

# Polling in a Proportional Representation System\*

Christos Mavridis<sup>†</sup> and Ignacio Ortuno-Ortín<sup>‡</sup>

## Abstract

We study the effects of opinion polls on elections results in proportional representation systems. Moderate voters have preferences over the vote shares received by the parties so that an agent's optimal voting decisions might depend on the other agents behavior. A voter's information about other voters' behavior can be improved through a series of opinion polls. We show that the mass of undecided voters decreases monotonically with the number of polls, but may not necessarily disappear. Voters who remain undecided have centrist ideologies. On average a series of polls brings the society closer to complete information even though specific polls may push the election result away from the complete information case or the maximum welfare one.

*Keywords:* opinion polls, proportional representation, undecided voters  
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## 1 Introduction

The effects that opinion polls have on the electorate have been studied extensively. The conclusion of the literature is that the publication of polls influences how the electorate votes [Myerson and Weber, 1993, Morton et al., 2015]. This influence is the reason why many countries have restrictions on how, when and if a poll should be published. Chung [2012] provides information about poll restrictions in 83 countries and finds existence of a blackout period for pre-election opinion polls in 38 of them. The restrictions can range from a ban on publishing polls for a specific time before the election, like in Italy (fifteen days) or in Greece (fifteen days until 2014, one day since), to an outright ban during

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<sup>†</sup>Lancaster University, Bailrigg, Lancaster LA1 4YX, UK. Email: c.mavridis@lancaster.ac.uk.

<sup>‡</sup>Department of Economics, Universidad Carlos III de Madrid. Calle Madrid 126, 28903 Getafe, Madrid, Spain. Email: iortuno@eco.uc3m.es.

the entire campaign period like in Singapore. In the last decade in many countries there have been public discussions on whether these types of restrictions should be posed or lifted. In 2012 India's Chief Election Commissioner stated that opinion polls and exit polls should both be banned.<sup>1</sup> A 2013 survey in the United Kingdom, where there is not a ban on publishing pre-election opinion polls, showed that three out of ten MPs supported the idea of such a ban.<sup>2</sup>

These restrictions and discussions show that the issue of the interaction between opinion polls and elections is important both from a theoretical and from a practical point of view. The motive behind such bans is usually to let the electorate vote "truly" without the influence of the results of opinion polls, an influence which is perceived as bad. The fear that polls may be biased, in which case the electorate might base its decision on false information, is often used to argue in favor of restrictions. Another argument is that basing the decision itself on the information generated by opinion polls might distort the election results away from the "true" preferences of the electorate.<sup>3</sup> On the other hand, opinion polls are useful inasmuch as they clear some uncertainty in the political environment leading to more efficient decision making by political actors. Examples of uncertainty include uncertainty about valence of the candidates as e.g. in Kendall et al. [2015] or on the actual policy-ideology of candidates as in Baron [1994].

In democracies with proportional representation system where parties compete for parliament seats, these seats can be seen as a measure of political power; a ruling party that controls enough seats to just secure a majority might be more moderate in its policies than a party that controls a larger fraction of the parliament. For this reason a moderate voter may choose not to vote for his favorite party if he feels that it may be too strong. Therefore a moderate voter who is happier with a more equal (or a particular) split of parliamentary seats would like to know how the rest of the voters will vote before he decides his own vote, implying that he would need to have some information about the distribution of the rest of the voters. This idea can be seen, for example, in McMurray [2017] or in Morton et al. [2015]. In an experimental study Ceci and Kain [1982] show that moderate voters, when instructed that the latest poll showed that a candidate "commanded a substantial lead over the other", tend to report that they would vote for the trailing one.

The goal of this paper is to examine the effects that opinion polls might have in the behavior of voters in proportional representation systems. In particular, we analyze under what conditions the publication of opinion polls during the electoral campaign lead to complete information and their welfare consequences. To do so, we construct a simple two-party proportional representation system with a continuum of voters and fixed policy proposals. It's assumed that the implemented policy is a combination of the parties' proposals with weights given by their vote shares. Voters care about the implemented policy, which

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<sup>1</sup>Opinion, exit polls have no scientific basis: CEC (April 28, 2012). IBNLive. Retrieved from: <http://ibnlive.in.com/news/opinion-exit-polls-have-no-scientific-basis-cec/252787-3.html>

<sup>2</sup>Eaton, David. Should pre-election opinion polls be banned? A third of MPs think so (November 13, 2013). NewStatesman. Retrieved from: <http://www.newstatesman.com/politics/2013/11/should-pre-election-opinion-polls-be-banned-third-mps-think-so>

<sup>3</sup>Morton et al. [2015] identification strategy exploits a 2005 French voting reform that came into place precisely to avoid the situations where exit polls were public before some voters went to vote.

implies that they have preferences over the vote shares received by the parties. Even though there is a continuum of voters we will assume that they behave as if they could have some (small) impact on the electoral outcome. In this case, an agent’s optimal voting decision might depend on the other agents behavior. Voters have incomplete information about each other’s preferences and therefore about their voting behavior. A voter’s information about other voters’ preferences can be improved through a series of polls. Polls provide information on the percentage of people who declare that they will vote for each party as well as the percentage of undecided people. It’s assumed that agents always respond honestly to the questions in the opinion polls. It turns out that the agents most likely to report that they do not know what they will vote yet are the more moderate ones.

The paper first shows under which conditions on the distribution of citizens’ preferences a sequence of polls will be able to reveal the complete information case. With every poll the undecided mass of agents decreases, as more citizens are sure of what to vote. After enough number of polls agents have (almost) complete information. However, for other distributions of voters’ preferences, after a number of polls the undecided mass of agents stops decreasing and additional polls cannot provide any more information. The paper also shows that the implemented policy and social welfare are both not monotonic on the number of polls: the publication of one last additional poll before the election may result in an implemented policy that is further away from the optimal policy or the policy that would be result under complete information.

To the best of our knowledge this is the only paper that examines polls and proportional representation together in a dynamic setting. It is also important to mention that this model provides three predictions: i) the positive correlation between undecidedness and centrist ideologies, ii) the non-monotonicity of vote shares with respects to the number of opinion polls and iii) the decrease of the mass of undecided voters when more polls are published. As explain below, each of these three predictions can be also generated by other models but we do not know of any model that predicts all three of them. In the empirical appendix we provide some empirical evidence supporting these predictions of the model.

## 1.1 Related Literature

Various works have examined how opinion polls affect voters and parties through the information they reveal. Whenever there is some uncertainty, an objective flow of information may help minimize it. Denter and Sisak [2015] show that poll results affect campaigning spending which in turn affect how voters vote. Bernhardt et al. [2009], Morgan and Stocken [2008] and Meirowitz [2005] show that polls are used to clear uncertainty over voters preferences, but at the same time this can give rise to strategic poll answering on behalf of the voters. McKelvey and Ordeshook [1985] show that polls create a “bandwagon” effect in favor of the leading party, while Goeree and Großer [2006] and Taylor and Yildirim [2010] show that the opposite “underdog” effect can appear, through the mobilization of the voters of the trailing party. In both of these papers it is argued that through polls the “wrong” side may win which would result in a welfare loss. Klor and Winter [2006] in an experiment show that in both close and lopsided elections polls have a bandwagon effect, but the welfare effect of the polls is ambiguous, and in a later paper [Klor and Winter, 2014] they use US

Gubernatorial elections data to show that the increase in participation in elections is greater for the supporters of the leading candidate. Großer and Schram [2010] find similar results. Herrmann [2014] examines polls under proportional representation and shows that voters act strategically, trying to anticipate the future coalitions after the election takes place. In his model there are three types of voters using the results of polls to vote strategically in order to influence the future coalitions, while in our model there is a continuum of voters whose voting intentions has a dynamic relationship with poll results.

