Dynamic Collective Choice with Endogenous Status Quo*

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Abstract

This paper analyzes an ongoing bargaining situation in which preferences evolve over time and the previous agreement becomes the next status quo, determining the payoffs until a new agreement is reached. We show that the endogeneity of the status quo induces perverse incentives that exacerbate the players' conflict of interest: Players disagree more often than they would if the status quo was exogenous. This leads to inefficiencies and status quo inertia. Under certain conditions, the endogenous status quo may lead the negotiations to a complete gridlock in which players never reach an agreement. Such gridlock can occur between players with arbitrarily similar preferences, provided they are sufficiently patient. When applied to the legislative setting, our model predicts polarized behavior, and explains why oftentimes legislators fail to react in a timely fashion to economic shocks.

1 Introduction

Despite a wide consensus on the necessity of reforms in important policy areas such as entitlements, immigration laws, or the tax code, the U.S. Congress has failed to act on these issues. The economic literature has proposed several explanations for the inability of legislators to reach seemingly beneficial agreements such as electoral calculus, vested interests, or uncertainty over the

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distributional impact of reforms.[1] This paper shows that gridlock and legislative inertia can be the strategic consequence of two features of policy making. First, the policies that govern entitlements, immigration, or taxation are typically continuing in nature. That is, they remain in effect until a new agreement is reached. Second, a constantly evolving environment requires these policies to be periodically revisited.

To understand how the combination of the continuing nature of policies and an evolving environment leads to excessive disagreement, consider the case of legislators negotiating over taxation. When the budget surplus is large and/or the returns to public spending are low, lower tax rates may be favored by all parties to ease the tax burden of the citizens. When deficits run high and/or the returns to public spending are high, all parties may agree to generate extra tax revenues to reduce the public debt or finance socially desirable programs. In other times, however, legislators may genuinely disagree on whether taxes should be raised or cut. If today's policy becomes the default for future negotiations, liberals may be reluctant to decrease taxes when that is desirable, out of fear that their conservative counterparts will veto tax increases when the need for high taxes arises. Similarly, anticipating future disagreement, conservatives may refuse to increase taxes when needed, out of fear that liberals will oppose tax cuts in the future.

We first formalize this insight in a model in which two players engage in an infinite sequence of choices over two alternatives, called L and R. At the beginning of each period, one alternative serves as the status quo. If both players agree to move away from the status quo, the new agreement is implemented. Otherwise, the status quo stays in place. In both cases, the implemented alternative determines the players' payoffs in this period and becomes the status quo for the next; that is, players operate under *the endogenous status quo* protocol. We assume that players' preferences over the alternatives change over time. They sometimes disagree, and whenever they do, one player (the rightist) prefers R and the other (the leftist) prefers L.

We show that indeed the endogeneity of the status quo combined with an evolving environment exacerbates the players' conflict of interest: in all stationary equilibria, the leftist player votes for L and the rightist player votes for R more often than they would if the policies were not continuing in nature or if there were no shocks to the preferences. In other words, players behave as if they were more *polarized* than they are. As a result, they may fail to reach an agreement even when

[1] See Section 2 and Drazen (2002) for a detailed review of this literature.

the status quo is Pareto dominated. The status quo stays in place too often, and the bargaining outcome is insufficiently responsive to the environment.

Our analysis reveals that players' equilibrium behavior is driven by a vicious cycle: players that behave in a polarized way disagree more often; more frequent disagreement in turn increases players' incentives to secure their preferred status quo, and thus increases their incentives to become even more polarized. Because of this vicious cycle, the polarizing effect of the endogenous status quo can be quite dramatic. We identify a class of payoff profiles such that for any profile in that class, if players are sufficiently patient then the negotiations can come to a complete gridlock in which players never reach an agreement, and the bargaining outcome is unresponsive to the evolution of preferences. Interestingly, this class of payoff profiles allows players' preferences to be arbitrarily similar.

When players bargain over more than two alternatives, the polarizing effect of the endogenous status quo can be accompanied by other equilibrium effects. These effects depend on the fine details of the model such as the process that governs the evolution of the state, or the bargaining procedure used to select the outcome in each period (with more than two alternatives, not all bargaining procedures are equivalent). The complex interaction of these effects makes it hard to isolate analytically the impact of the endogenous status quo on the equilibrium behavior. However, we identify two arguably relevant environments in which our results are qualitatively unchanged irrespective of the number of alternatives. In Section 4.2, we analyze a setting in which in each period, players use a bargaining procedure that favors incremental policy changes. Such a procedure captures the incremental nature of legislative policy making (Hayes 1992, Wildavsky 1992), and has been shown to generate predictions consistent with the actual decisions of monetary policy committees (Riboni and Ruge-Murcia, 2010). We show that under this procedure, the endogenous status quo generates polarization and status quo inertia as in the case of two alternatives. Moreover, in certain environments, the magnitude of these effects is independent of the space of alternatives. In Section 4.3, we allow for a large class of within-period bargaining procedures, but unlike in the $2-$alternative model in which players bargain once every period, we assume that preferences change smoothly and players can revise the status quo as soon as the environment renders it inadequate. This assumption is meant to reflect the fact that actual bargaining institutions typically allow the bargaining parties to propose policy changes whenever the situation prompts it. We show that in such environment, the equilibrium outcome exhibits status quo inertia.

Our model is institutionally sparse, but can shed light on some aspects of policy making in the U.S. Congress. It predicts that in policy areas governed by permanent legislations and subject to changing needs—such as mandatory spending, taxation or civil liberties and national security—legislators' voting behavior can exhibit an excessive degree of polarization which can lead to inefficient policy inertia. Clearly, our model cannot claim to explain all gridlock in Congress nor the dramatic increase in polarization that has been happening over the last decades. It does, however, suggests that the degree of polarization in legislators' voting record may overestimate their actual degree of ideological polarization. We discuss this issue in more detail in Section 5.

Our results also have implications for monetary policy making. In some countries, monetary policy is set by a committee with heterogeneous preferences and beliefs, and the interest rate stays the same until the committee agrees to change it (see Riboni and Ruge-Murcia 2008). Our results show that the endogeneity of the status quo can affect the ability of the committee to respond to economic shocks. Similarly, the renegotiations of ongoing contracts such as trade agreements, international treaties (e.g., for the World Trade Organization or the European Union), and financial or labor contracts are usually conducted in the presence of a binding previous agreement.[2] The impossibility of making these contracts fully contingent on all economic shocks and political events creates the need for frequent renegotiation. In the case of labor contracts, the intuition highlighted in this model suggests a possible explanation for wage stickiness: During a recession, lower wages may leave the workers and the employers better off, thanks to a lower risk of bankruptcy and layoffs. Yet, the workers maybe reluctant to agree on lower wages out of fear that the employers will refuse to increase them when the economy improves. Conversely, during a boom, higher wages may be necessary to retain skilled workers and improve morale. However, the employers may be reluctant to increase wages out of fear that the workers will refuse to reduce them when the situation deteriorates.

[2] In the case of international agreements, each party can always unilaterally violate the terms of the agreement. The consequences of that, however, are oftentimes severe enough that in practice the relevant default is the prevailing international agreement. Hence, in effect, countries negotiate under the endogenous status quo protocol.

