Strategic Profit Sharing Between Firms[1]

José Luis Ferreira[2]
Universidad Carlos III de Madrid

Roberts Waddle
Universidad Carlos III de Madrid

October, 2008

---

[1] Both authors gratefully acknowledge financial support from grant ECO2008-00977 / ECON, Ministerio de Ciencia e Innovación, Spain.

[2] Corresponding author. e-mail: jlferr@eco.uc3m.

Abstract

We introduce the possibility of unilaterally giving profits away to the rival in different oligopolistic contexts. We find that this strategy may be profitable in some circumstances. Although the strategy in itself may look unrealistic, we argue that it may be hidden behind a more complicated relation of the firms.

JEL classification: L13.

Key words: profit sharing, oligopoly.

1 Introduction

The study of oligopolies is a recurrent topic that attracts the attention of economists. Models of oligopolistic competition are studied both because of their increasing number of applications and because of the theoretical insights they provide. The applications include the fields of industrial organization, macroeconomics, public economics and international trade. The theoretical works deal with issues such as the analysis of different choices of strategic variables, the effects of dynamic considerations and, more recently and related to the present work, the role of cross-ownership (see, for example, FritzRoy and Kraft, 1986, Farrell and Shapiro, 1990, Malueg 1992 and Caminal and Vives, 1996.)

Along this last line of research, the present paper focuses on the particular strategy of firms that consists on unilaterally and voluntarily giving a part of their own profits to their rivals. Thus, in the first stage, firms decide simultaneously what part of their profits to give away to their rival and then, in the second stage, the equilibrium price and quantities are determined.

We use a general model to show the direct (negative) and indirect (may be positive) effects of this strategy, and, then, concentrate on the effects of its inclusion in some oligopolistic models (Cournot, Bertrand, Hotelling). We show that giving away profits is a rewarding strategy for firms in some of (but not all) the above oligopolistic competition models. Our analysis of the standard models shows that firms may have incentives to share profits in equilibrium in the Bertrand and Hotelling cases (at least for some values of the parameters that define the model) and not so much in the Cournot models. However, the use of the profit-sharing strategy does not depend on the relevant variables being strategic substitutes or complementary, although it seems that it works better in models of price competition.

In our model, the profit-giving strategy is performed directly. However, in real life, the strategy that makes the rival firm internalize, at least partially, one's well-being may be more indirect, and could be hidden behind a more complicated relation. After the presentation of the basic model, we suggest one such possibility, namely, the use of a joint venture.

In a game-theoretical setting, Jackson and Wilkie (2005) have characterized the outcomes of games when players can make binding offers of strategy-contingent side payments before the game is played. However, there are two main differences with our model. The first one is our unilateral-decision assumption. Contrary to their paper, our firms do not join together to precommit for side-payments before the game is played. Our firms behave non cooperatively and look for the optimal part of their profits to give away. The second one is our two-sides profit-sharing strategy; unlike in Jackson and

Wilkie (2005), in our model both firms share profits and the maximization is performed at the individual level.

The literature that integrates the notion of partial ownership and that is more connected to our work includes Malueg (1992) and Farrell and Shapiro (1990).

Malueg (1992) shows that, if firms interact repeatedly, increasing cross ownership may reduce the likelihood of collusion. A high level of cross ownership may even entail a lower likelihood of collusion than no-cross-ownership would. We differ from this article in two respects: First, Malueg's paper considers that each firm is exogenously entitled to a common fraction of the profit earned by the other firm. Second, our objective is just different. Malueg investigates whether increasing cross ownership among rivals increases the likelihood of collusion while we examine whether a firm is better off by sharing its profit to its rival. We do not consider the problem of collusion, which is left for future research.

Farrell and Shapiro (1990) study a one-way cross-ownership model where a big firm wants to acquire assets from another firm. Throughout a single-period Cournot oligopoly model, they show that, as the (exogenously given) degree of cross ownership among rivals increases, the equilibrium in the market becomes less competitive in the sense that aggregate output falls towards the monopoly level. Again, in our model, the degree of cross-ownership is endogenous and independently decided.

In this work we do not consider other models of oligopolistic competition, like the addition of a first stage of capacity building (like in Kreps and Scheinkman, 1983 or Moreno and Úbeda, 2006). Our aim to explore the issue of the profit sharing strategy already complicates the standard models to make it almost impossible to offer analytical results in most cases. The addition of more stages in the game will presumably complicate matters even more.

In Section 2 we present the basic model to show the direct (negative) and indirect (perhaps positive) effects of the profit-giving strategy. Section 3 presents two models of price competition with product differentiation. Section 4 studies the Cournot models both with homogeneous and heterogeneous goods. Finally, Section 5 deals with the standard Bertrand case with homogeneous goods. Section 6 concludes.

2 The basic model

Consider a duopolistic market in which two firms have s_1 and s_2 as their respective strategic variables. These variables can be price, quantity, location, or any other standard strategic variable in oligopoly theory. Assume now that, previous to the strategic choice of s_1 and s_2, Firm i can give part of its profits to Firm j, and that Firm j accepts. Let $\alpha_i \in [0,1]$ denote the part of the profit that Firm i gives to Firm j. The profit functions of Firm i can the be written as $\Pi_i(s_1(\alpha_1,\alpha_2), s_2(\alpha_1,\alpha_2))$. To simplify this exposition, we will assume interior solutions in all maximization problems. In the next section we provide examples where these assumptions are met, as well as examples of corner solutions.

In the second stage of the game Firm i maximizes $P_i = (1-\alpha_i)\Pi_i(s_i,s_j) + \alpha_j\Pi_j(s_i,s_j)$, whose first-order conditions for an interior solution give:

$$\alpha_i \frac{\partial \Pi_i(s_i,s_j)}{\partial s_i} - \alpha_j \frac{\partial \Pi_j(s_i,s_j)}{\partial s_i} = \frac{\partial \Pi_i(s_i,s_j)}{\partial s_i}. \tag{1}$$

The same condition is found for Firm j. The solution of the system thus defined, if it exists, defines the Nash equilibrium $s_1^*(\alpha_1,\alpha_2)$ and $s_2^*(\alpha_1,\alpha_2)$ of the second stage. From there, we can write the profits function $P_1(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2))$ that we use to solve the first stage of the game:

$$\max_{\alpha_i} \quad P_i = (1-\alpha_i)\Pi_i(s_i^*(\alpha_i,\alpha_j), s_j^*(\alpha_i,\alpha_j)) + \alpha_j\Pi_j(s_i^*(\alpha_i,\alpha_j), s_j^*(\alpha_i,\alpha_j)).$$

Using (1), the first-order conditions for an interior solution are given by

$$\frac{\partial P_i}{\partial \alpha_i} = -\Pi_i + \frac{\partial s_j^*}{\partial \alpha_i}\frac{\partial(\Pi_i + \Pi_j)}{\partial s_j} = 0,$$

which, together with a similar condition for Firm j, defines the system of equations that yields the equilibrium choice of α_1^* and α_2^* in the first stage.

