Dynamic Pivotal Politics[1]

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Abstract

We analyze a dynamic extension of a parsimonious model of lawmaking in which today's policy becomes tomorrow's status quo. We show that unlike in static models, policy makers' equilibrium voting behavior depends on the institutional environment and on policy makers' expectations about future economic and political shocks. Relative to sincere voting, the equilibrium behavior is more polarized and may exhibit an average policy bias. These voting distortions increase with the degree of consensus required by the institution, the volatility of the policy environment, and the expected ideological polarization of the future policy makers. Our analysis implies that the static approach underestimates the inertial effect of checks and balances such as the filibuster, overestimate the impact of institutional biases such as fiscally conservative budget procedures, and overlook the impact of political expectations.

1 Introduction

Theoretical models of lawmaking in the spirit of pivotal politics integrate policy makers' preferences and institutions to understand legislative outcomes (Krehbiel 1998, Brady and Volden 2006, Cox and McCubbins 2005, Chiou and Rothenberg 2009). These models typically assume that policy makers vote sincerely, at least for final passage votes. Sincere voting implies that institutions affect only agenda setting and how votes are aggregated. This independence of institutions and voting behavior allows theorists to analyze the role of institutions taking policy makers' voting behavior as fixed. Likewise, it allows empiricists to recover policy makers' preferences from roll call votes independently of the institutional settings (e.g., Poole and Rosenthal 1985, Heckman and Snyder 1997, Clinton, Jackman and Rivers 2004).

These models of lawmaking, however, are inherently static. They take the current status quo as exogenous and overlook the legislative process that led to it. In reality, the legislative process is inherently dynamic, with policy makers periodically revising policies to respond to changing circumstances. Thus, the current status quo is inherited from previous votes, and is therefore endogenous. As highlighted by the game theoretic literature (e.g., Baron, 1996; Kalandrakis, 2004; Duggan and Kalandrakis, 2012; Riboni and Ruge Murcia, 2008; Penn, 2009; Dziuda and Loeper, 2016; Buisseret and Bernhardt, forthcoming), the endogeneity of the status quo can distort policy makers' behavior in important ways.[1] The goal of this paper is to introduce dynamics into models of pivotal politics, and investigate how the endogeneity of the status quo affects the main conclusions of the static literature.

We analyze an infinite-horizon extension of an otherwise parsimonious model of lawmaking. A set of policy makers repeatedly chooses between two policies, and the policy implemented in a given period can be revised in the next period only if a decisive coalition under the prevailing institution agrees to do so. Policy makers' preferences over these two policies

[1] As has been highlighted by the lawmaking literature, the endogeneity of the status quo is not the only reason why legislators may not vote sincerely. We discuss this literature in Section 2.

are redrawn in each period in a stochastic fashion, but their relative ideological positions remain constant. The shocks to the preferences capture changes in the relative efficiency or popularity of the policies, for instance due to economic crises, changes in demographic trends, international events, or simply the vagaries of public opinion.

The equilibrium analysis reveals that under all institutions with some degree of checks and balances, policy makers do not vote sincerely. Specifically, policy makers' voting behavior is distorted in two systematic ways. First, consistently with the findings in Dziuda and Loeper (2016),[2] the voting behavior exhibits *strategic polarization*: relative to sincere voting, any two policy makers disagree more frequently. As a result, the government gridlocks more frequently. Second, policy makers may also exhibit an *average policy bias*: the voting behavior of the government as a whole can be tilted towards a particular policy on average. This effect affects the relative prevalence of liberal versus conservative policy reforms, relative to sincere voting.

The intuition for these distortions is as follows. For institutions with checks and balances, one can identify two distinct policy makers, whose agreement is needed for a policy change (Krehbiel 1998, Brady and Volden 2006). Following Krehbiel, we call these policy makers *pivots*. Since the status quo stays in place whenever the pivots disagree, policy makers have incentives to affect which policy is the future status quo by distorting their votes today. Rightist and leftist policy makers disagree on which policy should be the status quo. As a result, relative to their sincere preferences, the former vote more frequently for the conservative policy and the latter more frequently for the liberal policy, leading to an increase in gridlock. Moreover, this strategic polarization effect may not affect the two sides of the political spectrum symmetrically. Depending on who the pivots are, and thus depending on the states in which they disagree, the policy makers on average may prefer one policy to be the status quo. As a result, the government as a whole may exhibit a bias in the direction of this policy.

[2]Dziuda and Loeper (2016) analyzed only unanimity, while we allow for a broad class of institutions. We describe in detail how this paper relates to Dziuda and Loeper (2016) in Section 2.

One might argue that our theory is observationally indistinguishable from the static theory of voting, the only difference being that the static models treat policy makers' voting behavior as the sincere expression of their intrinsic preferences, whereas our model interprets it as a more complex equilibrium object. However, this apparent equivalence overlooks an important point. In the dynamic model, voting behavior is no longer independent of the institutions. This implies that when analyzing the impact of institutional changes, one cannot take as given the preferences induced from the current or past votes.

To see how the static and the dynamic models differ in their predictions of the impact of an institutional change, consider an increase in the size of coalitions required to implement a liberal policy change, e.g., an increase in the institutional hurdle to raise taxes. A static model makes two predictions. First, a more rightist policy maker becomes pivotal for a liberal policy change. This means that the pivots are more ideologically polarized, which increases gridlock. Second, since the policy maker pivotal for a conservative policy change remains the same, liberal reforms becomes less frequent relative to conservative ones. Our analysis shows that these two predictions need to be qualified. First, the strategic polarization effect implies that a static model *underestimates the inertial effect of this institutional change*. In our dynamic setting, the considered institutional change increases the likelihood of future gridlock, and thus increases policy makers' incentives to defend their preferred status quo for tomorrow, relative to voting for the policy that they sincerely prefer today. As a result, they vote in a more polarized way, increasing gridlock further. Second, the average policy bias effect implies that a static model *overestimates the policy bias of this institutional change*. In our dynamic setting, increasing the institutional hurdle to pass liberal reforms makes liberal policy makers less willing to adopt conservative reforms when needed, by fear of not being able to revert to more liberal policies once the environment changes. This decreases the incidence of conservative policies compared to the static model. We further show that this strategic effect can be so strong that the institutional bias in favor of conservative policies can actually increase the incidence of liberal policies in the long run.

The last finding is not of purely theoretical interest. In the last decades, several U.S. states have passed constitutional amendments that require a qualified majority to increase taxes. These amendments were designed to reduce spending and tax levels. In a similar spirit, the US Congress has voted every year between 1996 and 1999 on an amendment to require two-thirds supermajority to enact tax increases. Our results imply that such amendments can exacerbate the polarization of these legislatures, and thus severely limit their ability to react to political and economic shocks. More surprisingly, these amendments can even fail to achieve their primary goal of keeping taxes low.

Our model reveals another important determinant of policy makers' behavior which by assumption is absent from static models, namely policy makers' expectations. We first investigate the impact of expectations about the future policy environment. Specifically, we show that policy makers vote in a more polarized manner on issues on which their preferences are expected to be more volatile. That is, there is more gridlock precisely in domains in which the policy should respond quickly to frequent shocks. This finding implies that when designing institutions, checks and balances should be weaker for such policy domains. A prominent example is taxation: tax policies must react to varying factors such as the business cycle, investors' expectations about fiscal sustainability, or swings in voters' opinion. Our findings suggest that procedural rules that limit the power of the filibuster, such as the reconciliation process, can be particularly beneficial in such volatile policy areas.

Second, we investigate the impact of policy makers' expectations about the future political environment. To do so, we extend the model and allow for political turnover. We show that policy makers' voting behavior depends on their expectation about the composition of the next government because the latter determines the likelihood of future gridlock, and thus the incentives for policy makers to distort their vote and defend their preferred status quo. For example, if policy makers expect the next government to be united, the future status quo is unlikely to matter, so they vote more in line with their sincere preferences. Conversely, if they expect the next government to be divided, they bias their voting behavior significantly.

An important insight of our model is that the equilibrium voting behavior depends on the composition of the future, but not on the current government. In particular, a perfectly united government may vote strategically. For example, a united conservative government may decrease taxes even in times of high budget deficit in order to put themselves in a better negotiating position in case the future government is divided.

One of the puzzles of recent years has been an increase in polarization of policy makers in the U.S. Congress as inferred from their voting behavior (see, e.g., McCarty et al. 2008). The goal of this paper is to analyze lawmaking in the simplest dynamic setting; hence, we abstract from many aspects of lawmaking that may speak to this increase in polarization.[3] Nevertheless, our results suggest three possible factors that may have contributed to it. First, concurrently with the raise in measured polarization, the filibuster—once an infrequently used tool reserved for the most salient and controversial bills—has become a routine practice in American politics (see Table 1, and also Koger 2010, Chapter 5 of Binder and Smith 1997, or Wawro and Schickler 2006). Some political scientists and pundits blame the more frequent use of the filibuster on the increase in policy makers' ideological polarization (Krehbiel 1998, McCarty 2007). Our results suggest a possible reverse causality: the change in institutional practice de facto introduced more checks and balances, and hence made more extreme policy makers pivotal. This, according to our analysis, increase the incentives of all legislators to defend their preferred status quo, and thus should result in a greater degree of strategic polarization. Second, since the 1970s, divided governments have become increasingly common in the U.S.(e.g., Fiorina, 1994). Arguably, this should have affected policy makers' expectations about future gridlock. According to our results, such a change in expectations should have pushed policy makers to vote in a more polarized way, even during the times of united government. And finally, even if ideological polarization is indeed on the rise, our results suggest that the studies might have overestimated the magnitude of

[3] Political scientists have proposed several possible explanations for this phenomenon, such as electorate polarization, party primaries, or economic inequality (see, e.g., Barber and McCarty 2010, McCarty, Pool and Rosenthal 2008).

such increase. In the dynamic model, any increase in ideological polarization is magnified by an increase in strategic polarization.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the basic model. Section 4 first characterizes the equilibrium in the benchmark static game (Section 4.1) and then characterizes equilibria in the dynamic game (Section 4.2). Section 5 analyzes the impact of institutions on the equilibrium behavior. Section 6 analyzes how the behavior of policy makers varies with the expectations they hold about the future policy and political environment. Section 7 concludes. All proofs are in the appendix.

2 Related theoretical literature

Our paper can be viewed as a dynamic extension of the static models of lawmaking in the spirit of pivotal politics, such as Krehbiel (1998), Brady and Volden (2006), Cox and McCubbins (2005), or Chiou and Rothenberg (2009). We greatly simplify the legislative bargaining game played in each period in order to isolate the impact of forward looking considerations on policy makers' voting behavior. Our results show that legislators' policy preferences, as revealed by their votes, and institutions cannot be taken as two independent primitives.[4]

The paper closest to this one is Dziuda and Loeper (2016). Dziuda and Loeper (2016) analyze a similar model with two players and the unanimity rule. They show that equilibrium behavior exhibits strategic polarization. The current paper shows that strategic polarization extends to N players in a way that preserves their ideological ordering (Proposition 2), and that the voting distortions can also induce an average bias effect (Corollary 1). More importantly, by considering N players and by modelling explicitly players' information about shocks, the current paper can analyze how institutions and expectations affect equilibrium behavior and outcomes, which we view as its main contribution.

[4]Some papers (e.g., Chiou and Rothenberg 2009) assume that legislators' voting behavior depend on the degree of party discipline, but is still assumed to be independent of the formal institutions.

Strategic voting has been extensively studied in the political science literature in related but different contexts. The literature on binary voting trees has shown that even when considering a single policy period, forward-looking legislators may not vote sincerely over a sequence of amendments (see, e.g., Austen-Smith ,1987, or Anesi and Seidmann, 2014). Since we consider only two policies, we do not need to consider amendments, which allows us to abstract away from this type of voting calculus. Other authors have argued that it may be mutually beneficial and self-enforcing for legislators to exchange favors across time and policy issues (see, e.g., Clinton and Meirowitz, 2004, and the references therein). Our model rules out this type of log-rolling strategies by focusing on Markov equilibria. Instead, the departure from sincere voting comes solely from the dynamic linkage between today's policy and tomorrow's status quo.[5]

The game theoretic implications of the dynamic linkage between today's policy and tomorrow's status quo has been studied by a large and growing literature on dynamic legislative bargaining. This literature finds that the endogenous status quo can generate equilibrium distortions of various kinds (see, e.g., Baron, 1996; Kalandrakis, 2004; Roberts 2007; Penn 2009; Bowen et al. 2014; Baron and Bowen 2015) and can lead to inefficient outcomes (Anesi and Seidmann, 2015; Anesi and Duggan, forthcoming; Buisseret and Bernhardt, forthcoming). These papers differ from ours in that they assume constant preferences, and the only impetus for policy change is the changing identity of the proposer, the ability of the proposer to build new coalitions over time and/or the random arrival of new alternatives. We focus instead on situations in which the impetus for policy change comes solely from shocks to the policy environment, that is, shocks to the policy makers' preferences. In this respect, Riboni and Ruge-Murcia (2008), Zapal (2011) and Duggan and Kalandrakis (2012) are closer to our paper. These papers, however, do not identify the polarizing effect of the endogenous status quo and do not study how voting behavior vary with the voting rules and players' expectations of future shocks.

[5]Clinton (2012) argues that this dynamic linkage should be taken into account when jointly estimating legislators' ideologies, status quos and proposals.

Our paper relates to the literature that compares different voting rules on the basis of their ability to prevent socially detrimental policies and to enact socially desirable ones. Since Madison (1787) and Tocqueville (1835), political scientists have long recognized that simple majority rule can lead to a "tyranny of the majority" (Wicksell 1986, Buchanan and Tullock 1962, Riker 1962). However, supermajority requirements confer an unwarranted privilege to the status quo which can be socially detrimental (May 1952, Rae 1975, Guttman 1998, Tsebellis 2002, McGann 2004). Our paper complements this literature by showing that in a dynamic context, the voting rule affects not only the number of votes that is necessary to change the status quo, but also the incentives of each voter to approve a policy change. These incentives systematically exacerbate the inertial effect of supermajoritarian requirements and decrease the probability of passing good laws.