The implemented policy as seen as a compromise among different ideological sides, or power sharing, is not a new idea [Alesina and Rosenthal, 1996, Grossman and Helpman, 1996, Llavador, 2006, De Sinopoli and Iannantuoni, 2007, Saporiti, 2014, Matakos et al., 2015, Herrera et al., 2015, Matakos et al., 2016]. Ortuño-Ortín [1997] discusses a two-party setting where implemented policy is a convex combination of the two parties’ platforms generated using a continuous function of the parties’ shares in the election. Sahuguet and Persico [2006] point out that in a proportional system parties need to maximize their share (rather the probability of getting at least 50 per cent of the vote) and that the implemented policy is, is at least partly influenced by the minority party.

Herrera et al. [2015] in a proportional representation setting similar to the one of this paper show that the “marginal voter’s curse” has the effect that voters with low quality information abstain and not vote for fear of casting a vote for the wrong side. Finally, voters may choose to support the party that they are least ideologically close an idea which is shared by the concept of the “protest vote”, by which voters may want to punish, or control the power of their favorite party (see eg. Myatt [2015]).

The rest of the paper is organized as follows. In Section 2, we analyze the voting model. In Section 3, we provide empirical evidence regarding the influence of polls on the implemented policy, and the relationship between ideology and undecidedness. This evidence is consistent with the main findings of the theoretical model. In Section 4 we discuss the predictions of the model about implemented policy and social welfare. Finally, Section 5 concludes.

## 2 Model Description

We assume an uni-dimensional policy space  $D \subset \mathfrak{R}$ . The voters have single-peaked preferences over this policy space and their optimal policies are distributed over  $D$  according to a distribution function  $F(\cdot)$  which is assumed to be continuous, strictly increasing and differentiable with a corresponding density function  $f(\cdot)$ . We identify a voter’s type with her ideal policy. The utility of a voter of type  $x \in D$  from the implemented policy  $\hat{x}$  is given by the function  $v(|\hat{x} - x|)$ , i.e. it is a function of the absolute distance between  $x$  and the implemented policy  $\hat{x}$ . Function  $v(|\hat{x} - x|)$  is decreasing and concave in  $|\hat{x} - x|$  and it has a unique maximum at  $\hat{x} = x$  such that  $v(0) = 0$ . These mean that the utility function of a voter is symmetric around his optimal policy point where the utility is maximum and normalized to zero. There are two parties running in the election,  $L$  and  $R$ , each having a fixed policy position at 0 and 1 respectively. The actual implemented policy,  $x(p)$ , will be a compromise between those two policy positions, namely :

$$x(p) = (1 - p) \times 0 + p \times 1 = p,$$

$L$ share	Implemented policy	Decision
0.8	0.2	$R$
0.3	0.7	$L$

where  $p$  is the share of votes party  $R$  gets. This share is defined as  $p = \frac{m_R}{m_L + m_R}$ , with  $m_L$  and  $m_R$  being the voter masses that voted for  $L$  and  $R$  respectively, meaning that the parties' shares are calculated over the mass of voters that actually voted. If  $u$  is the mass of the citizens that decide not to vote then  $m_L + m_R + u = 1$ . The setting is similar to Ortuño-Ortín [1999]. Since  $x(p) = p$  we use  $p$  to describe the vote share received by  $R$  as well as the implemented policy.

Notice that  $p = \frac{m_R}{m_L + m_R}$  implies that the parties' shares of influence in the implemented policy are equal to their vote shares.<sup>4</sup> Even though this assumption is made for simplicity and clarity, all the results follow if we instead assume the more flexible weight function

$$\hat{p} = \frac{\left(\frac{m_R}{m_L + m_R}\right)^\gamma}{\left(\frac{m_R}{m_L + m_R}\right)^\gamma + \left(\frac{m_L}{m_L + m_R}\right)^\gamma}$$

with  $\gamma \geq 1$  and finite (as in, for example, Saporiti [2014], Matakos et al. [2015], Matakos et al. [2016]). Obviously, for  $\gamma = 1$ ,  $p = \hat{p}$ . As  $\gamma$  increases then the share of the bigger party is getting bigger, and that of the smaller party smaller. This captures the fact that some parliamentary systems, while mostly proportional, give some bonus seats to the biggest party or demand a certain percentage threshold to be passed before they assign seats to smaller parties.

Given that there is a continuum of agents no single agent can affect the implemented policy. We assume, however, that agents want to behave as if anybody could move the implemented policy in certain direction. Thus, if an agent  $x > p$  voted for  $L$ , after the outcome is revealed, she will regret the way she voted. Even though her vote couldn't change anything she voted in the "wrong direction". For this reason, each voter would like to know how the other voters are behaving before he chooses which party to vote for. Consider Table 1 where we analyze the decision of a voter located at 0.4. This voter is closer to 0 than to 1, but if he expects that too many voters will vote for the left party (first row) the implemented policy will be in his left and therefore he would like to vote for the right party.

## 2.1 Complete information equilibrium

If the distribution function  $F(\cdot)$  is common knowledge we can easily describe the behavior of voters. Let  $\sigma_x$  denote the vote strategy chosen by agent  $x$ . We assume that  $\sigma_x \in \{\sigma_x^L, \sigma_x^R, \sigma_x^{1/2}\}$ . The interpretation is that  $\sigma_x^L$  ( $\sigma_x^R$ ) means a vote for  $L$  ( $R$ ) and  $\sigma_x^{1/2}$  means that with probability 1/2 he votes for  $L$ . Let  $\sigma = (\sigma_x)_{x \in \mathbb{R}}$  be a list of strategies one for each type of agent and let  $p(\sigma)$  be the associated percentage of votes obtained by party  $R$ .

<sup>4</sup>Under this formulation an alternative interpretation of the voters' utility is that they have direct preferences over the parties' shares themselves.

**Definition 1.** A list of strategies  $\sigma = (\sigma_x)_{x \in \mathbb{R}}$  constitutes a Complete Information Voting Equilibrium (CIVE) if  $\forall x \in D$ :

- $\sigma_x = \sigma_x^L$  implies  $p(\sigma) > x$ ,
- $\sigma_x = \sigma_x^R$  implies  $p(\sigma) < x$ ,
- $\sigma_x = \sigma_x^{1/2}$  implies  $p(\sigma) = x$

The equilibrium concept is similar to the one in Gerber and Ortuno-Ortín [1998]. It is not difficult to see that our CIVE is a standard Strong Nash Equilibrium. An agent votes to move the implemented policy toward her ideal policy. Even though she cannot do it on her own she might believe that people with types close to hers will behave in the same way. At equilibrium there is no set of agents (with positive measure) that can deviate from their strategies and change the outcome in a beneficial way for all of them.

Notice that the equilibrium defines a cut-off point  $x^* = p(\sigma)$  that divides the electorate in two parts: all agents to the left (right) of the implemented policy vote for  $L$  ( $R$ ).

**Theorem 1.** *There always exists a unique Complete Information Voting Equilibrium  $\sigma^*$  characterized by the cut-off point  $x^* = 1 - F(x^*)$ .*

*Proof.* Proof. By continuity of  $F(\cdot)$  the point  $x^*$  exists, and by strict monotonicity this point is unique. Now, let  $x^*$  be the implemented policy. Then strategy profile  $\sigma^*$  implies that the voters to the left of  $x^*$  vote for  $L$  and the ones to the right of  $x^*$  vote for  $R$ . Notice that the mass of voters of type  $x^*$  is zero, so that the mass of voters of  $L$  is  $F(x^*)$  and the mass of voters of  $R$  is  $1 - F(x^*)$ . In turn, these masses give rise to the implemented policy  $x^* = 0 \times F(x^*) + 1 \times (1 - F(x^*)) = 1 - F(x^*)$ , which proves that an equilibrium always exists, and since  $x^*$  is unique there is only one equilibrium defined by  $x^* = 1 - F(x^*)$ .