This means that, if the equilibrium is found solving the first order conditions for an interior solution, we are in a situation where α_1^* and α_2^* are greater than zero. I.e., firms are willing to give away part of their profits to their rival. The last expression shows that sharing profits has two opposite effects. First, a direct or negative effect given by the first term $-\Pi_i$ and then, a strategic effect (positive in an interior solution) given by the second term, $\frac{\partial s_j^*}{\partial \alpha_i}\frac{\partial(\Pi_i+\Pi_j)}{\partial s_j}$. Hence, if the strategic effect is positive and large enough, firms may find profitable to give away profits to their rival.

So far, the profit-giving strategy has been performed directly. However, in real life, this strategy may be hidden behind a more complicated relation. To see this, consider the following simple case of a joint venture. Let $\beta_i \in [0,1]$ denote the part of its own profits that Firm i is willing to invest in a joint venture along with Firm j. The total investment in the joint venture is, then, given by $\beta_1\Pi_1 + \beta_2\Pi_2$. We will assume a simple joint venture activity with net profits given by $F = k\left(\beta_1\Pi_1 + \beta_2\Pi_2\right)$ where $k > 0$. Finally, we assume that each firm receives s_iF, where $s_i = \frac{\beta_i}{\beta_i+\beta_j}$. Consequently, we can write the new profit function of each firm as $P_i = (1-\beta_i)\Pi_i + s_ik\left(\beta_i\Pi_i + \beta_j\Pi_j\right)$ or $P_i = \left[1-(1-s_ik)\beta_i\right]\Pi_i + s_i\gamma\beta_j\Pi_j$. It is now straightforward to see that $(1-s_ik)\beta_i$ plays the role of α_i in the previous model, and that conditions on α_i can immediately be translated as conditions on β_i and on k for the profit-giving strategy to be profitable in equilibrium.

3 Models of oligopolistic competition with heterogenous goods

3.1 The Bertrand model

Let us apply the basic model to the Bertrand model with heterogeneous goods, where $p_i = 1 - q_i - \gamma q_j$ is the inverse demand function of Firm i. Then, profits are $\Pi_i(q_1, q_2) = p_iq_i(p_1, p_2)$, with $q_i = \frac{1}{1-\gamma^2}(1-\gamma-p_i+\gamma p_j)$. We assume that $\gamma^2<1$, that is, the own-price effect dominates the cross-price effect. In the second stage the first-order conditions with respect to p_i in the problem of maximizing $P_i = (1-\alpha_i)p_iq_i + \alpha_jp_jq_j$ with respect to q_i give:

$$p_i(p_j) = \frac{1-\gamma}{2} + \gamma\frac{1-\alpha_i+\alpha_j}{2(1-\alpha_i)}p_j.$$

By solving the system formed by $p_1 = p_1(p_2)$ and $p_2 = p_2(p_1)$, we have

$$p_i^*(\alpha_i, \alpha_j) = \frac{(1-\alpha_j)(1-\gamma)\left[2(1-\alpha_i) + \gamma(1-\alpha_i+\alpha_j)\right]}{4(1-\alpha_i)(1-\alpha_j) - \gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)}.$$

Substituting, in q_i:

$$q_i^*(\alpha_i, \alpha_j) = \frac{(2+\gamma)(1-\alpha_j)(1-\alpha_i) - \gamma(1+\gamma)\alpha_j(1-\alpha_j) - \gamma^2\alpha_i\alpha_j}{(1+\gamma)[4(1-\alpha_i)(1-\alpha_j) - \gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)]}.$$

In the first stage, profits have the form $P_i(\alpha_i, \alpha_j) = (1-\alpha_i)p_i^*q_i^* + \alpha_jp_j^*q_j^*$, $i, j = 1, 2$, $i \neq j$. The equilibrium choice of (α_1^*, α_2^*) (assuming interior solutions) is given by maximizing $P_i(\alpha_i, \alpha_j)$ with respect to α_i, and solving the system formed by the reaction functions $\alpha_1 = \alpha_1(\alpha_2)$, $\alpha_2 = \alpha_2(\alpha_1)$ to get (α_1^*, α_2^*). Once this is done, it is of interest to know whether there exist values of γ for which $(\alpha_1^*, \alpha_2^*) > (0, 0)$. However, the analytical expression for $\alpha_i^*(\gamma)$

in very complicated and it is better to rely on a numerical computation. Notice that, by the symmetry of the problem, $\alpha_1^* = \alpha_2^* = \alpha^*$.

For negative values of γ there is no profit-sharing equilibrium. Thus, we illustrate the equilibria for positive γ. Figure 1 shows the values for which the interior solution gives the equilibrium, and also the values of γ for which there is a corner solution $((\alpha_1, \alpha_2) = (0, 0))$ for the equilibrium.

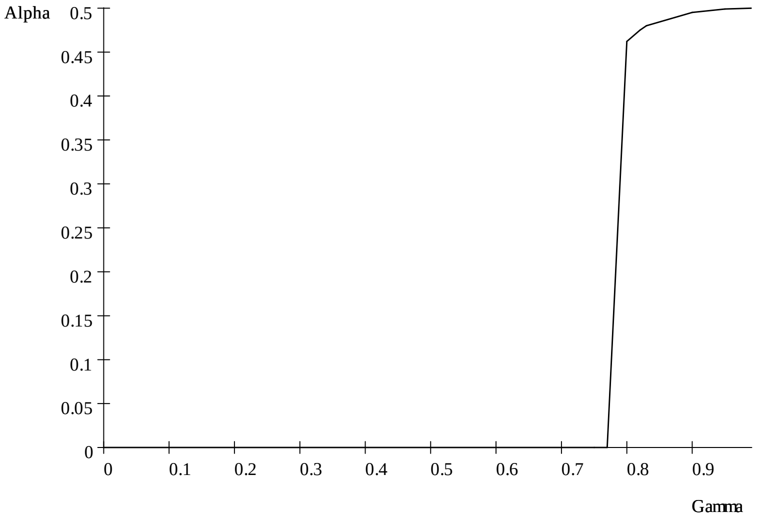
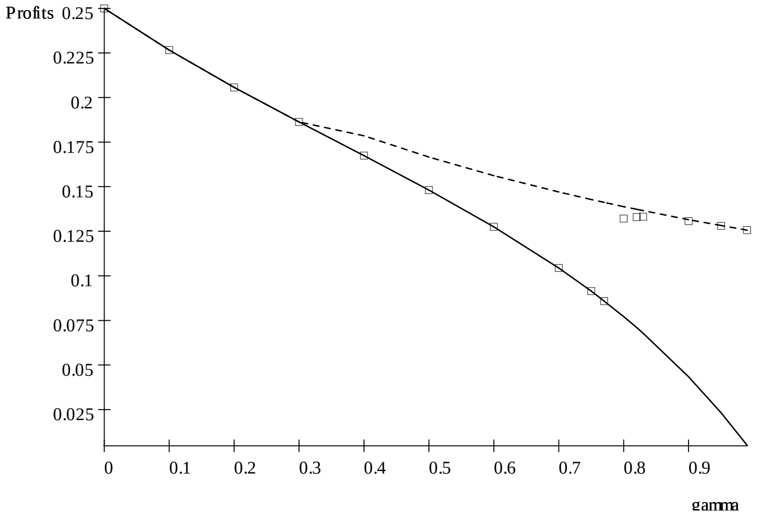


Figure 1: Equilibrium α as a function of γ

In Figure 2 profits $P_i(\alpha_1, \alpha_2)$ are shown for three cases, the equilibrium case with $(\alpha_1, \alpha_2) = (\alpha_1^*, \alpha_2^*)$ case of not-profit sharing with $(\alpha_1, \alpha_2) = (0, 0)$ and the case of collusion to share monopolistic profits with $(\alpha_1, \alpha_2) = (0.5, 0.5)$. The equilibrium values of (α_1^*, α_2^*) are given by the solution of the reaction functions $\alpha_1 = \alpha_1(\alpha_2)$, $\alpha_2 = \alpha_2(\alpha_1)$ whenever $\mathbf{P}_i(\alpha_1 = \alpha_2 = \alpha^*) > \mathbf{P}_i(\alpha_1 = \alpha_2 = 0)$. The differences between $\mathbf{P}_i(\alpha_1 = \alpha_2 = 0.5)$ and $\mathbf{P}_i(\alpha_1 = \alpha_2 = \alpha^*)$ provide an idea of how much it is to be gained if some sort of collusion is reached between the firms. These comparisons are interesting for a later discussion



.

