Gridlock and Inefficient Policy Instruments

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Abstract

Policy makers often regulate the economy using inefficient rather than efficient policy instruments. For example, externalities are typically regulated by quotas or standards even if Pigou taxes could have raised revenues and led to a Pareto improvement. This paper recognizes that, in many political systems, there are multiple veto players and the current policy defines the status quo to be used in the future. Thus, even if a player benefits from the efficient policy instrument today, it is anticipated that this instrument will be particularly hard to repeal once implemented. Less interventionist players, therefore, can prefer use of a Pareto dominated instrument that is easier to repeal when appropriate. Within a dynamic political economy model that captures this intuition, we also show, *inter alia*, that both relatively more and less interventionist players may propose inefficient policy interventions in equilibrium, and that access to a Pareto dominated policy instrument can be welfare improving as it mitigates inefficiencies due to legislative gridlock.

1 Introduction

Despite the sometimes considerable differences in perspective, economists often agree on the optimal policy instrument with which to address a particular problem. A well-known example is the regulation of environmental problems. Economists from the left to the right tend to recommend Pigou taxes because they are cost-effective, require little information, and offer a "double dividend" whereby emission taxes generate public revenues that allow governments to reduce other, distortionary, taxes.[1] It is thus 'a mystery', according to some economists, why (at the time of writing) the Republican Party in the US has blocked such a market-based policy, since doing so effectively led to the quantity controls on power plants introduced by the Obama administration in 2015.[2]

The gap between economically efficient and politically feasible policy interventions can be very costly. Minimizing these costs requires identifying second-best alternatives, that is, policy recommendations that are efficient subject to being politically feasible. Therefore, as is widely appreciated in the literature,[3] it is as important to understand the political realities constraining policy choice as it is to identify (politically unconstrained) first-best economic recommendations. In this paper, we develop a parsimonious political economy theory of the choice between using an efficient and an inefficient instrument to manage political or economic shocks.

The model rests on two characteristics. First, we recognize that policy-making frequently involves multiple pivotal players. Enacting or repealing regulation requires the consent of veto players who typically have different opinions on the desirability of the policy. Second, our model is dynamic, admitting an endogenous status quo. The policy agreed upon in one period becomes the status quo in the next. Combined, these two ingredients imply that the pivotal legislator for introducing any regulation is distinct from the legislator pivotal for any subsequent repeal of that regulation. Consequently, anticipating the potential loss of

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[1]Starting with Tullock (1967), there is a large literature in economics on the double dividend. While a "strong" version of it is controversial, the "weak" version—that the revenues reduce overall distortions compared to a setting without these revenues—is generally accepted. Only the weak version is required for the argument we make here. For surveys on the literature on the double dividend, see Bovenberg (1999), Sandmo (2000), Goulder (2002), or Jorgensen et al. (2013). In part because of the double dividend, all but four of fifty one prominent economists surveyed in 2011 agreed that a carbon tax would be the less expensive way to reduce carbon-dioxide emissions. (http://www.igmchicago.org/igm-economic-experts-panel/poll-results?SurveyID=SV_9Rezb430SESUA4Y)

[2]On this 'mystery,' see: http://www.nytimes.com/2015/07/01/business/energy-environment/us-leaves-the-markets-out-in-the-fight-against-carbon-emissions.html. The famous exception is British Columbia, which introduced a carbon tax in 2008. Although initially controversial, the tax has gained support from all important stakeholders thanks to the rebates in other taxes that the revenues permit (http://www.nytimes.com/2016/03/02/business/does-a-carbon-tax-work-ask-british-columbia.html?smid=pl-share&_r=0)

[3]See, for example, Dewatripont and Roland (1995).

political influence, the pivotal legislator for implementing a policy has an incentive not to introduce the regulation in the first place. Hence, there is more gridlock and status-quo bias in the dynamic model than in the static case.

The contribution of this paper is to show how gridlock varies systematically with the choice of policy instrument. In particular, an efficient and attractive regulatory instrument is difficult to repeal, even if the regulatory problem and the need for regulation is diminished. Anticipating the persistence of efficient instruments, the most reluctantly interventionist pivotal legislator may veto an appropriate policy intervention that would have been acceptable in a static environment. In contrast, a policy instrument that is considered less attractive by everyone, and which would thus have been Pareto dominated in a static setting, is easier to repeal should circumstances change. For this reason, the most efficient instrument might not be politically feasible, yet the less efficient instrument may still be approved by all veto players. In terms of the example above with emission taxes vs. quotas, it is exactly the benefit of the tax instrument—the double dividend—that makes the instrument hard to repeal once implemented and, therefore, less likely than the quota policy to be approved today.

The theory also sheds light on the mapping from the political system to the choice of policy instrument. If decisions require super-majorities or must pass multiple legislative chambers, then there is a larger set of veto players and the ideological distance between them is widened. Gridlock and status-quo bias can be severe in these circumstances, and it is then particularly likely that only the less efficient instrument will be approved. Furthermore, the inefficient instrument is more likely to be preferred and approved if the economic environment is volatile and future policy preferences are uncertain, since it is the relative ease with which inefficient interventions today may be repealed later if warranted.

These positive results are derived and formalized below. We also describe the equilibrium dynamic paths and predict, for example, that regulation is more likely to be replaced by no regulation than by another type of instrument. While a serious empirical investigation must await future research, note that our results can also be interpreted normatively: we find, in contrast to a static setting, conditions under which all veto players are better off in a dynamic environment by including an inefficient instrument as an available policy option. Likewise, all veto players may prefer a set of policy alternatives that does not include an efficient instrument, as the mere availability of such an alternative can lead to the sorts of intertemporal concerns sketched above and gridlock.

Environmental regulation is not the only example of an economically efficient intervention being foregone in favor of an inefficient, but apparently more politically feasible, policy. For example, taxation is argued to be a more efficient at curbing systemic risk, but we tend to see non-price regulation. Similarly, responding to fiscal need, we see government resorting to

increases in distortionary taxation instead of taxing negative externalities or removing inefficient loopholes. The prevalence of inefficient instruments has proved a puzzle, especially in view of the argument that political competition should eliminate inefficiencies (Becker (1976, 1983), Wittman (1989)). Consequently, there is a literature exploring a variety of reasons for the widespread use of inefficient policies. Unlike the current paper, however, the existing explanations focus, for the most part, on characteristics of policy-making other than the institutional logic of legislative choice *per se*. An exception is Spolaore (2004) who assesses the relative efficiency of policy interventions across three stylized organizational forms in a continuous time model. Individuals differ only in their preferences over policies: "for any possible [policy], there is always an agent who prefers to use a different instrument" (p.122). So although one of the organizational forms Spolaore considers is essentially the same as the form we use, *viz.* "checks and balances" whereby one agent proposes an intervention but others have a veto, legislators' policy preferences, unlike in our model, are fixed over time and every policy is efficient. Any inefficiencies that arise in Spolaore's framework are due to costly delays in enacting some intervention, i.e. gridlock with respect to which of several efficient policies to implement at any time, rather than to a strategic choice of an inefficient instrument.

Of necessity, explanations that consider policy choice by a unitary actor, whether a single legislator or a fully coordinated party or group, invoke aspects of the political economic environment other than legislative design. Coate and Morris (1995) and Acemoglu and Robinson (2001) both offer explanations of inefficient policy choices as arising from an incumbent legislator's efforts to retain office. Coate and Morris consider an environment in which voters are uncertain about both the extent to which the legislator is biased in favor of an interest group and the net social value of a potential public project that surely benefits the interest group. They then argue that, in equilibrium, biased politicians may choose to effect an inefficient transfer to the group by implementing the project when doing so is unwarranted. Acemoglu and Robinson (2001) point out, however, that an unbiased politician who wishes to approve the project when it is warranted to do so, could perfectly reveal her type by using lump-sum taxes to redistribute the wealth back from the subsidized group. Instead, these authors propose a model in which a given group currently enjoys political influence, but is threatened by declining support, structures transfers in a way that attracts newcomers to the group. They then demonstrate that such transfers are distortionary. In both of these two papers, inefficient instruments are used by the policymaker to increase the probability of staying in power. In contrast, inefficiency in our model arises regardless of how today's policy affects tomorrow's allocation of bargaining power.

A number of other papers also look to interest group influence as a source of policy inef-

ficiency. In particular, Tullock (1993), Grossman and Helpman (1994), Becker and Mulligan (2003) and Drazen and Limao (2008) argue in various ways that any resource transfer increases wasteful lobbying (rent-seeking) activity from the group of beneficiaries, leading to excessive redistribution. By committing itself to inefficient transfers, the government lowers the level of wasteful lobbying and the effective amount of transfer. But these papers do not explain why the government can commit to an inefficient instrument but not to a predetermined level of transfers. Similarly, Aidt (2003) claims that inefficient command-and-control instruments are more bureaucracy intensive and hence, to the extent that bureaucrats influence policy design and derive value from implementing policy, such interventions are favored by bureaucrats.

Focusing on the choice between taxes and quotas, Alesina and Passarelli (2014) and Masciandaro and Passarelli (2013) argue that, in a heterogenous society, quotas and taxes have different distributional consequences. Since the decision between them is made by the median voter, who does not internalize the costs and benefits to others, the choice of the instrument may not be socially optimal. However, both taxes and quotas are Pareto optimal in these papers. In contrast, our paper rationalizes the use of a Pareto suboptimal alternative.

At a technical level, our paper relates to the growing literature on dynamic legislative bargaining with an endogenous status quo. The seminal contributions here are Baron (1996) and Kalandrakis (2004).[4] Unlike our paper, these contributions analyze models in which (induced) policy preferences do not vary over time, the impetus for policy change coming solely from shocks to the allocation of bargaining power, or the fact that the same proposer seeks the approval of different coalitions over time. In contrast, the qualitative nature of policy inefficiencies in our model is, by and large, independent of the allocation of bargaining power. More closely related to this paper, Duggan and Kalandrakis (2012), Zapal (2011), Riboni and Ruge Murcia (2014), Dziuda and Loeper (2015a,b), and Bowen, Chen, Eraslan and Zapal (2015) consider models of dynamic bargaining with shocks to the preferences. These papers show that inefficiencies can occur and that they often take the form of policy inertia. Our paper complements this literature by analyzing the choice of policy instrument and in showing that an inefficient instrument can mitigate gridlock.

The next section illustrates the mechanism in a simple two-period example comparing emission quotas and taxes. Section 3 describes the more general model, analyzed in Section 4. Section 5 relaxes the assumption that the state of nature is distributed i.i.d. over time and

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[4]Subsequent contributions include Bernheim, Rangel and Rayo (2006), Diermeier and Fong (2011), Bowen Chen and Eraslan (2014), and Anesi and Seidmann (2015). Note that while we, as well as this literature, assume that the policy does not affect the state, a different strand of literature assumes that the policy is not the status quo but that it does influence future states (see Hassler et al., 2003, for example).

discusses the consequence of increasing the magnitude of uncertainty. Section 6 concludes.

2 Example: Prices vs. Quantities

In this section we develop a simple, stylized example to illustrate the key mechanism underlying the more general model introduced later, and to preview some of our results. The mechanism requires two periods and two pivotal players, or legislators, L and R, both of whom must approve any change in environmental policy for that change to be implemented. Legislators can move away from no intervention (n) by introducing either an emission quota (q) or an emission tax, i.e., a price instrument (p). While the emission reduction gives everyone a benefit θ, w_i and $w_i + e_i$ are the costs associated with p and q, respectively, for $i \in \{L, R\}$.

To illustrate the preceding preference parameterization, suppose that a representative firm can cut emissions from two to one unit, but the firm's cost of doing so is one. Let $\pi_i > 0$ be the weight that legislator $i \in \{L, R\}$ assigns to the firm's profit; in particular, assume $\pi_R > \pi_L$ so R is more business-friendly than L. With an emission tax, the tax must be (at least) of size one to induce the firm to cut emissions. In this case, the cost to the firm is two, because the firm cuts emissions by one unit and pays taxes on the remaining unit. The tax revenue of one is valuable to the government since other taxes can be reduced accordingly. If the value of public revenues equals some $\lambda > \max_i \pi_i$, the (net) cost of the tax is $w_i = 2\pi_i - \lambda$ while the cost of the quota is $\pi_i = w_i + e_i$, where $e_i \equiv \lambda - \pi_i$, $i \in \{L, R\}$. Since $e_i > 0$, both players prefer p to q in a static setting because of the 'double dividend' associated with the tax revenues. For the same reason, policy p is more likely to be introduced than q, since it is accepted by i whenever $\theta \geq w_i$, while q is accepted only when $\theta \geq w_i + e_i$. We can immediately make some simple observations:

Proposition 0 *Suppose there is only one period.*
(i) Emissions are regulated for a larger set of benefits θ when the policy menu is $\{n, p\}$ than when the menu is $\{n, q\}$.
(ii) With menu $\{n, p, q\}$, every player proposing regulation proposes p, and never q.
(iii) Both players prefer menu $\{n, p\}$ to $\{n, q\}$, while $\{n, p\}$ and $\{n, p, q\}$ are equivalent since q is never used.

As a comparison, suppose now that there are two periods and let $\delta \geq 0$ be the common discount factor. The status quo in the first period is n and while the first-period policy becomes the status quo in the second period. Since the legislators have different preferences, there are states in which the players agree on the need for a policy, and there are states

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in which they disagree. For simplicity, consider an environment in which, in each period, $\theta \in \{\underline{\theta}, \overline{\theta}\}$, where $\theta = \overline{\theta} > w_R + e_R$ with probability $1 - \rho$ and, with probability ρ, $\theta = \underline{\theta} \in (w_L, \min\{w_L + e_L, w_R\})$. Since $e_L > 0$ and $w_R > w_L$, $\theta = \underline{\theta}$ is clearly possible. By construction, if $\theta = \overline{\theta}$ both players prefer p to q, and q to n; if $\theta = \underline{\theta}$, however, player R prefers n to p, and p to q, while L prefers p to n, and n to q. Thus, if $\theta = \underline{\theta}$ in the second period, a first-period intervention using a quota would be repealed, but a first-period intervention using the price instrument remains in effect.

If the policy menu is $\{n, q\}$, so that p is not available, then q would be agreed on when $\theta = \overline{\theta}$ and n when $\theta = \underline{\theta}$, regardless of the status quo. If instead the policy menu is $\{n, p\}$ and the first-period policy remains the status quo in the next period, then R recognizes that, by accepting p in the first period, p remains in effect regardless of the second-period state. If, however, the first-period policy is n, the two players agree on p in the second period if and only if $\theta = \overline{\theta}$. Anticipating these possibilities, R accepts p in the first period only if the benefit of such a policy today is larger than the expected cost tomorrow:

$$\overline{\theta} - w_R \geq \delta\rho(w_R - \underline{\theta}) \Rightarrow \delta \leq \widehat{\delta}_{np} \equiv \frac{\overline{\theta} - w_R}{\rho(w_R - \underline{\theta})}.$$

Thus, if the future is sufficiently important (specifically, $\delta > \widehat{\delta}_{np}$), then R does not accept p even though R would have accepted the less efficient instrument, q. In this situation, the menu $\{n, q\}$ is clearly giving both players a higher utility than the menu $\{n, p\}$.

Suppose now that the policy menu is $\{n, p, q\}$, so that all three policies are available. If $\theta = \overline{\theta}$ in the second period, both players agree on policy p. If $\theta = \underline{\theta}$ in the second period, the players cannot agree on a change in policy if the status quo is either n, where R blocks change, or p, where L blocks change. However, if the first-period policy is q, then L prefers a change to p, while R prefers a change to n. Suppose $b_L \in [0, 1]$ is the probability that L has the authority to make a take-it-or-leave-it policy proposal in the second period, and $1 - b_L$ is the probability that R has authority to make such a proposal. Then, R prefers q to n in the first period only if the benefit of q today is larger than its expected cost tomorrow:

$$\overline{\theta} - w_R - e_R \geq \delta\rho b_L(w_R - \underline{\theta}) \Rightarrow \delta \leq \widehat{\delta}_{npq} \equiv \frac{\overline{\theta} - w_R - e_R}{\rho b_L(w_R - \underline{\theta})}$$

Hence, if $\delta > \widehat{\delta}_{npq}$, R would agree to q in the first period if $\theta = \overline{\theta}$ and p were not available, but not when p is also on the policy menu. To see why, first note that, since R prefers p to q in state $\underline{\theta}$ and there are no periods beyond the second, if the second-period state is $\underline{\theta}$ then R would agree to a proposal from L to change a status quo policy of q to p. And second, given the importance of the future $(\delta > \widehat{\delta}_{npq})$, the likelihood of the event that both $\theta = \underline{\theta}$
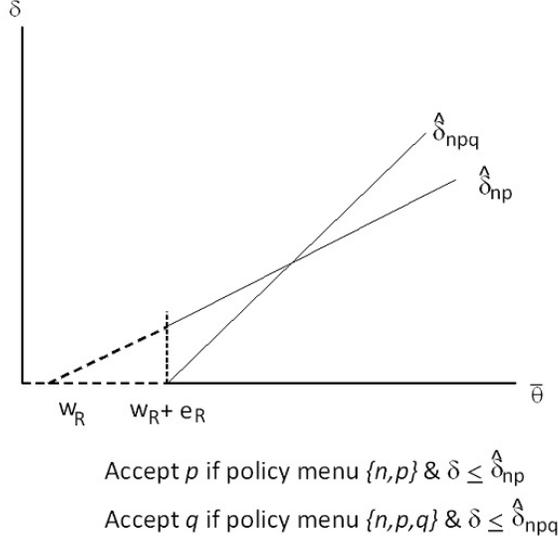
Accept *p* if policy menu *{n,p}* & $\delta \le \widehat{\delta}_{np}$

Accept *q* if policy menu *{n,p,q}* & $\delta \le \widehat{\delta}_{npq}$

Figure 1: R's period 1 acceptance sets for $\overline{\theta} > w_R + e_R$

and L has proposal power in the second period is sufficiently high, that the expected cost tomorrow of implementing q today exceeds any benefit to R of intervening at all when $\theta = \overline{\theta}$ in period one. Figure 1 illustrates $\widehat{\delta}_{npq}$ and $\widehat{\delta}_{np}$.

Summarizing the preceding discussion, we have

Proposition 1 *Suppose there are two periods and, in the first period, $\theta = \overline{\theta} > w_R + e_R$.*

(i) *Emissions are always regulated when the policy menu is $\{n, q\}$, but not when the menu is $\{n, p\}$ if $\delta > \widehat{\delta}_{np}$.*

(ii) *With menu $\{n, p, q\}$, both players always prefer and propose q rather than p if $\delta \in \left(\widehat{\delta}_{np}, \widehat{\delta}_{npq}\right)$, while no regulation can be approved if $\delta > \max\left\{\widehat{\delta}_{np}, \widehat{\delta}_{npq}\right\}$.*

(iii) *Both players prefer menu $\{n, q\}$ to $\{n, p\}$ if $\delta > \widehat{\delta}_{np}$, and both players prefer menu $\{n, q\}$ and $\{n, p, q\}$ if $\delta > \max\left\{\widehat{\delta}_{np}, \widehat{\delta}_{npq}\right\}$.*

In other words, even when every player prefers the price instrument in a one-period model, it may be politically impossible for both players to agree on using this instrument in a dynamic model since an efficient policy will be harder to repeal later on. Thus, both legislators may propose, and prefer, the less efficient emission quota. Note that it is exactly the additional *benefit* of p, the double dividend, that makes player R *unwilling* to approve it, since an attractive policy is harder to repeal. As part (iii) of the proposition states, it may not only be beneficial to include the inefficient instrument on the menu, it may also be

desirable to remove the most efficient instrument. The reason, as mentioned, is that R fears that by introducing q today, L may later pressure R to accept policy p in a situation where both R and L would have preferred n to q.

Since the length of a period (and thus the size of $1/\delta$) can be interpreted as the time it takes for θ to be drawn anew, Proposition 1 also suggests that when the state is more volatile (and changes even for short period lengths), the players are more likely to propose and prefer the quota rather than the tax.

The results above are both important and new to the literature, but they are derived in a stylized example which raises a number of questions. For instance, while instrument q is attractive today in part because the players will never use the inefficient instrument in the last period, what is the desirability of q in a dynamic model where there is no last period? Furthermore, while the example assumed that instrument p would never be repealed once implemented, why should the players agree on q in a more general model where both instruments can eventually be repealed? To answer these and other questions in depth, the following section generalizes the model to an infinite number of periods and considerably more general distribution of states.

3   A Dynamic Model of Instrument Choice

The dynamic game: Every period $t \in \mathbb{N}$ starts with some status quo $s_t \in \{n, p, q\}$. As a convenience, we assume $s_0 = n$ throughout and refer to policy n as no policy, while p and q, respectively, are the efficient and the inefficient policy instruments.[5] At the beginning of period t, both players observe the state of nature $\theta_t \in \mathbb{R}$. The realization of the state influences the desirability of an intervention in period t. The process $\{\theta_t : t \geq 0\}$ captures the evolution of the environment. For example, benefits (or costs) of intervention vary with the business cycle and the future environmental sensitivity may be unknown today. And insofar as the players in the game are interpreted as legislators, θ_t can be interpreted as voters' concern about regulation rather than the severity of the problem, per se. For simplicity, we assume $\{\theta_t : t \geq 0\}$ is distributed identically and independently over time according to some continuous cumulative distribution function F with full support on $\mathbb{R}$. Section 5 relaxes the i.i.d. assumption and permits correlation across periods.

After θ_t is observed, the players choose a new policy in the set $\{n, p, q\}$. The crucial assumption is that there are multiple veto players who must agree for a change to be enacted.

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[5]In what follows, we use the terms "policy instrument" and "intervention" interchangeably to refer exclusively to alternatives p and q. The term "policy" may refer to any of the available alternatives, including n. This looseness should cause no confusion. Also, we say that intervention p or q is "repealed" when that intervention is the status quo and players agree to replace it by n.

We refer to the most (least) interventionist player as L (R). Assume that one player can make a take-it-or-leave-it offer to the other regarding which policy intervention, if any, to implement: after θ_t is observed, player L or R is recognized as the *proposer* for period t. The recognition probability for player i is given by $b_i(\theta_t, s_t)$, perhaps dependent on θ_t and s_t. Throughout we assume that the recognition probability of each player is bounded away from zero: for all $i \in \{L, R\}$ and all (θ, s), $b_i(\theta, s) \in (\underline{b}, 1 - \underline{b})$ for some $\underline{b} > 0$. The proposer offers a policy $y_t \in \{n, p, q\}$. If the other player, the *veto-player*, accepts this proposal, then y_t is implemented. If the veto-player rejects the proposal, then the status quo s_t stays in place. The policy implemented in t, whether the proposal y_t or the status quo s_t, generates the flow payoff for that period, and becomes the status quo in $t + 1$.

