Quarterly Journal of Political Science, 2012, 8: 1–53

Federal Directives, Local Discretion and the Majority Rule*

Antoine Loeper

Departamento de Economía, Universidad Carlos III de Madrid, C./ Madrid, 126, 28903 Getafe (Madrid), Spain; aloeper@eco.uc3m.es

ABSTRACT

I consider a heterogeneous federal system in which policy coordination is desirable but underprovided in the absence of a federal intervention. To improve policy coordination, the federal layer can intervene by imposing bounds on local policies. These federal bounds define a restricted policy space within which local jurisdictions have residual discretion. I analyze a voting game in which the federal bounds are determined directly by the citizens via federal majority rule.

The voting equilibrium exhibits various forms of inefficiencies. When the distribution of voters' ideal policy is skewed in one direction, the federal bounds are biased in the opposite direction. When the gains from policy coordination are negligible, local discretion is too limited, and a majority of voters are worse-off with the federal intervention

* This research benefited from the financial support of the grant ECO2010-19596 from the Spanish "Ministerio de Ciencia e Innovación". I am grateful to the editor, two anonymous referees, Jacques Crémer, Helmut Cremer, Michel Lebreton, Carlo Prato, and seminar participants at Toulouse University, Laval University, Torcuato Di Tella University, the 2008 EEA congress in Milan, the 2008 Sylvaplana workshop in Political Economy, the 2009 ASSA meeting in San Francisco, and the 2009 LAGV meeting in Marseille for suggestions that helped me improve the paper. All remaining errors are mine.

MS submitted 7 December 2010; *final version received 5 July 2012*
ISSN 1554-0626; DOI 10.1561/100.00010096

than without. When policy coordination is more important, the federal intervention is supported by a majority of voters, but contrary to the received wisdom, it is socially worse than no intervention. Hence, the model shows that inadequate and excessively rigid federal interventions can emerge in a democratic federation without agency costs or informational imperfections.

Keywords: Federalism; decentralization; local discretion; subsidiarity; majority rule; uncovered set.

JEL Codes: H77, D72

Since the seminal work of Oates (1972), the economic literature on federalism investigates which policies are better handled at the local or at the central level. However, the functioning of actual federal systems does not fit this dichotomy. On many issues, legislative, policy making, and judicial authority is shared between several actors from different tiers (Rodden, 2004; Treisman, 2002). In this paper, I analyze the case in which the federal layer can impose bounds on local policies but local jurisdictions keep their residual sovereignty within the federal bounds. Examples of such power sharing arrangements between the local and the central level abound. In the E.U., most state laws are transpositions of European directives enacted by the European Commission. These directives impose some constraints but leave member states with a certain amount of leeway as to their implementation. In the U.S., the Sentencing Reform Act imposes sentencing ranges on state courts but grants them discretion within those ranges. In most democracies, judicial review and appellate supervision from central courts limit the discretion of local courts and legislatures. In the fiscal sphere, local taxes are often subject to minima and maxima set by the central level. Likewise, the Stability and Growth Pact boils down to a set of bounds on states' fiscal policies. These federal interventions essentially define a *discretionary space* within which states or local jurisdictions can set the policies that best fit their specific needs.

The presumed advantage of a partial restriction of local discretion through federal constraints is to combine the comparative advantages of decentralization (responsiveness to local specificities) and centralization (policy coordination). Moreover, a federal intervention which leaves some degree of freedom at the local level is more likely to obtain the unanimous approval of the members of the federation.[1]

However, citizens and local policy makers often complain that federal directives are insufficiently sensitive to local needs, and that, in contrast to the subsidiarity principle, the discretion devolved to the local level is too limited, even when the gains from policy coordination are negligible. In what follows, I refer to the former bias as the *preferences-matching* problem and to the latter as the *federal-encroachment* problem. From a public choice perspective, the usual suspects for these biases and inefficiencies are the vested interests, the desire of control, and the lack of information of federal policy makers (Brennan and Buchanan, 1980; Frey and Eichenberger, 1996; Weingast, 1995). As a result, it is often argued that more a democratic, bottom-up decision process at the federal level would be more responsive to the needs of the citizens, more efficient, and more likely to conform to the subsidiarity principle.

To investigate this claim, I analyze a model in which the federal intervention is determined directly by the citizens of the federation via majority rule. The federation is composed of a finite number of jurisdictions which I call states for concreteness. In each state, a unidimensional policy has to be implemented. The voters of different states have heterogeneous preferences but all voters of the federations care about the harmonization of policies across states. The federal intervention imposes an interval $[L, R]$ within which states can choose the policy that best meet the needs of their constituents. The span of this interval determines the degree of discretion left to the states. This class of federal interventions encompasses complete decentralization (if $[L, R]$ include the ideal policies of every state) and unitarian centralization (if $L = R$).

Citizens first vote at the federal level on the discretionary interval $[L, R]$ and then vote at the state level on their respective policies. In this model, institutional imperfections and agency costs are assumed away, so the federal intervention boils down to a mere *preferences-aggregation* mechanism that allows voters to express their preferences on the degree of discretion left to the local level. This stylized model is not an attempt to describe any

[1] The tension between local preference matching and policy coordination has first been highlighted by Oates (1972). The difficulty of achieving Pareto improvement via complete harmonization is extensively discussed in the fiscal federalism literature (Oates, 1999). Kanbur and Keen (1993) and Peralta and Ypersele (2006) show that imposing a policy range instead of a uniform policy allows Pareto improving harmonization.

actual federal system in detail. Instead, its goal is to provide a transparent framework to investigate whether majoritarian decision making at the federal level can lead to a satisfactory trade-off between coordination and flexibility.

I first analyze a federal voting procedure in which citizens vote successively on the size of the discretionary interval $\Delta = \frac{|R-L|}{2}$ (the degree of local discretion), and on its location $\Gamma = \frac{L+R}{2}$ (the policy guideline). Each stage of this voting game has a natural interpretation. Because Δ determines the maximal degree of coordination that the federal intervention can impose on the member states, the first stage vote on Δ can be viewed as a constitutional referendum on the degree of decentralization. The parameter Γ is a federal guideline from which states should not depart by more than Δ. Hence, the second stage vote on Γ can be viewed as a referendum on the policy orientation of the federal intervention. Alternatively, it can be interpreted as the election of a federal representative whose mandate is to coordinate state policies in a certain direction, given the degree of discretion Δ granted to the states in the first stage. Given this interpretation, it is natural to let voters vote first on Δ and then on Γ because the institutional and procedural rules that determine the degree of decentralization of a federation are more resistant to change than the identity of the federal representative.

This voting procedure is reminiscent of the existing political economy models of federalism in which citizens typically first vote on whether to centralize or decentralize a policy ($\Delta = 0$ or $\Delta = +\infty$), and then vote on the policy at the central or local level. It extends these models in that it allows for a continuum of degrees of decentralization. In line with this literature, the vote on the degree of local discretion Δ pits moderate versus extreme voters whereas the vote on the federal guideline Γ opposes rightist and leftist voters.[2]

The analysis of this voting procedure yields the following results. First, consistent with the aforementioned preferences-matching problem, in equilibrium, the federal guideline Γ varies negatively with the preferences of nonmedian states: as the voters' preference distribution become skewed in some direction, the socially optimal federal bounds move in the same

[2] Rightist and leftist voters are defined as voters whose ideal policy is to the right and the left, respectively, of the ideal policy of the median voter. Moderates and extreme voters are defined as voters whose ideal policy is close and far, respectively, from the ideal policy of the median voter.

direction while the equilibrium bounds move in the opposite direction. When preferences are sufficiently skewed, this bias makes the federal intervention socially detrimental, as compared to complete decentralization. Second, consistent with the aforementioned federal-encroachment problem, when coordination costs are small, the equilibrium degree of discretion Δ is too limited. As a result, the federal intervention is socially detrimental and leaves a majority of voters worse off. Third, as the need for coordination increases, the federal intervention receives the support of a majority of voters but contrary to the received wisdom, it is still socially detrimental.

These results show that rigid and inadequate federal directives, often blamed on dysfunctional federal institutions or the neglect of local specificities by federal bureaucrats, can also emerge from a neutral and democratic decision process. In particular, majority rule may not be compatible with the subsidiarity principle. One reason is that moderate voters have an incentive to abuse the federal coordination mechanism and impose most of the harmonization effort on the states that need the most flexibility. They do so in order to maximize policy coordination across the federation without restricting their own sovereignty.

However, the detrimental effects of the federal intervention goes beyond the usual tyranny of the majority (of moderate voters) on the minorities (of extreme voters) because the federal intervention can make a majority of voters worse off. The reason is that the sequential vote on Δ and on Γ are driven by two different sets of pivotal voters with conflicting incentives. It turns out that this equilibrium feature further reduces local discretion in equilibrium. To see why, observe that the proponents of coordination (moderate voters who push for a smaller Δ) have more homogeneous preferences than the proponents of local discretion (extreme right and left voters who push for a greater Δ), because the latter have diametrically opposite preferences when voting on Γ at the second stage. This lack of cohesion makes their induced preferences on Δ at the first stage less congruent than those of the proponents of coordination. Hence, this model formalizes the idea that in a federation composed of a homogeneous group of core states and a group of peripheral states with different centrifugal motives, the latter cannot form a cohesive opposition to the centripetal influence of the former.

In the last section of the paper, I investigate whether the above findings are specific to this voting procedure, or they are instead unavoidable consequences of majoritarian decision making. Because the set of possible federal interval $[L, R]$ is two dimensional, there is generically no Condorcet winner at the federal level. Therefore, the outcome of a majoritarian voting procedure on the discretionary interval $[L, R]$ depends on the details of the procedure. For instance, if citizens vote successively on L and R — instead of voting successively on $\Delta = \frac{|R-L|}{2}$ and on $\Gamma = \frac{L+R}{2}$ — the unique equilibrium outcome of that voting procedure is complete centralization. However, with the help of theoretical concepts from the social choice literature (namely, the uncovered set and the Banks set), I show that detrimental effects of the federal intervention hold unchanged for a large class of majoritarian procedures which includes issue by issue voting games, amendment voting games, and electoral competition games.

1 Related Literature

Starting with Oates (1972), the public economics literature on federalism literature analyzes the costs and benefits of complete centralization and decentralization.[3] Political economy models enrich these analyses and incorporate political imperfections at the local and federal levels.[4] The findings of this normative literature have been the guiding principles of the academic debate on the subsidiarity principle in the E.U. (Inman and Rubinfeld, 1998; Trachtman, 1992). However, this literature fails to account for the inefficiency of the allocation of responsibilities in existing federal systems (Alesina *et al.*, 2005; Weingast, 1995) and the complexity of the policy making arrangements between local and federal institutions (Chubb, 1985; Rodden, 2004; Treisman, 2002). To address these shortcomings, this paper assumes that decision rights are shared between the local and the central levels, and that the degree of decentralization is determined by a popular vote. The results show that democratic decision making at the federal level can be in conflict with the subsidiarity principle.

A number of political economy papers on federalism endogenize the vertical allocation of policy responsibilities (Bolton and Roland, 1997; Crémer and Palfrey, 1996; Ellingsen, 1998; Lockwood, 2002; Redoano and Scharf, 2004). In these models, citizens choose among the two extreme regimes of complete centralization and decentralization, whereas I allow for a continuum of degrees of (de)centralization. In Crémer and Palfrey (1996),

[3] See Epple and Nechyba (2004) or Oates (1999) for a review.
[4] See, among others, Besley and Coate (2003), Inman and Rubinfeld (1997), Lockwood (2002), or Seabright (1996). See also Oates (2005) for a review.

the vote on the architecture of the federation pits moderate versus extreme voters, as in this paper. However, preferences for centralization stem from electoral uncertainty rather than externalities. Crémer and Palfrey (1999) allows for various degrees of centralization, but the strategic response of the states to the federal intervention is not modelled formally. Instead, it is assumed that each state's policy is a weighted sum of the median ideal policy at the local and federal level. Hatfield and Padro-i-Miquel (2008) analyze a federation with a continuum of public goods in which each public good is allocated entirely to one layer of government. They show that the centralization of some but not all local public goods allows the capital-poor median voter to achieve some degree of redistribution without taxing capital too heavily.

In the political economy models of federal mandates (Crémer and Palfrey, 2000, 2006; Kanbur and Keen, 1993; Monheim-Helstroffer and Obidzinski, 2010) and dual provision of public goods (Epple and Romano, 2003; Hafer and Landa, 2007; Lulfesman, 2008), policy making authority is effectively shared by the local and the federal levels.[5] Because these papers consider free-riding rather than coordination problems, the federal intervention takes the form of a unidirectional constraint. For this reason, the conflict of interests between voters at the federal level has a different flavor than in this paper. Janeba (2006) proposes a model of ideological externalities in which the federal intervention is also modelled as an interval of discretion (see his Section 4.5). Because he considers only two types of voters, the voting equilibrium always entails complete policy uniformity.

Redoano and Scharf (2004) build a model in which political integration is more likely under representative democracy because strategic delegation allows voters to commit to a small federal government. In a more public choice perspective, some authors (Brennan and Buchanan, 1980; Frey and Eichenberger, 1996; Riker, 1964; Weingast *et al.*, 1981) argue on the contrary that representative democracy leads to excessive centralization because it allows self-interested local representatives to increase public spending.[6] This paper shows that excessive centralization can also arise without political representation.

---

[5] See also Brueckner (2009), Hooghe and Marks (2003) Joanis (2009), Schwager (1999), or Wilson and Janeba (2005).

[6] See Feld *et al.* (2008) and the references therein for empirical tests of this claim.

The collective choice problem on which this model is based has been introduced in Loeper (2011). This paper departs from Loeper (2011) in that the latter compares social welfare under complete centralization and decentralization. This coordination game is reminiscent of the beauty contests analyzed in the global game literature (Angeletos and Pavan, 2007; Dewan and Myatt, 2008; Morris and Shin, 2002) in that players share a common motive to be close to the action of the other players. However, the global game literature focuses on the tension between information asymmetries and coordination whereas this paper focuses on the tension between preference heterogeneity and coordination. Consequently, coordination in the beauty contests is achieved through communication rather than through binding rules as in this model.[7]

2 The Model

Throughout the paper, if x is a vector of $\mathbb{R}^N$, $\overline{x}$ denotes its mean $\frac{1}{N}\sum_n x_n$.

2.1 The Federation

I consider a federal system composed of an odd number N of jurisdictions that I call states for concreteness. For all $n \in \{1, \ldots, N\}$, the policy of state n is denoted by x_n and the welfare of its residents is given by:

$$U_n(x) = -|x_n - \theta_n|^2 - \frac{\beta}{N} \sum_{m \neq n} |x_n - x_m|^2. \tag{1}$$

The first term in Equation (1) corresponds to the intrinsic preferences of state n, i.e., whether its policy x_n meets the specific needs θ_n of its constituents. The profile of state types $\theta \in \mathbb{R}^N$ allows for heterogeneity across states. For simplicity, I rule out heterogeneity within state.[8] The second term captures the gains from policy coordination. Depending on the policy considered, it can embody the legal uncertainty, litigation costs, or sense

[7] In the literature on the theory of the firm, Alonso *et al.* (2007) and Rantakari (2008) analyze the role of strategic communication in a coordination game similar to this one.

[8] Intrastate preferences heterogeneity does not change the results of the paper if votes are aggregated at the federal level through the "one state one vote" rule, i.e., if votes are aggregated first at the state level via intrastate majority rule and then at the federal level via interstate majority rule (Crémer and Palfrey, 1996). The median voter in each state becomes its representative voter.

of unfairness generated by heterogeneous laws; the transaction costs and barriers to trade caused by a fragmented regulatory system; the duplication of fixed costs due to the lack of standardization of public services; or the barriers to mobility generated by incompatible school curricula and calendar.[9] The parameter $\beta > 0$ determines the magnitude of these externalities.

For welfare comparison, I use the utilitarian social welfare function $W = \sum_n U_n$. When a decision is taken by vote, votes are aggregated via majority rule: for any two outcomes $x, y \in \mathbb{R}^N$, x is (strictly) majority preferred to y if a majority of voters (strictly) prefer x to y.

By convention, $\theta_1 \leq \cdots \leq \theta_N$, and $\mu = \frac{N+1}{2}$ refers to the state with median preferences. The states n such that $\theta_n < \theta_\mu$ ($\theta_n > \theta_\mu$) are called *leftist* (*rightist*) states, although the type space should not necessarily be interpreted as a liberal/conservative ideological spectrum. I assume that no majority of voters have the same type, so all voters agree that some degree of harmonization is desirable but no majority agree on the direction of harmonization.

For the clarity of exposition, I occasionally restrict attention to a federation composed of three homogeneous groups of leftist, median, and rightist states.

Definition 1 *A federation is triadic if there exists $\lambda, \rho \in \{1, \ldots, \frac{N+1}{2}\}$ such that $\rho + \lambda > \frac{N+1}{2}$ and*

$$\theta_1 = .. = \theta_\lambda < \theta_{\lambda+1} = .. = \theta_\mu = .. = \theta_{N-\rho} < \theta_{N-\rho+1} = .. = \theta_N.$$

The conditions imposed on the number of leftist and rightist states λ and ρ in Definition 1 guarantee that no single group commands a majority.

A remark is in order about the setting of the model. I depart from the standard case of positive or negative spillovers and consider instead coordination externalities, i.e., externalities that are driven by the differences and incompatibilities between local policies (as in Baniak and Grajzl, 2011; Carbonara and Parisi, 2007; Garoupa and Ogus, 2006; or Loeper, 2011), for several reasons. First, coordination externalities are at the center of many economically

---

[9] On the cost of legal and regulatory heterogeneity, see, e.g., Ribstein and Kobayashi (1996), Kox *et al.* (2005) for an empirical analysis, and Marciano and Josselin (2002) for an edited volume of contributions on the topic. See Loeper (2011) and the references therein for more on the cost of policy heterogeneity in federal systems.

relevant issues in federal systems (among others, legal and regulatory harmonization, labor mobility, and standardization of technical norms). Second, the case of spillovers has already received a lot of attention in the federalism literature. Finally, with either positive or negative spillovers, the conflict of interest at the federal level typically pits low- versus high-demand voters.[10] This symmetric coalition structure fails to capture a fundamental aspect of federalism: the voters and states that push for more centralization are usually more homogeneous than those that push for more decentralization, because the centrifugal pressures of the latter group have typically diverse motives. Consistently, in this model, the voters who prefer more coordination are residents of *core* states (loosely defined as states whose type is relatively close to θ_μ) and the voters who prefer more local discretion are residents of *peripheral* states (loosely defined as states whose type is relatively distant from θ_μ).

2.2 Centralization and Decentralization

Under decentralization, each state n has unrestricted sovereignty on its policy x_n and maximizes the welfare of its constituents U_n, taking the other policies as given. The corresponding non-cooperative equilibrium x^{dec} is given by:

$$x_n^{\text{dec}} = \frac{\theta_n + \beta\overline{\theta}}{1+\beta}, \quad \text{for all } n = 1, \dots, N. \tag{2}$$

It is Pareto inefficient because voters do not internalize interstate coordination externalities, and thus choose policies which are too heterogeneous. For instance, the policy vector x^* which maximizes the social welfare function W is such that for each state n, $x_n^* = \frac{\theta_n + 2\beta\overline{\theta}}{1+2\beta}$, so x^* is a mean-preserving contraction of x^{dec}. Hence, a federal intervention that imposes some degree of coordination can in principle improve on decentralization.

Typically, the federalism literature compares decentralization to a centralized regime that imposes a uniform policy vector $x^c = (u, \dots, u)$ chosen by federation-wide majority rule. From Equation (1), induced preferences on uniform policies are single-peaked, so the median voter theorem implies that the centralized voting equilibrium is $x^c = (\theta_\mu, \dots, \theta_\mu)$.