Figure 2: Profits in
equilibrium: $\mathbf{P}_i\left(\alpha_1=\alpha_2=\alpha^*,\gamma\right)$ (boxed points),
non-sharing: $\mathbf{P}_i\left(\alpha_1=\alpha_2=0,\gamma\right)$ (solid line), and
collusion (half monopoly): $\mathbf{P}_i\left(\alpha_1=\alpha_2=0.5,\gamma\right)$ (dotted line)

The model of Bertrand competition with heterogeneous goods studied in this section has an equilibrium in which firms share profits for the values $\gamma\in[0.75,1)$.

3.2 The Cournot model

Consider now the case of Cournot with heterogenous goods, with demand functions given by $p_i=1-q_i-\gamma q_j$ $(i,j=1,2,\ i\neq j)$. Again assume, as usual, $\gamma^2<1$.

In the second stage, maximization of P_1 and P_2 with respect to q_1 and q_2, respectively yields reaction functions

$$q_i=\frac{(1-\alpha_i)-\gamma(1-\alpha_i+\alpha_j)q_j}{2(1-\alpha_i)},$$

and equilibrium quantities

$$q_i^*(\alpha_i,\alpha_j)=\frac{(1-\alpha_j)(2(1-\alpha_i)-\gamma(1-\alpha_i+\alpha_j))}{4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)}.$$

Substituting in $p_i=1-q_i-\gamma q_j$ we find the equilibrium prices

$$p_i^*(\alpha_i,\alpha_j) = \frac{2(1-\gamma)(1-\alpha_i)(1-\alpha_j)-\gamma^2\alpha_j(1-\alpha_j+\alpha_i)+\gamma(1-\alpha_j)(1-\alpha_i+\alpha_j)}{4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)}.$$

In the first stage, Firm i maximizes $P_i(\alpha_i,\alpha_j) = (1-\alpha_i)p_i^*q_i^* + \alpha_j p_j^* q_j^*$ with respect to α_i.

The equilibrium choice of (α_1^*,α_2^*) (assuming interior solutions) is given by the solution of the reaction functions $\alpha_1(\alpha_2)$ and $\alpha_2(\alpha_1)$. These expressions are quite complicated, and an explicit solution is not available. When proceeding as in the Bertrand case, our computations (not shown here) show that it never pays to give profits away. (However, if we allow for values of γ with $\gamma^2 > 1$, there are cases of profit sharing in equilibrium. For instance, if $\gamma = -1.09$ then $(\alpha_1^*,\alpha_2^*) = (0.47, 0.47)$ with profits $P_i\,(\alpha = 0.47, \gamma = -1.09) = 9.5$, greater than $P_i\,(\alpha = 0, \gamma = -1.09) = 1.21$.)

Figure 3 is the counterpart of Figure 2. However, contrary to the Bertrand case, it shows that equilibrium profits decrease with the value of γ, and that there is a very small difference in profits between the equilibrium case (no-sharing, $\alpha_1 = \alpha_2 = 0$) and the sharing of monopolistic profits ($\alpha_1 = \alpha_2 = 0.5$). Thus, in the Cournot case, there is little to gain from collusion.

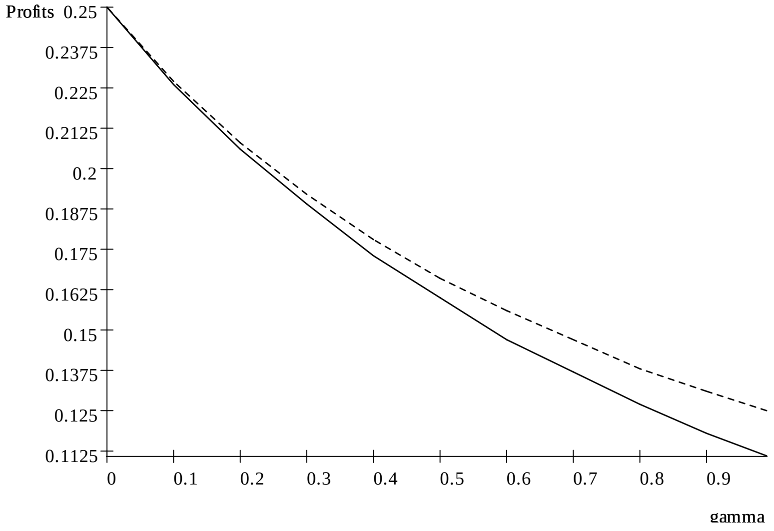


Figure 3: Profits in equilibrium: $\mathbf{P}_i\,(\alpha_1 = \alpha_2 = \alpha^* = 0, \gamma)$ (solid line), and collusion (half monopoly): $\mathbf{P}_i\,(\alpha_1 = \alpha_2 = 0.5, \gamma)$ (dotted line)

3.3 The Hotelling model

Consider a linear city that lies on the interval $[0, 1]$, and where consumers are uniformly distributed with density 1 along this interval. There are two firms which sell the same physical good, and that are located at the extremes of the city. Firm 1 is at $x = 0$ and Firm 2 at $x = 1$. Consumers incur a transportation cost $t > 0$ per unit of length. Thus, a consumer living at x incurs a cost of tx to go to Firm 1 and a cost of $t(1-x)$ to go to Firm 2. Consumers have unit demands, and will choose from which firm to buy minimizing the sum of the price and transportation cost. Firms compete in price, which they choose simultaneously.

A consumer who is indifferent between the two firms is located at $x = D_1(p_1, p_2)$, where x is given by equating costs; i.e., $p_1 + tx = p_2 + t(1-x)$. The firms' respective demands are $D_1(p_1,p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}$, and $D_2(p_1,p_2) = 1 - D_1$.