Payoffs: Policy makers tend to disagree on what the policy ought to be. Even if θ measures a stochastic but common component, such as the benefit from introducing some policy, policy makers may have different costs or thresholds associated with this: we thus assume player i prefers the policy p rather than the no intervention, policy n, if and only if $\theta \geq w_i$. Assume $w_L < w_R$.

In addition to the policy instrument p, players can intervene with a less efficient instrument, q. The cost to $i \in \{L, R\}$ of this instrument is $w_i + e_i$, where every $e_i > 0$. In a static setting, therefore, i prefers q to n only when $\theta \geq w_i + e_i$ and p is always preferred to q.[6]

For every policy $x \in \{n, p, q\}$ and $\theta \in \mathbb{R}$, let $U_i(\theta, x)$ denote the state-contingent flow payoff to pivotal player $i \in \{L, R\}$ in any period with policy x and state θ. It follows from the text above that the payoffs are parametrized as follows: for every $i \in \{L, R\}$ and all $\theta \in \mathbb{R}$,

$$
\begin{aligned}
U_i(\theta, p) - U_i(\theta, n) &= \theta - w_i, \\
U_i(\theta, p) - U_i(\theta, q) &= e_i.
\end{aligned}
\tag{1}
$$

The key simplifying assumption in specification (1) is that the difference between intervening with p rather than q for player i, namely $U_i(\theta, p) - U_i(\theta, q)$, is independent of θ. The players' static preferences, as a function of θ, are illustrated in Figure 2. Each player is infinitely lived and thus interested in maximizing its expected discounted payoff over the infinite horizon. The common discount factor is $\delta \in (0, 1)$.

Equilibrium: We denote the infinite horizon game by Γ and restrict attention to stationary Markov-perfect equilibria (referred to as simply the "equilibria" in what follows). A *stationary strategy* for player $i \in \{L, R\}$ specifies, for any period t and any history to that

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[6]Despite the interpretation of p and q in the earlier emissions example, hereafter we always use p (respectively, q) to denote the efficient (respectively, inefficient) instrument, leaving any substantive meaning beyond relative efficiency dependent on the application.

$$\overset{\longleftarrow n >_L p >_L q \longrightarrow}{} \quad \overset{\leftarrow p >_L n >_L q \rightarrow}{} \quad \overset{\longleftarrow p >_L q >_L n \longrightarrow}{}$$

$$w_R \qquad\qquad w_R + e_R$$

$$\theta$$

$$w_L \qquad\qquad w_L + e_L$$

$$\overset{\longleftarrow n >_R p >_R q \longrightarrow}{} \quad \overset{\leftarrow p >_R n >_R q \rightarrow}{} \quad \overset{\leftarrow p >_R q >_R n \rightarrow}{}$$
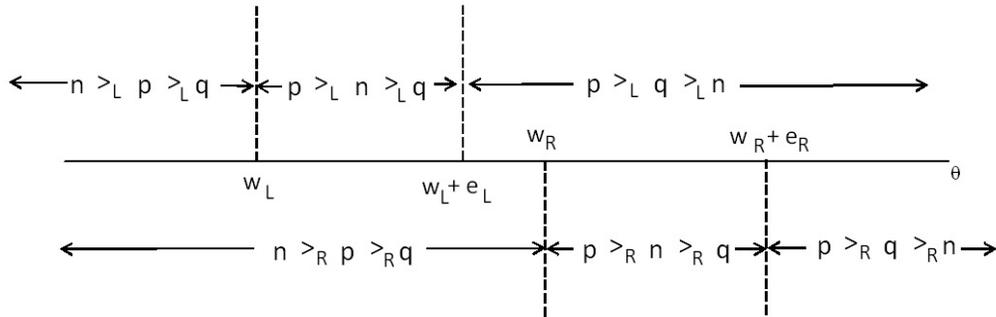
Figure 2: An example of policy preferences

period, two contingent actions. First, a policy proposal, conditional on being the proposer, that maps the realized state and status quo into a proposal; and second, conditional on not being the proposer, a veto decision that takes the realized state, the status quo and the proposal into a choice over accepting or rejecting the proposal.[7] We let σ_i denote i's stationary strategy pair (proposal, veto) and write $\sigma = (\sigma_L, \sigma_R)$.

Remarks on the assumptions: Before proceeding, it is worth discussing three features of the model, beginning with the assumption of two veto players. Multiple veto players are natural in politics. With a unicameral legislature taking decision under simple majority rule, the policy maker i with the median w_i would be the unique pivotal decision maker. However, bicameralism, supermajority requirements, or presidential veto power imply the existence of a set of veto-players, or pivotal policy makers, whose approval is necessary and sufficient to enact a policy change. In the case of a unicameral legislature taking decisions under a qualified majority $m \in \left(\frac{1}{2}, 1\right]$, player L is such that exactly a fraction m of the w_i's are larger than w_L and, for player R, exactly m of the w_i's are smaller than w_R. Thus, the degree of heterogeneity, or polarization, $w_R - w_L$, increases in the majority requirement m and player L is the more interventionist player: since $w_L < w_R$, L prefers policy p to n for a larger set of states (i.e. all states $\theta \geq w_L$) than player R.

We assume there is no transferable utility. If the players could make unlimited side-payments then only efficient instruments can be expected in equilibrium. In particular, policy p would be implemented in every period in which $\theta > (w_L + w_R)/2$, and policy n would be implemented otherwise. Although the assumption that side-payments are unavailable is predicated on the difficulty, or even legality in some polities, of enforcing such transfers, it should be seen as a theoretical limitation on the results.

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[7]Mixed strategies are admissible. More formally, writing ΔS for the set of probability distributions over a set S, i's proposal strategy takes $\mathbb{R} \times \{n, q, p\}$ into $\Delta\{n, q, p\}$; and i's veto strategy takes $\mathbb{R} \times \{n, q, p\}^2$ into $\Delta\{accept, reject\}$.

Finally, if the status quo were exogenously given at the start of every period, making any policy decision transient, the environment would be a sequence of static games. In this case, there would be a status quo bias if $\theta_t \in (w_L, w_R)$, while for larger (smaller) realizations of the state, the policy would be p (n). In this case, the inefficient policy q would neither be proposed nor agreed upon. It is less clear, however, what happens if the players could choose between an institution in which policy decisions are transient, or one in which policy decisions persist unless both players agree to a change. Party L prefers policy p as the status quo and would propose (and have accepted) a persistent policy if θ were large, while player R would propose (and have accepted) a transitory policy over time. In reality, there are many reasons for why policies may not be transitory, related to transaction costs or legal rights orthogonal to the model. In the following, therefore, we assume any chosen policy stays in place until it is actively changed.

4   Analysis

In the following subsections, we characterize equilibria in which players intervene only with the efficient instrument and equilibria in which they also intervene with the inefficient instrument; describe the conditions under which the inefficient instrument is used in all equilibria; demonstrate how political polarization is crucial for the use of inefficient instruments; and argue that both players may benefit from having the inefficient instrument available. It is useful to begin, however, with a basic property on the players' continuation payoffs for any equilibrium strategy profile.

For every stationary strategy profile σ, $i \in \{L, R\}$, $\theta \in \mathbb{R}$, and $x \in \{n, p, q\}$, let $V_i^\sigma (\theta, x)$ be the continuation value for player i of implementing policy x in some period $t \in \mathbb{N}$, conditional on $\theta_t = \theta$ and on continuation play σ.

Lemma 1 *Equilibria exist in* Γ. *Moreover, for any equilibrium strategy profile* σ, *there exists* $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \in \mathbb{R}^4$ *such that, for all* $i \in \{L, R\}$, $\theta \in \mathbb{R}$, *the continuation value function* V_i^σ *satisfies*

$$\begin{aligned} V_i^\sigma (\theta, p) - V_i^\sigma (\theta, n) &= \theta - w_i^\sigma, \\ V_i^\sigma (\theta, p) - V_i^\sigma (\theta, q) &= e_i^\sigma. \end{aligned} \tag{2}$$

All proofs are in the Appendix. By comparing (1) and (2), we see that players' continuation payoffs $V^\sigma = (V_L^\sigma, V_R^\sigma)$ have the same structure as their flow payoffs $U = (U_L, U_R)$, but with parameters $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma)$ instead of (w_L, w_R, e_L, e_R).[8] Call $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma)$ the continuation payoff parameters induced by σ. Generically, the profile $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma)$ differs from

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[8]This result relies on $\{\theta_t\}$ being i.i.d. and on the stationarity of σ as follows. If σ was nonstationary, the same expression would hold but the function V_i^σ and the parameter w_i^σ would have to be indexed by the

(w_L, w_R, e_L, e_R) because today's policy affects not only today's payoff, but also tomorrow's status quo policy. The difference between flow and continuation payoff parameters, therefore, captures players' forward-looking preferences over the next status quo.[9]

In Section 4.1, we characterize the equilibria of Γ in terms of their continuation payoff parameters, as $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma)$ (almost) uniquely determine players' behavior. In any period t, the veto-player i accepts (rejects) any proposal x for which $V_i^\sigma(\theta_t, x) > (<) V_i^\sigma(\theta_t, s_t)$. For instance, i accepts proposal p under status quo n (q) when $\theta_t > w_i^\sigma$ $(e_i^\sigma > 0)$ and rejects it when the reverse inequality holds. Likewise, the proposer i proposes the policy that gives i the greatest $V_i^\sigma(\theta_t, x)$ among the policies that are accepted.[10] Hence, the greater is w_i^σ, the more likely is player i to accept or (as appropriate) propose to intervene on the equilibrium path. The greater is e_i^σ, the more likely is player i to accept or (as appropriate) propose to intervene via p rather than via q.

4.1 Description of the equilibria

This subsection distinguishes between equilibria in which the inefficient intervention is never enacted from those in which it is enacted with positive probability. We also characterize these equilibria here but postpone discussing their existence to the next subsection.

Definition 1 *Let σ be an equilibrium strategy profile. Then, σ is an **instrument-efficient equilibrium** (EE) if q is implemented with probability-zero along the equilibrium path; σ is an **instrument-inefficient equilibrium** (IE) otherwise.*

Note that an instrument inefficient equilibrium is inefficient in a strong sense: q is implemented with positive probability even when it is not the status quo. Unlike in an instrument efficient equilibrium, therefore, the probability that q is implemented in any period does not vanish over time.

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period t from which the continuation payoff is computed. Likewise, if $\{\theta_t\}$ was not i.i.d., w_i^σ would have to be a function of θ.

[9]Duggan and Kalandrakis (2012) provide a very general existence result for dynamic bargaining games with endogenous status quo. But to apply their result directly here requires violating our assumption that $U(\theta, p) - U(\theta, q)$ is constant in θ. Although introducing some noise to the payoffs, to ensure the difference is not locally constant, and letting that noise tend to zero is possible, the result would be a correlated equilibrium that obscures the particular tradeoffs of interest here.

[10]The precise characterization of the equilibria of Γ is a bit more involved when a player i is indifferent between two alternatives for a nonnegligible set of states of nature, which happens when $e_i^\sigma = 0$. This possibility is taken into account in the Appendix, but for the sake of clarity, we abstract away from it when describing the equilibria of Γ in the main text.

Proposition 2 *If σ is an EE, then:*

$$e_L^\sigma \;>\; 0,\; e_R^\sigma \geq 0,\; and$$
$$w_L^\sigma \;<\; w_L < w_R < w_R^\sigma.$$

That $e_L^\sigma > 0$ and $e_R^\sigma \geq 0$ simply means that both players always get a greater continuation payoff from implementing p than from implementing q in an EE. As a result, they behave as if n and p were the only two policies available.[11] Proposition 2 further states that, when comparing n and p, dynamic considerations lead players to behave in a more polarized way relative to the static environment. Specifically, the inequality $w_L^\sigma < w_L$ states that forward-looking considerations lead the interventionist player L to bias her behavior, relative to L's primitive preference, in favor of the intervention p; conversely, $w_R^\sigma > w_R$ means that the laissez-faire player R's behavior is similarly biased in favor of no intervention (see Dziuda and Loeper (2015a) for a similar result in a two-alternative model).

To see the intuition for the result, suppose, for example, that p is the status quo for some period t and that the realization of θ_t is such that R, the laissez-faire player, would surely repeal p if R was the only policy-maker. But replacing p by n in Γ also requires acquiescence by the player least willing to overturn p for n, the interventionist player L. Consequently, anticipating L's relative reluctance to overturn an interventionist status quo, R prefers not to support an efficient intervention for some realizations of the state in which R would prefer otherwise. The intuition for L's relatively increased bias is symmetric.

For the preceding reasons, the equilibrium is inefficient even if players use only the instrument p. When $s_t = p$ and $\theta_t \in (w_L^\sigma, w_L)$, policy p remains in place despite both players receiving strictly higher payoffs from n in period t. Similarly, when $s_t = n$ and $\theta_t \in (w_R, w_R^\sigma)$, policy n remains despite both players receiving strictly higher payoffs from p in period t. However, this inefficiency can only take the form of status-quo inertia: although a status quo may stay in place when it ceases to be statically Pareto efficient, a change in the status quo can only occur if the change is statically Pareto efficient.[12]

Recalling that, in equilibrium σ, player i is indifferent between p and n at $\theta = w_i^\sigma$, and is likewise indifferent between q and n at $\theta = w_i^\sigma + e_i^\sigma$, the following result characterizes the main properties of instrument inefficient equilibria.

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[11] When $e_R^\sigma = 0$, and only then, R is indifferent between q and p and therefore may veto a change away from q when it is the status quo. However, once $s_t \in \{n, p\}$, then q is never implemented in any subsequent period.

[12] A sequence of policies $(x_t)_{t \in \mathbb{N}}$ is statically Pareto efficient if, in each period $t \in \mathbb{N}$, there is no other policy y that is strictly preferred by both players to x_t, keeping all other policies in the sequence unchanged.

Proposition 3 *If σ is an IE, then players disagree on the choice of instruments,*

$$\min\left\{e_L^\sigma, e_R^\sigma\right\} \leq 0 < \max\left\{e_L^\sigma, e_R^\sigma\right\};$$

and q is always repealed for a larger set of states than p,

$$\min\left\{w_L^\sigma, w_R^\sigma\right\} < \min\left\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\right\}.$$

Furthermore, an IE is one of three types:

IE-A : $\quad e_R^\sigma \leq 0 < e_L^\sigma,\ w_L^\sigma < w_R^\sigma$, *and* $w_L^\sigma + e_L^\sigma < w_R^\sigma,$

IE-B : $\quad e_R^\sigma \leq 0 < e_L^\sigma,\ w_L^\sigma < w_R^\sigma$, *and* $w_L^\sigma + e_L^\sigma \geq w_R^\sigma,$

IE-C : $\quad e_R^\sigma > 0 \geq e_L^\sigma$ *and* $w_L^\sigma > w_R^\sigma.$

An important property common to all types of IE is that e_R^σ and e_L^σ have opposite signs. The players thus disagree on which instrument they prefer to implement. Moreover, in any IE, the efficient intervention p is repealed and replaced by n when both players agree to do so, that is, when $\theta < \min\left\{w_L^\sigma, w_R^\sigma\right\}$, and it stays in place when the reverse inequality holds. Likewise, the inefficient intervention q is repealed and replaced by n when both players agree to do so, that is, when $\theta < \min\left\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\right\}$, and it stays in place when the reverse inequality holds. Thus, the inequality $\min\left\{w_L^\sigma, w_R^\sigma\right\} < \min\left\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\right\}$ in Proposition 3 implies that in any IE, the inefficient intervention q is repealed for a larger set of states than the efficient intervention p.[13] The fact that inefficient interventions are less "sticky" than efficient interventions mitigates R's concern with being able to repeal an intervention to the extent that R prefers to intervene via q rather than p, despite the lower flow payoff that q generates.

Now consider IE-A and IE-B equilibria. The inequalities $e_L^\sigma > 0$ and $e_R^\sigma \leq 0$ imply that player L prefers to intervene via the efficient policy instrument p whereas player R prefers to use the inefficient instrument q. For sufficiently high states θ, both players prefer to intervene in period t when the status quo is n although, for some more intermediate states, each player is willing to intervene only with its preferred instrument (p for L and q for R). Hence, if

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[13]This description of the mapping between the state and status quo and the policy outcome is valid only for pure strategy IE. The mapping is slightly different for a mixed strategy IE, which can happen when $e_i^\sigma = 0$ and $e_j^\sigma > 0$. However, a mixed IE differs from a pure IE only in that q can be replaced directly by p with positive probability for some states, and in any such states, p stays in place. So the claim that p stays in place for a larger set of states than q is true also for this type of equilibria. We consider either type of equililbria when proving our results in the Appendix, but for the sake of clarity, we focus on pure strategy IE when describing the intuition for our results.
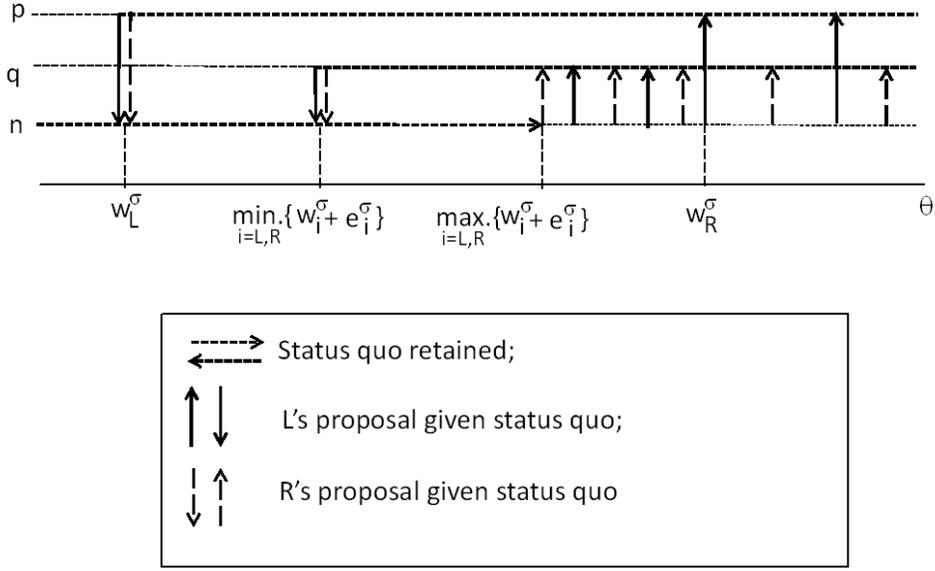
Figure 3: IE-A

$s = n$, an intervention is implemented for relatively large θ; whether it is implemented using p or q depends on the severity of the problem reflected in θ and the identity of the proposer. If θ is large, R suggests q and L approves, while L proposes p and R approves. If θ is smaller, two alternative situations may arise, as described by the difference between IE-A and IE-B. An IE-A is illustrated in Figure 3.

In this case, $w_L^\sigma + e_L^\sigma < w_R^\sigma$, implying $\max \{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\} < \max \{w_L^\sigma, w_R^\sigma\}$. In turn, this inequality implies that instrument q is accepted under a larger set of states than instrument p. Thus, when $\theta \in (\max \{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\}, w_R^\sigma)$, R prefers n to p but prefers q to n and so, if L wishes to implement some policy given a status quo n, then L must propose q here. For these states, L prefers q to n so R can propose, and L accepts, q. In other words, both players propose the inefficient policy for $\theta \in (\max \{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\}, w_R^\sigma)$. If $\theta < \max \{w_L^\sigma + e_L^\sigma, w_R^\rho + e_R^\sigma\}$, the status quo n persists.

Corollary 1 *For some states, any proposer will suggest (and the other player will accept) the inefficient instrument in any IE-A.*

When $\theta > w_R^\sigma$, both players prefer any type of intervention to remaining with a status quo n. Whether the intervention for such states is efficient or inefficient, therefore, depends on the player with proposal power. If L is the proposer, p is implemented and remains the status quo for a larger set of states than is the case if R is the proposer and q is implemented.

Now consider IE-B, illustrated in Figure 4. In this case we have $w_L^\sigma + e_L^\sigma > w_R^\sigma$, implying $\max \{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\} > \max \{w_L^\sigma, w_R^\sigma\}$. Thus, p is accepted under a larger set of states
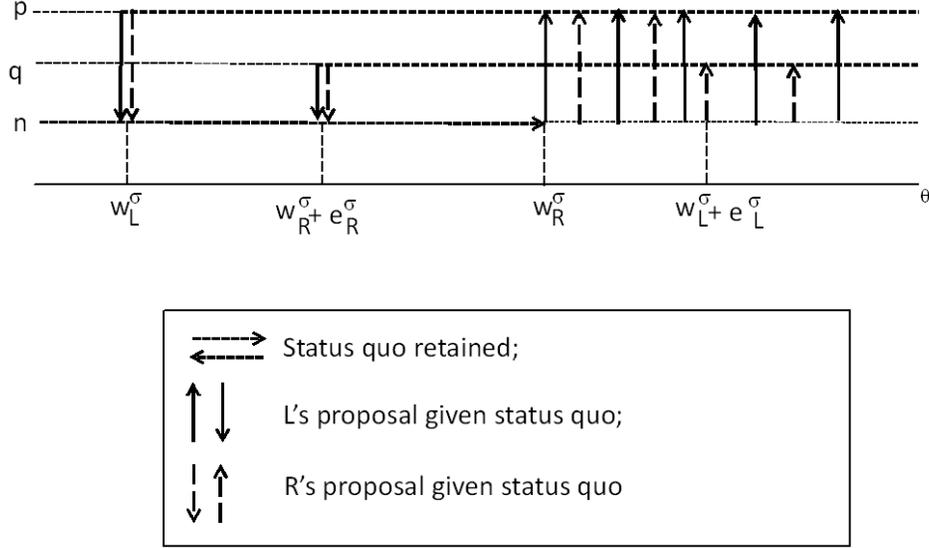
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Figure 4: IE-B

than policy q. Thus, when $\theta \in (w_R^\sigma, w_L^\sigma + e_L^\sigma)$, R prefers intervening with any instrument to maintaining a status quo n, but L strictly prefers n to the inefficient instrument q; hence p is implemented for any allocation of proposal power. And, similarly to the situation illustrated in Figure 3, when θ is large ($\theta > w_L^\sigma + e_L^\sigma$), both players strictly prefer any intervention to the status quo, so which particular instrument is adopted depends on the identity of the proposer, exactly as for the previous example.