The last step is to show that there is no other equilibrium. Suppose that there is an equilibrium implemented policy  $x'^* \neq x^*$ . The equilibrium strategy implies that everybody to the left (right) of  $x'$  vote for  $L$  ( $R$ ), giving rise to the implemented policy  $x'' = 1 - F(x')$ , but we have that  $x'' \neq x'$ , which leads to a contradiction.  $\square$

Notice that  $x^* = 1 - F(x^*)$  does not necessarily correspond to the median policy  $x^M$ ,  $F(x^M) = 1/2$ . The only case in which we obtain a "median voter outcome" is when  $x^M = 1/2$ .<sup>5</sup>

## 2.2 Polls

When the distribution of voters is common knowledge, voters can calculate  $x^* = 1 - F(x^*)$  and infer what they should vote. However, when the distribution is no longer known to the voters, we have incomplete information, and some voters do not have enough information to be certain which party they should vote for. For some other agents this will not be a problem. For example, any agent of type  $x < 0$  will always vote for  $L$  regardless what other agents do. Thus,

<sup>5</sup>Since in this case we would have  $\frac{1}{2} = 1 - F(\frac{1}{2})$ , so  $F(\frac{1}{2}) = \frac{1}{2}$ .

it is clear that all agents with type outside the interval  $[0, 1]$  have a dominant strategy. We will call agents with  $x < 0$  as *left partizans* and agents  $x > 1$  as *right partizans*. It will be useful to denote by  $\rho_L$  the mass of left partizans, and by  $\rho_R$  the mass of right partizans, i.e.  $\rho_L = F(0)$  and  $\rho_R = 1 - F(1)$ . We denote the agents in  $[0, 1]$  as *moderate*. These agents have no dominant strategies, but the only relevant information they need to calculate their optimal strategy is the share of votes that each party will get. It can be argued, then, that polls can provide this information. The problem is, however, that “Voting intentions may change from day to day as the voters’ perceptions evolve during a campaign, so that a poll, when published, may invalidate itself.” (Myerson and Weber [10, page 102]). The question then is to find under which circumstances the election under incomplete information and polls lead to the same outcome as the election outcome when the distribution of agents’ types is common knowledge. The answer to this question depends very much on the assumptions on voters’ beliefs and the way they use the information provided by the polls. Here we will adopt a very simple model to deal with those issues that will yield clear results. We believe, however, that the basic results are robust to more general specifications as we discuss below.

We suppose that there is a sequence of polls at periods  $t = 0, 1, 2, \dots, T - 1$ . The election takes place at period  $T$ . The question asked is: “what are you going to vote on the day of the election?” For  $t < T$  we write  $v_{x,t}^i$ ,  $i \in \{0, 1, \emptyset\}$  to denote the voting intention that agent  $x$  (if asked) declares in period  $t$ . Thus,  $v_{x,t}^0$  ( $v_{x,t}^1$ ) denotes that  $x$  will vote for  $L$  ( $R$ ) and  $v_{x,t}^\emptyset$  denotes that  $x$  doesn’t know yet. This possibility of “indecision” is essential in our model. This is also a very common feature in many real elections.

A poll at the end of period  $t$  is a vector  $P_t(\varepsilon_t, u_t)$  such that  $\varepsilon_t$  is the percentage of people that, during period  $t$ , announced the intention to vote for  $L$  and  $u_t$  the percentage of people who announced not to know yet which party they will vote for. Thus we see a poll as providing information on current intentions (in this sense a poll is not a prediction of the election outcome). We assume that  $P_t$  is a statistically perfect poll and appropriately samples and measures voting intentions at that period. Since we impose the condition that in the election day everybody votes for one of the parties<sup>6</sup> we may write the outcome of the election as  $\varepsilon_T \equiv \{\text{percentage of people who vote for } L\}$ .

At each period  $t = 0, 1, \dots, T - 1$  agent  $x$  has subjective beliefs on the outcome  $\varepsilon_T$  given by  $\beta_{x,t}$ . These beliefs are CDF on  $[0, 1]$ , i.e.  $\beta_{x,t}(y) = \text{Probability}\{\varepsilon_T \leq y\}$ . In general  $\beta_{x,t}$  will depend on the original beliefs  $\beta_{x,0}$  and the information provided by the polls  $P_{t'}, t' < t$ . Define

$$\begin{aligned} m_{x,t} &= \sup\{y : \beta_{x,t}(y) = 0\} \\ M_{x,t} &= \inf\{y : \beta_{x,t}(y) = 1\} \end{aligned}$$

We suppose that at period  $t = 0, 1, \dots, T - 1$ , agent  $x$  responds to a poll in the following way

$$v_{x,t} = \begin{cases} v_{x,t}^1 & \text{if } m_{x,t}0 + (1 - m_{x,t})1 < x \\ v_{x,t}^0 & \text{if } M_{x,t}0 + (1 - M_{x,t})1 \geq x \\ v_{x,t}^\emptyset & \text{otherwise} \end{cases} \quad (1)$$

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<sup>6</sup>As we explain below, our results do not depend on this condition.

Expression (1) just says that an agent answers that her vote will be for, say,  $R$  if the implemented policy can never be, according to her beliefs, to the right of her ideal policy. This is equivalent to saying that agents want to rule out the possibility of regret, i.e., the possibility that later on they do something different from what they had stated. Thus, agents only announce their vote intention when they are quite sure about it.

This may be seen as a very ad hoc feature of our model. We believe, however, that there is some intuitive justification for it. Moreover, the results in this paper are robust to changes that allow for more “flexible” rules. Thus, suppose that agents announce their vote intentions even though they are not completely sure about their final vote. The results would be similar to the ones provided here as long as agents believe that with a high enough probability they will vote for the same party they said they were going to vote.<sup>7</sup> Now we have to describe the way people vote at period  $T$ . There are several ways to model this decision problem. Here we will assume that voters follow the rule in (1).<sup>8</sup>

A type  $x$  agent will vote for  $L$  only if the expected implemented policy—according to her subjective beliefs—is to the right of  $x$ . In the case the expected implemented policy coincides with the ideal policy an agent will vote with probability 0.5 for  $L$  (this is not essential and we could, alternatively, assume that she doesn’t vote).

One might object against this type of behavior because (1) doesn’t depend on the utility function. It turns out that this is not a problem in our model. Agents who were not undecided at period  $T - 1$  will not be undecided at period  $T$  either, so their behavior wouldn’t change if we introduce the expected utility in (1). The problem arises with people who at period  $T$ —the election day—don’t know yet if the outcome will be to the right or the left of their ideal policies. In principle, it seems more reasonable to assume that they consider the expected utility rather than the expected outcome. This alternative, however, wouldn’t change the type of qualitative results provided in this paper and so we stick to (1) for simplicity. Our results are also robust to other decision criteria for the undecided agents on the day of the election, including the possibility of abstention.

Now we must specify the way beliefs may change through time. Let  $P_{t-1} = (\varepsilon_{t-1}, u_{t-1})$  be the information provided by the poll at the end of period  $t - 1$ . Then the beliefs at period  $t$  are such that for all  $x$

$$\begin{aligned} M_{x,t} &= \varepsilon_{t-1} + u_{t-1} \\ m_{x,t} &= \varepsilon_{t-1} \end{aligned} \tag{3}$$

Here we are saying that agents believe that the number of votes  $L$  ( $R$ ) will get

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<sup>7</sup>An alternative approach would assume that an agent announces her vote for the party that, for example, in expected terms is more likely to be the one she finally votes for. This, however, would rule out the possibility of undecided agents. Moreover, this approach requires more information than the one we assume here about the way agents change their beliefs  $\beta_{x,t}$

<sup>8</sup>Another, equivalent, rule is to assume that the relevant variable is the expected value of the implemented policy. Thus, at period  $T$  agent  $x$  votes in the following way

$$\sigma_x = \begin{cases} \sigma_x^0 & \text{if } \int_0^1 (x - \varepsilon) d\beta_{x,t}(\varepsilon) < 0 \\ \sigma_x^1 & \text{if } \int_0^1 (x - \varepsilon) d\beta_{x,t}(\varepsilon) > 0 \\ \sigma_x^{1/2} & \text{otherwise.} \end{cases} \tag{2}$$



cannot be less than  $\varepsilon_{t-1}$  (more than  $\varepsilon_{t-1} + u_{t-1}$ ). Notice that this restriction is compatible with the possibility that the ratio of votes in favor of  $L$  over votes in favor of  $R$  changes over time. The equalities in (4) also imply that, at period  $t$ , agents believe that with some positive probability (almost) all undecided agents will vote  $L$  and with some positive probability (almost) all undecided agents will vote  $R$ . In other words, this assumption captures the idea of a very imperfect information on the distribution of types. Since expression (4) is the same for all agents we can drop the subscript and write  $M_t$  and  $m_t$ .