To find the equilibrium in the second stage of the game, after a first moment of profit-sharing decisions, Firm i maximizes $P_i = (1-\alpha_i)\, p_i D_i\,(p_i, p_j) + \alpha_j p_j D_j\,(p_i, p_j)$. The first-order conditions with respect to p_i give:

$$p_i(p_j) = \frac{t}{2} + \frac{1-\alpha_i+\alpha_j}{2\,(1-\alpha_i)} p_j, i, j = 1, 2, i \neq j.$$

The solution of the system yields

$$p_i^*(\alpha_i, \alpha_j) = t\,(1-\alpha_j)\, \tfrac{3-3\alpha_i+\alpha_j}{(1-\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)}, \;\; i, j = 1, 2, i \neq j.$$

Substituting equilibrium prices in D_i, we have:

$$D_i(p_i^*, p_j^*) = \tfrac{1}{2}\tfrac{3-2\alpha_j}{3-\alpha_i-\alpha_j}.$$

Now, profits in the first stage are given by $P_i(\alpha_i, \alpha_j) = (1-\alpha_i)\, p_i^* D_i(p_1^*, p_2^*) + \alpha_j p_j^* D_j(p_1^*, p_2^*)$, or

$$P_i = \tfrac{t(1-\alpha_i)}{2(3-\alpha_i-\alpha_j)^2(1-\alpha_j-\alpha_i)} \left(2\alpha_j^3 - 8\alpha_j^2 - 3\alpha_j + 12\alpha_i\alpha_j - 2\alpha_i^2\alpha_j + 9 - 9\alpha_i\right).$$

Maximization with respect to α_i gives $\alpha_1 = \alpha_2 = \alpha^* = 0.15$, with profits $P_1^* = P_2^* = 0.6t$ and prices $p_1^* = p_2^* = 1.2t$. However, if firms decided not to share their profits, that is, if $\alpha^* = 0$, we find $P_1^* = 0.5t$.

4 Bertrand and Cournot models with homogenous goods

4.1 The Bertrand case

Consider two firms, 1 and 2, that compete *a la* Bertrand in a homogeneous market, and that each firm incurs in a cost c per unit of production. Let the market demand function be $q = D(p) = 1 - p$, and assume that firms do not have capacity constraints, and always supply the demand they face. The profit function for Firm i is:

$$\Pi_i = \begin{cases} (p_i - c)(1 - p_i) & if \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(1 - p_i) & if \quad p_i = p_j \qquad i, j = 1, 2 \ (i \neq j) \\ 0 & \quad otherwise. \end{cases}$$

Consider now two-stage game of profit-sharing. In the first stage of the game firms choose (α_1, α_2). In the second stage, firms select (p_1, p_2). Because of the discontinuity of the profit functions, the basic model is not a reference for Bertrand competition with homogenous goods, however, one may still ask whether the same general ideas apply. Indeed, in Waddle (2005) it is shown that any price between perfect competition and monopoly can be achieved, yielding positive profits to the industry. The ambiguity of the result and the difference in methodology suggest a separate treatment of the case. The result is reported here for the sake of completeness.

4.2 The Cournot case

Consider two firms, 1 and 2, that compete in a homogeneous market, and that have no production costs. Let the demand curve be given by $p = 1 - q_1 - q_2$ where q_i is Firm i's output. It is straightforward to see that the equilibrium in the second stage of the profit-giving game is $q_i^*(\alpha_i, \alpha_j) = \frac{1-\alpha_j}{3-\alpha_i-\alpha_j}$, with prices given by $p^*(\alpha_1, \alpha_2) = \frac{1}{3-\alpha_1-\alpha_2}$. Hence $P_i(\alpha_i, \alpha_j) = \frac{1-\alpha_i}{(3-\alpha_i-\alpha_j)^2}$, and $\frac{\partial P_i}{\partial \alpha_i} = \frac{-1-\alpha_i+\alpha_j}{(3-a_i-\alpha_j)^3} < 0$ for $\alpha_i, \alpha_j \in [0, 1]$, and, thus, the maximization problem of the first stage yields $\alpha_1^* = \alpha_2^* = 0$.

5 Discussion

Due to the complicated expressions, it is almost impossible to derive neat conditions that explain when profit sharing is a profitable equilibrium strategy. We can, however, provide some hints by examining the behavior of prices and profits under different scenarios.

In Section 3 we already saw that the differences in profits between the non-profit sharing case and the monopoly case where much higher for the Bertrand competition than for the Cournot competition. We can see this in more detail by fixing the value of γ and observing the changes in profits as the shares go from $\alpha_1 = \alpha_2 = 0$ to $\alpha_1 = \alpha_2 = 0.5$. Consider, then, the case of $\gamma = 0.9$, and make both firms choose the same sharing rules ($\alpha_i = \alpha_j = \alpha$), and show the changes in prices and profits as α changes.

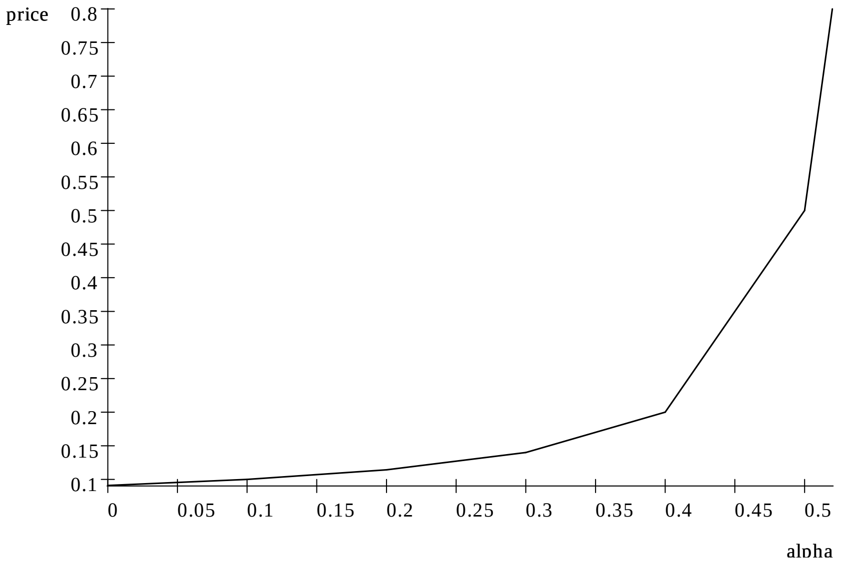


Figure 4

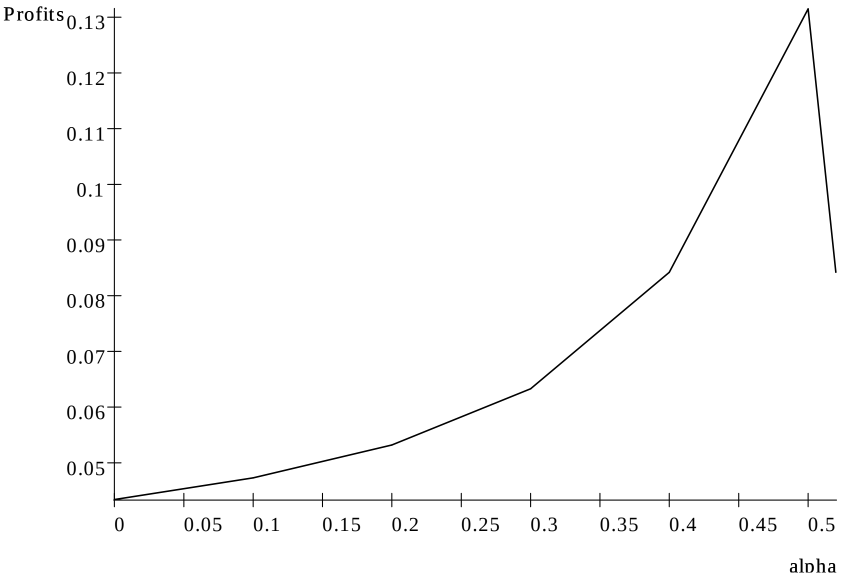


Figure 5

Now we do the same with the Cournot model for heterogenous goods. Recall that in this model profit sharing was not an equilibrium strategy. Figures 6 and 7 are the counterpart of Figures 4 and 5 (also for the case of $\gamma = 0.9$). Here we see an increase in price as $\alpha_1 = \alpha_2 = \alpha$ increase, but a very slow and moderate increase in profits up to the monopolistic case ($\alpha = 0.5$) followed by a sharp decline.

