Public Good Provision and Cooperation Among Representative Democracies\textsuperscript{1}

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Abstract

This paper analyzes the provision of international public goods by sovereign democracies. The contribution of each country is decided by a representative elected by majority rule at the country level. We compare the case in which representatives choose their contributions noncooperatively with the case in which they choose it through Coasian cooperation. Cooperation among representatives internalize cross-border externalities but induces strategic voters to elect representatives who care less about the public good, so as to reduce their share of the cost of provision. The former effect increases public good provision while the latter reduces it. We show that once voters’ incentives are taken into account, whether cooperation increases public good provision depends neither on voters’ preferences, nor on the magnitude of spillovers, nor on the size, bargaining power, or efficiency of each country. Instead, it depends only on the elasticity of the demand for the public good, and the degree of substitutability between countries’ contributions. Hence, the desirability of international cooperation depends mostly on the type of public good considered.

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1 Introduction

Cross border externalities and transnational public goods lead to inefficiencies and collective action failure when countries set their policies noncooperatively. In the absence of overarching political institutions, observers often call for greater coordination between national policy makers to internalize these externalities. However, despite the multiplication of international negotiations and summits, the supposed gains from international cooperation have arguably not fully materialized. Many global public goods such as the reduction of greenhouse gas emission, political asylum, disease eradication, fish stocks, or fiscal stimulus are still underprovided.

Since Coase (1960), economists have invoked transaction costs of various sorts to explain the inability of bargaining parties to reach mutually beneficial arrangements.\(^1\) This strand of literature focuses on the bargaining process and does not take into account the specificities of the decision process within each country. Others have argued that international policy coordination can exacerbate inefficiencies in national politics.\(^2\) This paper assumes away any inefficiencies in national politics or in the bargaining process, and focuses instead on the interaction between elections at the national level and cooperation at the international level.

In modern democracies, most decision are taken not by the voters, but by political representatives appointed by the voters. As Persson and Tabellini (1992) first pointed out, even if one abstracts away from political agency issues, this distinction has important consequences, because sophisticated voters can use elections as a strategic delegation mechanism. Several papers have shown that once voters’ incentives are taken into account, the impact of international cooperation on public good provision is ambiguous (Segendorff 1998, Gradstein 2004, Buchholz et al. 2005, Kempf and Rossignol 2013). On the one hand, cooperation helps national policy makers internalize cross-border spillovers. This direct effect increases public good provision. On the other hand, more public good requires greater contributions from

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\(^1\) Among others, commitment and enforcement problems (Williamson 1985, North 1990, or Acemoglu 2003) or imperfect information (Mailath and Postlewaite 1990, Harstad 2007) can lead to inefficiencies.

participating countries. Therefore, cooperation induce strategic voters to elect representatives who care less about the public good, so as to decrease their relative contribution. This electoral effect decreases public good provision. In this paper, we determine the main drivers of the magnitude of this electoral effect, and characterize the conditions under which it only mitigates, or completely offsets the direct effect of cooperation.

We consider a global public good model with two countries populated by a continuum of heterogeneous voters. The preferences of a given voter or candidate are characterized by a type that determines her trade-off between public good and private good consumption. Alternatively, this type can be interpreted as her “tax price” of the public good. Countries’ contributions are determined by a two-stage game. In the first stage—the electoral stage—each country elects its representative by majority rule. In the second stage—the policy-making stage—the elected representatives choose, cooperatively or noncooperatively, their contributions to the global public good. In the cooperative scenario, the representatives implement the generalized Nash bargaining solution with the noncooperative outcome as the bargaining default.

The main result is that whether cooperation increases public good provision in equilibrium depends neither on the distribution of voters’ preferences, nor on the magnitude of cross-border spillovers, nor on the relative size, efficiency, or bargaining power of each country. Instead, it depends only on the curvature of the demand for the public good, and on the degree of substitutability between countries’ contributions. In the canonical case of a pure public good whose demand has a constant tax-price elasticity, cooperation increases the level of public good if and only if this elasticity is greater than two.

The model further reveals that whether cooperation increases public good provision does not depend on how severe the underprovision of the public good is in the noncooperative equilibrium. For instance, greater cross-border externalities, or allowing for transfers between countries moves the efficiency frontier farther away from the noncooperative equilibrium. However, once voters’ incentives are taken into account, greater spillovers or monetary compensations do not make cooperation more likely to be beneficial.

To understand the intuition behind these results, consider voters’ incentives in the electoral stage. Voters trade off electing a public good lover to increase public good provision,
versus electing a public good skeptic to shift the cost of provision on the other country. As the elasticity of the public good demand decreases, the cooperative level of public good becomes less sensitive to the representatives’ type, so the latter affect mostly how the cost of provision is shared among the two countries. As a result, voters put a greater weight on reducing their share of the cost of provision, relative to increasing the level of public good, and thus appoint a representative with a lower demand for the public good. This explains why a more inelastic public good demand makes cooperation less likely to be beneficial. Likewise, greater spillovers increase the gains from cooperation in the policy-making stage, but require greater contributions from countries, which increase voters’ incentives to elect a low demand representative and shift the cost of provision on the other country. The latter, electoral effect offsets the former, direct effect, which explains why greater spillovers do not make cooperation more likely to be beneficial.

Our results can be related to the empirical literature on the demand for public goods. Interestingly, existing estimates of the tax-price elasticity vary greatly between public goods (see, e.g., Feldstein 1975, Brooks 2007). Hence, our results imply that the efficiency of interjurisdictional cooperation can differ importantly across types of public goods. Moreover, estimated elasticities are typically smaller than two (Wildasin 1989, Auten et. al. 2002). Therefore, this model suggests that for plausible specifications, strategic voting can severely offset the gains from Coasian cooperation between sovereign democracies. Hence, stronger forms of cooperation are required, such as pooling of sovereignty, or explicit cost sharing.

Our results also shed light on the debate over the structure of federal systems. Several competing principles have been invoked to determine the optimal allocation of policy responsibilities between central and local governments. One of them, “cooperative federalism,” states that federal policies should be negotiated by and “agreed to unanimously by the elected representatives from each of the lower tier governments” (Inman and Rubinfeld 1979). Our analysis suggests that cooperative federalism is not an adequate solution for the provision of local public goods with spillovers, and more coercive forms of centralization are needed. Moreover, contrary to received wisdom (Oates 1972), whether cooperative federalism dominates decentralization depends neither on the heterogeneity of local preferences nor on the magnitude of externalities, but on the elasticity of the demand for the public good.
The paper is organized as follows. Section 2 reviews the literature. Section 3 describes the pure public good model. Section 4 derives the main results. Section 5 analyzes the case of local public goods with spillovers. Section 6 considers the robustness of the results to a number of extensions, and Section 7 relates the results to the empirical literature on the demand for public goods.

2 Related Literature

Since Olson (1965), the literature on collective action and public good provision has shown that the inefficiency of noncooperative behavior is more severe—and thus the potential gains from cooperation are greater—when spillovers are large (Oates 1972, Sandler 1998), when countries’ contributions are closer substitutes (Hirshleifer 1983), or when preferences are homogeneous (Cornes 1993). In contrast, in our model, the distribution of spillovers and preferences do not matter, and cooperation is more likely to be beneficial when countries’ contributions are closer complements. This paper differs from that literature in that the latter focuses on the coordination failure between policy makers while this paper assumes that policy makers cooperate efficiently, and focuses instead on the coordination failure between voters of different jurisdictions.

A number of papers have shown that strategic delegation in elections can distort the outcome of Coasian cooperation between jurisdictions. Brueckner (2000) shows that one can restore efficiency by appropriately allocating the proposal power. Harstad (2008) focuses on the impact of transfers on electoral incentives (the role of transfers and bargaining power in this model is analyzed in Section 6.1). Kempf and Rossignol (2013) investigate the impact of equity considerations in international agreements on electoral incentives, but do not compare electoral equilibria with and without cooperation.\textsuperscript{3} In Section 6.2, we show that our results hold unchanged when representatives use the egalitarian bargaining solution as in Kempf and

\textsuperscript{3}Kempf and Rossignol (2013) compare the cooperative and noncooperative policy outcome assuming that the representatives elected in the cooperative scenario are also the policy makers in the noncooperative scenario. However, this comparison cannot be used to assess the impact of cooperation under strategic voting because in equilibrium, voters elect different representatives depending on whether they expect them to cooperate or not.
Rossignol (2013). Most closely related to this paper, Segendorff (1998), Gradstein (2004), and Buchholz et al. (2005) also investigate whether strategic delegation offsets the gains from international cooperation. Segendorff (1998) and Gradstein (2004) consider a symmetric, pure public good model, and a unilateral externality model with transfers, respectively. Our results show that their conclusion are driven by their particular specification for the cost structure of the public good (see the discussion of these papers in Sections 5 and 6.1). Buchholz et al. (2005) consider a symmetric environment with public bads and transfers, and show that strategic voting leads to an infinite amount of public bad as spillovers become global. The discrepancy between our results and theirs comes from the fact that they focus on symmetric equilibria, which exists only when spillovers are sufficiently small, so their result cannot be applied to the case of global spillovers. In Section 6.4, we show that once asymmetric equilibria are considered, their model yields the same results as ours.

This paper departs from the aforementioned contributions in that we focus on the empirically more relevant case in which international cooperation is carried out without monetary transfers, and political leaders stay in place in case of negotiation failure. More importantly, we consider a large class of public good environments, and characterize the conditions under which strategic voting makes cooperation detrimental. Our analysis singles out the elasticity of the demand for the public good as the main driver of the cost of strategic voting. Even though this parameter has received a lot of attention in the empirical literature (see Section 7), its central role has been overlooked by the theoretical literature on strategic delegation.

Some papers analyze the impact of strategic delegation on more institutionalized forms of cooperation (Chari et al. 1997, Cheikbossian 2000, Besley and Coate 2003, Redoano and Scharf 2004, Dur and Roelfsema 2005, Harstad 2010, Christiansen 2013). These models differ from ours in that cooperation is carried out through federal institutions, fiscal arrangements, and/or majoritarian decision making. In contrast to the case of voluntary bargaining, these forms of cooperation generate electoral incentives that lead to overprovision. Common fiscal pools induce voters to elect public good lovers to increase central spending in their preferred public good. Majoritarian decision making leads to expropriation by the winning coalition, and thus induces voters to elect representatives that are biased in favor of the public project so as to be included in the winning majority.
3 A Pure Public Good Model

We consider an economy composed of two countries. Each country $c \in \{1, 2\}$ is inhabited by a continuum of residents indexed by a set $I_c$. A voter $i \in I_c$ in country $c$ is denoted by $(i, c)$. Each country $c$ chooses the level of provision $x_c \geq 0$ of a public good, and finance this provision by national taxation. For expositional purpose, in this section, we first present the model in the canonical case of a pure, global public good: for a given vector of provision $x = (x_1, x_2)$, the level of public good consumed by the residents of either country is $g = x_1 + x_2$.

3.1 Preferences and the Demand for the Public Good

The preferences of each voter $(i, c)$ are parametrized by a type $p_{i,c} \geq 0$ and are derived from the following utility function:

$$U_c(p_{i,c}, x) = G(x_1 + x_2) - p_{i,c}x_c.$$  \hspace{1cm} (1)

The term $G(x_1 + x_2)$ in (1) corresponds to the utility she derives from public good consumption, while the term $-p_{i,c}x_c$ captures the utility loss due to the extra tax she pays to finance her country’s provision $x_c$. We assume that $G$ is twice continuously differentiable on $\mathbb{R}_+$, and for all $g > 0$, $G'(g) > 0$, $G''(g) < 0$, $\lim_{x \to 0} G''(x) = +\infty$, and $\lim_{x \to \infty} G'(x) = 0$.

Note that the parameter $p_{i,c}$ allows for several sources of heterogeneity. It can capture how much voter $(i, c)$ enjoys public good consumption relative to what she consumes with her after tax income, but also how much she contributes to the cost of public good provision relative to other tax payers in country $c$, or how efficient her country $c$ is in collecting taxes and using the receipts to provide the public good.

The representative of each country $c$ is selected among its population, and its type is denoted by $p_{r,c}$. The distribution of types within each country $(p_{i,c})_{i \in I_c}$ is assumed to have full support on $\mathbb{R}_+$, and the national median type $p_{m,c}$ is positive.

If country $c$ was the sole provider of the public good, and if voter $(i, c)$ could decide its policy, the resulting level of public good would be $(G')^{-1}(p_{i,c})$. Thus, one can interpret $p_{i,c}$ as the tax price of voter $(i, c)$ for the public good, and the function $D = (G')^{-1}$ as the public good demand function. Accordingly, for any profile of representatives’ types $p^r$, we refer to
the representative who has the smaller (greater) \( p^*_c \) as the high (low) demand representative, and likewise with \( p^*_m \) for the median voters.

To illustrate our results, we occasionally use the following specification for \( G \): for all \( g > 0 \) and all \( \varepsilon > 0 \) such that \( \varepsilon \neq 1 \),

\[
G(g) = \frac{\varepsilon}{\varepsilon - 1} g^{\frac{\varepsilon - 1}{\varepsilon}},
\]

and \( G = \ln \) when \( \varepsilon = 1 \). The public good demand induced by (2) is \( D(p) = p^{-\varepsilon} \). Hence, its tax price-elasticity is constant and equal to \(-\varepsilon\).

Some remarks about the specification in (1) are in order. It is general in some respects and restrictive in others. It is general in that it places minimal assumptions on the public good technology \( G \). More precisely, it allows for any continuous and decreasing public good demand function \( D \). As we shall see, this degree of freedom turns out to be the crucial one in our model. This specification also allows for any degree of preferences heterogeneity within and across countries via the type distribution. Moreover, differences in aggregate tax price \( \int_{i \in L_i} p_{i,c} \) across countries can reflect differences in population sizes, or in taxation and public good provision efficiency.

