about organic flourishing, we have yet to be given reasons for thinking that these judgements are of a factual nature. This is not the place to explore the proper understanding of such judgements in detail, but it is worth noting the attractions of a simple error theory. On this view, we mistakenly believe species to have natures of the kind that enable us to make judgements of the form Foot has alerted us to. We might go on to argue that the failure to reduce statements of these kinds to those of a more respectable biological variety does not show the existence of non-natural facts about species' natures, nor does it point us in the direction of an Aristotelian account of function. It shows, simply, that we are mistaken about the nature of the natural world. I have not argued in detail for such an error theory here, but I suggest that the problems faced by any account of the alleged facticity of claims about species' flourishing and species' natures boost the plausibility of such accounts considerably.[1]

University of Cambridge
Department of History and Philosophy of Science
Free School Lane, Cambridge CB2 3RH, UK
tml1000@cam.ac.uk

References

Foot, P. 1961. Goodness and choice. *Proceedings of the Aristotelian Society* Supp. Vol. 35: 45–60.

Foot, P. 2001. *Natural Goodness*. Oxford: Oxford University Press.

1 An earlier version of this article was presented at a meeting on 'La notion de fonction: des sciences de la vie à la technologie' at the Collège de France, Paris, in May 2008. I am grateful to the audience there for comments, and to Hallvard Lillehammer for discussion. Funding: This work was supported by the Leverhulme Trust and the Isaac Newton Trust.

Monty Hall drives a wedge between Judy Benjamin and the Sleeping Beauty: a reply to Bovens

LUC BOVENS AND JOSÉ LUIS FERREIRA

Bovens (2010) points out that there is a structural analogy between the Judy Benjamin problem (JB) and the Sleeping Beauty problem (SB). On grounds of this structural analogy, he argues that both should receive the same solution, viz. the posterior probability of the eastern region of the matrix in Table 1

Analysis Vol 70 | Number 3 | July 2010 | pp. 473–481 doi:10.1093/analys/anq020

Table 1. The common structure of the JB (roman script) and the SB (italics)

	B *Ta*	R *He*
Q *Mo*		
S *Tu*		X

should equal 1/3. Hence, $P^*(Red) = 1/3$ in the JB and $P^*(Heads) = 1/3$ in the SB.

Bovens's argument rests on a standard error in implementing Bayesian updating, which is spelled out in Shafer 1985. When we are informed of some proposition, we do not only learn the proposition in question, but also that we have learned the proposition as one of the many propositions that we might have learned. The information is generated by a *protocol,* which determines the various propositions that we might learn. We should then update not on the proposition in question, but rather on the fact that we learned this proposition as one of the many propositions that we might have learned.

A well-known application of this insight is the Monty Hall problem (MH) as Speed (1985: 276) points out in a discussion of Shafer 1985. As an illustration, let us apply Shafer's insight to the MH. In the MH, the contestant in a game show learns that there is a goat behind two of three doors *X*, *Y* and *Z* and a car behind one door. She is asked to pick one of the three doors. The contestant picks door *X*. Monty will then open one of the remaining doors, which he knows to have a goat behind it. Suppose Monty opens door *Y*. The contestant is then asked whether she wants to stick to the door she originally chose, i.e. door *X*, or whether she wants to switch to the other unopened door, i.e. door *Z*. Should the contestant switch doors, assuming that she wants to win the car rather than a goat?

If we naively update only on the content of the information, viz. that there is a goat behind door *Y*, then we reach the following conclusion:

$$(1) \quad P(CarX|GoatY) = \frac{P(GoatY|CarX)P(CarX)}{P(GoatY)} = \frac{1 \times 1/3}{2/3} = 1/2$$

Hence, $P(CarZ|GoatY) = 1/2$ as well and so, on this reasoning, it does not make any difference whether she does or does not switch doors. But, as is well-known, this reasoning is incorrect. We do not only learn that there is a goat behind door *Y*, we also learn that we learn this information as one of a range of possible items of information that Monty might have provided. If the protocol specifies that Monty *will* open one of the remaining doors with a goat behind it, then there are two items of information that the contestant might receive, viz. 'A goat is behind door *Y*' and the 'A goat is behind door *Z*'. Let us also specify that, as far as the contestant knows, Monty will randomize between doors *Y* and *Z* if both have goats behind them. We can now construct a table with conditional probabilities. Let '*INF*' be the variable that specifies the information provided by Monty and let '@' be the variable that specifies the actual location of the car. We construct the conditional probability table in Table 2.

In addition, the contestant has no reason to think one door more likely than another prior to Monty's information, i.e. $P(@ = CarX) = P(@ = CarY) = P(@ = CarZ) = 1/3$. We can now calculate:

$$P(@ = CarX|INF = GoatY)$$

$$= \frac{P(INF = GoatY|@ = CarX)P(CarX)}{P(INF = GoatY)} \qquad (2)$$

$$= \frac{1/2 \times 1/3}{1/3(1/2 + 0 + 1)} = 1/3$$

So $P(@=CarZ \mid INF = GoatY) = 2/3$ and hence the contestant should switch doors.

