro flows, and renewable energy, are monthly estimates of a representative year.

Demand has been assumed to be price-inelastic at the actual hourly demand levels that were observed in 2005. Furthermore, we have set the market-reserve price at 120€/MWh, below its explicit 180€/MWh level, with the aim of reflecting issues such as the threat of entry or regulatory intervention. Nevertheless, setting this cap at either 120 or 180€/MWh does not change the qualitative nature of the results.

[41] As it is by now well understood (Bushnell (2003)), firms could strategically shift hydro from peak to off-peak hours, thereby distorting the efficient use of hydro resources. A full analysis of this issue is out of the scope of this section. However, despite assuming competitive bidding for hydro units, hydro still affects firms' strategic decisions through its impact on their inframarginal output.

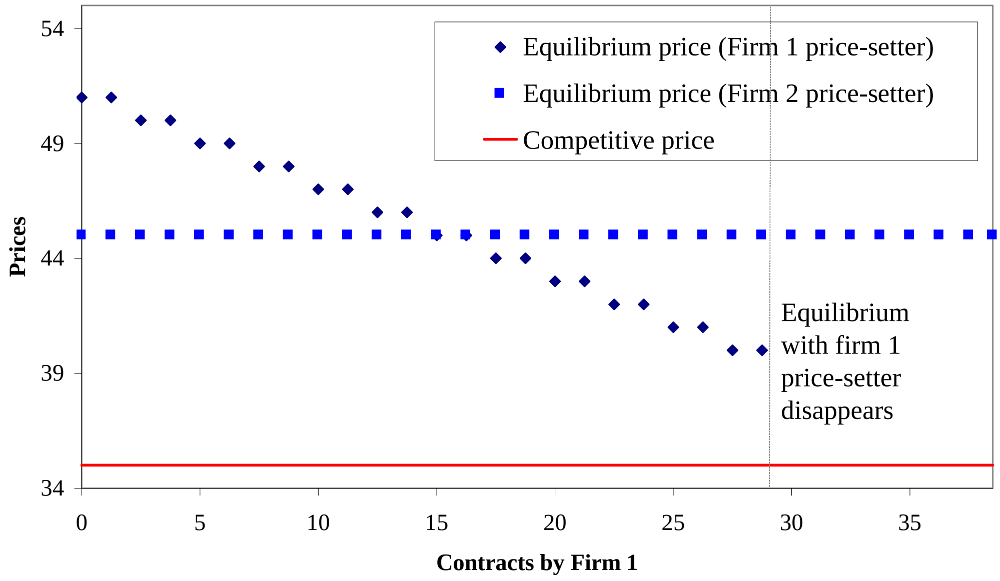

Figure 1: Forward contracts by firm 1 and equilibrium prices (asymmetric firms (55%,45%))