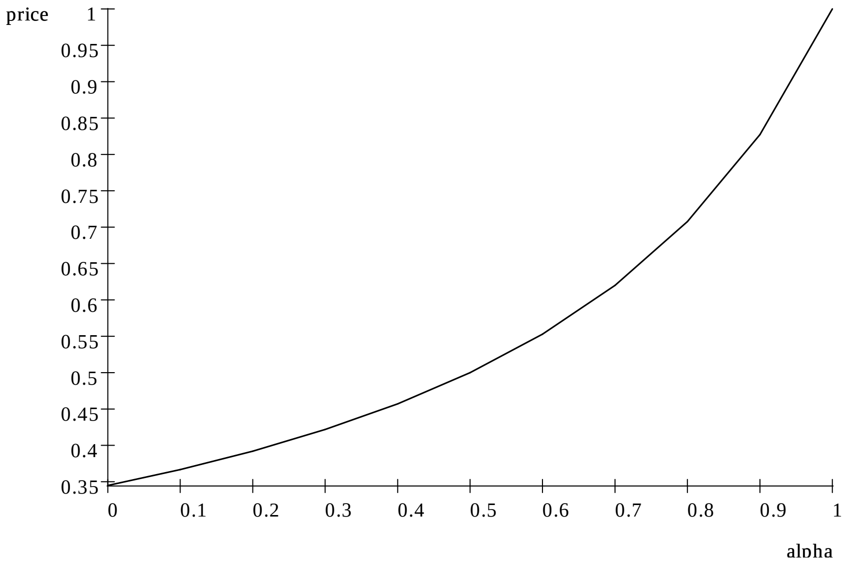


Figure 6: Changes in prices when α varies

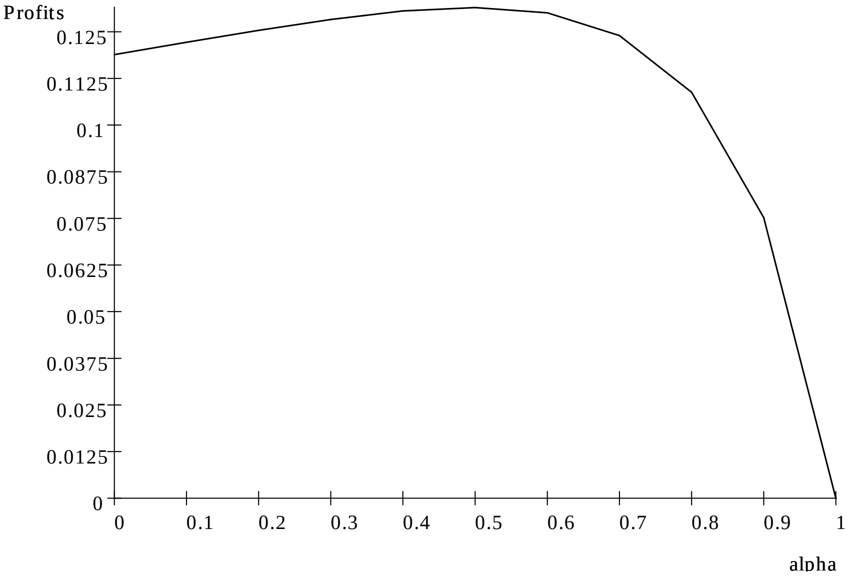


Figure 7: Changes in profits when α varies

The model of Hotelling shows a pattern similar to the Bertrand case with $\gamma = 0.9$. Here we show just the counterpart of Figures 5 and 7. This is shown in Figure 8.

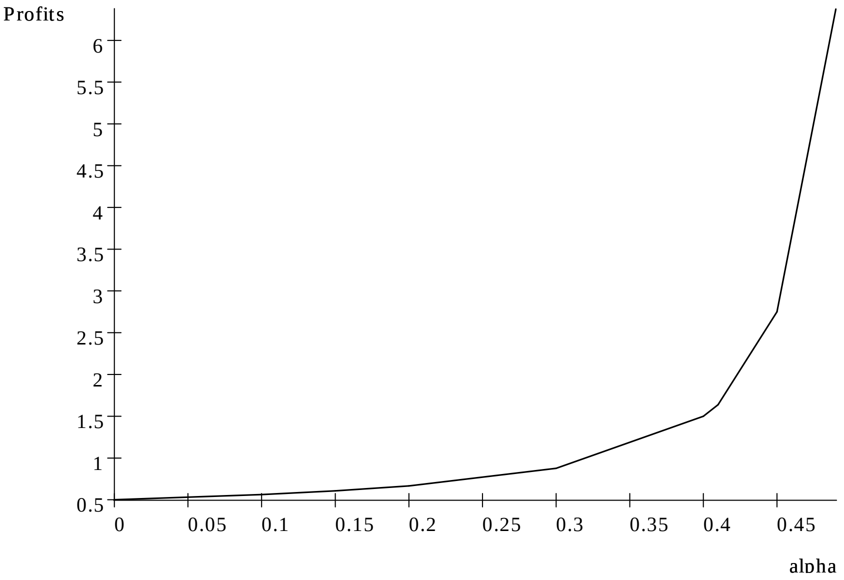


Figure 8 Hotelling: Changes in profits when α varies

All cases of equilibrium with positive sharing show patterns similar to the ones found in the above Bertrand and Hotelling examples. All cases of non-sharing in equilibrium are similar to the Cournot example. The main differences between these cases are (i) a rapid increase in profits as we move from $\alpha_i = \alpha_j = \alpha = 0$ to $\alpha_i = \alpha_j = \alpha = 0.5$, and (ii) a big difference between the oligopoly and monopoly profits This is more likely to occur when decision variables are strategic complementaries, and when competition is more aggressive.

6 Conclusion

This work has shown how two firms in a duopolistic market may be able to limit the competition through the profit-sharing strategy, thus increasing their profits. Using different oligopolistic models, we have brought to light some situations where giving away profits could be a rewarding strategy for firms. In fact we have analyzed models where the strategic variables were strategic substitutes and models where they were strategic complements, and have seen that in both situations, depending on the parameters of the model, profit-giving was sometimes an equilibrium, but not always.

In our model firms give profits directly to the other firm. In more realistic situations, this strategy may be performed indirectly, perhaps disguised behind some other strategy. If this is the case, our model still serves as a departing point to study these other cases, as one would expect the same qualitative results.