The specification in (1) is restrictive in two main respects. First, the term \( G(x_1 + x_2) \) in (1) assumes that countries make perfectly substitutable contributions to a pure public good, and this public good induces the same demand function in both countries. These assumptions are relaxed in Section 5. Second, the term \(-\tau_{i,c}x_c\) in (1) implicitly assumes that preferences are quasi-linear in after-tax income and that the tax price of each individual is constant over the relevant range of public good levels. This assumption can be viewed as a first order approximation which is justified if the cost of provision of the public good under consideration is a small share of the total budget of each country. Also, this assumption makes our results more comparable with the existing literature, since all papers on strategic delegation and public good provision consider quasi linear environments. Nevertheless, to investigate the impact of tax distortions or preferences that are nonlinear in after tax income, we relax that assumption in Section 9.
3.2 The Elections

To focus on the role of elections as delegation mechanisms, and abstract away from other electoral effects such as electoral competition or political agency, we model national elections via a simplified “citizen-candidate” model as in Persson and Tabellini (1992). Representatives are selected from the pool of voters, and any voter is willing to become the representative of her country if she receives a majority of votes. Candidates cannot make credible electoral promises (Alesina 1988). Therefore, once elected, policy makers maximize their own utility.

Voters are consequentialist and rational: they care only about the policy outcome and foresee how the type of their representative affect the policy outcome. To abstract away from the issue of voters’ miscoordination, we assume that the election winner in a given country is the citizen who is preferred by a majority of voters to any other candidate.\(^4\) The following remark implies then that the national median voters are pivotal in all elections.

**Remark 1** Voters’ preferences within each country \(c\) satisfy the intermediate preferences property of Grandmont (1978),\(^5\) so the majority preferences in country \(c\) over any pair of policy vectors coincide with the preferences of its median type \(p^m_c\).

Depending on the nature of the public good, voting on the type of the representative can be interpreted as choosing a candidate with a particular belief about the intrinsic value of the public good (e.g., a global warming skeptic, or a monetarist in the case of fiscal stimulus), a special inclination towards the polluting industry or the environmental lobbies, or a candidate who seek the support of voters from a particular income group, and thus from voters with a particular tax price.

\(^4\)One can model elections as a normal-form game in which the strategy of each voter is the individual for whom she votes, and the outcome is the policy vector induced by the type of the plurality winner. This game typically has multiple Nash equilibria (e.g., all voters voting for a given type). However, whenever a type is a Condorcet winner, there is a strong Nash equilibrium in which a majority of voters vote for a candidate of that type, and all strong Nash equilibria are outcome equivalent, consistently with our assumption.

\(^5\)This property is satisfied because the type \(p_{i,c}\) appears linearly in (1) (see Grandmont 1978).
4 Public Good Provision by Elected Representatives

4.1 Public Good Provision without Cooperation

In this section, we consider the benchmark case in which elected representatives choose their contributions noncooperatively. The game proceeds as follows. In the first stage—the electoral stage—the voters of each country elect a representative among themselves by majority rule, taking the representative of the other country as given. In the second stage—the policy-making stage—elected representatives choose their policy noncooperatively, taking the policy of the other country as given.

We first derive the subgame equilibrium in the policy-making stage. For any profile of representatives \( p^r \) elected in the first stage, the noncooperative policy equilibrium is:

\[
x^N(p^r) = \begin{cases} 
(D(p_1^r),0) & \text{if } p_1^r < p_2^r, \\
(0,D(p_2^r)) & \text{if } p_1^r > p_2^r,
\end{cases}
\]  

and if \( p_1^r = p_2^r \), there is a continuum of equilibria, but our results do not depend on which is selected in that case. Since public good contributions are perfect substitutes, (3) shows that the contribution of the high-demand representative completely crowds out the contribution of the low-demand representative, and the latter free rides in equilibrium.

Solving by backward induction, the equilibrium in the electoral stage is defined as follows:

**Definition 1** A vector of types \( p^r \) is a noncooperative electoral equilibrium (henceforth NEE) if for all \( p_1 \in \mathbb{R}_+ \), \( x^N(p^r) \) is majority preferred in country 1 to \( x^N(p_1, p_2^r) \), and for all \( p_2 \in \mathbb{R}_+ \), \( x^N(p^r) \) is majority preferred in country 2 to \( x^N(p_1^r, p_2) \).

Remark 1 implies that a CEE is a Nash equilibrium of the game in which national median voters control the type of their representatives \( p^r \), and whose outcome is \( x^N(p^r) \).

**Proposition 1** For any \( p_2^m > 0 \), there exists \( \underline{\rho}, \bar{\rho} \in \mathbb{R} \) such that \( 0 < \underline{\rho} < 1 < \bar{\rho} \) and

- If \( \frac{p_1^m}{p_2^m} \notin [\underline{\rho}, \bar{\rho}] \), in all NEE, the high-demand median voter is elected in her country, while the other country elects a low-demand representative and free rides. The level of public good is the same as if there were no strategic delegation,
b If \( \frac{p_1^m}{p_2^m} \in [\underline{p}, \overline{p}] \), two types of NEE coexist: the same as in case a), and another type in which the low-demand median voter is elected in her country, while the other country elects a low-demand representative and free rides. In the latter type, the level of public good is lower than if there were no strategic delegation.

The intuition behind Proposition 1 is the following. Suppose country 1 is free riding in the NEE. Country 2 is the sole contributor, so its median voter has no incentive to strategically delegate, and thus \( p_2^r = p_2^m \). From (3), to be the free rider, country 1 must elect a low-demand representative, that is, a type \( p_1^r \geq p_2^m \). This strategy is a best response to \( p_2^r = p_2^m \) whenever the median voter in country 1 prefers to free ride on country 2 rather than provide a greater amount of public good and let country 2 free ride. In the proof of Proposition 1, we show that this is the case whenever \( p_1^m \geq \rho p_2^m \) for some \( \rho < 1 \). In particular, when \( p_1^m \in [\rho p_2^m, p_2^m] \), country 1 has the high-demand median voter, but it can credibly commit to free ride by electing a low-demand representative. In this case, strategic delegation exacerbates the underprovision of the public good.

Note that Proposition 1 implies that political representation changes the nature of the game between the two countries: it turns a public good game with a unique equilibrium into a delegation game with multiple equilibria which is reminiscent of the “hawk-dove” game: both countries want to free ride, but if one country, say 1, commits to elect a sufficiently low demand representative (the hawkish strategy), the best response of country 2 is to elect its median voter (the dovish strategy) and let country 1 free-ride.

### 4.2 Does Cooperation Increases Public Good Provision?

Can international cooperation mitigate the free-riding problem that arises in the NEE? Cooperation helps policy makers internalize externalities, which increases public good provision. However, cooperation requires greater contributions from countries, which exacerbates voters’ incentives to elect low-demand representatives and shift the cost of provision on the other country. The latter effect decreases public good provision.

To assess the relative strength of these two effects, we modify the game analyzed in Section 4.1 by letting representatives cooperate in the policy-making stage. Specifically, for
a given profile of representative $p^r$ elected in the first stage, we assume that the outcome of the second stage, denoted by $x^B (p^r)$, is the generalized Nash bargaining solution among the representatives, where the default outcome is the noncooperative equilibrium $x^N (p^r)$ characterized in (3). Formally, $x^B (p^r)$ is the policy vector that maximizes

$$B (p^r, x) = \sum_{c=1,2} \pi_c \ln \left( U_c (p^r_c, x) - U_c (p^r_c, x^N (p^r)) \right),$$

where $\pi_c \in [0, 1]$ is the bargaining power of country $c$, with $\pi_1 + \pi_2 = 1$. Solving by backward induction, the equilibrium of the electoral game is defined as follows:

**Definition 2** A vector of types $p^r$ is a cooperative electoral equilibrium (henceforth CEE) if for all $p_1 \in \mathbb{R}_+, x^B (p^r)$ is majority preferred in country 1 to $x^B (p_1, p^r_2)$, and for all $p_2 \in \mathbb{R}_+$, $x^B (p^r)$ is majority preferred in country 2 to $x^B (p^r_1, p_2)$.

Remark 1 implies that a CEE is a Nash equilibrium of the game in which median voters control the type of their representatives $p^r$, and whose outcome is $x^B (p^r)$.

A few remarks about the way we model cooperation are in order. First, even though countries cooperate, they remain sovereign in that representatives are elected by their respective electorate, policies are financed at the national level, and the cooperative outcome leaves both representatives better off. Second, the Nash bargaining solution used in the second stage captures the unstructured and voluntary nature of Coasian cooperation between leaders of sovereign nations. We consider alternative forms of cooperation in Section 6.2 and allow for side-payments in Section 6.1. Finally, the bargaining default $x^N (p^r)$ implicitly assumes that elected representatives stay in power in case of negotiation breakdown, which is consistent with the observation that in practice, reelects are rarely held on the ground that representatives failed to reach an agreement.\(^6\)

The next proposition states our main result. It compares public good provision in the CEE and the NEE. Before stating the result, note that from Proposition 1, public good provision may vary across NEEs depending on which country elects the high-demand representative. To make the comparison more meaningful, we consider pairs of equilibria in which

\(^6\)For the case in which reelects are held in case of negotiation breakdown, see Segendorff (1998) and Gradstein (2004).
the same country elects the high-demand representative in the CEE and the NEE. Remark 2 in the appendix shows that when countries have similar bargaining power, this assumption is without loss of generality.\footnote{More precisely, Remark 2 shows that for any CEE, there is a NEE in which the same country elects the high-demand representative. Moreover, when the median voters of the two countries are not too homogeneous—i.e., when $p^{m}_1$ and $p^{m}_2$ are not too close—the country of the high-demand median voter always elects the high-demand representative both in the CEE and in the NEE.}

**Proposition 2** In any CEE, each country elects a representative whose type is lower than the type of its median voter, but no country completely free rides.

If $D$ denotes the public good demand function derived from $G$ (see Section 3.1), then for all $p^m$, $\pi$, for any pair of CEE and NEE in which the same country elects the high-demand representative, the level of public good is greater (smaller) in the CEE than in the NEE when $p \to |D'(p)| \times p^3$ is decreasing (increasing).

Proposition 2 states that cooperation reduces free-riding in that it forces both countries to make positive contribution, but it increases public good provision only when $|D'(p)|$ decreases faster than $1/p^3$, or equivalently, when $D(p)$ is “more convex” than $1/p^2$.\footnote{As shown in the proof of Proposition 2, our results hold unchanged when $|D'(p)| \times p^3$ is increasing or decreasing only on the interval $\left[\min\{p^m_1, p^m_2\}, \max\{p^m_1, p^m_2\}\right]$, where $p^r$ is the CEE.} Hence, once voters’ incentives are taken into account, the desirability of international cooperation is independent of the preferences, size, bargaining power, or efficiency of each country, as captured by $p^m$ and $\pi$. Instead, it depends only on the curvature of the public good demand function. Since $D = (G')^{-1}$, Proposition 2 states cooperation is beneficial if and only if the marginal utility of the public good decreases sufficiently rapidly.

One can readily apply Proposition 2 to any parametrized family of public good technology such as the isoelastic one, as specified in (2). In that case, $|D'(p)| \times p^3 = \varepsilon p^{2-\varepsilon}$, so Proposition 2 implies the following.

**Corollary 1** If the public good demand has a constant tax-price elasticity, in the electoral equilibrium, cooperation increases public good provision if and only if this elasticity is greater than 2 (in absolute value), irrespective of the distribution of preferences and bargaining power.
To understand the intuition behind these results, note that the extent to which strategic voters offsets the gains from cooperation depends on their incentives in the electoral stage. When choosing their representatives, voters trade off electing a high-demand type to increase public good provision, versus electing a low-demand type to shift the cost of provision on the other country. Corollary 1 states that the relative weight that voters attach to these two conflicting goals is driven by the tax-price elasticity of the demand for the public good.

To see why, consider the case in which the public good demand is very inelastic. In that case, the level of public good in the cooperative scenario is insensitive to the type of the representatives, so the latter affects mostly how the cost of provision is shared among the two countries. As a result, voters put a greater weight on reducing their share of the cost of provision, relative to increasing the level of public good. This electoral calculus leads them to appoint a low-demand representative, which decreases public good provision in the cooperative scenario. As the demand for the public good becomes more elastic, the level of public good becomes more sensitive to the type of the representative, so voters put a greater weight on increasing the level of public good in their electoral trade-off, which limits their incentives to strategically delegate to a low-demand representative. In contrast, as shown by Proposition 1, in the noncooperative scenario, voters’ incentives are independent of the tax-price elasticity.

To conclude this section, observe that our results characterize the impact of cooperation on public good provision. In Section 6.3, we show that under some conditions, the same results hold for the impact of cooperation on social welfare.

5 Local Public Goods with Spillovers

The model introduced in Section 3 assumes that countries make perfectly substitutable contributions to a pure and homogeneous public good. To investigate the role of these assumptions, in this section, we assume instead that each country provides a distinct public good whose benefit partially spills over to the other country. Formally, for any country $c \in \{1, 2\}$, if $c'$ denote the other country, the utility function of a voter $(i, c)$ is given by

$$U_c(p_{i,c}, x) = G_c(x_c) + \beta_{c',c} G_{c'}(x_{c'}) - p_{i,c} x_c,$$  

(5)
where $G_1$ and $G_2$ satisfy the same assumptions as in Section 3, and $\beta_{c',c} > 0$ captures the effect of the public good provided by country $c'$ on country $c$. Hence, spillovers can have arbitrary spatial patterns, countries’ contributions are imperfect substitutes, and each public good $x_c$ can induce a distinct demand function $D_c = (G'_c)^{-1}$. We refer to the specification (5) as the heterogeneous public good model. Most of the papers on strategic delegation in public good environments use particular cases of this specification.\footnote{See for instance Segendorff (1998), Cheikbossian (2000), Besley and Coate (2003), Gradstein (2004), Dur and Roelfsema (2005), or Kempf and Rossignol (2013).}

The NEE and the CEE are defined as in Section 3 (see Definitions 1 and 2). Let us first consider the noncooperative scenario. For a given profile of representatives $p^r$ elected in the first stage, the subgame equilibrium in the policy-making stage is

$$x^N(p^r) = (D_1(p^r_1), D_2(p^r_2)).$$

Equation (6) shows that policy makers do not internalize the cross-border externalities $\beta_{1,2}$ and $\beta_{2,1}$, but contrary to the homogeneous public good case, since countries contributions are not perfect substitutes, the contribution of the high-demand representative does not crowd out the contribution of the low-demand representative. Note also that from (6), in the policy-making stage, the contribution chosen by a representative is unaffected by the type of the other representative. Therefore, in the electoral stage, voters have no incentive to strategically delegate, and the unique NEE is $p^r = p^m$.

The following proposition determines the condition under which the CEE improves on the NEE.