Finally, consider IE-C. That equilibria of this sort can exist may be somewhat surprising. This is because $w_R^\sigma < w_L^\sigma$ implies that the interventionist player is willing to accept the efficient intervention *less* frequently than the laissez-faire player. The intuition for IE-C is as follows. If e_R is large enough (see Proposition 5 below), R prefers to intervene with p rather than q, if at all. As a result, R is concerned about the following situation: the status quo is n and the realization of θ sufficiently large that R has no choice but to accept a proposal q by L. The only way R can avoid such situations is by making the status quo n less likely, and thus by being biased against n when the status quo is p. If this bias is large enough, R can become more reluctant than L to repeal p, which explains why $w_R^\sigma < w_L^\sigma$. Anticipating R's reluctance to repeal p, L prefers to intervene via the less sticky instrument q, which rationalizes R's aforementioned concern with respect to having to accept a proposal q by L.

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4.2 Existence of the Equilibria

While the previous subsection described the equilibria, we now state that IE's often exist and that IE-A is the most natural one. As a start, Proposition 4 provides some relatively weak conditions on the environment for which IE exists for any payoff parameters (w_L, w_R, e_L, e_R). In particular, no matter how inefficient is the policy instrument q relative to p, all equilibria may be IE.

Proposition 4 *Let G be a c.d.f. with mean 0 and variance 1. For any (w_L, w_R, e_L, e_R), for all δ sufficiently close to 1, there exists a set of $m \in \mathbb{R}$ and $v > 0$ of positive Lebesgue measure such that, for $F(\theta) \equiv G\left(\frac{\theta - m}{v}\right)$, all equilibria of Γ are IE.*

The sufficient condition on the distribution of states here essentially requires that relatively little probability mass is concentrated in the tails. To see why, observe that if θ is likely to be either very large or very small, players would agree most of the time on whether or not to intervene. As a result, their preferences over which instrument to use would not be distorted by their expectation of future disagreements on when to repeal any intervention.

More specifically, recall Figure 3, above. In this case, player R's preference for implementing q instead of p increases with the likelihood of states in which q is repealed but a status quo p remains, that is, in states $\theta \in (w_L^\sigma, \min\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\})$. Similarly, because, all else equal, p generates a greater flow payoff for R, R's preference for implementing q instead of p decreases in the likelihood of states where both interventions persist, that is, in the relatively extreme states $\theta > w_R^\sigma$ in Figure 3. Thus, the condition on F used in Proposition 4 ensures, for the example of Figure 3, that states in $(\max\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\}, w_R^\sigma)$ are sufficiently likely, and states greater than w_R^σ sufficiently unlikely, to guarantee that an IE exists.[14]

With respect to *which* type of IE that exists, we can show that if an IE-C exists, then there also exists an equilibrium that is not IE-C. Formally, we have the following result.

Proposition 5 *If an IE-C exists, then also an IE-A, an IE-B, or an EE exists. Moreover, IE-C cannot exist if $e_L \geq e_R$ and IE-B cannot exist if $e_L \leq e_R$. Furthermore, if $\frac{|e_R - e_L|}{(w_R - w_L)} < (1 - \delta)^3$, any equilibrium is either EE or IE-A.*

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[14] A similar result for the exclusive existence of EE is also available. The only difference (*mutatis mutandis*) is that the choice of m for insuring only EE is distinct from that for insuring only IE; for both cases, it suffices to consider v vanishing small to insure that, most of the time, states are in the neighbourhood of m. Choosing $m = w_L + \eta$ for $\eta > 0$ and sufficiently small, implies players disagree between n and p in the typical state m, but agree that q is preferred to n. Hence, for high θ, R prefers intervening with q to limit the expected persistence of the intervention for states around m, yielding IE. On the other hand, by choosing $m \notin [w_L, w_R]$, players also agree between n and p in the typical state m, so R is unconcerned that efficient interventions persist and agrees to use p.

The second and third sentences of the proposition state that the IE is always IE-A if e_R and e_L are sufficiently close to each other. Moreover, IE-A is the only inefficient equilibrium for which existence is not dependent on the ranking of e_L and e_R. Given that whenever an IE-C, the substantively suspect equilibrium, exists, there also exists either an IE-A or an IE-B, we henceforth ignore IE-C when discussing any intuition for the results to follow.

4.3   The Effect of Polarization

This subsection describes how the use of inefficient instruments hinges on political polarization and other parameters characterizing the political environment.

Proposition 6 *Fix $\delta \in (0,1)$, the c.d.f. F and any allocation of bargaining power $b = (b_L, b_R)$. Then*

(i) For all (w_L, w_R), if all equilibria are IE for some (e_L, e_R), then all equilibria are IE for all (e'_L, e'_R) such that $e'_L \geq e_L$ and $e'_R \leq e_R$.

(ii) For every w_R, w_L and $e_L > 0$, there exists $\epsilon > 0$ such that all equilibria are IE for any $e_R \leq \epsilon$.

(iii) For all (e_L, e_R) and fixed mean ideology $(w_L + w_R)/2$, there exists an EE as $(w_R - w_L) \to 0$. Furthermore, all equilibria are EE as $(w_R - w_L) \to \infty$.

The comparative static in Proposition 6(i) on e_R is intuitive. If, for some (e_L, e_R), R is willing to accept the inefficient intervention q in exchange for an increase in the likelihood that the intervention is repealed in the future, R is also willing to accept q in such circumstances for lower degrees of inefficiency. The claim in Proposition 6(i) regarding changes in e_L, however, is less obvious, since an increase in e_L has two effects. On the one hand, L is more willing to repeal a status quo q when e_L is large. This, in turn, increases the strategic value of q for R. On the other hand, since a larger e_L implies that L's payoff from q is smaller, L's willingness to accept any proposal to implement q decreases as well. However, regardless of the value of e_L, for θ large enough, L prefers q to a status quo n, so R can be sure q is accepted and implemented on the equilibrium path.

It is informative to reformulate Proposition 6(i) in terms of player polarization. Recall that w_i and $w_i + e_i$ can be interpreted as the ideological position of player i on how often to intervene when using policy p and q, respectively. For a fixed (w_L, w_R), as e_L increases and e_R falls, the gap between the players' on policy p remains unaffected; for policy q, however, the difference between the two players decreases. Proposition 6(i) then says that as e_L increases and e_R falls, all else equal, players become more inclined to use the inefficient,

but more consensual, policy instrument q. In particular, the inefficient policy instrument is used because it is costly for player L; not because it is costly for player R. Thus, as part (ii) of the proposition states, the inefficient instrument is always chosen (for sufficiently high states) if e_R is small enough (where "small enough" depends on w_R, w_L and e_L).

Part (iii) of Proposition 6 concerns how the equilibria change with alterations in legislative polarization. Given a mean ideology, existence of an instrument-efficient equilibrium is assured for sufficiently small polarization. Since the players' preferences are essentially aligned in this case, muting any concerns with future status quo bias, the result is intuitive. On the other hand, when polarization is sufficiently large, the players are rarely aligned with respect to whether intervention is warranted. Thus, any policy is unlikely to be repealed and the players perceive the decision as permanent. In common with the players' behavior in a static setting, both agree to an efficient intervention if they agree to intervene at all. In sum, therefore, part (iii) of the proposition yields the following prediction.

Corollary 2 *If an inefficient instrument is used in any equilibrium, then political polarization cannot be too small or too large.*

Political polarization can, as observed earlier, be a function of the political system. With only one political chamber under simple majority rule, the median voter is pivotal in every decision and there can be no polarization between multiple pivotal legislators. With a supermajority requirement, however, there is almost always some ideological distance between the veto players. In this case, legislative players can use inefficient policy instruments to avoid gridlock.

4.4 The Value of Inefficient Instruments

Empirically, the menu of available policy instruments can be endogenous. For example, if environmental policy is chosen at the local level, or by a regulatory body, then the instruments that are available may be defined ex ante by the federal government and, in such cases, it is not at all clear whether an inefficient policy instrument would, or should, be made available to decision makers. By definition of the two interventions, efficient p and inefficient q, there is no social welfare gain to be had from the existence of q in a static environment; the question, then, is whether this holds for the dynamic setting.

Because players are free to ignore the inefficient instrument, having instrument q available seems at least welfare-irrelevant. And although instrument q can be abused by player R to the extent that R proposes q solely to improve R's future bargaining position, it turns out that q can have a positive strategic value when an efficient intervention is politically infeasible,

because of R's fears that an efficient policy will not be repealed when R prefers. The next result states that, under certain conditions, both players are strictly better off with access to the inefficient instrument q, than if they are constrained to choosing only between n and p. In other words, there are environments in which a statically inefficient policy instrument supports Pareto improvements in the dynamic game.

Let $\Gamma(n, p, q)$ denote the original game, $\Gamma(n, p)$ the game in which the inefficient instrument is unavailable, and $\Gamma(n, q)$ the game in which the efficient instrument is unavailable.

Proposition 7 *For any (w_L, w_R, e_L, e_R) and for all δ sufficiently close to one, there exists an F under which both players are strictly better off in any equilibrium of $\Gamma(n, p, q)$ than in any equilibrium of $\Gamma(n, p)$, and both players are strictly better off in any equilibrium of $\Gamma(n, q)$ than in any equilibrium of $\Gamma(n, p)$.*

The intuition for Proposition 7 is similar to that of Proposition 4. If the c.d.f. F puts enough weight on $(w_L, \min\{w_L + e_L, w_R\})$, then L and R are likely to disagree over n and p, but are unlikely to disagree over n and q. Consequently, in the game $\Gamma(n, p)$, sufficiently patient players become very biased in favor of the policy each prefers most on average. In particular, w_R^σ increases in δ, so R never agrees to intervene and the initial status quo n persists indefinitely. The availability of q, however, provides room for players to intervene for states $\theta > \min\{w_L + e_L, w_R + e_R\}$, since the likelihood of agreement on repealing the intervention is greater with q than with p. This greater flexibility leaves both players better-off in equilibria in $\Gamma(n, p, q)$ than in $\Gamma(n, p)$. Similarly, since it is harder, others things equal, to repeal an efficient intervention than an inefficient intervention, the same relative welfare property applies when comparing $\Gamma(n, q)$ to $\Gamma(n, p)$. Proposition 7 is silent on the comparison between $\Gamma(n, p, q)$ and $\Gamma(n, q)$, but we conjecture that the result of Proposition 1 holds in some environments. Namely, that there exist F under which both players prefer any equilibrium to $\Gamma(n, q)$ to any $\Gamma(n, p, q)$.

5 Persistent States

To this point, we have assumed that states, and thereby players' state-contingent preferences, are i.i.d. draws over time. However, in many applications, states might persist for several consecutive periods before a period of relative volatility. There may be periods of relatively stability when players do not expect to change their positions quickly, and there may also be periods in which new information about the desirability of an intervention arrives frequently, resulting in frequent revisions of the relevant policy preferences.[15] In this section, we ask

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[15] A painful example is the US Congressional response to the 2008 fiscal collapse. For some years before the 2008 fiscal collapse, the US economy was growing strongly and atypical Congressional economic interventions

how expectations about the volatility or persistence of states affect players' strategic policy decisions.

To capture the possibility (in an analytically tractable way) that states θ may or may not persist across periods, and that players' expectations regarding the persistence of the current state may vary over time, consider, for every period t, the tuple $(\theta_t, v_t) \in \mathbb{R} \times [0, 1]$. Assume the evolution of such tuples across periods satisfies the following transition property: for all t,

$$(\theta_{t+1}, v_{t+1}) = \begin{cases} (\theta_t, v_t) & \text{with probability } 1 - v_t \\ (\theta_{t+1}, v_{t+1}) \sim H & \text{with probability } v_t \end{cases}.$$

As before, θ_t is the underlying policy-relevant state. We interpret the additional variable v_t as a measure of volatility of the current policy-relevant state θ_t or, equivalently, of players' period t expectations over θ_{t+1}. In each period t, the state θ_t persists into period $t + 1$ with probability $(1 - v_t)$ and, for simplicity, we assume that the volatility v_t also persists into period $t + 1$; with probability v_t, θ_{t+1} and v_{t+1} are drawn according to some joint c.d.f. H. Thus, the volatility of future state-contingent policy preferences is redrawn if and only if the state-contingent policy preferences are redrawn.

Note that this evolution of the state collapses to the basic i.i.d. model if $v_t \equiv 1$ for all t. Similarly, $v_t \equiv 0$ for all t implies preferences never change and $v_t \equiv v \in (0, 1)$ for all t implies the degree of volatility is fixed.

The continuation payoff function V^σ is defined as follows. For every stationary strategy profile σ, let V_i^σ be the continuation payoff for player $i \in \{L, R\}$ of implementing policy $x \in \{n, p, q\}$ in some period $t \in \mathbb{N}$ with state $(\theta_t, v_t) = (\theta, v) \in \mathbb{R} \times [0, 1]$ until the state is redrawn, and play σ thereafter.

Lemma 2 *For any equilibrium strategy profile σ, there exist $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \in \mathbb{R}^4$ such that, for all $i \in \{L, R\}$, $\theta \in \mathbb{R}$, and $v \in [0, 1]$,*

$$\begin{aligned} (1 - \delta(1 - v))(V_i^\sigma(\theta, v, p) - V_i^\sigma(\theta, v, n)) &= \theta - (1 - v) w_i - v w_i^\sigma \\ (1 - \delta(1 - v))(V_i^\sigma(\theta, v, p) - V_i^\sigma(\theta, v, q)) &= (1 - v) e_i + v e_i^\sigma. \end{aligned} \tag{3}$$

Hence, as in the basic model, players continuation payoff functions V^σ have the same shape as their flow payoff functions in (1), but the flow payoff parameters w_i and e_i are replaced by $(1 - v) w_i + v w_i^\sigma$ and $(1 - v) e_i + v e_i^\sigma$, respectively.[16] As before, the difference be-

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were minimal. The fall of Lehman Brothers and the subsequent turmoil in much of the global economy led to serious Congressional disagreement regarding the appropriate level and duration of any extraordinary intervention, from whether to bail out banks or the car industry, to extensions of unemployment and welfare benefits.

[16]The common scaling factor $1 - \delta(1 - v)$ does not affect the sign of the expressions in (3), and thus does

tween the flow payoff parameters and the continuation payoff parameters, namely $v\left(w_i^\sigma - w_i\right)$ and $v\left(e_i^\sigma - e_i\right)$, captures the way players distort their equilibrium behavior relative to their flow payoff. Note that in this extended model, such distortion varies with v in a systematic way. When $v = 0$, players expect their preferences to remain constant in the future. In this case, each player behaves as in a one period-model and the difference in the continuation payoffs coincides with the difference in the static flow payoffs (1). As v increases, the (state-contingent) preferences are more likely to change in the future and, therefore, the status quo becomes more salient, driving a wedge between the difference in the continuation payoffs and the difference in the static flow payoffs.

Recall that the inefficient instrument q is implemented for some realizations of the policy-relevant state θ in some equilibrium σ when $V_i^\sigma\left(\theta, v, p\right) < V_i^\sigma\left(\theta, v, q\right)$ for some player i, whereas q is not implemented when the opposite inequality holds for both players. From (3), if $e_i^\sigma \geq 0$ for $i = L, R$, for all $v < 1$, there is no realization of θ at which $V_i^\sigma\left(\theta, v, p\right) < V_i^\sigma\left(\theta, v, q\right)$; hence, q is implemented with probability 0 on the equilibrium path. In environments where $e_i^\sigma < 0$ for some i, however, from (3), for v sufficiently close to 1, $V_i^\sigma\left(\theta, v, p\right) < V_i^\sigma\left(\theta, v, q\right)$ for some nonnegligible set of realizations of θ, so q is implemented with positive probability on the equilibrium path. The same logic as in Proposition 4 implies that for any profile of payoff parameters $\left(w_L, w_R, e_L, e_R\right)$, one can find a distribution H such that $e_i^\sigma < 0$ for some i, yielding the following result.

Proposition 8 *In any equilibrium σ, there exists $\bar{v} \in (0, 1]$ such that q is never implemented when the realization of volatility is such that $v < \bar{v}$ and q is implemented for a nonnegligible set of realizations of θ if $v > \bar{v}$. Moreover, for any $\left(w_L, w_R, e_L, e_R\right)$, for all δ sufficiently close to 1, there exists a distribution H with full support on $\mathbb{R} \times [0, 1]$ such that, for all equilibria, $\bar{v} < 1$.*

Proposition 8 states that q is implemented on the equilibrium path only in sufficiently volatile environments. Intuitively, when players expect the state to remain fairly stable over time $(v_t \leq \bar{v})$, strategic concerns regarding the possibility of conflict over repealing today's intervention tomorrow, say, are muted and any intervention is efficient. When the state is expected to be sufficiently volatile, however $(v_t > \bar{v})$, today's choice is likely to need revision in the next period, making salient exactly the sorts of strategic consideration underlying the use of inefficient interventions.

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not affect which policy parties prefer to implement on the equilibrium path.

6 Conclusion

The continued and widespread use of inefficient policy instruments in more-or-less democratic political systems is a puzzle. For example, while economists uniformly recommend regulating emissions with Pigou taxes, technology and quantity controls are the most adopted instruments in reality. Why would rational politicians agree on the use of Pareto dominated policy instruments? In this paper, we show that the puzzle can be understood without pointing to informational asymmetries, interest group influence, or differential distributional implications of alternative policy instruments among the electorate at large.

With a heterogeneous legislature and multiple veto players, inefficient policy instruments are politically easier than efficient instruments to repeal in dynamic environments subject to policy-relevant stochastic shocks. And since inefficient instruments are easier to repeal, heterogeneous veto players, differentiated only by the threshold shocks beyond which they judge some policy intervention to be warranted, can be more willing to agree on responding to a sufficiently severe downside shock with an inefficient instrument. As a consequence, inefficient instruments are more likely to be used in polarized political environments and for issues where the fundamentals are subject to change over time.

Our analysis and results hinge on a number of crucial assumptions. We assume that today's policy constitutes the status quo for the next period; that players cannot use side payments in decision-making; and that the player relatively reluctant to intervene in any period, is likely to be similarly reluctant in subsequent periods. Each of these assumptions is crucial for our results, but we also believe that they are reasonable and satisfied in most democracies: few policies come with expiration dates; explicit side payments are rare in politics; and parties that are left-wing today have been left-wing for decades.

We also assume that the policy decision is binary: the instrument is either imposed or it is not. This assumption is for simplicity, however, and we conjecture that our results would continue to hold if an emission quota or tax, for example, were any real number. Even if the policy is not binary, when the need to regulate decreases, it should still be easier to agree on scaling back an abatement requirement than a revenue-generating emission tax. Hence, a model with a continuum of intervention levels should predict that an inefficient intervention is frequently adjusted and fine-tuned. This theory's qualitative prediction seems to be consistent with reality, whereby technology standards and emission quotas are indeed frequently adjusted

It is beyond the scope of this paper to analyze in depth a model with non-binary policies. While we have tried to keep the model as general as possible in other respects, we believe future research should make more specific (rather than general) assumptions on the available

instruments and the legislative process. Specific assumptions frequently yield sharp testable predictions that can be taken to the data. Understanding better the political constraints on implementing efficient policies in specific policy areas is important.

7 Appendix

Throughout this appendix, for any player specific variable, the variable without the player subscript refers to the vector of this variable for each player. For instance, w refers to (w_L, w_R).

Notation 1 *For a given stationary Markov strategy profile σ, the policy outcome in some period $t \in \mathbb{N}$ with status quo $s_t \in \{n, p, q\}$ depends on the realization of θ_t, the identity of the proposer in period t, and possibly players' private randomization devices if the equilibrium is mixed. Let υ_t denote the random variable that encodes all this information. We refer to υ_t as the* state of the world *in period t. Let Υ denote the set of possible states of the world. Note that $\{\upsilon_t : t \in \mathbb{N}\}$ is i.i.d.. Let μ denote its probability distribution. For any state of the world $\upsilon \in \Upsilon$, $\theta(\upsilon)$ denotes the corresponds realization of the* state of nature.
For all $s, x \in \{n, p, q\}$, we denote by $\Upsilon^{\sigma}(s, x)$ the set of realizations of the state of the world for which status quo s leads to outcome x.

We first prove the following lemma, which is instrumental in proving Lemma 1.