We must now specify the beliefs agents have at period  $t = 0$ , when no poll has been conducted yet. We will assume –consistent with the idea that agents have a very imperfect information on  $F$ – that first period beliefs,  $\varepsilon_{x,0}$ , are such that only agents outside the interval  $(0, 1)$  will be sure about the party they will vote for. Therefore, we have that  $P_0 = (\varepsilon_0, u_0)$ , where  $\varepsilon_0 = F(0)$  and  $u_0 = F(1) - F(0)$ . Recall that we denote by  $\rho_L$  the mass of left partizans, and by  $\rho_R$  the mass of right partizans, i.e.  $\rho_L = F(0)$  and  $\rho_R = 1 - F(1)$ , so that we can also write the initial beliefs as  $P_0 = (\rho_L, 1 - \rho_R - \rho_L)$ .

We know that after publishing poll  $P_t = (\varepsilon_t, u_t)$  all agents know that the final implemented policy will lie in the interval  $[l_t, r_t] \subseteq [0, 1]$  where

$$\begin{aligned} l_t &= 1 - M_t \\ r_t &= 1 - m_t \end{aligned}$$

Indeed, if all the undecided agents at period  $t$  finally vote for party  $L$  the implemented policy would be  $l_t = 0 \times M_t + 1 \times (1 - M_t) = 1 - \varepsilon_{t-1} - u_{t-1}$ , and if all of them vote for party  $R$  the implemented policy would be  $r_t = 0 \times m_t + 1 \times (1 - m_t) = 1 - \varepsilon_{t-1}$ . Thus, after poll  $t$  is published the set of still undecided agents coincides with the interval  $[l_t, r_t]$  and, given all our above assumptions, when poll  $t + 1$  is conducted agents with  $x < l_t$  ( $x > r_t$ ) will respond that they will vote for  $L$  ( $R$ ), and agents  $x \in [l_t, r_t]$  will declare to be undecided.

We are interested in the sequence of polls and elections outcomes

$$P_0 = (\varepsilon_0, u_0), P_1 = (\varepsilon_1, u_1), \dots, P_{T-1} = (\varepsilon_{T-1}, u_{T-1}), \varepsilon_T$$

Recall that  $\varepsilon^*$  denotes the vote share obtained by  $L$  in the complete information case. If polls are good agregators of information we should observe  $\varepsilon_T = \varepsilon^*$ , or at least  $\lim_{T \rightarrow \infty} \varepsilon_T = \varepsilon^*$ . (By  $T \rightarrow \infty$  we mean that the number of polls, before the election day, goes to infinity). A sufficient condition for this to happen is that

$$\lim_{T \rightarrow \infty} (\varepsilon_{T-1}, u_{T-1}) = (\varepsilon^*, 0) \quad (4)$$

At period  $t = 0$  we have  $P_0 = (\varepsilon_0, u_0)$ . Now it is quite straightforward to calculate  $X_L^1$ , the set of types who at period  $t = 1$  announce to vote for  $L$

$$X_L^1 = \{x \leq l_1, \text{ where } l_1 = 1 - F(1)\} \quad (5)$$

In a similar way the set of types who announce to vote for  $R$  is

$$X_R^1 = \{x \geq r_1, \text{ where } r_1 = 1 - F(0)\} \quad (6)$$

Notice that (5) and (6) follow from the behavior described by (1). Agents at period  $t = 1$  believe that there is some positive probability that almost all

undecided agent at  $t = 0$  vote for  $L$ . Were that the case the implemented policy would be  $l_1$  and then all agents with ideal points to the left of such number should vote for  $L$  (a similar argument works for  $r_1$ ). For the rest of periods we can define in a similar way to  $X_L^1$  and  $X_R^1$  the set of types announcing their vote for  $L$  and the set of types announcing their vote for  $R$

$$\begin{aligned} X_L^t &= \{x \leq l_t, \text{ where } l_t = 1 - F(r_{t-1})\} \\ X_R^1 &= \{x \geq r_t, \text{ where } r_t = 1 - F(l_{t-1})\} \end{aligned} \quad (7)$$

Then the variables we need to analyze are  $l_t$  and  $r_t$ : at period  $t$  agents with type in the interval  $[l_t, r_t]$  are undecided and agents with type  $x < l_t$  ( $x > r_t$ ) will vote for  $L$  ( $R$ ). Thus, given initial conditions  $l_0 = 0$ ,  $r_0 = 1$  the dynamic process to consider is given by the following system of equations

$$\begin{aligned} l_t &= 1 - F(r_{t-1}) \\ r_t &= 1 - F(l_{t-1}) \end{aligned} \quad (8)$$

If after a number of polls  $s$  with each subsequent poll the interval of the undecided does not change, then the process has reached a steady state where  $l_s = l_{s+t} = l^*$  and  $r_s = r_{s+t} = r^*$ ,  $t \geq 1$ . Plugging the expression for  $r_{t-1}$  into the expression for  $l_t$  we get:

$$l_t = 1 - F(1 - F(l_{t-2})). \quad (9)$$

This is a second order difference equation with initial values  $l_0 = 0$  and  $l_1 = 1 - F(r_0) = 1 - F(1)$ .

Next, we will establish sufficient conditions for the sequence of polls to converge to the complete information outcome. First define

$$h(x) \equiv 1 - F(1 - F(x)), \quad x \in [0, 1].$$

Notice that the function  $h(x)$  captures the dynamic in (9)

**Theorem 2.** *If  $h'(x) = F'(1 - F(x))F'(x) < 1 \forall x \in [0, 1]$  then a sequence of polls will lead to the complete information outcome.*

*Proof.* Notice that the complete information outcome,  $x^*$ , is a fixed point of  $h(x)$ . Indeed, since  $x^* = 1 - F(x^*)$ , we have that  $h(x^*) = 1 - F(1 - F(x^*)) = 1 - F(x^*) = x^*$ . Therefore, if  $h(x)$  has a unique fixed point it will be the complete information outcome and the (probably infinite) sequence of polls will make  $l_t$  to converge to its complete information value. It's straightforward to see that in that case  $r_t$  also converges to the complete information value. Uniqueness of  $h(x)$  is not guaranteed however, and depends on the form of the distribution function  $F(\cdot)$ . Given that  $h(x)$  is continuous and increasing, and  $h(0) > 0$  it is clear that the condition  $h'(x) < 1$  in  $[0, 1]$  is enough to guarantee that there is a unique fixed point in  $[0, 1]$ .  $\square$

The following straightforward corollary establishes when the sequence of polls will converge to a fixed point that could be different from the one associated to the complete information one.