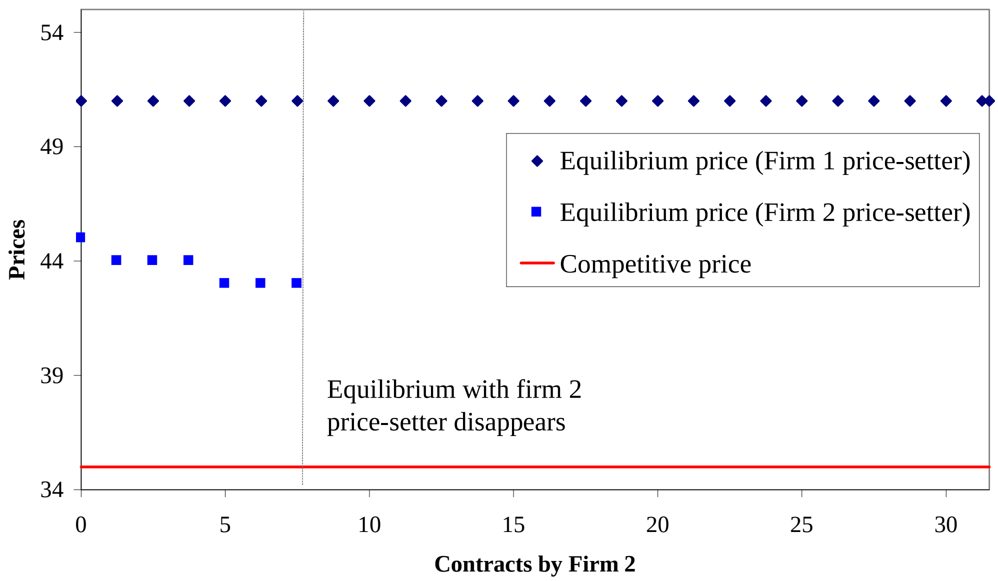

Figure 2: Forward contracts by firm 2 and equilibrium prices (asymmetric firms (55%,45%))

Note: In Figures 1 and 2 we have assumed $N = 2$. There are 200 units, 2 at each marginal cost level, $k = 1, ..., 100$, each owned by a different firm. Firm 1's units have capacity 1.10 while firm 2's units have capacity 0.9. Demand is price-inelastic, $D = 70$, so that $p^c = 35$ and $q_1^c = 38.5 > q_2^c = 31.5$.

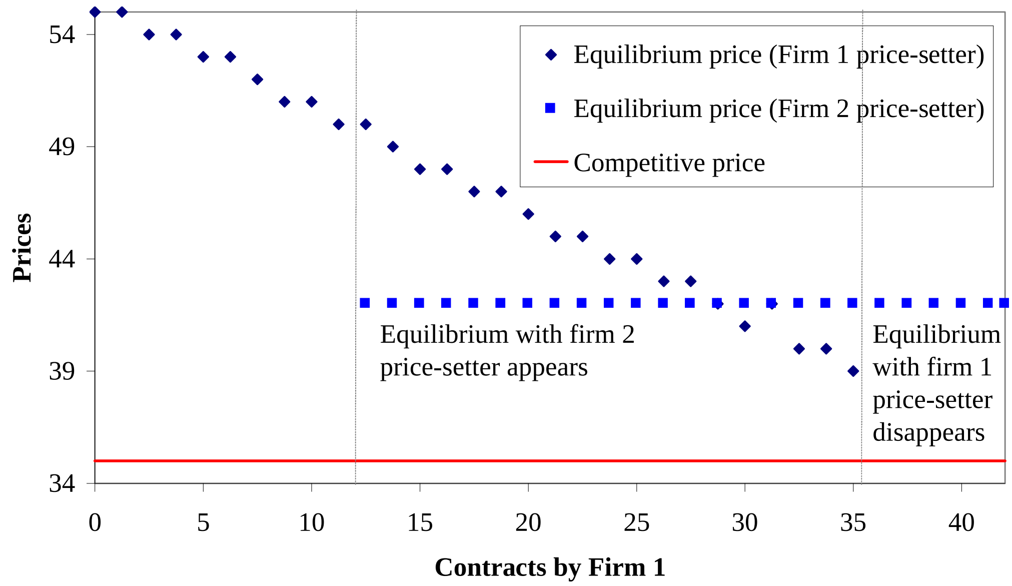

Figure 3: Forward contracts by firm 1 and equilibrium prices (asymmetric firms (60%,40%))