**Proposition 3** For all $c \in \{1, 2\}$, if $D_c$ denotes the public good demand function derived from $G_c$ (see Section 3.1), then for all $p^m$, $\beta$, and $\pi$, the level of public good $c$ is greater (smaller) in any CEE than in the NEE if $p \rightarrow |D'_c(p)| \times p^2$ is decreasing (increasing).
in that the public good technology is quite different in the two environments, and it leads to qualitatively different noncooperative policy equilibria—compare (3) with (6). Nevertheless, Propositions 2 and 3 show that in both environments, once voters’ incentives are taken into account, the impact of cooperation is driven mostly by the curvature of the demand for the public good. For the isoelastic specification in (2), Proposition 3 implies the following.

**Corollary 2** *If the demand for public good* \( c \in \{1, 2\} \) *has a constant tax-price elasticity, in the electoral equilibrium, cooperation increases the provision of public good* \( c \) *if and only if this elasticity is greater than 1 (in absolute value), irrespective of the distribution of preferences, bargaining power, and spillovers.*

The most noteworthy implication of Proposition 3 and Corollary 2 is that the impact of cooperation depends neither on the magnitude, nor on the symmetry of the spillovers. This result might seem counterintuitive, because the greater and the more unilateral the spillovers, the less efficient the noncooperative equilibrium (see, e.g., Sandler 1998), and thus the greater the potential gains from cooperation. The intuition for that result is that when spillovers increase, the internalization of externalities induced by cooperation increases countries’ contributions in the policy-making stage. Greater contributions in the policy-making stage in turn increases voters’ incentives to reduce their share of the cost of provision by appointing a low-demand representative in the electoral stage.

If country 1 and 2 are interpreted as local jurisdictions in a federal system, if the noncooperative scenario is interpreted as a decentralized regime, and if centralization takes the form of voluntary cooperation among local jurisdictions (“cooperative federalism”, as coined by Inman and Rubinfeld 1997), Proposition 3 can be viewed as a decentralization theorem in the spirit of Oates’ (1972). However, in sharp contrast to Oates’ decentralization theorem, the comparative advantage of centralization depends neither on the heterogeneity of local preferences, nor on the magnitude of externalities, but only on the curvature of the demand for the public good.\(^{10}\)

Note that in the heterogeneous public good model, the parameter \( \varepsilon \) in the isoelastic specification (2) also determines the degree of substitutability between countries’ contributions.

\(^{10}\)See also Besley and Coate (2003), Harstad (2007), or Loeper (2011) on spillovers and the optimal allocation of power in federal systems.
Hence, Corollary 2 states that cooperation is more likely to be beneficial when countries’ contributions are closer substitutes. This result contrasts with the finding of the collective action literature that the coordination failure is more severe with an additive supply aggregation technology—i.e., when countries contributions are perfect substitutes—than with the weakest link technology—i.e., when countries contributions are perfect complements (Hirshleifer 1983).

In the intermediate case in which the demand for the public good has a constant tax-price elasticity equal to 1—that is, when \( G \sim \ln \)—one can fully characterize the CEE.

**Proposition 4** If \( G_c \sim \ln \) for all \( c \in \{l, r\} \), there exists a unique CEE. In the CEE, the contribution of either country is the same as in the NEE.

Note that Proposition 4 implies that once electoral incentives are taken into account, the contribution of each country is independent of the allocation of bargaining power \( \pi \). The intuition for this result is as follows. In the policy-making stage, when the bargaining power of a country decreases, its contribution relative to that of the other country increases. A greater contribution in the policy-making stage increases the incentives of its voters to strategically elect a low demand representatives. When \( G \sim \ln \), these direct and electoral effects exactly offset each other. For more general specification of \( G \), the equilibrium outcome may depend on \( \pi \), but the above intuition explains why, as shown by Propositions 2 and 3, \( \pi \) does not affect whether cooperation is beneficial.

Segendorff (1998, Proposition 3) considers the special case \( G_1 = G_2 = -\exp, \beta_{1,2} = \beta_{2,1} = 1 \), and \( \pi_1 = \pi_2 = 1/2 \). Given this public good technology \( G \), \( |D'(p)| \times p^2 = p \), which is increasing. So Proposition 3 implies that public good provision is lower in any CEE than in the NEE, but this comparison hinges on this particular specification for \( G \).

6 Extensions

6.1 Transfers

Side payments are typically beneficial in bargaining situations because they increase the potential gains from cooperation. In this section, we investigate whether they remain beneficial
when negotiators are elected by strategic voters. To address this question, we modify the basic model of Section 3 and assume that each country can use its tax receipts either to finance the provision of the public good or to costlessly transfer money to the other country. The NEE and CEE are defined as in Section 4 (see Definitions 1 and 2), and representatives set their contributions and the transfers in the policy-making stage.

Clearly, no transfers are made in the noncooperative scenario, so the NEE’s characterization in Proposition 1 holds unchanged. In the cooperative scenario, the bargaining outcome at the policy-making stage \( x^B(p^r) \) depends on which country can provide the public good more efficiently. For all \( c \in \{1, 2\} \), let \( p_c \equiv \int_{i \in I_c} p_{i,c}di \) denote the aggregate cost of providing one unit of public good in country \( c \). Then if \( p_1 < p_2 \), in the Nash bargaining solution, all the public good is provided by country 1, and its cost of provision is partially paid by a transfer from country 2. If \( t_1 > t_2 \), the roles are reversed. In what follows, we focus on the (arguably most plausible) case in which the country that free rides in the NEE is the least efficient country.

Note that in the absence of transfers, the public good is provided by both countries in the Nash bargaining solution. Hence, transfers decrease the average cost of provision, and thus increase the level of public good provision in the policy-making stage. However, as the next proposition shows, transfers do not make cooperation more likely to increase public good provision in the electoral equilibrium.

**Proposition 5** The level of public good is greater (smaller) in the CEE than in the NEE under the same conditions as in Proposition 2.

To understand why transfers do not make cooperation more beneficial, note that in the absence of transfers, when the voters of the efficient country elect a low-demand representative, they shift the cost of provision on the other country, but by doing so, they also increase the average cost of provision. When transfers are available, the latter effect does not arise, because the public good is always provided by the efficient country. Therefore, transfers increase the incentives of voters in the efficient country to elect a low-demand representative. This electoral effect offsets the beneficial effect of transfers in the policy-making stage.

Note that Proposition 5 does not imply that transfers have no effect on the equilibrium
outcome. For instance, for a fixed \( p_1 \), as \( p_2 \to \infty \)—that is, as public good provision by country 2 becomes prohibitively costly—the impact of cooperation without transfers is negligible because country 2 has nothing to offer to country 1, so the CEE tends towards the NEE. However, the availability of transfers effectively reduce country 2’s cost of provision to the same level as country 1, thereby making cooperation possible irrespective of how large \( p_2 \) is. If furthermore \( |D'(p)| \times p^3 \) is increasing, Proposition 5 implies that the level of public good in the CEE is lower than in the NEE. So in the case of strong asymmetries, transfers can be socially detrimental. Hence, this model provides one possible explanation as to why transfers are rarely used in international negotiations.\(^{11}\)

Gradstein (2004, Proposition 2) considers a special case of Proposition 5 with \( G(g) = \sqrt{g} \), \( p_2 = +\infty \), \( \pi_1 = 1 \), and \( \pi_2 = 0 \). When \( G(g) = \sqrt{g} \), \( |D'(p)| \times p^3 \) is constant, so the level of public good provision is the same in the CEE as in the NEE. However, Proposition 5 shows that this comparison holds only for this particular public good technology \( G \).

### 6.2 Alternative Forms of Cooperation

This paper models cooperation between representatives via the generalized Nash bargaining solution. In this section, we discuss whether our results depend on this assumption.

Note first that the utility possibility set is convex, and the equilibrium at the bargain default \( x^N(p') \) is unique. Therefore, our results are unchanged if we assume instead that in the policy-making stage of the cooperative scenario, representative play one of the various games that have been proposed in the bargaining literature to provide game-theoretic foundations for the Nash bargaining solution (see, e.g., Muthoo 1999). Moreover, as can be seen from Propositions 2 and 3, whether cooperation increases public good provision does not depend on the allocation of bargaining power among representatives. Hence, the basic model covers a wide variety of cooperation.

Kempf and Rossignol (2013) argue that equity considerations play a crucial role in international negotiations, and assume accordingly that representatives implement the egalitarian bargaining solution instead of the Nash bargaining solution as in this paper. Their analysis

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\(^{11}\)See also Harstad (2007, 2010) on the value of transfers in international negotiations.
differs from ours also in that they compare the cooperative and noncooperative outcome fixing the type of the policy makers. Nevertheless, to see how such equity concerns affect our results, we modify the basic model of Section 3 and assume that the outcome in the policy-making stage of the cooperative scenario is the egalitarian bargaining solution. Formally, for any profile of representatives \( p^r \), the egalitarian bargaining solution \( x^E(p^r) \) is the only Pareto efficient policy vector for which the utility gains relative to the noncooperative outcome \( x^N(p^r) \) is the same for the two representatives.

**Proposition 6** Consider the basic model of Section 3 in which the CEE is defined as in Definition 2 with \( x^E \) replacing \( x^B \). Then proposition 2 holds unchanged.

Several papers on strategic delegation use the utilitarian bargaining solution to capture the gains form cooperation (Cheikbossian 2000, Besley and Coate 2003, and Dur and Roelfsema 2005). Formally, for any profile of representatives \( p^r \), the utilitarian bargaining solution \( x^U(p^r) \) is the policy vector that maximizes

\[
B(p^r, x) = \pi_1 U_1(p^r_1, x) + \pi_2 U_2(p^r_2, x).
\]  

(7)

Despite its analytical tractability, this bargaining solution is inadequate to model unstructured, voluntary negotiations because it violates the participation constraints of the bargaining parties, and it does not take into account the role of the status quo in the allocation of the bargaining surplus. The next proposition further shows that in the basic model of Section 3, this bargaining solution generically leads to the inexistence of a CEE.

**Proposition 7** Consider the basic model of Section 3 in which the CEE is defined as in Definition 2 with \( x^U \) replacing \( x^B \). Then for all \( \pi \) for which \( x^U \) is uniquely defined (which is generically the case), there is no CEE.

### 6.3 Cooperation and Social Welfare

Our results in Section 4 characterize the conditions under which cooperation mitigates the underprovision of the public good. Intuitively, cooperation should be socially desirable when it mitigates it and socially detrimental otherwise. But since the ideal level of public good
of a voter depends on her type, the impact of cooperation on social welfare depend on the
distribution of voters’ types. Moreover, if the two countries are not equally efficient at
providing the public good, cooperation can also affect the average cost of provision of the
public good. However, when the two countries are equally efficient, and when the distribution
of voters’ preferences within each country is not skewed, the following proposition shows
that cooperation is socially beneficial whenever it increases public good provision, and thus
whenever \( p \to p^3D'(p) \) is decreasing (see Proposition 2).

**Proposition 8** If the two countries are equally efficient at providing the public good, i.e., if
\( \int_{i \in I_1} p_{i,1}di = \int_{i \in I_2} p_{i,2}di \), and if in each country, the median tax price is equal to the average
tax price, then for any pair of NEE and CEE, the level of utilitarian social welfare in the
CEE is greater than in the NEE if and only if the level of public good in the CEE is greater
than in the NEE.

### 6.4 Public Bads

Some international negotiations attempt to reduce the production of a public bad rather than
increase the provision of a public good. To investigate how our results adapt to this case,
we consider the following variation of the basic model of Section 3. Each country \( c \in \{1, 2\} \)
chooses the level of consumption/production \( x_c \geq 0 \) of a good (e.g., energy), where \( x_c \) is
measured in terms of the average utility gains from the consumption/production of that
good in country \( c \). A vector of production \( x \) of this good generates a quantity of public bad
(e.g., pollution) \( b = x_1 + x_2 \). The utility of a voter \( (i, c) \) is

\[
U_c(p_{i,c}, x) = -B(x_1 + x_2) + p_{i,c}x_c,
\]

where \( B \) is such that \( B' > 0 \), \( B'' > 0 \), \( \lim_{x \to 0} B'(x) = 0 \), and \( \lim_{x \to \infty} B'(x) = +\infty \). The
NEE and CEE are defined as in Section 4.

If country \( c \) was the sole producer/consumer of the good, and if individual \( (i, c) \) could
decide its level of per capita consumption, the resulting level of public bad would be
\( x_c = (B')^{-1}(1/(p_{i,c})) \). Therefore, in the hypothetical situation in which voter \( (i, c) \) can produce
this public bad via a cost function \( B \) and sell it at a price \( p_{i,c} \), then \( S \equiv (B')^{-1} \) can be viewed
as her supply function.
The following proposition shows that, similarly to the public good case, cooperation is beneficial if and only if the supply function is sufficiently concave, or equivalently, if the marginal disutility of the public bad increases sufficiently rapidly.

**Proposition 9** For all \( p \) and \( \pi \), and for all CEE and NEE in which the same country elects the high-supply representative, the level of public bad is smaller (greater) in the CEE than in the NEE when \( p \to S'(p) \times p^3 \) is decreasing (increasing).

Note that when \( S \) has a constant elasticity \( \varepsilon > 0 \)—that is, when \( B(g) = \frac{\varepsilon}{\varepsilon+1} g^{\frac{\varepsilon}{\varepsilon+1}} \) and thus \( S(p) = p^\varepsilon \)—then \( S'(p) \times p^3 = p^{\varepsilon+2} \) which is increasing in \( p \) for all \( \varepsilon > 0 \). Hence, Proposition 9 implies that, contrary to the public good model, for the isoelastic specification, cooperation is always detrimental.

This model corresponds to the “global pollution” case in Buchholz et. al. (2005)—the limit case \( s \to 1 \) using their notations. These authors show that, unlike in this model, pollution can increase infinitely as the public bad becomes pure. The discrepancy between our results and theirs comes from the fact that they focus on symmetric equilibria, which do not exist when spillovers are large (see Remark 3 in the appendix).\(^{12}\)

### 6.5 Tax Distortions

The models analyzed in Sections 3 and 5 assume that for each voter \((i, c)\), the tax price for the public good \( p_{i,c} \) is independent of \( x_c \). This assumption could be violated if the tax levied to finance the public good generates distortions which grow more than proportionally with the level of taxation.