**Corollary 1.** *Let  $\underline{x}^*$  be the smallest fixed point of  $h(x)$ . If  $F'(1 - F(x))F'(x) < 1$  holds in  $[0, \underline{x}^*]$  then it is a sufficient condition for  $l_t$  to converge to  $\underline{x}^*$ .*

Therefore, if  $F'(1 - F(x))F'(x) < 1$  holds in  $[0, 1]$  then the complete information outcome is a unique fixed point of  $h(\cdot)$ , and a sequence of possibly infinite polls will converge to it. In absence of fixed point uniqueness Proposition 1 also tells us that if there is a fixed point  $\underline{x}^*$  such that  $\underline{x}^* < x^*$  with  $x^*$  being the fixed point corresponding to the complete information outcome, and if  $F'(1 - F(x))F'(x) < 1$  is satisfied in the interval  $[0, \underline{x}^*]$ , then we will have a mass of undecided voters located in  $[\underline{x}^*, \bar{x}^*]$ , with  $\bar{x}^* = 1 - F(\underline{x}^*)$ . In this case, the polls cannot lead to *complete information*, and there will always be some positive mass of undecided voters in  $[\underline{x}^*, \bar{x}^*]$ , no matter how many polls will be conducted afterwards.

Inequality  $F'(1 - F(x))F'(x) < 1$  may seem a little strange; however it can be intuitively explained in the case of a symmetric distribution of the undecided at period 0. It provides a sort of upper bound of the the maximum of the probability density function, or putting it differently, an implicit lower limit on the variance of the distribution. When most of the mass of the undecided is located close to the mean of the distribution, their influence in the election is potentially so large that there can be no improvement in information in the sense of having less undecided citizens.

To illustrate the insight of the Theorem , we provide a couple of examples with different citizen preference distributions. First, assuming that  $F$  is uniform on the interval  $D = [-\rho_L, 1 + \rho_R]$ , we have that:

$$F'(1 - F(x))F'(x) = \frac{1}{(1 + \rho_R + \rho_L)^2}. \quad (10)$$

Given  $\rho_R + \rho_L > 0$ , then  $\frac{1}{(1 + \rho_R + \rho_L)^2} < 1$ , therefore the sufficient condition for uniqueness and convergence hold for all  $x \in [0, 1]$ . On the other hand, an extreme case when the condition fails is provided by a symmetric triangle distribution with small masses of partizan voters. In fact, a simple numerical application shows that for  $\rho_L = 0.05$  and  $\rho_R = 0.07$  and the triangle distribution, polls cannot improve the information since at least 98.5 per cent of the voters in  $[0, 1]$  will remain undecided. In Section 4 we provide further examples of distributions for which the minimum undecided mass can be between these two extremes examined here.

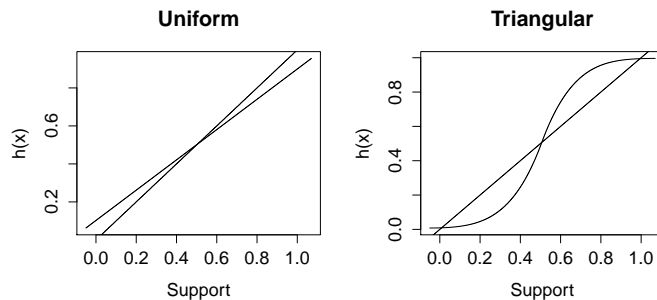


Figure 1: Examples of  $h(x)$

Figure 1 shows the plots of the  $h(\cdot)$  function for the two cases mentioned above. In the uniform case, there exists only the fixed point corresponding

to the complete information outcome, therefore, with enough polls, it can be achieved. On the contrary, in the triangular distribution case, we can see that there are three fixed points: the complete information one,  $x^*$ , in the middle, the left one,  $x_l$ , where  $l_t$  converges, and the third one  $x_r$ , which corresponds to the point  $r_t$  would converge to. It is easy to see that the condition for  $l_t$  to converge to the complete information outcome does not in fact hold. The two non-middle fixed points define the interval where the minimum possible mass of undecided citizens is located on.

In the Appendix, we examine an extension of the model that allows for some day-to-day political noise that can shift the ideological position of the citizens to the left or the right. The only significant difference is that in this case the polls can never help reach the complete information outcome, so there will always be some undecided voters. This extension provides some other interesting insights. In fact one implication is that the results of the model can follow even with potentially biased polls or with citizens lying, as long as the bias is relatively small and bounded. What this means is that if the results of polls are biased but this bias is a random variable that is drawn from a distribution of a known support, the voters can still calculate minimum and maximum implemented policies by taking into account the most extreme cases of bias possible. If the bias is relatively small the mechanism of the model still works with the only difference that in this case polls can never bring the society to complete information. On the other hand, if the bias is relatively big, polls are unable to offer any information to the undecided voters.

An important assumption that we have maintained so far is that agents respond honestly in polls. Even though it may be a strong assumption, a model with more strategic agents would be intractable. Even if some partizan voters have incentives to misrepresent their views, the discussion in the Appendix shows that our model can deal with this situation as long as the share of strategic agents is not too large. On the other hand, if all agents are strategic, then polls are not an accurate description of the electorate and in this sense they become useless as they provide no information to the undecided voters.

### 2.3 Three-party case

In this subsection we will briefly go over the three-party case. With three parties we assume that there is a party  $M$ , located between  $L$  and  $R$ . Moreover, we assume that there exists a mass of  $M$  partizans who, whatever the reason, have made up their mind and will vote for  $M$ . Given this the model still holds and no other voter except the  $M$  partizans vote for  $M$ . The reasoning is as follows. Since the implemented policy is a convex combination of all three policies, voting for the extreme parties has a greater effect on the implemented policy. As such, a voter that is interested in moving the implemented policy more towards his own optimal policy, say to the right, prefers to vote for  $R$  instead of  $M$ . This result can naturally be extended to the case of polls: no agent ever declares he will vote for  $M$ , with the exception of its own partizan voters.

### 3 Empirical Evidence

We use the January 2015 Greek election case to briefly go over some empirical evidence regarding the influence of the number of polls on the undecided vote and winner’s share. After the previous parliament failed to elect a new head of state constitutional provisions forced its dissolution. The announcement of the election took place on December 29 with the date set on January 25, making the campaigning period a little more than three weeks long. There were plenty of polls conducted by a number of public opinion polling organizations and companies. Most organizations had time to conduct four to five polls before January 25.

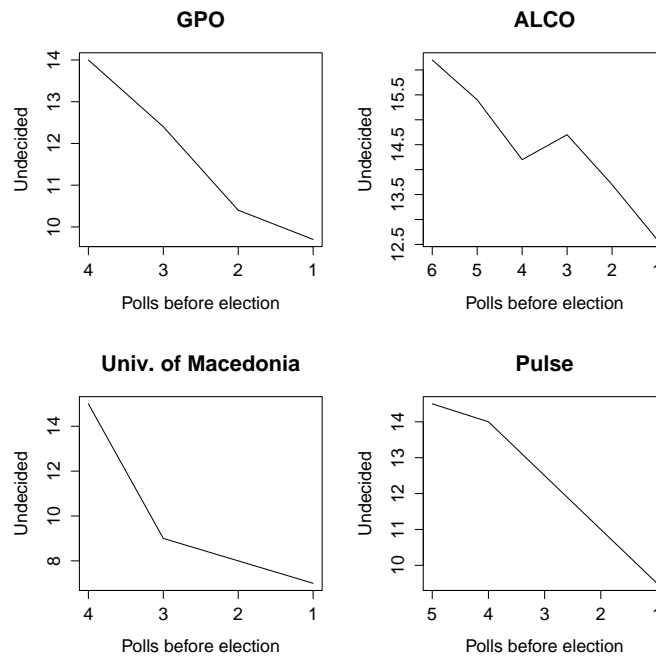


Figure 2: Polling for the January 2015 Greek general election

Figure 2 shows the percentages of undecided voters reported by four polling organizations:<sup>9</sup> GPO, ALCO, University of Macedonia Public Polling Unit and Pulse in chronological order. The  $x$ -axis in each of the graphs shows the number of polls conducted by each organization before the election took place.<sup>10</sup> Only polls conducted after the election were called are reported because at this moment the positions of the parties must be thought as given. Furthermore, under the assumption that respondents answered thoughtfully and given that the election was imminent, the answers are taken to represent the respondents’ true voting behavior.