Lemma 3 (Continuation Payoff) *Let σ and σ^* be two Markov strategy profiles, and let $V_i^{\sigma,\sigma^*}(\theta, x)$ denote the expected continuation payoff for player $i \in \{L, R\}$ from implementing policy $x \in \{n, p, q\}$ in period 0 conditional on $\theta(0) = \theta$, and on players playing σ in period 1 and playing σ^* from period 2 onwards. Then there exists $w_i^{\sigma,\sigma^*}, e_i^{\sigma,\sigma^*} \in \mathbb{R}$ such that, for all $\theta \in \mathbb{R}$,*

$$
\begin{aligned}
V_i^{\sigma,\sigma^*}(\theta, p) - V_i^{\sigma,\sigma^*}(\theta, n) &= \theta - w_i^{\sigma,\sigma^*}, \\
V_i^{\sigma,\sigma^*}(\theta, p) - V_i^{\sigma,\sigma^*}(\theta, q) &= e_i^{\sigma,\sigma^*}.
\end{aligned}
$$

Proof. Using the notations of the lemma, $V_i^{\sigma,\sigma^*}(\theta, p) - V_i^{\sigma,\sigma^*}(\theta, n)$ is given by the flow payoff gain from implementing p instead of n in $t = 0$, which is $\theta - w_i$, plus δ times the continuation payoff gain from period 1 onwards from having $s_1 = p$ instead of $s_1 = n$, given continuation play σ in $t = 1$ and σ^* in $t \geq 2$. If V^{σ^*} denotes the continuation value function for the strategy profile σ^* as defined at the beginning of Section 4, then using Notation 1, the above reasoning implies that

$$
\begin{aligned}
&V_i^{\sigma,\sigma^*}(\theta, p) - V_i^{\sigma,\sigma^*}(\theta, n) \\
&= \theta - w_i + \delta \sum_{x, y \in \{n, p, q\}} \left(\int_{\Upsilon^{\sigma}(p, x) \cap \Upsilon^{\sigma}(n, y)} \left(V_i^{\sigma^*}(\theta(\upsilon), x) - V_i^{\sigma^*}(\theta(\upsilon), y) \right) d\mu(\upsilon) \right).
\end{aligned}
$$

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Therefore, to obtain the desired expression for $V_i^{\sigma,\sigma^*}(\theta,p) - V_i^{\sigma,\sigma^*}(\theta,n)$, it suffices to set

$$w_i^{\sigma,\sigma^*} \equiv w_i - \delta \sum_{x,y \in \{n,p,q\}} \left(\int_{\Upsilon^\sigma(n,x) \cap \Upsilon^\sigma(p,y)} \left(V_i^{\sigma^*}(\theta(v),x) - V_i^{\sigma^*}(\theta(v),y) \right) d\mu(v) \right). \quad (4)$$

An analogous reasoning on the continuation payoff gain from implementing p instead of q implies that

$$e_i^{\sigma,\sigma^*} = e_i + \delta \sum_{x,y \in \{n,p,q\}} \left(\int_{\Upsilon^\sigma(p,x) \cap \Upsilon^\sigma(q,y)} \left(V_i^{\sigma^*}(\theta(v),x) - V_i^{\sigma^*}(\theta(v),y) \right) d\mu(v) \right). \quad (5)$$

■

Proof of Lemma 1. If we set $\sigma = \sigma^*$ in Lemma 3, we obtain the second claim of Lemma 1: the continuation payoffs for any strategy profile σ are given by (2) with $w^\sigma = w^{\sigma,\sigma}$ and $e^\sigma = e^{\sigma,\sigma}$. The first claim, namely that an equilibrium exists, follows from Lemma 6 below, which proves the existence of an equilibrium with particular properties (these properties will be useful in the proof of Proposition 5). Note that Lemmas 4, 5 and 6 only use the second claim of Lemma 1, which we have just proven, so there is no circularity. ■

In what follows, we say that a Markov strategy profile σ is stage undominated for some continuation payoff parameters $(w^*, e^*) \in \mathbb{R}^4$, if σ is a subgame perfect equilibrium of the game in which players play only one period of the infinite horizon game $\Gamma(n, p, q)$ and their payoff for that period is such that, for all $\theta \in \mathbb{R}$, $U_i(\theta, p) - U_i(\theta, n) = \theta - w_i^*$ and $U_i(\theta, p) - U_i(\theta, q) = e_i^*$.

Lemma 4 *A Markov strategy profile σ is an equilibrium if and only if σ is stage undominated for the continuation payoff parameters (w^σ, e^σ).*

Proof. By definition of the continuation payoff parameters (w^σ, e^σ) in (2), σ is stage undominated for the continuation payoff parameters (w^σ, e^σ) if and only if σ is stage undominated using the original definition of stage undomination in Baron and Kalai (1993). To complete the proof, observe that in the game $\Gamma(n, p, q)$, players play sequentially, so subgame perfection is equivalent to stage undomination in the sense of Baron and Kalai (1993). ■

The following lemma derives some properties of the parameters w^{σ,σ^*} and e^{σ,σ^*} introduced in Lemma 3, and considers the case in which σ is stage undominated in $t = 1$ given continuation play σ^*. These properties will help us characterize players' equilibrium continuation payoff — which corresponds to the case $\sigma = \sigma^*$ — and prove the existence of an equilibrium by considering σ as a function of σ^* and finding a fixed point of that mapping.

Lemma 5 (Best Response) *The parameters w_i^{σ,σ^*} and e_i^{σ,σ^*} introduced in Lemma 3 depend on the strategy profile σ^* only through the continuation payoff parameters $w_i^{\sigma^*}$ and $e_i^{\sigma^*}$, so we can write $w_i^{\sigma,\sigma^*} = W_i^\sigma\left(w_i^{\sigma^*}, e_i^{\sigma^*}\right)$ and $e_i^{\sigma,\sigma^*} = E_i^\sigma\left(w_i^{\sigma^*}, e_i^{\sigma^*}\right)$.*
Moreover, for any profile of continuation payoff parameters $(w^*, e^*) \in \mathbb{R}^4$,

(i) $w^* = W^\sigma(w^*, e^*)$ and $e^* = E^\sigma(w^*, e^*)$ if and only if $w^* = w^\sigma$ and $e^* = e^\sigma$.

(ii) If the actions prescribed by σ are stage undominated given continuation payoff parameters (w^*, e^*), then for all $i \in \{L, R\}$,

$$W_i^\sigma\left(w_i^*, e_i^*\right) = w_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)} \left(w_i^* - \theta(v)\right) d\mu(v) - \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,q)} e_i^* d\mu(v), \quad (6)$$

and

$$
\begin{aligned}
E_i^\sigma\left(w_i^*, e_i^*\right) &= e_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,q)} e_i^* d\mu(v) + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} \left(\theta(v) - w_i^*\right) d\mu(v) \quad (7) \\
&\quad + \delta \int_{\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)} \left(w_i^* + e_i^* - \theta(v)\right) d\mu(v).
\end{aligned}
$$

Proof. From (2), depending on the policies $x, y \in X$, the terms $V_i^{\sigma^*}(\theta(v), x) - V_i^{\sigma^*}(\theta(v), y)$ in (4) and (5) are equal to 0, $\pm e_i^{\sigma^*}$, $\pm\left(\theta(v) - w_i^{\sigma^*}\right)$, or $\pm\left(\theta(v) - w_i^{\sigma^*} - e_i^{\sigma^*}\right)$. Thus, they depend on σ^* only through $w_i^{\sigma^*}$ and $e_i^{\sigma^*}$, which proves the first claim of the lemma. Moreover, these terms are bounded by $\delta\left(E(|\theta|) + \left|w_i^{\sigma^*}\right| + \left|e_i^{\sigma^*}\right|\right)$ and, in the formula (4) and (5), they are integrated over disjoint sets of states of the world. Therefore, the mappings $\left(w_i^{\sigma^*}, e_i^{\sigma^*}\right) \to W_i^\sigma\left(w_i^{\sigma^*}, e_i^{\sigma^*}\right)$ and $\left(w_i^{\sigma^*}, e_i^{\sigma^*}\right) \to W_i^\sigma\left(w_i^{\sigma^*}, e_i^{\sigma^*}\right)$ are $\delta-$contractions.

To prove Part (i), note that if players expect continuation payoff parameters (w^σ, e^σ) in period 2 onwards and play σ in period 1, then by definition of (w^σ, e^σ), they expect continuation payoff (w^σ, e^σ) in period 0. This proves that $w^\sigma = W^\sigma(w^\sigma, e^\sigma)$ and $e^\sigma = E^\sigma(w^\sigma, e^\sigma)$. Conversely, suppose that $(w^*, e^*) \in \mathbb{R}^4$ is such that $w^* = W^\sigma(w^*, e^*)$ and $e^* = E^\sigma(w^*, e^*)$. Then (w_i^*, e_i^*) is a fixed point of the mapping $(\omega, \varepsilon) \to (W^\sigma(\omega, \varepsilon), E^\sigma(\omega, \varepsilon))$. As shown above, this mapping is a $\delta-$contraction. As such, it has a unique fixed point. Since (w^σ, e^σ) is a fixed point of that mapping, necessarily $(w^\sigma, e^\sigma) = (w^*, e^*)$.

To prove (7), let us consider the integral $\int_{\Upsilon^\sigma(p,x) \cap \Upsilon^\sigma(q,y)} \left(V_i^{\sigma^*}(\theta(v), x) - V_i^{\sigma^*}(\theta(v), y)\right) d\mu(v)$ inside the sum on the R.H.S. of (5) for all possible values of x and y, that is, for all possible outcomes x and y that replace status quo p and q, respectively. Observe first that this integral is 0 when $x = y$, so we only need to consider the cases $x \neq y$. Let us now consider $x = q$ and $y = n$. Since σ is stage undominated given continuation payoff parameters (w^*, e^*), for all $v \in \Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,n)$, both players must weakly prefer to implement q to p and n to q in

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period 1. Moreover, one of them must be indifferent between implementing n and q, because if both players strictly preferred to implement n to q in state v, then under status quo p, any veto player would accept n, so proposing n instead of q would be a profitable deviation for the proposer. Therefore, for all $v \in \Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,n)$, $\theta(v) - w_i^\sigma - e_i^\sigma = 0$ for some player i. Since F is atomless, this implies that $\mu(\Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,n)) = 0$. An analogous reasoning implies that $\mu(\Upsilon^\sigma(q,p) \cap \Upsilon^\sigma(p,n)) = 0$. Therefore, modulo a zero measure set of states of the world, status quo p and q lead to different outcomes x and y in period 1 if and only if one the following happens: (i) both status quos stay in place, i.e., $v \in \Upsilon^\sigma(q,q) \cap \Upsilon^\sigma(p,p)$, (ii) p stays in place whereas q is replaced by n, i.e., $v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$, (iii) p is replaced by n whereas q stays in place, i.e., $v \in \Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)$, or (iv) p is replaced by q and q is replaced by p, i.e., $v \in \Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,p)$. Therefore, using (2), the right-hand-side of (5) can be simplified as follows

$$
E_i^\sigma(w_i^*, e_i^*) = e_i + \delta \left(\begin{array}{l} \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,q)} e_i^* d\mu(v) + \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(v) - w_i^*) d\mu(v) \\ + \int_{\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)} (w_i^* + e_i^* - \theta(v)) d\mu(v) - \int_{\Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,p)} e_i^* d\mu(v) \end{array} \right).
$$

Note that if $\Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,p) \neq \emptyset$, stage undomination implies that $e_L^* = e_R^* = 0$. Substituting the latter equation into the above one, we obtain (7).

The proof of (6) follows a similar logic as the proof of (7) above. The details are as follows. Consider the integral $\int_{\Upsilon^\sigma(p,x) \cap \Upsilon^\sigma(n,y)} \left(V_i^{\sigma^*}(\theta(v),x) - V_i^{\sigma^*}(\theta(v),y) \right) d\mu(v)$ inside the sum on the R.H.S. of (4) for all possible values of x and y with $x \neq y$. Using the same steps as the ones we used when proving $\mu(\Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(q,n)) = 0$ and reversing the role of p and q, we obtain that for all v, $v \in \Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(n,q)$. An analogous reasoning implies that $\mu(\Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(n,p)) = 0$. Moreover, for all $v \in \Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(n,p)$, either player i must be indifferent between implementing n and p, so $\theta(v) - w_i^\sigma - e_i^\sigma = 0$. Since F is atomless, this implies that $\mu(\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(n,p)) = 0$. Therefore, modulo a zero measure set of states of the world, status quo p and n lead to different outcomes in period $t = 1$ if and only if one the following happens: (i) both status quo stay in place, i.e., $v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)$, (ii) p stays in place while n is replaced by q, i.e., $v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,q)$, or (iii) p is replaced by q while n stays in place, i.e., $\theta \in \Upsilon^\sigma(p,q) \cap \Upsilon^\sigma(n,n)$. Equation (6) follows from substituting these three cases into the right-hand side of (4) and using (2). ∎

Lemma 6 (Equilibrium Existence) *There exists an equilibrium such that* $w_L^\sigma < w_R^\sigma$ *and* $e_L^\sigma > 0$.

Proof. In this proof, we consider an arbitrary strategy profile σ whose continuation payoff parameters (w^σ, e^σ) satisfy the conditions of the lemma, construct a stage-undominated

response to σ, show that this mapping satisfies Brower's theorem and that, to any fixed point of this mapping, there corresponds an equilibrium with the desired properties.

Let σ be such that $e_L^\sigma \geq 0$ and $w_L^\sigma \leq w_R^\sigma$, and let $\pi \in [0,1]$ be such that $\pi = 0$ if $e_R^\sigma < 0$ and $\pi = 1$ if $e_R^\sigma > 0$. As will be clear from what follows, π will be used as a tie-breaking rule when player R is indifferent between implementing q and p, i.e., when $e_R^\sigma = 0$. Below, we construct a strategy profile $\phi(\pi, w^\sigma, e^\sigma)$ which is stage undominated given continuation payoff parameters (w^σ, e^σ). This construction is quite intuitive, but we describe it in detail below to show that $\phi(\pi, w^\sigma, e^\sigma)$ can be chosen to be continuous in $(\pi, w^\sigma, e^\sigma)$.

Veto player's strategy: In any Markov state in which the veto player $i \in \{L, R\}$ has the choice between n and p (q), ϕ prescribes i to choose n when $\theta \leq w_i^\sigma$ (when $\theta \leq w_i^\sigma + e_i^\sigma$), and p (q) otherwise. When the veto player has to choose between p and q, ϕ prescribes L to always choose p, and R to choose p with probability π and q with probability $1 - \pi$. This behavior is clearly stage undominated given continuation play σ because, by assumption, $e_L^\sigma \geq 0$, $\pi = 0$ when $e_R^\sigma < 0$ and $\pi = 1$ when $e_R^\sigma > 0$. Finally, when the status quo is proposed, the action prescribed by ϕ is irrelevant so ϕ prescribes both players to accept the proposal.

Proposer's strategy under status quo p: By assumption, $e_L^\sigma \geq 0$, and by construction, ϕ prescribes L always to reject proposal q. Therefore, proposal q is always weakly dominated by proposal p irrespective of the identity of the proposer. So we can restrict attention to proposals n or p.

When $\theta \leq w_L^\sigma$, since $w_L^\sigma \leq w_R^\sigma$, both players prefer to implement n to p and, by construction of ϕ, proposal n is accepted by both players. So we set ϕ to prescribe both players to propose n.

When $\theta > w_L^\sigma$, L strictly prefers to implement p to n and rejects proposal n. So we set ϕ to prescribe both players to propose p.

Proposer's strategy under status quo n: Since $w_R^\sigma \geq w_L^\sigma$, one can easily check that when $\theta \leq \min\{\max_i(w_i^\sigma + e_i^\sigma), w_R^\sigma\}$, there is no alternative that is accepted by one player and that gives a strictly greater continuation payoff than n to the other player. Therefore, we set ϕ to prescribe both players to propose n.

When $\max_i(w_i^\sigma + e_i^\sigma) < \theta \leq w_R^\sigma$ (see Figure 3), ϕ prescribes R to reject proposal p and R prefers to implement n to p, so proposal p is weakly dominated by proposal n for both players. Moreover, ϕ prescribes both players to accept proposal q and both players strictly prefer to implement q to n. Therefore, we set ϕ to prescribe both players to propose q.

When $w_R^\sigma < \theta \leq \max_i(w_i^\sigma + e_i^\sigma)$ (see Figure 4), the same configuration arises in which the role of q and p reversed, so we set ϕ to prescribe both players to propose p.

When $\theta > \max\{\max_i(w_i^\sigma + e_i^\sigma), w_R^\sigma\}$, both players accept q and p. So it is stage undomi-

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nated for proposer $i \in \{L, R\}$ to propose the policy that gives her the greatest continuation payoff, which is p if $e_i^\sigma \geq 0$ and q if $e_i^\sigma \leq 0$. Therefore, we set $\phi(\pi, \sigma)$ to prescribe L to propose p with probability 1, and R to propose p with probability π and q with probability $1 - \pi$.

L's proposal strategy under status quo q: Given how we have set ϕ in the Markov states of the veto player R, the continuation payoff gain for L of proposing p instead of q is πe_L^σ, which is nonnegative. So one can restrict attention to strategies in which L only proposes n or p. Note that the continuation payoff gain for L of proposing p instead of n is πe_L^σ when n is not accepted by R (i.e., when $\theta > w_R^\sigma + e_R^\sigma$), and it is $\pi (\theta - w_L^\sigma) + (1 - \pi)(\theta - w_L^\sigma - e_L^\sigma) = \theta - w_L^\sigma - \pi e_L^\sigma$ when n is accepted by R (i.e., when $\theta \leq w_R^\sigma + e_R^\sigma$).

When $\theta \leq \min\{w_L^\sigma + \pi e_L^\sigma, w_R^\sigma + e_R^\sigma\}$, from what precedes, n is accepted by R and L prefers proposal n to p, so we set ϕ to prescribe L to propose n.

When $\theta > \min\{w_L^\sigma + \pi e_L^\sigma, w_R^\sigma + e_R^\sigma\}$, either $\theta > w_R^\sigma + e_R^\sigma$, in which case proposal n is not accepted by R and thus yields outcome q, so L weakly prefers to propose p (since $e_L^\sigma \geq 0$), or $w_L^\sigma + \pi e_L^\sigma < \theta \leq w_R^\sigma + e_R^\sigma$, in which case R accepts n, but as argued above, since $\theta > w_L^\sigma + \pi e_L^\sigma$, L is better off proposing p than n, so we set ϕ to prescribe L to propose p.

R's proposal strategy under status quo q: By construction of ϕ, when the status quo is q proposals q and p are both accepted with probability 1 by L so, by definition of π, proposing p with probability π and q with probability $1 - \pi$ always weakly dominates any other proposal which mixes between q and p. In what follows, P^π refers to the latter proposal strategy. Thus, we can restrict attention to proposals n or P^π. The continuation payoff gain of proposing P^π instead of n is $\pi (\theta - w_R^\sigma) + (1 - \pi)(\theta - w_R^\sigma - e_R^\sigma) = \theta - w_R^\sigma - \pi e_R^\sigma$ when proposal n is accepted by L (i.e., when $\theta \leq w_L^\sigma + e_L^\sigma$), and it is πe_R^σ otherwise.

When $\theta \leq \min\{w_R^\sigma + \pi e_R^\sigma, w_L^\sigma + e_L^\sigma\}$, L accepts proposal n so, from what precedes, R prefers proposal P^π to n, so we set ϕ to prescribe R to propose n, which is stage undominated.

When $\theta > \min\{w_R^\sigma + \pi e_R^\sigma, w_L^\sigma + e_L^\sigma\}$, then either $\theta > w_L^\sigma + e_L^\sigma$, in which case proposal n is not accepted by L, so R weakly prefers proposal P^π because, by definition of π, $\pi e_R^\sigma \geq 0$; or $w_R^\sigma + \pi e_R^\sigma < \theta \leq w_L^\sigma + e_L^\sigma$, in which case proposal n is accepted by L but, as argued above, since $w_R^\sigma + \pi e_R^\sigma < \theta$, R prefers proposal P^π to proposal n. Thus, we set ϕ to prescribe P^π, which is stage undominated. This completes the definition of ϕ.

Recall that σ is such that $w_L^\sigma \leq w_R^\sigma$, $e_L^\sigma \geq 0$ and π is such that $\pi = 0$ when $e_R^\sigma < 0$ and $\pi = 1$ when $e_R^\sigma > 0$. By construction, $\phi(\pi, w^\sigma, e^\sigma)$ is stage undominated given continuation payoff (w^σ, e^σ). Therefore, if

$$(w^\sigma, e^\sigma) = \left(w^{\phi(\pi, w^\sigma, e^\sigma)}, e^{\phi(\pi, w^\sigma, e^\sigma)}\right),$$

then Lemma 4 implies that σ is an equilibrium. Using Lemma 5 Part (i), the above condition is equivalent to

$$(\pi, w^\sigma, e^\sigma) = \left(\pi, W^{\phi(\pi, w^\sigma, e^\sigma)}(w^\sigma, e^\sigma), E^{\phi(\pi, w^\sigma, e^\sigma)}(w^\sigma, e^\sigma)\right). \tag{8}$$

Thus, letting $\Phi(\pi, w^\sigma, e^\sigma)$ denote the R.H.S. of (8), to prove the lemma, it suffices to show that Φ has a fixed point (π^*, w^*, e^*) such that $w_R^* \leq w_R^*$, $e_L^* \geq 0$, $\pi^* = 0$ if $e_R^* < 0$ and $\pi^* = 1$ if $e_R^* > 0$. Hence, it is enough to show that Φ has a fixed point in D, where

$$D \equiv \left\{ (\pi^*, w^*, e^*) \in [0,1] \times [-B, B]^4 : w_L^* \leq w_R^*, \ e_L^* \geq 0, \ \pi^* = \left\{ \begin{array}{l} 0 \text{ if } e_R^* < 0 \\ 1 \text{ if } e_R^* > 0 \end{array} \right\} \right\},$$

and $B \equiv \frac{\max_i\{|w_i| + |e_i| + E(|\theta|)\}}{1-\delta}$ is a bound on continuation payoff parameters. Since D is a compact space, Brower's theorem implies the existence of a fixed point of Φ in D if Φ is continuous and $\Phi(D) \subseteq D$.

Let us first prove that Φ is continuous. One can easily see from the definition of ϕ that for all $x, y \in \{n, p, q\}$ and $(\pi^*, w^*, e^*) \in D$, the probability that $\phi(\pi^*, w^*, e^*)$ prescribes to replace status quo x by y in state θ is piece-wise constant in θ. Moreover, on each of the interval on which this probability is constant in θ, it is continuous in (π^*, w^*, e^*). Finally, the bounds of these intervals depend continuously on (π^*, w^*, e^*). Therefore, if we substitute $\sigma = \phi(\pi^*, w^*, e^*)$ in (6) and (7), the continuity of F implies that the integrals on the R.H.S. of these equations must be continuous in (π^*, w^*, e^*), which implies the continuity of Φ.

To complete the proof, it remains to show that $\Phi(D) \subseteq D$. To do so, the only nontrivial condition to check is that if $w_L^* \leq w_R^*$ and $e_L^* \geq 0$, then $W_L^{\phi(\pi^*, w^*, e^*)}(, w^*, e^*) \leq W_R^{\phi(\pi^*, w^*, e^*)}(w^*, e^*)$ and $E_R^{\phi(\pi^*, w^*, e^*)}(w^*, e^*) \geq 0$. In what follows, for notational convenience, $\phi(\pi, w^*, e^*)$ is denoted ϕ. Since ϕ is stage undominated given continuation payoff parameters (w^*, e^*), Lemma 5 Part (ii) implies that

$$\frac{E_i^\phi(w^*, e^*) - e_i}{\delta} = \int_{\Upsilon^\phi(p,p) \cap \Upsilon^\phi(q,q)} e_i^* d\mu(v) + \int_{\Upsilon^\phi(p,p) \cap \Upsilon^\phi(q,n)} (\theta(v) - w_i^*) d\mu(v)$$
$$+ \int_{\Upsilon^\phi(p,n) \cap \Upsilon^\phi(q,q)} (w_i^* + e_i^* - \theta(v)) d\mu(v).$$

By construction of ϕ, for all $v \in \Upsilon^\phi(p, n)$, $\theta(v) \leq w_i^*$, so $w_i^* + e_i^* - \theta(v) \geq e_i^*$. Substituting the latter inequality inside the last integral on the R.H.S. of the above equation, we obtain

$$\frac{E_i^\phi(w^*, e^*) - e_i}{\delta} \geq \int_{\left(\Upsilon^\phi(p,p) \cup \Upsilon^\phi(p,n)\right) \cap \Upsilon^\phi(q,q)} e_i^* d\mu(v) + \int_{\Upsilon^\phi(p,p) \cap \Upsilon^\phi(q,n)} (\theta(v) - w_i^*) d\mu(v).$$

By construction of ϕ, when $\theta < w_L^*$, status quo p is always replaced by n, so for all $v \in \Upsilon^\phi(p, p)$, $\theta(v) \geq w_L^*$. Therefore, the second integral on the R.H.S. of the above equation must be weakly positive for $i = L$. By assumption, $e_L^* \geq 0$ implying the first integral is also weakly positive; thus $E_L^\phi(w^*, e^*) \geq e_L > 0$.