Further, the fact that our findings show that the strategy of profit sharing is more likely to occur in models of price competition opens the door to investigate its influence in other scenarios of price competition, like in the literature of price leadership (cf. Dastidar and Furth, 2005, and Yano and Komatsubara, 2006.)

References

Caminal, P., Vives, X., 1996. Why market shares matter: Information-based theory. RAND Journal of Economics 27(2), 221-239.

Dastidar, K. G., Furth, D., 2005. Endogenous Price Leadership in a Duopoly: Equal Products, Unequal Technology, International Journal of Economic Theory 1, 189–210.

Farrell, J., Shapiro C., 1990. Asset ownership and market structure in oligopoly. RAND Journal of Economics 21, 275-292.

FitzRoy, F. R, Kraft F., 1986. Profitability and profit-sharing. Journal of Industrial Economics 35, 113-130.

Jackson, M. O., Wilkie S., 2005. Endogenous games and mechanisms: Side payments among players. Review of Economics Studies 72, 543-566.

Kreps, D., Scheinkman, J., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. Bell Journal of Economics 14, 326-337.

Malueg, D., 1992. Collusive behavior and partial ownership of rivals. International Journal of Industrial Organization 10, 27-34.

Moreno, D., Úbeda, L., 2006. Capacity precommitment and price competition yield the Cournot outcome. Games and Economic Behavior, 56: 323-332

Waddle, R., 2005. Strategic profit sharing between firms: the Bertrand model. Working paper 05-09, Economic Series, Universidad Carlos III de Madrid.

Yano, M., Komatsubara, T., 2006. Endogenous Price Leadership and Technological Differences, International Journal of Economic Theory 2, 365–383.

Appendix
Computations for Strategic Profit Sharing Between Firms

Section 3.1 The Bertrand model

First stage of the game: After substitution, the profit function for Firm i is given by

$$\begin{aligned}
P_i &= (1-\alpha_i)p_i^* q_i^* + \alpha_j p_j^* q_j^* \\
&= (1-\alpha_i)\tfrac{(1-\alpha_j)(1-\gamma)[2(1-\alpha_i)+\gamma(1-\alpha_i+\alpha_j)]}{[4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)]} \\
&\cdot \tfrac{1}{1+\gamma}\tfrac{(2+\gamma)(1-\alpha_j)(1-\alpha_i)-\gamma(1+\gamma)\alpha_j(1-\alpha_j)-\gamma^2\alpha_i\alpha_j}{[4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)]} \\
&+\alpha_j\tfrac{(1-\alpha_i)(1-\gamma)[2(1-\alpha_j)+\gamma(1-\alpha_j+\alpha_i)]}{[4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)]}\tfrac{1}{1+\gamma} \\
&\cdot \tfrac{(2+\gamma)(1-\alpha_j)(1-\alpha_i)-\gamma(1+\gamma)\alpha_i(1-\alpha_i)-\gamma^2\alpha_i\alpha_j}{[4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)]}
\end{aligned}$$

The first-order conditions with respect to α_i give:

$$\begin{aligned}
&[(1-\alpha_j)(1-\gamma)\{[(1-\alpha_i)(-4-\gamma)-\gamma(1-\alpha_i+\alpha_j)] \\
&[(2+\gamma)(1-\alpha_j)(1-\alpha_i)-\gamma(1+\gamma)\alpha_j(1-\alpha_j)-\gamma^2\alpha_i\alpha_j] \\
&-[(2+\gamma)(1-\alpha_j)+\gamma^2\alpha_j](1-\alpha_i)[2(1-\alpha_i)+\gamma(1-\alpha_i+\alpha_j)]\} \\
&+\alpha_j(1-\gamma)\{[-2(1-\alpha_j)-\gamma(1-\alpha_j+\alpha_i)+\gamma(1-\alpha_i)] \\
&[(2+\gamma)(1-\alpha_j)(1-\alpha_i)-\gamma(1+\gamma)\alpha_i(1-\alpha_i)-\gamma^2\alpha_i\alpha_j] \\
&-2[(2+\gamma)(1-\alpha_j)-2\gamma(1+\gamma)\alpha_i+\gamma(1+\gamma)+\gamma^2\alpha_j] \\
&(1-\alpha_i)[2(1-\alpha_j)+\gamma(1-\alpha_j+\alpha_i)]\}] \\
&[4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)]^2(1+\gamma) \\
&-4[4(1-\alpha_i)(1-\alpha_j)-\gamma^2(1-\alpha_i+\alpha_j)(1-\alpha_j+\alpha_i)] \\
&[-2(1-\alpha_j)+\gamma^2(a_i-\alpha_j)](1+\gamma)\{(1-\gamma)(1-\alpha_i)(1-\alpha_j) \\
&[2(1-\alpha_i)+\gamma(1-\alpha_i+\alpha_j)] \\
&[(2+\gamma)(1-\alpha_j)(1-\alpha_i)-\gamma(1-\gamma)\alpha_j(1-\alpha_j)-\gamma^2\alpha_i\alpha_j] \\
&+\alpha_j(1-\gamma)(1-\alpha_i)[2(1-\alpha_j)+\gamma(1-\alpha_j+\alpha_i)] \\
&[(2+\gamma)(1-\alpha_j)(1-\alpha_i)-\gamma(1-\gamma)\alpha_j(1-\alpha_j)-\gamma^2\alpha_i\alpha_j]\}=0
\end{aligned}$$

By symmetry, the interior solutions must satisfy $\alpha_1 = \alpha_2 = \alpha$

$$\begin{aligned}
&[(1-\alpha)(1-\gamma)\{[(1-\alpha)(-4-\gamma)-\gamma] \\
&[(2+\gamma)(1-\alpha)^2-\gamma(1+\gamma)\alpha(1-\alpha)-\gamma^2\alpha^2] \\
&-[(2+\gamma)(1-\alpha)+\gamma^2\alpha](1-\alpha)[2(1-\alpha)+\gamma]\} \\
&+\alpha(1-\gamma)\{[-2(1-\alpha)-\gamma+\gamma(1-\alpha)] \\
&[(2+\gamma)(1-\alpha)^2-\gamma(1+\gamma)\alpha(1-\alpha)-\gamma^2\alpha^2]
\end{aligned}$$

$$\begin{array}{l}
-2[(2+\gamma)(1-\alpha)-2\gamma(1+\gamma)\alpha+\gamma(1+\gamma)+\gamma^2\alpha] \\
(1-\alpha)[2(1-\alpha)+\gamma]\}]\left[4(1-\alpha)^2-\gamma^2\right]^2(1+\gamma)- \\
4[4(1-\alpha)^2-\gamma^2][-2(1-\alpha)]\,(1+\gamma)\left\{(1-\gamma)\,(1-\alpha)^2\right. \\
{[2(1-\alpha)+\gamma][(2+\gamma)(1-\alpha)^2-\gamma(1-\gamma)} \\
\alpha(1-\alpha)-\gamma^2\alpha^2]+\alpha(1-\alpha)\,(1-\gamma)\,[2(1-\alpha)+\gamma] \\
\left.[(2+\gamma)(1-\alpha)^2-\gamma(1-\gamma)\alpha(1-\alpha)-\gamma^2\alpha^2]\right\}=0
\end{array}$$

By solving the above equation, we find $\alpha=-\frac{1}{2}\gamma+1$, $\alpha=\rho$, $\alpha=1+\frac{1}{2}\gamma$,and $\alpha=1+\frac{1}{2}\gamma$ where ρ solves

$$\begin{array}{l}
(8+8\gamma)\,\alpha^5+(-52-48\gamma)\,\alpha^4+(-2\gamma^3+106\gamma+128)\,\alpha^3 \\
+\left(6\gamma^2-\gamma^4+9\gamma^3-110\gamma-152\right)\alpha^2 \\
+\left(-12\gamma^2+88+54\gamma-10\gamma^3\right)\alpha+3\gamma^3-20+6\gamma^2-10\gamma=0
\end{array}$$

For different values of γ we solve the above expression for ρ numerically. In this way we get the values for α in Figure 1 in Section 3. We only take into account the solution $\alpha=\rho$ since $\alpha=1$, $\alpha=1+\frac{1}{2}\gamma$, and $\alpha=1-\frac{1}{2}\gamma$ give negative values for the variables or a minimum for the objective function.