<sup>9</sup>The data was compiled from a number of news sources and the polling organizations own reports.

<sup>10</sup>Meaning that 1 signifies the last poll before the election, 2 the second to last poll before the election and so on.

From the results presented here, with the exception of the University of Macedonia results, “undecided” means any voter who when asked what he would vote he did not give a party as an answer, or put it differently he is classified as “undecided” if he answered any of the following: “I will vote blank”, “I will abstain”, or “I don’t know/ I haven’t decided yet.” In reporting their results for the parties and “undecided” shares, the University of Macedonia first discarded those who indicated they would vote blank or that they would abstain, therefore their “undecided” share includes only those who specifically answered they had not decided yet. However they do report what was the share of original answers that were blanks and abstentions. Figure 3 shows the

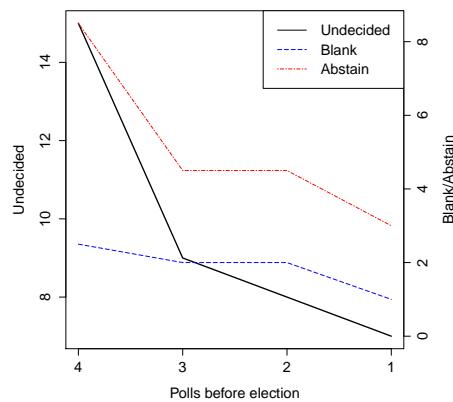


Figure 3: University of Macedonia polls

percentage of the undecided along with the percentage of blanks and abstentions. All graphs tell the same story: the percentage of the undecided voters, in any way we choose to define them, goes down with every single poll as the date of the election comes closer. In Figure 4 in the left panel we plot the frontrunner Syriza’s expected results of the four polling organizations together. When more than one organizations published results at the same day, we took the average of them. These results show, that abstracting from random noise, there is no monotonic effect of the number of polls on the predicted share of Syriza. This could imply the absence of a pure bandwagon or underdog effect in this election. At the same time, in the right panel, we plot the difference between the same Syriza shares depicted on the left one, and the ND (the second party in the election, leaving office) shares. The difference is positive, showing that Syriza was always the frontrunner, but it was not monotonic in the number of polls.

A finding of the theoretical model is that the citizens who are more likely to abstain are mostly the voters who are ideologically more moderate. We provide some evidence about that in the appendix. The important finding is that there is a positive relationship between being ideologically in the center and being undecided. This positive relationship along with the findings that more polls imply less undecided voters, and the non-monotonic effect of polls on the share of the winning party and its difference from share of its contender are all features that the theoretical model is able to accommodate.

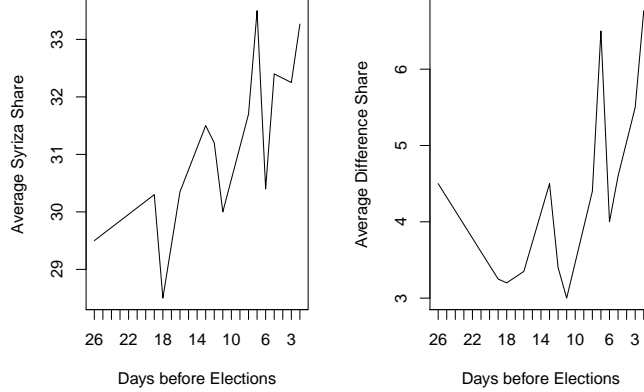


Figure 4: Syriza predicted share

## 4 Implemented Policy and Welfare Analysis

In this section we will discuss the influence of polls on the implemented policy and on welfare. From the preceding discussion it should be clear that the behavior of the undecided citizens on the day of the election does not affect the choice of the decided citizens: if after, say,  $k$  polls there is an election, then all citizens understand that the implemented policy will lie somewhere in  $[l_k, r_k]$  regardless of whether the undecided voters abstain, follow a decision criterion under uncertainty or simply randomize their choice. This is why the model is robust on the decision criterion used by the undecided citizens at the election day. For concreteness however we will assume in this section that the undecided citizens simply use the midpoint of  $[l_k, r_k]$  as a cutoff point:

$$x_U^k = \frac{l_k + r_k}{2}.$$

This means that all the undecided citizens to the left (right) of  $x_U^k$  vote for  $L$  ( $R$ ). Then, if there were an election right after the  $k$ -th poll we would have the implemented policy defined as follows:

$$x^k = 1 - F\left(\frac{l_k + r_k}{2}\right)$$

If the conditions for the complete information outcome hold then  $x^k$  will converge to  $x^*$ .

We assume that social welfare is represented by the utilitarian welfare function:

$$W(x) = \int_D v(|\hat{x} - x|) dF(x) \quad (11)$$

with  $\hat{x}$  being the implemented policy. Thus,  $W(x^k)$  is the social welfare that would be obtained if the election takes place after the  $k$ -th poll. In the case of

linear utility  $v(|\hat{x} - x|) = |\hat{x} - x|$ , the welfare function is strictly concave in  $\hat{x}$  as long as  $f(\hat{x}) > 0$  for all  $\hat{x} \in D$ , and it has a maximum at the point  $x^M$  that satisfies:

$$\frac{1}{2} = F(x^M)$$

i.e., at the median citizen. For  $v(|\hat{x} - x|)$  strictly concave in  $\hat{x}$  it is easy to see that this welfare function is also strictly concave in  $\hat{x}$  but its maximum will generally not correspond to the median citizen. For instance, in the quadratic case, the maximum occurs at the mean of the distribution. Focusing on the linear utility case, the welfare is increasing as the implemented policy is getting closer to the median citizen and decreasing when the implemented policy moves away from him. Notice that in case of complete symmetry, that is  $\rho_L = \rho_R$  and symmetric density function  $f(\cdot)$  around point 0.5, the implemented policy will stay at 0.5. In this case the maximum welfare possible is achieved no matter the number of polls and no matter the mass of undecided agents because with each poll each party gets the exact same mass of voters and they start with the same mass of partisans.

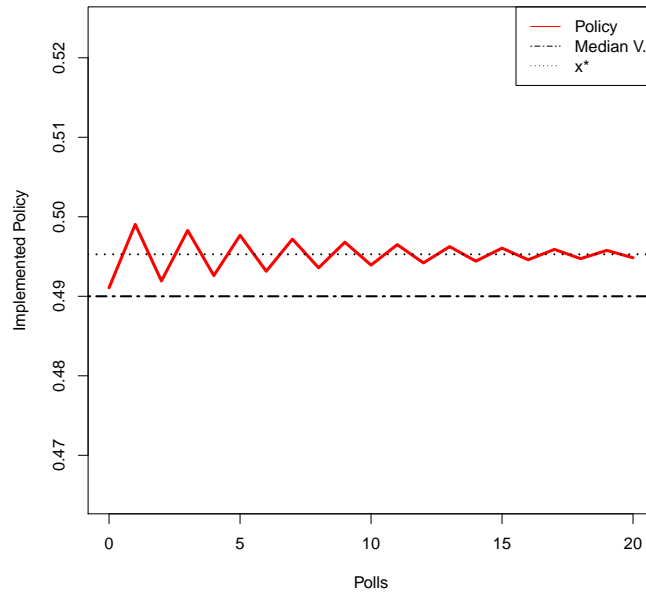


Figure 5: Uniform distribution example

The fact that welfare is strictly concave in the implemented policy can help us analyze the social welfare using the levels of the implemented policy rather than the levels of the welfare itself. In particular, if the density function is symmetric around the median, then welfare is a strictly increasing function of the distance of the implemented policy from the median voter. Take the case of



a uniform distribution of the voters and without loss of generality assume that  $\rho_L > \rho_R$  (since we have already discussed what happens in case of symmetric distribution and  $\rho_L = \rho_R$ ), an example of which, with  $\rho_L = 0.07$  and  $\rho_R = 0.05$  is depicted in Figure 5.