Several recent papers analyze the effect of voting rules in dynamic settings. Eraslan and Merlo (2002) analyze a divide the dollar game in which the size of the surplus to be allocated varies stochastically over time. They show that majorities lower than unanimity lead to inefficiencies because a current winning coalition may prefer to divide a small pie today by fear of not being in the winning coalition next period. Harstad (2005) considers a multilateral hold-up setting in which the cost of implementing a reform depends on a private, up-front investment. He shows that a greater supermajority requirement decreases the incentives to invest, because each party's is more likely to be in the winning coalition, and thus to be compensated for the cost of reform. Strulovici (2010) considers a dynamic voting model in which policy experimentation generates information which affects players' future preferences. In his model, a player's incentives to experiment depend on the voting rule, because a more conservative voting rule increases the likelihood that a new policy stays in place even if turns out to be detrimental to that voter. Acemoglu et al. (2015) analyzes a model in which today's collective decision affect the future allocation of power, and show that this dynamic linkage generate inefficiencies. In contrast to the aforementioned papers, in our model, the outcome of the vote in a given period does not affect future information, preferences, or bargaining power. It only affects the policy in place, and thus who is pivotal in the next period. In an environment with randomly generated alternatives and a single policy decision, Compte and Jehiel (2008) show that simple and qualified majority rules tend to generate less delay, whereas Dougherty and Edward (2012) show that the majority rule dominates the unanimity rule in terms of generating Pareto efficient outcomes. However, when allowing for strategic proposals, the finding of Dougherty and Edward (2012) reverses. Battaglini and Coate (2007, 2008) analyze a dynamic legislative model with targeted transfers in which unlike in our model, voting rules that require a higher degree of consensus lead to more efficient policies. This opposite finding arises because such rules limit majoritarian incentives to particularize benefits and collectivize costs as in Ferejohn et al. (1987) or Baron (1991). Moreover, legislators share the same preferences over the public good (or debt), so unlike in our model, its current level affect all legislators' continuation value uniformly.

3 The model

Players and policies

A set of policy makers $\mathcal{N} = \{1, ..., n, ..., N\}$ are in a relationship that lasts for infinitely many periods. In each period, they must decide which of the two policies L and R to implement, where L can be interpreted as a liberal policy such as a high tax rate, high public spending level, or high level of protection of domestic industries, and R as its conservative counterpart.

Payoffs

Each policy maker $n \in \mathcal{N}$ maximizes her expected discounted sum of period payoffs with discount factor $\delta \in (0, 1)$. Her payoff gain from implementing policy R instead of L in some period $t \in \mathbb{N}$ is denoted by $\theta_n(t)$. Hence, if $\theta_n(t)$ is positive (negative), policy maker n prefers the conservative policy R (the liberal policy L) in period t. One can view this specification as a stylized dynamic extension of the canonical spatial model where $\theta_n(t)$ is the peak of policy maker n's single-peaked utility function.[6] We refer to $\theta_n(t)$ as *the sincere preferences* of policy maker n in period t, and to $\theta(t) \circeq (\theta_1(t), ..., \theta_N(t))$ as the state of nature in period t.

The stochastic process $\{\theta(t) : t \in \mathbb{N}\}$ that governs the evolution of the state is assumed to be i.i.d. across periods, and distributed according to an integrable probability distribution P that satisfies the following property.

Assumption 1 *For all $t \in \mathbb{N}$, $\theta_1(t) < ... < \theta_N(t)$ with probability one, and for any two policy makers $n, m \in \mathcal{N}$ with $n < m$, $\theta_n(t) < 0 < \theta_m(t)$ with positive probability.*

Assumption 1 states that even though policy makers' preferences over a given issue evolve over time, their ideological rank order on that issue remains constant. Specifically, those with lower indices are more leftist than those with higher indices. Therefore, whenever two policy makers disagree on a given issue, the relatively more leftist on that issue always prefers L and the relatively more rightist on that issue prefers R. This assumption is in line with Poole and Rosenthal (1991), who find that "Congress as a whole may adapt by, for example, moving to protectionism when jobs are lost to foreign competition. But as such new items move onto the agenda, their cutting lines will typically be consistent with the preexisting, stable voting alignments".[7]

The following definition characterizes a simple environment in which Assumption 1 is satisfied. We will use this environment for some of the results.

Definition 1 (Common shock specification) *For all $n \in \mathcal{N}$ and $t \in \mathbb{N}$, $\theta_n(t) = \bar{\theta}_n + \varepsilon(t)$, where $\bar{\theta} \in \mathbb{R}^N$ is such that for all $n > m$, $\bar{\theta}_n > \bar{\theta}_m$, the process $\{\varepsilon(t) : t \in \mathbb{N}\}$ is i.i.d., the c.d.f. F of $\varepsilon(t)$ has mean* 0.

[6] To see this, note that it is equivalent to assuming that the payoff for n from implementing $x \in \{L, R\}$ in some period t is $-(x - \theta_n(t))^2$, with $R = -L = 1/4$.

[7] Assumption 1 is sufficient but not necessary for our results, though it simplifies the analysis considerably. Under a suitable equilibrium refinement, some degree of preference reversal would not alter our results. All we would need is that conditional on the preferences of any two policy makers disagreeing, the more leftist always prefers L in expectation and the more rightist always prefers R in expectation.

In the common shock specification, $\bar{\theta}_n$ captures the ideological position of n relative to the other policy makers, whereas $\varepsilon(t)$ captures the shock to the policy environment in period t. Thus $(\bar{\theta}_n)_{n\in\mathcal{N}}$ allows policy preferences to vary across policy makers whereas $\{\varepsilon(t) : t \in \mathbb{N}\}$ allows policy preferences to vary over time.

The legislative game

In each period t, a status quo $q(t) \in \{L, R\}$ is in place. First, $\theta(t)$ is realized and observed by all policy makers.[8] Then they choose whether to vote for the status quo or the alternative policy. If the set of policy makers who vote for the alternative policy is a winning coalition, this policy is implemented. Otherwise, the status quo stays in place. The implemented policy determines the payoffs for period t and becomes the status quo for the next period $q(t+1)$.

The institutions

In our stylized environment, the institutional arrangement is summarized by the set of winning coalitions needed for a policy change. We allow this set to depend on the identity of current status quo: for all $q \in \{L, R\}$, Ω_q denotes the set of winning coalitions when the status quo is q, Ω denotes (Ω_L, Ω_R), and $\Gamma(\Omega)$ denotes the game thus defined. We impose the following conditions on Ω.

Assumption 2 *For all $q \in \{L, R\}$,*

(i) *Monotonicity: if $C \in \Omega_q$ and $C \subset C'$, then $C' \in \Omega_q$,*

(ii) *Nonemptyness: $\mathcal{N} \in \Omega_q$,*

(iii) *Properness: if $C \in \Omega_q$ and $q' \neq q$, then $N \setminus C \notin \Omega_{q'}$.*

Conditions (i) to (iii) are standard in the literature on voting (see, e.g., Austen-Smith and Banks 1999). Monotonicity ensures that an additional vote in favor of an alternative can only change the outcome in favor of that alternative. Nonemptyness means that the

[8] This assumption is without loss of generality: we will see later that in equilibrium, each policy maker n conditions her action in period t only on $\theta_n(t)$; hence, whether she observes the preferences of the other policy makers is inconsequential.

institution is Paretian. Properness requires that if a coalition can change the policy in place, those outside this coalition cannot immediately reverse this change.

Assumption 2 allows for a large class of institutions. For instance, if we set $\Omega_R = \Omega_L = \Omega^M$ where

$$\Omega^M \doteq \{C \subseteq \mathcal{N} : |C| \geq M\}, \tag{1}$$

then Ω corresponds to a parliamentary system with a single chamber operating under a supermajority rule with threshold M. Assumption 2 is satisfied for all $M \geq (N+1)/2$. The two polar cases $M = (N+1)/2$ and $M = N$ correspond to simple majority rule and unanimity rule, respectively.

Assumption 2 allows also for nonanonymous institutions, such as the combination of simple majority rule and a veto player (or gate keeper) $v \in \mathcal{N}$:

$$\Omega_R = \Omega_L = \left\{C \subseteq \mathcal{N} : C \in \Omega^{(N+1)/2} \text{ and } v \in C\right\}.$$

Nonanonymous institutions are the de-facto rules in many democracies in which the legislative body uses simple majority but is subject to the negative agenda control of the majority leader, or the veto power of the president.[9]

Equilibrium

As standard for dynamic voting games, we look for Markov perfect equilibria in stage-undominated strategies (henceforth, equilibria) as defined in Baron and Kalai (1993). A Markov strategy for player n in $\Gamma(\Omega)$ maps the state θ and the status quo q into a probability distribution over votes. Stage undomination requires that in each period, each player votes for the policy that gives her the greater continuation payoff. This equilibrium refinement rules out pathological equilibria such as all players always voting for the status quo. We assume that when indifferent, a player votes for R, but this assumption is made only for expositional convenience and does not affect our results qualitatively.

[9] In the case of a presidential veto, the veto player v is not a member of the parliament, so the institution is actually characterized by $\Omega_R = \Omega_L = \left\{C \subseteq \mathcal{N} : C \backslash \{v\} \in \Omega^{(N+1)/2} \text{ and } v \in C\right\}$.

Comments on Assumptions

We abstract from many aspects of lawmaking to isolate the sole impact of introducing dynamics. In particular, our policy makers are purely policy motivated and we abstract from elections. We introduce elections in a reduced form in Section 6.2.

The policy space consists of two alternatives, which is clearly an abstraction, albeit a useful one. Since there is only one possible alternative to the status quo, this assumption implies that in each period, the bill under consideration is exogenous and cannot be amended. As a result, we do not need to specify the details of the procedures that determine how bills and amendments are chosen, and can completely characterize the political decision process by the collections of winning coalitions Ω. This allows us to focus on the difference between the static and dynamic models without keeping track of the differences among static models.[10] Note also that our results do not rely on the absence of "compromise" alternatives that lie between L and R. In a setting with two players and the unanimity rule, Dziuda and Loeper (2016) show that under reasonable assumptions on the bargaining procedure, similar voting distortions arise when players can chose from a one-dimensional space of alternatives, and there is no reason to suspect that the same would not hold in the setting of this paper.[11] Allowing for more than two alternatives, however, would come at a significant cost in terms of technical and expositional complexity, and would not allow for the richness of the results obtained in this paper.

We consider policies that are continuing in nature: policies that do not have an expiration date and remain in effect until a new agreement is reached. Many policies have this feature. In the U.S., this is the case of most tax policies (Gale and Orszag 2003) and most spending policies: by law, all mandatory spending is continuing and this spending category, which contains all entitlements, represents more than 2/3 of total federal budget (Levit and Austin

[10] For the same reason, most of the literature comparing voting rules considers binary decisions (e.g., May 1952, Rae 1969, Guttman 1998, Barbera and Jackson 2004, Messner and Polborn 2004).

[11] Specifically, Dziuda and Loeper (2016, Section 4.2) consider a bargaining procedure in which the status quo is replaced by successive incremental policy changes. This bargaining procedure has also been studied in related environment in Gradstein (1999), Riboni and Ruge Murcia (2010), and Acemoglu et al (2014).

2014). Similarly, labor laws and minimum wage legislation tends to be continuing in nature (Clinton 2012). Sunset provisions, i.e., clauses that attach an expiration date to a legislation, historically have been the exception rather than the norm (see Dziuda and Loeper, 2016, or Austen-Smith et al, 2016, and the references therein for possible explanations on why sunset provisions are relatively rare or irrelevant for the continuance of the policies beyond the expiration date).

4 Equilibrium

4.1 The static model

Since most of the lawmaking literature in the spirit of pivotal politics is static or assumes that policy makers consider each legislative period in isolation, let us first consider as a benchmark the game $\Gamma^0(\Omega)$ in which policy makers play only the first period $t = 0$ of $\Gamma(\Omega)$. In the unique equilibrium of this static game, each policy maker n votes sincerely. That is, she votes for R if $\theta_n(0) \geq 0$ and for L if $\theta_n(0) < 0$, irrespective of the status quo. Under such behavior, Assumption 1 implies that if a policy maker n votes for R, so do all more rightist policy makers $k \geq n$, and if she votes for L, so do more leftist policy makers $k \leq n$. Therefore, if n is such that

$$\{n, ..., N\} \in \Omega_L \text{ and } \{n+1, ..., N\} \notin \Omega_L, \tag{2}$$

then status quo L is replaced by R if $\theta_n(0) \geq 0$ and it stays in place if $\theta_n(0) < 0$. Thus, when the status quo is L, the equilibrium outcome always coincides with n's vote. Conditions (i) and (ii) in Assumption 2 guarantee that (2) characterizes a unique n, which we label $l(\Omega_L)$ and refer to as the *left pivot L*. Figure 1 illustrates how $l(\Omega_L)$ is determined. The *right pivot*,

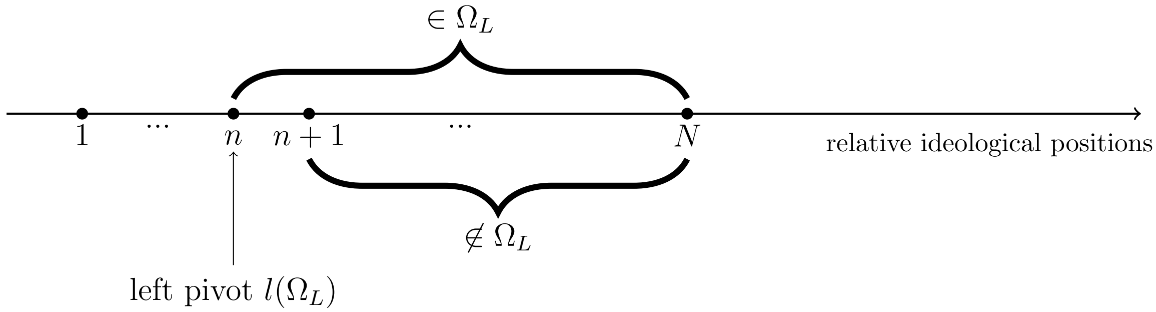

Figure 1: *Determination of the left pivot. Legislators are ordered on a line. Legislator n together with all of the more rightist legislators form a winning coalition under Ω_L, but not legislators that are strictly to the right of her.*

which we denote by $r(\Omega_R)$, is defined in a symmetric fashion by

$$\{1, ..., n\} \in \Omega_R \text{ and } \{1, ..., n-1\} \notin \Omega_R. \tag{3}$$

The following proposition summarizes the above findings.