2 Related literature

Inefficiency and inertia in dynamic policy making models

Policy makers' incentives (not) to adopt socially beneficial policy changes have been the study of numerous papers. Inefficient status quo inertia can arise if there is uncertainty over who benefits from the policy change (Fernandez and Rodrik, 1991; Strulovici, 2010), if the costs of policy change are borne disproportionately by those who initiate these changes (Alesina and Drazen, 1991), or if citizens respond to the current policy by undertaking private actions that make them more likely to oppose a policy change in the future (Glomm and Ravikumar, 1995; Krusell and Rios-Rull, 1996; Coate and Morris, 1999). In our model, it is neither the uncertainty over future preferences nor their dependence on past policies that matters, but the mere fact that they evolve.

Inefficiencies can also arise if the allocation of decision power evolves over time (Persson and Svensson, 1989; Alesina and Tabellini, 1990; Krusell and Rios-Rull, 1999; Azzimonti, 2011; and Battaglini and Coate, 2007, 2008) or if it depends on past decisions (Besley and Coate,1998; Acemoglu and Robinson, 2001; Cho, 2012; Acemoglu, Egorov and Sonin, 2008, 2012, 2013; Bai and Lagunoff, 2011; Baron, Diermeier and Fong, 2012; and Duggan and Forand, 2013). In contrast, the inefficiencies highlighted in our model arises even if the allocation of decision power is exogenous and constant over time.

Bargaining with an endogenous status quo in a static environment

Our paper complements the literature on dynamic bargaining with an endogenous status quo. Most of this literature considers environments in which preferences are static; the bargaining outcome changes over time because the identity of the proposer changes, or because the same proposer seeks the support of a different coalition. In our basic model with two alternatives and unanimity rule, these two channels do not play any role. Instead, the impetus for policy change comes solely from the evolution of preferences. In static environments with collective goods policies, this literature (Baron, 1996; Baron and Herron, 2003; and Zapal, 2011a) finds that the endogenous status quo has as a *moderating* effect. In contrast, we show that in a changing environment, the endogenous status quo has a *polarizing* effect.

The moderating effect highlighted by the aforementioned literature is similar to the effect that occurs in this model in the case of more than two alternatives (see Section 4.1). This effect is closely related to the observation initially made by Romer and Rosenthal (1978) that an extreme status quo increases the leverage of the proposer, and is thus detrimental to non-proposers. Therefore, with an endogenous status quo, implementing a moderate policy today can constrain the next proposer. Diermeier and Fong (2011) show that this effect can limit the bargaining power of a monopolistic proposer, and Bowen et al. (2014a) show that it mitigates the inefficiency due to political uncertainty. We contribute to that literature by showing that in a stochastic environment, under reasonable conditions, the moderating effect is dominated by the polarizing effect.

Since Kalandrakis (2004), a large literature has looked at the case of purely distributive policies.[3] These papers focus on the fairness of equilibrium allocations, and on the size and stability of the coalitions that support them. By virtue of analyzing collective goods policies under the unanimity rule, our paper has nothing to say on these issues. Instead, we focus on the responsiveness of policies to the environment and on polarization, on which the aforementioned papers are mute, as in the distributive setting, players' preferences are constant and always diametrically opposed.

In contrast to our model, the qualitative properties of the equilibrium outcome differ widely across Markov equilibria in the distributive settings. Markov perfection gives sharper predictions in our environment because policy changes need to be approved unanimously and transfers are ruled out. These two features limit the extent to which an unexpected Markov state can trigger a punitive subgame equilibrium that deters potential deviators.[4]

Bargaining with an endogenous status quo in an evolving environment

Even though dynamic bargaining with an endogenous status quo in a stochastic environment is at the center of many economically relevant situations, the existing literature on this topic is scarce. This may be a consequence of the intractability of these games. As Romer and Rosenthal (1978) showed in a static setup, the induced preferences over the status quo are typically not convex, which complicates the analysis of the multi-period extension.

To the best of our knowledge, only Riboni and Ruge-Murcia (2008), Zapal (2011b), Duggan and Kalandrakis (2012), and Bowen et al. (2014b) make progress on this front. Riboni and

[3] See, among others, Bernheim et al (2006), Kalandrakis (2010), Anesi (2010), Diermeier and Fong (2011), Bowen and Zahran (2012), Anesi and Seidmann (2012), Baron and Bowen (2013), and Richter (2013).

[4] Note that both transfers and nonunanimous bargaining are needed for Markov perfection to loose its bite. Indeed, Anesi and Seindman (2012) show in a purely distributive settings that under the unanimity rule, the equilibrium outcome is unique. Conversely, in Dziuda and Loeper (2014), we extend this model to an arbitrary number of players and show that our results hold essentially unchanged for a large class of nonunanimous voting rules.

Ruge-Murcia (2008) solve a two-period, two-state example with quadratic preferences, and use numerical solutions for a six-state, infinite horizon model. Zapal (2011b) considers a two-state environment with quadratic preferences and a continuum of alternatives. These authors recognize that the dynamic linkage may lead to inefficient policies. However, due to the stylized nature of their setting, they do not identify the systematic polarizing effect of the endogenous status quo, and do not disentangle it from the moderating effect of the particular bargaining procedure they use within each period—namely, monopolistic agenda setter—which can confound the polarizing effect (see our discussion in Sections 4.1 and 4.3.1).[5] Adding noise to the status quo, Duggan and Kalandrakis (2012) establish the existence of an equilibrium in a very general setting. Bowen et al. (2014b) show that the inefficiencies generated by the endogenous status can be eliminated if the status quo can be fully contingent on the state of nature.

Our paper differs from these contributions in that by focusing on two alternatives, we show that the polarizing and inertial effect arises in every equilibrium for a large class of environments, and characterize the determinants of their magnitude. We also investigate the conditions under which these effects are robust to allowing for more alternatives.

Riboni and Ruge-Murcia (2010) analyze a game similar to our $N-$alternative model under four different within-period bargaining procedures. For tractability, they assume myopic players, which severs the strategic link present in our model. One within-period bargaining procedure—the consensus procedure—is shown to lead to least frequent and most moderate policy changes. Using data from five central banks, Riboni and Ruge-Murcia (2010) find that all procedures, including the consensus procedure, underestimate the actual degree of status quo inertia. Consistently with this finding, in Section 4.2, we consider the consensus procedure and show that with forward looking players, the endogenous status quo can dramatically increase status quo inertia.[6]

Our paper is also related to Montagnes (2010) who shows that inefficiencies may arise in a

[5] Zapal further (2011b) shows that the endogenous status quo is dominated by another protocol in which players negotiate separately on the current policy and the future status quo.

[6] Our analysis suggests that making player forward looking in their empirical analysis should provide further evidence in favor of the consensus procedure (Riboni and Ruge-Murcia, 2010, have conjectured this after solving a two-period forward-looking example). The reason for this is that in our environment, status quo inertia increases with players' patience under the consensus procedure (see Proposition 7 together with Proposition 3), whereas patience does not affect players behavior under simple majority and dictatorhip, because under those procedures, the status quo is irrelevant. Patience may affect the degree of status quo inertia under the fourth procedure considered by Riboni and Ruge-Murcia (2010), the agenda-setter protocol. But our discussion in Section 4.1 and our numerical simluations in Section 4.3.1 suggests that under such a procedure, strategic considerations can also lead to status quo instability.

contracting environment in which the current contract serves as the default option in future negotiations. In an environment with present-biased legislators, Piguillem and Riboni (2013a, 2013b) show that the endogenous status quo is beneficial as it provides partial commitment. Casella (2005) and Jackson and Sonnenschein (2007) show that linking voting decisions across time allows voters to express their preference intensity. Our results suggest that despite its pervasiveness, the endogenous status quo is not an efficient way to elicit preference intensity.