Are there protocols on which the contestant has no reason to switch? Well, suppose that Monty just opens one of the remaining doors at random – it may or may not have the car behind it. On this protocol, the contestant may expect four possible items of information. We construct the conditional probability table in Table 3.

Table 2. The conditional probability table in the MH

P(INF|@)		@ =		
		CarX	*CarY*	*CarZ*
INF =	*GoatY*	1/2	0	1
	GoatZ	1/2	1	0

Table 3. The conditional probability table with Monty opening up the remaining doors at random

P(INF|@)		@ =		
		CarX	*CarY*	*CarZ*
INF=	*CarY*	0	1/2	0
	GoatY	1/2	0	1/2
	CarZ	0	0	1/2
	GoatZ	1/2	1/2	0

We calculate:

$$(3)\quad \begin{aligned} &P(@ = CarX|INF = GoatY) \\ &= \frac{P(INF = GoatY|@ = CarX)P(CarX)}{P(INF = GoatY)} \\ &= \frac{1/2 \times 1/3}{1/3(1/2 + 0 + 1/2)} = 1/2 \end{aligned}$$

Hence, on this protocol, it does not pay to switch doors. So, the moral of the MH is that the protocol is all important. Let us now investigate whether we can gain some mileage from this insight for the SB and the JB.

We structure the SB so that we can invoke the mechanism of protocols. Let *the structure of the game* be the proposition that awakenings can occur in all four world-time quadrants (*Ta-Mo, etc.*), except for *He-Tu*. In the original SB, Beauty learns this information on Sunday and retains it throughout. In Bovens's SB′ (2010), Beauty learns on Sunday that all world-time quadrants are possible and is then told the structure of the game upon awakening, i.e. she is told that *He-Tu* is actually ruled out. Beauty also knows that amnesia of this additional information is induced when she is put back to sleep. This variation, he argues, cannot make for a difference to the solution of the SB.

To bring in protocols, let us parse the structure of the game as follows. Suppose that on Su, Beauty is informed that one and only one world-time quadrant is impossible, but she is not told which one, and that she will be told upon awakening which world-time quadrant is impossible. Subsequently, when she awakens, she is informed that it is *He-Tu* that is impossible. She retains the information that she received on Su, but amnesia of the information provided upon awakening is induced. Let us call this the SB″. This variation cannot make a difference to the solution either. The information that Beauty has at her disposal is simply parsed up differently in the SB, in the SB′ and SB″. After being informed upon awakenings in both the SB′ and the SB″, she knows exactly the same in the SB.

Just as in the MH, we can now construct a conditional probability table representing Beauty's credences upon awakening in Table 4. @ is the variable that takes as its values the particular world-time quadrant that Beauty is in upon awakening.

Beauty's reasoning to construct this conditional probability table is as follows. All items of information are equally probable and hence my credence for each item is 1/4. If, however, the awakening were to occur in a *Ta-Mo* quadrant, then the information item ¬*Ta-Mo* cannot be forthcoming and so my conditional credence that I would receive this item is 0. But all other information items are still open and there is no reason to think that one is more likely to be forthcoming than the other. Hence, $P(INF = \neg Ta\text{-}Tu \mid @ = Ta\text{-}Mo) = \ldots = 1/3$.

Table 4. The conditional probability table for the SB″

P(INF|@)		@ =			
		Ta-Mo	*Ta-Tu*	*He-Mo*	*He-Tu*
INF=	¬*Ta-Mo*	0	1/3	1/3	1/3
	¬*Ta-Tu*	1/3	0	1/3	1/3
	¬*He-Mu*	1/3	1/3	0	1/3
	¬*He-Tu*	1/3	1/3	1/3	0

We calculate:

$$P^*(He) = P(@ = He\text{-}Mo|INF{=}He\text{-}Tu)$$

(4)
$$= \frac{P(INF = \neg He\text{-}Tu|@ = He\text{-}Mo)P(@ = He\text{-}Mo)}{P(INF = \neg He\text{-}Tu)}$$

$$= \frac{1/3 \times 1/4}{1/4(1/3 + 0 + 1/3 + 1/3)} = 1/3$$

So when Beauty is informed of the structure of the game in the SB″, she will update her credence for *He* to 1/3. How the information is parsed does not make any difference to Beauty's credence. Hence, 1/3 should also be Beauty's posterior credence for *Heads* in the SB.

