

Divide the Dollar, A Model of Interregional Redistributive Politics

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Abstract

We develop a dynamic political model of dividing a fixed amount of money among n districts. There are two political parties that make proposals on such divisions and compete in each district. Each district elects a representative to a legislature. Each party is governed by its representatives who are elected to the legislature. Voters are myopic and there is a slight incumbency advantage. We show that if all districts are of the same size then they all get the same share in the long run.

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1 Introduction

This paper develops a dynamic model of interregional distributive politics. We conceive of a society consisting of n identical districts. The society shall divide a fixed resources among the districts. This problem is the good old problem of dividing a dollar - a standard zero sum game. The division of the resources is done through the political process. Voters in each district elect a representative of the legislature. There are two parties in the society, which compete in every district. We assume that the party which gets a majority of representatives in the legislature has the power to implement the division it prefers. Each representative is assumed to be interested only in the share of the resources his own district will get, so the representatives are policy motivated but *district-egoistic*.

The question in which we are interested is what will happen in the long run. It is well known that the one shot problem of dividing the dollar is not well behaved. Our question is whether reasonable political institutions will lead to exploitation of certain districts, or whether in the long run all districts tend to get equal treatment. The result of the model is the latter, the long run average of the share of the resources that each district gets is the same for all districts.

The model rests on four important assumptions: 1. Political *parties are governed by successful politicians*, those politicians who are representatives in the legislature. Since these politicians typically change over time, this means that the objective of a party will change over time. 2. The representatives of the different districts who are member of the legislature have equal bargaining power in the formation of their party's policy. 3. All else equal voters have a

slight preference for the *incumbent*. 4. The voters and politicians are *myopic*, they base their decisions in a period on utility maximization in this period.

Each of these assumptions may be questioned of course. We have made them since we believe they have some descriptive value. In particular, the assumption of myopia may seem controversial. One might argue that today's politics may (and in the model will) influence events in the future. While this may be true, the logical step then is to solve for the full-fledged Markov perfect equilibrium. On the other hand, this is assuming a lot of rationality on part of the voters, and one may have doubts whether this has much descriptive value. Perhaps a reasonable case lies somewhere in-between myopia and a Markov perfect equilibrium. We have chosen to consider the case where the voters are myopic, and postpone the other case to future research. It should be considered as a benchmark case, but, we believe, an interesting benchmark case. In Section 3 we also provide some results for the case in which voters are myopic but political parties care about the current and future period.

In each period, one of the parties will win a majority of districts, this party's policy will be implemented and the party will be the incumbent of the next period. We assume that the party's policy in the next period is the same as the one it won the elections with. The other party, the challenger, proposes a new policy - a new division of the dollar. Importantly, this policy is chosen by those members of the legislature who represent this party (see von Beyme[7]). This is the assumption that a party is governed by successful politicians, they will propose a policy which is the best possible for them - them alone - subject to the constraint, that the party has to win a majority

in the legislature in order to implement its policy. A new election is held in each district and the new winner implements its policy. This continues ad infinitum. In each period the model works as if the challenger is a kind of Stackelberg follower. Not surprisingly, there is a huge advantage to being a follower in this model, the challenger always wins the election. Its winning strategy is to persuade sufficiently many of the districts who voted for the other party in the last period by offering them slightly more - ϵ more because of the incumbency advantage - than the incumbent offered them, and share the rest among the representatives of the party who are in the legislature. The districts which will be offered something are the “cheapest to buy”, those districts who got less among the winning districts in the last period, these are the “switching” districts¹. Switching districts suffer in the short run, since their representatives will not participate in the formulation of the parties strategies. So, in the short run they may end up getting small amounts, but these amounts will increase at least by ϵ at each period. In the long run the ϵ 's build up. Because of this the switching districts eventually become as well off as the other districts. The dynamics of the model then enter a second phase, where each district gets almost the double of its equalitarian share every second period, namely when the party to which the districts representative belongs formulate policy and win the election. In this way the long run average share of all districts become equal.

Hence, the conclusion seems to be that although the problem of dividing the dollar potentially is very badly behaved, voters are very myopic, and representatives are district-egoistic, the political process assumed in this paper

¹We could also call them “marginal” districts as opposed to the “safe” districts.

leads to fair sharing. No district will be exploited in the long run.