Since ϕ is stage undominated given continuation payoff parameters (w^*, e^*), Lemma 5 Part (ii) implies

$$\frac{W_i^\phi(w^*, e^*) - w_i}{\delta} = \int_{v \in \Upsilon^\phi(p,p) \cap \Upsilon^\phi(n,n)} (w_i^* - \theta(v)) d\mu(v) - \int_{v \in \Upsilon^\phi(p,p) \cap \Upsilon^\phi(n,q)} e_i^* d\mu(v),$$

so

$$\frac{W_R^\phi(w^*, e^*) - W_L^\phi(w^*, e^*) - (w_R - w_L)}{\delta}$$

$$= \int_{v \in \Upsilon^\phi(p,p) \cap \Upsilon^\phi(n,n)} (w_R^* - w_L^*) d\mu(v) - \int_{v \in \Upsilon^\phi(p,p) \cap \Upsilon^\phi(n,q)} (e_R^* - e_L^*) d\mu(v).$$

If $e_R^* \leq 0$, then $e_R^* \leq e_L^*$. Substituting this inequality, $w_L^* \leq w_R^*$, and $w_L < w_R$, into the above equation, we obtain $W_R^\phi(w^*, e^*) > W_L^\phi(w^*, e^*)$. If $e_R^* > 0$, then by construction of ϕ, $\Upsilon^\phi(n, q) = \emptyset$ so the above equation also implies that $W_R^\phi(w^*, e^*) > W_L^\phi(w^*, e^*)$. ∎

Proof of Proposition 2. Proposition 2 follows from Lemma 2 part A) below. ∎

Lemma 7 (Necessary and Sufficient Conditions for EE)

A) Let σ be an EE. Then $e_L^\sigma > 0$, $e_R^\sigma \geq 0$, $w_L^\sigma < w_L < w_R < w_R^\sigma$, and for all $i \in \{L, R\}$,

$$w_i^\sigma = w_i + \delta \int_{w_L^\sigma}^{w_R^\sigma} (w_i^\sigma - \theta) dF(\theta). \tag{9}$$

B) For any EE σ, there exists an EE σ' such that $w^{\sigma'} = w^\sigma$ and for all $i \in \{L, R\}$,

$$e_i^{\sigma'} = e_i + \delta \int_{w_L^\sigma}^{\min\{w_L^\sigma + e_L^{\sigma'}, w_R^\sigma\}} b_R(\theta, q)(\theta - w_i^\sigma) dF(\theta). \tag{10}$$

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C) *Reciprocally, if there exists $(w^*, e^*) \in \mathbb{R}^4$ which satisfy $e_R^* \geq 0$ and the same conditions as $\left(w^\sigma, e^{\sigma'}\right)$ in (9) and (10), then there exists an EE σ such that $(w^\sigma, e^\sigma) = (w^*, e^*)$.*

Proof. Part A:

Let σ be an arbitrary EE.

Step A1: $w_L^\sigma < w_R^\sigma$ and for all $i \in \{L, R\}$,

$$w_i^\sigma = w_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)} (w_i^\sigma - \theta(\upsilon)) \, d\mu(\upsilon). \tag{11}$$

From Lemma 4, σ prescribes stage undominated actions given continuation play σ. Therefore, Lemma 5 Part (i) implies that $w_i^\sigma = W_i^\sigma(w_i^\sigma, e_i^\sigma)$, and Lemma 5 Part (ii) further implies that $W_i^\sigma(w_i^\sigma, e_i^\sigma)$ is given by (6) where $(w_i^*, e_i^*) = (w_i^\sigma, e_i^\sigma)$. Moreover, since σ is an EE, $\mu(\Upsilon^\sigma(n, q)) = 0$. Substituting the latter equality into (7), we obtain (11).

Taking differences across players in (11) and solving for $w_R^\sigma - w_L^\sigma$, we obtain $w_R^\sigma - w_L^\sigma = \frac{w_R - w_L}{1 - \delta\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n))}$, which proves $w_L^\sigma < w_R^\sigma$.

Step A2: $e_L^\sigma \geq 0$ and $e_R^\sigma \geq 0$

Suppose that $e_i^\sigma < 0$ for some $i \in \{L, R\}$. Then in any period t in which $s_t = n$, the proposer is i, and $\theta_t > \max_{k \in \{L,R\}} \{w_k^\sigma + e_k^\sigma\}$, stage undomination implies that the other player j must accept proposal q. Since q is the outcome that gives the greatest continuation payoff to i in that state, the only stage undominated action for i is to propose q, which contradicts the assumption that σ is an EE.

Step A3: modulo a zero measure set, $\Upsilon^\sigma(p, p) = \{\upsilon \in \Upsilon : \theta(\upsilon) > w_L^\sigma\}$ and $\Upsilon^\sigma(n, n) = \{\upsilon \in \Upsilon : \theta(\upsilon) < w_R^\sigma\}$.

For all $\upsilon \in \Upsilon$ such that $\theta(\upsilon) > w_L^\sigma$, L strictly prefers to implement p to n, so stage undomination implies that $\upsilon \notin \Upsilon^\sigma(p, n)$. Since σ is an EE, $\mu(\Upsilon^\sigma(p, q)) = 0$. By definition of Υ^σ, $(\Upsilon^\sigma(p, x))_{x \in \{n,p,q\}}$ is a partition of Υ, so necessarily, $\upsilon \in \Upsilon^\sigma(p, p)$. Conversely, for all $\upsilon \in \Upsilon$ such that $\theta(\upsilon) < w_L^\sigma$, from Step A1, $\theta(\upsilon) < w_R^\sigma$, so both players strictly prefer to implement n to p. Therefore, stage undomination implies that $\upsilon \notin \Upsilon^\sigma(p, p)$. Since F is atomless, the set of υ such that $\theta(\upsilon) = w_L^\sigma$ has probability 0, which completes the proof of the first equality in Step A3.

For all $\upsilon \in \Upsilon$ such that $\theta(\upsilon) < w_R^\sigma$, R strictly prefers to implement n to p, so stage undomination implies that $\upsilon \notin \Upsilon^\sigma(n, p)$. Since σ is an EE, $\mu(\Upsilon^\sigma(n, q)) = 0$. Since $(\Upsilon^\sigma(n, x))_{x \in \{n,p,q\}}$ is a partition of Υ, necessarily, $\upsilon \in \Upsilon^\sigma(n, n)$. Conversely, for all $\upsilon \in \Upsilon$ such that $\theta(\upsilon) > w_R^\sigma$ and, from Step A1, $\theta(\upsilon) > w_L^\sigma$, both players strictly prefer to implement p to n. Therefore, stage undomination implies that $\upsilon \notin \Upsilon^\sigma(n, n)$. The second equality in Step A3 follows then from the fact that F is atomless.

Step A4: (w_L^σ, w_R^σ) satisfies (9) and $w_L^\sigma < w_L < w_R < w_R^\sigma$.

From Step A3, modulo a zero measure set, $\Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(n, n)$ is equal to the set of $\upsilon \in \Upsilon$ such that $\theta(\upsilon) \in (w_L^\sigma, w_R^\sigma)$. Substituting this equality into (11), we obtain (9). Step A1 and the assumption that F has full support imply

$$\int_{w_L^\sigma}^{w_R^\sigma} (w_L^\sigma - \theta) \, dF(\theta) < 0 < \int_{w_L^\sigma}^{w_R^\sigma} (w_R^\sigma - \theta) \, dF(\theta),$$

so (9) implies in turn that $w_L^\sigma < w_L < w_R < w_R^\sigma$.

Step A5: $e_L^\sigma > 0$

Let us first prove that $\mu(\Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(q, q)) = 0$. Suppose by contradiction that the latter probability is positive. Then for some $\upsilon \in \Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(q, q)$, both players strictly prefer to implement n to p and one player weakly prefers to implement q to n, so $\min_i \{w_i^\sigma + e_i^\sigma\} \leq \theta(\upsilon) < w_L^\sigma$ and, therefore, $\min_i \{w_i^\sigma + e_i^\sigma\} < w_L^\sigma$, a contradiction with Step A1 and A2. From Lemma 4, σ prescribes stage undominated actions given continuation play σ. So Lemma 5 Part (i) implies that $e_i^\sigma = E_i^\sigma(w_i^\sigma, e_i^\sigma)$, and Lemma 5 Part (ii) further implies that $E_i^\sigma(w_i^\sigma, e_i^\sigma)$ is given by (7) with $(w_i^*, e_i^*) = (w_i^\sigma, e_i^\sigma)$. Substituting $\mu(\Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(q, q)) = 0$ into (7), we obtain

$$e_i^\sigma = e_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,q)} e_i^\sigma \, d\mu(\upsilon) + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma) \, d\mu(\upsilon). \qquad (12)$$

From Step A3, for almost all $\upsilon \in \Upsilon^\sigma(p, p)$, $\theta(\upsilon) > w_L^\sigma$, so (12) implies that $e_L^\sigma \geq e_L > 0$.

Step A6: $\Upsilon^\sigma(q, q) \subseteq \Upsilon^\sigma(p, p)$

For all $\upsilon \in \Upsilon^\sigma(q, q)$, stage undomination implies that one player weakly prefers to implement q to n, so $\theta(\upsilon) \geq \max \{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\}$. From Step A1, A2, and A5, this implies that $\theta(\upsilon) > w_L^\sigma$ and so, as shown in Step A3, $\upsilon \in \Upsilon^\sigma(p, p)$.

Part B:

This step basically shows that for any EE σ, there exists an "equivalent" EE σ' such that under status quo q, σ' prescribes players to play pure strategies which never implement q irrespective of the status quo, and which lead to the same path of play as σ under initial status quo n. We construct this strategy profile σ' as the limit of a sequence of strategy profiles $(\sigma^k)_{k \in \mathbb{N}}$ which we define recursively as follows.

Step B1: construction of the sequence of strategy profiles $(\sigma^k)_{k \in \mathbb{N}}$ such that for all $k \in \mathbb{N}$, $w^{\sigma^k} = w^\sigma$.

We set $\sigma^0 = \sigma$. Suppose now that we have constructed $\sigma^0, ..., \sigma^k$ for some $k \in \mathbb{N}$. Consider first a Markov state in which the veto player moves. We set σ^{k+1} to prescribe the same

actions as σ if neither the status quo nor the proposal is q. When comparing p and q, we set σ^{k+1} to prescribe the veto player to vote in favor of p. Finally, when comparing n and q, we set σ^{k+1} to prescribe the veto player to vote in favor of q if and only if $\theta(\upsilon) > w_i^\sigma + e_i^{\sigma^k}$. Now consider the Markov states in which the proposer moves. We set σ^{k+1} to prescribe the same actions as σ when the status quo is not q. When the status quo is q, σ^{k+1} prescribes L to propose n when $\theta \leq w_L^\sigma$, and p when $\theta > w_L^\sigma$, and σ^{k+1} prescribes R to propose n when $\theta \leq \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}$ and p when $\theta > \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}$.

By construction, for all $k \in \mathbb{N}$, σ^k prescribes the same actions as σ when neither the status quo nor the proposal is q. Since σ is an EE, this implies that q is never implemented on the path of play of σ^k, and therefore that $w^{\sigma^k} = w^\sigma$.

Step B2: statements of the properties satisfied by $\left(\sigma^k\right)_{k\in\mathbb{N}}$

In the following steps, we show by induction on k that for all $k \in \mathbb{N}$, σ^k satisfies the following properties: (i) the actions prescribed by σ^k are stage undominated given continuation play σ^{k-1}, (ii) $0 < e_L^{\sigma^k} \leq e_L^{\sigma^{k-1}}$, $e_R^{\sigma^k} \geq e_R^{\sigma^{k-1}} \geq 0$, and (iii) for all $i \in \{L, R\}$,

$$e_i^{\sigma^k} = e_i + \delta \int_{w_L^\sigma}^{\min\left\{w_L^\sigma + e_L^{\sigma^{k-1}}, w_R^\sigma\right\}} b_R(\theta, q)(\theta - w_i^\sigma) \, dF(\theta). \tag{13}$$

Note first that since $\sigma^0 = \sigma$, from step A2 and A5, we have $e_L^{\sigma^0} > 0$ and $e_R^{\sigma^0} \geq 0$, so property (ii) is satisfied, which is the only condition we have to check for $k = 0$. In what follows, we assume that for some $k \in \mathbb{N}$, for all all $k' = 1, ..., k$, $\sigma^{k'}$ satisfies properties (i), (ii), and (iii), and prove that σ^{k+1} satisfies the same properties.

Step B3: σ^{k+1} *satisfies property* (i).

Consider first a Markov state in which the veto player moves and neither the status quo nor the proposal is q. From Step B1, $w^{\sigma^k} = w^\sigma$, so it is stage undominated for the veto player to play σ (or equivalently σ^{k+1}) given continuation play σ^k.

Consider now a Markov state in which the veto player $i \in \{L, R\}$ must choose between p and q. By the induction hypothesis, $e_i^{\sigma^k} \geq 0$, so it is stage undominated for i to vote for p, given continuation play σ^k.

In the Markov states in which the veto player i must choose between n and q, it is stage undominated for i to vote for q if and only if $\theta(\upsilon) > w_i^\sigma + e_i^{\sigma^k}$, as prescribed by σ^{k+1}.

Consider now a Markov state in which proposer $i \in \{L, R\}$ moves and the status quo is not q. Since σ^{k+1} coincides with σ on such Markov states, and since σ is an EE, the only potentially profitable deviations we need to rule out are deviations in which i proposes q. Suppose by contradiction that this deviation is profitable for i given continuation play σ^k. From the induction hypothesis, $e_i^{\sigma^k} \geq 0$, so p gives a weakly greater continuation payoff than

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q to i. Since q is a profitable deviation, the outcome prescribed by σ^k cannot be p, so it must be n, and i must strictly prefer implementing q to n. Since $e_i^{\sigma^k} \geq 0$, she must also strictly prefer implementing p to n, and since p is not the outcome prescribed by σ^k, the status quo must be n, and the veto player j must veto proposal p under status quo n. For the deviation to be profitable to i, j must also accept proposal q under status quo n. Since $e_j^{\sigma^k} \geq 0$, this implies that j is indifferent between implementing n, p, and q, so $\theta = w_j^\sigma + e_j^{\sigma^k}$. By construction of σ^k, we have assumed that in such states of nature, j vetoes proposal q, a contradiction.

Consider then the Markov states in which the status quo is q and proposer L moves. Since $e_L^{\sigma^k} \geq 0$, for all $\theta \leq w_L^\sigma$, we have $\theta \leq w_L^\sigma + e_L^{\sigma^k}$ so, given continuation play $e_L^{\sigma^k}$, n is the alternative that gives L the greatest continuation payoff and, since $w_L^\sigma < w_R^\sigma$, we have $\theta < w_R^\sigma \leq w_R^\sigma + e_R^{\sigma^k}$; by construction of σ^{k+1}, therefore, R accepts proposal n. Hence, it is stage undominated for L to propose n when $\theta \leq w_L^\sigma$, as prescribed by σ^{k+1}. When $\theta > w_L^\sigma$, p is the alternative that gives L the greatest continuation payoff, and R accepts it. So it is stage undominated for L to propose p, as prescribed by σ^{k+1}.

Consider finally the Markov states in which the status quo is q and proposer R moves. When $\theta \leq w_R^\sigma$, we have $\theta \leq w_R^\sigma + e_R^{\sigma^k}$ so, given continuation play σ^k, n is the alternative that gives R the greatest continuation payoff and σ^{k+1} prescribes L to accept proposal n. So when $\theta \leq \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}$, it is stage undominated for R to propose n. When $\theta > w_L^\sigma + e_L^{\sigma^k}$, σ^{k+1} prescribes L to veto n, so n leads to outcome q, which gives R a weakly smaller continuation payoff than proposing p. Therefore, it is stage undominated for R to propose p. When $\theta > w_R^\sigma$, p is the alternative that gives the greatest continuation payoff to R and, since $\theta > w_R^\sigma > w_L^\sigma$, σ^{k+1} prescribes L to accept p, it is also stage undominated for R to propose p. Thus, we have shown that it is stage undominated for R to propose n when $\theta \leq \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}$, and p when $\theta > \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}$, as prescribed by σ^{k+1}.

Step B4: σ^{k+1} satisfies property (iii).

By definition, $e_i^{\sigma^{k+1}}$ is the relative gain in continuation payoff of implementing p instead of q given continuation play $e_i^{\sigma^k}$. It is equal to e_i plus δ times the expected gain from having status quo p instead of q in the next period. To compute the latter expected gain, note that by construction of σ^k, when $\theta < w_L^\sigma$, status quo q and p both lead to outcome n; when $\theta > w_R^\sigma$, status quo q and p both lead to outcome p; when $\theta \in (w_L^\sigma, w_R^\sigma)$ and L is the proposer, status quo q and p both lead to outcome p; when $\theta \in \left(\min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}, w_R^\sigma\right)$ and R is the proposer, status quo q and p both lead to outcome p. Thus, in all the aforementioned cases, the expected gain in continuation payoff from having status quo p instead of q is 0. Modulo a zero measure states of nature, the only remaining case to consider is when R is the proposer and $\theta \in \left(w_L^\sigma, \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}\right)$. In this case, status quo q leads to outcome n

while status quo p stays in place and the gain in continuation payoff from having status quo p instead of q is $\theta - w_i^{\sigma^k} = \theta - w_i^\sigma$, which proves property (iii).

Step B5: σ^{k+1} satisfies property (ii).

From Step B4 and the induction hypothesis, both σ^k and $\sigma^k + 1$ satisfy property (iii), so

$$e_i^{\sigma^{k+1}} - e_i^{\sigma^k} = -\delta \int_{\min\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\}}^{\min\{w_L^\sigma + e_L^{\sigma^{k-1}}, w_R^\sigma\}} b_R(\theta, q)(\theta - w_i^\sigma) \, dF(\theta). \tag{14}$$

From the induction hypothesis, σ^k satisfy property (ii), so

$$w_L^\sigma \le \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\} \le \min\left\{w_L^\sigma + e_L^{\sigma^{k-1}}, w_R^\sigma\right\} \le w_R^\sigma.$$

The above inequalities imply that the right-hand side of (14) is negative for $i = L$ and positive for $i = R$, which proves that $e_L^{\sigma^{k+1}} \le e_L^{\sigma^k}$ and $e_R^{\sigma^{k+1}} \le e_R^{\sigma^k}$. Finally, (13) implies $e_L^{\sigma^{k+1}} \ge e_L > 0$.

Step B6: $\left(\sigma^k\right)_{k \in \mathbb{N}}$ has a limit σ' which satisfies the properties stated in part B) of the lemma.

We have shown by induction that for all $k \in \mathbb{N}$, σ^k satisfies property (ii), so $\left(e_L^{\sigma^k}\right)_{k \in \mathbb{N}}$ is decreasing and bounded below. As such, it converges to some limit e_L^∞. By construction, σ^{k+1} depends only and continuously on $e_L^{\sigma^k}$ (see Step B1), so $\left(\sigma^k\right)_{k \in \mathbb{N}}$ converges as well to some limit σ'. Since for all $k \in \mathbb{N}$, σ^k satisfies property (i), by continuity, the actions prescribed by σ' must be stage undominated given continuation play σ'. Taking the limit in (13), we obtain that σ' satisfies (10), $e_L^{\sigma'} > 0$ and $e_R^{\sigma'} \ge 0$. By construction of $\left(\sigma^k\right)_{k \in \mathbb{N}}$, σ' never leads to outcome q irrespective of the status quo, as needed.

Part C:

In this part, we assume that there exists $(w^*, e^*) \in \mathbb{R}^4$ which satisfies $e_R^* \ge 0$, and the same conditions as w^σ and $e^{\sigma'}$ in (9) and (10), and we construct an EE σ such that $(w^\sigma, e^\sigma) = (w^*, e^*)$. Let σ be a strategy profile which is stage undominated when players expect continuation payoff parameters (w^*, e^*) and such that, whenever a player i is indifferent between q and p given continuation payoff (w^*, e^*), σ prescribes i to break the indifference in favor of p. The strategy profile $\phi(\pi, w^\sigma, e^\sigma)$ constructed in the proof of Lemma 6, with parameters $(\pi, w^\sigma, e^\sigma) = (1, w^*, e^*)$ satisfies these properties.

One can see from (10) that $e_L^* \ge e_L$, so $e_L^* > 0$. Together with the assumption that $e_R^* \ge 0$ and the definition of σ, this implies that q is never implemented on the path of play of σ. Therefore, to complete the proof, it suffices to show that σ is an equilibrium. From Lemma 4, it is equivalent to show that σ prescribes stage undominated actions given continuation payoff parameters (w^σ, e^σ). Note that, by construction of σ, this is the case if $(w^\sigma, e^\sigma) = (w^*, e^*)$.

From Lemma 5 Part (i), $(w^\sigma, e^\sigma) = (w^*, e^*)$ if $w^* = W^\sigma(w^*, e^*)$ and $e^* = E^\sigma(w^*, e^*)$. To complete the proof, we show below that the latter two equations are satisfied.

Step C1: $w_L^* < w_R^*$ and $w^* = W^\sigma(w^*, e^*)$.