Proposition 1 *The unique equilibrium of* $\Gamma^0(\Omega)$ *is sincere voting: each policy maker* $n \in \mathcal{N}$ *votes for* R *if and only if* $\theta_n(0) \geq 0$. *If the status quo is* L *(*R*), policy maker* $l(\Omega_L)$ *(*$r(\Omega_R)$*) is* pivotal *in the sense that the equilibrium outcome always coincides with her vote.*

In the classic static models of lawmaking, the ideological positions of the pivots determine a set of policies that are never altered, called *gridlock* (Krehbiel 1998). In our model, each policy may be replaced if a current shock makes it sufficiently unappealing. Hence, we adapt the canonical definition of gridlock is as follows.

Definition 2 *For a given institution* Ω *and strategy profile* σ, *we say that gridlock occurs in state* $\theta \in \mathbb{R}^N$ *if the actions prescribed by* σ *are such that whenever* $\theta(t) = \theta$, *then* Ω *prescribes that* $q(t)$ *stays in place.*

From Proposition 1, gridlock occurs in state θ when the pivots' sincere preferences disagree in that state, in which case $l(\Omega)$ prefers L and $r(\Omega)$ prefers R. So in the equilibrium of the static game $\Gamma^0(\Omega)$, the set of gridlock state is

$$G^0 = \{\theta : \theta_{l(\Omega_L)} < 0 \leq \theta_{r(\Omega_R)}\}. \tag{4}$$

The pivots play a crucial role in the rest of the analysis, so it is instructive to discuss who they are as a function of the institution. Assumption 1 together with conditions (i) and (iii) in Assumption 2 imply that $l(\Omega_L) \leq r(\Omega_R)$, which means that the left pivot is more leftist than the right pivot. When $l(\Omega_L) = r(\Omega_R)$, we shall say that Ω is *quasi-dictatorial:* the equilibrium outcome of $\Gamma^0(\Omega)$ is the same as if the unique pivot were the dictator. Quasi-dictatorial institutions clearly include dictatorship, but also nondictatorial institutions such as simple majority rule: when N is odd, then $l(\Omega_L) = r(\Omega_R) = (N+1)/2$. When $l(\Omega_L) < r(\Omega_R)$, we shall say that Ω is *non quasi-dictatorial.* Examples of non quasi-dictatorial institutions include supermajority requirements, bicameral governments, or unicameral governments with a presidential veto.

4.2 The dynamic model

Let us now turn to the dynamic game $\Gamma(\Omega)$. Proposition 2 extends the main findings of Dziuda and Loeper (2016) to this richer environment.

Proposition 2 *For any equilibrium of $\Gamma(\Omega)$, there exists $d = (d_1, ..., d_N) \in \mathbb{R}^N$ such that each policy maker $n \in \mathcal{N}$ votes for R if and only if $\theta_n + d_n \geq 0$. If Ω is quasi-dictatorial, then the unique equilibrium is sincere voting, i.e., $d = (0, ..., 0)$. If Ω is non quasi-dictatorial, in any equilibrium,*

(i) $d_1 < .. < d_N$,

(ii) $d_{l(\Omega_L)} < 0 < d_{r(\Omega_R)}$.

As in the static game $\Gamma^0(\Omega)$, in every period t in which the status quo is L (R), policy maker $l(\Omega_L)$ $(r(\Omega_R))$ is pivotal in the sense that for all realizations of $\theta(t)$, the equilibrium *outcome coincides with her vote.*

Proposition 2 has three noteworthy implications. First, policy makers vote sincerely in $\Gamma(\Omega)$ only when Ω is quasi-dictatorial. When Ω is non quasi-dictatorial, they vote as if the preferences of each policy maker n were $\theta_n(t) + d_n$ instead of $\theta_n(t)$. We call the term $\theta_n(t) + d_n$ her *strategic preferences* in period t, and d_n her *voting distortion*.

Second, when Ω is non quasi-dictatorial, the voting distortions d have a polarizing effect. To see this, note that for any non quasi-dictatorial Ω, for all $n > m$,

$$(\theta_n(t) + d_n) - (\theta_m(t) + d_m) = (\theta_n(t) - \theta_m(t)) + (d_n - d_m) > \theta_n(t) - \theta_m(t) > 0, \qquad (5)$$

with probability 1, where the first inequality follows from Proposition 2 part (*i*) and the last inequality follows from Assumption 1. The distance between the strategic preferences of policy makers n and m on the left-hand side of (5) can be interpreted as their degree of *strategic polarization*. Hence, (5) states that under any non quasi-dictatorial rule, the degree of strategic polarization between any two policy makers n and m is greater than the degree of their ideological polarization $\theta_n(t) - \theta_m(t)$.

Finally, Proposition 2 has important implications for policy dynamics. In any equilibrium of $\Gamma(\Omega)$, the pivots have the same decisive role as in $\Gamma^0(\Omega)$, so gridlock occurs in state θ when the pivots' strategic preferences disagree in that state. In such states, $l(\Omega)$ votes for L and $r(\Omega)$ votes for R. Therefore, in any equilibrium d of $\Gamma(\Omega)$, the set of gridlock state is

$$G = \{\theta : \theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \leq \theta_{r(\Omega)} + d_{r(\Omega)}\}. \qquad (6)$$

When comparing (4) to (6), Proposition 2 part (*ii*) implies that the set of gridlock states is greater— in the inclusion sense—in any equilibrium of $\Gamma(\Omega)$ than in the equilibrium of $\Gamma^0(\Omega)$. Thus, the strategic polarization effect that arises in the dynamic game exacerbates gridlock: policy change occurs less frequently than predicted by the static model.[12]

[12]Part (*iii*) of Assumption 2 excludes decision rules in which a coalition C of policy makers can change

To understand how the dynamic calculus of policy makers can lead to strategic polarization, consider first the concrete example of legislators voting over entitlements. Suppose that due to a change in demographic trends or growth projections, the sustainability of the public debt deteriorates to the point that the left pivot would sincerely prefer to curtail entitlements. Foreseeing that the implemented policy will become the status quo for future negotiations, she will be reluctant to approve this policy reform, out of fear that the right pivot vetoes a return to more generous benefits once the country's fiscal outlook improves. As a result, she may strategically vote against the cut in entitlements. Similarly, the right pivot may be reluctant to increase entitlements in times of low public debt and favorable demographics, out of fear that the left pivot vetoes a return to fiscal discipline when the situation deteriorates.

The more formal intuition for Proposition 2 is as follows. In $\Gamma(\Omega)$, policy makers' votes determine not only today's policy, as in the static game $\Gamma^0(\Omega)$, but also tomorrow's status quo. The identity of the status quo matters only in gridlock states, in which case the status quo stays in place. Hence, if a policy maker expects gridlock to occur tomorrow, then she has an incentive to distort her vote today in favor of the alternative that she prefers conditional on tomorrow's gridlock. From Assumption 1, the more rightist a policy maker, the greater her relative payoff gain from implementing R instead of L. Therefore, more rightist policy makers have (weakly) more rightist voting distortions, consistently with part (i). The latter property implies that policy makers' strategic preferences are rank-ordered as their sincere preferences. As a result, the same players as in $\Gamma^0(\Omega)$ are pivotal. If Ω is quasi-dictatorial, then $l(\Omega_L) = r(\Omega_R)$. In this case, from (6), there is no gridlock, so the policy makers have no incentive to distort their voting behavior. This explains why $d = (0, ..., 0)$ is the unique equilibrium of $\Gamma(\Omega)$. If Ω is non quasi-dictatorial, then $l(\Omega_L) < r(\Omega_R)$, so in all gridlock

the policy, and another coalition C' of policy makers disjoint from C can reverse this policy change (e.g., a minority rule). Such rules are rarely used in practice, and in our model, they would lead to multiple policy changes within a single period because under such rules, $l(\Omega_L) > r(\Omega_R)$. Nevertheless, it is interesting to note that if we allowed for such rules but restricted policy makers to change policy only only once per period, our results would be overturned. policy makers would behave in a more moderate way and gridlock would decrease. We thank Salvador Barbera for pointing this out to us.

states $l(\Omega_L)$ prefers L and $r(\Omega_R)$ prefers R. This explains part (ii). The strict inequalities from part (i) follow then from the assumption that $l(\Omega_L)$ and $r(\Omega_R)$ disagree with positive probability (see Assumption 1).

Equation (7) below gives the equilibrium condition that characterizes the voting distortions. It formalizes the above intuition that the voting bias of a policy maker reflects her expected preferences conditional on gridlock.

Corollary 1 *For any equilibrium d of $\Gamma(\Omega)$, for all $n \in \mathcal{N}$,*

$$d_n = \frac{\delta \int_{\theta \in G} \theta_n dP(\theta)}{1 - \delta P(G)}, \tag{7}$$

so

$$\frac{1}{N} \sum_{n \in \mathcal{N}} d_n = \frac{\delta \int_{\theta \in G} \left(\frac{1}{N} \sum_{n \in \mathcal{N}} \theta_n \right) dP(\theta)}{1 - \delta P(G)}. \tag{8}$$

As shown in (5), the voting distortions imply that policy makers' strategic preferences are farther away from one another than are their sincere preferences. Corollary 1 further shows that on top of this strategic polarization effect, the voting distortions can also induce an *average policy bias effect*. To see this, note that in general, the right-hand side of (8) does not need to be 0, which means that policy makers' average strategic preferences $\frac{1}{N} \sum_{n \in \mathcal{N}} (\theta_n + d_n)$ can be more rightist or more leftist than their average sincere preferences $\frac{1}{N} \sum_{n \in \mathcal{N}} \theta_n$. Whereas the strategic polarization effect increases the probability of gridlock, this average bias effect affect the relative likelihood of liberal versus conservative policy change, and thus the relative prevalence of policies L and R.

Proposition 2 and Corollary 1 imply a fundamental difference between the insights from the static and the dynamic models. At first sight, the predictions of these models seem observationally equivalent. In both models, each policy maker votes for the policy that is closer to her ideal point in a given state, the only difference being that in the static theory, this ideal point reflects her sincere preference $\theta_n(t)$, whereas in the dynamic theory the ideal point $\theta_n(t) + d_n$ is also affected by strategic, forward looking considerations. However, this apparent equivalence overlooks an important point. First and foremost, the static approach treats policy makers' preferences as an independent primitive. In contrast, Proposition 2 implies that these preferences are equilibrium objects, and as such, are endogenous to the political decision process Ω. Hence, when considering the impact of institutional changes, one cannot take as given the preferences as revealed by the current legislative behavior, but has to understand how these strategic preferences will change with Ω. Second, policy makers' strategic preferences depend crucially on their expectations about the future, as can be seen from (7). Hence, changes in these expectations, even when not accompanied by any observable changes in the current institution or environment, are likely to affect current behavior.

Having established that in a dynamic model, institutions and expectations about future economic and political environment shape policy makers' behavior, it is important to better understand the mapping between the former and the latter. We do this in Sections 5 and 6.

Before we move on, let us note that when Ω is non quasi-dictatorial, $\Gamma(\Omega)$ may have multiple equilibria, which makes the comparative statics problematic. In the appendix (see Proposition 10), however, we prove that there exists an equilibrium in which the degree of strategic polarization between any two policy makers is smallest. The comparative statics in the following sections are derived for this equilibrium, which we denote by $d(\Omega)$.[13]

5 Institutions and voting distortions

5.1 Institutions and strategic polarization

In this section we investigate in more detail how the equilibrium voting behavior depends on the details of the voting institution. To this end, we use the following partial order on

[13] Using arguments similar to those in Dziuda and Loeper (2016), one can show that the equilibrium with the smallest degree of strategic polarization is also Pareto best and minimizes in the inclusion sense the set of gridlock states across equilibria. It is also worth noting that the comparative statics derived in the rest of the paper holds as well for the equilibrium with the largest degree of strategic polarization.

institutions.

Definition 3 *Let Ω and Ω' be two institutions. We say that Ω' is requires a greater consensus than Ω if $\Omega'_L \subseteq \Omega_L$ and $\Omega'_R \subseteq \Omega_R$.*

In words, Ω' requires a greater consensus than Ω if changing the status quo requires the approval of a greater set of policy makers under Ω' than under Ω. That is, any winning coalition under Ω' is also a winning coalition under Ω, but not necessarily the reverse. In the case of supermajoritarian institutions, rules that use a higher supermajority threshold require a greater consensus. Likewise, adding a veto player, and more generally adding checks and balances increase the degree of consensus required. By definition of $l(\Omega_L)$ and $r(\Omega_R)$, an institution that requires greater consensus leads to more extreme pivots. Formally, if Ω' requires a greater consensus than Ω, then

$$l(\Omega'_L) \leq l(\Omega_L) \leq r(\Omega_R) \leq r(\Omega'_R).^{14} \tag{9}$$

The following proposition shows that the degree of strategic polarization increases monotonically with the degree of consensus required by the institution.