Snyder[6] develops a model in which the policy proposed by each party is chosen by the set of party members who hold office. The party that wins a majority of seats in the legislature will implement its proposed policy. It is shown that party proposals will not converge. The intuition is that “...all incumbent legislators prefer this situation to one where the platforms converge, because divergence greatly improves their own chances of reelection” (p.202). Our model also presents this feature since the challenger always proposes a policy quite different from the one implemented by the incumbent. There are, however, many important differences between the two models. For example, in Snyder [6] there is no incumbent advantage, the analysis focuses on a stable legislature instead of on an evolving dynamic process, and the incumbent can chose a different policy from the one previously implemented. Moreover, Snyder [6] does not consider a distributional zero sum game problem between districts. Lindbeck and Weibull [4] study a problem of redistribution of given resources between groups of individuals (a zero sum game) in a two political party model. Even though these groups of individuals can be seen as the districts in our model their approach is quite different from the one developed in this paper. They analyze existence of equilibrium in a one period game and assume that each of the political parties tries to maximize its expected plurality. McKelvey and Riezman [5] analyze an infinitely repeated divide-the-dollar game in which the legislators may chose a seniority system for the current session and then proceed to a division of the dollar among the districts. They also obtain that districts outside the winning coalition get nothing. In contrast to our approach, McKelvey and Riezman only consider

one legislature and, what is more important, the relevant players are *individual legislators*, while we develop a dynamic *partisan* model of distributive politics. Jackson and Moselle [3] study a legislative voting game building on the seminal paper of Baron and Ferejohn [1]. Legislators can form political parties and the policy space contains an ideological and a distributive dimension. They, however, consider a one period game and focus on the interaction between the ideological and distributive dimensions. Finally, Chari, Johnes and Marimon [2] also consider a game where voters in different districts vote for congressmen who will benefit their district. Chari et al., however focus on explanations for split voting and do not consider the dynamic evolution over time.

2 The model

There is an odd number of districts, n . Each district has the same population and within a district all voters are identical. There is one dollar to be divided among the districts in each period t , $t = 0, 1, 2, 3, \dots$

There are two parties a and b , which compete in all districts. The parties have representatives in each district, and in each period there is an election where each district elects a representative. The elected representatives form the legislature. The legislature will sit for one period, then a new election is held and so forth. We assume that the party which has majority in the legislature in a period implements its announced policy. A policy is a division of the dollar, for example a policy for party a in period t is $x_{at} = (x_{at}^1, \dots, x_{at}^n)$, where $\sum_{i=1}^n x_{at}^i = 1$ and $x_{at}^i \geq 0$. When no confusion is possible we suppress

the subscript t .

The political process is supposed to work in the following way. In each period, before the election, the party with majority in the legislature is the incumbent, the other is the challenger. At the initial period $t = 0$, party a has majority and party a 's policy x_{a0} is implemented. In period 1, party a is the incumbent, we assume that if party a wins the election in period 1 it will again implement its policy from the last period x_{a0} . Hence, once a party has decided on a policy it sticks to this policy as long as it is the incumbent. An alternative interpretation is that voters will judge party a on its merits; regardless of what it says, it will be expected to continue the policy voters already have seen it implemented. The challenger, party b , proposes an alternative policy, x_b . In each district voters can vote on either party. We will assume that voters care about their own district only. Voters in district i derive utility from the resources directed to district i . All voters have the same utility function $u(x^i)$, $u' > 0$, $u'' < 0$. Voters judge policies by how much they benefit the district, but there is also a slight incumbency advantage. If e.g. party a is the incumbent, voters in district i vote for party b if and only if $x_b^i \geq x_a^i + \epsilon$, where $\epsilon > 0$ (and small). In this sense voters vote sincerely. As will be seen below this (slight) incumbency advantage—which we see as realistic—is very important for our result. Let $n(x_a, x_b; p)$ be the number of districts which vote for party a when policies are (x_a, x_b) and party p is the incumbent. The winner of the election implements its policy and become the incumbent of the next period.

A key assumption is that the policy of a challenging party is chosen by the party's members of the legislature in a given period. Of course these

members may change over time and in this way the party's preferences and policy may change over time. We will use the language that a *district belongs* to a party in a period, when the district voted for this party in the previous period. Each representative is assumed to represent the voters of his district, he derives utility from the resources directed to his district only. In this way the party's members have different interests, we assume that the bargaining within a party results in the policy which maximizes the sum of utilities of the members of the party in the legislature. Thus if B is the (non-empty) set of districts belonging to b , i.e. the districts that voted b in the last election, party b proposes the policy x_b which is the solution to

$$\begin{aligned} \max_x \sum_{i \in B} u_i(x^i) \\ \text{s.t. } n(x_a, x; a) < \frac{n}{2} \end{aligned} \tag{1}$$

In principle, one could imagine that it was better for the party to aim at letting party a win the election and implement its policy. However, as will be clear from the proof of the following Lemma, this is not so. Intuitively, the reason is that the amount of resources to be distributed is the same regardless of which party is in power and it always pays to take resources from (most of) the districts that voted for the other party in the previous election.