[10] It should be noted that in the case of joint provision of public goods with progressive taxation, coalitions of poor and rich voters versus moderate income voters can emerge in equilibrium (Epple and Romano, 2003; Hafer and Landa, 2007).

2.3 The Federal Intervention

In this simple setup, unitarian centralization is never socially better than decentralization (see Proposition 2 in Loeper, 2011). The present paper analyzes a more flexible federal coordination mechanism which limits states' policy to an interval $[L, R]$ within which states have residual control. Citizens first vote at the federal level on the discretionary interval — the details of the voting procedure are specified in the following sections — and then vote at the state level on their respective policy within the federal bounds $[L, R]$. This class of federal interventions can accommodate different degrees of local discretion (via the range of the interval) and different policy orientations (via the location of the interval on the preferences spectrum).

The power sharing arrangement induced by this coordination mechanism is consistent with the observation that in many cases, federal institutions coordinate local jurisdictions by imposing broad constraints instead of micromanaging local policies or implementing a complex scheme of contingent rewards and punishments. The federal authority simply specifies what policies, regulations, and laws the states can or cannot implement, but leaves them residual sovereignty.

This admittedly stylized class of federal interventions provides a tractable and transparent framework to analyze the trade-off between the need for coordination and flexibility. It is nevertheless inspired by existing institutions. In the U.S., the Sentencing Reform Act provides a grid of sentencing ranges to state courts for fines and jail times for each offense category. Its goal is to "provide certainty and fairness" while "avoiding unwarranted sentencing disparities" and "maintaining sufficient flexibility".[11] Many central governments impose lower and upper bounds on the tax rates set by subnational governments.[12] The imposition of both a lower and an upper federal constraints is due to the diversity of the externalities generated by local fiscal policies.[13] From a normative perspective, this mechanism could also be used

---

[11] U.S. Code, Chapter 58, Section 991: United States Sentencing Commission; establishment and purposes.

[12] Revelli (2010) documents the effect of federal minima and maxima on vehicle registration tax, electricity taxation, and waste management surcharge in Italy. See Joumard and Kongsrud (2003) for more examples of this type of federal intervention.

[13] Cross border effects of fiscal policies include fiscal competition, terms of trade, environmental spillovers, barriers to trade, or macroeconomic destabilization. See Joumard and Kongsrud (2003) or Peralta and Ypersele (2006).

to coordinate public spending in a monetary union without fiscal union: the upper bound would prevent states from abusing the implicit guarantee of a future bailout while the lower bound would prevent states from free-riding on the fiscal stimulus of the others.

Some federal interventions do not explicitly set an interval of discretion, but are *de facto* equivalent to the imposition of a space of discretion within which local governments have residual policy making authority. For instance, many federal grants are contingent on state compliance with federal regulatory goals, or the adoption of specific policy requirement (Chubb, 1985; Oates, 1999). In many democracies, judicial review and appellate supervision from federal courts, such as the U.S. supreme court or the European Court of Justice, limits the discretion of state courts and legislatures by allowing certain decisions and rulings out others (Howard, 1981). European directives leave member states some leeway for their transposition and implementation, and often specify an interval of time for implementation.[14] In a similar spirit, many nations have a core school curriculum that specifies a set of goals for student achievement but grants some degree of discretion to localities and schools to organize course system, to hire and train teachers, or to choose textbooks. In Germany, the supreme court's decision in 2003 gives some limited discretion to the Länder on the regulation of religious signs in public schools (Janeba, 2006).

2.4 The State Policy Equilibrium

The next proposition characterizes the policy equilibrium at the state level for a given interval of discretion $[L, R]$. A state equilibrium is a policy vector x such that each state $n = 1, \ldots, N$ choose its policy x_n within the federal bounds $[L, R]$ to maximize the welfare of its voters U_n, taking the policies of the other states $(x_p)_{p \neq n}$ as given.

Proposition 1 *For all $L \leq R$, there is a unique state equilibrium which is denoted by $x(L, R)$. If $l(L, R)$ and $r(L, R)$ denote the equilibrium number of*

[14] Article 249 of the Treaty establishing the European Community: "A directive shall be binding, as to the result to be achieved, upon each Member State to which it is addressed, but shall leave to the national authorities the choice of form and methods."

states constrained by the left and the right bound,[15] *respectively, then*

$$\begin{cases} \text{for } n = 1, \ldots, l(L,R),\ x_n(L,R) = L \text{ and } L > \frac{\theta_n + \beta \overline{x}(L,R)}{1+\beta}, \\ \text{for } n = l(L,R)+1, \ldots, N - r(L,R),\ x_n(L,R) = \frac{\theta_n + \beta \overline{x}(L,R)}{1+\beta}, \\ \text{for } n = N - r(L,R)+1, \ldots, N,\ x_n(L,R) = R \text{ and } R < \frac{\theta_n + \beta \overline{x}(L,R)}{1+\beta}. \end{cases}$$

Proposition 1 states an intuitive result: at the unique state equilibrium, the $l(L,R)$ states whose policy is constrained by the left bound L are the $l(L,R)$ most leftist states, and the $r(L,R)$ states whose policy is constrained by the right bound R are the $l(L,R)$ most rightist states. For the remaining states, their unconstrained ideal policy $\frac{\theta_n + \beta \overline{x}(L,R)}{1+\beta}$ lies in between the federal bounds.

3 Voting on the Federal Intervention

To analyze the vote at the federal level on the interval of local discretion $[L,R]$, I first consider in this section the following sequential voting game:

1. The federation first votes on the size of the discretionary interval $\Delta = \frac{R-L}{2}$.
2. The federation then votes on the location of the discretionary interval $\Gamma = \frac{L+R}{2}$.
3. Finally, each state votes on its respective policy within $[\Gamma - \Delta, \Gamma + \Delta]$.

In stages 1 and 2, votes are aggregated via federal majority rule, and in stage 3, they are aggregated via majority rule within each state. In the sequel, I refer to the above voting procedure as the (Γ, Δ) voting procedure.

A couple of remarks are in order. In this environment, there is no single canonical way to aggregate voters' preferences over the set of possible federal intervention $\{(L,R) \in \mathbb{R}^2 L \leq R\}$. For instance, one can let voters vote on the right and left bounds L and R sequentially. Alternatively, one can let each voter (or each state representative) propose an interval of local

[15] For a given policy vector $x \in [L,R]^N$, I say that a state n is constrained by the left bound L at x if the policy x_n^* that maximizes $U_n(x_1, \ldots, x_{n-1}, x_n^*, x_{n+1}, \ldots, x_N)$ over $\mathbb{R}$ is such that $x_n^* < L$. It is constrained by the right bound R if $x_n^* > R$.

discretion, and have all voters vote sequentially on these proposals via an amendment procedure. It turns out that the choice of the voting mechanism is not without loss of generality, because different voting procedures can result in different equilibrium outcomes. In the last section, I discuss in greater detail the sensitivity of the results to the specification of the voting procedure. I show that even though the equilibrium bounds L and R depend on the way votes are aggregated, the welfare effects of the federal intervention are qualitatively similar for a large class of voting procedures. Nevertheless, I believe that a systematic analysis of the (Γ, Δ) voting procedure is of independent interest for several reasons.

First, the decomposition into the two dimensions Γ and Δ has a natural interpretation which goes beyond the stylized, one dimensional environment in which the model is cast. The vote on the size of the discretionary interval Δ can be interpreted as a referendum on the degree of decentralization. Admittedly, in actual federal systems, decentralization is a complex phenomenon that can take many forms (Ebel and Yilmaz, 2002; Treisman, 2002; Rodden, 2004). However, in this simple model, the only role of the federal layer is to coordinate state policies, and Δ determines the maximal degree of policy coordination it can impose on the states. Therefore, Δ is a natural proxy for the relative power of the federal layer. The vote on Δ can be viewed as a constitutional decision because the coercive power of the federal institutions depends to a large extent on the authority, the legitimacy, and the resources conferred to them by constitutional rules, by organic laws, or by constitutional doctrines.[16]

The parameter Γ is a federal guideline from which states cannot deviate by more than Δ. Hence, the second stage vote on Γ can be viewed a referendum on the general policy orientation of the federal intervention. Alternatively, it can also be interpreted as the election by direct suffrage of a federal representative (e.g., the head of the European Commission, or the chief

[16] The constitutional rules and doctrines that affect the influence of the federal layer include the jurisdiction and mandate of the federal institutions (e.g., what policy areas can be subject to federal supervision), the degree of fiscal autonomy of local jurisdictions (what tax bases are controlled by the federal level, and what policy areas can be indirectly regulated via conditional grants, see Baker, 2001), the selection procedure of local policy makers (Treisman, 2002), the representation of states' interests at the federal level (Inman and Rubinfeld, 1997), or the sanction mechanism when a state violates a federal law (Buiter, 2003). The legal and administrative resources granted to the federal institutions also determine their influence because they affect the credibility of their coercive levers (e.g., how many cases the federal court can handle in a given year, how closely federal laws and programs can be monitored).

justice of the federal judiciary) whose mandate is to coordinate state policies in a certain direction, given the degree of discretion Δ granted to the states.[17]

Second, this voting mechanism is a plausible model of a perfectly democratic federal system. To see why, notice that the (Γ, Δ) voting procedure basically assumes that the two dimensions Δ and Γ can be considered separately in the decision process, and that voters view Δ as non-amendable when they consider changes in Γ. The above interpretation of Δ and Γ shows that these two dimensions correspond to different rules and procedural rights in the political process, and thus, to distinct decisions. Moreover, Δ is akin to a constitutional decision, whereas Γ is closer to an executive decision. For this reason, it is reasonable to assume that Δ is more resistant to changes than Γ, and thus that voters view Δ as fixed when they vote on Γ.

Third, the voting behavior generated by the (Γ, Δ) voting procedure is in line with (informal) empirical evidence and the above interpretation of Γ and Δ. As the analysis reveals, the vote on Δ opposes moderate versus extreme voters. This pattern is consistent with the observation that the political parties at the center of the political spectrum are typically more supportive of economic and political integration whereas extreme parties generally favor protectionism and national sovereignty.[18] The vote on Γ pits rightist versus leftists voters. This pattern is consistent with the observation that the elections of federal representatives usually divide voters along the conservative/liberal spectrum. Admittedly, no federal system has been designed entirely by popular referenda and elections as it is the case in that model. The architecture of a federation is often the result of historical accidents, initial conditions, power grabbing between central and local politicians, and judicial decisions by non-elected judges. This model is nevertheless relevant because it sheds light on how voters, when consulted, can affect the vertical allocation of power.

---

[17] Consistently with this interpretation, a number of emipircal studies have found that the influence of the federal institutions depend not only on the allocation of authority in the decision process, but also on the interests and ideology of the federal policy makers (Moe, 1982; Weingast and Moran, 1983). For instance, the latter can choose which federal program to monitor more closely (Chubb, 1985). This interpretation is also consistent with the view that even though federal courts do not create explicit rules of law, they exercise their leadership by setting broad decisional trends to signal their inclination (Baum, 1977).

[18] In the E.U., this observation also holds at the state level. The countries that push for more political integration are typically rich social democracies that share a long common history (France, Germany, Luxembourg, Belgium, and the Netherlands) while the countries that refuse to join the E.U., to adopt the Euro, or that oppose further political integration include countries as diverse as the U.K., ex-soviet satellites, Sweden, and Switzerland.

Finally, the (Γ, Δ) voting procedure follows the existing political economy models of federalism, in which citizens first vote on the architecture of the federation, and then vote on the policies at the central and/or local level.[19] It adds to these models a continuum of degrees of decentralization, but the qualitative features of the equilibrium are consistent with previous studies (Bolton and Roland, 1997; Ellingsen, 1998; Lockwood, 2002): as Proposition 6 shows, in equilibrium, the degree of decentralization Δ increases as preferences become more heterogeneous and decreases as the need for policy coordination β increases.

3.1 Notations and Equilibrium Concept

For convenience, I collect below all the notations used to analyze the (Γ, Δ) voting procedure:

Notation 1 *In this section, for any function f of the federal bounds (L, R), I write $f(\Gamma, \Delta)$ as a shortcut for $f(\Gamma - \Delta, \Gamma + \Delta)$. In particular,*

- $x(\Gamma, \Delta)$ *is the state equilibrium defined in Proposition* 1,
- $l(\Gamma, \Delta)$ *and* $r(\Gamma, \Delta)$ *are the number of states constrained by the left and right bound, respectively, at the state equilibrium* $x(\Gamma, \Delta)$,
- $V_n(\Gamma, \Delta)$ *is the welfare of state* $n \in \{1, \ldots, N\}$ *at the state equilibrium* $x(\Gamma, \Delta)$.

The game induced by the (Γ, Δ) voting procedure is solved by backward induction. At each stage, the voting equilibrium concept is the traditional Condorcet winner criterion. When a Condorcet winner does not exist, I use the more permissive notion of local majority rule equilibrium introduced by Kramer and Klevorick (1973).

Definition 2 *At the third stage, for all (Γ, Δ), the subgame equilibrium is $x(\Gamma, \Delta)$. At the second stage, for any $\Delta \geq 0$, the set of subgame equilibria, denoted by $\mathbf{\Gamma}(\Delta)$, is the set of Γ such that for all $\Gamma' \in \mathbb{R}$, $x(\Gamma, \Delta)$ is majority preferred to $x(\Gamma', \Delta)$. At the first stage, a federal equilibrium is a pair (Γ^e, Δ^e) such that $\Gamma^e \in \mathbf{\Gamma}(\Delta^e)$ and for all $\Delta \geq 0$ and all $\Gamma \in \mathbf{\Gamma}(\Delta)$, $x(\Gamma^e, \Delta^e)$ is majority preferred to $x(\Gamma, \Delta)$. A local federal equilibrium is a pair (Γ^e, Δ^e)*

[19] The models of local and federal provision of public good follow a similar timing (Crémer and Palfrey, 1999; Hafer and Landa, 2007; Lulfesman, 2008; Hatfield and Padro i Miquel, 2008).

such that $\Gamma^e \in \mathbf{\Gamma}(\Delta^e)$, *and there exists a neighborhood* N *of* Δ^e *such that for some* $\Delta \in N$ *and* $\Gamma \in \mathbf{\Gamma}(\Delta)$, $x(\Gamma, \Delta) \neq x(\Gamma^e, \Delta^e)$,[20] *and for any such* (Γ, Δ), $x(\Gamma^e, \Delta^e)$ *is majority preferred to* $x(\Gamma, \Delta)$.

3.2 Voting on the Federal Guideline $\mathbf{\Gamma}$

Note that for a fixed Δ, $l(\Gamma, \Delta)$ and $r(\Gamma, \Delta)$ have discontinuous jumps in Γ; these jumps induce convex or concave kinks in $V_n(\Gamma, \Delta)$, so $V_n(\Gamma, \Delta)$ is typically not single-peaked in Γ. Nevertheless, a Condorcet winner always exists at the second stage.

Proposition 2 *For all* $\Delta \geq 0$, *the second-stage equilibria* $\mathbf{\Gamma}(\Delta)$ *(see Definition 2) are the most preferred* Γ *of the voters of the median state. For all* $\Gamma \in \mathbf{\Gamma}(\Delta)$, *the policy of the median state is unconstrained, and with the conventions in Notation 1,*

$$\Gamma = \theta_\mu + \frac{l(\Gamma, \Delta) - r(\Gamma, \Delta)}{l(\Gamma, \Delta) + r(\Gamma, \Delta)}\Delta. \tag{3}$$

Proposition 2 suggests that, in equilibrium, the federal guideline is not positively responsive to the preferences of peripheral states: the farther from θ_μ the types of the leftist states are, relative to the types of the rightist states, the greater l relative to r, so the greater $\frac{l-r}{l+r}$, and from Equation (3), the more rightist the federal guideline. The intuition for this bias is the following: when they choose Γ, the voters of the median state determine which states are constrained by the federal bounds. This has two consequences. First, they vote for a guideline Γ which lets them choose the policy they prefer. Roughly speaking, this motive corresponds to the term θ_μ in Equation (3). Second, conditional on being unconstrained by the federal bounds, they want to maximize policy coordination across the federation. To do so, they choose a Γ which constrains the most extreme states. This motive explains the term $\frac{l-r}{l+r}\Delta$ in Equation (3).

Observe that for a given degree of local discretion Δ, the socially optimal federal guideline is increasing in the type of constrained states and thus leans towards the most extreme side of the preferences distribution.[21]

---

[20] This requirement rules out trivial equilibria in which states policies $x(\mathbf{\Gamma}(\Delta), \Delta)$ are locally constant in Δ, which could arise for Δ sufficiently large.

[21] For instance, for any $n \leq l(\Gamma, \Delta)$, $x(\Gamma, \Delta)$ is locally constant in θ_n, so $\frac{\partial^2 W(U(x(\Gamma,\Delta)))}{\partial \theta_n \partial \Gamma} = \frac{\partial}{\partial \Gamma}(\frac{\partial U_n}{\partial \theta_n})(x(\Gamma, \Delta))$. Lemma 2 in the appendix shows that $\frac{\partial U_n}{\partial \theta_n}$ is increasing in Γ. Therefore, from Topkis theorem, the socially optimal Γ must be increasing in θ_n.

Proposition 2 suggests that the equilibrium guideline does exactly the opposite. To state this result formally, the following definition is needed:

Definition 3 *A profile of type θ is skewed to the right (left) if for all $n \neq \mu$, $\frac{\theta_n+\theta_{2\mu-n}}{2} > \theta_\mu$ $\left(\frac{\theta_n+\theta_{2\mu-n}}{2} < \theta_\mu\right)$.*

In words, a distribution of preferences is skewed to the right if rightist states are more extreme than leftist states. In this case, the socially optimal guideline Γ^* lies to the right of the median type θ_μ but as the following proposition shows, the pivotal voters prefer to bias Γ toward the moderate (i.e., left) side of the preferences spectrum. This shows that the preference-matching problem mentioned in the introduction can emerge in a neutral and open democratic decision process without institutional imperfections.

Proposition 3 *The set of second-stage equilibria $\mathbf{\Gamma}(\Delta)$ (see Definition 2) is weakly decreasing in nonmedian types for the strong set order: if θ, and θ' are such that $\theta_\mu = \theta'_\mu$ and for all $n \neq \mu$, $\theta_n \leq \theta'_n$, if $\Gamma \in \mathbf{\Gamma}^\theta(\Delta)$ and $\Gamma' \in \mathbf{\Gamma}^{\theta'}(\Delta)$, then $\max(\Gamma, \Gamma') \in \mathbf{\Gamma}^\theta(\Delta)$ and $\min(\Gamma, \Gamma') \in \mathbf{\Gamma}^{\theta'}(\Delta)$.*

Moreover, if θ is skewed to the right (left), then for all $\Gamma \in \mathbf{\Gamma}(\Delta)$, $\Gamma \leq \theta_\mu$ ($\Gamma \geq \theta_\mu$).

Proposition 3 derives comparative statics for the equilibrium correspondence $\mathbf{\Gamma}(\Delta)$ with respect to the strong set order because $\mathbf{\Gamma}(\Delta)$ might not be single-valued. But when $\mathbf{\Gamma}(\Delta)$ is single-valued (which is true for almost all θ and Δ), Proposition 3 simply states that the equilibrium guideline $\mathbf{\Gamma}(\Delta)$ is decreasing in $(\theta_n)_{n\neq\mu}$, whereas as argued above, the socially optimal guideline is increasing in $(\theta_n)_{n\neq\mu}$.