By construction of σ, q is never implemented on the path of play of σ, so $\Upsilon^\sigma(n, q) = \emptyset$ and σ is stage undominated given continuation payoff parameters (w^*, e^*). By Lemma 5 Part (ii) implies that

$$W_i^\sigma(w^*, e^*) = w_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)} (w_i^* - \theta(v)) \, d\mu(v). \tag{15}$$

As in Step A1, taking differences across players in (9) and solving for $w_R^* - w_L^*$, we obtain that $w_L^* < w_R^*$. By construction, σ is stage undominated given continuation payoff parameters (w^*, e^*) so the same reasoning as in Step A3 implies that, modulo a zero measure set, $\Upsilon^\sigma(p, p) = \{v \in \Upsilon : \theta(v) > w_L^*\}$ and $\Upsilon^\sigma(n, n) = \{v \in \Upsilon : \theta(v) < w_R^*\}$. Together with (15), this implies that

$$W_i^\sigma(w^*, e^*) = w_i + \delta \int_{w_L^*}^{w_R^*} (w_i^* - \theta) \, dF(\theta).$$

Since w^* satisfies (9), the above equation implies that $w_i^* = W_i^\sigma(w_i^*, e_i^*)$.

Step C2: modulo a zero measure set of states of the world, $\Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n)$ is the set of $v \in \Upsilon$ such that $\theta(v) \in (w_L^*, \min\{w_L^* + e_L^*, w_R^*\})$ and R is the proposer.

Let $v \in \Upsilon$. Case 1: v is such that $\theta(v) < w_L^*$. Then from Step C1, $\theta(v) < w_R^*$, so both players get a strictly greater continuation payoff from n than from p, which implies that $v \notin \Upsilon^\sigma(p, p)$, and thus $v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n)$.

Case 2: v is such that $\theta(v) > w_R^*$. Then from Step C1, $\theta(v) > w_L^*$, so both players get a strictly greater continuation payoff from p than from n, which implies that $v \notin \Upsilon^\sigma(q, n)$, and thus $v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n)$.

Case 3: v is such that $\theta(v) \in (w_L^*, w_R^*)$ and L is the proposer. Since $\theta(v) > w_L^*$, L strictly prefers to implement p to n and, since σ prescribes L to behave as if L strictly prefers p to q, L proposes p whenever it is accepted. By construction of σ, R always accepts p under status quo q, so $v \in \Upsilon^\sigma(q, p)$, which implies that $v \notin \Upsilon^\sigma(q, n)$ and thus $v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n)$.

Case 4: v is such that $\theta(v) \in (\min\{w_L^* + e_L^*, w_R^*\}, w_R^*)$ and R is the proposer. In that case, $\theta(v) > w_L^* + e_L^*$, so σ prescribes L to reject n under status quo q, so $v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n)$.

Case 5: v is such that $\theta(v) \in (w_L^*, \min\{w_L^* + e_L^*, w_R^*\})$ and R is the proposer. As argued in Step C1, $\theta(v) > w_L^*$ implies that $v \in \Upsilon^\sigma(p, p)$. Since $\theta(v) < w_R^*$, $\theta(v) < w_R^* + e_R^*$, so R gets a greater continuation payoff from n than from p or q. Since $\theta(v) < w_L^* + e_L^*$, σ prescribes L to accept proposal n under status quo q, so the only stage undominated action for R is to propose n. Therefore, $v \in \Upsilon^\sigma(q, n)$, so $v \in \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n)$.

Since F is continuous, modulo a zero measure set, cases 1 to 5 above form a partition of

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Υ. Step C2 follows then from the observation that modulo a zero measure set, cases 1 to 5 above cover all possible states of the world, and $\upsilon \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$ only in case 5.

   Step C3: $e^* = E^\sigma(w^*, e^*)$.

By construction of σ, q is never implemented on the path of play of σ, so $\Upsilon^\sigma(q,q) = \emptyset$. By construction, σ is stage undominated given continuation payoff parameters (w^*, e^*). Successively using Lemma 5 Part (ii), $\Upsilon^\sigma(q,q) = \emptyset$ and Step C2, we obtain

$$
\begin{aligned}
E_i^\sigma(w^*, e^*) &= e_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^*) \, d\mu(\upsilon) \\
&= e_i + \delta \int_{w_L^*}^{\min\{w_L^* + e_L^*, w_R^*\}} b_R(\theta, q)(\theta - w_i^*) \, dF(\theta).
\end{aligned}
$$

Since e_i^* satisfies (10), the above equation implies that $e_i^* = E_i^\sigma(w^*, e^*)$. ■

   The following two lemmas derive some properties that are common to all IE, and to all equilibria, respectively.

Lemma 8 (Properties of IE) *Let σ be an IE, let Λ denote $\arg\min_{i \in \{L,R\}} \{w_i^\sigma\}$ and let ϱ denote $\arg\max_{i \in \{L,R\}} \{w_i^\sigma\}$. Then $e_\varrho^\sigma \leq 0 < e_\Lambda < e_\Lambda^\sigma$, $w_\Lambda^\sigma < w_\varrho^\sigma$, and $w_\Lambda^\sigma < \min_{i \in \{L,R\}} \{w_i^\sigma + e_i^\sigma\}$.*

   Proof. Let σ be an arbitrary IE.
   Step 1: $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) > 0$.
Since σ is an equilibrium, the actions prescribed by σ are stage undominated given continuation payoff (w^σ, e^σ). Using Lemma 5 Parts (i) and (ii) for $(w_i^*, e_i^*) = (w_i^\sigma, e_i^\sigma)$, we obtain

$$
\begin{aligned}
e_i^\sigma &= E_i^\sigma(\sigma, w^\sigma, e^\sigma) \\
&= e_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,q)} e_i^\sigma \, d\mu(\upsilon) + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma) \, d\mu(\upsilon) \quad (16) \\
&\quad + \delta \int_{\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)} (w_i^\sigma + e_i^\sigma - \theta(\upsilon)) \, d\mu(\upsilon).
\end{aligned}
$$

In any state $\upsilon \in \Upsilon^\sigma(p,n)$, each player i must prefer implementing n to p so $\theta(\upsilon) \leq w_i^\sigma$ and, therefore, $w_i^\sigma + e_i^\sigma - \theta(\upsilon) \geq e_i^\sigma$. Substituting the latter inequality in the above equation, we obtain

$$
e_i^\sigma \geq e_i + \delta \left(\int_{(\Upsilon^\sigma(p,p) \cup \Upsilon^\sigma(p,n)) \cap \Upsilon^\sigma(q,q)} e_i^\sigma \, d\mu(\upsilon) + \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma) \, d\mu(\upsilon) \right).
$$

Regrouping the terms in factor of e_i^σ yields

$$e_i^\sigma \geq \frac{e_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(v) - w_i^\sigma) \, d\mu(v)}{1 - \delta\mu((\Upsilon^\sigma(p,p) \cup \Upsilon^\sigma(p,n)) \cap \Upsilon^\sigma(q,q))}. \tag{17}$$

Suppose $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) = 0$. Then from (17), $e_L^\sigma > 0$ and $e_R^\sigma > 0$, that is, both players always strictly prefer to implement p to q. But then q is never implemented on the equilibrium path, which is impossible since σ is an IE.

Step 2: $w_\Lambda^\sigma < \min_{i \in \{L,R\}} \{w_i^\sigma + e_i^\sigma\}$.
For all $v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$, both players weakly prefer to implement n to q, i.e., $\theta(v) \leq w_i^\sigma + e_i^\sigma$, and at least one player weakly prefers to implement p to n, i.e., $\theta(v) \geq w_i^\sigma$, so

$$\min_{i \in \{L,R\}} w_i^\sigma \leq \theta(v) \leq \min_{i \in \{L,R\}} (w_i^\sigma + e_i^\sigma).$$

From step 1, $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) > 0$. Therefore, the above inequalities must hold strictly for some $v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$, inmplying $\min_{i \in \{L,R\}} w_i^\sigma < \min_{i \in \{L,R\}} (w_i^\sigma + e_i^\sigma)$ as needed.

Step 3: $e_\varrho^\sigma \leq 0$ and $e_\Lambda^\sigma > e_\Lambda > 0$.
For all $v \in \Upsilon^\sigma(p,p)$, at least one player weakly prefers to implement p to n and, since $w_\Lambda^\sigma \leq w_\varrho^\sigma$, we must have $\theta(v) - w_\Lambda^\sigma \geq 0$. Since F is continuous, the latter inequality must be strict for almost all $v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$. Substituting this inequality into (17) and using $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) > 0$, we obtain $e_\Lambda^\sigma > e_\Lambda > 0$. As argued in Step 1, since σ is an IE, we cannot have $e_L^\sigma > 0$ and $e_R^\sigma > 0$, so $e_\varrho^\sigma \leq 0$.

Step 4: $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$.
Let $v \in \Upsilon^\sigma(q,q)$. Since σ is an equilibrium, in state $\theta(v)$, one player must weakly prefer to implement q to n. Thus, $\theta(v) \geq \min_{i \in \{L,R\}} \{w_i^\sigma + e_i^\sigma\}$. From Step 2, this implies that $\theta(v) > \min_{i \in \{L,R\}} \{w_i^\sigma\}$, in which case one player must strictly prefer to implement p to n and $v \notin \Upsilon^\sigma(p,n)$. From Step 3, $e_\Lambda^\sigma > 0$ so $v \notin \Upsilon^\sigma(p,q)$. Since $(\Upsilon^\sigma(p,x))_{x \in \{n,p,q\}}$ is a partition of Υ, this implies that $v \in \Upsilon^\sigma(p,p)$, as needed.

Step 5: $w_\Lambda^\sigma < w_\varrho^\sigma$.
From Step 3, $e_\varrho^\sigma \leq 0$, so (17) implies that $\int_{v \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(v) - w_\varrho^\sigma) \, d\mu(v) < 0$. Therefore, there exists $\Upsilon^o \subset \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$ such that $\mu(\Upsilon^o) > 0$ and, for all $v^o \in \Upsilon^o$, $\theta(v^o) - w_\varrho^\sigma < 0$. That is, in all states in $v^o \in \Upsilon^0$, ϱ strictly prefers to implement n to p. Since $\Upsilon^o \subset \Upsilon^\sigma(p,p)$ and since σ is an equilibrium, Λ must weakly prefer to implement p to n in state v^o, i.e., $\theta(v^o) - w_\Lambda^\sigma \geq 0$, which implies $w_\Lambda^\sigma < w_\varrho^\sigma$. $\blacksquare$

Lemma 9 (Properties common to EE and IE) *For any equilibrium σ,* $\max_{i \in \{L,R\}} \{e_i^\sigma\} >$

0, $\min_{i\in\{L,R\}}\{w_i^\sigma\} < \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$, and

$$e_i^\sigma = e_i + \delta\left(\int_{\Upsilon^\sigma(q,q)} e_i^\sigma d\mu(\upsilon) + \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma)d\mu(\upsilon)\right), \quad (18)$$

$$w_i^\sigma = w_i + \delta\left(\int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)} (w_i^\sigma - \theta(\upsilon))d\mu(\upsilon) - \int_{\Upsilon^\sigma(n,q)} e_i^\sigma d\mu(\upsilon)\right). \quad (19)$$

Moreover, $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$, $\Upsilon^\sigma(p,q) = \emptyset$, and $\Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p)$.

Proof. *Step 1: if σ is an EE, then $\max_{i\in\{L,R\}}\{e_i^\sigma\} > 0$, $\min_{i\in\{L,R\}}\{w_i^\sigma\} < \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$, (18), and $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$.*
Step A6 and Equation (12) in the proof of Lemma 7 imply $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ and (18), respectively. The inequalities $\max_{i\in\{L,R\}}\{e_i^\sigma\} > 0$ and $\min_{i\in\{L,R\}}\{w_i^\sigma\} < \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$ follow readily from $w_L^\sigma < w_R^\sigma$, $e_L^\sigma > 0$, and $e_R^\sigma \geq 0$, as established in Lemma 7 part A).
Step 2: if σ is an IE, then $\max_{i\in\{L,R\}}\{e_i^\sigma\} > 0$, $\min_{i\in\{L,R\}}\{w_i^\sigma\} < \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$, (18), and $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$.
The inequalities $\max_{i\in\{L,R\}}\{e_i^\sigma\} > 0$ and $\min_{i\in\{L,R\}}\{w_i^\sigma\} < \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$ follow from Lemma 8. The inclusion $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ follows from Step 4 of the proof of Lemma 8. By definition of Υ^σ, $\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(p,p) = \emptyset$, so $\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q) = \emptyset$. Substituting the latter identities into (16), we obtain (18).
Step 3: If σ is an equilibrium, then $\Upsilon^\sigma(p,q) = \emptyset$, and $\Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p)$.
From Steps 1 and 2, $\max_{i\in\{L,R\}}\{e_i^\sigma\} > 0$, that is, one player always strictly prefer to implement p to q, so $\Upsilon^\sigma(p,q) = \emptyset$. To show that $\Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p)$ note, that in any state $\upsilon \in \Upsilon^\sigma(n,q)$, both players weakly prefer to implement q to n. Hence, $\theta(\upsilon) \geq \max_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$. From Steps 1 and 2, this implies that $\theta(\upsilon) > \min_{i\in\{L,R\}}\{w_i^\sigma\}$, that is, one player strictly prefers to implement p to n, and therefore $\upsilon \notin \Upsilon^\sigma(p,n)$. Since $\Upsilon^\sigma(p,q) = \emptyset$, and since $(\Upsilon^\sigma(p,x))_{x\in\{n,p,q\}}$ form a partition of Υ, necessarily, $\upsilon \in \Upsilon^\sigma(p,p)$, as needed.
Step 4: If σ is an equilibrium then it satisfies (19).
If σ is an equilibrium, then from Lemma 4, the actions prescribed by σ are stage undominated given continuation payoff (w^σ, e^σ). Successively using Lemma 5 Part (i) and Part (ii) for $(w_i^*, e_i^*) = (w_i^\sigma, e_i^\sigma)$, we obtain

$$w_i^\sigma = W_i(\sigma, w^\sigma, e^\sigma) = w_i + \delta\left(\int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)} (w_i^\sigma - \theta(\upsilon))d\mu(\upsilon) - \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,q)} e_i^\sigma d\mu(\upsilon)\right).$$

From Step 3, $\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,q) = \Upsilon^\sigma(n,q)$. Substituting the last equality into the above equation gives (19). ∎

Proof of Proposition 3. Let σ be an IE. That $e_L^\sigma \leq 0 < e_R^\sigma$ or $e_R^\sigma \leq 0 < e_L^\sigma$, and that $\min\{w_L^\sigma, w_R^\sigma\} < \min\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\}$, follow readily from Lemma 8. The fact that q is repealed for a larger set of states than p follows from the inclusion $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ established in Lemma 9.

We now prove the characterization of the three kinds of IE. Using the notations of Lemma 8, suppose first that σ is such that $\Lambda = L$, $\varrho = R$ and $w_L^\sigma + e_L^\sigma < w_R^\sigma$. Then Lemma 8 implies that σ satisfies all the properties of an IE-A. Suppose now that $\Lambda = L$, $\varrho = R$ and $w_L^\sigma + e_L^\sigma \geq w_R^\sigma$. Then Lemma 8 implies that σ satisfies all the properties of an IE-B. Finally, the only remaining possibility is that $\Lambda = R$ and $\varrho = L$. In that case, once again, Lemma 8 implies that σ satisfies all the properties of an IE-C. ∎

Proof of Proposition 4. For all $\delta \in (0,1)$, let

$$M_\delta \equiv \left(w_L, \min\left\{w_L + (1-\delta)e_L, w_R - \frac{(1-\delta)e_R}{\delta\underline{b}}\right\}\right),$$

where $\underline{b}$ is the lower bound on $b_R(.,.)$ (see Section 3). Observe first that, for all $\delta \in (0,1)$, $w_L < w_L + (1-\delta)e_L$ and, for δ sufficiently close to 1, $w_L < w_R - \frac{(1-\delta)e_R}{\delta\underline{b}}$ so M_δ is not empty. Therefore, for all $\delta < 1$ sufficiently close to 1, there exist a closed set $\bar{M}_\delta \subset M_\delta$ with non empty interior. In what follows, we prove the following claim: for any such δ, there exists $\bar{v} > 0$ such that for all $v < \bar{v}$ and $m \in \bar{M}_\delta$, all equilibria are IE. This claim completes the proof because the set of (m,v) such that $m \in \bar{M}_\delta$ and $v \in (0, \bar{v})$ is clearly of positive Lebesgue measure.

Suppose the claim is false. Then there exists $\delta \in (0,1)$, two sequences $(m_k)_{k \in \mathbb{N}}$ and $(v_k)_{k \in \mathbb{N}}$, and a sequence of strategy profiles $(\sigma(k))_{k \in \mathbb{N}}$ such that $v_k \to 0$ and such that, for all $k \in \mathbb{N}$, $m_k \in \bar{M}_\delta$ and $\sigma(k)$ is an EE for the c.d.f. $F_k(\theta) \equiv G\left(\frac{\theta - m_k}{v_k}\right)$. From Lemma 7 part B), we can assume that $\sigma(k)$ satisfies (10). Since $\bar{M}_\delta$ is compact, we can assume w.l.o.g. that m_k tends to some limit $m \in \bar{M}_\delta$. Since $w_L^{\sigma(k)}, w_R^{\sigma(k)}, e_L^{\sigma(k)}$, and $e_R^{\sigma(k)}$ are bounded by $\max_{i \in \{L,R\}}\left(\frac{|w_i| + |e_i| + E_{m_k,v_k}(|\theta|)}{1-\delta}\right)$, and since $E_{m_k,v_k}(|\theta|) \to |m|$, we can also assume w.l.o.g. that $\left(w_L^{\sigma(k)}, w_R^{\sigma(k)}, e_L^{\sigma(k)}, e_R^{\sigma(k)}\right)$ tends to some limit $(w_L^\infty, w_R^\infty, e_L^\infty, e_R^\infty) \in \mathbb{R}^4$.

From Lemma 7 part A), for all $k \in \mathbb{N}$, $w_L^{\sigma(k)} < w_L < w_R < w_R^{\sigma(k)}$ and

$$w_i^{\sigma(k)} = w_i + \delta \int_{w_L^{\sigma(k)}}^{w_R^{\sigma(k)}} \left(w_i^{\sigma(k)} - \theta\right) dF_{m_k,v_k}(\theta). \tag{20}$$

As $v_k \to 0$, the support of F_k becomes increasingly concentrated around m. Since $w_L^{\sigma(k)} < w_L < m < w_R < w_R^{\sigma(k)}$, if we let $k \to \infty$ in (20), we get $w_i^\infty = w_i + \delta\left(w_i^\infty - m\right)$. Solving for w_i^∞, we obtain

$$w_i^\infty = w_i + \frac{\delta}{1-\delta}\left(w_i - m\right). \tag{21}$$

By construction, $\sigma\left(k\right)$ satisfies (10), so

$$
\begin{aligned}
e_i^{\sigma(k)} &= e_i + \delta \int_{w_L^{\sigma(k)}}^{\min\left\{w_L^{\sigma(k)}+e_L^{\sigma(k)},w_R^{\sigma(k)}\right\}} b_R\left(\theta,q\right)\left(\theta - w_i^{\sigma(k)}\right) dF_k\left(\theta\right) \\
&= e_i + \delta\left(m - w_i^{\sigma(k)}\right) \int_{w_L^{\sigma(k)}}^{\min\left\{w_L^{\sigma(k)}+e_L^{\sigma(k)},w_R^{\sigma(k)}\right\}} b_R\left(\theta,q\right) dF_k\left(\theta\right) \\
&\quad +\delta \int_{w_L^{\sigma(k)}}^{\min\left\{w_L^{\sigma(k)}+e_L^{\sigma(k)},w_R^{\sigma(k)}\right\}} b_R\left(\theta,q\right)\left(\theta - m\right) dF_k\left(\theta\right).
\end{aligned} \tag{22}
$$

Let π^∞ denote the limit of $\int_{w_L^{\sigma(k)}}^{\min\left\{w_L^{\sigma(k)}+e_L^{\sigma(k)},w_R^{\sigma(k)}\right\}} b_R\left(\theta,q\right) dF_k\left(\theta\right)$. If we let $k \to \infty$ in (22), we obtain

$$e_i^\infty = e_i + \delta\pi^\infty\left(m - w_i^\infty\right). \tag{23}$$

Adding w_i^∞ on each side of (23) and using (21), we obtain

$$w_L^\infty + e_L^\infty = w_L + e_L + \frac{\delta\left(\pi^\infty - 1\right)}{1-\delta}\left(m - w_L\right).$$

Since $m > w_L$, the R.H.S. of the above equation is increasing in π^∞ and, as $\pi^\infty \geq 0$, the above equation implies that $w_L^\infty + e_L^\infty \geq \frac{w_L+(1-\delta)e_L-\delta m}{1-\delta}$. Since $m \in M_\delta$, $w_L + (1 - \delta)e_L > m$. Combining the last two inequalities, we obtain $w_L^\infty + e_L^\infty > m$. Since $w_L^\infty < m < w_R^\infty$, this implies that $w_L^\infty < m < \min\left\{w_L^\infty + e_L^\infty, w_R^\infty\right\}$. The latter inequalities together with the definition of π^∞ imply that $\pi^\infty \in [\underline{b}, 1 - \underline{b}]$. Substituting (21) into (23), we obtain

$$e_R^\infty = e_R + \delta\pi^\infty\left(m - w_R^\infty\right) = e_R - \frac{\delta\pi^\infty}{1-\delta}\left(w_R - m\right).$$

Since $m \in M_\delta$, $w_R - m > \frac{(1-\delta)e_R}{\delta\underline{b}}$ and the above equation implies $e_R^\infty < e_R\left(1 - \frac{\pi^\infty}{\underline{b}}\right)$ and, since $\pi^\infty \geq \underline{b}$, $e_R^\infty < 0$. But from Proposition 2, $e_R^{\sigma(k)} \geq 0$. Therefore, $e_R^\infty < 0$ contradicts the fact that $\sigma\left(k\right)$ is an EE for all k. $\blacksquare$

Lemma 10 *For all $x \in \{n, p, q\}$, and all strategy profile σ, define $\Upsilon^\sigma\left(x, \{n, q\}\right) \equiv \Upsilon^\sigma\left(x, n\right)\cup$*

$\Upsilon^{\sigma}(x, q)$ *and*

$$D(\sigma) \equiv (1 - \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, n)))(1 - \delta\mu(\Upsilon^{\sigma}(q, q)))$$
$$- \delta^{2}\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, n))\mu(\Upsilon^{\sigma}(n, q)).$$

Then if σ is an equilibrium, we have $(1 - \delta)^{2} \leq D(\sigma) \leq 1$ and

$$
\begin{aligned}
w_{R}^{\sigma} - w_{L}^{\sigma} &= \tfrac{(1 - \delta\mu(\Upsilon^{\sigma}(q, q)))[w_{R} - w_{L}] - \delta\mu(\Upsilon^{\sigma}(n, q))[e_{R} - e_{L}]}{D(\sigma)}, \\
e_{R}^{\sigma} - e_{L}^{\sigma} &= \tfrac{(1 - \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, n)))[e_{R} - e_{L}] - \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, n))[w_{R} - w_{L}]}{D(\sigma)}, \\
w_{R}^{\sigma} + e_{R}^{\sigma} - w_{L}^{\sigma} - e_{L}^{\sigma} &= \tfrac{(1 - \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, \{n, q\})))[w_{R} - w_{L}] + (1 - \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, \{n, q\})))[e_{R} - e_{L}]}{D(\sigma)}.
\end{aligned}
\tag{24}
$$

Proof. Subtracting (19) for $i = R$ from (19) for $i = L$, and doing the same for (18), we get

$$
\begin{cases}
w_{R}^{\sigma} - w_{L}^{\sigma} = w_{R} - w_{L} + \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, n))(w_{R}^{\sigma} - w_{L}^{\sigma}) - \delta\mu(\Upsilon^{\sigma}(n, q))(e_{R}^{\sigma} - e_{L}^{\sigma}) \\
e_{R}^{\sigma} - e_{L}^{\sigma} = e_{R} - e_{L} + \delta\mu(\Upsilon^{\sigma}(q, q))(e_{R}^{\sigma} - e_{L}^{\sigma}) - \delta\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, n))(w_{R}^{\sigma} - w_{L}^{\sigma})
\end{cases}.
$$

The above equations can be viewed as a linear system in $w_{R}^{\sigma} - w_{L}^{\sigma}$ and $e_{L}^{\sigma} - e_{R}^{\sigma}$. Straightforward algebra shows that its solution is given by the first two lines of (24).