Proposition 3 *If Ω' requires a greater consensus than Ω, then for all $n, m \in \mathcal{N}$ such that $n > m$, and all t,*

$$0 \leq [\theta_n(t) + d_n(\Omega)] - [\theta_m(t) + d_m(\Omega)] \leq [\theta_n(t) + d_n(\Omega')] - [\theta_m(t) + d_m(\Omega')].$$

If furthermore Ω is non quasi-dictatorial, if $l(\Omega_L) \neq l(\Omega'_L)$ or $r(\Omega_R) \neq r(\Omega'_R)$, and if for each n, the distribution of θ_n has full support,[15] *then all inequalities above are strict.*

The intuition for Proposition 3 is as follows. From (9), as the degree of consensus required to change the policy increases, ideologically more extreme policy makers become

[14]To see this, note that by definition of $l(\Omega_L)$, $\{l(\Omega_L)+1, ..., N\} \notin \Omega_L$, and since $\Omega'_L \subseteq \Omega_L$, $\{l(\Omega'_L)+1, ..., N\} \notin \Omega'_L$, so $l(\Omega'_L) \leq l(\Omega_L)$.

[15]By full support, we mean that for all $a < b$, the probability that $\theta_n \in (a, b)$ is strictly positive.

pivots. Hence, the probability that the pivots disagree in the future goes up. The increased probability of future disagreement increases the incentives of each policy maker to defend her preferred policy conditional on gridlock and clearly, policy makers with stronger preferences for R over L respond more strongly to that increase in the probability of future gridlock.

The following proposition further shows that when the policy makers n and m in Proposition 3 are the two pivots, then requiring a greater consensus not only pushes their strategic preferences farther away from each other, but it unambiguously pushes them in opposite directions.

Proposition 4 *If Ω' requires greater consensus than Ω, then*

$$d_{l\left(\Omega_L'\right)}\left(\Omega'\right) \leq d_{l\left(\Omega_L'\right)}\left(\Omega\right) \leq d_{l\left(\Omega_L\right)}\left(\Omega\right) \leq 0 \leq d_{r\left(\Omega_R\right)}\left(\Omega\right) \leq d_{r\left(\Omega_R'\right)}\left(\Omega\right) \leq d_{r\left(\Omega_R'\right)}\left(\Omega'\right). \qquad (10)$$

If Ω is non quasi-dictatorial, if $l\left(\Omega_L\right) \neq l\left(\Omega_L'\right)$ or $r\left(\Omega_R\right) \neq r\left(\Omega_R'\right)$, and if for each n, the distribution of θ_n has full support, then all inequalities in (10) are strict.

Proposition 4 and its consequences for gridlock in the common-shock case are illustrated in Figure 2 below. Proposition 4 states that an institution that requires a greater consensus exacerbates the strategic polarization between the two pivots for two reasons. First, from (9), more extreme decision makers become pivots, and from Proposition 2, for a fixed institution Ω, more extreme policy makers are more strategically polarized. This effect explains the four inner inequalities in (10). Second, the degree of strategic polarization between any two given voters increases, which explains the two outer inequalities in (10).

An important consequence of Proposition 4 is that the static and the dynamic approaches lead to different conclusions on the impact of institutional change, and this difference comes precisely from the aforementioned second effect. To see why, note that when assessing the impact of an institutional change from Ω to Ω', the static approach assumes that policy makers' past voting behavior under Ω, as given by their strategic preferences $\left(\theta_n\left(t\right) + d_n\left(\Omega\right)\right)_{n\in\mathcal{N}}$, is sincere, and thus that they would continue voting according to this revealed ideology

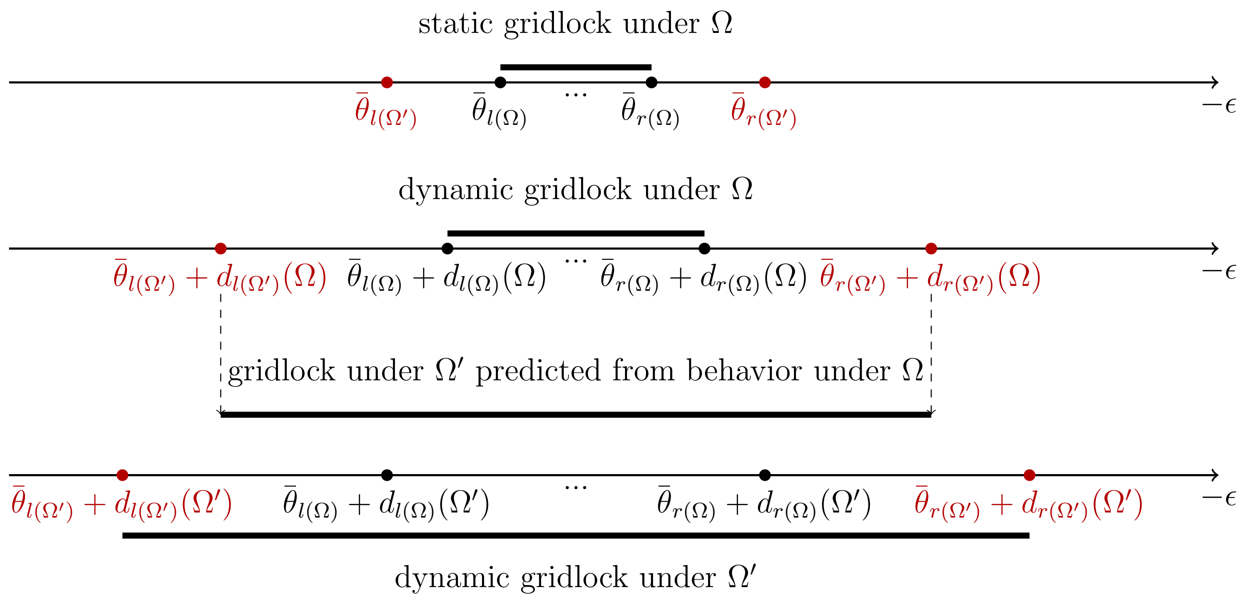

Figure 2: *The top picture illustrates static gridlock under Ω for the common-shock specification. The middle picture illustrates the gridlock predicted by static models when the institution changes to Ω'. The bottom picture illustrates the actual gridlock under Ω'*

$(\theta_n(t) + d_n(\Omega))_{n \in \mathcal{N}}$ after implementing Ω'. Given the new pivots $l(\Omega')$ and $r(\Omega')$, the static approach would then conclude that gridlock under Ω', as defined in Definition 2, would occur when

$$\theta_{l(\Omega')}(t) + d_{l(\Omega')}(\Omega) \leq 0 < \theta_{r(\Omega')}(t) + d_{r(\Omega')}(\Omega).$$

The dynamic approach implies instead that the set of gridlock states under Ω' will be

$$\theta_{l(\Omega')}(t) + d_{l(\Omega')}(\Omega') \leq 0 < \theta_{r(\Omega')}(t) + d_{r(\Omega')}(\Omega').$$

The two outer inequalities in (10) imply therefore that the static approach underestimates the inertial effect of replacing Ω by Ω'. Specifically, it takes into account the fact that the new pivots are farther away from each other on the ideological spectrum, but it does not take into account the fact these new pivots are less likely to agree under Ω' than under Ω.

These results provide potentially testable implications. Consider an institutional reform which increases the degree of consensus required for policy changed, such as adding a second chamber, allowing presidential veto, or replacing simple with a qualified majority. Then any two policy makers who are in office before and after the reform takes place should exhibit a greater degree of polarization in their voting behavior after the reform is implemented.

Proposition 3 also speaks to the impact of filibuster in the U.S. Senate. Pundits often argue that the U.S. Senate should abandon the filibuster since the increased ideological polarization makes it hard to gather 60 votes on virtually any important legislation. Proposition 3 implies that indeed abandoning the institution of the filibuster should decrease gridlock in the U.S. Congress, not only because the approval of fewer senators would be required to pass a policy change, but also because the voting behavior of all U.S. congressmen should become less polarized. This logic also suggests an alternative interpretation of the correlation between the more frequent use of the filibuster and the growing polarization among the U.S. legislators. The filibuster was once an infrequently used tool reserved for the most important and controversial bills, but since the 70s it has become a routine practice in American politics. At the same time, polarization in Congress started increasing. Pundits tend to interpret the rise in the use of the filibuster as a tactic used by a more ideologically polarized Senate to stifle reforms. Our results suggest a possible alternative explanation. The new norm of systematically using the filibuster de-facto shifted the pivotality in the U.S. Congress—not only in the U.S. Senate—to more ideologically polarized policy makers. Hence, Proposition 3 predicts that this de-facto institutional change should have resulted in more polarized voting behavior and an increase in gridlock.

5.2 Biased institutions

Some democratic institutions use different voting rules depending on the policy change under consideration. For example, in 16 U.S. states, a tax increase requires the approval of a qualified majority in each house, whereas a tax cut can be approved by a simple majority. Similarly, in the U.S. budget process, the Byrd Rule essentially requires a filibuster-proof majority to pass bills that raise the deficit, whereas a simple majority is sufficient to pass bills that lower it.[16] The explicit goal of these fiscally conservative institutions was to limit the growth of the public sector and the public debt. Indeed, within a static framework, increasing a hurdle for a policy change in one direction decreases the probability that such policy change occurs. However, the analysis below shows that once the strategic effects highlighted in this paper are taken into account, a fiscally conservative voting rule can induce a more liberal voting behavior on average, and can even lead to more liberal policies. This insight is consistent with the empirical finding that the effect of such biased voting rules on the level of state taxes in the US has been quite modest, if any.[17] It is also consistent with the observation that the Byrd rule did not prevent the federal debt from growing steadily since its implementation.

Definition 4 *An institution Ω is biased in favor of policy R if $l(\Omega) + r(\Omega) > N$.*

To see why the inequality in Definition 4 corresponds to an institution biased for policy R, observe that from Proposition 2, in equilibrium, status quo L is replaced by R in state θ if and only all policy makers in $\{l(\Omega), ..., N\}$ vote for R. Conversely, status quo R is overturned in state θ if and only if all policy makers in $\{1, ..., r(\Omega)\}$. So the condition $l(\Omega) + r(\Omega) > N$ means that de facto, a greater coalition is needed to repeal R than to repeal L. For instance, if Ω requires a supermajority with threshold M_q under each status quo $q \in \{L, R\}$, then Ω is biased in favor of R if and only if $M_L < M_R$.

Proposition 5 below identifies a class of environments in which the government as a whole has no ideological bias, but when operating under a rule biased in favor of R, the average voting bias effect implies that the government votes as if it was in favor of L on average.

[16] The U.S. federal budget process is governed by the Congressional Budget Act of 1974, which allows policy makers to bypass the filibuster via the reconciliation process. This act was amended in 1985 and 1990 by the Byrd Rule to restrict the use of reconciliation (and therefore restore the use of the filibuster) against provisions that increase the deficit beyond the years covered by the reconciliation measure.

[17] For the period 1980-2008, in the average state with no supermajority requirement, taxes as a share of personal income have been between 9.7 percent and 10.9 percent, while in the seven states with supermajority requirements, they have been between 9.7 percent to 10.8 percent (Leachman, Johnson, Grundman, 2012, and Jordan and Hoffman, 2009). Knight (2000) reports that in 1995, among the continental states, states with supermajoritarian requirements had identical average effective tax rates of 7.13% as states without such requirements. Using fixed effects models, Knight (2000) and Besley and Case (2003) find that supermajoritarian requirements reduce taxes by only about \$50 per capita.

Proposition 5 *Consider the common shock specification in Definition 1 where* $\left(\bar{\theta}_n\right)_{n\in\mathcal{N}}$ *and* $\varepsilon(t)$ *are symmetrically distributed around* 0. *If* Ω *is biased in favor of* R, *then the average voting distortion is biased in favor of L, that is,* $\frac{1}{N}\sum_n d_n < 0$.

Under the conditions of Proposition 5, when Ω is unbiased, L and R receive the same expected number of votes. Proposition 5 states that when Ω is biased in favor of R, the expected number of votes for L is greater. The intuition for that result is that policy makers anticipate that it may be hard to replace R with L in the future, and are therefore more inclined to vote for L today.

Proposition 5 raises a question whether the raise in the expected number of votes for L may be large enough to undo the institutional bias in favor of R. The example below shows that the answer is affirmative.

Example: Consider the common shock specification in Definition 1, where $\varepsilon(t)$ follows a standard normal distribution. Unlike in Proposition 5, however, assume that the government is ideologically biased toward L, with the median policy maker's ideological position being $\bar{\theta}_{(N+1)/2} = -0.5$. Under the simple majority rule $\left(\Omega^{(N+1)/2}, \Omega^{(N+1)/2}\right)$, the median voter is always pivotal, and there are no voting distortions; hence, the probability that L is implemented in any period is $\Pr(-0.5+\varepsilon<0) = \Phi(0.5) = 0.7$. Suppose now that we increase the hurdle to implement L. The median policy maker remains pivotal for the policy change from L to R,whereas a more rightist policy maker becomes the right pivot, $\bar{\theta}_r > -0.5$. Hence, the probability that R is replaced by L indeed goes down, and is equal to $\Phi\left(-\bar{\theta}_r - d_r\right) < \Phi\left(-\bar{\theta}_r\right) < \Phi(0.5)$. However, the median policy maker distorts her voting behavior in favor of L, $d_{(N+1)/2} < 0$, so the probability that L is replaced by R also goes down, $1 - \Phi\left(-\bar{\theta}_{(N+1)/2} - d_{(N+1)/2}\right) < 1 - \Phi(0.5)$. Hence, in terms of taxes, making a rule biased against tax increases makes both tax increases and decreases less likely.