Increasing marginal tax distortions can be most easily introduced in the heterogeneous public good model by replacing the term \(-p_{i,c}x_d\) in (5) by \(-p_{i,c}T(x_c)\) for some strictly increasing and convex function \( T \):

\[
U_c(p_{i,c}, x) = G_c(x_1) + \beta_{c'c}G_{c'}(x_{c'}) - p_{i,c}T(x_c),
\]

This specification can be mapped into the original model by expressing countries’ policies in terms of level of taxation instead of level of provision. Specifically, if we use the change of

\(^{12}\)Bucholtz et al. (2005) also differ from this paper in that they allow for monetary transfers. However, as argued in Section 6.1, this assumption does not change the results.
variable $y_c = T(x_c)$ for all $c \in \{l, r\}$ and $x_c \geq 0$, then (9) becomes identical to (5) except that $G$ is replaced by $G \circ T^{-1}$, which yields the following result:

**Proposition 10** For all $c \in \{1, 2\}$, if $D^T_c$ denotes the public good demand function derived from the public good technology $G^T_c \equiv G_c \circ T^{-1}$, then for the specification (9), for all $p^m$, $\beta$, and $\pi$, the level of public good $c$ is greater (smaller) in any CEE than in the NEE if $p \rightarrow |(D^T_c)'(p)| \times p^2$ is decreasing (increasing).

Note that the demand function $D^T_c$ has a different interpretation from $D_c$: $D^T_c(p)$ is the most preferred level of provision of public good $c$ for a voter with tax price $p$, whereas $D^T_c(p)$ is the level of tax that she would pay at this level of provision. Intuitively, when tax payments increase more than linearly with the level of provision—that is, when $T$ is convex instead of linear as in the basic model—$D^T$ should be less elastic, because as $p$ decreases and as the level of provision increases, the price of an additional unit of public good increases more rapidly, so public good demand increases less rapidly. Thus, Corollary 2 suggests that increasing tax distortions should make cooperation less likely to be beneficial.

This intuition can be formalized in the following environment. Suppose that for all $g \geq 0$, $T(g) = g + \alpha g^2$, so the case $\alpha = 0$ corresponds to the basic model of Section 3. Simple calculus shows (see Remark 4 in the appendix) that if $G_c$ is such that $|(D_c)'(p)| \times p^2$ is increasing, then $|(D^T_c)'(p)| \times p^2$ is also increasing. Therefore, from Proposition 10, if cooperation is detrimental for $\alpha = 0$, cooperation must also be detrimental for all $\alpha > 0$.

### 6.6 Equilibrium Existence

Propositions 2 and 3 hold for any CEE, but do not prove the existence of such equilibria. Existence of an electoral equilibrium in the cooperative scenario is difficult to ascertain because for the general specification considered in (1) and (5), the Nash bargaining solution does not have a closed-form solution, which makes it difficult to verify that voters’ payoff are quasi-concave in their representatives’ types. Nevertheless, Proposition 4 shows that in the heterogeneous public good case, when $G \sim \ln$—i.e., when the demand for public good has a tax-price elasticity equal to 1—a CEE exists for any distribution of bargaining power, spillovers, and types.
For general specifications of the function $G$, one can prove the existence of a CEE in mixed strategies in the heterogeneous public good model. Suppose that voters are expected utility maximizer with Bernouilli utility function given by (5). A mixed CEE consists of two probability distributions $\mu_1$ and $\mu_2$ over $\mathbb{R}_+$ such that for each $c \in \{1, 2\}$, any representative’s type $p^*_c$ in the support of $\mu_c$ is majority-preferred in country $c$ to any other type, given the probability distribution $\mu_{c'}$ of the type $p^*_{c'}$ elected in the other country. Hence, Definition 2 corresponds to a mixed CEE with degenerate distributions $\mu_1$ and $\mu_2$.\(^{13}\)

The following proposition extends Proposition 3 to mixed CEE.

**Proposition 11** A mixed CEE always exists. Let $D$ denotes the public good demand function derived from $G$ (see Section 3.1). For all $p^m$ and $\pi$, if $|D'(p)| \times p^2$ is decreasing (increasing), then for all mixed CEE $(\mu_1, \mu_2)$, all $c \in \{1, 2\}$ and all $p^*_c$ in the support of $\mu_c$, there exists $p^*_{c'}$ in the support of $\mu_{c'}$ such that the level public good in country $c$ is greater (smaller) in $x^B(p')$ than in the NEE.

In the homogeneous public good model, the equilibrium existence issue is further complicated by the discontinuity of the noncooperative equilibrium as given by (3) at any $p'$ such that $p^*_1 = p^*_2$. In the cooperative scenario, this discontinuity can give incentives to the voters of the country of the high demand representative to elect a type slightly higher than the other country so as to become the free-rider at the bargaining default, and thus decrease their contribution at the bargaining outcome. Intuitively, such deviations are profitable only when $p^*_1$ and $p^*_2$ are sufficiently close, and thus when $p^m_1$ and $p^m_2$ are sufficiently close. The

\(^{13}\)Since the electoral stage represents a game between two electorates rather than two unitary players, the interpretation of mixed equilibria is not the same as in standard normal form games. Nevertheless, such mixed strategy equilibria can be purified in the spirit of Harsanyi (1973) as follows: the payoff of a voter $(i, c)$ from a profile of elected representative $p'$ is $U_c(p_{i, c}, x^B(p')) + \epsilon_c(p^*_c)$, where $(\epsilon_c(p^*_c))_{p^*_c \in \mathbb{R}}$ is a random shock common to all voters in country $c$, and $\epsilon_1$ and $\epsilon_2$ are independent. The realization of $\epsilon_c(p^*_c)$ can be interpreted as the intrinsic popularity in country $c$ of the candidate with type $p^*_c$. Since voters’ payoff are still linear in the type, Remark 1 still holds, and for any realization of $\epsilon_c(p^*_c)$, the majority preferences in country $c$ on lotteries over $p'$ coincide with the preferences of the median type $p^m_c$. One can then choose $\epsilon_c$ such that for almost all its realizations, there is a unique best response $p^*_c$ for $p^m_c$ given $\mu_{c'}$, and such that the probability distribution of these best response induced by $\epsilon_c$ is $\mu_c$. If $(\mu_1, \mu_2)$ is a CEE, then since $p^m_c$ is indifferent between all types $p^*_c$ in the support of $\mu_c$, these popularity shocks can be chosen arbitrarily small.
following proposition confirms this intuition when $G(g) \sim \sqrt{g}$—i.e., when the public good demand has an elasticity equal to 2.

**Proposition 12** When $G(g) \sim \sqrt{g}$, there exists a CEE if and only if \( \frac{m_1}{p_1^2} \neq \left( \frac{1+\pi_1}{3} : \frac{3}{1+\pi_2} \right) \).

7 **Concluding Remarks**

It is instructive to relate our results to the empirical literature on the demand for public goods. Most empirical studies have found a relatively small tax-price elasticity of the public good demand, typically lower than one (see, e.g., Wildasin 1989) but estimates greater than two have been found for some particular public goods (see DelRossi and Inman 1999). Together with Corollaries 1 and 2, these studies suggest that strategic voting can greatly undermine the gains from interjurisdictional Coasian cooperation, but its efficiency can vary substantially across types of public goods.

It should be noted that most of these estimates are for local public goods. Tax-price elasticities for global public goods are more difficult to estimate because the effect of institutional or cultural differences across countries is harder to factor out from the variations of public good demands. For public goods of national or international scope, existing estimates are based on individual donations. They are not equivalent to tax-price elasticities, because charitable donations reflect a desire for warm-glow which is of smaller magnitude when individual contributions take the form of mandatory taxation. Nevertheless, it is interesting to note that, as for the case of local public goods, these estimates are typically around or below one (see, e.g., Auten et. al. 2002, and the references therein), and more importantly, they vary greatly between public goods (Feldstein 1975, Brooks 2007).

When empirical estimates of tax-price elasticities are not available, our results suggest the following rule of thumb: the negative effect of strategic voting on international cooperation is more severe for public goods whose marginal return decreases more rapidly. Policy expertise can then be used to assess this rate of decrease for a given public good, say fundamental research, disease eradication, or global warming.
8 Appendix

To keep the algebra tractable, it will be convenient to renormalize the utility function in (1) as follows:

\[ U_{i,c}(x) = \theta_{i,c} G(x_1 + x_2) - x_c, \]

where \( \theta_{i,c} = 1/p_{i,c} \). Since the Nash equilibrium concept and the Nash bargaining solution used in this paper are invariant through affine transformation of the utility function, this renormalization is w.l.o.g. Throughout the appendix, we use the above specification (except in the proof of Propositions 5 and 8, in which the cardinal representation of \( U_{i,c} \) matters), and the following notations.

**Notation 1** The median voter of any country \( c \in \{1, 2\} \) is denoted by \( i^m_c \), and its type is denoted by \( \theta^m_c \).

For all \( \theta^r \in \mathbb{R}^2_+ \), all \( x \in \mathbb{R}^2_+ \), and all \( c \in \{1, 2\} \), \( \Delta_c(\theta^r, x) = U_c(\theta^r_c, x) - U_c(\theta^r, x^N(\theta^r)) \) denotes the welfare gains for the representative of country \( c \) at \( x \) relative to the noncooperative equilibrium \( x^N(\theta^r) \).

For all \( p > 0 \), \( D(p) \equiv (G^r)^{-1}(p) \) and for all \( t > 0 \), \( \phi(t) \equiv -\frac{1}{t^2} D'(\frac{1}{t}) \).

**Proof of Proposition 1.** Let \( \bar{\theta}^r \) be a NEE. Necessarily, \( \bar{\theta}^r_1 \neq \bar{\theta}^r_2 \). To see this, suppose that \( \bar{\theta}^r_1 = \bar{\theta}^r_2 \). From (3), for at least one country \( c \), \( x^N(\bar{\theta}^r_c) > 0 \), and by lowering \( \bar{\theta}^r_c \), \( i^m_c \) keeps the public good level unchanged, but lowers its contribution to 0, a profitable deviation.

Suppose now to fix ideas that \( \bar{\theta}^r_1 < \bar{\theta}^r_2 \). In that case, for all \( \theta^r_1 \leq \bar{\theta}^r_1 \), \( (\theta^r_1, \bar{\theta}^r_2) \) is also a NEE. To see this, observe that from (3), \( x^N(\bar{\theta}^r) = x^N(\theta^r_1, \bar{\theta}^r_2) \), so if \( \bar{\theta}^r \) is a NEE, \( i^m_1 \) has no profitable deviation at the strategy profile \( (\theta^r_1, \bar{\theta}^r_2) \). Also, from (3), for all \( \theta^r \in \mathbb{R}^2_+ \), the level of public good at \( x^N(\theta^r) \) is \( \max(D(1/\theta^r_1), D(1/\theta^r_2)) \), which is weakly increasing in \( \theta^r_1 \), while \( x^N(\theta^r) \) is weakly decreasing in \( \theta^r_1 \). Therefore, for all \( \theta^r \in \mathbb{R}^2_+ \) such that \( \theta^r_1 \leq \bar{\theta}^r_1 \), \( i^m_2 \) weakly prefers \( x^N(\theta^r_1, \bar{\theta}^r_2) \) to \( x^N(\theta^r_1, \bar{\theta}^r_2) \), and since \( \bar{\theta}^r \) is a NEE, \( i^m_2 \) also weakly prefers \( x^N(\bar{\theta}^r_1, \theta^r_2) \) to \( x^N(\bar{\theta}^r_1, \theta^r_2) \). By transitivity, \( i^m_2 \) weakly prefers \( x^N(\bar{\theta}^r) = x^N(\theta^r_1, \bar{\theta}^r_2) \) to \( x^N(\theta^r_1, \bar{\theta}^r_2) \), so \( (\theta^r_1, \bar{\theta}^r_2) \) is a NEE. The previous reasoning implies that there exists a NEE such that \( \bar{\theta}^r_1 < \bar{\theta}^r_2 \) if and only if \( (0, \bar{\theta}^r_2) \) is a NEE. Moreover, \( i^m_2 \) has no profitable deviation at \( (0, \bar{\theta}^r_2) \) if and only if \( \theta^r_2 = \bar{\theta}^r_2 \) maximizes \( U_2(\theta^m_2, x^N(0, \theta^r_2)) \). One can easily check that the unique solution of the
latter program is \( \bar{\theta}_2^r = \theta_2^m \). Hence, we have shown that if \( \bar{\theta}^r \) is a CEE in which country 1 free rides (i.e., \( \bar{\theta}_1^r < \bar{\theta}_2^r \)), then \( \bar{\theta}_2^r = \theta_2^m \), and \( (0, \theta_1^m) \) is also a CEE, and it leads to the same policy outcome. When furthermore \( \theta_1^m \leq \theta_2^m \), from (3), this outcome is the same as \( x^N (\theta^m) \).

Reciprocally, consider the strategy profile \( (0, \theta_2^m) \). From what precedes, \( \bar{\theta}_2^m \) has no profitable deviation. Voter \( \bar{\theta}_1^m \) has a profitable deviation if and only if \( \theta_1^m = \theta_2^m \) is a profitable deviation, which is the case if and only if \( \theta_2^m > \theta_2^m \) and

\[
U_1 \left( \theta_1^m, \left(D \left( \frac{1}{\theta_1^m} \right), 0 \right) \right) > U_1 \left( \theta_1^m, \left(0, D \left( \frac{1}{\theta_2^m} \right) \right) \right),
\]

or equivalently,

\[
G \left( D \left( \frac{1}{\theta_1^m} \right) \right) - \frac{1}{\theta_1^m} D \left( \frac{1}{\theta_1^m} \right) - G \left( D \left( \frac{1}{\theta_2^m} \right) \right) > 0.
\]

The left-hand side of the above inequality is negative when \( \theta_1^m = \theta_2^m \), and since \( G' \circ D (p) = p \), its derivative w.r.t. \( \theta_1^m \) is \( D (1/\theta_1^m) / (\theta_1^m)^2 \), which is positive. This shows that for a given \( \theta_2^m \), there exists \( \rho > 1 \) such that \( (0, \theta_2^m) \) is a NEE if and only if \( \theta_1^m \leq \rho \theta_2^m \).