3 The $2-$alternative model

3.1 The model

Two players, l and r, are in a relationship that lasts for infinitely many periods. There are two alternatives, $X = \{L, R\}$. In each period t, a status quo $q(t) \in \{L, R\}$ is in place, and players vote simultaneously on which alternative to adopt. If both players vote for the same alternative, this alternative is implemented. If they disagree, this period's status quo $q(t)$ is implemented. The implemented alternative $x(t)$, be it the new agreement or the status quo, determines the players' payoff in t and becomes the status quo for the next period $t+1$.

The payoff of player $k \in \{l, r\}$ in period t from alternative $x \in X$ is denoted by $U_k(\theta(t), x)$, where $\theta(t)$ denotes the state of nature in that period. To capture the dynamic nature of the environment, we assume that $\{\theta(t) : t \geq 0\}$ is a stationary Markov process on an arbitrary (finite or infinite) state space Θ with sigma algebra $\mathcal{S}$. That is, the state $\theta(t)$ depends on the past only through last period's state $\theta(t-1)$, and the distribution of $\theta(t)$ conditional on $\theta(t-1)$ is independent of t. Note that this assumption does not rule out deterministic processes. Throughout, θ denotes an arbitrary realization of $\theta(t)$, and P_θ denotes the probability distribution of $\theta(t+1)$ conditional on $\theta(t) = \theta$. The state $\theta(t)$ is observed in t by both players before they cast their votes. We assume that for all $x \in X$, $\theta \to U_k(\theta, x)$ is bounded and measurable w.r.t. $\mathcal{S}$. Players maximize the expected discounted sum of payoffs and $\delta \in (0, 1)$ is the discount factor.

In this game, denoted by Γ^{en}, the strategic link across periods comes solely from the continuing nature of the policies. To characterize the impact of the endogenous status quo on the equilibrium behavior and outcomes, we will compare Γ^{en} to the game Γ^{ex}, which differs from Γ^{en} only in that in every period t, the status quo is exogenously fixed at some $q(t) \in X$, irrespective of players' past actions.

As is customary in dynamic voting games with an infinite horizon, we look for Markov perfect equilibria in stage-undominated strategies as defined in Baron and Kalai (1993).[7] A Markov strategy for player k in Γ^{ex} or Γ^{en}, denoted by σ_k, maps in each period t the current state $\theta(t)$ and status quo $q(t)$ into a probability distribution over votes. Stage undomination amounts to assuming that in each period, each player votes for the alternative that gives her the greater continuation payoff. It rules out pathological equilibria such as both players always voting for the status quo. Throughout the paper, the term equilibrium refers to this equilibrium concept unless stated otherwise.

Additional assumptions and notations

Definition 1 *A payoff function $U : \Theta \times X \to \mathbb{R}$ is more leftist than another payoff function U' (or equivalently, U' is more rightist than U) if for all $\theta \in \Theta$,*

$$U(\theta, R) - U(\theta, L) \leq U'(\theta, R) - U'(\theta, L).$$

Throughout, we assume that U_r is more rightist than U_l, and we occasionally refer to player r and l as the rightist and the leftist player, respectively. This assumption has a natural interpretation in political economy applications: players can be unambiguously ranked on the ideological spectrum. Note, however, that it imposes no restrictions on the preference distribution of a single player, nor on the severity of the conflict of interest between players: both players might prefer the same alternative for an arbitrarily large subset of Θ.

For all $\theta \in \Theta$, let $U_k(\theta) \doteq U_k(\theta, R) - U_k(\theta, L)$ be the relative period payoff for player k from alternative R as compared to L in a period in which the state is θ. We call $U_k(\theta)$ player k's *current preference,* as the sign of $U_k(\theta)$ determines her preference for the current period without taking into account the consequences of today's bargaining outcome on future periods. We denote $U = (U_l, U_r)$.[8]

[7] Stage-undominated Markov perfect equilibria, or variants thereof, are used in almost all of the infinite-horizon models cited in this paper. Baron and Ferejohn (1989) show that requiring only subgame perfection leads to a folk theorem. As shown in Baron and Kalai (1993), stage-undominated Markov perfect equilibria have a focal point property that derives from their simplicity. Markov perfection in the legislative sphere can be justified on the grounds that the game is played by a sequence of legislators who are never certain to be reelected. In such cases, the institutional memory required for more sophisticated nonstationary equilibria involving infinitely nested punishment strategies may be inappropriate. See Baskhar et al (2013) for a formalization of this argument, and Krehbiel (1991) for a critical discussion of the prevalence of folk-theorem like cooperation among legislators.

[8] Throughout the paper, for any pair of functions or scalars f_l, f_r, we denote $f = (f_l, f_r)$.

To illustrate our results, sometimes we will use the following family of payoff profiles in which each player k has single-peaked preferences with a peak equal to the state θ plus a player-specific shift b_k. This shift can be interpreted as her ideological bias.

Definition 2 *When $\Theta \subseteq \mathbb{R}$ and $X \subset \mathbb{R}$, a profile of payoff functions (U_l, U_r) is* quadratic *if for all $k \in \{l, r\}$, $\theta \in \Theta$, and $x \in X$, $U_k(\theta, x) = -(x - (\theta + b_k))^2$ for some $b_l, b_r \in \mathbb{R}$ such that $b_l < b_r$.*

A few comments about the model are in order. First, we analyze a two-player game which requires unanimity for changing the status quo, but in Section 5, we explain how our results can be extended to an arbitrary number of players and a large class of voting rules.

Second, we assume that in each period the outcome is decided by a simultaneous vote, but with two alternatives and two players, most within-period bargaining procedures are equivalent.[9] This assumption will need to be relaxed when we allow for more alternatives.

Third, to assess the role of the endogenous status quo, we compare Γ^{en} to the game with the exogenous status quo protocol Γ^{ex}. The exogenous status quo protocol is the simplest dynamic bargaining protocol which severs the link between today's agreement and tomorrow's status quo. Hence, this benchmark protocol allows us to show in a transparent way that it is the continuing nature of policies that leads to polarization. The comparison between Γ^{en} and Γ^{ex} is also of an independent interest, as the exogenous status quo protocol is probably the most commonly observed alternative to the endogenous status quo in actual bargaining environments. See Section 5 for examples of policies legislated under an exogenous status quo.

Finally, in this model, today's bargaining outcome affects the future only via tomorrow's status quo. In a more general model, it could also affect tomorrow's states of nature. We abstract from this dependence to isolate the dynamic linkage of the endogenous status quo in a transparent way. In a more general model, other equilibrium effects could arise, but it should be clear from the analysis below that the effect identified in our paper would remain.

[9] For instance, using standard equilibrium concepts, equilibrium outcomes are the same when players vote simultaneously, sequentially, when they make take-it-or-leave-it offers, or when they make several alternating offers in each bargaining period.