This figure shows how the implemented policy moves with every poll, and how far the implemented policy is from the median voter and the complete information outcome  $x^*$ . Since the welfare moves in a parallel way to the distance from the implemented policy to the median, the figures shows that neither the implemented policy nor welfare are monotonic in the number of polls. In fact there are polls that are welfare reducing: the first poll takes the implemented policy away from the median voter but also away from the complete information outcome. We have already seen that with the uniform distribution the complete information outcome can be reached. However this example shows that given a number of polls an additional one is not always better from a social point of view.

In the next example we will focus on a left-skewed Beta distribution with parameters  $\alpha = 1.4$  and  $\beta = 1.1$  and  $\rho_L = \rho_R = 0.05$ . In this case we do not have convergence to the complete information outcome: there will always be undecided voters and their mass cannot be lower than 67 per cent. See Figure 6.

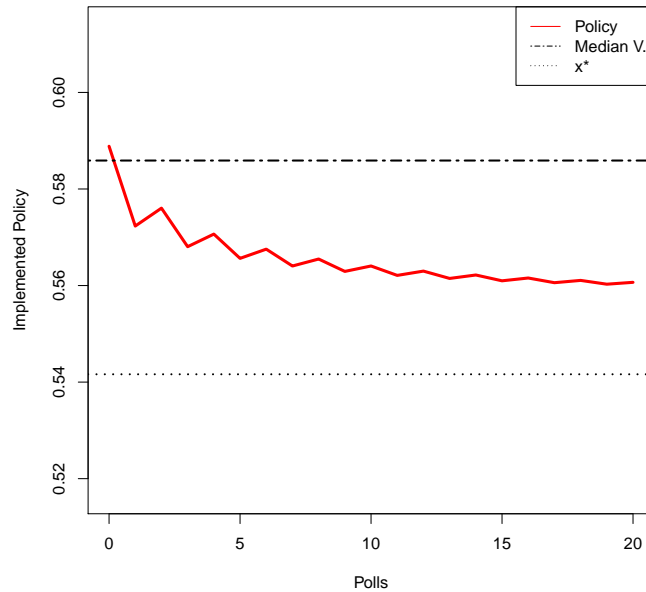


Figure 6: Beta distribution example,  $\eta = \theta = 0.05$

The implemented policy is decreasing on average, getting closer to the full information outcome, but it will never converge to it. The implemented policy converges to  $x = 0.56$ , however the complete information outcome is only at

$x^* = 0.54$ : the distribution is skewed to the left so there are relatively more undecided citizens located in ideologies close to  $R$ . The polls cannot reveal information to the more centrist undecided citizens. Welfare is on average decreasing with more polls, as the implemented policy is getting farther from the median citizen. The polls cannot lead to complete information and on average more polls make the citizens worse-off. Furthermore we can see that if there are not many polls conducted then the welfare has high variability from one poll to the other.

Summing up, if the conditions for the convergence to the complete information hold, then we know that a sequence of polls can lead us to the complete information outcome. However, even in this case, both the implemented policy and the citizen welfare are not necessarily monotonic in the number of polls.

An interesting question is if it is beneficial for the society to have plenty of polls before the election or whether there should be restrictions on their publication. The fact is that there is no way to know a priori if the polls will be welfare increasing or decreasing. There is an infinite number of possible distributions, and in some of them the welfare will be on average increasing and in some others on average decreasing. What we can say however, is that with more polls, and as the implemented policy is converging, the variability of the welfare will decrease and the outcome will tend to the complete information one as much as possible.

## 5 Conclusions

In this paper we examine a proportional representation voting model with polls in which the distribution of voter preferences is unknown. The model captures three actual qualitative features, that to the best of our knowledge cannot all three be replicated by any other model. First, the lack of monotonicity of the share of the leading party in the number of polls, second, the mass of undecided voters is decreasing in the number of polls, and third, there is a positive correlation between having a centrist ideology and being undecided. The most important finding of the paper is that although each poll weakly decreases the mass of undecided voters, a sequence of polls may not always be able to decrease this mass to zero and reveal the complete information outcome to the citizens. On top of that, there might be polls that actually push the society away from the complete information case: if the last poll before the election is such a poll, and we are interested in having an election result as close to the complete information as possible, this poll should not have happened. On the other hand, this bad influence can be remedied with a new extra poll. However without information about how the distribution of preferences looks like we cannot know if the final extra poll before the election has a beneficial effect or not. However, if the number of polls is sufficiently large the outcome approaches as much as possible to the complete information case.

## A Appendix

### A.1 Position Uncertainty

Even though the citizens have preferences over the implemented policy, it can often be the case that their opinions can change from day to day, due to random political noise. The assumption that this section maintains is that, while a citizen's ideological position can change due to random shocks, this position will not stray to far away from his initial ideological position. We will model this by assuming that the optimal policy for a voter with initial position  $x$  can move randomly around a small interval  $[x - \epsilon, x + \epsilon]$ , with  $0 < \epsilon < \min\{\rho_L, \rho_R\}$ . This is the only difference with the previous model. That is, using the distribution of voter optimal policies we have previously defined as a reference, each voter's optimal policy can change from day to day, moving randomly in this small interval. For simplicity we assume that partizan voters do not get the shock.

The timing of the new set-up is as follows. At time  $t$  the citizens calculate  $[l_t, r_t]$ , a poll takes place to which the citizens respond truthfully and then a shock takes place. Afterwards we can have the election, or we start over with a new poll. The shock is not necessarily common to all voters. Now at the beginning of the game, each voter  $x$  is aware of their initial optimal policy and the interval  $[x - \epsilon, x + \epsilon]$  that their optimal policy can fall into.

These random shocks from day to day, obviously affect the distribution as they are moving masses of voters around. There are two interesting "perturbed" distributions: the one that results if *all* voters receive the most extreme negative shock meaning that all voters move the most they can towards  $L$ , which we call  $F_{-\epsilon}(\cdot)$  and one that results if *all* voters receive the most extreme positive shock (the one that moves all the most towards  $R$ ):  $F_{\epsilon}(\cdot)$ .

Before the first poll we know that  $l_0 = 0$  so the implemented policy cannot be lower than 0, and  $r_0 = 1$ , so the implemented policy cannot be higher than 1. Everybody then responds truthfully. Now however, the set of types that during the second poll announce that they will vote for  $L$  becomes:

$$X_L^1 = \{x \leq l'_1, \text{ where } l'_1 = 1 - F(1) - \epsilon\}$$

And similarly for  $R$ :

$$X_R^1 = \{x \geq r'_1, \text{ where } r'_1 = 1 - F(0) + \epsilon\}.$$

Citizens who are in  $[0, \epsilon]$  cannot announce they will vote for  $L$ , because they understand that their shock might push them to the other side of 0. On the other hand, citizens who are in  $[1 - \epsilon, 1]$  cannot announce they will vote for  $R$ . From this example it should be clear that the interval where undecided voters fall in each period is simply:  $[l_t - \epsilon, r_t + \epsilon]$ .

The dynamic process is now:

$$l_t = 1 - F(r'_{t-1})$$

$$r_t = 1 - F(l'_{t-1}).$$

The proof of existence and uniqueness of the voting equilibrium follows the same lines as before and is therefore skipped. Rewriting the previous two equations and plugging the second one into the other we get:

$$l_t = 1 - F(1 - F(l_{t-2} - \epsilon) + \epsilon). \quad (12)$$

If similarly to Section 2 we define the function:

$$h_\epsilon(x) = 1 - F(1 - F(x - \epsilon) + \epsilon), \quad (13)$$

we see that all the conditions for the existence of a fixed point still hold. Now however, there is no fixed point that corresponds to full information in the sense we have defined it previously. In other words, there does not exist a fixed point where all the voters have made up their minds.