Let us now turn to the JB. What protocol might yield the information $R \rightarrow \neg S$? Consider the following protocol. The informer assesses the region R and only the region R in order to exclude one area – i.e. to indicate an area where Judy cannot possibly be. For example, she may have access to technology that permits her to provide exactly one quadrant in R that yields a true negative for the presence of Judy. If this is the case, then the protocol may yield two items of information, viz. $INF = R \rightarrow \neg S$ or $INF = R \rightarrow \neg Q$. If Judy is not in R, then, from Judy's perspective, there is a 50–50 chance that the technology will provide one of these two true negatives. Let @ be the variable which takes the actual location of SB as its values, so @ = *BQ, BS, RQ* or *RS*. We spell out the conditional probabilities in Table 5.

We calculate:

$$P^*(R) = P(@ = RQ|INF = R \rightarrow \neg S)$$

(5)
$$= \frac{P(INF = R \rightarrow \neg S|@ = RQ)P(@ = RQ)}{P(INF = R \rightarrow \neg S)}$$

$$= \frac{1 \times 1/4}{1/4(1/2 + 1/2 + 0 + 1)} = 1/2$$

So we see that, although the SB and the JB have *structural* similarities, careful attention to the *protocol* teaches us that $P^*(He) = 1/3$, whereas $P^*(R) = 1/2$.

Table 5. The conditional probability table for the JB

P(INF|@)		@ =			
		BQ	*BS*	*RQ*	*RS*
INF =	$R \rightarrow \neg S$	1/2	1/2	1	0
	$R \rightarrow \neg Q$	1/2	1/2	0	1

One might ask, is it possible to spell out alternative protocols so that $P^*(He) = 1/2$ and $P^*(R) = 1/3$? We can do so and, indeed, it will be instructive to evaluate such protocols.

Let us start with the SB. Consider the following protocol. Suppose that Beauty is put to sleep on Sunday and suppose she is told that a fair coin has been flipped, that there will be awakenings on both Monday and Tuesday independently of the coin flip, and that amnesia will be induced after an awakening. Furthermore, when she awakens someone will inform her of one time quadrant *in a Heads world* in which she is *not* located. Further, suppose that she is being informed that she is not in *He-Tu*. Then the SB protocol described by these conditional probabilities, spelled out in Table 6, precisely mirrors the JB protocol in Table 5.

We can calculate that $P^*(He) = 1/2$. But this is far away from the original SB. This protocol captures a game that has a radically different structure than the original SB.

The situation is less univocal in the JB. Let us first consider two protocols on which $P^*(R) = 1/3$. Suppose that the informer examines all quadrants and will provide JB with one true negative, i.e. one quadrant in which Judy is not. We spelled out the conditional probability table for this protocol in Table 7. Or suppose that the informer examines *RS* and is able to provide her with either a true positive or a true negative. We spelled out the conditional probability table for this protocol in Table 8. In each case, Judy does receive the information that $\neg RS$.

Table 7 is structurally analogous to Table 4 and so $P^*(R) = 1/3$. For Table 8, we calculate

$$
\begin{aligned}
P^*(R) &= P(@{=}RQ|INF{=}\neg RS) \\
&= \frac{P(INF = \neg RS|@ = RQ)P(@ = RQ)}{P(INF = \neg RS)} \qquad (6) \\
&= \frac{1 \times 1/4}{1/4(1+1+1+0)} = 1/3
\end{aligned}
$$

So, there do exist protocols on which $P^*(R) = 1/3$. But how plausible are these protocols, given the original formulation of the JB? In the original JB puzzle, Judy receives information of the form ‘If you are in *R*, then the

Table 6. The conditional probability table when Beauty is informed of one *Heads* world in which she is not located

P(INF|@)		@ =			
		Ta-Mo	*Ta-Tu*	*He-Mo*	*He-tu*
INF =	¬*He-Mo*	1/2	1/2	1	0
	¬*He-Tu*	1/2	1/2	0	1

Table 7. The conditional probability table when Judy is informed of one out of four quadrants in which she is not located

P(INF|@)		@ =			
		BQ	*BS*	*RQ*	*RS*
INF =	¬*BQ*	0	1/3	1/3	1/3
	¬*BS*	1/3	0	1/3	1/3
	¬*RQ*	1/3	1/3	0	1/3
	¬*RS*	1/3	1/3	1/3	0

Table 8. The conditional probability table when Judy is informed whether she is or is not located in *RS*

P(INF|@)		@ =			
		BQ	*BS*	*RQ*	*RS*
INF =	*RS*	0	0	0	1
	¬*RS*	1	1	1	0

probability that that you are in *Q* is *p*' (van Fraassen 1982: 377–78). A limiting case of this information is 'If you are in R, then the probability that you are in *Q* is 1', or, in other words, 'If you are in *R*, then you are not in *S*'. Now, the choice of the conditional as a mode of expressing the information carries a conversational implicature that one has checked *R*, rather than the whole area (as in the protocol underlying Table 7) or rather than *RS* (as in the protocol underlying Table 8). For this reason, the protocol underlying Table 5 is more in line with the choice of the conditional in the informer's message.