3.2 Equilibrium analysis

As a benchmark, we first look at the game with an exogenous status quo Γ^{ex}. Consider player k in period t. Since the bargaining outcome implemented in t has no impact on the subgame starting in the next period $t+1$, player k votes for R if and only if

$$U_k(\theta(t)) \geq 0.^{10} \tag{1}$$

Hence, Γ^{ex} has a unique equilibrium in which players simply vote for the best alternative according to their current preferences.

Consider now the same situation in the game with an endogenous status quo Γ^{en}. The alternative x implemented in t affects player k's payoff in t, and since x becomes the status quo in $t+1$, it also affects the subgame that starts in $t+1$. Let $W_k^\sigma(\theta, x)$ be the expected value of that continuation game, conditional on $\theta(t) = \theta$ and on the strategy profile σ being played after t. Let $W_k^\sigma(\theta) \doteq W_k^\sigma(\theta, R) - W_k^\sigma(\theta, L)$. One can interpret W_k^σ as the relative expected gain for player k from having R instead of L as the next period's status quo. With a slight abuse of notation, we call W_k^σ *the continuation value* of player k.

Stage undomination requires that in each period, players vote as if they were pivotal. Hence, in Γ^{en}, player k votes for R in t if and only if

$$U_k(\theta(t)) + \delta W_k^\sigma(\theta(t)) \geq 0. \tag{2}$$

Comparing (1) and (2), one can see that the effect of the endogenous status quo on equilibrium behavior is completely captured by W_k^σ. If $W_k^\sigma(\theta)$ is positive (negative) for all $\theta \in \Theta$, then in Γ^{en} player k votes for R in a larger (smaller) set of states than in Γ^{ex}.

The first proposition characterizes the effect of the endogenous status quo on equilibrium behavior.

Proposition 1 *In any equilibrium, for all $\theta \in \Theta$, $W_l^\sigma(\theta) \leq 0 \leq W_r^\sigma(\theta)$. Hence, in Γ^{en} player l votes for L and player r votes for R for a larger set of states (in the inclusion sense) than in Γ^{ex}.*

[10] We derive all our results without placing any restrictions on how players vote when indifferent. However, when explaining the intuition in the text, we adopt the convention that they vote for R.

Proof. All proofs are in the Appendix. ■

The intuition for Proposition 1 is as follows. The next period's status quo affects the next period's bargaining outcome only if players disagree in that period, in which case the status quo stays in place. Under our assumption that U_r is more rightist than U_l, when players disagree, r prefers R while l prefers L. Hence, player r prefers R as the future status quo while player l prefers L as the future status quo, which explains $W_l^\sigma(\theta) \leq 0 \leq W_r^\sigma(\theta)$. Therefore, player l is willing to sacrifice some of her payoff in the current period by distorting her voting behavior in favor of L in order to secure L as the next period's status quo. Similarly, player r is willing to sacrifice some of her current payoff to secure R as the next period's status quo.

Since U_r is more rightist than U_l, from (1) we see that already under an exogenous status quo, player r votes for R in a larger (in the inclusion sense) set of states than player l. Hence, Proposition 1 implies that the endogenous status quo has *a* polarizing effect: it exacerbates the conflict of interest between the leftist and rightist player. In what follows, we refer to this equilibrium behavior as *strategic polarization.*

Since the endogenous status quo induces players to vote to different alternatives in a larger set of states, it leads to more status quo inertia.

Corollary 1 *In any equilibrium of* Γ^{en}*, the set of states in which the status quo stays in place is greater in the inclusion sense than in the equilibrium of* Γ^{ex}*.*

An important implication of Corollary 1 is that the equilibrium outcomes may be Pareto inefficient even in the static sense.[11] That is, there may be states in which both players would benefit from changing the policy without changing the status quo for the next period, but since no player can commit to such a temporary change, one player prefers to stay at the current status quo.

It is instructive to note that the magnitude of the strategic polarization identified in Proposition 1 and Corollary 1 is the result of a vicious cycle in which players' behaviors feed on themselves. To see this, observe that as player l distorts her voting behavior in favor of L, players become less likely to reach an agreement in each period. This increase in the probability of future disagreement in

[11] A sequence of policies $(x(t))_{t\in\mathbb{N}}$ is statically efficient if in each period $t' \in \mathbb{N}$, there is no other policy y that is strictly preferred by both players to $x(t')$, keeping the policies in the other periods unchanged. It is dynamically efficient (or fully efficient) if there is no other *sequence* of policy $(y(t))_{t\in\mathbb{N}}$ that is strictly preferred by both players. Hence, static efficiency is a weak form of efficiency.

turn makes player r more willing to defend her preferred status quo R, and thus makes her distort her behavior in favor of R even further.

Because of this strategic complementarity, there can be multiple equilibria which differ in their degree of strategic polarization. Since a more polarized behavior from a given player is detrimental to the other player, these equilibria can be Pareto ranked. We will say that for any two strategy profiles σ and σ', σ' is more polarized than σ if for all $\theta \in \Theta$, $W_l^{\sigma'}(\theta) \leq W_l^{\sigma}(\theta) \leq 0 \leq W_r^{\sigma}(\theta) \leq W_r^{\sigma'}(\theta)$.

Proposition 2 *There exists a least (most) polarized equilibrium, and this equilibrium is Pareto best (worst) among all equilibria.*

We conclude this section with three comments. First, note that the only assumption we impose on players' payoff is that U_l is more leftist than U_r. Hence, the direction in which the endogenous status quo biases players' behavior only depends on players' *relative* ideological positions. A player with a given U_k is biased in favor of L if she plays against a more rightist opponent, but she becomes biased in favor of R if she plays against a more leftist opponent. The intuition behind this result is that since the status quo matters only when players disagree, players' preferences over the next status quo are given by their policy preferences conditional on disagreement, which depend only on players' relative ideology.

Second, we have established so far that if the endogenous status quo protocol distorts players' behavior relative to the exogenous status quo protocol, then these distortions increase disagreement and create status quo inertia. We have not established, however, that these distortions are strict; i.e., that the equilibrium of Γ^{ex} is not an equilibrium of Γ^{en}. However, for the equilibrium of Γ^{ex} to be also an equilibrium of Γ^{en}, the process $\{\theta(t) : t \geq 0\}$ must satisfy quite restrictive conditions. To see this, consider a sequence of states in which both players' current preferences favor R, but l's preferences for R are vanishingly weak. Then in Γ^{ex}, both players vote for R in all such states while in Γ^{en}, if the probability that players' preferences disagree in the next period is not vanishingly small in the limit, l will find it profitable to vote for L instead of R. In Section 3.4 we show that this is the case, for example, when payoffs are quadratic (see Definition 2) and the state is i.i.d. over time.

Third, in this simple model, the inefficiency generated by the endogenous status quo arises only if preferences evolve over time. To see this, observe that if players' preferences were fixed, then in Γ^{en}, each player would vote according to her current preferences like in Γ^{ex}. The status quo could be replaced only in the first period, and the constant bargaining outcome would be trivially Pareto optimal. Hence, it is the combination of the endogenous status quo and the evolving environment that generates polarization and inefficient status quo inertia.

3.3 The determinants of strategic polarization and gridlock effect

In this section, we investigate the main drivers of strategic polarization.