A symmetric argument shows that for a given \( \theta_2^m \), there exists \( \rho < 1 \) such that there exists a NEE \( \bar{\theta}^r \) in which country 2 free rides (i.e., \( \bar{\theta}_1^r > \bar{\theta}_2^r \)) if and only if \( \theta_1^m = \theta_2^m \) and \( \theta_1^m \geq \rho \theta_2^m \). When furthermore \( \theta_1^m \geq \theta_2^m \), from (3), for all such equilibria, \( x^N (\bar{\theta}^r) = x^N (\theta^m) \).

**Lemma 1** If \( \theta_1^r > \theta_2^r > 0 \), the Nash bargaining solution \( x^B (\theta^r) \) as defined in Section 4.2 is such that \( x_1^B (\theta^r) > 0 \), and \( x_2^B (\theta^r) > 0 \). It is given by

\[
\begin{align*}
x_1^B (\theta^r) &= (\pi_2 \theta_1^r - \pi_1 \theta_2^r) \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) + \pi_1 D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) + \pi_2 D \left( \frac{1}{\theta_1^r} \right) \\
x_2^B (\theta^r) &= (\pi_1 \theta_2^r - \pi_2 \theta_1^r) \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) + \pi_2 \left( D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - D \left( \frac{1}{\theta_1^r} \right) \right) \\
&= (\pi_1 \theta_2^r - \pi_2 \theta_1^r) \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) + \pi_2 \left( D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - D \left( \frac{1}{\theta_1^r} \right) \right)
\end{align*}
\]

(10)

The level of public good at \( x^B (\theta^r) \) is

\[
x_1^B (\theta^r) + x_2^B (\theta^r) = D \left( \frac{1}{\theta_1^r + \theta_2^r} \right).
\]

(11)

Using Notation 1, the marginal effect of the representatives’ types on the welfare of their median voters is

\[
\begin{align*}
\frac{\partial u_1 (\theta_1^m, x^B (\theta^r))}{\partial \theta_1^r} &= (\theta_1^m - \theta_1^r) \phi (\theta_1^r + \theta_2^r) - (\pi_1 \phi (\theta_1^r) + \pi_2 \phi (t)) \theta_2^r \\
\frac{\partial u_2 (\theta_2^m, x^B (\theta^r))}{\partial \theta_2^r} &= (\theta_2^m - \theta_2^r) \phi (\theta_1^r + \theta_2^r) - (\pi_1 \phi (\theta_1^r) + \pi_2 \phi (t)) \theta_2^r
\end{align*}
\]

(12)

for some \( t \in (\theta_1^r, \theta_2^r) \).
Proof. From (4), \( B(\theta^r, x) \) is concave in \( x \), so \( x^B(\theta^r) \) is characterized by the F.O.C. \( \partial B / \partial x = 0 \), which can be rewritten as follows: for all \( c \in \{1, 2\} \),

\[
\left( \frac{\pi_1 \theta_1'}{\Delta_1(\theta^r, x^B)} + \frac{\pi_2 \theta_2'}{\Delta_2(\theta^r, x^B)} \right) G'(x_1^B + x_2^B) = \frac{\pi_c}{\Delta_c(\theta^r, x^B)}.
\]

which implies that

\[
\frac{\pi_1}{\Delta_1(\theta^r, x^B)} = \frac{\pi_2}{\Delta_2(\theta^r, x^B)}.
\]

Dividing (13) by \( \pi_c / \Delta_c(\theta^r, x^B) \) and substituting (14), we obtain \( (\theta_1^r + \theta_2^r) G'(x_1^B + x_2^B) = 1 \), which implies (11).

If \( \theta_1^r > \theta_2^r \), then from (3), \( x^N(\theta^r) = \left( D \left( \frac{1}{\theta_1^r} \right), 0 \right) \). Using (11), this implies that

\[
\left\{ \begin{array}{l}
\Delta_1(\theta^r, x^B) = \theta_1^r \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) - x_1^B + D \left( \frac{1}{\theta_1^r} \right) \\
\Delta_2(\theta^r, x^B) = \theta_2^r \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) - x_2^B
\end{array} \right.
\]

Substituting the above relations into (14), we obtain

\[
\pi_2 x_2^B - \pi_1 x_1^B = (\pi_2 \theta_2^r - \pi_1 \theta_1^r) \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) + \pi_2 D \left( \frac{1}{\theta_1^r} \right). \tag{15}
\]

Note that (11) and (15) form a linear system in \((x_1^B, x_2^B)\), whose solution is given by (10).

Using Notation 1, simple calculus yields that for all \( u > 0 \), \( (D(1/u))' = \phi(u) u \) and \( (G \circ D(1/u))' = \phi(u) \), so differentiating (10) and simplifying, we obtain

\[
\left\{ \begin{array}{l}
\frac{\partial x_1^B}{\partial \theta_1^r} = \pi_2 \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) + \phi(\theta_1^r + \theta_2^r) \theta_1^r + \pi_1 \phi(\theta_1^r) \theta_2^r \\
\frac{\partial x_2^B}{\partial \theta_2^r} = \pi_1 \left( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) \right) + \phi(\theta_1^r + \theta_2^r) \theta_2^r
\end{array} \right.
\]

From the mean value theorem, \( G \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - G \circ D \left( \frac{1}{\theta_1^r} \right) = \theta_2^r \phi(t) \) for some \( t \in (\theta_1^r, \theta_1^r + \theta_2^r) \).

Substituting the latter equality into the above two equations, we obtain

\[
\left\{ \begin{array}{l}
\frac{\partial x_1^B}{\partial \theta_1^r} = \pi_2 \phi(t) \theta_2^r + \phi(\theta_1^r + \theta_2^r) \theta_1^r + \pi_1 \phi(\theta_1^r) \theta_2^r \\
\frac{\partial x_2^B}{\partial \theta_2^r} = \pi_1 \phi(t) \theta_2^r + \phi(\theta_1^r + \theta_2^r) \theta_2^r
\end{array} \right. \tag{16}
\]

Using (11), the welfare of \( i_c^m \) at \( x^B(\theta^r) \) is \( \theta_c^m g \circ D \left( \frac{1}{\theta_1^r + \theta_2^r} \right) - x_c^B(\theta^r) \), so

\[
\frac{\partial U_c(\theta_c^m, x^B(\theta^r))}{\partial \theta_c^r} = \theta_c^m \phi(\theta_1^r + \theta_2^r) - \frac{\partial x_c^B(\theta^r)}{\partial \theta_c^r}. \tag{17}
\]

Substituting (16) into (17), we obtain (12).
**Proof of Proposition 2.** Let \( \theta^r \) be a CEE, and assume w.l.o.g. that \( \theta^r_1 \geq \theta^r_2 \). Necessarily, \( \theta^r_1 \neq \theta^r_2 \), because as explained in the proof of Proposition 1, if \( \theta^r_1 = \theta^r_2 \), one of the two median voter can discontinuously increase its payoff at the bargaining default, and thus at the bargaining outcome by slightly undercutting the type of the other country. Therefore, \( \theta_1 > \theta^r_2 \). Moreover, \( \theta^r_1 > 0 \) because the best response to \( \theta^r_2 = 0 \) is \( \theta^m_1 \). Finally, under our assumption on \( G, \phi \) is continuous and positive on \( \mathbb{R}^+ \), so (12) yields

\[
\lim_{\theta^r_2 \to 0} \frac{\partial U_2(\theta^m_2, x^B(\theta^r))}{\partial \theta^r_2} = \theta^m_2 \phi(\theta^r_1) > 0,
\]

which implies that \( \theta^r_2 > 0 \), so from Lemma 1, \( x^B_1(\theta^r) > 0 \) and \( x^B_2(\theta^r) > 0 \).

Under the assumption of Proposition 2, country 1 also elects the high demand representative in the NEE. So the public good level in the NEE and the CEE is \( D(\theta^m_1, \theta^r_1 + \theta^r_2) \) and \( D(\theta^m_1, \theta^r_1 + \theta^r_2) \), respectively. Hence, the former is smaller than the latter if and only if \( \theta^m_1 \leq \theta^r_1 + \theta^r_2 \).

From Remark 1, \( i^m_1 \) is pivotal in country 1 when voting on \( \theta^r_1 \). Since \( \theta^r_1 > 0 \) and since \( \theta^r \) is a CEE, necessarily, \( \partial U_1(\theta^m_1, x^B(\theta^r)) / \partial \theta^r_1 = 0 \). From (12), this implies that

\[
\theta^r_1 - \theta^m_1 = -\frac{\pi_1 \phi(\theta^r_1) + \pi_2 \phi(t)}{\phi(\theta^r_1 + \theta^r_2)} \theta^r_2,
\]

so

\[
\theta^r_1 + \theta^r_2 - \theta^m_1 = \frac{\phi(\theta^r_1 + \theta^r_2) - \pi_1 \phi(\theta^r_1) - \pi_2 \phi(t)}{\phi(\theta^r_1 + \theta^r_2)} \theta^r_2
\]

(18)

Since \( \phi \) is positive, we see that \( \theta^r_1 + \theta^r_2 \) is greater (smaller) than \( \theta^m_1 \) if the numerator of the right hand-side of the above equation is positive (negative). Since \( \theta^r_1 < t < \theta^r_1 + \theta^r_2 \), the numerator is positive (negative) if \( \phi \) is increasing (decreasing), or equivalently, if \( p \to p^3 |D'(p)| \) is decreasing.

Since for all \( c \in \{1, 2\} , \theta^r_c > 0 , \partial U_c(\theta^m_c, x^B(\theta^r)) / \partial \theta^r_c = 0 \). Using (12), these F.O.C. yield

\[
\begin{cases}
\theta^r_1 = \theta^m_1 - \frac{\pi_2 \phi(t) + \pi_1 \phi(\theta^r_1)}{\phi(\theta^r_1 + \theta^r_2) + \pi_1 \phi(t)} \theta^m_2 \\
\theta^r_2 = \frac{\phi(\theta^r_1 + \theta^r_2)}{\phi(\theta^r_1 + \theta^r_2) + \pi_1 \phi(t)} \theta^m_2
\end{cases},
\]

(19)

which shows that median voters elect lower types than themselves. \( \blacksquare \)

The following remark justifies the assumption in Proposition 2 that the same country elects the high demand representative in the NEE and in the CEE.
Remark 2 When $\pi_1$ and $\pi_2$ are sufficiently close to 1/2, in any CEE $\theta^r$, the high demand median voter elects the high demand representative. That is, $\theta^m_1 > \theta^m_2$ implies $\theta^r_1 > \theta^r_2$. Moreover, there also exists a NEE in which the high demand median voter elects the high demand representative. This is the case for all NEE and all CEE when $\theta^m_1$ is sufficiently small or large relative to $\theta^m_2$.

Proof. Let $\theta^r$ be a CEE. As argued in the proof of Proposition 2, w.l.o.g., we can assume that $\theta^r_1 > \theta^r_2$. From (19),

$$\frac{\theta^m_1}{\theta^m_2} = \frac{(\phi(\theta^r_1 + \theta^r_2) + \pi_1 \phi(t)) \theta^m_1 - (\pi_2 \phi(t) + \pi_1 \phi(\theta^r_1)) \theta^m_2}{\phi(\theta^r_1 + \theta^r_2) \theta^m_2}.$$  

Substituting $\theta^r_1 > \theta^r_2$ in the above equality, we obtain

$$\frac{\theta^m_1}{\theta^m_2} > \frac{\phi(\theta^r_1 + \theta^r_2) + \pi_2 \phi(t) + \pi_1 \phi(\theta^r_1)}{\phi(\theta^r_1 + \theta^r_2) + \pi_1 \phi(t)} > 1 + \frac{\pi_1 \phi(\theta^r_1)}{\phi(\theta^r_1 + \theta^r_2) + \pi_1 \phi(t)} + \min \left(0, \frac{\pi_2 - \pi_1}{\pi_1} \right).$$

The fraction on the right-hand-side of the above inequality is bounded away from 0 in a neighborhood of $\pi_1 = 1/2$. This implies that when $\pi_1$ and $\pi_2$ are sufficiently close to 1/2, $\theta^m_1 > \theta^m_2$, and from Proposition 1, there exists a NEE in which country 1 elects the high demand representative. When furthermore $\theta^m_1$ is sufficiently large relative to $\theta^m_2$, from proposition 1, in all NEE, country 1 elects the high demand representative. ■

Proof of Proposition 3. As in the pure public good model, we renormalize w.l.o.g. the utility function in (5) as follows:

$$U_{i,c}(x) = \theta_{i,c} (G_c(x_c) + \beta_{c,c} G_c(x_{c'})) - x_c.$$

Step 1: For all $\theta^r_2 \geq 0$, if $\hat{\theta}^m_1$ is preferred by $i^m_1$ to any other $\theta^r_1$ given $\theta^r_2$, then $\hat{\theta}^r_1 > 0$.

For all $\theta^r_2 \geq 0$, $x^B(0, \theta^r_2) = x^N(0, \theta^r_2)$. Since $x^N_1$ and $x^N_2$ depend only on $\theta^r_1$ and $\theta^r_2$, respectively, by definition of $x^N$, $i^m_1$ prefers $x^N(\theta^m_1, \theta^r_2)$ to $x^N(0, \theta^r_2)$, and by definition of $x^N$, $i^m_1$ prefers $x^B(\theta^m_1, \theta^r_2)$ to $x^N(\theta^m_1, \theta^r_2)$. By transitivity, $i^m_1$ prefers $x^B(\theta^m_1, \theta^r_2)$ to $x^B(0, \theta^r_2)$, so $\theta^r_1 = 0$ cannot be a best response to $\theta^r_2$ for $i^m_1$.

