MICROECONOMÍA II. 2004-2005. PRUEBA 2.

$F(K,L) = (K^2+LK)^{1/2}$

(a) $F(tK,tL) = \left((tK)^2 + (tL)(tK)\right)^{1/2} = (t^2K^2 + t^2LK)^{1/2} = t\,(K^2+LK)^{1/2}$

$= tF(K,L)$ para todo $t>1$ → Rendimientos constantes de escala

(b) $w=1$ $r=4$ corto plazo $\bar{K}=1$

$q = (K^2+LK)^{1/2} = (1+L)^{1/2} \rightarrow L = q^2-1$

Los costes totales a CP son:

$C(q) = wL + r\bar{K} = (q^2-1) + 4 = 3+q^2$

$CM(q) = 2q$ $\quad CMe(q) = \frac{3+q^2}{q} = 3/q + q$ (donde $CVMe = q$; $CFMe = 3/q$)

(c) $\begin{cases} p = CM \text{ si } p \geq CVMe\,min \\ q = 0 \text{ si } p < CVMe\,min \end{cases}$

Para calcular CVMe min igualamos $CM = CVMe$

$2q = q \rightarrow q = 0 \rightarrow CVMe\,min = 0$

El precio mínimo de entrada es cero.

Entonces $p = CM$ si $p \geq 0$

$p = q \rightarrow \boxed{q = p}$ función de oferta individual a CP

(d) Para calcular los costes totales a LP resolvemos: $\min_{L,K} wL + rK$ sujeto a $F(K,L)=q$

Condiciones de optimalidad

(C1) $q = (K^2+LK)^{1/2}$

(C2) $RMST = \frac{w}{r} \rightarrow RMST = \frac{PM_L}{PM_K} = \frac{\frac{1}{2}(K^2+LK)^{-1/2}\,K}{\frac{1}{2}(K^2+LK)^{-1/2}(2K+L)} = \frac{K}{2K+L}$

(C1) $q = (K^2+LK)^{1/2}$

(C2) $\frac{K}{2K+L} = \frac{1}{4} \rightarrow 4K = 2K+L \rightarrow 2K = L$

Sustituyendo (C2) en (C1):

$q = (K^2 + 2K^2)^{1/2} = (3K^2)^{1/2} = K\sqrt{3} \rightarrow K^* = q/\sqrt{3}$

$L^* = 2K^* = \frac{2q}{\sqrt{3}}$

Los costes totales a LP son:

$C(q) = wL^* + rK^* = \frac{2q}{\sqrt{3}} + 4\frac{q}{\sqrt{3}} = \frac{6q}{\sqrt{3}} = 2\sqrt{3}\,q$

$CM(q) = 2\sqrt{3}$ $\quad CMe(q) = 2\sqrt{3}$