From Lemma 9, $\Upsilon^{\sigma}(q, q) \subset \Upsilon^{\sigma}(p, p)$ and, by definition of Υ^{σ}, $\Upsilon^{\sigma}(q, n)$ and $\Upsilon^{\sigma}(q, q)$ are disjoint. Thus, using the notations of the Lemma,

$$\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, n)) + \mu(\Upsilon^{\sigma}(q, q)) = \mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, \{n, q\})). \tag{25}$$

From Lemma 9, $\Upsilon^{\sigma}(n, q) \subset \Upsilon^{\sigma}(p, p)$ and, by definition of Υ^{σ}, $\Upsilon^{\sigma}(n, n)$ and $\Upsilon^{\sigma}(n, q)$ are disjoint, so

$$\mu(\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, n)) + \mu(\Upsilon^{\sigma}(n, q)) = \mu\Upsilon^{\sigma}(p, p) \cap (\Upsilon^{\sigma}(n, \{n, q\})). \tag{26}$$

Adding up the first two lines of (24) and substituting (25) and (26) into the corresponding expression for $w_{R}^{\sigma} + e_{R}^{\sigma} - w_{L}^{\sigma} - e_{L}^{\sigma}$, we obtain the third line of (24).

The inequality $D(\sigma) \leq 1$ is obvious from the definition of $D(\sigma)$. Let us now prove $D(\sigma) \geq (1 - \delta)^{2}$. Observe that $\Upsilon^{\sigma}(q, q)$ and $\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, n)$ are disjoint. So $\Upsilon^{\sigma}(q, q)$ is included into the complement of $\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(q, n)$. Likewise, $\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, n)$ and $\Upsilon^{\sigma}(n, q)$ are disjoint, so $\Upsilon^{\sigma}(p, p) \cap \Upsilon^{\sigma}(n, n)$ is included into the complement of $\Upsilon^{\sigma}(n, q)$.

Therefore,

$$
\begin{aligned}
D\left(\sigma\right) &= \left(1-\delta\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(n,n\right)\right)\right)\left(1-\delta\mu\left(\Upsilon^{\sigma}\left(q,q\right)\right)\right) \\
&\quad -\delta^{2}\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(q,n\right)\right)\mu\left(\Upsilon^{\sigma}\left(n,q\right)\right) \\
&\geq \left(1-\delta\left(1-\mu\left(\Upsilon^{\sigma}\left(n,q\right)\right)\right)\right)\left(1-\delta\left(1-\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(q,n\right)\right)\right)\right) \\
&\quad -\delta^{2}\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(q,n\right)\right)\mu\left(\Upsilon^{\sigma}\left(n,q\right)\right) \\
&= \left(1-\delta\right)\left(1-\delta+\delta\left(\mu\left(\Upsilon^{\sigma}\left(n,q\right)\right)+\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(q,n\right)\right)\right)\right) \\
&\geq \left(1-\delta\right)^{2}.
\end{aligned}
$$

■

Proof of Proposition 5. To prove the first claim in Proposition 5, simply note that the equilibrium σ constructed in Lemma 6 is such that $w_{L}^{\sigma}<w_{R}^{\sigma}$, so there always exists an equilibrium that is not an IE-C.

Step 1: if $e_{R}-e_{L}<\frac{(1-\delta)^{3}}{\delta}\left(w_{R}-w_{L}\right)$ (which is satisfied in particular when $e_{R}\leq e_{L}$), then for all equilibria σ, $w_{L}^{\sigma}<w_{R}^{\sigma}$, and hence σ cannot be IE-C.

Successively using (24), $e_{R}-e_{L}\leq\frac{(1-\delta)^{3}}{\delta}\left(w_{R}-w_{L}\right)$ and $\left(1-\delta\right)^{2}\leq D\left(\sigma\right)\leq1$ (see Lemma 10), we have

$$
\begin{aligned}
w_{R}^{\sigma}-w_{L}^{\sigma} &= \frac{\left(1-\delta\mu\left(\Upsilon^{\sigma}\left(q,q\right)\right)\right)}{D\left(\sigma\right)}\left(w_{R}-w_{L}\right)-\frac{\delta\mu\left(\Upsilon^{\sigma}\left(n,q\right)\right)}{D\left(\sigma\right)}\left(e_{R}-e_{L}\right) \\
&> \left(1-\delta\right)\left(w_{R}-w_{L}\right)-\frac{\delta\mu\left(\Upsilon^{\sigma}\left(n,q\right)\right)}{D\left(\sigma\right)}\frac{\left(1-\delta\right)^{3}}{\delta}\left(w_{R}-w_{L}\right) \\
&\geq \left(1-\delta\right)\left(w_{R}-w_{L}\right)-\frac{\delta}{\left(1-\delta\right)^{2}}\frac{\left(1-\delta\right)^{3}}{\delta}\left(w_{R}-w_{L}\right)\geq0.
\end{aligned}
$$

Step 2: if $e_{R}-e_{L}>-\left(1-\delta\right)^{3}\left(w_{R}-w_{L}\right)$ (which is satisfied in particular when $e_{R}\geq e_{L}$), then for all equilibria, $w_{L}^{\sigma}+e_{L}^{\sigma}<w_{R}^{\sigma}+e_{R}^{\sigma}$, and hence σ cannot be IE-B.

Successively using (24), $e_{R}-e_{L}>\left(1-\delta\right)^{3}\left(w_{R}-w_{L}\right)$ and $\left(1-\delta\right)^{2}\leq D\left(\sigma\right)\leq1$ (see Lemma 10), we have

$$
\begin{aligned}
&w_{R}^{\sigma}+e_{R}^{\sigma}-w_{L}^{\sigma}-e_{L}^{\sigma} \\
&= \frac{1-\delta\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(q,\{n,q\}\right)\right)}{D\left(\sigma\right)}\left(w_{R}-w_{L}\right)+\frac{1-\delta\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(n,\{n,q\}\right)\right)}{D\left(\sigma\right)}\left(e_{R}-e_{L}\right) \\
&\geq \left(\frac{1-\delta\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(q,\{n,q\}\right)\right)}{D\left(\sigma\right)}-\frac{1-\delta\mu\left(\Upsilon^{\sigma}\left(p,p\right)\cap\Upsilon^{\sigma}\left(n,\{n,q\}\right)\right)}{D\left(\sigma\right)}\left(1-\delta\right)^{3}\right)\left(w_{R}-w_{L}\right) \\
&\geq \left(\left(1-\delta\right)-\frac{1}{\left(1-\delta\right)^{2}}\left(1-\delta\right)^{3}\right)\left(w_{R}-w_{L}\right)\geq0.
\end{aligned}
$$

If σ is an IE-B, then $e_R^\sigma \leq 0 < e_L^\sigma$ and, from above, $w_L^\sigma + e_L^\sigma < w_R^\sigma + e_R^\sigma$. Together, these inequalities imply that $w_L^\sigma < w_L^\sigma + e_L^\sigma < w_R^\sigma + e_R^\sigma \leq w_R^\sigma$, contradicting the definition of an IE-B.

Step 3: if $|e_R - e_L| < (1 - \delta)^3 (w_R - w_L)$, then any equilibrium must be EE or IE-A. Let σ be an equilibrium. If σ is an EE, we are done. If σ is an IE then, since $(1 - \delta)^3 < \frac{(1-\delta)^3}{\delta}$, Step 1 implies that σ cannot be IE-C and Step 2 implies that σ cannot be IE-B. By elimination, σ can only be IE-A. ∎

Proof of Proposition 6. *Step 0: Notations.*

Throughout the proof, we fix δ, b, and F. Observe that the condition (9) depends on (w, e) only through w, so we let $W(w)$ denote the set of $w^\sigma \in \mathbb{R}^2$ that satisfy (9) for $i = L, R$. For any $w^\sigma \in \mathbb{R}^2$ and $e_L \in \mathbb{R}$, condition (10) for $i = L$ can viewed as a fixed point in e_L^σ :

$$e_L^\sigma = e_L + \delta \int_{w_L^\sigma}^{\min\{w_L^\sigma + e_L^\sigma, w_R^\sigma\}} b_R(\theta, q)(\theta - w_L^\sigma) \, dF(\theta). \tag{27}$$

Note that (27) depend on (w, e) only through w^σ and e_L, so we let $E_L(w^\sigma, e_L)$ denote the set of solutions e_L^σ to (27). Since the R.H.S. of (27) is continuous and bounded in e_L^σ, $E_L(w^\sigma, e_L)$ is not empty and closed, and we let $\underline{e_L^\sigma}(w^\sigma, e_L)$ denote its smallest element . Finally, condition (10) for $i = R$ gives

$$e_R^\sigma = e_R + \delta \int_{w_L^\sigma}^{\min\{w_L^\sigma + e_L^\sigma, w_R^\sigma\}} b_R(\theta, q)(\theta - w_R^\sigma) \, dF(\theta), \tag{28}$$

and we let $e_R^\sigma(w^\sigma, e_L^\sigma, e_R)$ denote the R.H.S. of (28).

Step 1: for all $w^\sigma \in W(w)$, $\underline{e_L^\sigma}(w^\sigma, e_L)$ is weakly increasing in e_L, and $e_R^\sigma(w^\sigma, e_L^\sigma, e_R)$ is weakly increasing in e_R, and weakly decreasing in e_L^σ.

Note that that the R.H.S. of (27) is continuous in (e_L, e_L^σ) and weakly increasing in e_L. From Villas Boas (1997, Theorem 1), the smallest solution $\underline{e_L^\sigma}(w^\sigma, e_L)$ to the fixed point condition (27) must also be weakly increasing in e_L. The comparative statics on $e_R^\sigma(.)$ follow readily from (28) and the fact that for all $w^\sigma \in W(w)$, $w_L^\sigma < w_R^\sigma$ (see Step A1 in the proof of Lemma 7).

Step 2: no EE exists if and only if for all $w^\sigma \in W(w)$, $e_R^\sigma\left[w^\sigma, \underline{e_L^\sigma}(w^\sigma, e_L), e_R\right] < 0$.

From Lemma 7, an EE exists if and only if there exists $(w^\sigma, e^\sigma) \in \mathbb{R}^4$ which satisfy the same condition as w^σ and $e^{\sigma'}$ in (9) and (10), and such that $e_R^\sigma \geq 0$. If we use the notations of Step 0, this means that an EE exists if and only if there exists $w^\sigma \in W(w)$ and $e_L^\sigma \in E_L(w^\sigma, e_L)$ such that $e_R^\sigma(w^\sigma, e_L^\sigma, e_R) \geq 0$. From Step 1, this is the case if and only if $e_R^\sigma\left(w^\sigma, \underline{e_L^\sigma}(w^\sigma, e_L^\sigma), e_R\right) \geq 0$ for some $w^\sigma \in W(w)$, as needed.

Step 3: Proof of Parts (i) and (ii) of Proposition 6.

Since an equilibrium can only be an EE or an IE, all equilibria are IE if and only if no EE exists. Therefore, Part (i) follows then from Step 2, together with the comparative statics established in Step 1.

We now prove Part (ii). Let $w^\sigma \in W(w)$. As shown in Step A1 in the proof of Lemma 7, $w_L^\sigma < w_R^\sigma$. Therefore, one can easily see from (27) that $\underline{e}_L^\sigma(w^\sigma, e_L) \geq e_L > 0$. Finally, (28) implies that

$$\lim_{e_R \to 0} e_R^\sigma\left(w^\sigma, \underline{e}_L^\sigma(w^\sigma, e_L), e_R\right) = \delta \int_{w_L^\sigma}^{\min\left\{w_L^\sigma + \underline{e}_L^\sigma(w^\sigma, e_L), w_R^\sigma\right\}} b_R(\theta, q)(\theta - w_R^\sigma)\, dF(\theta).$$

Since F has full support and $b_R(\theta, q) > \underline{b}$ for all $\theta \in \mathbb{R}$, the integral on the R.H.S. of the above equation is strictly negative. From step 2, this implies that, for e_R sufficiently small, no EE exists and all equilibria are IE.

Step 4: for any (e_L, e_R), as $w_R - w_L \to 0$, there exists an EE.

Suppose this is false. From step 2, this implies that there exists a sequence $(w^k)_{k \in \mathbb{N}}$ such that $w_R^k - w_L^k \to 0$ and for all $k \in \mathbb{N}$, for all $\hat{w}^k \in W(w^k)$, $e_R^\sigma\left(\hat{w}^k, \underline{e}_L^\sigma(\hat{w}^k, e_L), e_R\right) < 0$. Taking differences across players in (9), we obtain that

$$\hat{w}_R^k - \hat{w}_L^k = \frac{w_R^k - w_L^k}{1 - \delta\left(F(\hat{w}_R^k) - F(\hat{w}_L^k)\right)} \leq \frac{w_R^k - w_L^k}{1 - \delta}.$$

Therefore, $\hat{w}_R^k - \hat{w}_L^k \to 0$ as $k \to \infty$. Since $\hat{w}^k \in W(w^k)$, $\hat{w}_R^k > \hat{w}_L^k$ and consequently (28) implies

$$e_R^\sigma\left(\hat{w}^k, \underline{e}_L^\sigma(\hat{w}^k, e_L), e_R\right) \geq e_R + \delta \int_{\hat{w}_L^k}^{\hat{w}_R^k} b_R(\theta, q)(\theta - \hat{w}_R^k)\, dF(\theta).$$

Since $\hat{w}_R^k - \hat{w}_L^k \to_{k \to \infty} 0$ and F is continuous, the above inequality implies

$$e_R^\sigma\left(\hat{w}^k, \underline{e}_L^\sigma(\hat{w}^k, e_L), e_R\right) \to_{k \to \infty} e_R > 0,$$

contradicting the assumption that $e_R^\sigma\left(\hat{w}^k, \underline{e}_L^\sigma(\hat{w}^k, e_L), e_R\right) < 0$.

Step 5: Proof of Part (iii) of Proposition 6.

The case $w_R - w_L \to 0$ is treated in Step 4. Let us now consider the case $w_R - w_L \to +\infty$. Suppose by contradiction that Part (iii) is false in that case. Then there exists $e \in (0, +\infty)^2$, $m \in \mathbb{R}$ and two sequences $(w^k)_{k \in \mathbb{N}}$ and $(\sigma(k))_{k \in \mathbb{N}}$ such that $w_R^k - w_L^k \to +\infty$ and. for all $k \in \mathbb{N}$, $w_L^k + w_R^k = m$ and $\sigma(k)$ is an IE for the flow payoff parameters (e, w^k).

Since $w_R^k - w_L^k \to +\infty$, Equation (24) and inequality $D(\sigma) \geq (1-\delta)^2$ in Lemma 10 imply that, for k sufficiently large,

$$w_R^{\sigma(k)} - w_L^{\sigma(k)} \geq (1-\delta)\left(w_R^k - w_L^k\right) - |e_R - e_L|.$$

Hence, $w_R^{\sigma(k)} - w_L^{\sigma(k)} \to +\infty$ and, therefore, $w_R^{\sigma(k)} > w_L^{\sigma(k)}$ for k sufficiently large. Since $\sigma(k)$ is an IE, the latter inequality, together with Lemma 8, implies that, for k sufficiently large, $e_L^{\sigma(k)} > 0 \geq e_R^{\sigma(k)}$. By the same token, Lemma 10 implies that for k sufficiently large,

$$w_R^{\sigma(k)} + e_R^{\sigma(k)} - w_L^{\sigma(k)} - e_L^{\sigma(k)} \geq (1-\delta)\left(w_R^k - w_L^k\right) - |e_R - e_L|.$$

Hence, $w_R^{\sigma(k)} + e_R^{\sigma(k)} - w_L^{\sigma(k)} - e_L^{\sigma(k)} \to +\infty$. Together with $e_L^{\sigma(k)} > 0 \geq e_R^{\sigma(k)}$, this implies that for k sufficiently large,

$$w_L^{\sigma(k)} < w_L^{\sigma(k)} + e_L^{\sigma(k)} < w_R^{\sigma(k)} + e_R^{\sigma(k)} \leq w_R^{\sigma(k)}. \tag{29}$$

Let $v \in \Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)$. Since $\sigma(k)$ is an equilibrium, in state v, both players must weakly prefer to implement n to q and one player must prefer to implement p to n. Thus, $\min_{i \in \{L,R\}} w_i^{\sigma(k)} \leq \theta(v) \leq \min_{i \in \{L,R\}} \left(w_i^{\sigma(k)} + e_i^{\sigma(k)}\right)$ and (29) implies

$$\Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n) \subset \left\{v \in \Upsilon : w_L^{\sigma(k)} \leq \theta(v) \leq w_L^{\sigma(k)} + e_L^{\sigma(k)}\right\}. \tag{30}$$

From (30), for all $v \in \Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)$, $\theta(v) - w_L^{\sigma(k)} \leq e_L^{\sigma(k)}$. Substituting this inequality into (18), we obtain

$$e_L^{\sigma(k)} \leq e_L + \delta\mu\left(\Upsilon^{\sigma(k)}(q,q)\right) e_L^{\sigma(k)} + \delta\mu\left(\Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)\right) e_L^{\sigma(k)}$$

and therefore that $e_L^{\sigma(k)} \leq e_L/(1-\delta)$. Since $e_L^{\sigma(k)} > 0$, the latter inequality implies that $e_L^{\sigma(k)}$ is bounded. From (29) and (30), for all $v \in \Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)$, $0 \geq \theta(v) - w_R^{\sigma(k)} \geq w_L^{\sigma(k)} - w_R^{\sigma(k)}$. The preceding inequality and (30) imply that

$$\int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(v) - w_R^\sigma)\, d\mu(v) \geq \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} \left(w_L^{\sigma(k)} - w_R^{\sigma(k)}\right) d\mu(v)$$

$$\geq \int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)} + e_L^{\sigma(k)}} \left(w_L^{\sigma(k)} - w_R^{\sigma(k)}\right) dF(\theta).$$

Substituting the above inequality into (18), we obtain

$$e_R^{\sigma(k)} \;\geq\; e_R + \delta\mu\left(\Upsilon^{\sigma(k)}(q,q)\right)e_R^{\sigma(k)} \tag{31}$$

$$+\delta\int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}} w_L^{\sigma(k)}dF(\theta) - \delta\int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}} w_R^{\sigma(k)}dF(\theta).$$

Since $e_L^{\sigma(k)}$ is bounded and $w_L^{\sigma(k)} \to -\infty$ as $k \to \infty$, the integrability of F implies that the term $\int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}} w_L^{\sigma(k)}dF(\theta)$ in (31) tends to 0. If we can show that $w_R^{\sigma(k)} \sim \left|w_L^{\sigma(k)}\right|$ then, by the same token, $\int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}} w_R^{\sigma(k)}dF(\theta)$ tends to 0 and so (31) implies that, for k sufficiently large, $e_R^{\sigma(k)} > 0$. This last inequality, together with $e_L^{\sigma(k)} > 0$ and the assumption that $\sigma(k)$ is an IE, yields the desired contradiction. To complete the proof of Step 5, it suffices to show that $w_R^{\sigma(k)} \sim \left|w_L^{\sigma(k)}\right|$.