Step 2: For all $\theta^r_2 \geq 0$, if $\hat{\theta}^r_1$ is majority preferred in country 1 to any other $\theta^r_1$, then $x^B(\hat{\theta}^r_1, \theta^r_2)$ is greater (smaller) than $x^N(\theta^m)$ if $\frac{\partial^2 B_{x_1 x_2}}{\partial x_1 \partial x_2} = \frac{1}{(\theta^r_1)^2} \frac{\partial^2 B_{x_2}}{\partial x_2 \partial \theta_1}$ evaluated at $\theta^r = (\hat{\theta}^r_1, \theta^r_2)$ and $x = x^B(\theta^r_1)$ is positive (negative).
Remark 1 still holds in this environment, so the majority preferences in country 1 coincide with the preferences of $i_1^m$. From step 1, $\bar{\theta}_1^r > 0$, so

$$\frac{\partial [i_1^m(x_1^B(x_1^r), x_1^r)]}{\partial \bar{\theta}_1^r} (\bar{\theta}_1^r, \theta_2^r) = 0,$$

or equivalently

$$\bar{\theta}_1^m \bar{G}_1'(x_1^B(\bar{\theta}_1^r, \theta_2^r)) = 1 - \frac{\partial x_2^B}{\partial \bar{\theta}_1^r} (\bar{\theta}_1^r, \theta_2^r) \bar{\theta}_1^m \beta_{2,1} G_2'(x_2^B(\bar{\theta}_1^r, \theta_2^r)).$$

Comparing the above equation with (6), we see that since $\bar{G}_1'$ is decreasing, $x_1^B(\bar{\theta}_1^r, \theta_2^r) - x_1^N(\theta^m)$ is of the same sign as $\frac{\partial x_2^B}{\partial \bar{\theta}_1^r} (\bar{\theta}_1^r, \theta_2^r) / \frac{\partial \bar{\theta}_1^r}{\partial \bar{\theta}_1^r} (\bar{\theta}_1^r, \theta_2^r)$. Since $x_1^B(\theta^r)$ is increasing in $\theta_1^r$, this shows that $x_1^B(\bar{\theta}_1^r, \theta_2^r) - x_1^N(\theta^m)$ is of the same sign as $\frac{\partial x_2^B}{\partial \bar{\theta}_1^r} (\bar{\theta}_1^r, \theta_2^r)$. Differentiating the F.O.C. $\frac{\partial B}{\partial \bar{\theta}_1^r} (\theta^r, x_1^B(\theta^r)) = 0$ w.r.t. $\theta_1^r$, we obtain that at $(\theta^r, x_1^B(\theta^r))$,

$$\left[ \frac{\partial^2 B}{(\partial x_1)^2 (\partial x_2)} - \left( \frac{\partial^2 B}{\partial x_1 \partial x_2} \right)^2 \right] \frac{\partial x_2^B}{\partial \theta_1^r} = \frac{\partial^2 B}{\partial x_1 \partial x_2} \frac{\partial^2 B}{\partial x_1 \partial \theta_1^r} - \frac{\partial^2 B}{(\partial x_1)^2} \frac{\partial^2 B}{\partial x_2 \partial \theta_1^r}. \quad (20)$$

Since $B$ is strictly concave, the term in bracket on the left-hand side of (20) is positive, as needed.

**Step 3:** for all $\theta^r \in \mathbb{R}^2_{++}$, $\frac{\partial^2 B}{\partial x_1 \partial \bar{\theta}_1^r} - \frac{\partial^2 B}{(\partial x_1)^2} \frac{\partial^2 B}{\partial x_2 \partial \theta_1^r}$ evaluated at $(\theta^r, x_1^B(\theta^r))$ is positive (negative) if $u_\phi(u)$ is increasing (decreasing) in $u$.

Using Notation 1, for all $x \in \mathbb{R}^N_+$ and $c \in \{1, 2\}$, if $c'$ denote the other country,

$$\Delta_c(\theta^r, x) = \theta^r_c \left( G_c(x_c) + \beta_{c,c'} G_{c'}(x_{c'}) \right) - x_c$$

$$-\theta^r_c \left( G_c \circ D_c \left( \frac{1}{\theta^r_c} \right) + \beta_{c,c'} G_{c'} \circ D_{c'} \left( \frac{1}{\beta_{c,c'} \theta^r_{c'}} \right) \right) + D_c \left( \frac{1}{\theta^r_c} \right),$$

so

$$\frac{\partial \Delta_c}{\partial x_c} = \theta^r_c G'_{c'}(x_{c'}) - 1 \quad \text{and} \quad \frac{\partial \Delta_c}{\partial x_{c'}} = \theta^r_c \beta_{c,c'} G'_{c'}(x_{c'}), \quad (21)$$

and

$$\begin{cases} 
\frac{\partial B}{\partial x_1} = \pi_1 \frac{\theta^r_1 G'(x_1) - 1}{\Delta_1(\theta^r, x)} + \pi_2 \frac{\theta^r_2 \beta_{1,2} G_1'(x_1)}{\Delta_2(\theta^r, x)} \\
\frac{\partial B}{\partial x_2} = \pi_1 \frac{\theta^r_1 \beta_{2,1} G_2'(x_2)}{\Delta_1(\theta^r, x)} + \pi_2 \frac{\theta^r_2 G_2'(x_2) - 1}{\Delta_2(\theta^r, x)}.
\end{cases} \quad (22)$$

Using (22), the F.O.C. $\frac{\partial B}{\partial x} = 0$ that characterizes $x^B(\theta^r)$ is:

$$\begin{cases} 
\pi_1 \frac{1 - \theta^r_1 G_1'(x_1)}{\Delta_1(\theta^r, x)} = \pi_2 \frac{\theta^r_2 \beta_{1,2} G_1'(x_1)}{\Delta_2(\theta^r, x)} \\
\pi_1 \frac{\theta^r_1 \beta_{2,1} G_2'(x_2)}{\Delta_1(\theta^r, x)} = \pi_2 \frac{1 - \theta^r_2 G_2'(x_2)}{\Delta_2(\theta^r, x)}.
\end{cases} \quad (23)$$

\(^{14}\text{Since it is quite intuitive, the formal proof of this comparative statics is omitted for the sake of brevity.}

It is available from the author upon request.
Differentiating (22) and using (21), we obtain
\[
\frac{\partial^2 B}{\partial x_1 \partial x_2} = \frac{\pi_1 \theta_1^2 \beta_2 G'_2 (x_2) (1 - \theta_1 G'_1 (x_1))}{(\Delta_1 (\theta^r, x))^2} + \frac{\pi_2 \theta_2^2 \beta_1 G'_1 (x_1) (1 - \theta_2 G'_2 (x_2))}{(\Delta_2 (\theta^r, x))^2}.
\]
Using (23), the above expressions evaluated at \((\theta^r, x^B (\theta^r))\) becomes
\[
\frac{\partial^2 B}{\partial x_1 \partial x_2} = \frac{\pi_1 \theta_1^2 \beta_2 G'_2 (x_2) (1 - \theta_1 G'_1 (x_1))}{(\Delta_1 (\theta^r, x))^2},
\]

Differentiating (22) and using (21) and \(\frac{\partial \Delta_2}{\partial \theta^r_1} (\theta^r, x)\), we obtain
\[
\frac{\partial^2 B}{\partial x_2 \partial \theta^r_1} = \frac{\beta_2 G'_2 (x_2) (\Delta_1 (\theta^r, x) - \theta_1 \frac{\partial \Delta_1}{\partial \theta^r_1} (\theta^r, x))}{(\Delta_1 (\theta^r, x))^2} - \frac{\pi_2 (1 - \theta_2 G'_2 (x_2)) \theta_2^2 \beta_1 \phi_1 (\theta^r_1)}{(\Delta_2 (\theta^r, x))^2}
\]
Using (23) and \(\Delta_1 (\theta^r, x) - \theta_1 \frac{\partial \Delta_1}{\partial \theta^r_1} (\theta^r, x) = D_1 \left( \frac{1}{\theta^r_1} \right) - x_1\), the above expressions evaluated at \((\theta^r, x^B (\theta^r))\) becomes
\[
\frac{\partial^2 B}{\partial x_2 \partial \theta^r_1} = \frac{\beta_2 G'_2 (x_2) (D_1 \left( \frac{1}{\theta^r_1} \right) - x_1)}{(\Delta_1 (\theta^r, x))^2} - \frac{\pi_2 (1 - \theta_2 G'_2 (x_2)) \theta_2^2 \beta_1 \phi_1 (\theta^r_1)}{(\Delta_2 (\theta^r, x))^2}.
\]
From (25),
\[
\frac{\pi_1^2 \phi_1 (\theta^r_1) (1 - \theta^r_1 G'_1 (x_1))^2}{\pi_2 G'_1 (x_1) (\Delta_1 (\theta^r, x))^2} = \frac{\pi_1 \frac{\Delta_1 (\theta^r, x) - \theta_1 \frac{\partial \Delta_1}{\partial \theta^r_1} (\theta^r, x)}{(\Delta_1 (\theta^r, x))^2} (1 - \theta_1 G'_1 (x_1))}{(\Delta_1 (\theta^r, x))^2} - \frac{1 - \theta_1 G'_1 (x_1)}{(\Delta_1 (\theta^r, x))^2} \frac{\partial^2 B}{\partial x_2 \partial \theta^r_1}
\]
Differentiating (22) and using (21), we obtain
\[
\frac{\partial^2 B}{(\partial x_1)^2} = \frac{\pi_1 \theta_1 G''_1 (x_1)}{G'_1 (x_1) \Delta_1 (\theta^r, x)} - \frac{(1 - \theta_1 G'_1 (x_1))^2}{(\Delta_1 (\theta^r, x))^2} + \frac{\pi_2 \theta_2 \beta_2 G''_2 (x_2)}{(\Delta_2 (\theta^r, x))^2}
\]
Using successively (23) and (24), the above expression evaluated at \((\theta^r, x^B (\theta^r))\) becomes
\[
\frac{\partial^2 B}{(\partial x_1)^2} = \frac{\pi_1 G''_1 (x_1)}{G'_1 (x_1) \Delta_1 (\theta^r, x)} - \frac{1 - \theta_1 G'_1 (x_1)}{(\Delta_1 (\theta^r, x))^2} \frac{\partial^2 B}{\partial x_2 \partial \theta^r_1}.
\]
Differentiating (22) and using \(\frac{\partial \Delta_2}{\partial \theta^r_1} (\theta^r, x)\), we get
\[
\frac{\partial^2 B}{\partial x_1 \partial \theta^r_1} = \frac{\pi_1 G'_1 (x_1) \Delta_1 (\theta^r, x) + (1 - \theta_1 G'_1 (x_1)) \frac{\partial \Delta_1}{\partial \theta^r_1} (\theta^r, x)}{(\Delta_1 (\theta^r, x))^2} + \frac{\pi_2 \theta_2 \beta_2 G'_1 (x_1) \theta_2 \beta_2 \phi (\theta^r_1)}{(\Delta_2 (\theta^r, x))^2}.
\]
Using successively (23) and (26), the above expression evaluated at \((\theta^r, x^B(\theta^r))\) becomes

\[
\frac{\partial^2 B}{\partial x_1 \partial \theta_1^r} = \pi_1 \frac{G'_1(x_1) \Delta_1(\theta^r, x) + (1 - \theta_1^r G'_1(x_1)) \frac{\partial \Delta}{\partial \theta_1}(\theta^r, x)}{(\Delta_1(\theta^r, x))^2} + \frac{\pi_2^2 \phi(\theta_1^r) (1 - \theta_1^r G'_1(x_1))^2}{\pi_2 G'_1(x_1)(\Delta_1(\theta^r, x))^2}
\]

\[
= \frac{\pi_1}{\Delta_1(\theta^r, x) \theta_1^r} - \frac{1 - \theta_1^r G'_1(x_1)}{\theta_1^r \beta_{2,1} G''(x_2) \partial x_2 \partial \theta_1^r}.
\]

Using (27) and (28), we get that at \((\theta^r, x^B(\theta^r))\),

\[
\frac{\partial^2 B}{\partial x_1 \partial x_2 \partial x_1 \partial \theta_1^r} - \frac{\partial^2 B}{\partial x_1 \partial \theta_1^r} = \frac{\pi_1 G''_1(x_1)}{G'_1(x_1)^2} \frac{(\partial^2 B)}{\Delta_1(\theta^r, x) G'_1(x_1)} \left( \frac{\partial^2 B}{\partial x_1 \partial x_2 G''_1(x_1)} - \frac{\partial^2 B}{\partial x_2 \partial \theta_1^r} \right).
\]

If we substitute (24) and (25) on the right-hand side of the above equation, after simplification, we obtain:

\[
\left( \frac{\partial^2 B}{\partial x_1 \partial x_2 \partial x_1 \partial \theta_1^r} - \frac{\partial^2 B}{\partial x_1 \partial \theta_1^r} \right) \theta^r, x^B(\theta^r)) = \frac{\pi_1^2 \beta_{2,1} G'_1(x_2) (1 - \theta_1^r G'_1(x_1)) (-G''_1(x_1))}{\pi_2 (\Delta_1(\theta^r, x))^3 G'_1(x_1)} \left( \frac{(G'_1(x_1))^2}{-G''_1(x_1)} - \frac{x_1 - D_1}{G'_1(x_1)} - \frac{1}{G'_1(x_1)} \phi_1(t)ight).
\]

Note that \(\frac{G'_1(x_1)}{-G''_1(x_1)} = \frac{1}{G'_1(x_1)} \phi_1 \left( \frac{1}{G'_1(x_1)} \right)\), and since \((D_1(1/u))' = \phi_1(u) u\), the mean value theorem implies

\[
x_1 - D_1 \left( \frac{1}{\theta_1^r} \right) = D_1 \left( G'_1(x_1) \right) - D_1 \left( \frac{1}{\theta_1^r} \right) = \left( \frac{1}{G'_1(x_1)} - \theta_1^r \right) \phi_1(t) t,
\]

for some \(t \in (\theta_1^r, 1/G'_1(x_1))\). Therefore, (29) can be simplified as follows:

\[
\frac{\partial^2 B}{\partial x_1 \partial x_2 \partial x_1 \partial \theta_1^r} - \frac{\partial^2 B}{\partial x_1 \partial \theta_1^r} = \frac{1}{G'_1(x_1)} \phi_1 \left( \frac{1}{G'_1(x_1)} \right) - \pi_2 t \phi_1(t) - \pi_1^2 \phi_1(\theta_1^r).
\]

From (23), \(\theta_1^r G'' \left( x_1^B(\theta^r) \right) < 1 \) and \(\Delta_1 \left( \theta^r, x^B(\theta^r) \right) > 0\), so the denominator on the left-hand side of (30) is positive. Therefore, the numerator has the same sign as the right-hand side of (30). Since \(\pi_1 + \pi_2 = 1 \) and \(\theta_1^r < t < 1/G'(x_1)\), this implies that \(\frac{\partial^2 B}{\partial x_1 \partial x_2 \partial x_1 \partial \theta_1^r} - \frac{\partial^2 B}{\partial x_1 \partial \theta_1^r} \) evaluated at \((\theta^r, x^B(\theta^r))\) is positive (negative) if \(u \phi_1(u)\) is increasing (decreasing), or equivalently, if \(|D_1'|(p) \times p^2|\) is decreasing (increasing), which proves step 3.