Figure 3 below depicts the invariant probability of policy L; i.e., the probability that in a distant future the policy is L, as a function of $\bar{\theta}_r$ for the biased voting rule (solid

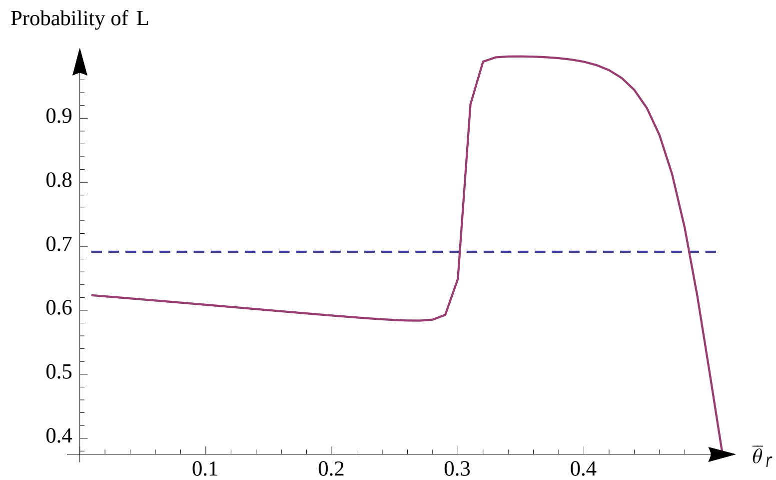

Figure 3: *Probability that L is in place in the long run a function of the bias of the voting rule in favor of R.*

line) and for the simple majority (dashed line). We see that increasing the bias in favor of R, which is equivalent to increasing $\bar{\theta}_r$, does not monotonically increase the long-run prevalence of R, and it can even make L more likely to be implemented in the long run.[18]

6 The Role of Expectations

The intuition for the equilibrium distortions provided in Section 4.2 highlights the role of policy makers' expectations about the policy environment (how preferences over policies evolve) and political environment (who the pivots may be). In this section, we investigate formally the equilibrium impact of these expectations. We first look at how policy makers' voting behavior depends on the expected volatility of their preferences. Second, we allow for political turnover and investigate how policy makers' expectation about the composition of future governments affect their current voting behavior.

[18]Increasing the bias in favor of R initially decreases the probability of L because the ideological polarization of the pivots is relatively low; hence, their voting distortions are low as well. For $\bar{\theta}_r \approx 0.3$, the least strategically polarized equilibrium changes discontinuously with $\bar{\theta}_r$, and the strategic distortion of the median policy maker in favor of L is large enough to overcome the bias of the voting rule in favor of R. As $\bar{\theta}_r$ increases even further, the right pivot's preferences for R become stronger and dominate the strategic distortion of the median policy maker in favor of L.

6.1 Volatility of the environment

The state θ can be interpreted as the current economic situation, state of knowledge on the issue voted upon, or citizens' current sentiment. In some areas, the state is likely to change more frequently than in others. Hence, the process $\{\theta(t) : t \in \mathbb{N}\}$ may vary across issues. In this section, we investigate how strategic polarization and gridlock depend on policy maker's expectations about the frequency with which the state changes.

To this end, we relax the assumption that the state is i.i.d. over time. Instead, we assume that in each period $t \geq 1$, conditional on $\theta(t-1)$, $\theta(t) = \theta(t-1)$ with probability $1 - v$, and with probability v, $\theta(t)$ is redrawn independently of the previous states of nature from a probability distribution P which is independent of t. Thus, P captures the shocks to the environment, and $v \in [0,1)$ measures the volatility of the environment.

We have established so far that non quasi-dictatorial institutions increase policy gridlock. Such gridlock is bound to be particularly detrimental in environments in which frequent policy changes are a desirable response to a volatile environment. In the next proposition we show that unfortunately, it is in such environments that strategic polarization, and hence gridlock, tend to be greater.

Proposition 6 *Let Ω be a non quasi-dictatorial institution, let $v, v' \in (0,1)$ be such that $v < v'$, and let d and d' be the profiles of equilibrium voting distortions corresponding to v and v' respectively. Then*

$$d'_{l(\Omega_L)} < d_{l(\Omega_L)} < 0 < d_{r(\Omega_R)} < d'_{r(\Omega_R)}.$$

When the environment is static, that is, when $v = 0$, then $d = (0, .., 0)$.

The intuition for Proposition 6 is as follows. As the volatility of the environment increases, the policy implemented today is more likely to require a revision tomorrow; hence, securing a favorable status quo becomes more important relative to implementing the desirable policy today.

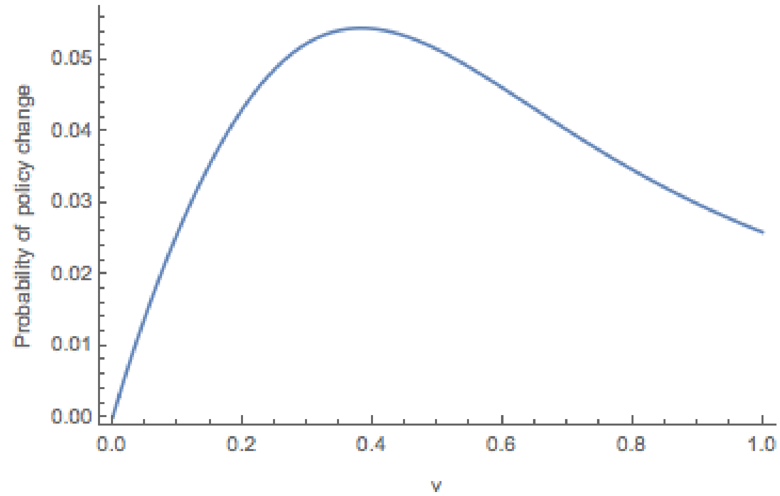

Figure 4: *Probability that policy changes from one period to another as a function of preference volatility.*

Proposition 6 suggests that the impact of the volatility of the environment on the likelihood of policy change is ambiguous. As the environment becomes more volatile, more frequent shocks prompt more frequent policy changes, but policy makers become strategically more polarized, and are thus less likely to agree on a policy response after a shock. Figure 4 above demonstrates that the latter effect may dominate, and policy persistence may be greater in environments that require more frequent policy reforms.

Figure 4 considers again the common shock specification from Definition 1 with an institution Ω such that the pivots are symmetric, i.e., $\bar{\theta}_{r(\Omega_R)} = -\bar{\theta}_{l(\Omega_L)} = 0.5$. For this environment, figure 4 depicts policy volatility—measured by the probability that the policy changes in the next period—as a function of the volatility of the environment v. Initially, these two volatilities move together. This is because for low volatility, the strategic effect is small. However, as volatility becomes large, the strategic effect dominates and the policy becomes more persistent.

Tax, welfare, or trade are prominent examples of volatile policy domains. The goal of most tax reforms oscillate between stimulating the economy (i.e., lowering tax rates) and reducing the deficit (i.e., increasing tax rates). These reforms are prompted by recurring shocks such as business cycles, changes in fiscal needs, or the vagaries of public opinion.

Likewise, since WWII, welfare policies have had to adapt to the emergence of new social risks, growing inequalities, and changing demographics (Hacker 2004). Trade policies must also react to sectorial shocks, domestic pressures, and the international environment. Proposition 6 provides a rationale for procedural rules which limit the degree of consensus required in such areas. Examples of such rules include the reconciliation process, which limits the use of the filibuster for fiscal policies, or the fast track authority, which limits the ability of Congress to veto the decisions of the president for trade policies (Koger 2010, Howell and Moe 2016).

6.2 Political turn-over and electoral expectations

The basic model assumes that policy makers stay in office indefinitely, and thus that the identity of the two pivots remains unchanged throughout the game. In reality, elections imply political turnover, and policy makers are uncertain about who the next pivots will be. Moreover, expectations about the identity of the future pivots may vary with which policy makers are up for reelections, who their challengers are, whether midterm or presidential elections are on the horizon, and the latest opinion polls. In this section, we investigate formally how policy makers' expectations about future political environment shape their voting behavior.

To do so, we extent the model as follows. Let $\mathcal{N} = \{1, ..., N\}$ denote the set of political positions (e.g., presidency, congressional seats) that have to be filled, and $\mathcal{C} = \{1, ..., C\}$ the set of potential candidates for these positions. For all $t \in \mathbb{N}$, $e(t)$ is a vector of length N describing the electoral outcome in period t. The institution Ω specifies the subsets of political positions that are winning coalitions under each status quo, independently of which candidates are currently appointed to these positions, and Ω is assumed to satisfy Assumption 2.

As in the basic model, the preferences of each candidate $c \in \mathcal{C}$ are given by an i.i.d. process $\{\theta_c(t) : t \in \mathbb{N}\}$. In line with Assumption 1, we assume that the rank order of the candidates' preferences stay constant over time, but we slightly relax that assumption to allow some candidates to have the same preferences. Formally, for all $c, c' \in \mathcal{C}$, if $c \leq c'$, then for all t, $\theta_c(t) \leq \theta_{c'}(t)$ almost surely. To introduce electoral uncertainty, we assume that the electoral outcome $\{e(t) : t \in \mathbb{N}\}$ is a stochastic process. To allow policy makers' expectations to vary over time, we assume that in each period t, before casting their votes, all policy makers observe a signal $s(t)$ drawn from an arbitrary set S, and $s(t)$ is correlated with the outcome of the next elections $e(t+1)$. To retain the stationarity of the model, we assume that $\{(s(t), e(t+1)) : t \in \mathbb{N}\}$ is i.i.d. and independent of $\{\theta(t) : t \in \mathbb{N}\}$. That is, the signal gives information only about the electoral outcome in the next period, and electoral outcomes are independent of the policy preferences.

To keep this extension in line with the basic model, we assume that each candidate $c \in \mathcal{C}$ cares about the implemented policy independent of whether she is in office. We acknowledge that this assumption may not always hold, but the results are likely to hold under alternative assumptions.[19]

Proposition 7 *In any equilibrium, there exists a profile of functions* $(d_1, ..., d_C) \in \left(\mathbb{R}^S\right)^{\mathcal{C}}$ *such that for all* $t \in \mathbb{N}$, *a candidate* $c \in \mathcal{C}$ *elected in period* t *votes for* R *if and only if* $\theta_c(t) + d_c(s(t)) \geq 0$. *For all realizations* $s \in S$ *of the signal,* $d_1(s) \leq ... \leq d_C(s)$.

Proposition 7 is a natural extension of Proposition 2, but reveals an interesting point overlooked by the latter. The voting distortions do not depend on which candidates are currently in office, $e(t)$. Instead, they depend on policy makers' current expectations about who will be in office tomorrow, $s(t)$. To see why, recall that policy makers distort their votes in order to influence which alternative prevails in case of *future* gridlock, and the latter depends only on who will be elected tomorrow, not on who is elected today. Note, however, that even though the electoral outcome $e(t)$ does not affect how a given policy maker votes

[19] If candidates care about policies only when in office, then their patience δ decreases to reflect the probablity of electoral defeat. Conditionally on being reelected, however, a policy maker may still face uncertainty over who fills the remaining positions, and hence, the identity of the pivots.

in period t, it affects the policy outcome in period t, because it determines who is elected in that period.

To illustrate the applicability of this extension, consider a simple case in which candidates can be of one of two types, say Republicans and Democrats, and suppose that these two types disagree in a static sense with positive probability. In this case, an electoral outcome $e(t)$ can lead to one of three cases: either there exists a winning coalition of the elected Democrats, or there exists a winning coalition of the elected Republicans, or no winning coalition of the same type exist. The first two cases correspond to a united government, whereas the last case corresponds to a divided government.

Proposition 8 *For all $s \in S$, let $\Pr(D|s)$ denote the probability that conditional on $s(t) = s$, the government is divided in period $t+1$, i.e., that no winning coalition is of the same type. Then in any equilibrium, there exists $d_{Dem} < 0 < d_{Rep}$ such that for all $c \in \{Dem, Rep\}$ and all $s \in S$, $d_c(s) = d_c \Pr(D|s)$.*

Thus, Democrats distort their voting behavior in favor of L and Republicans in favor of R. Moreover, the voting distortions depend not on whether the current government is divided, but on the probability that it is divided in the future, and are linearly increasing in the latter. The intuition for that result is as follows. When the government is united, a winning coalition of Democrats or of Republicans can unilaterally implement any policy irrespective of the current status quo. Conversely, when the government is divided, with positive probability, no policy receives a bipartisan support, in which case either status quo stay in place. Hence, if the current policy makers expect a united government in the future, then they do not care about the future status quo and vote sincerely, whereas if they expect a divided government in the future, they are biased in favor of their most preferred status quo, which is L for democrats and R for Republicans.

A careful analysis of the differences between united and divided governments in a dynamic setting is beyond the scope of this paper. It is interesting, however, to note that Proposition 8 implies policy distortions even under united government. To see that, consider a united

Republican government that inherited high tax rates from the previous government. Suppose further that a high level of public debt makes leaving those rates unchanged statically preferable for both parties. Republicans may nevertheless decrease the tax rates in order to assure that they face a favorable status quo should they want to negotiate a tax cut with Democrats in the next period.

The next proposition generalizes Proposition 8 to an arbitrary number of types. Before we state it, let us define formally who the pivots are in this environment. Given the electoral outcome $e(t)$ in period t, one can reorder the political positions from the position held by the most leftist appointee to the position held by the most rightist appointee, keeping unchanged the role of each position in Ω. One can then define the left and right pivots as in Section 4.1. Proposition 7 implies that almost surely, $\theta_1(t) + d_1(s(t)) \leq ... \leq \theta_C(t) + d_C(s(t))$, that is, more rightist appointees vote in a more rightist way, so for any realization of $e(t)$, the outcome of the vote under status quo L and R coincides with the vote of the left and right pivots, respectively, as in the basic model.

Proposition 9 *For all $(c_l, c_r) \in \mathcal{C}^2$ and $s \in S$, let $\Pr(c_l, c_r|s)$ denote the probability that, conditional on $s(t) = s$, the left and right pivots in period $t+1$ are candidates c_l and c_r, respectively. Then for any equilibrium $(d_1, ..., d_C) \in (\mathbb{R}^S)^{\mathcal{C}}$, for all $c \in \mathcal{C}$, there exists $(D_c(c_l, c_r))_{1 \leq c_l < c_r \leq C} \in \mathbb{R}^{\frac{C(C+1)}{2}}$ such that for all $s \in S$,*

$$d_c(s) = \sum_{1 \leq c_l < c_r \leq C} \Pr(c_l, c_r|s) \times D_c(c_l, c_r).$$

Moreover, $D_c(c_l, c_r)$ weakly decreases as c_l moves closer to c from either side and weakly increases as c_r moves closer to c from either side.