An interesting question is how the degree of local discretion Δ affects this preference-matching problem. From Equation (3), the magnitude of the bias $|\mathbf{\Gamma}(\Delta) - \theta_\mu|$ is given by $|\frac{l-r}{l+r}|\Delta$. For simplicity, let us focus on the generic case in which the equilibrium guideline $\mathbf{\Gamma}(\Delta)$ is unique except at its (finitely many) discontinuity points $D_1, \ldots, D_I$.[22] On each interval $]D_i, D_{i+1}[$, the number of right and left constrained states $r(\mathbf{\Gamma}(\Delta), \Delta)$ and $l(\mathbf{\Gamma}(\Delta), \Delta)$ are constant in Δ, so the bias $|\frac{l-r}{l+r}|\Delta$ increases in Δ. However, the term $|\frac{l-r}{l+r}|$ can have upward or downward jumps in Δ at the discontinuity point D_i. In general, one can show that the term $|\frac{l-r}{l+r}|$ takes its maximal value for Δ

[22] See Lemma 6 in the appendix for a proof of the genericity of this case.

sufficiently large and its minimal value for Δ sufficiently small.[23] In the special case of a triadic federation, one can further show that $|\frac{l-r}{l+r}|$ is always increasing in Δ.

Proposition 4 *In a triadic federation* (*see Definition* 1), *the degree of local discretion exacerbates the preference-matching problem*: *for any selection* $(\Gamma(\Delta))_{\Delta\geq 0}$ *of second stage equilibria*, $|\Gamma(\Delta)-\theta_\mu|$ *is increasing in* Δ.

Note that Δ determines how strictly federal directives are enforced, so Proposition 4 states a quite intuitive result: as the federal supervision of states become more permissive, the pivotal voters at the federal level have more incentives to strategically bias the political orientation of the federal intervention so as to restrict the leeway of the states which whom they have the strongest disagreement.

3.3 Voting on the Degree of Local Discretion $\boldsymbol{\Delta}$

Let me now turn to the first stage of the voting game. Notice that as Δ varies, the number of states constrained by each federal bounds changes, so Equation (3) implies that the second stage equilibria $\boldsymbol{\Gamma}(\Delta)$ jump discontinuously, possibly nonmonotonically in Δ. As a consequence, induced preferences on Δ are neither single-peaked, nor order-restricted, nor continuous. Therefore, a federal equilibrium may not exist at the first stage.[24] As shown below, it always exists in triadic federations. In the general case, I restrict attention to local federal equilibria (see Definition 2). Because this equilibrium concept is more permissive, equilibrium multiplicity can be a problem. Fortunately, it turns out that in this model, it is sufficiently discriminating to derive clear results.

Proposition 5 *A local federal equilibrium exists* (*see Definition* 2).[25] *If* (Γ^e,Δ^e) *is a local federal equilibrium, then* $\Delta^e > 0$ *and* $l(\Gamma^e,\Delta^e) + r(\Gamma^e,\Delta^e) \geq \frac{N+1}{2}$: *there is a positive degree of local discretion but at least a majority of states are constrained.*

---

[23] To see this, observe that as $\Delta\to\infty$, either only rightist states or only leftist are constrained, so $|\frac{l-r}{l+r}|=1$. As $\Delta\to 0$, all states but the median state are constrained, so $l=r=\frac{N-1}{2}$ and $|\frac{l-r}{l+r}|=0$.

[24] Crémer and Palfrey (2006) and Denzau and Mackay (1981) provide examples of nonexistence of a global voting equilibrium in a similar setup.

[25] Observe that since induced preferences on Δ are not continuous, we cannot resort to the general existence theorem of Kramer and Klevorick (1973).

Contrary to the second stage vote on the federal guideline Γ, which pits leftist versus rightist voters, the first stage vote on the degree of local discretion Δ cuts across the left–right cleavage and opposes coalitions of extreme versus moderate voters. Therefore, the voters of the median state are not pivotal at the first stage and cannot impose their most preferred policy $\Delta = 0$. Hence, an opposition between moderate and extreme voters at the first stage guarantees a positive degree of local discretion.

To see why a majority of states must be constrained in equilibrium, it is helpful to notice the following: the second stage equilibrium $\mathbf{\Gamma}(\Delta)$ is generically unique except at its (finitely many) discontinuity points $D_1, \ldots, D_I$, and as shown in the appendix (see Lemma 6), on each interval $]D_i, D_{i+1}[$,

$$\begin{cases} \frac{\partial[x_n(\mathbf{\Gamma}(\Delta),\Delta)]}{\partial\Delta} < 0, & \text{for } n = 1, \ldots, l(\mathbf{\Gamma}(\Delta), \Delta), \\ \frac{\partial[x_n(\mathbf{\Gamma}(\Delta),\Delta)]}{\partial\Delta} = 0, & \text{for } n = l(\mathbf{\Gamma}(\Delta), \Delta) + 1, \ldots, N - r(\mathbf{\Gamma}(\Delta), \Delta), \\ \frac{\partial[x_n(\mathbf{\Gamma}(\Delta),\Delta)]}{\partial\Delta} > 0, & \text{for } n = N - r(\mathbf{\Gamma}(\Delta), \Delta) + 1, \ldots, N. \end{cases} \tag{4}$$

Equation (4) shows that as the degree of local discretion decreases, the policies of the unconstrained states are unaffected, and the policies of the constrained states are forced to move closer to that of the unconstrained states. Hence, the harmonization efforts are borne entirely by the constrained states, and the unconstrained states unanimously benefit from less discretion. For this reason, as long as they are a majority, unconstrained states form a winning coalition of free riders that push for less local discretion.

The next proposition characterizes the federal equilibrium for a triadic federation.

Proposition 6 *In a triadic federation* (*see Definition* 1), *there is a unique local federal equilibrium* (*see Definition* 2). *It is furthermore a global federal equilibrium, and it is given by*

$$\Delta^e = \min\{D^o_\lambda, D^o_\rho, D_\lambda, D_\rho\} \quad \textit{and} \quad \Gamma^e = \theta_\mu + \frac{\lambda - \rho}{\lambda + \rho}\Delta^e,$$

where

$$D^o_\lambda = \sqrt{\frac{(\lambda+\rho)(N+\beta\lambda+\beta\rho)}{(\beta+1)^2\rho(N+\beta\rho)}}\frac{\theta_\mu - \theta_1}{2}, \quad D^o_\rho = \sqrt{\frac{(\lambda+\rho)(N+\beta\lambda+\beta\rho)}{(\beta+1)^2\lambda(N+\beta\lambda)}}\frac{\theta_N - \theta_\mu}{2},$$

$$D_\lambda \equiv \frac{\lambda+\rho}{\rho + \frac{\beta}{N}(\lambda^2+\rho\lambda+N\rho)}\frac{\theta_\mu - \theta_1}{2}, \qquad D_\rho \equiv \frac{\lambda+\rho}{\lambda + \frac{\beta}{N}(\rho^2+\lambda\rho+N\lambda)}\frac{\theta_N - \theta_\mu}{2}.$$

In particular, the equilibrium degree of local discretion Δ^e *is increasing in* $|\theta_\mu - \theta_1|$ *and in* $|\theta_N - \theta_\mu|$*, and decreasing in* β*.*

Consistently with Propositions 2 and 5, in equilibrium, the $N - \rho - \lambda$ median states are unconstrained, but a majority of states are constrained (the ρ rightists states and the λ leftists states). Consistently with Proposition 3, the greater the proportion of leftist voters as compared to rightist voters (i.e., the greater $\frac{\lambda-\rho}{\lambda+\rho}$), the more rightist is the federal guideline Γ^e. The last point of Proposition 6 shows that the equilibrium degree of local discretion Δ^e increases as preferences become more heterogeneous and decreases as the need for policy coordination β increases, as expected. This result suggests that the (Γ, Δ) voting procedure allows voters to reveal, at least to some extent, their preferences on the trade-off local discretion versus policy coordination.

The intuition behind Proposition 6 is as follows: D^o_λ and D^o_ρ are the degrees of local discretion below which leftist and rightist states are constrained, respectively. As shown in Equation (4), $\Delta > \min\{D^o_\lambda, D^o_\rho\}$ cannot be an equilibrium because at Δ, the voters of unconstrained states form a majority who push for less local discretion. D_λ and D_ρ are the ideal degrees of discretion of the citizens of leftist and rightist states, respectively, conditional on being constrained by the federal bounds. Therefore, $\Delta > \min\{D_\lambda, D_\rho\}$ cannot be an equilibrium because the voters of the median states always prefer a lower Δ and either rightist or leftist states are constrained but their voters nevertheless prefer less discretion. Likewise, at $\Delta < \min\{D^o_\lambda, D^o_\rho, D_\lambda, D_\rho\}$, both rightist and leftist states are constrained and their voters prefer more discretion. Therefore, the only equilibrium candidate is $\Delta = \min\{D^o_\lambda, D^o_\rho, D_\lambda, D_\rho\}$.

4 Welfare Analysis

In this section, I derive the welfare properties of the federal intervention at the voting equilibria of the (Γ, Δ) voting procedure, as a function of the preference distribution θ and the magnitude of externalities β.

4.1 Skewed Preference Distributions

Propositions 2 and 3 show that when the preference distribution is skewed, a preferences-matching problem arises: the federal intervention is biased towards the wrong side of the preference spectrum. The next proposition further shows that this bias makes the federal intervention socially detrimental irrespective of the magnitude of externalities.

Proposition 7 *For all $\beta > 0$, if the preference distribution is sufficiently skewed (i.e., if $\frac{|\theta_1 - \theta_\mu|}{|\theta_N - \theta_\mu|}$ is sufficiently small or sufficiently large), then for all degree of local discretion $\Delta \geq 0$ and all equilibrium guideline $\Gamma \in \mathbf{\Gamma}(\Delta)$, the federal intervention (Γ, Δ) is socially worse than decentralization.*

Proposition 7 shows that the welfare properties of a federal intervention depend to a large extent on the skewness of the distribution of local preferences. It contrasts with Oates Decentralization Theorem (Oates, 1972), which states that the welfare gains of centralization hinges on the heterogeneity of local preferences and the magnitude of externalities. Observe that this results holds for all degrees of local direction Δ, and thus also holds if Δ is exogenous or if Δ is chosen with a different procedure.

4.2 Small Externalities

Proposition 5 implies that in equilibrium, the pivotal voters on the Δ dimension are necessarily from a state constrained by the federal bounds. When they choose Δ, they internalize neither the benefits of harmonization for the unconstrained states, nor the benefits of flexibility for the other constrained states. For small coordination costs, the former is negligible compared to the latter, so the pivotal voters, together with the voters of the other unconstrained states, have an incentive to excessively constrict the leeway of peripheral states to secure negligible coordination gains. As the following proposition shows, the resulting federal intervention is excessively prescriptive (part a) and socially detrimental (part b). This show that the federal encroachment problem mentioned in the introduction can arise even when the federal intervention is determined directly by the voters.

Proposition 8 *For all θ, when β is sufficiently small, for any local federal equilibrium (Γ^e, Δ^e),*

(*a*) *the equilibrium discretionary policy space $[\Gamma^e - \Delta^e, \Gamma^e + \Delta^e]$ is strictly included in the socially optimal discretionary policy space $[\Gamma^* - \Delta^*, \Gamma^* + \Delta^*]$,*

(*b*) *the federal intervention* (Γ^e, Δ^e) *is socially worse than decentralization,*
(*c*) *a majority of voters are worse off under the federal intervention* (Γ^e, Δ^e) *than under decentralization.*

Proposition 8 part (c) further suggests that the above intuition (i.e., a majority of moderate voters who restrict the leeway of a minority of extreme voters) is not the only cause of the excessive rigidity of the federal intervention. Indeed, the tyranny of the majority cannot explain why the federal intervention makes a majority of voters worse off — contrary to the case of federal mandates or dual provision of public goods (Crémer and Palfrey, 2000; Hafer and Landa, 2007; Lulfesman, 2008). That result is somewhat counter-intuitive because the class of federal interventions I consider encompasses decentralization and all decisions are taken by majority rule. However, the pivotal voters at the first stage are not the median voters as in the second stage, and because the two types of pivotal voters have conflicting incentives, their choices may leave a majority of voters worse off.

Interestingly, it turns out that the strategic interactions between the two different types of pivotal voters restricts local discretion beyond what a standard tyranny-of-the-majority argument predicts. To see why, notice that a change in Δ has two distinct effects. First, it mechanically shrinks or widen the interval of discretion $[\Gamma - \Delta, \Gamma + \Delta]$. Second, a change in Δ also shifts the interval $[\Gamma - \Delta, \Gamma + \Delta]$ via the second stage equilibrium $\mathbf{\Gamma}(\Delta)$. With respect to the first, direct effect, peripheral states unanimously prefer more discretion. With respect to the second, strategic effect, their interests collide because at the second stage, rightist and leftist states have diametrically opposed preferences on Γ. As a result, some peripheral states find it advantageous to vote for less discretion so as to shift the federal guideline in their favored direction. In words, the heterogeneity of peripheral states makes their incentives at the first voting stage less aligned than that of core states, so the former cannot form a cohesive opposition to the centripetal influence of the latter.

In the case of a triadic federation (see Definition 1) with $\lambda = \rho$, the rigidity of the federal intervention takes a simple form: under the notations of Proposition 6, as $\beta \to 0$, $D^o_\lambda < D_\lambda$, $D^o_\rho < D_\rho$, and $(D^o_\lambda, D^o_\lambda)$ tends to $(\frac{\theta_\mu - \theta_1}{\sqrt{2}}, \frac{\theta_N - \theta_\mu}{\sqrt{2}})$, so

$$\lim_{\beta \to 0} \Delta^e = \frac{1}{\sqrt{2}} \min\{\theta_\mu - \theta_1, \theta_N - \theta_\mu\},$$

whereas conditional on $\Gamma^e = \theta_\mu$, the socially optimal degree of decentralization is

$$\lim_{\beta \to 0} \Delta^* = \max\{\theta_\mu - \theta_1, \theta_N - \theta_\mu\}.$$

The comparison of the above two expressions shows that in equilibrium, the degree of local discretion is excessively restricted by a factor of at least $\sqrt{2}$. Moreover, consistent with Proposition 7, this problem is exacerbated when the preference distribution is skewed (i.e., if $\theta_\mu - \theta_1 \neq \theta_N - \theta_\mu$).

4.3 Large Externalities

As argued in the previous section, the voters pivotal on the Δ dimension internalize neither the benefits of harmonization for the unconstrained states nor the benefits of flexibility for the other constrained states. When the magnitude of externalities increases, these two effects are of comparable magnitude, so whether the incentives of the pivotal voters at the first stage result in too little or too much discretion depend on the details of the preference distribution.[26]

However, irrespective of the first stage equilibrium Δ^e, Propositions 3 and 7 state that the equilibrium guideline Γ^e at the second stage is typically not socially optimal. As the next proposition shows, this preference matching problem more than offset any potential coordination gains from the federal intervention, and the latter is generically socially detrimental.

Proposition 9 *If $\bar{\theta} \neq \theta_\mu$, when β is sufficiently large, for any local federal equilibrium* (Γ^e, Δ^e),

(*a*) *the federal intervention* (Γ^e, Δ^e) *is socially worse than decentralization,*
(*b*) *a majority of voters prefer the federal intervention* (Γ^e, Δ^e) *to decentralization.*

[26] For instance, in the case of a federation with three states and a symmetric preference distribution (i.e., $\theta_2 - \theta_1 = \theta_3 - \theta_2 = \Delta_\theta$), the optimal guideline is $\Gamma^* = \theta_2$ and the socially optimal policy vector can be implemented by setting $\Delta^* = \frac{\Delta_\theta}{1+2\beta}$. From Proposition 6, at the unique FE, $\Gamma^e = \theta_2$, and for β sufficiently large, Δ^e is given by $D_\lambda = \frac{\Delta_\theta}{1+5\beta/3}$, so

$$[\Gamma^* - \Delta^*, \Gamma^* + \Delta^*] \subset [\Gamma^e - \Delta^e, \Gamma^e + \Delta^e].$$

Conversely, in a federation with symmetric types in which a majority but not all states have the same type, at the optimum, $\Gamma^* = \theta_\mu$ and $\Delta^* > 0$ but at the FE, $\Gamma^e = \theta_\mu$ and $\Delta^e = 0$ so

$$[\Gamma^e - \Delta^e, \Gamma^e + \Delta^e] \subset [\Gamma^* - \Delta^*, \Gamma^* + \Delta^*].$$

Observe that contrary to the received wisdom, Proposition 8 part (a) shows that the federal intervention is socially detrimental even when externalities are arbitrarily large and preferences are arbitrarily homogeneous. Proposition 8 part (b) further shows that, despite its inefficiency, the federal mechanism always receives the support of a majority of voters. The latter result is reminiscent of the delimitation problem highlighted in Crémer and Palfrey (2000): the federal intervention can be approved by a popular referendum even when it is not desirable.

The intuition behind Proposition 9 goes as follows: as the need for coordination β increases, the equilibrium degree of local discretion Δ^e tends to 0, which accelerates the convergence of state policies as compared to decentralization. This effect can be socially beneficial. However, a vanishingly small degree of local discretion Δ^e vests the voters of the median state with almost dictatorial powers when they vote on Γ at the second stage: they can force all the peripheral states to set their policy arbitrarily close to their ideal policy θ_μ. Hence, under the federal intervention, the policy vector becomes insensitive to the needs of nonmedian voters. On the contrary, under decentralization, as β increases, state policies converge towards each other at a lower than optimal rate, but policy convergence is driven by the need of each state to coordinate with *all* other states. The resulting equilibrium depends on the entire profile of type θ, and thus is more sensitive to the preferences of peripheral states.

4.4 The Timing of the Voting Procedure

The timing of the (Γ, Δ) voting procedure implicitly assumes that voters view Δ as fixed when voting on Γ. Alternatively, one can ask what happens if the voters' decisions on both dimensions can be revised equally easily. A reasonable prediction in that case is the equilibria of the voting game in which Γ and Δ are voted upon separately but simultaneously: a federal equilibrium is a pair (Γ^e, Δ^e) such that for all $\Gamma \in \mathbb{R}$, $x(\Gamma^e, \Delta^e)$ is majority preferred to $x(\Gamma, \Delta^e)$, and for all $\Delta \geq 0$, $x(\Gamma^e, \Delta^e)$ is majority preferred to $x(\Gamma^e, \Delta)$. The main results for the (Γ, Δ) procedure (Propositions 5, 7, 8, and 9) extend to this voting procedure as follows.

Proposition 10 *When Γ and Δ are voted upon simultaneously, a federal equilibrium always exists. Any federal equilibrium (Γ^e, Δ^e) is majority* *preferred to decentralization and is such that $\Delta^e > 0$. However,*

(a) *when θ is sufficiently skewed, any federal equilibrium is socially worse than decentralization,*
(b) *for almost all θ, when β is sufficiently small, any federal equilibrium is socially worse than decentralization,*
(c) *for almost all θ, when β is sufficiently large, any federal equilibrium is socially worse than decentralization.*

Thus, the welfare results are essentially unchanged, with the exception that the federal intervention is now approved by a majority of voters even when $\beta \to 0$ (c.f. Proposition 8 part (c)). The reason is that contrary to the case of sequential voting, the voters who are pivotal on the Δ dimension have no incentives to strategically reduce the degree of discretion so as to affect the outcome of the vote on Γ. This implies that the delimitation problem is even more severe in this case: there is always a popular demand for the federal intervention even though it is likely to be socially detrimental.

5 General Federal Voting Procedures

In this section, I investigate whether the results stated in the previous sections are specific to the (Γ, Δ) voting procedure, or are instead deeply rooted in majoritarian decision making. To do so, I keep the structure of the federal coordination mechanism unchanged — that is, the common interval of local discretion $[L, R]$ — and the assumption that the federal intervention is determined directly by the voters via majority rule, but I consider alternative procedures to aggregate votes at the federal level.