Lemma 1 *Assume that the districts belonging to the incumbent all got a strictly positive share. For ϵ sufficiently small, the challenger always wins. Furthermore, it wins in exactly $\frac{n+1}{2}$ districts. The districts who do not vote for the challenger will get zero resources in that period.*

Proof. W.l.o.g. assume that party a is the incumbent and let x_a be the already implemented policy. Let the set of districts belonging to b be B , and

the number of such districts $|B|$. The amount of resources directed to these districts by party a 's policy is $\sum_{i \in B} x_a^i < 1$. The districts which belong to a (the districts that voted a in the last period) hence share $1 - \sum_{i \in B} x_a^i$. Pick the $\frac{n+1}{2} - |B|$ districts who belong to a and who got least from a 's policy. At maximum they got in total

$$\frac{\frac{n+1}{2} - |B|}{\frac{n+1}{2}} \left(1 - \sum_{i \in B} x_a^i \right)$$

The rest of the a districts got at least

$$\frac{|B|}{\frac{n+1}{2}} \left(1 - \sum_{i \in B} x_a^i \right) > 0.$$

Consider the policy of giving ϵ more to each of the $\frac{n+1}{2} - |B|$ districts that voted for a and got least from a 's policy. Then they will vote now for party b . The remaining resources of the districts that belong to a amount to at least,

$$\frac{|B|}{\frac{n+1}{2}} \left(1 - \sum_{i \in B} x_a^i \right) - \left(\frac{n+1}{2} - |B| \right) \epsilon \quad (2)$$

and for ϵ sufficiently small this expression is strictly positive. The amount of resources in 2 can be distributed among the districts in B , such that they all get more than x_a^i and hence will vote for b again.

Clearly, it's a waste of resources, from the point of view of the districts in B , to make more a districts vote for party b , therefore b wins in exactly $\frac{n+1}{2}$ districts and the rest of the districts get zero resource. ■

The Lemma already tells quite a lot about the dynamic evolution of this game. In each period, the challenger will win in $\frac{n+1}{2}$ districts. Furthermore, these are the only districts which will get anything in that period. The remaining n districts will get nothing². These losing districts' representatives

²This must be understood, for example, as if these districts get the minimum given some possible legal or social constraints on the division of the resources among districts.

will form the challenging party in the next period. Given they are offered zero from the incumbent party, it is clear that the solution to (1) will be to buy the district among the incumbent's that got least. Hence, offer this district ϵ more than it will get from the other party, and share the remaining resources equally. Thus in each period, all districts belonging to the incumbent receive the same amount of resources except possibly the district which shifted in that period. This district may get very little. Let x_t^s be the resources diverted to the switching district in period t , then the other incumbent districts each gets

$$\frac{1 - x_t^s}{\frac{n-1}{2}}$$

In the next period, if

$$x_t^s < \frac{1 - x_t^s}{\frac{n-1}{2}}, \quad (3)$$

then district s is still the cheapest to buy and it will be offered $x_t^s + \epsilon$ by the challenger. This process will continue in every period t in which (3) is fulfilled. But as time passes the ϵ 's will build up. Eventually, at time t' say, the district will get at least as much as the other districts. At this time

$$\frac{1 - x_{t'}^s}{\frac{n-1}{2}} + \epsilon \geq x_{t'}^s \geq \frac{1 - x_{t'}^s}{\frac{n-1}{2}}$$

If the last inequality is strict, then another district will be bought by the challenger, all of the incumbents' other districts get the same, so they have an equal chance of getting bought. Now the dynamic enters a new phase. The district which switched will get *more* than the other districts. Hence, in the next period it will not be the cheapest district to buy, and hence not the switching district³. Now the switching district will change every period. If

³We could say, using the standard terminology in Political Science, that such a district

we assume that *districts are chosen at random when they are equally cheap to buy*, we then get the result that as time tends to infinity, then by law of large numbers, each district will be the switching district in $\frac{1}{n}$ of the periods. This means that we have

Theorem 2 *In the long run all districts get the same share of the resources, i.e.*

$$\lim_{t \rightarrow \infty} \frac{\sum_{\tau=1}^t x_{\tau}^i}{t} = \frac{1}{n} \text{ for all } i.$$

Thus every district gets its fair share in the long run. It is also important to analyze the behavior of the shares of the switching and non-switching districts at each period. One can easily show that the share of the switching district converges to

$$\frac{1 + \epsilon \left(\frac{n-1}{2} \right)}{\frac{n+1}{2}}$$

and the share of any other district in the winning coalition (i.e. voting for the incumbent) converges to

$$\frac{1 - \epsilon}{\frac{n+1}{2}}$$

If the incumbent advantage ϵ is large, it makes a big difference to be the switching district. In this case, however, the phase in which each period has a different switching district is reached very soon. When ϵ is small, on the contrary, the same district can be the switching one for many consecutive periods but the difference in payoffs with the other districts is also small.

is now a “safe district.”