Proposition 9 characterizes how the expectation of a policy maker c affects her voting behavior. Specifically, her bias is more rightist the closer she expects to be to the next right pivot c_r and the farther she expects to be from the next left pivot c_l. The intuition for this comparative statics is as follows. As in the basic model, the willingness of a policy maker to distort her vote in favor of R relative to L reflects her preferences over these policies conditional on the pivot disagreeing. The more similar she expects to be to the right pivot and the more dissimilar she expects to be to the left pivot, the more likely she is to prefer R whenever the former votes for R and the latter votes for L.

Proposition 9, like all of the previous results, is derived in a stylized model whose goal is to add dynamics to standard models of lawmaking and isolate the impact of this addition from other confounding factors; hence one has to exercise caution when applying it to the real world. Nevertheless, it is instructive to derive its implications in particular situations in order to illustrate its difference with the static approach.

Consider, for instance, a U.S. presidential election in which the Republican candidate is expected to win with a coattail effect in the concurrent congressional elections. In this environment, because of the coattail effect usually waning in the midterm elections, the expectations held before the presidential election about the ideology of the pivots right after the election are likely to be more rightist than the expectations held in the first two years of the presidency about the ideology of the pivots after the incoming midterm elections. Proposition 9 implies then that moderate policy makers—including the legislator who becomes the left pivot during the first two years of the presidency—will vote in a more rightist way during the first two years of the presidency than during the two years before. This means that a government under a Republican president may be more successful in replacing a liberal status quo (i.e., L) by a conservative policy (i.e., R) in his first two years not only because the ideology of the current president and Congress is more conservative, but also because their expectation of a liberal shift in the next midterm elections increases their incentives to vote for a conservative policy today.

And finally, Proposition 9 suggests that the effects of the 1986 Supreme Court decisions on redistricting may be more nuanced than predicted by the static literature. Theoretically, redistricting increases the expected ideological polarization of future delegations from the affected states. Suppose that redistricting happens in an extremely conservative state with the current representatives being to the right of the current right pivot. The expected right shift of some of the representatives from this state is not going to affect who the future pivots are, but the shift of the remaining representatives to the left increases the probability that the future right pivot or even both pivots are drawn from more leftist candidates (see Shotts, 2003, for a discussion of how redistricting affects national legislature). Proposition 9 implies then that redistricting affects expectations in a way that makes moderate policy makers become more rightist in their voting behavior.

7 Conclusions

The findings of this paper could be useful for institutional design. As we discussed in Section 1, most modern democracies have a system of check and balances. Admittedly, these checks and balances are not designed to smooth the decision process. Rather, their role is to limit agency costs and the problem of the tyranny of the majority. Our model shows, however, that when checks and balances are introduced in a decision process, they tend to make policy makers more polarized, which can greatly exacerbate their inherently inertial effect. These distortions are most severe in policy domains in which the current agreement serves as a default for future negotiations—as only then voting distortions occur—and in policy domains in which fairly frequent response to exogenous shocks is desirable. Hence, it is important to study institutional solutions that can mitigate these distortions without neutralizing the intended effects of checks and balances. One promising possibility is to use automatic indexation rules that tie the default to some verifiable variable that is correlated with the state of nature (see Weaver, 1988, and Bowen et. al., 2017).

The parsimonious nature of the model presented in this paper can be viewed as a limitation as well as an advantage. A tractable dynamic model provides an opportunity to study issues that are naturally considered in dynamic settings. For example, our model is a natural starting point to explore the incidence and impact of sunset provisions. Sunset provisions are clauses that determine an expiration date of a legislation (Gersen 2007). As such, they limit the endogeneity of the status quo, and hence may limit the polarizing effect identified in this paper. In this light, understanding why sunset provisions are not used more frequently, and why their use changes over time is a natural next step for research.

8 Tables

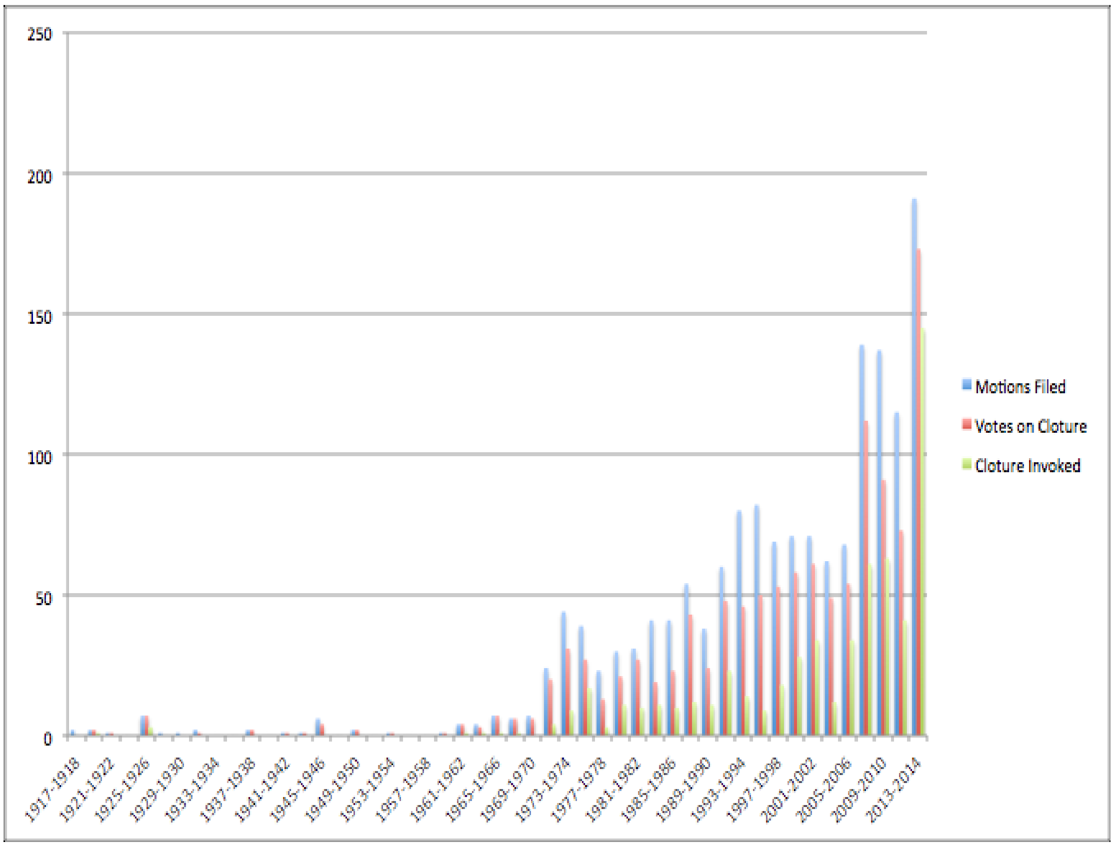

Table 1: *The prevalence of the filibuster in the U.S. Senate. Source: U.S. Senate*

9 Appendix

In the interest of space, we prove the results of Sections 4 and 5 for the more general model of Section 4 in which in each period $t \geq 1$, conditional on $\theta(t-1)$, $\theta(t) = \theta(t-1)$ with probability $1-v$, and with probability v, $\theta(t)$ is redrawn independently of the previous states of nature from a probability distribution P which is independent of t. This corresponds to the basic model for $v = 1$.

Throughout the appendix, we use the following notations:

Notation 1 *For all institutions* Ω, $v \in [0,1]$, *and* $d \in \mathbb{R}^N$, *we denote*

$$\Theta(\Omega, d) = \left\{\theta \in \mathbb{R}^N : \theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \text{ and } \theta_{r(\Omega_R)} + d_{r(\Omega_R)} \geq 0\right\} \tag{11}$$

and

$$H_n(v, \Omega, d) = \frac{\delta v}{1 - \delta(1 - v)} \int_{\Theta(\Omega,d)} (\theta_n + d_n)\, dP(\theta). \tag{12}$$

We omit v *and* Ω *in the argument of* l, r, Θ *and* H *when these parameters are clear from the context.*

Proof of Proposition 1. The proof is straightforward and hence omitted. ■

Lemma 1 *A strategy profile* σ *is an equilibrium of the dynamic game* $\Gamma(\Omega)$ *if and only if there exists* $d \in \mathbb{R}^N$ *such that* $d = H(v, \Omega, d)$ *and each policy maker* $n \in \mathcal{N}$ *votes for* R *in period* t *if and only if* $\theta_n(t) + d_n \geq 0$. *Moreover, in any such equilibrium,* $d_1 \leq ... \leq d_N$.

Proof. Let σ be an equilibrium. Since σ is Markov, in every t, it maps $(\theta(t), q(t))$ into a distribution over votes $\{L, R\}$. Let $V_n^\sigma(\theta, q)$ be expected continuation value of policy maker n from period $(t+1)$ onwards, conditional on $\theta(t) = \theta$, on $q(t+1) = q$, and on continuation play σ. Stage undomination (together with our tie-breaking rule) requires that in t, σ prescribes n to vote for R if and only if $\theta_n(t) + \delta V_n^\sigma(\theta(t), R) \geq \delta V_n^\sigma(\theta(t), L)$. Since this inequality does not depend on $q(t)$, we conclude that the actions prescribed by σ depend on the current state but not on the current status quo. This immediately implies that in σ players revise the policy only when $\theta(t)$ is redrawn.

For all $q \in \{L, R\}$, let $W_n^\sigma(q)$ be the expected continuation value of the game for player n from period $(t+1)$ onwards, conditional on on $\theta(t+1)$ being redrawn (but not on $\theta(t+1)$'s realization), on $q(t+1) = q$, and on continuation play σ. Note that since σ is Markov, and since $\theta(t+1)$ is redrawn, $W_n^\sigma(q)$ does not depend on the realization of previous states. The difference in the expected stream of payoffs for player n from implementing policy R versus L in period t can then be expressed as follows:

$$\theta_n(t) + \sum_{k=1}^{\infty} \delta^k (1-v)^{k-1} \left((1-v)\theta_n(t) + vW_n^\sigma(R)\right) - \sum_{k=1}^{\infty} \delta^k (1-v)^{k-1} vW_n^\sigma(L) \tag{13}$$
$$= \frac{1}{1-\delta(1-v)}\theta_n(t) + \frac{\delta v}{1-\delta(1-v)}\left(W_n^\sigma(R) - W_n^\sigma(L)\right).$$

In period t, if (14) is nonnegative (negative), then R (L) is the unique stage-undominated action for player n. Hence from (14), σ must prescribe n to vote for R if and only if

$$\theta_n(t) \geq -\delta v\left(W_n^\sigma(R) - W_n^\sigma(L)\right) \doteq -d_n^\sigma. \tag{14}$$

Therefore, for any equilibrium σ, there exists $d^\sigma \in \mathbb{R}^n$ such that n votes for R if and only if $\theta_n(t) + d_n^\sigma \geq 0$, and d_n^σ is given by (14).[20]

Part (*iii*) of Assumption 2 together with the already established fact that σ does not depend on the current status quo imply that that the state space can be grouped into three sets: a set of states in which R is implemented irrespective of the status quo, a set of states in which L is implemented irrespective of the status quo, and a set of states in which either status quo stays in place. Let Θ^σ denote the last set. Note that $q(t+1)$ affects the policy outcome in $t+1$ only when $\theta(t+1) \in \Theta^\sigma$, and for all $\theta(t+1) \in \Theta^\sigma$, the status quo stays in place. Hence, $W_n^\sigma(R) - W_n^\sigma(L)$ is simply the expectation of (14) over all $\theta \in \Theta^\sigma$, so

$$W_n^\sigma(R) - W_n^\sigma(L) = \int_{\theta\in\Theta^\sigma} \left[\frac{\theta_n}{1-\delta(1-v)} + \frac{\delta v}{1-\delta(1-v)}\left(W_n^\sigma(R) - W_n^\sigma(L)\right)\right] dP(\theta). \tag{15}$$

[20] Technically, the strategy does not have to specify the behavior for θ that are not in the support of $P(\theta)$, so if the support of θ is not connected, the thresholds may not be uniquely determined. We ignore this issue for simplicity, as the resulting multiplicity does not reverse any of our findings but only complicates the exposition.

Isolating $W_n^\sigma(R) - W_n^\sigma(L)$ from (15) and plugging it into (14), we obtain $d_n^\sigma = \frac{\delta v \int_{\theta \in \Theta^\sigma} \theta_n dP(\theta)}{1-\delta(1-v)-\delta v P(\Theta^\sigma)}$. Using Assumption 1, we can then conclude that $d_1^\sigma \leq ... \leq d_N^\sigma$.

Assumption 1 together with $d_1^\sigma \leq ... \leq d_N^\sigma$ implies that $\theta(t) + d^\sigma$ also satisfies Assumption 1. Since n votes for R if and only if $\theta_n(t) + d_n \geq 0$, the same reasoning as in Section 4.1 establishes that players $l(\Omega_L)$ and $r(\Omega_R)$ are pivotal when the status quo is L and R, respectively. Hence, using (11), we have $\Theta^\sigma = \Theta(\Omega, d)$. Substituting this and (14) into (15), we obtain that $d_n^\sigma = H_n(v, \Omega, d)$.

Conversely, let d be a fixed point of H, and let σ be the strategy profile in which each player n votes for R if and only if $\theta(t) + d_n \geq 0$. As shown above, the unique stage-undominated best response for n in some period t to σ is to vote for R if and only if $\theta_n(t) + d_n^\sigma \geq 0$, where d_n^σ is given by (14). To prove that σ is an equilibrium, we need to show that $d_n = d_n^\sigma$ for each $n \in N$.