Finally, by replacing $\alpha_1=\alpha_2=\alpha$ in the expression for $P_1(\alpha,\alpha)$, we find

$$\begin{array}{l}
P_1(\alpha,\alpha) \\
=\frac{1-\gamma}{1+\gamma}\frac{1-\alpha}{(2(1-\alpha)-\gamma)^2}\left((2+\gamma)\,(1-\alpha)^2-\gamma\,(1+\gamma)\,\alpha\,(1-\alpha)-\gamma^2\alpha^2\right) \\
+\frac{1}{2(1-\alpha)+\gamma}.
\end{array}$$

This is the expression used to compute the values in Figure 2.

Section 3.2 The Cournot model

In the first stage of the game Firm i maximizes $P_1(\alpha_1,\alpha_2)=(1-\alpha_1)p^*q_1^*+\alpha_2p^*q_2^*$, which, after substituting p^*,q_1^*, and q_2^* can be written as

$$\begin{array}{l}
P_1=(1-\alpha_1)\,\frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_2(1-\alpha_2+\alpha_1)+\gamma(1-\alpha_2)(1-\alpha_1+\alpha_2)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \\
\cdot\frac{(1-\alpha_2)(2(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2))}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \\
+\alpha_2\frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_1(1-\alpha_1+\alpha_2)+\gamma(1-\alpha_1)(1-\alpha_2+\alpha_1)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \\
\cdot\frac{(1-\alpha_1)(2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1))}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}
\end{array}$$

The first-order conditions with respect to α_1 give:

$$\begin{array}{l}
\frac{dP_1}{d\alpha_1}=0: \\
\left(\left(\gamma^2\alpha_2\,(2\alpha_1-\alpha_2)+(1-\alpha_2)\,(-\gamma\alpha_2-2\alpha_1-2\alpha_1\gamma)\right)\right. \\
(1-\alpha_2)\,(2\,(1-\alpha_1)-\gamma\,(1-\alpha_1+\alpha_2))+(1-\alpha_2)\,(-2+\gamma)
\end{array}$$

$$\begin{aligned}
&(1-\alpha_1)\left(2\left(1-\gamma\right)\left(1-\alpha_1\right)\left(1-\alpha_2\right)-\gamma^2\alpha_2\left(1-\alpha_2+\alpha_1\right)+\gamma\left(1-\alpha_2\right)\left(1-\alpha_1+\alpha_2\right)\right)\\
&+\left(\alpha_2\left(-2\left(1-\gamma\right)\left(1-\alpha_2\right)-\left(\gamma^2+\gamma\right)\left(1-\alpha_1+\alpha_2\right)-\gamma^2\alpha_1-\gamma\left(1-\alpha_1\right)\right)\right)\\
&(1-\alpha_1)\left(2\left(1-\alpha_2\right)-\gamma\left(1-\alpha_2+\alpha_1\right)\right)+\left(-2\left(1-\alpha_2\right)+\gamma\left(1-\alpha_1+\alpha_2\right)-\alpha_1\gamma\left(1-\alpha_1\right)\right)\\
&\left(\alpha_2\left(2\left(1-\gamma\right)\left(1-\alpha_1\right)\left(1-\alpha_2\right)-\gamma^2\alpha_1\left(1-\alpha_1+\alpha_2\right)+\gamma\left(1-\alpha_1\right)\left(1-\alpha_2+\alpha_1\right)\right)\right)\\
&\left(4\left(1-\alpha_1\right)\left(1-\alpha_2\right)-\gamma^2\left(1-\alpha_1+\alpha_2\right)\left(1-\alpha_2+\alpha_1\right)\right)^2\\
&-4\left(4\left(1-\alpha_1\right)\left(1-\alpha_2\right)-\gamma^2\left(1-\alpha_1+\alpha_2\right)\left(1-\alpha_2+\alpha_1\right)\right)\left(-2\left(1-\alpha_2\right)+\gamma^2\left(\alpha_1-\alpha_2\right)\right)\\
&\left(\left(1-\alpha_1\right)\left(2\left(1-\gamma\right)\left(1-\alpha_1\right)\left(1-\alpha_2\right)-\gamma^2\alpha_2\left(1-\alpha_2+\alpha_1\right)+\gamma\left(1-\alpha_2\right)\left(1-\alpha_1+\alpha_2\right)\right)\right.\\
&+\left(1-\alpha_2\right)\left(2\left(1-\alpha_1\right)-\gamma\left(1-\alpha_1+\alpha_2\right)\right)\\
&+\alpha_2\left(-2\left(1-\gamma\right)\left(1-\alpha_2\right)-\left(\gamma^2+\gamma\right)\left(1-\alpha_1+\alpha_2\right)-\gamma^2\alpha_1-\gamma\left(1-\alpha_1\right)\right)\\
&\left.\left(1-\alpha_1\right)\left(2\left(1-\alpha_2\right)-\gamma\left(1-\alpha_2+\alpha_1\right)\right)\right)=0
\end{aligned}$$

By symmetry, the interior solutions must satisfy $\alpha_1=\alpha_2=\alpha$

$$\begin{aligned}
&\tfrac{\partial P_1}{\partial\alpha_1}\rfloor_{\alpha_1=\alpha_2=\alpha}=0:\\
&\left(\gamma^2\alpha^2+\left(1-\alpha\right)\left(-3\alpha\gamma-2\alpha\right)\right)\left(1-\alpha\right)\left(2-2\alpha-\gamma\right)+\left(1-\alpha\right)^2\left(-2+\gamma\right)\\
&\left(2\left(1-\gamma\right)\left(1-\alpha\right)^2-\gamma^2\alpha+\gamma\left(1-\alpha\right)\right)+\\
&\alpha^2\left(-2\left(1-\gamma\right)\left(1-\alpha\right)-\gamma^2-\gamma-\gamma^2\alpha-\gamma\left(1-\alpha\right)\right)\\
&\left(1-\alpha\right)\left(2-2\alpha-\gamma\right)\left(-2+2a+\gamma-\alpha\gamma\left(1-\alpha\right)\right)\\
&\left(2\left(1-\gamma\right)\left(1-\alpha\right)^2-\gamma^2\alpha+\gamma\left(1-\alpha\right)\right)\left(4\left(1-\alpha\right)^2-\gamma^2\right)^2\\
&-4\left(4\left(1-\alpha\right)^2-\gamma^2\right)\left(-2+2\alpha\right)\\
&\left(\left(1-\alpha\right)\left(2\left(1-\gamma\right)\left(1-\alpha\right)^2-\gamma^2\alpha+\gamma\left(1-\alpha\right)\right)+\left(1-\alpha\right)\left(2-2\alpha-\gamma\right)\right.\\
&+\alpha\left(-2\left(1-\gamma\right)\left(1-\alpha\right)-\gamma^2-\gamma-\gamma^2\alpha-\gamma\left(1-\alpha\right)\right)\\
&\left.\left(1-\alpha\right)\left(2-2\alpha-\gamma\right)\right)=0
\end{aligned}$$