Summing (18) and (19) across players and collecting the terms in factor of $w_L^{\sigma(k)} + w_R^{\sigma(k)}$ and $e_L^{\sigma(k)} + e_R^{\sigma(k)}$, we obtain:

$$\begin{pmatrix} 1 - \delta\mu\left(\Upsilon^{\sigma(k)}(p,p)\cap\Upsilon^{\sigma(k)}(n,n)\right) & \delta\mu\left(\Upsilon^{\sigma(k)}(n,q)\right) \\ \delta\mu\left(\Upsilon^{\sigma(k)}(p,p)\cap\Upsilon^{\sigma(k)}(q,n)\right) & 1 - \delta\mu\left(\Upsilon^{\sigma(k)}(q,q)\right) \end{pmatrix} \begin{pmatrix} w_L^{\sigma(k)} + w_R^{\sigma(k)} \\ e_L^{\sigma(k)} + e_R^{\sigma(k)} \end{pmatrix}$$

$$= \begin{pmatrix} m - \delta\int_{\Upsilon^{\sigma(k)}(p,p)\cap\Upsilon^{\sigma(k)}(n,n)} \theta(v)\,d\mu(v) \\ e_L + e_R + \int_{\Upsilon^{\sigma(k)}(p,p)\cap\Upsilon^{\sigma(k)}(q,n)} \theta(v)\,d\mu(v) \end{pmatrix}.$$

From Lemma 10, the determinant of that system, which is $D(\delta)$, is bounded away from 0 as $k \to \infty$. Moreover, all the coefficients of the above system are bounded. Therefore, the solution $w_L^{\sigma(k)} + w_R^{\sigma(k)}$ must be bounded as $k \to \infty$. Since $w_L^{\sigma(k)} \to -\infty$, this implies that $w_R^{\sigma(k)} \sim \left|w_L^{\sigma(k)}\right|$, as needed. ∎

Proof of Proposition 7. Let $\{\varepsilon(t) : t \geq 0\}$ be a sequence of i.i.d., real valued random variables with mean 0, variance 1, and c.d.f. G with full support. For all $a \geq 0$, $\pi \in (0,1)$, $\theta_{low}, \theta_{high} \in \mathbb{R}$, consider the following distribution for $\{\theta(t) : t \geq 0\}$: in each period, with probability $1 - \pi$, $\theta(t) = \theta_{low} + a\varepsilon(t)$, and with the remaining probability π, $\theta(t) = \theta_{high} + a\varepsilon(t)$. Thus, the c.d.f. F_a of $\theta(t)$ is given as follows: for all $a > 0$ and all $\theta \in \mathbb{R}$,

$$F_a(\theta) = (1 - \pi)G\left(\frac{\theta - \theta_{low}}{a}\right) + \pi G\left(\frac{\theta - \theta_{high}}{a}\right),$$

and F_0 is defined as follows: $F_0(\theta) = 0$ if $\theta < \theta_{low}$, $F_0(\theta) = 1 - \pi$ if $\theta \in [\theta_{low}, \theta_{high})$, and $F_0(\theta) = 1$ if $\theta \geq \theta_{high}$. For all $a \geq 0$ and all $X \subseteq \{n, p, q\}$, let $\Gamma(a, X)$ denote the game Γ in which the set of available policies is X and the c.d.f. is F_a. Throughout, we assume

that the parameters π, θ_{low}, and θ_{high} satisfy the following conditions (whose meaning will be clarified as we use them):

$$w_L < \theta_{low} < \min_{i\in\{L,R\}} \{w_i + e_i\} \leq \max_{i\in\{L,R\}} \{w_i + e_i\} < \theta_{high}, \tag{32}$$

$$\theta_{low} + \delta\pi \left(\theta_{high} - \theta_{low}\right) < \min_i \{w_i + e_i\}, \tag{33}$$

$$\pi < \frac{w_R - (1-\delta)\,\theta_{high} - \delta\theta_{low}}{\delta\left(\theta_{high} - \theta_{low}\right)}, \tag{34}$$

$$\pi < \frac{w_L + (1-\delta)\,e_L - \theta_{low}}{\delta\left(\theta_{high} - \theta_{low}\right)}, \tag{35}$$

$$\theta_{high} - w_R + \frac{\delta}{1-\delta}\left[(1-\pi)\,\theta_{low} + \pi\theta_{high} - w_R\right] < \frac{\theta_{low} - w_R - e_R}{1 - \delta\left(1 - (1-\pi)\,b_R\left(\theta_{low}, q\right)\right)}. \tag{36}$$

Below, we prove the proposition for the degenerate distribution F_0, and then show that by continuity, the proposition must also be true for the full support distribution F_a for $a > 0$ sufficiently small.

Step 0: For all δ sufficiently close to 1, there exist $\pi > 0$, θ_{low}, and θ_{high} which satisfy conditions (32), (34), (35) and (36).

Step 0 follows from the simple observation that, for all $\theta_{high} > \max\{w_L + e_L, w_R + e_R\}$, if we set $\pi = 0$ and $\theta_{low} = w_L$, conditions (33) and (35) are satisfied for all $\delta \in (0,1)$ and conditions (34) and (36) are satisfied for δ sufficiently close to 1. Since these inequalities are strict, by continuity, they are also satisfied for some $\pi > 0$, $w_L < \theta_{low}$ and $\delta \in (0,1)$ and, for these parameter values, (32) is also satisfied.

Step 1: $\Gamma\left(0, \{n, p\}\right)$ has a unique equilibrium $\sigma\left(0, \{n, p\}\right)$. In that equilibrium, the initial status quo n stays in place in all future periods.

Let $\sigma\left(0, \{n, p\}\right)$ be an equilibrium of $\Gamma\left(0, \{n, p\}\right)$. From (32), $w_L < \theta_{low} < \theta_{high}$ so p gives a strictly greater flow payoff than n to L in both states. Therefore, if $s(t) = p$, in all subsequent periods, any equilibrium of $\Gamma\left(0, \{n, p\}\right)$ must prescribe L to veto any policy change and the subsequent equilibrium path must stay at p forever. So implementing p in period t yields a flow payoff of $\theta(t) - w_i$ today and an expected payoff of $(1-\pi)\left(\theta_{low}\right) + \pi\theta_{high} - w_i$ in all future periods. Note also that implementing n yields an expected continuation payoff of at least 0 to R, because R can always unilaterally impose to stay at n forever. Therefore, R prefers implementing n rather than p when $\theta(t) = \theta_{high}$ as long as

$$0 > \theta_{high} - w_R + \frac{\delta}{1-\delta}\left[(1-\pi)\left(\theta_{low}\right) + \pi\theta_{high} - w_R\right].$$

Simple algebra shows that the above equation is equivalent to (34). When (34) is satisfied, R also prefers implementing n rather than p when $\theta(t) = \theta_{low}$, which proves Step 1.

Step 2: $\Gamma\left(0,\{n,q\}\right)$ *has a unique equilibrium* $\sigma\left(0,\{n,q\}\right)$. *In that equilibrium, L and R unanimously implement n when* $\theta\left(t\right) = \theta_{low}$ *and q when* $\theta\left(t\right) = \theta_{high}$.

From (32), for all $i \in \{L, R\}$, $\theta_{low} < w_i + e_i < \theta_{high}$ and players have the same strict preferences between n and q in all states: when $\theta\left(t\right) = \theta_{low}$, n gives a strictly greater flow payoff than p to both players, whereas when $\theta\left(t\right) = \theta_{high}$, q gives a strictly greater flow payoff than n to both players. So both players proposing and accepting n in state θ_{low} and q in state θ_{high} is clearly an equilibrium.

Let us now derive the conditions that guarantee that it is the only equilibrium. To simplify notations, we assume that $w_L + e_L \leq w_R + e_R$; the proof in the other case is identical with the role of L and R reversed. Since there are only two alternatives, $\Gamma\left(0,\{n,q\}\right)$ is strategically equivalent to the game analyzed in Dziuda and Loeper (2015a). Their Proposition 1 implies that, for all equilibria σ,

$$w_L^\sigma + e_L^\sigma \leq w_L + e_L < w_R + e_R \leq w_R^\sigma + e_R^\sigma.$$

The above inequalities, together with (32), imply that L (R) always strictly prefers to implement q (n) in state θ_{high} (θ_{low}). So if the status quo is q (n) in state θ_{high} (θ_{low}), the status quo stays in place with probability 1. Let c_{low}^σ (c_{high}^σ) denote the probability before knowing the identity of the proposer that in state θ_{high} (θ_{low}), status quo n (q) is replaced by policy q (n). To complete the proof of Step 2, it suffices to show that $c_{low}^\sigma = c_{high}^\sigma = 1$.

Note that, since σ is an equilibrium, a necessary condition for $c_{low}^\sigma < 1$ $(c_{high}^\sigma < 1)$ is that L (R) is willing to implement q (n) in state θ_{low} (θ_{high}), that is, $w_L^\sigma + e_L^\sigma \leq \theta_{low}$ $(w_R^\sigma + e_R^\sigma \geq \theta_{high})$. The continuation payoff gain of implementing q instead of n in a period t is given by the flow payoff of having q instead of n today, plus the continuation value of having q instead of n tomorrow, given the equilibrium path of play defined by c_{low}^σ and c_{high}^σ. Therefore, we can write

$$\begin{aligned}\theta\left(t\right) - w_i^\sigma - e_i^\sigma \;=\;\; &\theta\left(t\right) - w_i - e_i + \\ &\delta\left[\left(1-\pi\right)\left(1 - c_{low}^\sigma\right)\left(\theta_{low} - w_i^\sigma - e_i^\sigma\right) + \pi\left(1 - c_{high}^\sigma\right)\left(\theta_{high} - w_i^\sigma - e_i^\sigma\right)\right].\end{aligned}$$

Solving for $w_i^\sigma + e_i^\sigma$, we obtain

$$w_i^\sigma + e_i^\sigma = \frac{w_i + e_i - \delta\left(1-\pi\right)\left(1 - c_{low}^\sigma\right)\theta_{low} - \delta\pi\left(1 - c_{high}^\sigma\right)\theta_{high}}{1 - \delta\left(1-\pi\right)\left(1 - c_{low}^\sigma\right) - \delta\pi\left(1 - c_{high}^\sigma\right)}. \tag{37}$$

From (32), $\theta_{low} < w_i + e_i < \theta_{high}$. Straightforward calculus shows then that the R.H.S. of (37) is decreasing in c_{low}^σ and increasing in c_{high}^σ. Therefore, a necessary condition for

$w_L^\sigma + e_L^\sigma \leq \theta_{low}$ (which is necessary for $c_{low}^\sigma < 1$) is that the latter inequality is satisfied when $w_L^\sigma + e_L^\sigma$ is given by the R.H.S. of (37) with $c_{low}^\sigma = 1$ and $c_{high}^\sigma = 0$; that is

$$\frac{w_L + e_L - \delta\pi\theta_{high}}{1 - \delta\pi} \leq \theta_{low}.$$

Simple algebra shows that the above condition is the negation of (33). Thus, we have shown that if (33) is satisfied, for all equilibria σ, $c_{low}^\sigma = 1$. Likewise, a necessary condition for $w_R^\sigma + e_R^\sigma \geq \theta_{high}$ (which is necessary for $c_{high}^\sigma < 1$) is that the latter inequality is satisfied when $w_R^\sigma + e_R^\sigma$ is given by the R.H.S. of (37) with $c_{low}^\sigma = 1$ and $c_{high}^\sigma = 1$, that is, $w_R + e_R \geq \theta_{high}$ which is violated from (32). Thus, we have shown that if (32) and (37) are satisfied, then for all equilibria, $c_{low}^\sigma = c_{high}^\sigma = 1$, as needed.

   Step 3: $\Gamma(0, \{n, p, q\})$ *has a unique equilibrium* $\sigma(0, \{n, p, q\})$, *and this equilibrium is outcome equivalent to the equilibrium* $\sigma(0, \{n, q\})$ *of* $\Gamma(0, \{n, q\})$.

From (32), p gives a strictly greater flow payoff than n and q to L in both states. Therefore, if $s(t) = p$, in all subsequent periods, any equilibrium of $\Gamma(0, \{n, p, q\})$ must prescribe L to veto any policy change, and the subsequent equilibrium path must stay at p forever. Note also that implementing n yields an expected continuation payoff of at least 0 to R. Therefore, as explained in Step 1, (34) implies that if $s(t) = n$, in either state of nature, R strictly prefers to implement n than p.

Consider a Markov state in which $\theta(t) = \theta_{low}$, the status quo is q, and R has proposed n, and L must decide whether to accept n. If L accepts n, L gets an expected continuation payoff of at least 0. If L refuses n, q is implemented instead and L gets at most the flow payoff from q in the current period, i.e., $\theta_{low} - w_L - e_L$, plus the continuation payoff from having his most preferred policy p in all subsequent periods, i.e., $\frac{\delta}{1-\delta}[(1 - \pi)(\theta_{low}) + \pi\theta_{high} - w_L]$. So if we assume that

$$0 > \theta_{low} - w_L - e_L + \frac{\delta}{1 - \delta}[(1 - \pi)(\theta_{low}) + \pi\theta_{high} - w_L],$$

then $\sigma(0, \{n, p, q\})$ must prescribe L to accept proposal n when $s(t) = q$ and $\theta(t) = \theta_{low}$. Simple algebra shows that the above equation is equivalent to (35).

Consider now a Markov state in which the state is $\theta \in \{\theta_{low}, \theta_{high}\}$, the status quo is q, L has proposed p, and suppose that $\sigma(0, \{n, p, q\})$ prescribes R to accept that proposal. If R accepts p, p is implemented in all future periods, and R gets an expected continuation payoff of

$$\theta - w_R + \frac{\delta}{1 - \delta}[(1 - \pi)\theta_{low} + \pi\theta_{high} - w_R]. \tag{38}$$

Consider now the following deviation from $\sigma(0, \{n, p, q\})$: R refuses p, imposes the status quo q until the first period $t' > t$ such that $\theta(t') = \theta_{low}$ and R is the proposer (which occurs with probability $(1 - \pi) b_R (\theta_{low}, q)$), R proposes n in that period t' (which we just showed will be accepted by L), and R vetoes any departure from status quo n in all subsequent periods (and thus gets continuation payoff 0). The expected continuation payoff from this deviation is

$$\sum_{t' \geq t} \delta^{t'-t} \left(1 - (1 - \pi) b_R (\theta_{low}, q)\right)^{t'-t} (\theta_{low} - w_R - e_R) = \frac{\theta_{low} - w_R - e_R}{1 - \delta (1 - (1 - \pi) b_R (\theta_{low}, q))}. \tag{39}$$

Condition (36) implies that (39) is greater than (38) for any $\theta \in \{\theta_{low}, \theta_{high}\}$, and thus that this deviation is profitable. Therefore, $\sigma(0, \{n, p, q\})$ cannot prescribe R to accept proposal p under status quo q. Thus, we have shown that in any equilibrium, p is implemented neither when $s(t) = n$ nor when $s(t) = q$. Since the initial status quo is n, p is never implemented on the equilibrium path. So $\sigma(0, \{n, p, q\})$ must be strategically equivalent to an equilibrium of $\Gamma(0, \{n, q\})$. Step 2 completes the proof of Step 3.

Step 4: both L and R are strictly better off in the unique equilibrium of $\Gamma(0, \{n, p, q\})$ and $\Gamma(0, \{n, q\})$ than in the unique equilibrium of $\Gamma(0, \{n, p\})$.

In the unique equilibrium of $\Gamma(0, \{n, q\})$, either L or R could unilaterally decide to stay forever at the initial status quo n as in the unique equilibrium of $\Gamma(0, \{n, p\})$. But from Step 2, both strictly prefer to alternate between n and q, so both must be strictly better off. To complete the proof of step 4, note that from Step 3, the unique equilibrium of $\Gamma(0, \{n, p, q\})$ is outcome equivalent to the unique equilibrium of $\Gamma(0, \{n, q\})$.

Step 5: for $a > 0$ sufficiently small, L and R are strictly better off in any equilibrium of $\Gamma(a, \{n, p, q\})$ and $\Gamma(a, \{n, q\})$ than in any equilibrium of $\Gamma(a, \{n, p\})$.

Suppose Step 5 is false. Then there exists a sequence $a^k \to 0$ and a corresponding sequence of equilibria $\sigma(a^k, \{n, p, q\})$ of $\Gamma(a^k, \{n, p, q\})$ and $\sigma(a^k, \{n, p\})$ of $\Gamma(a^k, \{n, p\})$ such that one player is weakly better off in $\sigma(a^k, \{n, p\})$ than in $\sigma(a^k, \{n, q\})$ (the proof for the case in which it is an equilibrium of $\Gamma(a, \{n, p\})$ that is preferred by one player to some equilibrium of $\Gamma(a, \{n, q\})$ is identical and is omitted for brevity). Let $w^{\sigma(a^k, \{n, p, q\})}$ and $e^{\sigma(a^k, \{n, p, q\})}$ denote the continuation payoff parameters of $\sigma(a^k, \{n, p, q\})$, and let $w^{\sigma(a^k, \{n, p\})}$ denote the continuation payoff parameters of $\sigma(a^k, \{n, p\})$, which we define as for the game $\Gamma(a, \{n, p, q\})$ (see Lemma 1). Note that once we fix the flow payoff parameters (w, e), δ, as well as π, θ_{low} and θ_{high}, the equilibrium parameters $w^{\sigma(a^k, \{n, p, q\})}$, $e^{\sigma(a^k, \{n, p, q\})}$ and $w^{\sigma(a^k, \{n, p\})}$ are bounded. So we can assume that they have a finite limit as $a^k \to 0$. By continuity, these

limits define an equilibrium of $\Gamma\left(0,\{n,p,q\}\right)$ and $\Gamma\left(0,\{n,p\}\right)$, respectively.[17] By continuity again, one player must be weakly better off in the equilibrium of $\Gamma\left(0,\{n,p,q\}\right)$ than in the equilibrium of $\Gamma\left(0,\{n,p\}\right)$, a contradiction with Step 4. ∎

Proof of Lemma 2.

We use the same notations as in Notation 1, with the exception that a state of the world $\upsilon\in\Upsilon$ now also includes the realization of the volatility v. For any strategy profile σ, for all $\upsilon\in\Upsilon$ and $s\in\{n,p,q\}$, let $X^{\sigma}\left(\upsilon,s\right)$ denote the policy outcome in a period in which the state of the world is υ and the status quo is s. By definition of V_i^{σ},

$$
\begin{aligned}
& V_i^{\sigma}\left(\theta,v,p\right) - V_i^{\sigma}\left(\theta,v,n\right) \\
= & \ \theta - w_i + \delta\left\{
\begin{array}{c}
\left(1-v\right)\left(V^{\sigma}\left(\theta,v,p\right)-V^{\sigma}\left(\theta,v,n\right)\right)+ \\
v\int_{\upsilon\in\Upsilon}\left(V^{\sigma}\left(\theta,v,X^{\sigma}\left(\upsilon,p\right)\right)-V^{\sigma}\left(\theta,v,X^{\sigma}\left(\upsilon,n\right)\right)\right)d\mu\left(\upsilon\right)
\end{array}
\right\} \\
= & \ \frac{\theta - \left(1-v\right)w_i - v\left(w_i - \delta\int_{\upsilon\in\Upsilon}\left(V^{\sigma}\left(\theta,v,X^{\sigma}\left(\upsilon,p\right)\right)-V^{\sigma}\left(\theta,v,X^{\sigma}\left(\upsilon,n\right)\right)\right)d\mu\left(\upsilon\right)\right)}{1-\delta\left(1-v\right)}.
\end{aligned}
$$

Note that if we set w_i^{σ} equal to the term inside the brackets on the numerator of the above fraction, then w_i^{σ} depends neither θ nor on v, which proves the first line of (3). The proof for the second line of (3) follows an analogous argument and is omitted for brevity. ∎

Proof of Proposition 8. The existence of $\bar{v}$ follows readily from Lemma 2. Let us now prove the second claim. Suppose by contradiction that it is false. Then for some δ arbitrarily close to 1, for any H with full support on $\mathbb{R}\times[0,1]$, there exists an equilibrium σ such that $e_L^{\sigma}\geq 0$ and $e_R^{\sigma}\geq 0$. For any such δ, we can choose a sequence of c.d.f. $\left(H_k\right)_{k\in\mathbb{N}}$ such that, for all $k\in\mathbb{N}$, $H_k\left(v,\theta\right)=G_k\left(v\right)F\left(\theta\right)$. In this last expression, the c.d.f. F of the marginal distribution of θ is fixed and the c.d.f. G_k of the marginal distribution of v tends to the degenerate distribution which puts probability 1 on $v=1$ as $k\rightarrow\infty$. Let $\sigma\left(k\right)$ be an equilibrium for these parameter values. By assumption, we can choose $\sigma\left(k\right)$ such that $e_L^{\sigma(k)}\geq 0$ and $e_R^{\sigma(k)}\geq 0$. Since w, e and δ are fixed, $w^{\sigma(k)}$ and $e^{\sigma(k)}$ are bounded, so we can extract a subsequence such that $w^{\sigma(k)}$ and $e^{\sigma(k)}$ converge. Since $e_L^{\sigma(k)}\geq 0$ and $e_R^{\sigma(k)}\geq 0$, one can easily check from (3) that for all $v<1$, for almost all θ, for any two distinct policies $x,y\in\{n,p,q\}$, $V_i^{\sigma(k)}\left(\theta,v,x\right)\neq V_i^{\sigma(k)}\left(\theta,v,y\right)$, that is, players are not indifferent between implementing any two policies. Thus, for all $v<1$, players have unique and pure stage undominated actions at any Markov states for almost all θ. If $e_i^{\sigma(k)}=0$ for some player i, then for $v=1$, player i might be indifferent between implementing p and q for all $\theta\in\mathbb{R}$.

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[17]Note that in principle, the limit of these equilibria could be a correlated equilibrium, since the realization of the noise $\varepsilon\left(t\right)$ provides a coordination device to the players. However, the arguments we have used in Steps 1-3 are essentially an iterative deletion of strictly dominated strategies, so the existence of a correlation device does not allow for any other equilibrium.

But we can assume w.l.o.g. that in this case, $\sigma(k)$ prescribes i to behave as if she strictly prefers implementing p to q. Since G_k puts probability 0 on $v = 1$, this deviation from $\sigma(k)$ does not affect the continuation payoff parameters $w^{\sigma(k)}$ and $e^{\sigma(k)}$, and it is therefore still an equilibrium.

Given this restriction on $\sigma(k)$, for all $v \in [0,1]$, the parameters $w^{\sigma(k)}$ and $e^{\sigma(k)}$ uniquely pin down the equilibrium behavior prescribed by $\sigma(k)$ for almost all $\theta \in [0,1]$. Moreover, they do so in a continuous way in the sense that, for all $v \in [0,1]$, the set of realizations of θ for which a given action is prescribed to the veto player or the proposer depend continuously on $w^{\sigma(k)}$ and $e^{\sigma(k)}$ (for instance, veto player i must veto proposal p under status quo n when $\theta < (1-v)w_i + vw_i^\sigma$ and accept it when the reverse inequality holds). Since $w^{\sigma(k)}$ and $e^{\sigma(k)}$ have a limit, this implies that for all $v \in [0,1]$, $\sigma(k)$ has a limit as $k \to \infty$ and, by continuity, this limit must be an equilibrium for the game with the limit distribution $H_\infty(v, \theta)$, which puts probability 1 on $v = 1$. This game is equivalent to the game considered in Proposition 4. Since δ can be chosen arbitrarily close to 1, and since F can be chosen arbitrarily, we obtain a contradiction with Proposition 4. ∎

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