Equation (12) implies that any solution to $d_n = H(v, \Omega, d)$ is increasing in n, and thus by the same reasoning as above, $\Theta^\sigma = \Theta(\Omega, d)$. So $W_n^\sigma(R) - W_n^\sigma(L)$ must still satisfy (15). Substituting $\Theta^\sigma = \Theta(\Omega, d)$ and the definition of d^σ in (14) on both sides of (15), we obtain that $d_n^\sigma = \frac{\delta v}{1-\delta(1-v)} \int_{\Theta(\Omega,d)} (\theta_n + d_n^\sigma) dP(\theta)$. A simple contraction argument shows that this equation has a unique solution d_n^σ, and since by construction d_n is also a solution to that equation, necessarily $d_n = d_n^\sigma$. ■

Proof of Proposition 2. By Lemma 1, in any equilibrium σ, each player n votes for R in some period t if an only if $\theta_n(t) + d_n \geq 0$, where d is such that $d = H(v, \Omega, d)$.[21]

If Ω is quasi-dictatorial, then $l(\Omega_L) = r(\Omega_R)$, so using (11), $\Theta(\Omega, d) = \emptyset$ for all $d \in \mathbb{R}^N$, so $d = H(v, \Omega, d) = (0, ..., 0)$.

Suppose now that Ω is not quasi-dictatorial. From (11) and (12), $H_{l(\Omega_L)}(v, \Omega, d) \leq 0 \leq H_{r(\Omega_R)}(v, \Omega, d)$, so $d_{l(\Omega_L)} \leq 0 \leq d_{r(\Omega_R)}$. So $T' \doteq \left\{\theta \in \mathbb{R}^N : \theta_{l(\Omega_L)} + d_{l(\Omega_L)} < 0 \text{ and } \theta_{r(\Omega_R)} + d_{r(\Omega_R)} > 0\right\}$ contains $T'' \doteq \left\{\theta \in \mathbb{R}^N : \theta_{l(\Omega_L)} < 0 \text{ and } \theta_{r(\Omega_R)} > 0\right\}$. Since $l(\Omega_L) < r(\Omega_R)$, Assumption 1

[21] As mentioned in Section 2, Proposition 2 parallers the result of Dziuda and Loeper (2016) for two players and the unanimity rule, but because of technical differences between these two papers, proving this result independently is easier than adapting the proof od Dziuda and Loeper (2016).

implies that $P(T'') > 0$, and hence $P(T') > 0$. Since $T' \subset \Theta(\Omega, d)$ and $P(T') > 0$, we have

$$d_{r(\Omega_R)} = H_{r(\Omega_R)}(v, \Omega, d) \geq \frac{\delta v}{1 - \delta(1 - v)} \int_{T'} \left(\theta_{r(\Omega_R)} + d_{r(\Omega_R)}\right) dP(\theta) > 0.$$

An analogous reasoning implies that $d_{l(\Omega_L)} < 0$, which proves property (ii). Isolating d_n from $d_n = H_n(v, \Omega, d)$, we obtain

$$d_n = \frac{\int_{\Theta(\Omega,d)} \theta_n dP(\theta)}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d))}.$$

Setting $v = 1$ in the above equation delivers the expression from Corollary 1From what precedes, $P(\Theta(\Omega, d)) > P(T') > 0$, so Assumption 1 implies that the integral on the numerator of the above equation is strictly increasing in n, which proves property (i). ■

Lemma 2 *Let $(\leq, \geq)_\Omega$ be the partial order on $\mathbb{R}^N$ defined as follows: for all $d, d' \in \mathbb{R}^N$, $d'(\leq, \geq) d$ if $d'_{l(\Omega_L)} \leq d_{l(\Omega_L)}$ and $d'_{r(\Omega_R)} \geq d_{r(\Omega_R)}$. Then the mapping $d \to H(v, \Omega, d)$ is monotonic in the order $(\leq, \geq)_\Omega$.*

Proof. Fix Ω, and since its fixed, let us omit it as the argument in Θ, H, and l and r in this proof. Let $d, d' \in \mathbb{R}^N$ be such that $d'_l \leq d_l$ and $d_r \leq d'_r$. Then using (11), $\Theta(d) \subseteq \Theta(d')$, so

$$H_l(d'_l, d'_r) - H_l(d_l, d_r) = \frac{\delta v}{1 - \delta(1 - v)} \left(\int_{\Theta(d)} (d'_l - d_l) dP(\theta) + \int_{\Theta(d') \setminus \Theta(d)} (\theta_l + d'_l) dP(\theta) \right).$$

The integrands of the two integrals on the right-hand side of the above equation are negative on their domains of integration, so $H_l(d'_l, d'_r) \leq H_l(d_l, d_r)$. A similar proof shows that $H_r(d_l, d_r) \leq H_r(d'_l, d'_r)$. ■

Proposition 10 *There exists an equilibrium d^* of $\Gamma(\Omega)$ such that for any other equilibrium d',*

$$d'_{l(\Omega_L)} \leq d^*_{l(\Omega_L)} \leq 0 \leq d^*_{r(\Omega_R)} \leq d'_{r(\Omega_R)}, \tag{16}$$

and for all $n > m$, $0 \leq d_n^* - d_m^* \leq d_n' - d_m'$.

Proof. Let $\|\theta\| \doteq \max_{n \in \mathcal{N}} \int |\theta_n| \, dP(\theta)$, and for all $d \in \mathbb{R}^N$, let $\|d\| \doteq \max_{n \in \mathcal{N}} |d_n|$. From (12), for all $d \in \mathbb{R}^N$, $\|H(d)\| \leq \frac{\delta v}{1-\delta(1-v)} (\|\theta\| + \|d\|)$. So if $d = H(d)$, then $\|d\| \leq \frac{\delta v}{1-\delta(1-v)} (\|\theta\| + \|d\|)$, and so $\|d\| \leq \frac{\delta v}{1-\delta} \|\theta\|$. Therefore, all the fixed points of $d \to H(d)$ are in $D \doteq \left\{ d \in \mathbb{R}^N : \|d\| \leq \frac{\delta v}{1-\delta} \|\theta\| \right\}$ and $H(D) \subseteq D$. Since D is a complete lattice for the order $(\leq, \geq)_\Omega$ defined in Lemma 2, Tarski's theorem implies then that the set of fixed points of $d \to H(d)$—and thus the set of equilibria of $\Gamma(\Omega)$—is a complete lattice. As such, it has a minimal element d^* for the order $(\leq, \geq)_\Omega$, which by definition of $(\leq, \geq)_\Omega$ satisfies (16) for any other fixed point d'.

Using (11), (16) implies that $\Theta(\Omega, d^*) \subseteq \Theta(\Omega, d')$. Since $d^* = H(d^*)$ and $d' = H(d')$, the latter inclusion implies that for all $n > m$,

$$\begin{aligned} & ((d_n' - d_m') - (d_n^* - d_m^*)) \frac{1 - \delta(1-v)}{\delta v} \\ = & \int_{\Theta(\Omega, d^*)} (d_n' - d_m') - (d_n^* - d_m^*) \, dP(\theta) + \int_{\Theta(\Omega, d') \setminus \Theta(\Omega, d^*)} (\theta_n + d_n' - (\theta_m + d_m')) \, dP(\theta), \end{aligned}$$

which implies that

$$\begin{aligned} & (d_n' - d_m') - (d_n^* - d_m^*) \\ = & \frac{1}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d^*))} \int_{\Theta(\Omega, d') \setminus \Theta(\Omega, d^*)} (\theta_n + d_n' - (\theta_m + d_m')) \, dP(\theta). \end{aligned} \tag{17}$$

By Proposition 2, the integral in (17) is nonnegative, so $d_n' - d_m' \geq d_n^* - d_m^*$. ■

Proof of Proposition 4. When Ω is quasi-dictatorial, from Proposition 2, the four inner inequalities in (10) hold with equality. When Ω is not quasi-dictatorial, Proposition 2 part (*i*) and (*ii*) imply that the four inner inequalities in (10) hold strictly.

Before proving the two outer inequalities in (10), let us first prove that

$$d_{l(\Omega_L')}(\Omega') \leq d_{l(\Omega_L)}(\Omega) \text{ and } d_{r(\Omega_R)}(\Omega) \leq d_{r(\Omega_R')}(\Omega'). \tag{18}$$

Let $\hat{d} \in \mathbb{R}^N$ be such that $d_{l(\Omega'_L)}(\Omega') \leq \hat{d}_{l(\Omega_L)}$ and $\hat{d}_{r(\Omega_R)} \leq d_{r(\Omega'_R)}(\Omega')$. Together with Assumption 1, these inequalities imply that $\Theta\left(\Omega, \hat{d}\right) \subseteq \Theta(\Omega', d(\Omega'))$. Using successively Lemma 1, the latter inclusion, $d_{l(\Omega'_L)}(\Omega') \leq \hat{d}_{l(\Omega_L)}$, and Assumption 1, we have

$$\begin{aligned} d_{l(\Omega'_L)}(\Omega') &= \frac{\delta v}{1-\delta(1-v)} \int_{\Theta(\Omega', d(\Omega'))} \left(\theta_{l(\Omega'_L)} + d_{l(\Omega'_L)}\right) dP(\theta) \\ &\leq \frac{\delta v}{1-\delta(1-v)} \int_{\Theta\left(\Omega, \hat{d}\right)} \left(\theta_{l(\Omega'_L)} + d_{l(\Omega'_L)}\right) dP(\theta) \\ &\leq \frac{\delta v}{1-\delta(1-v)} \int_{\Theta\left(\Omega, \hat{d}\right)} \left(\theta_{l(\Omega'_L)} + \hat{d}_{l(\Omega'_L)}\right) dP(\theta) = H_{l(\Omega'_L)}\left(v, \Omega, \hat{d}\right). \end{aligned}$$

An analogous argument shows that $d_{r(\Omega'_R)}(\Omega') \geq H_{r(\Omega'_R)}(v, \Omega, d)$. Thus, if we denote $\hat{D} = \left\{\hat{d} \in \mathbb{R}^N : d_{l(\Omega'_L)}(\Omega') \leq \hat{d}_{l(\Omega_L)} \text{ and } \hat{d}_{r(\Omega_R)} \leq d_{r(\Omega'_R)}(\Omega')\right\}$, the two previous inequalities imply that $H(v, \Omega, d) \subseteq \hat{D}$. Since the mapping $\hat{d} \to H(v, \Omega, d)$ is monotonic in the order $(\leq, \geq)_\Omega$ defined in Lemma 2, and since $\hat{D}$ is a complete lattice for that order, Tarski's theorem implies that $\hat{d} \to H(v, \Omega, d)$ admits a fixed point in $\hat{D}$. Clearly, its smallest fixed point, which is $\left(d_{l(\Omega_L)}, d_{r(\Omega_R)}\right)$ by definition, must also be in be in $\hat{D}$. This proves (18).

We are now ready to prove the two outer inequalities in (10). We only prove $d_{r(\Omega'_R)}(\Omega) \leq d_{r(\Omega'_R)}(\Omega')$, the argument for $d_{l(\Omega'_L)}(\Omega') \leq d_{l(\Omega'_L)}(\Omega)$ is analogous. Assumption 1 and (18) imply that $\Theta(\Omega, d(\Omega)) \subseteq \Theta(\Omega', d(\Omega'))$. Using successively Lemma 1 and the latter inclusion, we get

$$\begin{aligned} & \left(d_{r(\Omega'_R)}(\Omega') - d_{r(\Omega'_R)}(\Omega)\right) \frac{1-\delta(1-v)}{\delta v} \\ = & \int_{\Theta(\Omega', d(\Omega'))} \left(\theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega')\right) dP(\theta) - \int_{\Theta(\Omega, d(\Omega))} \left(\theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega)\right) dP(\theta) \\ = & \int_{\Theta(\Omega, d(\Omega))} \left(d_{r(\Omega'_R)}(\Omega') - d_{r(\Omega'_R)}(\Omega)\right) dP(\theta) + \int_{\Theta(\Omega', d(\Omega')) \setminus \Theta(\Omega, d(\Omega))} \left(\theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega')\right) dP(\theta). \end{aligned}$$

Isolating $d_{r(\Omega'_R)}(\Omega') - d_{r(\Omega'_R)}(\Omega)$, we obtain

$$d_{r(\Omega'_R)}(\Omega') - d_{r(\Omega'_R)}(\Omega) = \frac{\int_{\Theta(\Omega', d(\Omega')) \setminus \Theta(\Omega, d(\Omega))} \left(\theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega')\right) dP(\theta)}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d(\Omega)))}. \tag{19}$$

By definition of $\Theta(\Omega', d(\Omega'))$, $\theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega') \geq 0$ for all $\theta \in \Theta(\Omega', d(\Omega'))$, so the integral in (19) is nonnegative, and thus $d_{r(\Omega'_R)}(\Omega') \geq d_{r(\Omega'_R)}(\Omega)$. To complete the proof, it remains to show that when $(l(\Omega_L), r(\Omega_R)) \neq (l(\Omega'_L), r(\Omega'_R))$, this inequality holds strictly, or equivalently, that the integral in (19) is strictly positive.