As usual with multidimensional policy spaces, there is no Condorcet winner on the set of possible federal interventions $F = \{(L, R) \in \mathbb{R}^2 \colon L \leq R\}$.[27] As is well known, in the absence of a Condorcet winner, the equilibrium outcome of a voting game depends on the details of the voting procedure. Even within the class of issue by issue voting procedures — of which the (Γ, Δ) voting procedure is a particular case — the way the space of alternative

[27] As shown in Propositions 6 and 10, Condorcet winners can exists at all stages of a sequential or simultaneous voting game, because such procedures implicitly limit amendments to a one-dimensional subset of F at each stage. When amendment power is unrestricted, it is easy to show that any proposal in F is majority dominated by some other alternative in F.

F is divided into issues matters. In other words, the outcome of the vote depends on how the questions are framed to the voters.[28] To illustrate this phenomenon, let me first define formally the class of issue-by-issue voting procedures.

An issue-by-issue voting procedure is a pair (I, f) where $I \subset \mathbb{R}^2$ and f is a map from I to F. The two dimensions of I are the two issues voters are called to vote upon, and for all $(i_1, i_2) \in I$, the federal bounds are given by $(L, R) = f(i_1, i_2)$. I assume that I is two-dimensional because the set of alternative F is also two-dimensional. To guarantee voters' sovereignty, f is required to be onto: for all $(L, R) \in F$, there exists $(i_1, i_2) \in I$ such that $f(i_1, i_2) = (L, R)$.

Voters first vote at the federal level on i_1, then on i_2, and each state vote on its respective policy within the interval $f(i_1, i_2)$. The voting game induced by (I, f) is solved by backward induction and the equilibrium concept is the same as for the (Γ, Δ) voting procedure (see Definition 2).[29]

This class of voting games encompasses the (Γ, Δ) voting procedure: i_1 is Γ and i_2 is Δ, $I = \mathbb{R} \times \mathbb{R}^+$ and $f(i_1, i_2) = (i_1 - i_2, i_1 + i_2)$. Alternatively, one can let voters vote successively on each of the federal bound. In that case, i_1 is L, i_2 is R, $I = F$ and f is the identity mapping. The following proposition shows that the equilibrium of the (L, R) voting procedure is qualitatively different from that of the (Γ, Δ) voting procedure.

Proposition 11 *If L and R are voted upon sequentially in any order, the unique federal equilibrium is equivalent to complete centralization*: $L = R = \theta_\mu$.[30] *It is socially worse than decentralization.*

Proposition 11 shows formally that the descriptive feature of the federal equilibrium depend on the details of the voting procedure. The intuition behind Proposition 11 is that contrary to the (Γ, Δ) voting procedure, the (L, R) voting procedure opposes rightist to leftist voters at both voting

---

28 Kramer (1972) and Shepsle (1979) were the first to show that the details of the voting procedure can affect the outcome of the vote when no Condorcet winner exists. Ferejohn and Krehbiel (1987) make a similar point in the case of a two-dimensional budgetary decision.

29 At the third stage, for any (i_1, i_2), the subgame equilibrium is the state equilibrium $x(f(i_1, i_2))$ characterized in Proposition 1. At the second stage, for any first-stage outcome i_1, i_2^e is a subgame equilibrium if $(i_1, i_2^e) \in I$ and for all i_2 such that $(i_1, i_2) \in I$, $x(i_1, i_2^e) \succeq x(i_1, i_2)$. Finally, a federal equilibrium is a pair (i_i^e, i_2^e) such that i_2^e is a subgame equilibrium following i_i^e, and for any (i_1, i_2) such that i_2 is a subgame equilibrium following i_1, $x(i_i^e, i_2^e) \succeq x(i_1, i_2)$.

30 In the appendix, I show that this result holds even if L and R are voted upon simultaneously as in Proposition 10.

stages. This makes the voters of the median state pivotal at both stages and allows them to secure their most preferred policy vector. Nevertheless, the outcomes of the two voting procedures share some similarities. In particular, the federal intervention is socially detrimental, and as $\beta \to 0$, it leaves a majority of voters worse-off. A natural question is whether these negative results hold for a larger class of voting procedures.

5.1 The Uncovered Set

To investigate what properties of the federal intervention are inherent to majoritarian decision making, I use the concept of the uncovered set introduced in Fishburn (1977) and Miller (1980).

Definition 4 *For any two elements (L^1, R^1) and (L^2, R^2) of F, (L^1, R^1) is said to cover (L^2, R^2) if (L^1, R^1) is strictly majority preferred to (L^2, R^2) and if for all $(L, R) \in F$, if (L^2, R^2) is majority preferred to (L, R), then (L^1, R^1) is strictly majority preferred to (L, R).*

The uncovered set is the set of $(L, R) \in F$ which are covered by no $(L', R') \in F$.[31]

The uncovered set plays a central role in positive political theory for the following reason: as Miller (1980) and McKelvey (1986) first showed, it contains the equilibria of standard majoritarian decision procedures under various behavioral assumptions, from sophisticated behavior in a legislative environment, to electoral competition in a large electorate, to cooperative behavior in small committees. More formally, the uncovered set contains the non-cooperative equilibria of the amendment voting procedures with exogenous or endogenous agenda (Miller, 1980; McKelvey, 1986; Dutta *et al.*, 2004),[32] the support of the equilibria of two-candidate electoral competition games (McKelvey, 1986; Banks *et al.*, 2002), and

---

[31] It should be noted that the literature has used various definitions of the covering relation (McKelvey, 1986; Penn, 2006), depending on the particular strategic situation that the covering relation was meant to capture. The definition I use is the strongest one: if (L^1, R^1) covers (L^2, R^2) according to Definition 4, (L^1, R^1) also covers (L^2, R^2) according to the other definitions that have been proposed in the literature. Therefore, the uncovered set in Definition 4 contains the alternative definitions of the uncovered set. A larger uncovered set makes my results more general, because it contains the equilibrium outcome of a larger class of voting procedures, and any property that holds for all element of the uncovered sets of Definition 4 also holds on a smaller set.

[32] Amendment voting, also called voting by successive elimination, is commonly used in modern legislatures. It compares pairs of alternatives sequentially via majority rule according to a

the von Neumann–Morgenstern solutions to the cooperative game induced by the majority rule (McKelvey, 1986).[33] As the next proposition shows, the uncovered set also contains the equilibria of all issue by issue voting procedures.

Proposition 12 *If* (i_1^e, i_2^e) *is a federal equilibrium for some issue-by-issue decomposition* (I, f), *then* $f(i_1^e, i_2^e)$ *is in the uncovered set.*

For the sake of brevity, in what follows, I refer to all of the above voting procedures as *democratic procedures*. It is important to note that these procedures assume away institutional imperfections or agency costs between voters and policy makers. So the negative welfare results that I derive are the results of majoritarian forces only.

5.2 Majority Rule, Federal Interventions and Welfare

The next proposition generalizes the previous welfare results in the case of a skewed preference distribution (Proposition 7), and of large externalities (Proposition 9).[34]

Proposition 13 *If* (i) *the preference distribution is sufficiently skewed* $(i.e.,$ *if* $\frac{|\theta_\mu - \theta_1|}{|\theta_\mu - \theta_N|}$ *is sufficiently small or sufficiently large*$)$, *or if* (ii) β *is sufficiently large and* $\bar{\theta} \neq \theta_\mu$, *then for all* (L, R) *in the uncovered set,* $x(L, R)$ *is socially worse than decentralization. Moreover, there exists a majority of voters who prefer* $x(L, R)$ *to decentralization for any* (L, R) *in the uncovered set.*

Proposition 13, together with the properties of the uncovered set, imply that when preferences are sufficiently skewed, or when externalities are sufficiently large, any democratic voting procedures at the federal level is socially

certain agenda. The winner of a given round is paired against the next alternative on the agenda.

[33] It should be noted that in Miller (1980), the set of alternatives is finite and in McKelvey (1986), preferences are quasi-concave. These assumptions are violated in this model, because F is infinite and the induced preferences over (L, R) have convex kinks. However, the result on electoral competition in McKelvey (1986) (Theorem 3) do not use the quasi-concave assumption (see also Banks *et al.*, 2002, Theorem 4). As for the case of amendment procedures, Penn (2006, Theorem 1) shows without using the quasi-concavity assumption that equilibrium outcomes must be in the Banks Set, which is included in the uncovered set.

[34] It should be noted that strictly speaking, Proposition 13 does not imply Propositions 7 and 9 because Proposition 7 holds even when Δ is not determined by majority rule, and Proposition 9 is proved under the more permissive solution concept of local majority rule equilibrium.

worse than decentralization. In line with Proposition 9, Proposition 13 also highlights a delimitation problem: a democratic federal intervention receives the support of a majority of voters, even though it is socially detrimental. Moreover, the identity of the supporters does not depend on the particular outcome in the uncovered set, so a majority of voters support the intervention even if each voter expect her least preferred outcome in the uncovered set.

As externalities become smaller and preferences more symmetric, the uncovered set grows larger, and it is thus harder to derive results that hold uniformly over the uncovered set as in Proposition 13. Nevertheless, it can be shown that the federal encroachment effect described in Proposition 8 — that is, when $\beta \to 0$, majority rule leads to an excessively constraining federal intervention — is not an artefact of the (Γ, Δ) voting procedure. To see this, observe first that, as Proposition 11 shows, the federal encroachment can arise in an even more extreme form with other issue-by-issue voting procedures. Moreover, excessively prescriptive federal interventions are not specific to issue-by-issue voting procedures. As the next proposition shows, complete centralization can also be the outcome of an amendment voting procedure for any value of β. More precisely, the proposition states that complete centralization is always included in the Banks Set, which was shown to coincide with the set of equilibrium outcomes of amendment voting games with an exogenous agenda (Banks, 1985) and with an endogenous agenda (Dutta *et al.*, 2004). This result is interesting because contrary to issue-by-issue voting, amendment voting does not impose any structure on the set of alternatives, and in the case of an endogenous agenda, it allows all voters to make proposals in an unrestricted way.

Proposition 14 *Let G be a finite subset of $\mathbb{R}$ such that $\theta_\mu \in G$, and let $F_G = F \cap G^2$, then the Banks Set of F_G contains (θ_μ, θ_μ).*

Proposition 14 discretizes the set of alternatives F because there is no unambiguous way to have voters vote sequentially over an infinite agenda.[35] But because the discretization is arbitrary, it amounts to assume that state policies can only take a finite but arbitrarily large number of values, which is arguably a reasonable assumption.

[35] Banks (1985) restricted attention to a finite set of alternative, and as Dutta *et al.* (2005) put it, "an uncontroversial definition for the Banks Set has not been given for the case where X is infinite."

6 Conclusion

I consider a federation in which the federal layer coordinates member states by imposing bounds on their policies. These federal constraints define a discretionary policy space whose span determines the degree of discretion left to the states. The size of the discretionary space (the degree of decentralization) and its location on the political spectrum (the political orientation of the federal intervention) are determined directly by the citizens via federal majority rule.

In equilibrium, the federal bounds are negatively responsive to voters' preferences, and local discretion is too limited when the need for coordination is negligible. The federal intervention is too prescriptive not only because of the usual problem of the tyranny of the majority (of moderate voters) against the minority (of extreme voters) but also because extreme voters are less homogenous than moderate voters, so the former cannot form a cohesive opposition to the centripetal influence of the latter. In contrast to the received wisdom, as externalities become more severe, the federal intervention is still socially detrimental but it nevertheless receives the support of a majority of voters. The same delimitation problem arises when the preference distribution is sufficiently skewed. These negative welfare results are shown to hold for a large class of majoritarian voting procedures. Hence, this model suggest that agency costs or informational imperfection between the local and the federal level are not the only causes of excessively rigid and socially detrimental federal interventions. They can also emerge from a perfectly democratic decision process.

7 Appendix

Throughout the appendix, V^{dec} refers to the profile of welfare under decentralization.

Lemma 1 proves Proposition 1. Lemma 2 gives conditions under which majority preferences on $\{L, R\}$ aggregate transitively.

Lemma 1 *For all (L, R), the state equilibrium $x(L, R)$ characterized in Proposition 1 is unique and $x_n(L, R)$ is weakly increasing in θ, L, R and n. For any affine map $L(.)$ and $R(.)$, $x(L(\lambda), R(\lambda))$ is continuous, piecewise affine in λ and*

$$\frac{\partial \overline{x}(L, R)}{\partial L} = \frac{(1+\beta)l}{N + \beta(l+r)} \text{ and } \frac{\partial \overline{x}(L, R)}{\partial R} = \frac{(1+\beta)r}{N + \beta(l+r)}. \tag{5}$$

Proof: The first-order condition of the maximization program of each state gives that a state equilibrium is a fixed point of the best-response function:

$$f_n(x) = \max\left(L, \min\left(R, \frac{\theta_n + \beta\overline{x}}{1+\beta}\right)\right), \quad \forall n. \tag{6}$$

Because f is a contraction for the sup norm on $[L, R]^N$, the state equilibrium exists and is unique. From Equation (6), $x_n(L, R)$ is weakly increasing in n. Because f is weakly increasing in x and in (θ, L, R), Villas-Boas (1997, Theorem 4) implies that its fixed point $x(L, R)$ is weakly increasing in (θ, L, R).

Observe that for all $A, B \subset N$, the set of $\{L, R\}$ such that the constraint $L \leq \frac{\theta_n+\beta\overline{x}}{1+\beta}$ is binding for $n \in A$ and the constraint $\frac{\theta_n+\beta\overline{x}}{1+\beta} \leq R$ is binding for $n \in B$ is a convex subset of $\mathbb{R}^2$, because if x and x' are solutions of Equation (6) for (L, R) and (L', R'), respectively, then $\alpha x + (1-\alpha)x'$ is solution for $\alpha(L, R) + (1-\alpha)(L', R')$. The implicit function theorem implies that $x(L, R)$ is differentiable on the interior of these convex sets, and because there is a finite number of subsets A and B, for all affine maps $L(.)$ and $R(.)$, $x(L(\lambda), R(\lambda))$ is piecewise affine in λ. To obtain Equation (5), I differentiate Equation (6) w.r.t. L and R, sum over n, and solve for $\frac{\partial\overline{x}(L,R)}{\partial L}$ and $\frac{\partial\overline{x}(L,R)}{\partial R}$. ■

Lemma 2 *For all $L \leq L', R \leq R'$, $V_n(L', R') - V_n(L, R)$ is weakly increasing in n. In particular, majority preferences between $[L, R]$ and $[L', R']$ coincide with the preferences of the voters of the median state.*

Proof: Observe that the induced utility function $V_n(L, R)$ of state n can be written as

$$W_n(t_n, x) = \max_{y \in [L,R]}\left(-|y - t_n|^2 - \frac{\beta}{N}\sum_{m=1}^{N}|y - x_m|^2\right), \tag{7}$$

for $x = x(L, R)$ and $t_n = \theta_n$. Let $y^*(t_n, x)$ be the maximizer of Equation (7). From the envelope theorem, for all $t_n \in \mathbb{R}$, $\frac{\partial W_n}{\partial t_n} = 2(y^*(t_n, x) - t_n)$, so

$$V_n(L, R) - V_m(L, R) = \int_{\theta_n}^{\theta_m} 2(y^*(t, x(L, R)) - t)dt,$$

which in turns implies

$$V_n(L', R') - V_n(L, R) - (V_m(L', R') - V_m(L, R)) = 2\int_{\theta_n}^{\theta_m}\begin{pmatrix} y^*(t, x(L', R')) \\ -y^*(t, x(L, R)) \end{pmatrix} dt. \tag{8}$$

Observe that if $W(t_n, x, y)$ denotes the maximand of Equation (7), for all m, $\frac{\partial^2 W}{\partial y \partial x_m} > 0$. Hence W is supermodular in (y, x_m) and Topkis theorem implies that y^* is weakly increasing in x. From Lemma 1, if $L' \geq L$ and $R' \geq R$ then $x(L', R') \geq x(L, R)$, so the integrand in the right-hand side of Equation (8) is non-negative, which proves the first part of the lemma.

To prove the second part, notice that if the median voters strictly prefer (L', R') to (L, R), so do the voters of the states $n \geq \mu$. ∎

Lemma 3 *For all $L \leq R$, the state equilibrium $x(L, R)$ is equal to $x^{\text{dec}}(t)$ (see Equation (2)) where t is given by $t_n = \max(\underline{t}, \min(\overline{t}, \theta_n))$ for all n, with*

$$\begin{cases} \underline{t} = (1+\beta)L - \beta\overline{x}(L, R), \\ \overline{t} = (1+\beta)R - \beta\overline{x}(L, R). \end{cases} \tag{9}$$

At the decentralized equilibrium, for all $m \neq n$,

$$\frac{\partial V_n^{\text{dec}}}{\partial \theta_p} = \frac{2\beta}{N(1+\beta)}(\theta_n - x_p^{\text{dec}}). \tag{10}$$

Proof: The map $\theta \to x^{\text{dec}}(\theta)$ defined in Equation (2) can be inverted as follows: $\theta_n = (1+\beta)x_n^{\text{dec}} - \beta\overline{x^{\text{dec}}}$. To obtain the profile of type t in Equation (9), I substitute $x^{\text{dec}} = x(L, R)$ in the previous expression and use Equation (6).

Observe that for all n, the welfare under decentralization is given by $V_n^{\text{dec}} = W_n(\theta_n, x^{\text{dec}}(\theta))$ where W is defined in Equation (7) with $[L, R] = [\theta_1, \theta_N]$. The envelope theorem for $p \neq n$ gives

$$\begin{aligned} \frac{\partial V_n^{\text{dec}}}{\partial \theta_p} &= \frac{2\beta}{N} \sum_{m \neq p} (x_n^{\text{dec}} - x_m^{\text{dec}}) \frac{\partial x_m^{\text{dec}}}{\partial \theta_p} + \frac{2\beta}{N}(x_n^{\text{dec}} - x_p^{\text{dec}}) \frac{\partial x_p^{\text{dec}}}{\partial \theta_p}, \\ &= \frac{2\beta}{N(1+\beta)} \left[\sum_{m \neq p} (x_n^{\text{dec}} - x_m^{\text{dec}}) \frac{\beta}{N} + (x_n^{\text{dec}} - x_p^{\text{dec}}) \left(1 + \frac{\beta}{N}\right) \right], \end{aligned} \tag{11}$$

where $\frac{\partial x_m^{\text{dec}}}{\partial \theta_p}$ and $\frac{\partial x_p^{\text{dec}}}{\partial \theta_p}$ are derived from Equation (2). To obtain Equation (10), I substitute the first-order condition of Equation (7), i.e., $\frac{\beta}{N} \sum_m (x_n^{\text{dec}} - x_m^{\text{dec}}) = \theta_n - x_n^{\text{dec}}$, in Equation (11). ∎

Proof of Proposition 2. *Step 1: the voters of the median state are pivotal.* $\Gamma - \Delta$ and $\Gamma + \Delta$ are increasing in Γ, so Lemma 2 implies that the majority preferences on Γ coincide with the preferences of the median state, so $\mathbf{\Gamma}(\Delta)$ are the most preferred Γ of the voters of the median state.