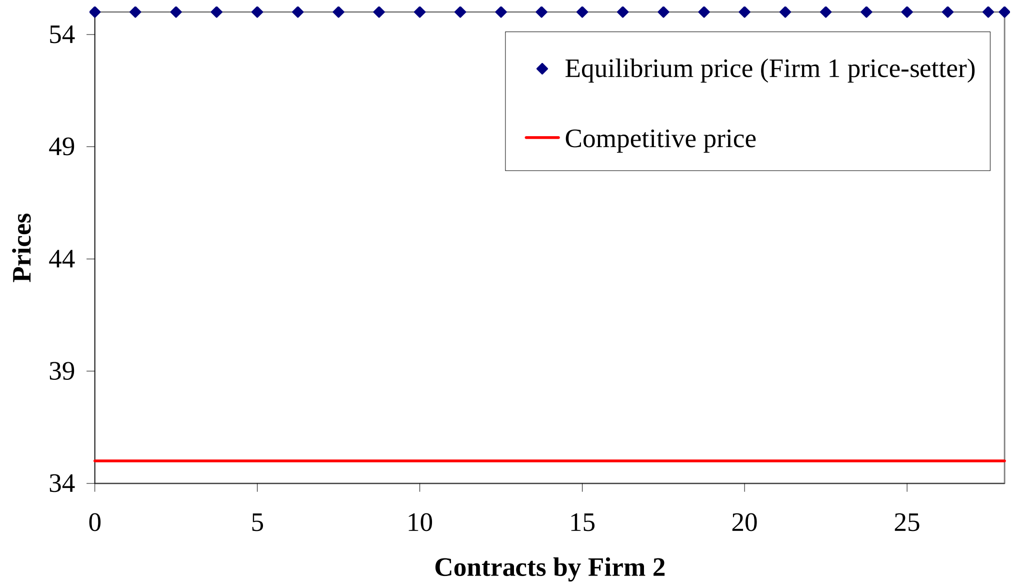

Figure 4: Forward contracts by firm 2 and equilibrium prices (asymmetric firms (60%,40%))

Note: In Figures 3 and 4 we have assumed $N = 2$. There are 200 units, 2 at each marginal cost level, $k = 1, ..., 100$, each owned by a different firm. Firm 1's units have capacity 1.20 while firm 2's units have capacity 0.8. Demand is price-inelastic, $D = 70$, so that $p^c = 35$ and $q_1^c = 42 > q_2^c = 28$.

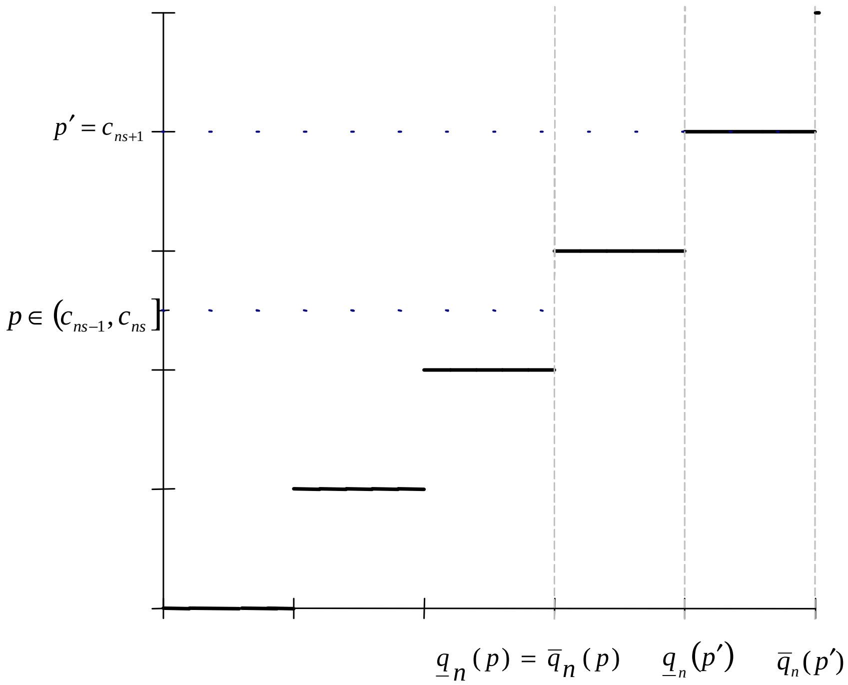

Figure 5: An example of a firm's marginal cost function and the quantities $\underline{q}_n(p)$ and $\bar{q}_n(p)$