Proposition 3 follows then immediately from steps 1 to 3 and Definition 2. ■

**Proof of Proposition 4.**
If $G_1 \sim G_2 \sim \ln$, then for all $u > 0$, $u\phi_c(u) = 1$. From (30), this means that for all $\theta^r$, at $(\theta^r, x^B(\theta^r))$, $rac{\partial^2 B}{\partial x_1 \partial x_2}$ is equal to $\frac{\partial^2 B}{\partial x_1 \partial \theta_1}$, so from (20), for all $\theta_1^r > 0$, $\frac{\partial x^B_2}{\partial \theta_1^r}(\theta^r) = 0$, and $x^B_2$ is constant in $\theta_1^r$ on $\mathbb{R}_+$. By the maximum theorem, $x^B_1$ is continuous in $\theta_1^r$, and $x^B_1(\theta^r) \to 0$ ($+\infty$) as $\theta_1^r \to 0$ ($+\infty$). Hence, $\theta_1^r \to x^B_1(\theta^r)$ is onto on $\mathbb{R}_+$. Therefore, voters choosing their respective representative at the electoral stage is strategically equivalent to them choosing their respective policy directly. Together with Remark 1, this shows that $\theta^r$ is a CEE if and only if $x^B(\theta^r)$ is a Nash equilibrium of a two person game in which the median voter of each country choose their respective policies. The equilibrium of that game is $x^N(\theta^m)$, as given by (6).

**Proof of Proposition 5.** Suppose that the most efficient country is country 1. When transfers are available, country 1 produces all the public good and is compensated by a transfer from country 2. Thus, the availability of transfers amounts to let country 2 use the technology of country 1 in the policy stage of the cooperation scenario. Thus, in terms of payoff, it is equivalent to assume that transfers are not available, but in the cooperative scenario, the tax price of voters in country 2 are uniformly scaled down by $p_1/p_2$. This transformation of tax prices does not apply in the NEE and in the bargaining default of the CEE, but in these two cases, since country 1 provides all the public good (as in Proposition 2, we assume that the same country elects the high demand representative in the CEE and the NEE), the outcome does not depend on the tax prices in country 2. Therefore, the outcome of the NEE and the CEE is the same as if transfers were not available but the tax price of the voters of country 2 were scaled down by $p_1/p_2$ in all cases. This implies that (19) is still valid, since $\theta^m_2$ does not appear in it. As argued in the proof of Proposition 2, (19) implies that public good provision is greater (smaller) in the CEE than in the NEE if $|D'(p)|p^3$ is decreasing (increasing).

**Proof of Proposition 6.** The Nash bargaining solution $x^B(\theta^r)$ is Pareto efficient. Moreover, from (14), when $\pi_1 = \pi_2$, $\Delta_1(\theta^r, x^B(\theta^r)) = \Delta_2(\theta^r, x^B(\theta^r))$. Therefore, $x^B(\theta^r)$ coincides with $x^E(\theta)$ for that particular $\pi$. To conclude, note that Proposition 2 does not depend on $\pi$. $\blacksquare$
Proof of Proposition 7. Maximizing (7) yields

\[ x^U (\theta^r) = \begin{cases} 
  \left( D \left( \frac{\pi_1}{\pi_1 + \pi_2} \right), 0 \right) & \text{if } \pi_1 < \pi_2 \\
  \left( 0, D \left( \frac{\pi_2}{\pi_1 + \pi_2} \right) \right) & \text{if } \pi_1 > \pi_2 
\end{cases}, \tag{31} \]

and a non unique solution for \( \pi_1 = \pi_2 \). From (31), if \( \pi_1 > \pi_2 \) is constant in \( \theta_1^r \) while \( x^U_2 (\theta^r) \) is strictly increasing in \( \theta_1^r \). Therefore, the welfare of \( r^m \) induced by \( x^U (\theta^r) \) is strictly increasing in \( \theta_1^r \), so \( r^m \) does not have a best response. A symmetric argument holds for \( r^m_2 \) when \( \pi_1 < \pi_2 \). \( \blacksquare \)

Proof of Proposition 8. Let \( m_c \) denote the population of each country \( c \in \{1, 2\} \) and \( p_c \) its average tax price. Under the hypothesis of the Proposition, \( m_1 p_1 = m_2 p_2 = m_1 p^m_1 \), so the level of utilitarian welfare at a policy vector \( x \) is given by

\[ S (x) = (m_1 + m_2) G (x_1 + x_2) - m_1 p_1 x_1 - m_2 p_2 x_2 = (m_1 + m_2) G (g) - m_1 p^m_1 (g), \]

where \( g = x_1 + x_2 \). Let \( g^* = D \left( \frac{m_1}{m_1 + m_2} p^m_1 \right) \) denote the socially optimal level of public good. Suppose to fix ideas that country 1 elects the high-demand representative in the NEE. Then in the NEE, the level of public good is \( g^{NEE} = D (p^m_1) \), which is smaller than \( g^* \). From (11), in any CEE \( p^r \), the level of public good is \( g^{CEE} = D \left( \frac{p^r_1 p^r_2}{p^r_1 + p^r_2} \right) \), and from Proposition 2, \( p^r_1 > p^m_1 \) and \( p^r_2 > p^m_2 = \frac{m_1}{m_2} p^m_1 \). Therefore, \( \frac{p^r_1 p^r_2}{p^r_1 + p^r_2} > \frac{m_1}{m_2 + m_1} p^m_1 \), and so \( g^{CEE} < g^* \). To conclude the proof, note that the function \( g \to (m_1 + m_2) G (g) - m_1 p^m_1 g \) is concave, with a maximum at \( g^* \), so \( S (g^{NEE}) \leq S (g^{CEE}) \) if and only if \( g^{NEE} \leq g^{CEE} \). \( \blacksquare \)

Proof of Proposition 9. One can readily check that all the steps in the proof of Lemma 1, and Propositions 1 and 2 hold unchanged with \( G \) replaced by \( B \) and \( D \) replaced by \( S \). So if \( \theta^r \) is a CEE such that \( \theta_1^r \leq \theta_2^r \) (the proof in the other case is identical), the level of public bad in the NEE and the CEE are \( S \left( \frac{1}{\theta_1^r} \right) \) and \( S \left( \frac{1}{\theta_1^r + \theta_2^r} \right) \), respectively. Hence, the latter is smaller than the former if and only if \( \theta_1^r + \theta_2^r \geq \theta_1^m \). The formula (18) is still valid. However, since \( \phi (t) = \frac{-1}{B''(B')^{-1}(\frac{1}{2})} t^3 \), \( \phi \) is negative, so (18) implies that \( \theta_1^r + \theta_2^r \) is greater (smaller) than \( \theta_1^m \) if \( |\phi| \) is increasing (decreasing), or equivalently if \( S' (p) \times p^3 \) is decreasing (increasing). \( \blacksquare \)

The following remark shows that, as claimed in Section 6.4, the symmetric equilibria characterized in Buchholz et. al. (2005) typically do not exist when spillovers are large.

34
Remark 3 Consider the same model as in Section 6.4 with partial and symmetric spillovers and transfers, as in Buchholz et al. (2005). So the utility of a voter with type \( \theta_{i,c} \) is given by:

\[
U_c(\theta_{i,c}) = -\theta_{i,c}B(x_1 + \beta x_2) - x_c + t_c,
\]

where \( \beta \in (0, 1) \), \( t_c \) is the transfer received by country \( c \), with \( t_1 + t_2 = 0 \), and \( \pi_1 = \pi_2 \). Suppose furthermore that \( S \equiv (B')^{-1} \) does not increase exponentially, that is, \( \limsup_{p \to \infty} \frac{S(2p)}{S(p)} < \infty \)—which is the case for instance if \( S \) is isoelastic or if \( B(x) \) is a polynomial of any order \( n > 1 \).

Then there exists \( \bar{\beta} < 1 \) such that for all \( \beta \geq \bar{\beta} \), for every symmetric profile of preferences for the median voters \( (\theta^m, \theta^m) \), there exists no symmetric CEE.

**Proof.** Suppose by contradiction that Remark 3 is false. Then there exists a sequence \( \beta_n \to 1 \) such that for all \( n \in \mathbb{N} \), \( \beta_n < 1 \) and there exists a symmetric CEE for \( \beta = \beta_n \), which we denote \( (\theta(\beta), \theta(\beta)) \).

As shown in Buchholz et al (2005), the CEE \( (\theta(\beta), \theta(\beta)) \) must satisfy their first order condition (12). As shown in the proof of their Proposition 4, this implies that \( \theta(\beta) \leq (1 - \beta^2) \theta^m \), and thus that \( \theta(\beta_n) \to 0 \). At a symmetric CEE, transfers are equal to 0 and the policy vector must satisfy their first order condition (6), which yields

\[
x^B(\theta(\beta), \theta(\beta)) = \frac{1}{1 + \beta} \left( S \left( \frac{1}{(1 + \beta) \theta(\beta)} \right), S \left( \frac{1}{(1 + \beta) \theta(\beta)} \right) \right).
\]

Therefore, the welfare of \( i^m_1 \) in the CEE is

\[
U_1(\theta^m, x^B(\theta(\beta), \theta(\beta))) = -\theta^m B \left( S \left( \frac{1}{(1 + \beta) \theta(\beta)} \right) \right) + \frac{1}{1 + \beta} S \left( \frac{1}{(1 + \beta) \theta(\beta)} \right) \geq -\theta^m B \left( S \left( \frac{1}{2 \theta(\beta)} \right) \right). \tag{32}
\]

Let \( \theta^d > 0 \) and consider the deviation from \( \theta^c_1 = \theta(\beta) \) to \( \theta^c_1 = \theta^d \) for \( i^m_1 \), while keeping \( \theta^c_2 = \theta(\beta) \). For \( \beta \) close to 1, \( \theta(\beta) \) is negligible relative to \( \theta^d \), so the noncooperative policy equilibrium with these representatives is given by \( x^N(\theta^d, \theta(\beta)) = (0, S(1/\theta(\beta))) \). Since utils are transferable, the bargaining equilibrium with these representatives \( x^B(\theta^d, \theta(\beta)) \) must maximizes the sum of the representatives’ utility. That is, \( x^B(\theta^d, \theta(\beta)) \) must solve

\[
\max_{x_1 \geq 0, x_2 \geq 0} -\theta^d B(x_1 + \beta x_2) - \theta(\beta) B(x_2 + \beta x_1) + x_1 + x_2.
\]
Note that the solution \( x^B \) to this program is unique, and is such that \( x_1^B \leq (B')^{-1} (1/\theta^d) \) and \( x_2^B \leq S \left( 1/ (\beta_0 \theta^d) \right) / \beta \), so \( x^B (\theta^d, \theta (\beta)) \) is bounded as \( \beta \to 1 \). Since \( \pi_1 = \pi_2 \), the transfers \( t_1 (\theta^d, \theta (\beta)) \) and \( t_2 (\theta^d, \theta (\beta)) \) must equalize the gains from cooperation for the representatives, so

\[
U_1 (\theta^d, x^B (\theta^d, \theta (\beta))) + t_1 (\theta^d, \theta (\beta)) - U_1 (\theta^d, x^N (\theta^d, \theta (\beta)))
= U_2 (\theta (\beta), x^B (\theta^d, \theta (\beta))) + t_2 (\theta^d, \theta (\beta)) - U_2 (\theta (\beta), x^N (\theta^d, \theta (\beta))).
\]

From what precedes, \( U_1 (\theta^d, x^B (\theta^d, \theta (\beta))) \) and \( U_2 (\theta (\beta), x^B (\theta^d, \theta (\beta))) \) are bounded, and since \( t_1 (\theta^d, \theta (\beta)) = -t_2 (\theta^d, \theta (\beta)) \), the above equality implies that

\[
t_1 (\theta^d, \theta (\beta)) = U_1 (\theta^d, x^N (\theta^d, \theta (\beta))) - U_2 (\theta (\beta), x^N (\theta^d, \theta (\beta))) + O (1)
= -\theta^d B \left( \beta S \left( \frac{1}{\theta (\beta)} \right) \right) + \theta (\beta) B \left( S \left( \frac{1}{\theta (\beta)} \right) \right) - S \left( \frac{1}{\theta (\beta)} \right) + O (1)
\geq -\theta^d B \left( S \left( \frac{1}{\theta (\beta)} \right) \right) - S \left( \frac{1}{\theta (\beta)} \right) + O (1).
\]

Therefore, the welfare of \( i^m \) induced by \( x^B (\theta^d, \theta (\beta)) \) and \( t_1 (\theta^d, \theta (\beta)) \) is such that

\[
U_1 (\theta^m, x^B (\theta^d, \theta (\beta))) + t_1 (\theta^d, \theta (\beta)) \geq -\theta^d B \left( S \left( \frac{1}{\theta (\beta)} \right) \right) - S \left( \frac{1}{\theta (\beta)} \right) + O (1). \tag{33}
\]

By comparing (32) with (33), we see that the deviation from \( \theta^r = (\theta (\beta_n), \theta (\beta_n)) \) to \( \theta^r = (\theta^d, \theta (\beta_n)) \) is profitable to \( i^m \) for \( n \) sufficiently large if

\[
\liminf_{n \to \infty} \frac{\theta^d B \left( S \left( \frac{1}{\theta (\beta_n)} \right) \right) + S \left( \frac{1}{\theta (\beta_n)} \right)}{\theta^m B \left( S \left( \frac{1}{2 \theta (\beta_n)} \right) \right)} < 1. \tag{34}
\]

Since \( \lim_{x \to +\infty} B' (x) = +\infty, x \) is negligible relative to \( B (x) \) as \( x \) tends to \( \infty \). Since \( \lim_{t \to 0} S \left( 1/ t \right) = \infty \), this implies that the second term on the numerator of (34) is negligible relative to the first term as \( n \to \infty \). Therefore, if \( B \) is such that

\[
\limsup_{p \to -\infty} \frac{B \left( S \left( 2p \right) \right)}{B \left( S \left( p \right) \right)} < \infty, \tag{35}
\]

then (34) holds for some \( \theta^d > 0 \), so \( (\theta (\beta_n), \theta (\beta_n)) \) cannot be a CEE for \( n \) sufficiently large.