Step 2: at the most preferred Γ of *the voters of the median state, their policy is unconstrained.* With the notation of Lemma 3, for all θ, Δ and for almost all Γ at which the median state is unconstrained,

$$\begin{aligned}\frac{\partial V_\mu}{\partial \Gamma}(\Gamma,\Delta) &= \frac{\partial[V_\mu^{\text{dec}}((\max(\underline{t},\min(\overline{t},\theta_n)))_n)]}{\partial \Gamma} \\ &= \sum_{n>N-r(L,R)} \frac{\partial[V_\mu^{\text{dec}}(t)]}{\partial t_n}\frac{\partial \overline{t}}{\partial \Gamma} + \sum_{n\leq l(L,R)} \frac{\partial[V_\mu^{\text{dec}}(t)]}{\partial t_n}\frac{\partial \underline{t}}{\partial \Gamma}. \quad (12)\end{aligned}$$

The substitution of Equation (5) in Equation (9) implies

$$\frac{\partial \overline{t}}{\partial \Gamma} = \frac{\partial \underline{t}}{\partial \Gamma} = 1+\beta-\beta\frac{(1+\beta)(l+r)}{N+\beta(l+r)} = \frac{(1+\beta)N}{N+\beta(l+r)}. \quad (13)$$

The substitution of Equations (10) and (13) in Equation (12) implies that for almost all Γ at which the median state is unconstrained,

$$\begin{aligned}\frac{\partial V_\mu}{\partial \Gamma}(\Gamma,\Delta) &= \frac{\beta}{N(1+\beta)}\left[l\times(\theta_\mu-\Gamma+\Delta)\frac{\partial \underline{t}}{\partial \Gamma} + r\times(\theta_\mu-\Gamma-\Delta)\frac{\partial \overline{t}}{\partial \Gamma}\right] \\ &= \frac{\beta[l\times(\theta_\mu-\Gamma+\Delta)+r\times(\theta_\mu-\Gamma-\Delta)]}{N+\beta(l+r)}. \quad (14)\end{aligned}$$

From Equation (14), for almost all such Γ, $\frac{\partial V_\mu}{\partial \Gamma} > 0$ whenever $\Gamma+\Delta < \theta_\mu$ and $\frac{\partial V_\mu}{\partial \Gamma} < 0$ whenever $\Gamma-\Delta > \theta_\mu$. The same is true a fortiori when the median state is constrained by the federal bounds.[36] So the most preferred guideline Γ^* of the voters of the median state is such that $\theta_\mu \in [\Gamma^*-\Delta, \Gamma^*+\Delta]$. For all Γ, $\overline{x}(\Gamma,\Delta) \in [\Gamma-\Delta,\Gamma+\Delta]$, so necessarily, $\frac{\theta_\mu+\beta\overline{x}(\Gamma^*,\Delta)}{1+\beta} \in [\Gamma^*-\Delta,\Gamma^*+\Delta]$, so the median state is unconstrained.

Step 3: for all $\Gamma \in \mathbf{\Gamma}(\Delta)$, $\Gamma = \theta_\mu + \frac{l-r}{l+r}\Delta$. Observe that $l(\Gamma,\Delta)$ and $r(\Gamma,\Delta)$ are weakly increasing and decreasing, respectively. So from Equation (14), at any point Γ^d of discontinuity of l or r, $\frac{\partial V_\mu}{\partial \Gamma}$ has an upward jump.

[36] For instance, if the constraint $x_\mu \leq \Gamma+\Delta$ is binding, then $\Gamma+\Delta < \theta_\mu$ so the right-hand side of Equation (14) is positive. Using the envelope theorem on Equation (7) for $x = x(\Gamma,\Delta)$, $\frac{\partial V_\mu}{\partial \Gamma}$ is given by Equation (14) plus the Lagrange multiplier of the constraint $x_\mu \leq \Gamma+\Delta$, which is positive. Therefore, $\frac{\partial V_\mu}{\partial \Gamma} > 0$.

Hence, the kinks of V_μ are all convex so V_μ is maximized at a differentiability point. To conclude, observe that from Equation (14), $\frac{\partial V_\mu}{\partial \Gamma} = 0$ implies $\Gamma = \theta_\mu + \frac{l-r}{l+r}\Delta$. ■

Proof of Proposition 3. *Step 1*: $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ *is weakly decreasing in* θ_n *for all* $n \neq \mu$ *and for almost all* Γ *such that at* (Γ, Δ), *the median state is unconstrained.* From Equation (14), for all $n \neq \mu$, $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ depends on θ_n only through r or l. Therefore, $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ is weakly decreasing in θ_n if the right-hand side of Equation (14) is decreasing in r and increasing in l. If $A(l,r)$ refers to the right-hand side of Equation (14),

$$A(l, r+1) - A(l,r) = \beta \frac{N\left(\theta_\mu - \Gamma - \Delta\right) - 2\beta l \Delta}{\left(N + \beta\left(l+r\right)\right)\left(N + \beta(l+r+1)\right)}. \tag{15}$$

For a fixed (Γ, Δ) and $(\theta_n)_{n\neq\mu}$, because l is decreasing in θ_μ, the numerator of Equation (15) is increasing in θ_μ. Let t_μ be the largest θ_μ at which state μ is unconstrained. Because the median state is unconstrained in equilibrium, from what precedes, to prove that $A(l,r)$ is weakly decreasing in r, it suffices to prove that the numerator of Equation (15) is negative at $\theta_\mu = t_\mu$. From Equation (6), at $\theta_\mu = t_\mu$, $x_\mu = \frac{\theta_\mu + \beta\overline{x}}{1+\beta} = \Gamma + \Delta$, so

$$t_\mu - \Gamma - \Delta = \beta(\Gamma + \Delta - \overline{x}). \tag{16}$$

Moreover,

$$\overline{x} \leq \frac{(N-l)\times(\Gamma+\Delta) + l \times (\Gamma - \Delta)}{N}.$$

The substitution of the above inequality in Equation (16) implies $t_\mu - \Gamma - \Delta \leq 2\beta\frac{l\Delta}{N}$, which shows that the right-hand side of Equation (15) is negative. Similar algebra shows that $A(l,r)$ is decreasing in l.

Step 2: $\mathbf{\Gamma}^\theta(\Delta)$ *is weakly decreasing in* θ_n *in the strong set order sense for* $n \neq \mu$. This follows directly from Step 1, Topkis Theorem and Proposition 2.

In what follows, t is a profile of type which is skewed to the right and t^s is defined by $t^s_n = \frac{t_n + (2t_\mu - t_{2\mu-n})}{2}$ for all n.

Step 3: *if there exists* $G \in \mathbf{\Gamma}^t(\Delta)$ *such that* $G > t_\mu$, then $2\theta_\mu - G \in \mathbf{\Gamma}^t(\Delta)$. By construction, t^s is symmetric around t_μ (i.e., for all n, $t^s_n = 2t^s_\mu - t^s_{2\mu-n}$), $t^s_\mu = t_\mu$ and $t^s_n < t_n$ for $n \neq \mu$. By symmetry, $\mathbf{\Gamma}^{t^s}(\Delta)$ is symmetric around t_μ so there exists $G' \in \mathbf{\Gamma}^{t^s}(\Delta)$ such that $G' \leq t_\mu$. Because $G' < G$ and $t^s \leq t$, from Step 2, $G' \in \mathbf{\Gamma}^t(\Delta)$ and $G \in \mathbf{\Gamma}^{t^s}(\Delta)$. By symmetry, $2\theta_\mu - G \in \mathbf{\Gamma}^{t^s}(\Delta)$ and because $2\theta_\mu - G \leq G$, Step 2 implies that $2\theta_\mu - G \in \mathbf{\Gamma}^t(\Delta)$.

Step 4: *if there exists $G \in \mathbf{\Gamma}^t(\Delta)$ such that $G > t_\mu$, then for almost all Γ in $[2\theta_\mu - G, G]$*, $l^{t^s}(\Gamma,\Delta) = l^t(\Gamma,\Delta)$ and $r^{t^s}(\Gamma,\Delta) = r^t(\Gamma,\Delta)$. From Equation (14), $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ depends on $(\theta_n)_{n\neq\mu}$ only through l and r. As shown in Step 1, $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ is increasing in l and decreasing in r. Because $t^s \leq t$,

$$l^{t^s}(\Gamma,\Delta) \geq l^t(\Gamma,\Delta) \text{ and } r^{t^s}(\Gamma,\Delta) \leq r^t(\Gamma,\Delta), \tag{17}$$

which implies that for almost all Γ, $\frac{\partial V_\mu^{t^s}}{\partial \Gamma}(\Gamma,\Delta) \leq \frac{\partial V_\mu^{t}}{\partial \Gamma}(\Gamma,\Delta)$, with a strict inequality when one of the inequalities in Equation (17) is strict. From Step 3, the median state is indifferent between $\Gamma = G$ and $\Gamma = 2\theta_\mu - G$ at $\theta = t$ and $\theta = t^s$, i.e. $\int_{2\theta_\mu - G}^{G} \frac{\partial V_\mu^\theta}{\partial \Gamma}(\Gamma,\Delta)d\Gamma = 0$ for $\theta \in \{t, t^s\}$. From what precedes, this implies that the inequalities in Equation (17) hold with equality for almost all Γ in $[2\theta_\mu - G, G]$.

Step 5: *for all $G \in \mathbf{\Gamma}^t(\Delta)$, $G \leq t_\mu$*. Suppose $G > t_\mu$. For all $\theta \in \mathbb{R}^N$ and $n < \mu$, let $\Gamma_n^\theta(\Delta)$ be a value of Γ at which l jumps from $n-1$ to n, i.e., $\frac{\theta_n + \beta\overline{x}(\Gamma_n,\Delta)}{1+\beta} = \Gamma_n - \Delta$. Because $\overline{x}$ and θ_n are greater at $\theta = t$ than at $\theta = t^s$, strictly so for θ_n, $\Gamma_n^{t^s}(\Delta) < \Gamma_n^t(\Delta)$. Likewise, one can show that the Γ at which r jumps from $n-1$ to n is strictly greater at $\theta = t$ than at $\theta = t^s$. Together with Step 4, this implies that l and r are constant on $[2\theta_\mu - G, G]$. From Proposition 3, $G = 2\theta_\mu - G = \theta_\mu + \frac{l-r}{l+r}\Delta$ so $G = \theta_\mu$. ■

The next lemma characterizes $\mathbf{\Gamma}(\Delta)$ in the case of a triadic federation.

Lemma 4 *In a triadic federation* (*see Definition* 1),

$$\Delta > \min\{D_\lambda^o, D_\rho^o\} \Rightarrow \mathbf{\Gamma}(\Delta) = \begin{cases} \{\theta_\mu - \Delta\} & \text{if } D_\lambda^o < D_\rho^o \\ \{\theta_\mu + \Delta\} & \text{if } D_\lambda^o > D_\rho^o \\ \{\theta_\mu - \Delta, \theta_\mu + \Delta\} & \text{if } D_\lambda^o = D_\rho^o. \end{cases},$$

$$\Delta < \min\{D_\lambda^o, D_\rho^o\} \Rightarrow \mathbf{\Gamma}(\Delta) = \left\{\theta_\mu + \frac{\lambda - \rho}{\lambda + \rho}\Delta\right\}.$$

For $\Delta \in]0, \min\{D_\lambda^o, D_\rho^o\}[$, $U_1(\theta, \mathbf{\Gamma}(\theta,\Delta), \Delta)$ and $U_N(\theta, \mathbf{\Gamma}(\theta,\Delta), \Delta)$ are single-peaked in Δ with a peak at D_λ and D_ρ, respectively.

Proof: From Proposition 2, for all Δ, there are three possible equilibria at the second stage: either $(l,r) = (0,\rho)$ and $\Gamma = \theta_\mu - \Delta$, or $(l,r) = (\lambda, 0)$ and $\Gamma = \theta_\mu + \Delta$, or $(l,r) = (\lambda,\rho)$ and $\Gamma = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta$.

Step 1: derivation of the state equilibrium when $(l,r) = (0,\rho)$ *and* $\Gamma = \theta_\mu - \Delta$: In this case, $x(\theta_\mu - \Delta, \Delta)$ as characterized in Equation (6) is given by

$$\begin{cases} x_1 = \frac{\theta_1 + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} = \frac{(N+\beta\lambda+\beta\rho)\theta_1 + (\beta^2\rho + N\beta - \beta\lambda)\theta_\mu}{(\beta+1)(N+\beta\rho)} \\ x_\mu = \frac{\theta_\mu + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} = \frac{(N+\beta^2\rho+N\beta-\beta\lambda+\beta\rho)\theta_\mu + \beta\lambda\theta_1}{(\beta+1)(N+\beta\rho)} \\ x_N = \theta_\mu \end{cases}, \tag{18}$$

where the second equality is simply the solution of the system described by the first equality. This solution is possible at Δ iff the leftist states are indeed unconstrained at $(\Gamma = \theta_\mu - \Delta, \Delta)$, i.e.,

$$x_1 \geq \theta_\mu - 2\Delta \Leftrightarrow \Delta \geq D' \equiv \frac{N + \beta\lambda + \beta\rho}{2(\beta+1)(N+\beta\rho)}(\theta_\mu - \theta_1).$$

In this case, the substitution of Equation (18) in Equation (1) implies

$$V_\mu(\theta_\mu - \Delta, \Delta) = -\frac{\beta\lambda(N + \beta\lambda + \beta\rho)}{N(\beta+1)^2(N+\beta\rho)}(\theta_\mu - \theta_1)^2. \tag{19}$$

Step 2: derivation of the state equilibrium when $(l,r) = (\lambda,0)$ *and* $\Gamma = \theta_\mu + \Delta$: In this case, $x(\theta, \theta_\mu + \Delta, \Delta)$ is given by

$$\begin{cases} x_1 = \theta_\mu \\ x_\mu = \frac{\theta_\mu + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} = \frac{(N+\beta^2\lambda+N\beta-\beta\rho+\beta\lambda)\theta_\mu + \beta\rho\theta_N}{(\beta+1)(N+\beta\lambda)} \\ x_N = \frac{\theta_N + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} = \frac{(N+\beta\rho+\beta\lambda)\theta_N + (\beta^2\lambda + N\beta - \beta\rho)\theta_\mu}{(\beta+1)(N+\beta\lambda)} \end{cases}. \tag{20}$$

The substitution of Equation (20) in Equation (1) implies

$$V_\mu(\theta_\mu + \Delta, \Delta) = -\frac{\beta\rho(N + \beta\lambda + \beta\rho)}{N(\beta+1)^2(N+\beta\lambda)}(\theta_N - \theta_\mu)^2. \tag{21}$$

From Proposition 2, for any Δ, $\mathbf{\Gamma}(\Delta)$ are the most preferred Γ of the voters of the median state. From Equations (19) and (21), they strictly prefer $\Gamma = \theta_\mu - \Delta$ to $\Gamma = \theta_\mu + \Delta$ if and only if

$$\lambda(N+\beta\lambda)(\theta_\mu - \theta_1)^2 < \rho(N+\beta\rho)(\theta_N - \theta_\mu)^2 \Leftrightarrow D^o_\lambda < D^o_\rho. \tag{22}$$

Step 3: derivation of the state equilibrium when $(l,r) = (\lambda,\rho)$ *and* $\Gamma = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta$: In this case, $x(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta)$ is given by

$$\begin{cases} x_1 = \theta_\mu - \frac{2\rho}{\lambda+\rho}\Delta, \\ x_\mu = \theta_\mu, \\ x_N = \theta_\mu + \frac{2\lambda}{\lambda+\rho}\Delta. \end{cases} \tag{23}$$

Notice that this solution is possible at Δ if and only if states 1 and N are indeed constrained at $(\Gamma,\Delta)=(\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta,\Delta)$, i.e.,

$$\frac{\theta_1+\frac{\beta}{N}(\lambda x_1+(N-\lambda-\rho)x_\mu+\rho x_N)}{1+\beta}\leq\Gamma-\Delta\quad\text{and}$$
$$\frac{\theta_N+\frac{\beta}{N}(\lambda x_1+(N-\lambda-\rho)x_\mu+\rho x_N)}{1+\beta}\geq\Gamma+\Delta,$$

which is equivalent to

$$\Delta\leq D''\equiv\frac{\lambda+\rho}{2\,(1+\beta)}\min\left\{\frac{\theta_\mu-\theta_1}{\rho},\frac{\theta_N-\theta_\mu}{\lambda}\right\}.$$

The substitution of Equation (23) in Equation (1) implies that at $(\Gamma,\Delta)=(\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta,\Delta)$,

$$\begin{cases}V_1=-\left(\theta_1-\theta_\mu+\frac{2\rho}{\lambda+\rho}\Delta\right)^2-\frac{\beta}{N}\left((N-\lambda-\rho)\left(\frac{2\rho}{\lambda+\rho}\right)^2+4\rho\right)\Delta^2,\\ V_\mu=-4\frac{\beta}{N}\frac{\lambda\rho}{\lambda+\rho}\Delta^2,\\ V_N=-\left(\theta_N-\theta_\mu-\frac{2\lambda}{\lambda+\rho}\Delta\right)^2-\frac{\beta}{N}\left((N-\lambda-\rho)\left(\frac{2\rho}{\lambda+\rho}\right)^2+4\lambda\right)\Delta^2.\end{cases}\tag{24}$$

Step 4: *derivation of the second stage equilibrium*: from Equations (19) and (24), the voters of the median state prefer $\Gamma=\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta$ to $\Gamma=\theta_\mu-\Delta$ if and only if

$$\Delta\leq\sqrt{\frac{(\lambda+\rho)\,(N+\beta\lambda+\beta\rho)}{4\rho(\beta+1)^2\,(N+\beta\rho)}}(\theta_\mu-\theta_1)=D^o_\lambda.$$

A symmetric reasoning shows that the voters of the median state prefer $\Gamma=\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta$ to $\Gamma=\theta_\mu-\Delta$ if and only if $\Delta\leq D^o_\rho$. Together with Equation (22), this proves that if $D^o_\lambda\leq D^o_\rho$, for $\Delta>D^o_\lambda$, $\mathbf{\Gamma}(\Delta)=\{\theta_\mu-\Delta\}$ and for $\Delta<D^o$, $\mathbf{\Gamma}(\Delta)=\{\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta\}$. Likewise, if $D^o_\lambda\geq D^o_\rho$, for $\Delta>D^o_\rho$, $\mathbf{\Gamma}(\Delta)=\{\theta_\mu+\Delta\}$ and for $\Delta<D^o_\rho$, $\mathbf{\Gamma}(\Delta)=\left\{\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta\right\}$. Simple algebra shows that the feasibility constraints, i.e. $D''\geq\min\{D^o_\lambda,D^o_\rho\}\geq D'$, are always satisfied,[37] which completes the proof of the first part of the lemma.

[37] Suppose to fix ideas that $D^o_\lambda\leq D^o_\rho$. If $\frac{|\theta_\mu-\theta_1|}{\rho}\leq\frac{|\theta_N-\theta_\mu|}{\lambda}$, $D''\geq D^o_\lambda\geq D'$ can be rewritten as

$$\sqrt{\frac{(\lambda+\rho)(N+\beta\rho)}{(N+\beta(\lambda+\rho))\rho}}\geq 1\geq\sqrt{\frac{(N+\beta(\lambda+\rho))\rho}{(\lambda+\rho)(N+\beta\rho)}},$$

Step 5: *derivation of the induced preferences of state* 1 *and* N *on* Δ: if $\Delta < \min\{D^o_\lambda, D^o_\rho\}$, from Equation (24), $\frac{\partial U_1(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta)}{\partial \Delta} = 0$ iff

$$-\frac{4\rho}{\lambda+\rho}\left(\theta_1 - \theta_\mu + \frac{2\rho}{\lambda+\rho}\Delta\right) - \frac{2\beta}{N}\left((N-\lambda-\rho)\Big(\frac{2\rho}{\lambda+\rho}\Big)^2 + 4\rho\right)\Delta = 0,$$

the solution of which is $\Delta = D_\lambda$. Likewise, $\frac{\partial V_N\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)}{\partial \Delta} = 0$ iff $\Delta = D_\rho$. ■

Proof of Proposition 4. Lemma 4 shows that $|\mathbf{\Gamma}(\Delta) - \theta_\mu|$ is single-valued and increasing in Δ on $[0, \min\{D^o_\lambda, D^o_\rho\}[$ and on $]\min\{D^o_\lambda, D^o_\rho\}, +\infty[$. To complete the proof, it suffices to notice that $\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta$ is closer to θ_μ than $\theta_\mu \pm \Delta$. ■

Lemmas 5 and 6 show that $\mathbf{\Gamma}(\Delta)$ has a finite number of discontinuity points and characterize its behavior at the continuity and discontinuity points. Lemmas 7 and 8 derive some necessary and sufficient conditions of the local federal equilibrium.