**Proposition 1.** *The complete information outcome cannot be achieved with uncertainty in preferences.*

*Proof.* Suppose not. Then there exists an  $s$  such that  $l'_s = r'_s = l'_{s+1} = r'_{s+1} = l'_* = r'_*$ . Then we have to have:

$$l_* - \epsilon = r_* + \epsilon$$

or,

$$l_* = r_* + 2\epsilon$$

which is a contradiction as  $l_*$  cannot be greater than  $r_*$ .  $\square$

We can also see that the results of Section 2 follow immediately using:

$$F'(1 - F(x - \epsilon) + \epsilon)F'(x - \epsilon) < 1 \quad (14)$$

The natural next question is to find the fixed point that corresponds to the “second best”, where the mass of the undecided voters is as low it can get. An obvious candidate fixed point is a point where  $l_* = r_* = x^*$ , that is, the point that there is no uncertainty about the position of the implemented policy anymore. Then, we will have  $l'_* = x^* - \epsilon$  and  $r'_* = x^* + \epsilon$ .

**Proposition 2.** *There is no fixed point of  $h_\epsilon(\cdot)$  of the form  $l_* = r_* = x^*$ .*

*Proof.* Suppose that time  $s$  is the first time we observe  $x_L^s = x_R^s$ . Then the mass of undecided voters becomes  $F(r_s + \epsilon) - F(l_s - \epsilon)$ , and we have:  $l_{s+1} = (F(l_s - \epsilon) + F(r_s + \epsilon) - F(l_s - \epsilon)) \times 0 + (1 - F(r_s + \epsilon)) \times 1 = 1 - F(r_s + \epsilon)$ . Similarly  $r_{s+1} = 1 - F(l_s - \epsilon)$ , showing that  $l_{s+1} \neq r_{s+1}$ .  $\square$

Proposition 2 tells us that there cannot be a situation where there is no uncertainty about the implemented policy, which further implies that the ideological distance between the last voter to vote for  $L$  and the first voter to vote for  $R$  cannot be less than  $2\epsilon$ . The exact distance cannot be pinned down without first knowing the distribution function.

What we take from the preceding analysis is that the added uncertainty of preferences makes not only the full information outcome completely unattainable, but also puts a lower bound on the measure of undecided voters. As  $\epsilon$  is getting smaller, more and more people are voting and we are getting closer and closer to full information.

If  $\epsilon$  becomes too big ( $\epsilon > 1$ ) then polls essentially break down: only the first poll improves the information of the individuals because the only citizens to ever respond to the polls are the partizan ones.

## A.2 Further Empirical Analysis

The World Values Survey<sup>11</sup> aims to provide insight on the political, social, economic, and in general life attitudes of citizens from 59 countries. To this end, it utilizes an extensive questionnaire. Using a measure of Undecidedness the interest lies in showing that there is positive correlation between that and being a moderate voter. In particular, the specification to be estimated is the following:

$$\begin{aligned} Undecided_i = & \beta_0 + \beta_1 Center_i + \beta_2 NeverNational_i \\ & + \beta_3 NeverLocal_i + \beta_4 NoTrust_i + \epsilon_i \end{aligned} \quad (15)$$

Where  $i$  refers to a respondent and *Undecided* is a variable that takes the value 1 if the respondent indicated that, in case there was an election the next day, he either would not know what to vote, he would vote null or blank, or not vote at all, and 0 otherwise. In the survey, Question 95 asks the interviewees to state where they position themselves in the Left-Right political spectrum, giving a number from 1 to 10, with 1 being Left and 10 being Right. The answers to this question constitute the variable *Ideology*. Using the stated *Ideology* we constructed a series of variables called *Center* as follows: *Center56* assigns the value 1 if the respondent declared he places himself on 5 or 6 (*Ideology* = 5 or *Ideology* = 6) and 0 otherwise, and similarly for *Center4567* and *Center345678*. The variables *NeverNational* and *NeverLocal* take the value 1 if the respondent indicated that he never votes in national or local elections respectively and 0 otherwise. *NoTrust* is a variable that takes the value 1 if the respondent answered that he does not trust political parties in the country and 0 otherwise. *NeverNational* and *NeverLocal* are included to control for voters that (according to their own answer) would never vote in elections, no matter who was running. Similarly, *NoTrust* controls for voters that are in general mistrustful towards parties which can make them vote less. Using the full country data and after dropping observations for which there were no valid answers for the variables examined, there are 53734 observations.

Table 2 depicts the results of the least squares estimations of (15) using the three different *Center* definitions and with and without Country fixed effects. Country fixed effects are implemented by simply adding a dummy variable for each country, the coefficients of which are not reported in the table for the economy of space. Since these estimations are of linear probability models, the results of columns (1) to (3), are found by employing weighted least squares, using the fitted values of a first step ordinary least squares estimation to construct appropriate weights.<sup>12</sup> The last three columns use a heteroskedasticity-robust variance-covariance matrix for calculating the standard errors because the WLS method was not appropriate.<sup>13</sup> The results of each column tell the same story: voters that never vote in elections are naturally more likely to be Undecided.

More interestingly, the results show that belonging ideologically to the center implies a higher probability of being Undecided. Columns (1) and (4) define the

<sup>11</sup>WORLD VALUES SURVEY Wave 6 2010-2014 OFFICIAL AGGREGATE v.20141107. World Values Survey Association ([www.worldvaluessurvey.org](http://www.worldvaluessurvey.org)). Aggregate File Producer: Asep/JDS, Madrid, SPAIN

<sup>12</sup>More precisely, each model was estimated by OLS first. Then we calculated the estimated standard deviation as follows:  $\hat{\sigma}_i = (\hat{y}_i(1 - \hat{y}_i))^{1/2}$ , provided that  $\forall i \ 0 < \hat{y}_i < 1$ . Then the WLS estimation was conducted by simply using  $1/\hat{\sigma}_i$  as weights.

<sup>13</sup>Since not all fitted values of the initial OLS were such that  $0 < \hat{y}_i < 1$

Table 2: LS Estimation Results of Specification (15)

	(1)	(2)	(3)	(4)	(5)	(6)
Constant	0.078*** (0.003)	0.071*** (0.003)	0.068*** (0.003)	0.136*** (0.023)	0.118*** (0.023)	0.108*** (0.023)
Center56	0.097*** (0.004)			0.080*** (0.003)		
Center4567		0.077*** (0.003)			0.075*** (0.003)	
Center345678			0.059*** (0.004)			0.071*** (0.004)
NeverNational	0.018* (0.008)	0.017* (0.008)	0.017* (0.008)	0.067*** (0.007)	0.066*** (0.007)	0.066*** (0.007)
NeverLocal	0.177*** (0.008)	0.180*** (0.008)	0.183*** (0.008)	0.148*** (0.007)	0.150*** (0.007)	0.153*** (0.007)
NoTrust	0.108*** (0.003)	0.110*** (0.003)	0.114*** (0.003)	0.057*** (0.003)	0.058*** (0.003)	0.059*** (0.003)
Observations	53,734	53,734	53,734	53,734	53,734	53,734
Fixed Effects	No	No	No	Country	Country	Country
R <sup>2</sup>	0.235	0.234	0.233	0.241	0.240	0.237
Adjusted R <sup>2</sup>	0.235	0.234	0.233	0.240	0.239	0.236
Unimplemented multicol						

center in the strictest way. There we see that being in the center, *ceteris paribus*, increases the probability of being Undecided by 9.7 and 8 per cent respectively. As the definition of the center is expanded, the effect of being in the center on the probability of being undecided is still positive and significant but falls monotonically. This shows that it is indeed the more ideologically central voters that are more likely to be undecided.

We have implemented various robustness checks and this basic result holds. These checks are available upon request.

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