From (9), $(l(\Omega_L), r(\Omega_R)) \neq (l(\Omega'_L), r(\Omega'_R))$ implies that $r(\Omega_R) < r(\Omega'_R)$ or $l(\Omega_L) > l(\Omega'_L)$. Suppose $r(\Omega_R) < r(\Omega'_R)$, the proof in the other case is analogous. Let

$$T \doteq \left\{\theta \in \mathbb{R}^n : \theta_{l(\Omega'_L)} + d_{l(\Omega'_L)}(\Omega') < 0 \ \& \ \theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega') > 0 \ \& \ \theta_{r(\Omega'_R)} + d_{r(\Omega_R)}(\Omega) < 0\right\}. \tag{20}$$

The first two inequalities in (20) imply that $T \subseteq \Theta(\Omega', d(\Omega'))$. From Assumption 1, the last inequality in (20) implies that $\theta_{r(\Omega_R)} + d_{r(\Omega_R)}(\Omega) < 0$, and thus that T has no intersection with $\Theta(\Omega, d(\Omega))$. Therefore, $T \subseteq \Theta(\Omega', d(\Omega')) \backslash \Theta(\Omega, d(\Omega))$. Since $\theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega') > 0$ for all $\theta \in T$, the integral in (19) is positive if $P(T) > 0$. From Proposition 2, $d_{l(\Omega'_L)}(\Omega') \leq 0 \leq d_{r(\Omega'_R)}(\Omega)$, so since $l(\Omega'_L) < r(\Omega'_R)$, Assumption 1 implies that with probability 1, $\theta_{l(\Omega'_L)} + d_{l(\Omega'_L)}(\Omega') < \theta_{r(\Omega'_R)} + d_{r(\Omega'_R)}(\Omega)$. Therefore, the first inequality in (20) is redundant with the last, so $T = \left\{\theta \in \mathbb{R}^n : -d_{r(\Omega'_R)}(\Omega') < \theta_{r(\Omega'_R)} < -d_{r(\Omega_R)}(\Omega)\right\}$. As shown before, $d_{r(\Omega'_R)}(\Omega') \geq d_{r(\Omega'_R)}(\Omega) > d_{r(\Omega_R)}(\Omega)$, so under our assumption that $\theta_{r(\Omega'_R)}(t)$ has full support, $P(T) > 0$, as needed. ■

Proof of Proposition 3. Using the same steps as for the derivation of (19), we obtain that for all $n > m$,

$$[d_n(\Omega) - d_m(\Omega)] - [d_n(\Omega') - d_m(\Omega')] = \frac{\int_{\Theta(\Omega', d(\Omega')) \backslash \Theta(\Omega, d(\Omega))} (\theta_n + d_n(\Omega') - (\theta_m + d_m(\Omega'))) \, dP(\theta)}{\frac{1-\delta(1-v)}{\delta v} - P(\Theta(\Omega, d(\Omega)))}.$$

From Assumption 1 and Proposition 2, the integrand of the above integral is positive, so $d_n(\Omega) - d_m(\Omega) \leq (d_n(\Omega') - d_m(\Omega'))$ as needed. Strict inequality follows if the probability of the set in the integrand $P(\Theta(\Omega', d(\Omega')) \backslash \Theta(\Omega, d(\Omega))) > 0$, which we have already established in the proof of Proposition 4. ■

Proof of Proposition 5. From (8) and $\theta_n(t) = \bar{\theta}_n + \varepsilon(t)$, we have

$$\frac{1}{N}\sum_{n\in\mathcal{N}} d_n = \frac{\delta P(G)}{1-\delta P(G)}\left[\frac{1}{N}\sum_{n\in\mathcal{N}}\bar{\theta}_n + E[\varepsilon|G]\right] = \frac{\delta P(G)}{1-\delta P(G)}E[\varepsilon|G],$$

so the sign of the average distortion is the same as the sign of $E[\varepsilon|G]$. Suppose, by contradiction, that $E[\varepsilon|G] = \frac{\int_{-\bar{\theta}_r-d_r}^{-\bar{\theta}_l-d_l}\varepsilon dF(\varepsilon)}{P(G)} \geq 0$. Then, by the symmetry of F, the limits of the integrand in the numerator must form an interval skewed to the right, that is, $-\bar{\theta}_l - d_l - \bar{\theta}_r - d_r \geq 0$. Moreover, since $\left(\bar{\theta}_n\right)_{n\in\mathbb{N}}$ is symmetrically distributed around 0, and since Ω is biased in favor of R, $\bar{\theta}_r + \bar{\theta}_l > 0$. The last two inequalities imply that $d_r + d_l < 0$. But from (7),

$$d_l + d_r = \frac{\delta\int_{-\bar{\theta}_r-d_r}^{-\bar{\theta}_l-d_l}\left(\bar{\theta}_l+\bar{\theta}_r+2\varepsilon\right)dF(\varepsilon)}{1-\delta\int_{-\bar{\theta}_r-d_r}^{-\bar{\theta}_l-d_l}dF(\varepsilon)} > 0,$$

which is a contradiction. ■

Proof of Proposition 6. Fix Ω, and since its fixed, let us omit it as the argument in Θ, H, and l and r. Using the notations of the proposition, let $\hat{d}\in\mathbb{R}^2$ be such that

$$d_l' \leq \hat{d}_l \leq 0 \text{ and } 0 \leq \hat{d}_r \leq d_r'. \tag{21}$$

Applying H to (21) and using Lemma 2 we obtain

$$\begin{cases} H_l(v, d_l', d_r') \leq H_l\left(v, \hat{d}_l, \hat{d}_r\right) \leq H_l(v,0,0) = 0 \\ H_r(v, d_l', d_r') \geq H_r\left(v, \hat{d}_l, \hat{d}_r\right) \geq H_r(v,0,0) = 0 \end{cases}.$$

Using successively Lemma 1, $\frac{\partial H_n}{\partial v} = \frac{1-\delta}{(1-\delta(1-v))v}H_n$, and the above inequalities, we obtain

$$\begin{cases} d_l' = H_l(v', d_l', d_r') < H_l(v, d_l', d_r') \leq H_l\left(v, \hat{d}_l, \hat{d}_r\right) \leq 0, \\ d_r' = H_r(v', d_l', d_r') > H_r(v, d_l', d_r') \geq H_r\left(v, \hat{d}_l, \hat{d}_r\right) \geq 0 \end{cases} \tag{22}$$

If we denote $\hat{D} = \left\{\hat{d}\in\mathbb{R}^n : d_l' \leq \hat{d}_l \leq 0 \text{ and } 0 \leq \hat{d}_r \leq d_r'\right\}$, (22) shows that $H\left(v, \hat{D}\right) \subseteq \hat{D}$.

Since the mapping $\hat{d} \to H\left(v, \hat{d}\right)$ is monotonic in the order $(\leq, \geq)_\Omega$ defined in Lemma 2, and since $\hat{D}$ is a complete lattice for that order, Tarski's theorem implies that $\hat{d} \to H\left(v, \hat{d}\right)$ admits a fixed point in $\hat{D}$. Clearly, its smallest fixed point, which is (d_l, d_r) by definition, must also be in $\hat{D}$. This means that $d_l' \leq d_l$ and $d_r \leq d_r'$. To show that these inequalities are strict, we need to show that (d_l', d_r') cannot be such that $(d_l', d_r') = (H_l, H_r)(v, d_l', d_r')$, which follows from (22).

When $v = 0$, (12) implies $H(v, d) \equiv (0, ..., 0)$, and thus that $d = (0, ..., 0)$. ■

Proof of Propositions 7, 8, and 9. Let σ be an equilibrium of the extension considered in Section 6.2. Since σ is Markov, in every t, σ maps the payoff relevant variables $(\theta(t), s(t), e(t), q(t))$ into a distribution over votes $\{L, R\}$. By assumption, it follows that $\{(s(t), e(t+1)) : t \in \mathbb{N}\}$ and $\{\theta(t) : t \in \mathbb{N}\}$ are both i.i.d., and independent of each other, so the realization of $e(t)$ and $\theta(t)$ affect the continuation game from period $t+1$ onwards only via $q(t+1)$. So let $V_c^\sigma(s, q)$ denote expected continuation value for a candidate $c \in \mathcal{C}$ from period $(t+1)$ onwards, conditional on $s(t) = s$, on $q(t+1) = q$, and on continuation play σ. Stage undomination (together with our tie-breaking rule) implies that in t, σ prescribes c to vote for R if and only if

$$\theta_c(t) + \delta V_c^\sigma(s(t), R) \geq \delta V_c^\sigma(s(t), L);$$

equivalently, if and only if $\theta_c(t) + d_c^\sigma(s(t)) \geq 0$ with $d_c^\sigma(s(t)) \equiv \delta(V_c^\sigma(s(t), R) - V_c^\sigma(s(t), L))$, which proves the first claim of Proposition 7.

As in the basic model, for a given realization of $(\theta(t+1), e(t+1), s(t+1))$, one of three cases can occur in $t+1$: either σ prescribes a winning coalition of elected candidates to vote for R, in which case R is implemented in $t+1$ irrespective of $q(t+1)$, or σ prescribes a winning coalition of elected candidates to vote for L, in which case L is implemented in $t+1$ irrespective of $q(t+1)$, or the status quo $q(t+1)$ stays in place in $t+1$. Let Σ^σ denote the set of realizations of $(\theta(t+1), e(t+1), s(t+1))$ that correspond to the last case. In those cases, the continuation payoff gain for c of having status quo R instead of L is $\theta_c(t+1) + \delta\left(V_c^\sigma(s(t+1), R) - V_c^\sigma(s(t+1), L)\right)$. Therefore, if $Q(.)$ denotes the probability distribution of $s(t)$, we have that for all $s \in S$,

$$V_c^\sigma(s,R) - V_c^\sigma(s,L)$$
$$= \sum_{e\in\mathcal{C}^{\mathcal{N}}} \left\{ \Pr(e(t+1) = e | s(t) = s) \int_{\theta, s' : (\theta, e, s') \in \Sigma^\sigma} [\theta_c + \delta(V_c^\sigma(s', R) - V_c^\sigma(s', L))] \, dP(\theta) \, dQ(s') \right\}. \tag{23}$$

Let $c, c' \in \mathcal{C}$ be such that $c < c'$. By assumption, $\theta_c(t) \leq \theta_{c'}(t)$ with probability 1. Together with (23) and the definition of $d_c^\sigma(.)$, this implies that for all $s \in S$, $d_c^\sigma \leq d_{c'}^\sigma$, which proves the second claim of Proposition 7.[22] Therefore, for all $s \in S$ and all θ, $\theta_c + d_c^\sigma(s) \leq \theta_{c'} + d_{c'}^\sigma(s)$. As a result, in every period t, the policy outcome coincides with the vote of the pivot under the current status quo, where pivots are defined like in the static model with the caveat that we put more ideologically rightist candidates in political positions with a higher rank. If $l(e)$ and $r(e)$ denote these two pivots for a given realization $e \in \mathcal{C}^{\mathcal{N}}$—since Ω is fixed in this section, we omit it from the notations—then

$$\Sigma^\sigma = \left\{ \theta, e, s : \theta_{l(e)} + d_{l(e)}^\sigma(s) < 0 \leq \theta_{r(e)} + d_{r(e)}^\sigma(s) \right\}.$$

Therefore, using the notations of Proposition 9, we can rewrite (23) as follows:

$$d_c^\sigma(s) = \delta \sum_{1 \leq c_l < c_r \leq C} \Pr(c_l, c_r | s) \int_{\theta, s' : \theta_{c_l} + d_{c_l}^\sigma(s') < 0 \leq \theta_{c_r} + d_{c_r}^\sigma(s')} [\theta_c + \delta d_c^\sigma(s')] \, dP(\theta) \, dQ(s'), \tag{24}$$

which proves the first claim of Proposition 9 for

$$D_c(c_l, c_r) \equiv \delta \int_{\theta, s' : \theta_{c_l} + d_{c_l}^\sigma(s') < 0 \leq \theta_{c_r} + d_{c_r}^\sigma(s')} [\theta_c + \delta d_c^\sigma(s')] \, dP(\theta) \, dQ(s'). \tag{25}$$

[22] For a more detailed explanation of this step, see the proof of Proposition 1, especially footnote 22, in Dziuda and Loeper (2016).

We now prove the claim in Proposition 9 that $D_c(c_l, c_r)$ weakly increases as c_r moves closer to c from either side. From (24), for all $c_l < c_r < c'_r$,

$$\begin{aligned} D_c(c_l, c'_r) - D_c(c_l, c_r) &= \delta \int_{\theta, s': \theta_{c_l} + d^\sigma_{c_l}(s') < 0 \leq \theta_{c'_r} + d^\sigma_{c'_r}(s')} [\theta_c + \delta d^\sigma_c(s')] \, dP(\theta) \, dQ(s') \\ &\quad - \delta \int_{\theta, s': \theta_{c_l} + d^\sigma_{c_l}(s') < 0 \leq \theta_{c_r} + d^\sigma_{c_r}(s')} [\theta_c + \delta d^\sigma_c(s')] \, dP(\theta) \, dQ(s') \\ &= \delta \int_{\theta, s': \theta_{c_r} + d^\sigma_{c_r}(s') < 0 \leq \theta_{c'_r} + d^\sigma_{c'_r}(s')} [\theta_c + \delta d^\sigma_c(s')] \, dP(\theta) \, dQ(s'). \end{aligned}$$

If $c \leq c_r$, then $\theta_{c_r} + d^\sigma_{c_r}(s') < 0$ implies $\theta_c + d^\sigma_c(s') < 0$ so the integrand on the R.H.S. of the above equation is negative on its domain of integration. Therefore, $D_c(c_l, c'_r) \leq D_c(c_l, c_r)$. If $c'_r \leq c$, then $\theta_{c_r} + d^\sigma_{c_r}(s') \geq 0$ implies $\theta_c + d^\sigma_c(s') \geq 0$ so the integrand on the R.H.S. of the above equation is weakly positive on its domain of integration. Therefore, $D_c(c_l, c'_r) \geq D_c(c_l, c_r)$.

The proof that $D_c(c_l, c_r)$ weakly decreases as c_l moves closer to c from either side is analogous and is omitted for brevity.

To conclude, note that the formula for $d_c(s)$ in Proposition 8 is a straightforward corollary of Proposition 9 where for all $c \in \{Dem, Rep\}$, $d_c \equiv D_c(Dem, Rep)$ and $\Pr(D|s) \equiv \Pr(Dem, Rep|s)$. To complete the proof, it suffices to show that $d_{Dem} < 0 < d_{Rep}$. Since there are only two types and since we have assumed that they disagree in a static sense with positive probability, the proof is analogous to the argument used to show that $d_l < 0 < d_r$ in the proof of Proposition 2. We omit it for brevity. ■

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