Lemma 5 *With the following notations*

$$\begin{cases} \boldsymbol{L}(\Delta) = \{l(\Gamma, \Delta) : \Gamma \in \mathbf{\Gamma}(\Delta)\}, \\ \boldsymbol{R}(\Delta) = \{r(\Gamma, \Delta) : \Gamma \in \mathbf{\Gamma}(\Delta)\}, \end{cases}$$

there exists $D_1, \ldots, D_I$ *(with the convention that* $D_0 = 0$, $D_{I+1} = +\infty$*) such that for all* $i = 0, \ldots, I$, $(\boldsymbol{L}, \boldsymbol{R})(\Delta)$ *is constant on* $]D^i, D^{i+1}[$ *and for all* $i = 1, \ldots, I$,

$$\left\{\lim_{\Delta \nearrow D^i} (\boldsymbol{L}, \boldsymbol{R})(\Delta), \lim_{\Delta \searrow D^i} (\boldsymbol{L}, \boldsymbol{R})(\Delta)\right\} \subset (\boldsymbol{L}, \boldsymbol{R})(D^i). \tag{25}$$

---

which is satisfied since $\frac{N+\beta x}{x}$ is decreasing in x and $\lambda + \rho > \rho$. If $\frac{|\theta_\mu - \theta_1|}{\rho} > \frac{|\theta_N - \theta_\mu|}{\lambda}$, $D'' \geq D^o_\lambda \geq D'$ holds when

$$\frac{|\theta_N - \theta_\mu|}{|\theta_\mu - \theta_1|} \geq \sqrt{\frac{(N + \beta(\lambda+\rho))\lambda^2(N+\beta\lambda)}{(\lambda+\rho)(N+\beta\lambda)\rho(N+\beta\rho)}},$$

which is true since $D^o_\lambda \leq D^o_\rho$ implies $\frac{|\theta_N - \theta_\mu|}{|\theta_\mu - \theta_1|} \geq \sqrt{\frac{\lambda(N+\beta\lambda)}{\rho(N+\beta\rho)}}$, and from above, $\sqrt{\frac{(N+\beta(\lambda+\rho))\lambda}{(\lambda+\rho)(N+\beta\lambda)}} < 1$.

Proof: From Proposition 2, $(\boldsymbol{L}, \boldsymbol{R})(\Delta) = \arg\max_{(l,r)} V_\mu(\theta_\mu + \frac{l-r}{l+r}\Delta, \Delta)$. From Lemma 1, for all (l, r), $V_\mu(\theta_\mu + \frac{l-r}{l+r}\Delta, \Delta)$ is piecewise quadratic in Δ. Because (l, r) can only take a finite number of values, $(\boldsymbol{L}, \boldsymbol{R})(\Delta)$ must be piecewise constant. The Berge maximum theorem implies that $(\boldsymbol{L}, \boldsymbol{R})(\Delta)$ is upper hemi-continuous in Δ, which implies Equation (25). ■

Lemma 6 *The induced preferences of the voters of the median state on* $x(\boldsymbol{\Gamma}(\Delta), \Delta)$ *are decreasing in* Δ. *With the notations of Lemma* 5, *for almost all type profile* θ, $\boldsymbol{\Gamma}(\Delta)$ *is single-valued except at its finitely many discontinuity points* $D_1, \ldots, D_I$. *Whenever* $\boldsymbol{\Gamma}(\Delta)$ *is single-valued on some* $]D^i, D^{i+1}[$, *the unconstrained states* $\{l+1, \ldots, N-r\}$ *have decreasing preferences over* Δ *on* $]D^i, D^{i+1}[$ *and*

$$\begin{cases} x_n(\boldsymbol{\Gamma}(\Delta), \Delta) = \theta_\mu - \frac{2r}{l+r}\Delta, & \text{for } n \leq l, \\ x_n(\boldsymbol{\Gamma}(\Delta), \Delta) = \theta_\mu + \frac{2l}{l+r}\Delta, & \text{for } n > N-r, \\ \frac{\partial[x_n(\boldsymbol{\Gamma}(\Delta),\Delta)]}{\partial\Delta} = 0, & \text{for } l < n \leq N-r. \end{cases} \tag{26}$$

Proof: Suppose that $\boldsymbol{\Gamma}(\Delta)$ is not single-valued on $]D^i, D^{i+1}[$, then one can slightly perturb the type of a state which is constrained in one second stage equilibrium but not at the other so as to make the median state not indifferent anymore between the two guidelines. This shows that for almost all θ, $\boldsymbol{\Gamma}(\Delta)$ is single-valued except at its discontinuity points.

Suppose $\boldsymbol{\Gamma}(\Delta)$ is single-valued on $]D^i, D^{i+1}[$. Equation (26) for $n \leq l$ and $n > N-r$ is a corollary of Proposition 2. For $n \in \{l+1, \ldots, N-r\}$, from Proposition 1,

$$x_n(\boldsymbol{\Gamma}(\Delta), \Delta) = \frac{\theta_n + \beta\overline{x}(\boldsymbol{\Gamma}(\Delta), \Delta)}{1+\beta}.$$

I differentiate with respect to Δ and substitute Equation (5) to get

$$\frac{\partial[x_n(\boldsymbol{\Gamma}(\Delta), \Delta)]}{\partial\Delta} = \frac{\beta}{1+\beta}\frac{(1+\beta)(l+r)\frac{l-r}{l+r} + (1+\beta)(r-l)}{N+\beta(l+r)} = 0.$$

The substitution of Equation (26) in Equation (1) shows that unconstrained states prefer less discretion on $]D^i, D^{i+1}[$.

From Proposition 2, the median voters are indifferent between all elements of $\boldsymbol{\Gamma}(\Delta)$ and their policy is unconstrained at (Γ, Δ) for any $\Gamma \in \boldsymbol{\Gamma}(\Delta)$. From what precedes, this implies that the median voters always prefer less discretion. ■

Lemma 7 *If* (Γ^e, Δ^e) *is a local federal equilibrium, then* $\mathbf{\Gamma}(\Delta)$ *is single valued on* $]\Delta^e - \varepsilon, \Delta^e[$ *for some positive* ε *and* $\Gamma^e = \lim_{\Delta \nearrow \Delta^e} \mathbf{\Gamma}(\Delta)$.

Proof: Suppose that $\mathbf{\Gamma}(\Delta)$ is not single-valued on $]\Delta^e - \varepsilon, \Delta^e[$. From Equation (25), $\mathbf{\Gamma}(\Delta^e)$ is not single-valued. Let $\Gamma^o \in \mathbf{\Gamma}(\Delta^e)$ such that $\Gamma^o \neq \Gamma^e$, say $\Gamma^o < \Gamma^e$ for concreteness. From Proposition 2, $\Gamma^o = \theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^e$ where $(l^o, r^o) = (l, r)(\Gamma^o, \Delta^e)$ and the voters from the median state are indifferent between (Γ^o, Δ^e) and (Γ^e, Δ^e). From Lemma 2, the voters from leftist states strictly prefer (Γ^o, Δ^e) to (Γ^e, Δ^e). By continuity, they strictly prefer $(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon'), \Delta^e - \varepsilon')$ for some small $\varepsilon' > 0$. From Lemma 6, so do the median voters. From Lemma 5, $\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon') \in \mathbf{\Gamma}(\Delta^e - \varepsilon')$, hence $(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon'), \Delta^e - \varepsilon')$ is a valid deviation from (Γ^e, Δ^e) and (Γ^e, Δ^e) cannot be a local federal equilibrium.

If $\Gamma^e \neq \lim_{\Delta \nearrow \Delta^e} \mathbf{\Gamma}(\Delta)$, the same reasoning shows that $(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon), \Delta^e - \varepsilon)$ is majority preferred to (Γ^e, Δ^e) where $(l^o, r^o) = (l, r)(\Gamma^o, \Delta^e)$ and $\Gamma^o = \lim_{\Delta \nearrow \Delta^e} \mathbf{\Gamma}(\Delta)$. ∎

Lemma 8 *With the notations of Lemma* 5, (Γ^e, Δ^e) *is a local federal equilibrium if and only if there exists* $i > 0$ *such that one of the following is true*:

(*a*) $\Delta^e \in]D^{i-1}, D^i[$, $\mathbf{\Gamma}(\Delta)$ *is single-valued on* $]D^{i-1}, D^i[$, $\mathbf{\Gamma}(\Delta^e) = \{\Gamma^e\}$ *and majority preferences on* Δ *are strictly single peaked on* $]D^{i-1}, D^i[$ *with a peak at* Δ^e.

(*b*) $\Delta^e = D_i$, $\mathbf{\Gamma}(\Delta)$ *is single-valued on* $]D^{i-1}, D^i[$, $\Gamma^e = \lim_{\Delta \nearrow D_i} \mathbf{\Gamma}(\Delta)$ *and majority preferences on* Δ *are increasing on* $]D^{i-1}, D^i[$.

Proof: If (a) is satisfied, (Γ^e, Δ^e) is clearly a local federal equilibrium. Reciprocally, suppose that (Γ^e, Δ^e) is a local federal equilibrium and $\Delta^e \in]D^{i-1}, D^i[$. From Lemmas 7 and 5, $\mathbf{\Gamma}(\Delta)$ is single-valued on $]D^{i-1}, D^i[$. Therefore, the induced preferences of all voters on $x(\mathbf{\Gamma}(\Delta), \Delta)$ are well-defined, quadratic and concave, and not flat by definition of a local federal equilibrium. Therefore, majority preferences are strictly quasi-concave on $]D^{i-1}, D^i[$, and the conclusion follows from the median voter theorem.

If (Γ^e, Δ^e) is a local federal equilibrium and $\Delta^e = D^i$, from Lemma 7 and $\mathbf{\Gamma}(\Delta)$ is single-valued on $]D^{i-1}, D^i[$. Suppose majority preferences are not increasing on $]D^{i-1}, D^i[$. As argued earlier, majority preferences are quasi-concave on $]D^{i-1}, D^i[$, so they must be decreasing on $]D^i - \varepsilon, D^i[$ for some

$\varepsilon > 0$. From Lemma 7, $\Gamma^e = \lim_{\Delta \nearrow D^i} \mathbf{\Gamma}(\Delta)$ and from what precedes, (Γ^e, Δ^e) cannot be a local federal equilibrium.

Reciprocally, suppose (b) is satisfied, then by assumption, there exists $\varepsilon > 0$ such that for all $\Delta \in]\Delta^e - \varepsilon, \Delta^e[$, (Γ^e, Δ^e) is majority preferred to $(\mathbf{\Gamma}(\Delta), \Delta)$. For all $\Gamma \in \mathbf{\Gamma}(\Delta^e)$, the median voter is indifferent between (Γ^e, Δ^e) and (Γ, Δ^e) so from Lemma 2, (Γ^e, Δ^e) is majority preferred to (Γ, Δ^e). Finally, let $\Delta^k \searrow \Delta^e$ and $\Gamma^k \in \mathbf{\Gamma}(\Delta^k)$. One can restrict attention to sequences such that $(l, r)(\Gamma^k, \Delta^k) = (l^o, r^o)$ for all k and some (l^o, r^o). Because $\Delta^k > \Delta^e$, Lemma 6 implies that the median state strictly prefer (Γ^e, Δ^e) to (Γ^k, Δ^k). From Equation (3), $\Gamma^o \equiv \lim \Gamma^k$ exists. Suppose to fix ideas that $\Gamma^e < \Gamma^o$.[38] For k sufficiently large, $\Gamma^e \pm \Delta^e < \Gamma^k \pm \Delta^k$ so from Lemma 2 and what precedes, (Γ^e, Δ^e) is strictly preferred to (Γ^k, Δ^k) by leftist states. Finally (Γ^e, Δ^e) is majority preferred to (Γ^k, Δ^k) for k sufficiently large. ■

Proof of Proposition 5. As $\Delta \to 0$, from Proposition 2, all states n such that $\theta_n \neq \theta_\mu$ must be constrained and $\mathbf{\Gamma}(\Delta)$ is single valued. In this case, one can easily see from Equation (1) that $V_n(\mathbf{\Gamma}(\Delta), \Delta)$ must be increasing in a neighborhood of $\Delta = 0$.[39] For a majority of states, $\theta_n \equiv \theta_\mu$, so $\Delta = 0$ cannot be a local federal equilibrium. With the notations of Lemma 5, because $\mathbf{\Gamma}(\Delta)$ is single valued on $]D^0, D^1[$, majority preferences are quasi-concave on $]D^0, D^1[$. If they are increasing on $]D^0, D^1[$, Lemma 8 implies that D^1 is a local federal equilibrium. If they are not increasing, from what precedes, they must be single-peaked and Lemma 8 implies that there exists a local federal equilibrium in $]D^0, D^1[$.

From Lemma 8, at any local federal equilibrium (Γ^e, Δ^e), a majority of voters have preferences which are weakly increasing in Δ on $]\Delta^e - \varepsilon, \Delta^e[$. From Lemma 6, these voters are from states which are constrained by the federal bounds, which proves that a majority of states must be constrained at (Γ^e, Δ^e). ■

Proof of Proposition 6. For the sake of brevity, I assume throughout the proof that $D^o_\lambda > D^o_\rho$. The cases $D^o_\lambda < D^o_\rho$ and $D^o_\lambda = D^o_\rho$ can be derived identically. On $]D^o_\rho, +\infty[$, from Lemma 4, the state equilibrium is constant (see Equation (18)) and equal to $x(\theta_\mu + D^o_\rho, D^o_\rho)$. Because

[38] As can be seen from the proof of Lemma 5, since $\mathbf{\Gamma}(\theta, \Delta)$ is single-valued on $]\Gamma^e - \varepsilon, \Gamma^e[$, $\Gamma^e \neq \Gamma^o$.

[39] The first-order effect on $-|x_n - \theta_n|^2$ is positive while the first-order effect on $-\frac{\beta}{N} \sum_{m \neq n} |x_n - x_m|^2$ is zero.

$\lim_{\Delta \nearrow D^o_\rho} \mathbf{\Gamma}(\Delta) \neq \theta_\mu + D^o_\rho$, from Lemma 7, $(\Gamma, \Delta) = (\theta_\mu + D^o_\rho, D^o_\rho)$ is not a local federal equilibrium. From Lemma 7, the only local federal equilibrium candidates are therefore $\left\{\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right): \Delta \leq D^o_\rho\right\}$. On $[0, D^o_\rho[$, the preferences of all voters on $x(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta)$ are quadratic and concave in Δ. Therefore, majority preferences are quasi-concave on $[0, D^o_\rho[$. Moreover, they are either single-peaked or strictly increasing on $[0, D^o_\rho[$.[40]

Case 1: majority preferences are increasing on $[0, D^o_\rho[$. From Lemma 7 $(\Gamma^e, \Delta^e) \equiv \left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho} D^o_\rho, D^o_\rho\right)$ is the only local federal equilibrium. By assumption, (Γ^e, Δ^e) is majority preferred to $\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)$ for $\Delta < D^o_\rho$. From Proposition 2, the voters of the median state are indifferent between (Γ^e, Δ^e) and $(\theta_\mu + D^o_\rho, D^o_\rho)$, from Lemma 2 the rightist and leftist voters have opposite preferences between these two alternatives, so (Γ^e, Δ^e) is majority preferred to $(\theta_\mu + D^o_\rho, D^o_\rho)$. This shows that (Γ^e, Δ^e) is a Condorcet winner among all (Γ, Δ) such that $\Gamma \in \mathbf{\Gamma}(\Delta)$.

Case 2: majority preferences are single-peaked on $[0, D^o_\rho[$. From Lemma 4, $\Delta^e = \min\{D_\lambda, D_\rho\}$ and $\Delta^e \leq D^o_\rho$. From Lemma 7, $\left(\Gamma^e = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta^e, \Delta^e\right)$ is the unique local federal equilibrium. By assumption, (Γ^e, Δ^e) is majority preferred to $\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)$ for $\Delta \leq D^o_\rho$. From Lemma 6, the voters of the median state strictly prefer (Γ^e, Δ^e) to $(\theta_\mu + D^o_\rho, D^o_\rho)$. Moreover, because $\Delta^e \leq D^o_\rho$,

$$\theta_\mu + \frac{\lambda - \rho}{\lambda + \rho}\Delta^e \pm \Delta^e < \theta_\mu + D^o_\rho \pm D^o_\rho,$$

so from Lemma 2 (Γ^e, Δ^e) is majority preferred to $(D^o_\rho, \theta_\mu + D^o_\rho)$. Hence, (Γ^e, Δ^e) is a Condorcet winner among all (Γ, Δ) such that $\Gamma \in \mathbf{\Gamma}(\Delta)$.

To prove the last point, it suffices to notice that D^o_λ, D^o_ρ, D_λ, and D_ρ are all decreasing in β and increasing in $|\theta_\mu - \theta_1|$ and in $|\theta_\mu - \theta_N|$. ∎

Proof of Proposition 7. The substitution of Equation (2) in Equation (1) yields

$$\begin{cases} V^{\text{dec}}_n = -\left(\frac{\beta}{1+\beta}(\theta_n - \bar{\theta})\right)^2 - \frac{\beta}{N}\sum_m \left(\frac{1}{1+\beta}(\theta_n - \theta_m)\right)^2, \\ W^{\text{dec}} = -N\frac{2\beta+\beta^2}{1+2\beta+\beta^2}\text{var}(\theta). \end{cases} \tag{27}$$

Let $(\theta^k)_{k\in\mathbb{N}}$ be such that $\frac{|\theta^k_N - \theta^k_\mu|}{|\theta^k_1 - \theta^k_\mu|} \to \infty$. Without loss of generality, I can renormalize the type space and the policy space for each $k \in \mathbb{N}$ so that

[40] They cannot be decreasing to the right of $\Delta = 0$ from Equation 24.

θ_1^k and θ_μ^k are constants, and thus for all $n \leq \mu$, θ_n^k is bounded, but $\theta_N^k \to +\infty$. Along this sequence $(\theta^k)_{k\in\mathbb{N}}$, the voters of the median state can always choose Γ to make sure that $x^{\theta^k}(\Gamma,\Delta)$ is bounded over k and Δ (e.g., $\Gamma = \theta_\mu - \Delta$). From Proposition 2, the voters of the median state are always pivotal on Γ, so $x^{\theta^k}(\mathbf{\Gamma}^{\theta^k}(\Delta),\Delta)$ must be bounded over k and Δ, otherwise $\liminf_k V_\mu^{\theta^k}(\mathbf{\Gamma}^{\theta^k}(\Delta),\Delta) = -\infty$. This implies that for all $n \in N$, $V_n^{\theta^k}(\Gamma,\Delta) = -(\theta_n^k)^2 + O(1)$, so $W^{\theta^k}(\Gamma,\Delta) \leq -Nvar(\theta^k) + O(1)$, and from (27), $W^{\theta^k}(\Gamma,\Delta) < W^{\theta^k,\mathrm{dec}}$ for k sufficiently large. ■

Lemma 9 *As $\beta \to 0$, for any selection of local federal equilibrium $(\Gamma^\beta,\Delta^\beta)$, for all constrained states n, $|x_n^\beta(\Gamma^\beta,\Delta^\beta) - \theta_n|$ is bounded away from* 0.