Since strategic polarization is driven by the anticipation of future disagreement, it should be affected by the degree of players' conflict of interest and by their patience. Players whose current preferences are more likely to disagree, and who care more about the future should be more willing to sacrifice today's payoff to obtain their preferred status quo for tomorrow's negotiations. The next proposition formalizes this intuition.

Proposition 3 *Let (U_l, U_r) and (U_l', U_r') be two profiles of payoff functions such that U_l' is more leftist than U_l and U_r' is more rightist than U_r (in the sense of Definition 1), and let δ and δ' be two discount factors such that $\delta' \geq \delta$. If σ and σ' denote the Pareto best (worse) equilibria for the parameters U_l, U_r, and δ, and for U_l', U_r' and δ', respectively, then σ' is more polarized than σ.*

Since strategic polarization leads to status quo inertia, Proposition 3 implies that more patient and more ideologically polarized players are less likely to agree on Pareto improving policies after a shock to the environment has made the status quo suboptimal.

The next proposition shows that the impact of the endogenous status quo on equilibrium behavior can be quite dramatic: under some conditions, there exists a gridlock equilibrium in which players never reach an agreement, and hence the bargaining outcome is totally unresponsive to the evolution of the environment.

Proposition 4 *Let σ^g be the strategy profile in which player l always votes for L and player r always votes for R. Then σ^g is an equilibrium if and only if for any $\theta \in \Theta$,*

$$E_{\theta(0)=\theta}\left[\sum_{t=0}^{\infty}\delta^t U_l\left(\theta\left(t\right)\right)\right] \leq 0 \leq E_{\theta(0)=\theta}\left[\sum_{t=0}^{\infty}\delta^t U_r\left(\theta\left(t\right)\right)\right]. \qquad (3)$$

Condition (3) requires that for any initial state, player r's (l's) expected payoff from implementing R in all future periods is greater (smaller) than her expected payoff from implementing L in all future periods.

Note that whenever σ^g is an equilibrium, it is necessarily most polarized, so from Proposition 2, it is also Pareto worst. Hence, Proposition 3 implies that if gridlock is an equilibrium for some environment, then it remains an equilibrium when players become more patient or more ideologically polarized. The following corollary, however, demonstrates that the degree of ideological polarization needed for gridlock does not have to be large.

Corollary 2 *Suppose that the process $\{\theta(t) : t \geq 0\}$ is uniformly ergodic*[12] *and that*

$$E\left[U_l\left(\tilde{\theta}\right)\right] < 0 < E\left[U_r\left(\tilde{\theta}\right)\right], \tag{4}$$

where $\tilde{\theta}$ is a random variable distributed according to the stationary distribution of $\{\theta(t) : t \geq 0\}$. Then there exists $\bar{\delta} < 1$ such that for all $\delta \geq \bar{\delta}$, σ^g is an equilibrium.[13]

Corollary 2 implies that as long as (4) is satisfied, even players with arbitrarily similar preferences (i.e., $\left|E\left[U_l\left(\tilde{\theta}\right)\right] - E\left[U_r\left(\tilde{\theta}\right)\right]\right|$ arbitrarily small) may disagree forever if they are sufficiently patient. Note that condition (4) requires that players' payoffs are *not identical*, but it includes some environments in which players' current preferences always *favor the same alternative*. For example, consider an i.i.d. process with two equally likely states θ_R and θ_L such that $U(\theta_R) = (1, 2)$ and $U(\theta_L) = (-2, -1)$. In this environment, for both players, R is best in state θ_R, and L is best in state θ_L, but for $\delta > \frac{2}{3}$, Condition (3) is satisfied, so gridlock is an equilibrium.[14]

Clearly, in the last example, players always agreeing on the Pareto efficient policy is an equilibrium as well. The next section, however, shows an example of an environment in which gridlock is the unique equilibrium for sufficiently patient players.

[12] A stationary Markov chain with transition function f is uniformly ergodic if as $n \to \infty$, $f^n\left(\mu^0\right)$ converges to its unique stationary distribution uniformly over all initial distributions μ^0 (see Meyn and Tweedie 1993, Chapter 16). This condition is satisfied for instance when the Markov process is i.i.d. When Θ is finite, a (generic) sufficient condition for uniform ergodicity is that the transition matrix has all its entries strictly positive. Similar sufficient conditions can be derived when Θ is compact (see, e.g., Theorem 16.2.5 in Meyn and Tweedie 1993).

[13] Note that the order of the qualifiers matters. There may not exist $\bar{\delta} < 1$ such that for all $\delta \geq \bar{\delta}$, gridlock is an equilibrium independent of how close $E\left[U_l\left(\tilde{\theta}\right)\right]$ and $E\left[U_r\left(\tilde{\theta}\right)\right]$ are to 0.

[14] We thank an anonymous referee for suggesting this example to us.

3.4 An example with quadratic preferences and i.i.d. shocks

In this section, we illustrate our results in the following simple environment. The profile of payoff functions U is quadratic (see Definition 2), $R = 1$, $L = -1$, and the state is drawn in an i.i.d. fashion across periods.

For the quadratic payoff specification, the current preferences are given by $U_k(\theta) = 4(\theta + b_k)$. Substituting this expression in (1) and in (2), we obtain that under an exogenous status quo, each player k votes for R if and only if $\theta \geq -b_k$, while under an endogenous status quo, player k votes for R if and only if $\theta \geq -b_k - \frac{\delta}{4}W_k(\theta)$. Since today's state does not affect future states, $W_k(\theta)$ is independent of θ. Therefore, in the equilibria of Γ^{ex} and Γ^{en}, each player k plays a cutoff strategy with the cutoff $-b_k$ and $-b_k - \frac{\delta}{4}W_k$, respectively.

Denote $p_k = \frac{\delta}{4}W_k$. The above implies that in the game with an endogenous status quo, players behave as if the status quo were exogenous but their ideological biases were $b + p$ instead of b. From Proposition 1, in any equilibrium of Γ^{en}, $p_l \leq 0 \leq p_r$. Hence, since $b_l < b_k$, the strategic polarization as captured by p moves players farther apart on the ideological spectrum.

It is instructive to see how W_k, and thus p_k, are determined. Recall that W_k measures the expected gain for player k in period t from having R instead of L as the status quo in $t+1$. The status quo in $t+1$ matters only if players' votes disagree in that period, that is, when $\theta(t+1) \in \left[-b_r - \frac{\delta}{4}W_r, -b_l - \frac{\delta}{4}W_l\right)$, in which case the status quo stays in place. For any such $\theta(t+1)$, the continuation payoff from implementing R relative to L is $U_k(\theta(t+1)) + \delta W_k$. Hence, in equilibrium, W_k must satisfy

$$W_k = \int_{-b_r - \frac{\delta}{4}W_r}^{-b_l - \frac{\delta}{4}W_l} \left(U_k(\theta) + \delta W_k\right) dP(\theta).$$

Figure 1 below depicts the equilibrium p_r in the symmetric case in which $b_r = -b_l = 0.6$ and $\theta(t)$ is drawn uniformly on $\Theta = [-2, 2]$ (by symmetry, $p_l = -p_r$).For $\delta < 0.7$, the equilibrium is unique and the corresponding p_k strictly increases in δ. When $\delta \geq 0.7$, condition (3) is satisfied, so the gridlock equilibrium exists (which corresponds to $p_r = 1.4$). For $\delta \in (0.7, 0.83)$, there are three equilibria, and for $\delta \geq 0.83$, only the gridlock equilibrium remains. In that case, under an exogenous status quo players reach an agreement with probability 0.7 (when $-b_l \leq \theta < -b_r$), but they never do so under an endogenous status quos.