3 Forward looking parties

In this section we relax the assumption that parties are myopic, i.e. the assumption that they care only about the current period. Voters, however, will remain myopic and behave in the same way as in the previous sections. Here a representative cares about today and tomorrow resources. Thus, the utility of representative of district i at period t is given by

$$u(x_t^i) + \beta u(x_{t+1}^i) \quad (4)$$

where β is the time discount factor. The analysis in the previous sections corresponds to the case $\beta = 0$.

The optimal proposal by the challenger, say b , at time t , x_{bt} , depends on the incumbent's policy, x_{at-1} , and on the strategy party a will play at period $t + 1$. Say that, at equilibrium, the optimal proposal is given by $x_{bt} = \phi_t(x_{at-1})$. We don't need to be specific about the equilibrium concept to solve this dynamic game. The only important thing to note now is that to find out its optimal proposal a party has to take into account the consequences in the current and in the next period. Thus, given the incumbent's strategy and policy x_{at-1} , party b proposes the solution to

$$\begin{aligned} \max_{x_t} \sum_{i \in B} u(x_t^i) + \beta u(\phi_{t+1}^i(x_t)) \\ \text{s.t. } n(x_{a,t-1}, x_t; a) < \frac{n}{2} \end{aligned} \quad (5)$$

This maximization problem is similar to the one in (1) but now the party has to take into account the consequences of its choice of platform in the current and future periods⁴. I.e., the challenger knows that the implemented

⁴In principle, this maximization problem is not well defined: the feasible policies could

policy in period $t + 1$ will depend on its proposal for period t . It is difficult to get general results characterizing the solution to this game. There are, however, some important observations that can be made comparing this model and the “myopic” one.

The two main differences between the optimal proposal here and the optimal proposal in the previous model are: 1) It might be the case that the challenger buys a district by increasing its share by an amount of resources greater than ϵ . This might be optimal if, by doing that, the switching district obtains a higher share than the one obtained by the challenger’s districts. In this way, one (or several) of such challenger’s districts will be the switching one in the next period. 2) The optimal proposal might be such that the challenger wins in more than $\frac{n+1}{2}$ districts. Thus, it is not always the case that the challenger buys the minimum number of districts to win the election. The reason is also clear. It could be optimal for the challenger to offer a higher share to a larger number of the incumbent districts since that will imply that in the next period a larger number of its own districts will become the switching ones.

Hence, one might conclude that our original model was not robust to the introduction of some slight concern for the future. However, this is not the case. When the parameter β is positive and small enough the optimal proposals coincide with the ones in the myopic model. To see this, we just have to notice that if the optimal proposals don’t coincide in both models form an open set. The incumbent might want to offer an amount of resources to the switching district greater than the resources proposed to its own districts but as close as possible to it. However, this is just a technical problem without much economic meaning and, to get rid of it, we will assume that there is an smallest possible unit, say a dollar.

the switching districts have to obtain more in this model and the challenger districts less than in the myopic case. However, it is clear that for β small enough this cannot be optimal. Thus, our results are robust to a slight relaxing of the myopia assumption. Moreover, one can also show that for values of β high enough so that the the myopic model behaves in a different way than the one in this section, if the optimal proposal is always such that the challenger wins with $\frac{n+1}{2}$ districts, in the long run we get the same result as in the myopic case: all districts get the same expected share. The intuition is simple. In the periods in which the switching district gets an increment of ϵ the model behaves as in the myopic case. The only possible divergence between the two models happens when the challenger increases the share of the switching district by an amount greater than ϵ . But in this case, as mentioned above, the switching district gets a larger share than the one obtained by the challenger's districts. Thus, in the next period there will be a new switching district and the dynamics is exactly the same as the one described in the explanation of Theorem 2.

4 Concluding remarks

In this paper we have developed a partisan model of distribution of a fixed amount of resources among districts. Evidently, this is at best a simplifying assumption, in the real world transfers among districts are used for public goods, personal transfers and public consumption. These transfers are acquired by society at a cost, they are the proceeds of taxation, and as is well-known, taxation is distortionary. An extension of the model would take

these things serious and will be the subject of further research.

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