Proof: To show that for all constrained states n, $|x_n^\beta(\Gamma^\beta,\Delta^\beta) - \theta_n|$ is bounded away from 0 as $\beta \to 0$, I proceed by contradiction: in what follows, β^k is a sequence such that $\beta^k \to 0$ and (Γ^k,Δ^k) is a sequence of local federal equilibrium for the profile of preferences (θ,β^k) such that $x_m^{\beta^k}(\Gamma^k,\Delta^k) \to \theta_m$ for some state m (independent of k) which is constrained for all k, and $(\boldsymbol{L},\boldsymbol{R})$ refer to the map defined before Lemma 5. I assume w.l.o.g. that for all k, m is constrained by the right bound.

Step 1: there exists a subsequence of β^k such that $(\Gamma^k,\Delta^k) \to (\Gamma^o,\Delta^o)$, for all k, $l^{\beta^k}(\Gamma^k,\Delta^k) = l^o$ and $r^{\beta^k}(\Gamma^k,\Delta^k) = r^o$ for some Γ^o, Δ^o, l^o and r^o, and $\theta_m = \Gamma^o + \Delta^o$. Because $(\boldsymbol{L},\boldsymbol{R})$ can only take a finite number of values and (Γ^k,Δ^k) is bounded, the limit exists. To see the last point, observe that for all k, $\bar{x}^{\beta^k}(\Gamma^k,\Delta^k) \leq \Gamma^k + \Delta^k$. So Proposition 1 implies that $\theta_m \geq x_m^{\beta^k}(\Gamma^k,\Delta^k)$, so $\theta_m \geq \Gamma^o + \Delta^o$. Because m is constrained by the right bound, Proposition 1 implies further that $\theta_m > \Gamma^k + \Delta^k$, so $\theta_m \geq \Gamma^o + \Delta^o$.

Step 2: there exists a subsequence of β^k such that $l^{\beta^k}(\Gamma,\Delta^k)$ and $r^{\beta^k}(\Gamma,\Delta^k)$ converge pointwise to two step functions $l(\Gamma)$ and $r(\Gamma)$ such that $\lim_{\Gamma\searrow\Gamma^o} r(\Gamma) \leq r^o - 1$ *and* $\lim_{\Gamma\searrow\Gamma^o} l(\Gamma) \geq l^o$. Observe that for a given k, $l^{\beta^k}(\Gamma,\Delta^k)$ and $r^{\beta^k}(\Gamma,\Delta^k)$ are functions of Γ which are piecewise constant, bounded and have a bounded number of points of discontinuities as $k \to \infty$. The existence of $l(\Gamma)$ and $r(\Gamma)$ follows from Bolzano–Weierstrass. From Step 1, for all $\Gamma > \Gamma^o$, for k sufficiently large, $\Gamma^k < \Gamma$ so $\lim_{k\to\infty} l^{\beta^k}(\Gamma,\Delta^k) \geq l^o$. From Step 1 again, $\theta_m = \Gamma^o + \Delta^o$ so for k sufficiently large, $r^{\beta^k}(\Gamma,\Delta^k) \leq N - m$. Because m is constrained for all k, $N - m + 1 \leq r^o$.

Step 3: *there exists* $\epsilon > 0$ *and* $c > 0$*such that*

$$V_\mu^{\beta^k}(\Gamma^k - \epsilon, \Delta^k) - V_\mu^{\beta^k}(\Gamma^k, \Delta^k) = c\beta^k + o(\beta^k) \tag{28}$$

From Equation (14), for all $\epsilon > 0$,

$$\begin{aligned} & V_\mu^{\beta^k}(\Gamma^k + \epsilon, \Delta^k) - V_\mu^{\beta^k}(\Gamma^k, \Delta^k) \\ & = \frac{\beta^k}{N} \int\limits_{\Gamma^k}^{\Gamma^k+\epsilon} \begin{pmatrix} l^{\beta^k}(\Gamma, \Delta^k)(\theta_\mu - \Gamma + \Delta^k) \\ + r^{\beta^k}(\Gamma, \Delta^k)(\theta_\mu - \Gamma - \Delta^k) \end{pmatrix} d\Gamma + o(\beta^k). \end{aligned} \tag{29}$$

From Proposition 2, $\Gamma^k \to \theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^o$. Together with Step 2 and the dominated convergence theorem, this implies that

$$\begin{aligned} & \lim_{k\to\infty} \int_{\Gamma^k}^{\Gamma^k+\epsilon} [l^{\beta^k}(\Gamma, \Delta^k)(\theta_\mu - \Gamma + \Delta^k) + r^{\beta^k}(\Gamma, \Delta^k)(\theta_\mu - \Gamma - \Delta^k)] d\Gamma \\ & = \int_0^\epsilon \begin{bmatrix} l\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^o + \varepsilon\right) \times \left(\frac{2r^o}{l^o + r^o}\Delta^o - \varepsilon\right) \\ + r\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^o + \varepsilon\right) \times \left(-\frac{2l^o}{l^o + r^o}\Delta^o - \varepsilon\right) \end{bmatrix} d\varepsilon \\ & \geq \int_0^\epsilon \left(-(l^o + r^o + 1)\varepsilon + \frac{2l^o}{l^o + r^o}\Delta^o\right) d\varepsilon. \end{aligned} \tag{30}$$

Simple calculus shows that Equation (30) is positive for ϵ sufficiently small, which, together with Equation (29), completes the proof of Step 3.

To conclude, observe that Step 3 implies that $V_\mu^{\beta^k}(\Gamma^k - \epsilon, \Delta^k) > V_\mu^{\beta^k}(\Gamma^k, \Delta^k)$ for some k and some $\epsilon > 0$, which contradicts Proposition 2. ■

Proof of Proposition 8. Lemma 9 implies that as $\beta \to 0$, the left bound of the discretionary interval $\Gamma^\beta - \Delta^\beta$ is bounded above $\theta_1 + \varepsilon$ and that the right bound $\Gamma^\beta + \Delta^\beta$ is bounded below by $\theta_N - \varepsilon$ for some $\varepsilon > 0$ independent of β; it should be clear from Equation (1) that as $\beta \to 0$, the optimal interval of discretion tends to $[\theta_1, \theta_N]$, which proves part (a).

Lemma 9 also implies that for all constrained states n, for any selection of local federal equilibrium $(\Gamma^\beta, \Delta^\beta)_{\beta>0}$, $V_n^\beta(\Gamma^\beta, \Delta^\beta)$ is bounded away from 0 as $\beta \to 0$, and for all unconstrained states n, $V_n^\beta(\Gamma^\beta, \Delta^\beta) \to 0$. From Equation (27), for all $n \in N$, $V_n^{\mathrm{dec},\beta} \to 0$, which proves part (b). ■

Proof of Proposition 9. From Equation (27), as $\beta \to \infty$,

$$\lim_{\beta\to\infty} V_n^{\mathrm{dec},\beta} = -(\theta_n - \overline{\theta})^2. \tag{31}$$

Let $(\Gamma^\beta, \Delta^\beta)_{\beta>0}$ be a selection of local federal equilibrium. From Equation (6), as $\beta \to \infty$, for all n, $x_n^\beta(\Gamma^\beta, \Delta^\beta) - \overline{x}^\beta(\Gamma^\beta, \Delta^\beta) \to 0$. Because a majority of states must be constrained at any local federal equilibrium, $\Delta^\beta \to 0$. From Proposition 2, $\theta_\mu \in [\Gamma^\beta - \Delta^\beta, \Gamma^\beta + \Delta^\beta]$, so $\overline{x}^\beta(\Gamma^\beta, \Delta^\beta) \to \theta_\mu$. From Equation (6), as $\beta \to \infty$, $x_n^\beta(\Gamma^\beta, \Delta^\beta) = \overline{x}^\beta(\Gamma^\beta, \Delta^\beta) + \frac{\theta_n - \overline{x}^\beta(\Gamma^\beta, \Delta^\beta)}{\beta} + o\left(\frac{1}{\beta}\right)$. I substitute the previous equality in Equation (1) to obtain

$$\lim_{\beta\to\infty} V_n^\beta(\Gamma^\beta, \Delta^\beta) = -(\theta_n - \theta_\mu)^2. \tag{32}$$

Suppose to fix ideas that $\theta_\mu < \overline{\theta}$. Equations (31) and (32) imply that the voters of all states $n \leq \mu$ prefer $x(\Gamma^\beta, \Delta^\beta)$ to x^{dec} as $\beta \to \infty$, which proves part (a). The summation of Equations (31) and (32) over n yields $\lim_{\beta\to\infty} \sum_n V_n^{\text{dec},\beta} > \lim_{\beta\to\infty} \sum_n V_n^\beta(\Gamma^\beta, \Delta^\beta)$, which proves part (b). ■

Proof of Proposition 10. *Step 1: a federal equilibrium exists.* Let $\Delta^e > |\theta_N - \theta_1|$ so that only rightist or only leftist states can be constrained. Given Δ^e, Proposition 2 implies that the Condorcet winner on the Γ dimension is $\Gamma^e = \theta_\mu \pm \Delta^e$. Suppose $\Gamma^e = \theta_\mu + \Delta^e$ to fix ideas. Because $\Gamma^e > \theta_N$, for all $\Delta \geq 0$, rightist states are not constrained and the state equilibrium $x(\Gamma^e, \Delta)$ is equivalent to $x(\Gamma^e + (\Delta^e - \Delta), \Delta^e)$. By construction of Γ^e, it is not majority preferred to $x(\Gamma^e, \Delta^e)$.

Step 2: any federal equilibrium (Γ^e, Δ^e) *is majority preferred to decentralization, and* $\Delta^e > 0$. Observe that the majority of states always prefer to constrain at least the most extreme states than to constrain none, so $x(\Gamma^e, \Delta^e) \neq x^{\text{dec}}$. For Δ sufficiently large, $x(\Gamma^e, \Delta)$ is equivalent to x^{dec}. By construction, it is not majority preferred to $x(\Gamma^e, \Delta^e)$. Suppose $\Delta^e = 0$, then $\Gamma^e = \theta_\mu$, but then one can easily see from Equation (1) that (Γ^e, ε) is majority preferred to $(\Gamma^e, 0)$ for some $\varepsilon > 0$.

Step 3: proof of part (a). Immediate from Proposition 7.

Step 4: proof of part (b). With the notations of Lemma 3, for almost all Δ,

$$\begin{aligned}
\frac{\partial V_m(\Gamma, \Delta)}{\partial \Delta} &= \frac{\partial [V_m^{\text{dec}}((\max(\underline{t}, \min(\overline{t}, \theta_n)))_n)]}{\partial \Delta} \\
&= \sum_{n > N - r(L,R)} \frac{\partial [V_m^{\text{dec}}(t)]}{\partial t_n} \frac{\partial \overline{t}}{\partial \Delta} + \sum_{n \leq l(L,R)} \frac{\partial [V_m^{\text{dec}}(t)]}{\partial t_n} \frac{\partial \underline{t}}{\partial \Delta}, \\
&= \frac{2\beta}{N(1+\beta)} \left[r(L,R)(\theta_m - R) \frac{\partial \overline{t}}{\partial \Delta} + l(L,R)(\theta_m - L) \frac{\partial \underline{t}}{\partial \Delta} \right].
\end{aligned} \tag{33}$$

From Equation (6), $0 \leq \frac{\partial \bar{x}(L,R)}{\partial R} \leq 1$ so from Equation (9), for almost all Δ, $\frac{\partial \bar{t}}{\partial \Delta} > 0$ and $\frac{\partial \underline{t}}{\partial \Delta} < 0$. Therefore, the term in bracket in the right-hand side of Equation (33) is always negative when $\theta_m \in [\Gamma - \Delta, \Gamma + \Delta]$, which shows that $\frac{\partial V_m(\Gamma,\Delta)}{\partial \Delta}$ is negative for all states m such that $\theta_m \in [\Gamma - \Delta, \Gamma + \Delta]$. Therefore, in the generic case in which all state types are distinct, as $\beta \to 0$, in any federal equilibrium, at most a bare majority of states are unconstrained, which is clearly socially worse than decentralization.

Step 5: proof of part (c). Suppose that at the federal equilibrium, only rightist states are constrained. Then from Proposition 2, the right bound $\Gamma + \Delta$ is equal to θ_μ, and the left bound is inconsequential. As $\beta \to \infty$, one can easily see from Proposition 1 that all state policies converge to θ_μ. By symmetry, the same is true if only leftist states are constrained. If both rightist and leftist states are constrained, this means that $\Delta^e \to 0$ as $\beta \to \infty$. Therefore, from Proposition 2, $\Gamma^e \to \theta_\mu$, and $x(\Gamma^e, \Delta^e) \to (\theta_\mu, \ldots, \theta_\mu)$. Finally, the same argument as in the proof of Proposition 9 shows that the federal equilibrium is socially worse than decentralization. ■

Proof of Proposition 11. Suppose for concreteness that citizens vote first on L and then on R. From Lemma 2, the second stage equilibria are the most preferred R of the median state, which exists by continuity of $x(L, R)$.

At the first stage, for any $L \geq \theta_\mu$, one can see from Equation (1) that the most preferred L of the median state is $L = R$. Hence, from Lemma 2, the median states and all leftist states prefer $L = \theta_\mu$ to any $L > \theta_\mu$ at the first stage.

Suppose now that $L < \theta_\mu$. I first show that in this case, the most preferred R of the median state at the second stage are such that $L < R \leq \theta_\mu$. For all L, R such that $L < \theta_\mu \leq R$, the median state is unconstrained at the state equilibrium and from Lemma 3, $V_\mu(L,R) = V_\mu^{\mathrm{dec}}((\max(\underline{t}, \min(\bar{t}, \theta_n)))_n)$ so for almost all R,

$$\begin{aligned} \frac{\partial V_\mu(L,R)}{\partial R} &= \sum_{n > N - r(L,R)} \frac{\partial [V_\mu^{\mathrm{dec}}(t)]}{\partial t_n} \frac{\partial \bar{t}}{\partial R} + \sum_{n \leq l(L,R)} \frac{\partial [V_\mu^{\mathrm{dec}}(t)]}{\partial t_n} \frac{\partial \underline{t}}{\partial R} \\ &= \frac{2\beta}{N(1+\beta)} \left[r(L,R)(\theta_\mu - R) \frac{\partial \bar{t}}{\partial R} + l(L,R)(\theta_\mu - L) \frac{\partial \underline{t}}{\partial R} \right]. \quad (34) \end{aligned}$$

From Equation (6), $0 \leq \frac{\partial \bar{x}(L,R)}{\partial R} \leq 1$ so from Equation (9), for almost all R, $\frac{\partial \bar{t}}{\partial R} > 0$ and $\frac{\partial \underline{t}}{\partial R} < 0$. Therefore, if $L < \theta_\mu$, for almost all $R \geq \theta_\mu$, $\partial V_\mu / \partial R \leq 0$. Hence, given $L < \theta_\mu$, the most preferred R of the median state is such that

$R \leq \theta_\mu$. Clearly, the median state prefers $L = R = \theta_\mu$ to $L \leq R \leq \theta_\mu$, and from Lemma 2, so do all rightist states.

In the simultaneous case, from Lemma 3, the median voters are pivotal on L and R so for any $(L, R) \neq (\theta_\mu, \theta_\mu)$, a change of either L or R can make them better off. Proposition 2 in Loeper (2011) implies that the resulting uniform policy is socially worse than decentralization. ■

Proof of Proposition 12. Let (i_1^e, i_2^e) be a federal equilibrium of (I, f), and let $(L, R) \in F$. Because f is onto, there exists $(i_1, i_2) \in I$ such that $f(i_1, i_2) = (L, R)$. If i_2 is a subgame equilibrium following i_1, then because (i_1^e, i_2^e) is a federal equilibrium, $f(i_1^e, i_2^e)$ is majority preferred to $f(i_1, i_2)$, so $f(i_1, i_2)$ does not cover $f(i_1^e, i_2^e)$. Suppose now that i_2 is not a subgame equilibrium following i_1. Let i_2' be a subgame equilibrium following i_1. Then $f(i_1, i_2')$ is majority preferred to $f(i_1, i_2)$, and because (i_1^e, i_2^e) is a federal equilibrium, $f(i_1^e, i_2^e)$ is majority preferred to $f(i_1, i_2')$. Therefore, the implication in Definition 4 does not hold, and $f(i_1, i_2)$ does not cover $f(i_1^e, i_2^e)$. ■

Proof of Proposition 13. For the sake of brevity, I use the symbol $\succeq$ for the majority preference relation throughout this proof. I first deal with case (i). Fix β and let $(\theta^k)_{k\in\mathbb{N}}$ be such that $\frac{|\theta_\mu^k - \theta_1^k|}{|\theta_\mu^k - \theta_N^k|} \to \infty$. W.l.o.g., I renormalize the policy and the type space so that θ_μ^k and θ_1^k are constant and $|\theta_\mu^k - \theta_N^k| \to 0$.

Step 1: if $(L^k, R^k)_{k\in\mathbb{N}}$ and $(L'^k, R'^k)_{k\in\mathbb{N}}$ are such that $L^k \to \theta_\mu$ and for all $k \in \mathbb{N}$, $(L'^k, R'^k) \succeq^{\theta^k} (L^k, R^k)$, then $L'^k \to \theta_\mu$. Because $L^k \to \theta_\mu$ and $|\theta_\mu^k - \theta_N^k| \to 0$, $x^{\theta^k}(L^k, R^k) \to (\theta_\mu, \ldots, \theta_\mu)$ so for all $n \geq \mu$, $V_n^{\theta^k}(L^k, R^k) \to 0$. Because $(L'^k, R'^k) \succeq^{\theta^k} (L^k, R^k)$, for some $p \geq \mu$, $V_p^{\theta^k}(L'^k, R'^k) \geq V_p^{\theta^k}(L^k, R^k)$ so necessarily, $V_p^{\theta^k}(L'^k, R'^k) \to 0$. Because θ_1^k is bounded away from θ_μ^k, this implies that $L'^k \to \theta_\mu$.

Step 2: if $(L'^k, R'^k)_{k\in\mathbb{N}}$ and $(L''^k, R''^k)_{k\in\mathbb{N}}$ are such that for all $k \in \mathbb{N}$, $(L''^k, R''^k) \succeq^{\theta^k} (L'^k, R'^k)$ and $(L'^k, R'^k) \succeq^{\theta^k} (\theta_\mu, \theta_\mu)$, then $L''^k \to \theta_\mu$. From Step 1, $L'^k \to \theta_\mu$. I apply Step 1 again to (L''^k, R''^k) and (L'^k, R'^k) to get $L''^k \to \theta_\mu$.

Step 3: if $(L''^k, R''^k)_{k\in\mathbb{N}}$ is such that for all $k \in \mathbb{N}$, (L''^k, R''^k) is uncovered for the preference profile θ^k, then for all $n \geq \mu$, $x^{\theta^k}(L''^k, R''^k) \to (\theta_\mu, \ldots, \theta_\mu)$ and $V_n^{\theta^k}(L''^k, R''^k) \to 0$. By assumption, (L''^k, R''^k) is not covered by $(L, R) = (\theta_\mu, \theta_\mu)$. This means that either $(L''^k, R''^k) \succeq^{\theta^k} (\theta_\mu, \theta_\mu)$, or $(L''^k, R''^k) \succeq^{\theta^k} (L'^k, R'^k)$ and $(L'^k, R'^k) \succeq^{\theta^k} (\theta_\mu, \theta_\mu)$ for some $(L'^k, R'^k) \in F$.

In the first case, Step 1 implies that $L''^k \to \theta_\mu$. In the second case, Step 2 also implies that $L''^k \to \theta_\mu$, which proves Step 3.