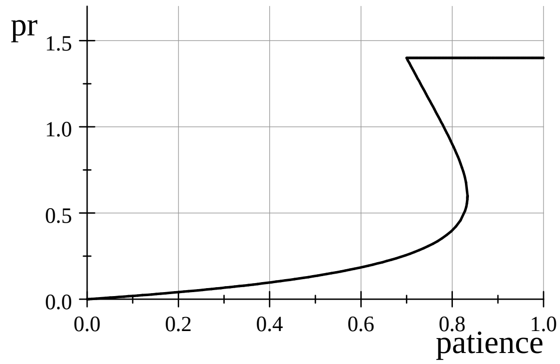


Figure 1: Equilibrium p_r as a function of δ.

4 The $N-$alternative model

The $2-$alternative model allows us to isolate in a transparent way the polarizing effect of the endogenous status quo. In this section, we argue that under reasonable conditions, this effect is robust to the number of alternatives.

4.1 Polarization and moderation: Intuition

With an arbitrary number of alternatives, it is still true that the status quo matters only when players disagree, so players' equilibrium behavior still depends on which policies they prefer when they disagree. However, players' preferences conditional on disagreement are harder to determine, because there is more than one way to disagree. Moreover, the consequences of some types of disagreement depend on the within-period bargaining procedure (i.e., the procedure that is used to select the bargaining outcome in each period).

First, as in the $2-$alternative model, players might disagree about which policies are better than the status quo. For example, in the context of legislative bargaining over tax policies, the conservatives may want to decrease tax rates while the liberals may want to increase them. In such disagreement states, the status quo stays in place irrespective of the within-period bargaining procedure. Hence, a higher (lower) status quo results in a higher (lower) tax rate. When the status quo is endogenous, the anticipation of such disagreement biases the conservatives in favor of lower and the liberals in favor of higher taxes. This is the same *polarizing effect* as in the case of two alternatives.

However, players may also agree that a certain subset of policies is better than the status quo, but disagree about the ranking of these alternatives. In such disagreements, the bargaining outcome might not vary monotonically with the status quo. To see this, consider again the case of legislators bargaining over taxes, and suppose that a shift in investors' sentiment over the sustainability of the deficit makes increasing taxes desirable. In that case, all legislators may want to increase taxes, but the liberals will want to increase them more. If the status quo tax rates are excessively low given the needs, and if the liberals can make a take-it-or-leave-it offer, they will propose a greater tax increase than what is optimal for the conservatives, and the conservatives will have no choice but to accept it. In this case, a slightly higher status quo would have constrained the bargaining power of the liberals and resulted in a lower tax rate. When the status quo is endogenous, the anticipation of this scenario biases the conservatives against low taxes. Thus, the endogenous status quo can also have a *moderating effect*.

The two effects have opposite implications in terms of the dynamics of the bargaining outcome. The polarizing effect induces players to disagree more often than their current preferences do, and thus leads to status quo inertia. In contrast, the moderating effect biases players' behavior towards alternatives that are typically favored by the opponent. Therefore, it may induce a player to agree on a policy change even if her current preferences do not favor it; that is, a new policy may be approved even if it is Pareto dominated by the status quo. This means that moderation can lead to status quo instability. Hence, whether the endogenous status quo induces policy inertia as in the $2-$alternative model depends on which of these two effects dominates. In Sections 4.2 and 4.3, however, we identify two arguably relevant environments in which the polarizing effect dominates and status quo inertia prevails.

In Section 4.2, we assume that within each period, the bargaining outcome is selected through the consensus procedure (Gradstein 1999, Dal Bo 2006, Riboni and Ruge-Murcia 2010): players revise the status quo through a series of incremental changes. We show that under this within-period bargaining procedure, the polarizing effect dominates. To understand this result, note that from the above discussion, the polarizing effect occurs irrespective of the within-period bargaining procedure. For the moderating effect to occur, however, the bargaining procedure must favor more drastic policy changes whenever players agree on the direction of the change but not its magnitude. By the virtue of being incremental, the consensus procedure favors less drastic policy changes, thereby eliminating the moderating effect.

There are reasons to believe that the consensus procedure is a reasonable description of the negotiations that occur in some decision-making bodies. First, since the seminal work of Lindblom (1959) and Wildavsky (1992), a large body of literature on policy processes has found that public decision making in pluralist societies tends to follow an incremental approach which is in line with the consensus procedure.[15] Second, Riboni and Ruge-Murcia (2010) have found that in the case of monetary policy committees, the consensus procedure delivers outcomes that are consistent with empirical evidence.[16]

In Section 4.3, we allow for a general class of within-period bargaining procedures, but consider instead an environment in which preferences evolve smoothly over time and players are given the opportunity to revise the status quo arbitrarily frequently. We show that in such environments, status quo inertia prevails in equilibrium. To understand why, note that from the above discussion, to generate the type of disagreement that leads to moderation, the preferences must change drastically from one period to the next: players must agree on low taxes at some point in time, but both must prefer to increase them substantially when the next chance to renegotiate arises. Such events do not occur when players can revise the policy agreement as soon as the environment prompts it. We believe that the latter assumption is empirically relevant because in actual bargaining situations, bargaining parties are rarely constrained on when they can revise the status quo.[17]

4.2 Bargaining with the consensus procedure

The set of alternatives $X = \{x_1, x_2, ..., x_N\}$ is a subset of $\mathbb{R}$, with $x_1 < ... < x_N$. As in the $2-$alternative model, we assume that the profile of payoff functions $(U_k(\theta, x))_{k\in\{l,r\},\theta\in\Theta,x\in X}$ is bounded, and that player r is more rightist than player l in the sense that naturally extends

[15] This strand of literature typically compares a rational, comprehensive policy making model in which all possible alternatives are considered before a policy change is chosen to an incremental bargaining model in which non incremental policy changes are attainable only through a series of incremental changes and conclude in favor of the latter. See, e.g., Hayes 1992.

[16] See Section 2 for a detailed discussion of how our results relate to the empirical evidence in Riboni and Ruge-Murcia (2010).

[17] Suppose for instance that legislators cut taxes after an economic contraction, but quickly thereafter, the markets lose faith in the ability of the government to repay its debts. In that scenario, the legislators are unlikely to wait for the next ordinary legislative session to start negotiating over how to react to the new situation. Similarly, since 1981 the Federal Open Market Committee have held eight regularly scheduled meetings each year at intervals of five to eight weeks. If circumstances require, however, members may be called on to participate in a special meeting. For example, during the financial crisis the committee met eleven times in 2007 and fourteen in 2008. Source http://www.federalreserve.gov/monetarypolicy/fomchistorical2008.htm

Definition 1: For all $\theta \in \Theta$ and all $x, y \in X$ such that $x < y$, $U_l(\theta, y) - U_l(\theta, x) \leq U_r(\theta, y) - U_r(\theta, x)$.