Is there no protocol that would warrant the use of the conditional and that would yield $P^*(R) = 1/3$? Imagine the following scenario. Suppose that the informer is *intent* on checking the area *R*. However, due to heavy cloud cover, he can get no information about *RQ*. He is able to report a false positive or a false negative about *RS*. If this is the protocol then we are back to Table 8 and indeed $P^*(R) = 1/3$. Now, arguably, it might not be unnatural for the informer to say 'If you are in *R*, then you are not in *S*' in this case, since he was indeed intent on checking R and this comes through in the choice of the conditional in his message.

But, as we know from everyday life, much misunderstanding is due to misreading conversational implicatures. So what is Judy supposed to do? In the absence of a clear protocol, the problem is simply underdetermined. Judy may have a subjective probability distribution over alternative protocols – some yielding $P^*(R) = 1/3$ and some yielding $P^*(R) = 1/2$. If this is so, then, given her credences, she will need to calculate a weighted average and $P^*(R)$ will take on a determinate value in the range [1/3, 1/2]. Or she may face radical uncertainty with respect to the protocols. In this case, there is no more to be said than that the problem is underdetermined and that $P^*(R)$ has upper bound 1/2 and lower bound 1/3. And in the face of limited uncertainty with respect to protocols, Judy can determine a more narrow range of values within [1/3, 1/2].

What makes an appeal to protocols so inviting is that it provides us not only with a correct treatment of the SB and the JB, but also with an error theory of all the confusion in this area. Simply recall the early confusion around the MH. The MH is a case in which the relevance of the protocol is straightforward and still, the erroneous solution of $P^*(CarX) = 1/2$ in the actual MH was much argued for due to a complete disregard for protocols. What underlies all the confusion about the SB and the JB is the same disregard for protocols.

Bovens 2010 has the virtue of recognizing a certain similarity between the SB and the JB. But it fails to recognize the dissimilarity between underlying protocols. Protocols are expressed in conditional probability tables that spell out the probability of coming to learn various propositions conditional on the actual state of the world. The principle of total evidence requires that we not update on the content of the proposition learned but rather on the fact that we learn the proposition in question. Now attention to protocols drives a wedge between the SB and the JB. We have shown that the solution to a close variant of the SB which involves a clear protocol is $P^*(He) = 1/3$ and since Beauty has precisely the same information at her disposal in the original SB at the time that she is asked to state her credence for *Heads*, the same solution should hold. The solution to the JB, on the other hand, is dependent on Judy's probability distribution over protocols. One reasonable protocol yields $P(R) = 1/2$, but Judy could also defend alternative values or a range of values in the interval [1/3, 1/2] depending on her probability distribution over protocols.[1]

1 José Luis Ferreira gratefully acknowledges grant number 2009/00055/001 from the Spanish Ministry of Education. We are grateful for comments from Richard Bradley and Wolfgang Spohn.

Department of Philosophy, Logic and Scientific Method
London School of Economics and Political Science
London, WC2A2AE
UK
L.Bovens@LSE.ac.uk

Department of Economics
Universidad Carlos III de Madrid
Madrid, 126, 28903 Getafe
Spain
jlferr@eco.uc3m.es

References

Bovens, L. 2010. Judy Benjamin is a Sleeping Beauty. *Analysis* 70: 23–26.

Shafer, G. 1985. Conditional probability. *International Statistical Review* 53: 261–77.

Speed, T.S. 1985. Discussion of paper by G. Shafer. *International Statistical Review* 53: 276.

van Fraassen, B. 1982. A problem for relative information minimizers in probability kinematics. *British Journal for the Philosophy of Science* 32: 375–79.

Rational constraints and the Simple View

EZIO DI NUCCI

According to the Simple View of intentional action, I have *intentionally* switched on the light only if I *intended* to switch on the light. The idea that intending to φ is necessary for intentionally φ-ing has been challenged by Bratman (1984, 1987) with a counter-example in which a videogame player is trying to hit either of two targets while knowing that she cannot hit both targets. When a target is hit, the game finishes. And if both targets are about to be hit simultaneously, the game shuts down. The player knows that she cannot hit both targets, but still she concludes that, given her skills, the best strategy is to have a go at each target at the same time. Suppose she hits target 1. It seems obvious that she has hit target 1 intentionally. But, Bratman argues, she could not have intended to hit target 1. Since the scenario is perfectly symmetrical, had the player intended to hit target 1, she would have also had to intend to hit target 2. But the player knows that she cannot hit both targets.

Analysis Vol 70 | Number 3 | July 2010 | pp. 481–486 doi:10.1093/analys/anq032