To conclude the proof of case (i), suppose that for all $k \in \mathbb{N}$, there exists (L''^k, R''^k) which is uncovered for the preference profile θ^k and such that $x^{\theta^k,\text{dec}} \succeq x^{\theta^k}(L''^k, R''^k)$. From Equation (27), for all $n \geq \mu$, $V_n^{\theta^k,\text{dec}} \leq -\frac{\beta}{N}\left(\frac{1}{1+\beta}(\theta_n^k - \theta_1^k)\right)^2$, which is bounded away from 0. This contradicts Step 3. Therefore, there exists K such that for all $k \geq K$, for the preference profile θ^k, the voters of all states $n \geq \mu$ prefer any uncovered (L, R) to decentralization. Step 3 implies that $x^{\theta^k}(L^k, R^k) \to (\theta_\mu, \ldots, \theta_\mu)$, so the summation of $V_n^{\theta^k}(L^k, R^k)$ over n yields $W^{\theta^k}(L^k, R^k) \leq -N\text{var}(\theta^k) + o(1)$. Together with Equation (27), this shows that $W^{\theta^k,\text{dec}} \geq W^{\theta^k}(L^k, R^k)$ for k sufficiently large.

I now turn to case (ii). Fix θ and consider a sequence $\beta^k \to \infty$.

Step 0: *if* $x^{\beta^k}(L^k, R^k)$ converges to *some* $x \in \mathbb{R}^N$, *then* $x = (l, \ldots, l)$ *for some* $l \in \mathbb{R}$, *and for all* $n \in N$, $V_n^{\beta^k}(L^k, R^k) \to -(l - \theta_n)^2$. Equation (6) implies that

$$x_n^{\beta^k}(L^k, R^k) = \overline{x}^{\beta^k}(L^k, R^k) + \frac{\theta_n - \overline{x}^{\beta^k}(L^k, R^k)}{\beta^k} + o\left(\frac{1}{\beta^k}\right). \qquad (35)$$

If $x^{\beta^k}(L^k, R^k) \to x$, then $\overline{x}^{\beta^k}(L^k, R^k) \to \bar{x}$, so Equation (35) implies $x^{\beta^k}(L^k, R^k) \to (\bar{x}, \ldots, \bar{x})$. The substitution of Equation (35) in Equation (1) yields $V_n^{\beta^k}(L^k, R^k) \to -(\overline{x} - \theta_n)^2$.

Step 1: if $(L^k, R^k)_{k\in\mathbb{N}}$ *and* $(L'^k, R'^k)_{k\in\mathbb{N}}$ *are such that* $x(L^k, R^k) \to (\theta_\mu, \ldots, \theta_\mu)$ *and for all* $k \in \mathbb{N}$, $(L'^k, R'^k) \succeq^{\beta^k} (L^k, R^k)$, *then* $x(L'^k, R'^k) \to (\theta_\mu, \ldots, \theta_\mu)$. Let x be a limit of a subsequence of $x^{\beta^k}(L'^k, R'^k)$. From Step 0, $x = (l, \ldots, l)$ *for some* $l \in \mathbb{R}$, and in the limit, voters' preferences over (L'^k, R'^k) and (L^k, R^k) coincide with their preferences over $(l, \ldots, l)$ and $(\theta_\mu, \ldots, \theta_\mu)$ when $l \neq \theta_\mu$. The median voter theorem and the hypothesis $(L'^k, R'^k) \succeq^{\beta^k} (L^k, R^k)$ imply then that $l = \theta_\mu$. From the Bolzano Weierstrass theorem, $x^{\beta^k}(L'^k, R'^k)$ must converge to $(\theta_\mu, \ldots, \theta_\mu)$.

Step 2: *if* $(L'^k, R'^k)_{k\in\mathbb{N}}$ *and* $(L''^k, R''^k)_{k\in\mathbb{N}}$ *are such that for all* $k \in \mathbb{N}$, $(L''^k, R''^k) \succeq^{\beta^k} (L'^k, R'^k)$ *and* $(L'^k, R'^k) \succeq^{\beta^k} (\theta_\mu, \theta_\mu)$, *then* $x(L''^k, R''^k) \to (\theta_\mu, \ldots, \theta_\mu)$. From Step 1, $(L'^k, R'^k) \to (\theta_\mu, \theta_\mu)$. I apply Step 1 again to (L''^k, R''^k) and (L'^k, R'^k) to get $x(L''^k, R''^k) \to (\theta_\mu, \ldots, \theta_\mu)$.

Step 3: *if* $(L''^k, R''^k)_{k\in\mathbb{N}}$ *is such that for all* $k \in \mathbb{N}$, (L''^k, R''^k) *is uncovered for* β^k, *then* $x(L''^k, R''^k) \to (\theta_\mu, \ldots, \theta_\mu)$ *and for all* $n \in N$,

$V_n^{\beta^k}(L''^k, R''^k) \to -(\theta_\mu - \theta_n)^2$. The proof of $x(L''^k, R''^k) \to (\theta_\mu, \ldots, \theta_\mu)$ uses Step 1 and Step 2 exactly as in the proof of case (i). Step 0 implies then that $V_n^{\beta^k}(L''^k, R''^k) \to -(\theta_\mu - \theta_n)^2$.

To conclude the proof of case (ii), observe that Step 3 is the analog to Equation (32) in the proof of Proposition 9, so the same argument applies. The identity of the supporters of the federal intervention depend on whether $\bar{\theta} \lesseqgtr \bar{\theta}$ but not on $(L''^k, R''^k)_{k\in\mathbb{N}}$. ■

Proof of Proposition 14. A chain C in F_G is a subset of F_G such that majority preferences are transitive on C. A chain is said to be externally stable if for all $(L, R) \notin F_G$, there exists $(L', R') \in F_G$ that is strictly majority preferred to (L, R). By definition (Banks 1985), the Banks Set is the set of majority preferred elements of all externally stable chains.

Consider the following subsets of F_G:

$$C_L = \{(L, \theta_\mu)\colon L \in G \text{ and } L \leq \theta_\mu\},$$
$$C_R = \{(\theta_\mu, R)\colon R \in G \text{ and } R \geq \theta_\mu\}.$$

For any two elements (L, R) and (L', R') of $C_L \cup C_R$, either $L \leq L'$ and $R \leq R'$ or the reverse inequalities hold. Therefore, Lemma 2 implies that on $C_L \cup C_R$, majority preferences coincide with the preferences of the voters of the median state, which are transitive. So $C_L \cup C_R$ is a chain and its majority preferred element is the most preferred alternative of the median voters, which is (θ_μ, θ_μ).

To show that $C_L \cup C_R$ is externally stable, let $(L, R) \in F_G \setminus (C_L \cup C_R)$. If (θ_μ, θ_μ) is strictly majority preferred to (L, R), then I am done. Suppose instead that (L, R) is majority preferred to (θ_μ, θ_μ). Necessarily, $L \leq \theta_\mu \leq R$. To complete the proof, it suffices to show that (θ_μ, R) or (L, θ_μ) is strictly majority preferred to (L, R). To fix ideas, suppose that $\overline{x}(L, R) \geq \theta_\mu$ (and thus that $\theta_\mu < R$), the proof in the case $\overline{x}(L, R) \leq \theta_\mu$ is symmetric. Because $\theta_N > \theta_\mu$, Proposition 1 implies that $x_N(L, R) > \theta_\mu$, so $r(L, \theta_\mu) > 1$. As shown in the proof of Proposition 11, (L, θ_μ) is preferred to (L, R) by the voters of state μ, and this preference is strict because $r(L, \theta_\mu) > 1$.[41] From Lemma 2, this implies that (L, θ_μ) is also strictly preferred to (L, R) by the voters of all sates $n < \mu$. ■

[41] To see why the preference must be strict, observe that for all R' close enough to θ_μ, $l(\theta_\mu, R') > 1$. Using the notations in the proof of Proposition 11, for almost all R', $\frac{\partial \bar{t}}{\partial R}(L, R') > 0$ and $\frac{\partial \underline{t}}{\partial R}(L, R') < 0$, and since $L < \theta_\mu < R'$, Equation (34) implies that $\frac{\partial V_\mu}{\partial R}(L, R') < 0$.

References

Alesina, A., I. Angeloni, and L. Schuknecht. 2005. "What Does the European Union Do?" *Public Choice* 123(3): 275–319.

Alonso, R., W. Dessein, and N. Matouschek. 2007. "When Does Coordination Require Centralization?" *American Economic Review* 98(March): 145–179.

Angeletos, G.-M. and A. Pavan. 2007. "Efficient Use of Information and Social Value of Information." *Econometrica* 75(4): 1103–1142.

Baker, L. A. 2001. "The Spending Power and the Federalist Revival." *Chapman Law Review* 4(1): 195–230.

Baniak, A. and P. Grajzl. 2011. "Interjurisdictional Linkages and the Scope for Institutional Harmonization." *Review of Law and Economics* forthcoming.

Banks, J. S. 1985. "Sophisticated Voting Outcomes and the Covering Relation." *Social Choice and Welfare* 1: 295–306.

Banks, J. S., J. Duggan, and M. Le Breton. 2002. "Bounds for Mixed Strategy Equilibria and the Spatial Model of Elections." *Journal of Economic Theory* 103(1): 88–105.

Baum, L. 1977. "Judicial Impact as a Form of Policy Implementation." In *Public Law and Public Policy*, John A. Gardiner, (ed.), New York: Praeger.

Besley, T. and S. Coate. 2003. "Centralized versus Decentralized Provision of Local Public Good: a Political Economy Approach." *Journal of Public Economics* 87(12): 2611–2637.

Bolton, P. and G. Roland. 1997. "The Breakup of Nations: A Political Economy Analysis." *The Quarterly Journal of Economics* 112(4): 1057–1090.

Brueckner, J. K. 2009. "Partial Fiscal Decentralization." *Regional Science and Urban Economics* 39(1): 23–32.

Buiter, W. H. 2003. "How to Reform the Stability and Growth Pact." *Central Banking* 13(3): 49–58.

Carbonara, E. and F. Parisi. 2007. "The Paradox of Legal Harmonization." *Public Choice* 132(3, 4): 367–400.

Crémer, J. and T. R. Palfrey. 1996. "In or Out? Centralization by Majority Vote." *European Economic Review* 40: 43–60.

Crémer, J. and T. R. Palfrey. 1999. "Political Confederation." *American Political Science Review* 93(1): 69–83.

Crémer, J. and T. R. Palfrey. 2000. "Federal Mandates by Popular Demand." *Journal of Political Economy* 108(5): 905–927.

Crémer, J. and T. R. Palfrey. 2006. "An Equilibrium Voting Model of Federal Standards with Externalities." *Journal of Public Economics* 90(10): 2091–2106.

Denzau, A. T. and R. J. Mackay. 1981. "Structure-Induced Equilibria and Perfect-Foresight Expectations." *American Journal of Political Science* 25(4): 762–779.

Dewan, T. and D. P. Myatt. 2008. "The Qualities of Leadership: Direction, Communication, and Obfuscation." *American Political Science Review* 102(3): 351–368.

Dutta, B., M. O. Jackson, and M. Le Breton. 2004. "Equilibrium Agenda Formation", *Social Choice and Welfare* 23(1): 21–57.

Dutta, B., M. O. Jackson, and M. Le Breton. 2005. "The Banks Set and the Uncovered Set in Budget Allocation Problems." In *Social Choice and Strategic Decisions, Essays in Honor of Jeff Banks*, David Austen Smith and John Duggan, (eds.), Series *Studies in Choice and Welfare*, Springer Verlag, pp. 163–199.

Ebel, R. D. and S. Yilmaz. 2002. "On the Measurement and Impact of Fiscal Decentralization." *World Bank Policy Research Working Paper* 2809. http://ssrn.com/abstract=636092.

Ellingsen, T. 1998. "Externalities vs. Internalities: a Model of Political Integration." *Journal of Public Economics* 68(2): 251–268.

Epple, D. N. and T. J. Nechyba. 2004. "Fiscal Decentralization." In *Handbook of Regional and Urban Economics*, J. Vernon Henderson and Jacques Francois Thisse, (eds.), first edition, vol. 4. Amsterdam: Elsevier.

Epple, D. N. and R. E. Romano. 2003. "Collective Choice and Voluntary Provision of Public Goods." *International Economic Review* 44(2): 545–572.
Feld, L., C. A. Schalteggerb, and J. Schnellenbach. 2008. "On Government Centralization and Fiscal Referendums." *European Economic Review* 52(4): 611–645.
Fishburn, P. C. 1977. "Condorcet Social Choice Functions." *SIAM Journal of Applied Mathematics* 33: 469–489.
Frey, B. S. and R. Eichenberger. 1996. "To Harmonize or to Compete? That's not the Question." *Journal of Public Economics* 60(3): 335–349.
Ferejohn, J. A. and K. Krehbiel. 1987. "The Budget Process and the Size of the Budget." *American Journal of Political Science* 31(2): 296–320.
Garoupa, N. and A. Ogus. 2006. "A Strategic Interpretation of Legal Transplants." *The Journal of Legal Studies* 35(2): 339–363.
Hafer, D. and C. Landa. 2007. "Public Goods in Federal Systems." *Quarterly Journal of Political Science* 2(3): 253–275.
Hatfield, J. W. and G. Padro i Miquel. 2008. "A Political Economy Theory of Partial Decentralization." *Journal of the European Economic Association* 10(3): 605–633.
Hooghe, L. and G. Marks. 2003. "Unraveling the Central State, but How? Types of Multi-Level Governance." *The American Political Science Review* 97(2): 233–243.
Howard, J. W. 1981. *Courts of Appeals in the Federal System*. Princeton, New Jersey: Princeton University Press.
Inman, R. P. and D. L. Rubinfeld. 1997. "The Political Economy of Federalism." In *Perspectives on Public Choice: A Handbook*, Dennis C. Mueller, (ed.), New York, Cambridge: Cambridge University Press.
Inman, R. P. and D. L. Rubinfeld. 1998. "Subsidiarity and the European Union." *NBER Working Paper* 6556. http://www.nber.org/papers/w6556.pdf (May 1998).
Janeba, E. 2006. "Moral Federalism." *Contributions to Economic Analysis and Policy* 5(1), article 32: 1558–1558.
Joanis, M. 2009. "Intertwined Federalism: Accountability Problems under Partial Decentralization." *GREDI working paper* 08–22. http://pages.usherbrooke.ca/gredi/wpapers/GREDI-0822.pdf.
Joumard, I. and P. Mathis Kongsrud. 2003. "Fiscal Relations across Government Levels." *OECD Economics Department Working Papers*, 375. http://www.oecd.org/dataoecd/21/34/33638994.pdf.
Kanbur, R. and M. Keen. 1993. "Jeux Sans Frontieres: Tax Competition and Tax Coordination When Countries Differ in Size." *American Economic Review* 83(4): 877–892.
Kox, H., A. Lejour, and R. Montizaan. 2005. "The Free Movement of Services Within the EU." *CPB Netherlands Bureau for Economic Policy Analysis*, document 69. http://www.cpb.nl/en/publication/free-movement-services-within-eu.
Kramer, G. H. 1972. "Sophisticated Voting Over Multi-dimensional Choice Spaces." *Journal of Mathematical Sociology* 2(2): 165–181.
Kramer, G. H. and A. K. Klevorick. 1973. "Existence of a 'Local' Cooperative Equilibrium in a Class of Voting Games." *The Review of Economic Studies* 41(4): 539–547.
Lockwood, B. 2002. "Distributive Politics and the Cost of Centralization." *The Review of Economic Studies* 69(2): 313–337.
Loeper, A. 2011. "Coordination in Heterogeneous Federal Systems." *Journal of Public Economics* 95(7, 8): 900–912.
Lulfesman, C. 2008. "Collective and Private Provision of Public Goods in Democracy." Unpublished manuscript.
Marciano, A. and J.-M. Josselin, (eds.). 2002. *The Economics of Harmonizing European Law*. Northampton, Massachusets: Edward Elgar Publishing.
McKelvey, R. D. 1986. "Covering, Dominance, and Institution-Free Properties of Social Choice." *American Journal of Political Science* 30(2): 283–314.
Miller, N. R. 1980. "A New Solution Set for Tournaments and Majority Voting: Further Graph-Theoretical Approaches to the Theory of Voting." *American Journal of Political Science* 24(1): 68–96.

Moe, T. M. 1982. "Regulatory Performance and Presidential Administration." *American Journal of Political Science* 26(2): 197–224.

Monheim-Helstroffer, J. and M. Obidzinski. 2010. "Optimal Discretion in Asylum Lawmaking." *International Review of Law and Economics* 30(1): 86–97.

Morris, S. and H. S. Shin. 2002. "Social Value of Public Information." *American Economic Review* 92(5): 1521–1534.

Oates, W. E. 1972. *Fiscal Federalism*. New York: Harcourt Brace Jovanovich.

Oates, W. E. 1999. "An Essay on Fiscal Federalism." *Journal of Economic Literature* 37(3): 1120–1149.

Oates, W. E. 2005. "Toward a Second-Generation Theory of Fiscal Federalism." *International Tax and Public Finance* 12(August): 349–373.

Penn, E. M. 2006. "Alternate Definitions of the Uncovered Set and Their Implications." *Social Choice and Welfare* 27(1): 83–87.

Peralta, S. and T. van Ypersele. 2006. "Coordination of Capital Taxation Among Asymmetric Countries." *Regional Science and Urban Economics* 36(6): 708–726.

Rantakari, H. 2008. "Governing Adaptation." *The Review of Economic Studies* 75(4): 1257–1285.

Redoano, M. and K. A. Scharf. 2004. "The Political Economy of Policy Centralization: Direct versus Representative Democracy." *Journal of Public Economics* 88(3, 4): 799–817.

Revelli, F. 2010. "Tax Mix Corners and Other Kinks." Unpublished manuscript.

Riker, W. H. 1964. *Federalism: Origin, Operation, Significance*. Boston: Little, Brown and Company.

Rodden, J. A. 2004. "Comparative Federalism and Decentralization—On Meaning and Measurement." *Comparative Politics* 36(4): 481–500.

Schwager, R. 1999. "Administrative Federalism and a Central Government with Regionally Based Preferences." *International Tax and Public Finance* 6(2): 165–189.

Seabright, P. 1996. "Accountability and Decentralization in Government: An Incomplete Contract Model." *European Economic Review* 40(1): 61–89.

Shepsle, K. A. 1979. "Institutional Arrangements and Equilibrium in Multidimensional Voting Models." *American Journal of Political Science* 23(1): 27–59.

Shepsle, K. A. and B. R. Weingast. 1984. "Uncovered Sets and Sophisticated Voting Outcomes with Implications for Agenda Institutions." *American Journal of Political Science* 28(1): 49–74.

Trachtman, J. P. 1992. "L'Etat, C'est Nous: Sovereignty, Economic Integration and Subsidiarity." *Harvard International Law Journal* 33(2): 459–473.

Treisman, D. 2002. "Defining and Measuring Decentralization: A Global Perspective." Unpublished manuscript.

Villas-Boas, J. M. 1997. "Comparative Statics of Fixed Points." *Journal of Economic Theory* 73(1): 183–198.

Weingast, B. R. 1995. "The Economic Role of Political Institutions: Market-Preserving Federalism and Economic Development." *Journal of Law, Economics, and Organization* 11(1): 1–31.

Weingast, B. R. and M. J. Moran. 1983. "Bureaucratic Discretion or Congressional Control? Regulatory Policymaking by the Federal Trade Commission." *Journal of Political Economy* 91(5): 765–800.

Weingast, B. R., K. A. Shepsle, and C. Johnsen. 1981. "The Political Economy of Benefits and Costs: A Neoclassical Approach to Distributive Politics." *Journal of Political Economy* 89(4): 642–664.

Wilson, J. D. and E. Janeba. 2005. "Decentralization and International Tax Competition." *Journal of Public Economics* 89(7): 1211–1229.