The game differs from the one analyzed in Section 3 only in that in each period, the bargaining outcome is decided using the consensus procedure. That is, each period is divided into multiple stages. In the first stage, the proposer $pr \in \{l, r\}$ (whose possibly random identity can depend on the state θ) can play $left$ or $right$. If she plays $right$ ($left$), then in the subsequent stages of that period, players vote via the unanimity rule on how far to the right (left) they should incrementally move the status quo. Formally, if the status quo is x_n, players vote on whether to move to x_{n+1} (x_{n-1}). If one of them votes *no*, the game stops. If both vote *yes*, x_{n+1} (x_{n-1}) becomes the temporary agreement for the next stage. In the next stage, players vote on whether to move to x_{n+2} (x_{n-2}), and so on. Bargaining continues until one player plays *no*, or until the temporary agreement is x_N (x_1). The last temporary agreement x becomes the bargaining outcome for that period, players obtain payoffs $(U_k(\theta, x))_{k \in \{l,r\}}$, and the game moves to the next period with x as the new status quo. The discounting $\delta \in (0, 1)$ occurs only between periods. We call this game Γ_c^{en}. Let Γ_c^{ex} denote the game which differs from Γ_c^{en} in that the status quo in each period t is exogenously fixed at some $q(t)$.

As in the 2−alternative model, the effect of the endogenous status quo on equilibrium behavior is captured by the value function $q \to W_k^\sigma(\theta, q)$. If $W_k^\sigma(\theta, q)$ is increasing (decreasing) in q, player k favors replacing x_n with x_{n+1} in more (fewer) states than under the exogenous status quo. Hence, we will say that an equilibrium with a strategy profile σ is polarized if for all $\theta \in \Theta$, $W_r^\sigma(\theta, q)$ is weakly increasing in q and $W_l^\sigma(\theta, q)$ is weakly decreasing in q.[18]

Proposition 5 *If Θ is finite, there exists an equilibrium of Γ_c^{en} that is polarized.*

We cannot ascertain that all equilibria are polarized, because if the proposer is indifferent between choosing $left$ and $right$ for some status quos, she can impose a nonmonotonic mapping between the status quo and the outcome. As explained in Section 4.1, this implies that the rightist (leftist) player may not necessarily prefer more rightist (leftist) status quos.

In Assumption 1 below, we define a large class of environments in which such indifferences occur with negligible probability and show that in these environments, all equilibria are polarized. Assumption 1 requires that players' payoffs depend on a stochastic parameter $\theta_1(t)$ that is arbitrarily

[18] Note that for $|X| = 2$, if we set $x_1 = L$ and $x_2 = R$, this definition of strategic polarization is equivalent to the definition used in Section 3.

correlated across time, and on a shock $\theta_2(t)$ that smoothly perturbs the payoffs (condition (i)) but does not affect the distribution of future payoffs (condition (ii)).

Assumption 1 *In all periods t, the state can be decomposed into two coordinates $\theta(t) = (\theta_1(t), \theta_2(t))$ such that*

(i) conditioning on $\theta_1(t)$ and $\theta_1(t+1)$, $(U_k(\theta_1(t+1), \theta_2(t+1), x))_{k\in\{l,r\},x\in X}$ has an absolutely continuous distribution on $\mathbb{R}^{\{l,r\}\times X}$;

(ii) conditioning on $\theta_1(t)$, $\theta_1(t+1)$ and $\theta_2(t+1)$ are independent of $\theta_2(t)$.

Note that the payoff perturbation induced by $\theta_2(t)$ can have an arbitrarily small support. So the environment described in Assumption 1 can be viewed as an infinitesimal perturbation of the environment in which the (arbitrary) Markov process $\{\theta_1(t) : t \geq 0\}$ is the only payoff relevant state. Alternatively, if $\{\theta_1(t) : t \geq 0\}$ is deterministic, Assumption 1 encompasses the case in which players' payoffs are independently distributed over time.

Proposition 6 *If Assumption 1 is satisfied, then all equilibria are polarized. If furthermore Θ_1 is finite, an equilibrium exists.*

4.2.1 The case of quadratic preferences and random walk

Propositions 5 and 6 show that players' equilibrium behavior exhibits strategic polarization, but are silent on how the set of alternatives X affects the magnitude of polarization. In this section, we analyze a reasonable environment in which it is independent of X and is exactly the same as in the $2-$alternative model.

The profile of payoff functions U is quadratic (see Definition 2), and the process $\{\theta(t) : t \in \mathbb{N}\}$ is a random walk on $\Theta = \mathbb{R}$: for all $t \in \mathbb{N}$, $\theta(t+1) = \theta(t) + \nu(t)$, where the random variables $(\nu(t))_{t\in\mathbb{N}}$ are i.i.d and integrable.[19] We further assume that each $\nu(t)$ admits a probability density function f that is single-peaked with a peak at 0.

Let us first analyze the equilibrium behavior with an exogenous status quo. Once the direction is chosen, say *right*, whenever voting between a temporary status quo x_n and a change to x_{n+1},

[19] The careful reader will note that since $\Theta = \mathbb{R}$, quadratic U violates the assumption that U is bounded over Θ. However, the results stated in this section are derived without this assumption.

the unique stage-undominated action for each player is to vote *yes* if and only if x_{n+1} is closer to her peak than x_n. Given this behavior at the voting stages, a stage-undominated action for the proposer is to propose the direction of her peak, and all stage-undominated actions are outcome equivalent.

Consider a period with state θ and status quo x_n. Since the payoffs are quadratic, the strategy profile described above moves the policy to the right if both players prefer x_{n+1} to x_n, which happens when $\theta \geq \frac{x_{n+1}+x_n}{2} - b_l$. Likewise, the policy moves to the left if both players prefer x_{n-1} to x_n, which happens when $\theta < \frac{x_{n-1}+x_n}{2} - b_r$.[20] Hence, the status quo x_n stays in place if

$$\theta \in \left[\frac{x_n + x_{n-1}}{2} - b_r, \frac{x_{n+1} + x_n}{2} - b_l\right). \tag{5}$$

Proposition 7 *There exist $p_l < 0$ and $p_r > 0$ such that for all finite $X \subset \mathbb{R}$, the equilibrium of Γ_c^{ex} with players' biases $b + p$ is an equilibrium of the game Γ_c^{en} with players' biases b.*

Proposition 7 states that with an endogenous status quo, there exists an equilibrium in which players behave as if the status quo were exogenous but their biases were $b + p$ instead of b. Since $b_l < b_r$ and $p_l < 0 < p_r$, the strategic polarization again moves players further apart on the ideological spectrum. Together with (5), Proposition 7 implies that the status quo x_n stays in place when

$$\theta \in \left[\frac{x_n + x_{n-1}}{2} - (b_r + p_r,) \frac{x_{n+1} + x_n}{2} - (b_l + p_l)\right). \tag{6}$$

By comparing (5) and (6), we see that as in the 2−alternative model, the endogenous status quo increases status quo inertia, more so the greater the polarizing effect p.

Note that Proposition 7 states that p_l and p_r are independent of X. Hence, the endogenous status quo generates the same magnitude of polarization and status quo inertia independent of the number and similarity of the available alternatives. Moreover, since for $|X| = 2$ the equilibria of Γ_c^{en} are outcome equivalent to the equilibria of Γ^{en} analyzed in Section 3.2, Proposition 7 implies that the polarizing effect is exactly as in the 2−alternative model. Hence, consistent with the intuition laid out in Section 4.1, there is no moderating effect in this setting.