To conclude the proof, we show that (35) is satisfied whenever \( S (p) \) does not increase exponentially. Note first that since \( S' \) is positive and increasing,

\[
\frac{p}{2} \left( S (p) - S (0) \right) = \frac{p}{2} \int_0^p S' (q) \, dq \leq \int_0^p q S' (q) \, dq \leq p \int_0^p S' (q) \, dq = p \left( S (p) - S (0) \right).
\]
Since \( (B(S(p)))' = pS'(p) \), \( B(S(p)) = B(0) + \int_0^p qS'(q) \, dq \), so
\[
\frac{B(S(2p))}{B(S(p))} \leq \frac{B(0) + 2p(S(2p) - S(0))}{B(0) + \frac{p}{2} (S(p) - S(0))}.
\]
The right hand-side of the above inequality is bounded when \( S(2p)/S(p) \) is bounded, as needed. Finally, note that if \( B(x) \) is a polynomial of order \( n > 1 \), then \( S(p) \sim p^n/(n+1) \), so \( S(2p)/S(p) \to_{p \to \infty} 2^{n/(n+1)} \), and if \( S \) is isoeelastic, then \( S(2p)/S(p) = 2^c \). □

**Remark 4** Consider the model with tax distortion discussed in Section 6.5 in which for all \( g \geq 0 \), \( T(g) = g + \alpha g^2 \). If \( G \) is such that \( |D'(p)| \) is increasing in \( p \), then \( (G^T) = G \circ T^{-1} \) is such that \( |(D^T)'(p)| \) is increasing in \( p \).

**Proof.** Since \( D = (G')^{-1} \), simple calculus shows that \( p \to |D'(p)| \) is increasing if and only if \(-G''/(G')^2 \) is increasing if and only if \( 1/G' \) is convex. Elementary algebra yields that for all \( t \geq 0 \), \( T^{-1}(t) = (-1 + \sqrt{1 + 4\alpha t}) / 2\alpha \), so \( (G^T)(t) = G((-1 + \sqrt{1 + 4\alpha t}) / 2\alpha) \) and therefore,
\[
-\frac{(G^T)''(t)}{((G^T)'(t))^2} = -\frac{G''(-1 + \sqrt{1 + 4\alpha t})}{(G'(-1 + \sqrt{1 + 4\alpha t}))^2} + \frac{2\alpha}{\sqrt{1 + 4\alpha t}} \frac{1}{G'(-1 + \sqrt{1 + 4\alpha t})}.
\]
From what precedes, \(-G''/(G')^2 \) is increasing, so the first term on the right-hand side of the above equation is increasing in \( t \). Thus, \(- (G^T)'' / (G^T)' \) is increasing whenever the second term is also increasing. Using the change of variable \( g = \frac{-1 + \sqrt{1 + 4\alpha t}}{2\alpha} \), the second term is equal to \( \frac{1}{g^{(g+1/2\alpha)G'(g)}} \). Since \( \frac{g}{g+1/2\alpha} \) is increasing, \( (g+1/2\alpha)G'(g) \) is increasing if \( \frac{1}{gG'(g)} \) is increasing. From what precedes, \( \frac{1}{gG'(g)} \) is convex, so the slope \( \left( \frac{1}{g-0} \right) \left( \frac{1}{G'(x)} - \frac{1}{G'(0)} \right) \) must be increasing, which means that \( \frac{1}{gG'(g)} \) is increasing, as needed. □

**Proof of Proposition 11.** Since the Bernoulli utility function of each voter, as given by (5), is linear in her type \( \theta_{i,c} \), the expected utility is also linear in \( \theta_{i,c} \), so Remark 1 still holds for preferences over policy lotteries. Therefore, as for pure CEE, the mixed CEE are the mixed Nash equilibria of the two-player game in which the median voter of each country \( c \) controls the type \( \theta_c^r \) of her representative and her payoff is \( U_c \left( \theta_c^m, x^B(\theta^r) \right) \). One can clearly restrict the strategy space of each median voter to a compact subset of \( \mathbb{R}_+ \). Moreover, since \( U_c(\theta_c^r, x) \) is strictly concave in \( x \) for all \( \theta_c^r, x^B(\theta^r) \) is unique, and from Berge’s theorem, it is
continuous in $\theta^r$. Existence of a (possibly mixed) Nash equilibrium of this delegation game follows then from the Debreu-Fan-Glicksberg theorem (see, e.g., Fudenberg and Tirole 1991 p. 35).

Let $(\mu_1, \mu_2)$ be a mixed strategy CEE, and let $\Pi_1 (\theta_1^r, \mu_2)$ denote the expected payoff of $i_1^m$ for a given representative’s type $\theta_1^r$ and a given distribution $\mu_2$ of the representative’s type in country 2. For all $\bar{\theta}_1^r$ in the support of $\mu_1$, $\frac{\partial \Pi_1}{\partial \theta_1^r} (\bar{\theta}_1^r, \mu_2) = 0$, so

$$\int \left[ \theta_1^m G'_1 \left( x_1^B \left( \bar{\theta}_1^r, \bar{\theta}_2^r \right) \right) + \frac{\partial x_2^B}{\partial \theta_1^r} (\bar{\theta}_1^r, \bar{\theta}_2^r) \theta_1^m \beta_{2,1} G'_2 \left( x_2^B \left( \bar{\theta}_1^r, \bar{\theta}_2^r \right) \right) - \frac{\partial x_1^B}{\partial \theta_1^r} (\bar{\theta}_1^r, \bar{\theta}_2^r) \right] d\mu_2 (\bar{\theta}_2^r) = 0.$$ 

The above equation implies that for some $\bar{\theta}_2^r$ in the support of $\mu_2$, the integrand must be nonpositive, and since $\partial x_1^B / \partial \theta_1^r > 0$,

$$G'_1 \left( x_1^B \left( (\bar{\theta}_1^r, \bar{\theta}_2^r) \right) \right) \leq \frac{1}{\theta_1^m} - \frac{\partial x_2^B}{\partial \theta_1^r} (\bar{\theta}_1^r, \bar{\theta}_2^r) \theta_1^m \beta_{2,1} G'_2 \left( x_2^B \left( \bar{\theta}_1^r, \bar{\theta}_2^r \right) \right).$$

If $|D_1^r (p)| \times p^2$ is decreasing, then as shown in steps 2 and 3 in the proof of Proposition 3, $\partial x_2^B / \partial \theta_1^r$ is positive, so the right-hand side of the above inequality is smaller than $1/\theta_1^m$.

Since $G'_1$ is decreasing, this implies that $x_1^B \left( \bar{\theta}_1^r, \bar{\theta}_2^r \right) \geq (G'_2)^{-1} (1/\theta_1^m) = x_1^N \left( \bar{\theta}_1^r, \bar{\theta}_2^r \right)$. The proof when $|D_1^r (p)| \times p^2$ is increasing is analogous.

**Proof of Proposition 12.** Suppose $G (g) = 2 \sqrt{g}$. Using Notation 1, for all $u > 0$, $G (D (u)) = 2u$ and $\phi (u) = 2$. Substituting these identities into Lemma 1, we obtain that for all $\theta^r \in \mathbb{R}_+^2$ such that $\theta_1^r < \theta_2^r$,

$$x^B \left( \theta^r \right) = \left( \left( \theta_1^r + \theta_2^r \right)^2 - (1 + \pi_1) \theta_2^r \right)^2.$$ 

and

$$\begin{cases} 
U_1 \left( \theta_1^m, x^B \left( \theta^r \right) \right) = 2 \theta_1^m (\theta_1^r + \theta_2^r) - (\theta_1^r + \theta_2^r + (1 + \pi_1) \theta_2^r)^2; \\
U_2 \left( \theta_2^m, x^B \left( \theta^r \right) \right) = 2 \theta_2^m (\theta_1^r + \theta_2^r - (1 + \pi_1) \theta_2^r)^2.
\end{cases} \tag{36}$$

Let $\bar{\theta}^r$ be a CEE such that $\theta_1^r > \bar{\theta}_2^r$. From (19),

$$\bar{\theta}^r = \left( \frac{\theta_1^m - \theta_1^m}{1 + \pi_1} \frac{\theta_2^m}{1 + \pi_2} \right). \tag{37}$$

Substituting (37) into (36), we obtain

$$\begin{cases} 
U_1 \left( \theta_1^m, x^B \left( \bar{\theta}^r \right) \right) = (\theta_1^m)^2 + \frac{1}{1 + \pi_1} (\theta_2^m)^2; \\
U_2 \left( \theta_2^m, x^B \left( \bar{\theta}^r \right) \right) = 2 \theta_2^m \theta_2^m - \frac{1}{1 + \pi_1} (\theta_2^m)^2.
\end{cases} \tag{38}$$
The above equation shows that the equilibrium payoff of countries 1 and 2 are decreasing and increasing in $\pi_1$, respectively, as needed.

The initial assumption $\bar{\theta}_1^r > \bar{\theta}_2^r$ is satisfied at (37) if and only if $\frac{\theta_m}{\bar{\theta}_2^r} > \frac{2}{1+\pi_1}$. In what follows, we assume that $\frac{\theta_m}{\bar{\theta}_2^r} > \frac{2}{1+\pi_1}$, and determine under which condition $\bar{\theta}_r^r$ is a CEE. Note that if $i^m_2$ has a profitable deviation $\theta^r_c$ such that $\theta^r_c = \bar{\theta}_-^r$, then for $\varepsilon > 0$ sufficiently small, $\theta^r_c - \varepsilon$ is also a profitable deviation, because the discontinuity in the bargaining default at $\theta^r_c = \bar{\theta}_-^r$ leads to an upward jump in the payoff of the voters of country $c$. So we can restrict attention to deviations such that $\theta^r_c \neq \bar{\theta}_-^r$.

From (36), for all $c \in \{1, 2\}$, $U_c (\theta^m_c, x^B (\theta^r_c))$ is concave in $\theta^r_c$ on $\{\theta^r_c \in \mathbb{R}^2_+ : \theta^r_1 > \theta^r_2\}$. Therefore, the F.O.C. of the CEE (which yields (37)) is sufficient on that domain. That is, at $\bar{\theta}_r$, there is no profitable deviation $\theta^r_1 (\theta^r_2)$ for $i^m_1 (i^m_2)$ such that $\bar{\theta}_1^r > \bar{\theta}_2^r$ ($\bar{\theta}_1^r > \bar{\theta}_2^r$).

Given $\bar{\theta}_1^r$, $i^m_2$ clearly has no incentive to choose a type $\theta^r_2$ such that $\theta^r_2 > \bar{\theta}_2^r$, because such a deviation lowers her payoff at the bargaining default (country 2 is not free-riding anymore) as compared to that of country 1, and thus lowers her payoff at the bargaining outcome. Consider now a deviation of $i^m_1$ such that $\theta^r_1 < \bar{\theta}_1^r$. The welfare of $i^m_1$ at the bargaining outcome $x^B (\theta^r_1, \bar{\theta}_2^r)$ is given by the expression symmetric to (36):

$$U_1 (\theta^m_1, x^B (\theta^r_1)) = 2\theta^m_1 (\theta^r_1 + \bar{\theta}_2^r) - (1 + \pi_2)(\theta^r_1)^2. \tag{39}$$

From (39), $U_1 (\theta^m_1, x^B (\theta^r_1, \bar{\theta}_2^r))$ is concave in $\theta^r_1$ on $[0, \bar{\theta}_2^r]$. The unconstrained maximum of the expression on the right-hand side of (39) is $\hat{\theta}_1^r = \frac{1}{1+\pi_2} \theta^m_1$. Using the assumption $\frac{\theta_m}{\bar{\theta}_2^r} > \frac{2}{1+\pi_1}$ and (37), we have

$$\hat{\theta}_1^r \geq \frac{2}{1+\pi_2} \frac{\theta^m_1}{1+\pi_1} = \frac{2}{1+\pi_2} \bar{\theta}_2^r \geq \bar{\theta}_2^r.$$  

This shows that the supremum of $U_1 (\theta^m_1, x^B (\theta^r_1, \bar{\theta}_2^r))$ w.r.t. $\theta^r_1$ on $[0, \bar{\theta}_2^r]$ is equal to the expression on the right-hand side of (39) evaluated at $\theta^r_1 = (\bar{\theta}_2^r, \bar{\theta}_2^r)$. Substituting the latter expression and (37) into (39), we get

$$\sup_{\theta^r_1 \in [0, \bar{\theta}_2^r]} U_1 (\theta^m_1, x^B (\theta^r_1, \bar{\theta}_2^r)) = \frac{4\theta^m_1 \theta^m_1}{1+\pi_1} - \frac{(1+\pi_2)(\theta^m_1)^2}{(1+\pi_1)^2}. \tag{40}$$

The comparison of (38) and (40) reveals that $i^m_1$ has no profitable deviation $\theta^r_1$ on $[0, \bar{\theta}_2^r]$ if and only if

$$3 \left( \frac{\theta^m_2}{\theta^m_1} \right)^2 - 4 (1+\pi_1) \frac{\theta^m_2}{\theta^m_1} + (1+\pi_1)^2 \geq 0.$$  

39
The left-hand side of the above inequality is a polynomial of order two in \( \frac{\theta^m}{\bar{\theta}^r} \) whose roots are \( \frac{1+\pi_1}{3} \) and \( 1 + \pi_1 \). Therefore, under our assumption that \( \frac{\theta^m}{\bar{\theta}^r} < \frac{1+\pi_1}{2} \), the above inequality is satisfied if and only if \( \frac{\theta^m}{\bar{\theta}^r} \leq \frac{1+\pi_1}{3} \).

Summing up, we have shown that if \( \bar{\theta}^r \) is a CEE such that \( \bar{\theta}_1^r > \bar{\theta}_2^r \), then \( \bar{\theta}^r \) is given by (37), and reciprocally, this strategy profile is a CEE if and only if \( \frac{\theta^m}{\bar{\theta}^r} \leq \frac{1+\pi_1}{3} \). A symmetric argument shows that there exists a CEE \( \bar{\theta}^r \) such that \( \bar{\theta}_1^r < \bar{\theta}_2^r \) if and only if \( \frac{\theta^m}{\bar{\theta}^r} \leq \frac{1+\pi_2}{3} \). As argued in the proof of Proposition 2, there is no CEE \( \bar{\theta}^r \) such that \( \bar{\theta}_1^r = \bar{\theta}_2^r \).

References