By solving the above equation, we find as solutions $\alpha=1$, $\alpha=\rho$, $\alpha=1+\frac{1}{2}\gamma$, and $\alpha=1-\frac{1}{2}\gamma$ where ρ solves

$$\begin{aligned}
&\left(4-4\gamma\right)\alpha^4+\left(-32+28\gamma+4\gamma^2\right)\alpha^3\\
&+\left(72-44\gamma-11\gamma^2-\gamma^3\right)\alpha^2\\
&+\left(-64+20\gamma+10\gamma^2\right)\alpha-3\gamma^2+20=0
\end{aligned}$$

Solving numerically for different values of γ, we get Tables 1 and 2. We only take into account $\alpha=\rho$ since the values $\alpha=1$, $\alpha=1+\frac{1}{2}\gamma$, and $\alpha=1-\frac{1}{2}\gamma$ yield negative values for the variables or correspond to a minimum. The tables show that the strategy of profit-giving is not worth.

$\boldsymbol{\gamma<0}$	$\boldsymbol{\alpha}$	$\mathbf{P}_i\left(\alpha,\gamma\right)$	$\mathbf{P}_i\left(0,\gamma\right)$
−0.97	*0.51*	*−24.12*	*0.94*
−0.71	*0.6*	*−1.63*	*0.6*
−0.61	*0.64*	*−1.01*	*0.51*
−0.27	*0.8*	*−0.27*	*0.33*
0	*1*	*0*	*0.25*

Tab. 1

$\boldsymbol{\gamma > 0}$	$\boldsymbol{\alpha}$	$\mathbf{P}_i\,(\alpha,\gamma)$	$\mathbf{P}_i\,(0,\gamma)$
0	1	0	0.25
0.27	0.94	0.121	0.19
0.47	0.9	0.115	0.16
0.79	0.84	0.106	0.12
1	0.80	0.101	0.111

Tab. 2

By replacing $\alpha_1 = \alpha_2 = \alpha$ in the expression for profits we find

$$P_1^*(\alpha,\alpha) = (1-\alpha)^2\, \frac{2(1-\gamma)(1-\alpha)^2-\gamma^2\alpha+\gamma(1-\alpha)}{4(1-\alpha)^2-\gamma^2}\, \frac{2-2a-\gamma}{4(1-\alpha)^2-\gamma^2}$$
$$+\alpha\frac{2(1-\gamma)(1-\alpha)^2-\gamma^2\alpha+\gamma(1-\alpha)}{4(1-\alpha)^2-\gamma^2}\,(1-\alpha)\,\frac{2-2a-\gamma}{4(1-\alpha)^2-\gamma^2}$$
$$=(1-\alpha)\,\frac{2(1-\gamma)(1-\alpha)^2-\alpha\gamma+\gamma(1-\alpha)}{(2(1-\alpha)+\gamma)^2(2(1-\alpha)-\gamma)}$$

This is the expression we use to compute the values in Figure 3.

Section 3.3 The Hotelling model

In the second stage of the game Firm i solves:

$$Max_{p_1} \quad P_1 = (1-\alpha_1)p_1\left(\tfrac{1}{2}+\tfrac{p_2-p_1}{2t}\right)+\alpha_2 p_2\left(\tfrac{1}{2}-\tfrac{p_2-p_1}{2t}\right)$$

The first-order conditions with respect to p_1 give

$$(1-\alpha_1)\left(\tfrac{1}{2}+\tfrac{p_2-p_1}{2t}\right)+\tfrac{\alpha_2 p_2}{2t}=0$$

or

$$p_1(p_2)=\tfrac{t}{2}+\tfrac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)}p_2$$

and by analogy

$$p_2(p_1)=\tfrac{t}{2}+\tfrac{1-\alpha_2+\alpha_1}{2(1-\alpha_2)}p_1$$

Solving the system, one finds the equilibrium prices

$$p_i^*(\alpha_i,\alpha_j)=t\,(1-\alpha_j)\,\frac{3-3\alpha_i+\alpha_j}{(1-\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)},\, i,j=1,2, i\neq j$$

and then

$$D_i(p_i^*,p_j^*)=\tfrac{1}{2}\tfrac{3-2\alpha_j}{3-\alpha_i-\alpha_j},\, i,j=1,2, i\neq j$$

Using the above expressions we can write the profits function as

$$\begin{aligned}&P_i = (1-\alpha_i)\,\tfrac{1}{2}\tfrac{3-2\alpha_j}{3-\alpha_i-\alpha_j}t\,(1-\alpha_j)\,\tfrac{3-3\alpha_i+\alpha_j}{(1-\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)}\\ &+\alpha_j\tfrac{1}{2}\tfrac{3-2\alpha_i}{3-\alpha_i-\alpha_j}t\,(1-\alpha_i)\,\tfrac{3-3\alpha_j+\alpha_i}{(1-\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)}\\ &= \tfrac{t(1-\alpha_i)}{2(3-\alpha_i-\alpha_j)^2(1-\alpha_j-\alpha_i)}\\ &\cdot\left(2\alpha_j^3-8\alpha_j^2-3\alpha_j+12\alpha_i\alpha_j-2\alpha_i^2\alpha_j+9-9\alpha_i\right)\end{aligned}$$

In the first stage of the game Firm i solves

$$Max_{\ \alpha_i}\ P_i(\ \alpha_i,\, \alpha_j),$$

whose first-order conditions with respect to α_i give:

$$\tfrac{1}{2}t\,(-3\alpha_i+3+\alpha_j)\,\tfrac{2\alpha_j^4-16\alpha_j^3+4\alpha_i\alpha_j^3-16\alpha_i\alpha_j^2+37\alpha_j^2+2\alpha_i^2\alpha_j^2+12\alpha_i\alpha_j-26\alpha_j+3-3\alpha_i^2}{(-3+\alpha_i+\alpha_j)^3(-1+\alpha_j+\alpha_i)^2} = 0$$

By symmetry, the interior solutions must satisfy $\alpha_1 = \alpha_2 = \alpha$. After simplification, the first order conditions are:

$$-\tfrac{1}{2}\left(4\alpha^3-10\alpha^2+8\alpha-1\right)t = 0$$

By solving the above equation, we find $\alpha^* = 0.15$, and, then, $P_i^* = \frac{1}{2}(1-\alpha)\frac{t}{1-2\alpha^*} = 0.6t$. However, if firms decide not to share their profit, that is, if $\alpha = 0$, we find $P_1 = 0.5t$.