Figure 2 depicts the evolution of the equilibrium outcome in $\Gamma_c^{en}(\{L, M, R\}, b)$ (solid lines) and

[20] Recall that in the text we adopt the convention that players vote for the more rightist alternative when indifferent.

$\Gamma_c^{ex}(\{L, M, R\}, b)$ (dashed lines). The horizontal lines represent the set of states in which a given status quo stays in place, and the arrows represent the cutoff states at which the policy switches.

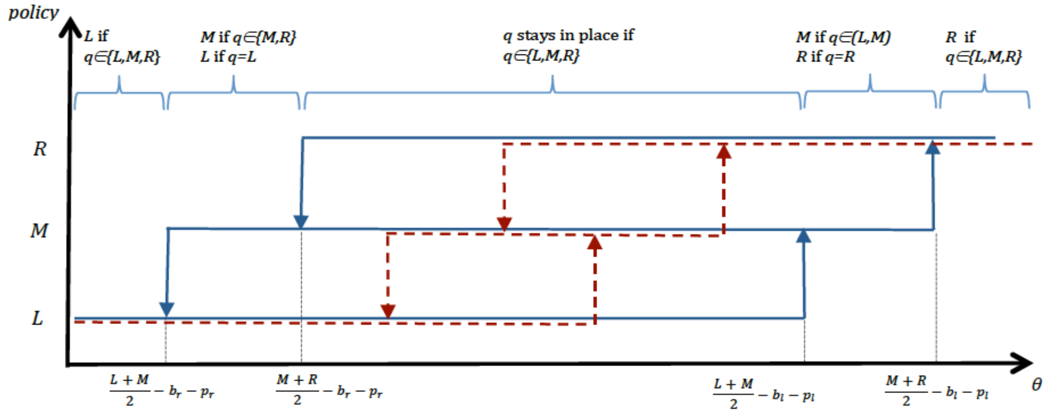


Figure 2: The evolution of the policy

4.3 Bargaining in a continuously changing environment

In this section, we formalize the intuition laid out in Section 4.1 that when preferences evolve continuously and players can bargain as soon as the status quo becomes obsolete, the moderation effect becomes negligible, and thus status quo inertia prevails.

The set of alternatives X is finite. Time is continuous, but players bargain only every $\Delta > 0$ units of time. As before, at each bargaining time $t \in \{0, \Delta, 2\Delta, ..\}$, a status quo $q(t)$ is in place, players observe the current state $\theta(t)$, and they determine the bargaining outcome by playing some (possibly state-dependent and stochastic) within-period bargaining procedure. Its outcome determines players' flow payoff for the period $[t, t + \Delta)$, and the status quo for the next bargaining time $q(t + \Delta)$. The within-period bargaining procedure used in state $\theta \in \Theta$ and status quo $q \in X$ is denoted by $\wp(\theta, q)$. The state $\theta(t)$ follows a continuous-time stochastic Markov process on Θ. The flow payoff of player $k \in \{l, r\}$ in state θ from alternative $x \in X$ is denoted by $u_k(\theta, x)$, which we assume is uniformly bounded. Players maximize the expected discounted sum of flow payoffs, and $\rho > 0$ is the discount factor. We denote this game by $\Gamma_{\wp}^{en}(\Delta)$.

Note that the models analyzed in Sections 3 and 4.2 are special cases of this model, where Δ is fixed, P_θ is the distribution of $\theta(t + \Delta)$ conditional on $\theta(t) = \theta$, $\delta = e^{-\rho\Delta}$, the payoff function is given by

$$U_k(\theta, x) = E_{\theta(0)=\theta}\left[\int_0^{\Delta} (u_k(\theta(t), x))\, \rho e^{-\rho t} dt\right], \tag{7}$$

and $\wp$ is the voting protocol or the consensus protocol, respectively.

4.3.1 A numerical example with the agenda-setter procedure

Suppose $X = \{-1, 0, 1\}$, the profile of flow payoffs is quadratic (see Definition 2), and $\{\theta(t) : t \in \mathbb{N}\}$ is a standard Brownian motion on $\Theta = \mathbb{R}$. Note that conditional on $\theta(t)$, $\theta(t + \Delta)$ is distributed according to $N\left(\theta(t), \sqrt{\Delta}\right)$, and if we set $U_k(\theta, x) = E_{\theta(0)=\theta}\left[\int_0^{\Delta} (u_k(\theta(t), x))\, \rho e^{-\rho t} dt\right]$, then $U_k(\theta, x)$ is quadratic (modulo a constant). Hence, if $\wp = c$ denotes the consensus procedure, then the game $\Gamma_c^{en}(\Delta)$ is equivalent to the game analyzed in Section 4.2.1 with $\delta = e^{-\rho\Delta}$ and f being the density of the normal distribution with $\mu = 0$ and $\sigma = \sqrt{\Delta}$.

Consider now the agenda-setter procedure, denoted by $\wp = r$, in which r makes a take-it-or-leave-it offer to player l in each period. By analogy with the equilibrium of $\Gamma_c^{en}(\Delta)$ characterized in Proposition 7, we postulate the following cutoff equilibrium of $\Gamma_r^{en}(\Delta)$: for each pair of consecutive alternatives, x_n and x_{n+1}, there exists $p_{x_n,x_{n+1}}$ and p_{x_{n+1},x_n} such that when $q = x_n$, r proposes a change to the right when $\theta \geq \frac{x_n + x_{n+1}}{2} - \left(b_l + p_{x_n,x_{n+1}}\right)$, and when $q = x_{n+1}$, r proposes a change to the left when $\theta < \frac{x_n + x_{n+1}}{2} - \left(b_r + p_{x_{n+1},x_n}\right)$. All proposals of r are accepted by l.[21] Note that this equilibrium of $\Gamma_r^{en}(\Delta)$ exhibits less polarization than the equilibrium of $\Gamma_c^{en}(\Delta)$ if $p_{x_n,x_{n+1}} > p_l$ and $p_{x_{n+1},x_n} < p_r$, where p_l and p_r are players' degree of strategic polarization in the equilibrium of $\Gamma_c^{en}(\Delta)$ characterized in Proposition 7.

The intuition laid out in Section 4.1 suggests that under the procedure $\wp = r$, the moderating effect should bias player l in favor of a moderate status quo $q = 0$ relative to a extreme leftist one $q = -1$ (and vice versa for player r), i.e., $p_{-1,0} > p_l$ and $p_{0,-1} < p_r$. This is because when both players wants to move the policy to the right but disagree by how much, a very leftist status quo $q = -1$ allows the proposer r to force l to accept a drastic change of policy to $x = 1$, while a moderate status quo $q = 0$ does not. However, the moderating effect should vanish as $\Delta \to 0$, i.e., $p_{-1,0} - p_l$ and $p_{0,-1} - p_r$ should tend to 0, because as players can revise the policy sufficiently frequently, the probability that they agree on -1 in some period but both prefer 1 in the next is

[21] This is only a partial description of the equilibrium, but is sufficient for the point made in this example.

negligible.

Figure 3 plots our numerical estimation of (p_l, p_r) (solid) and $(p_{-1,0}, p_{0,-1})$ (dotted) as a function of Δ for $\rho = 0.001$. As intuited, the moderating effect arises in $\Gamma_r^{en}(\Delta)$, but vanishes as $\Delta \to 0$.[22] Interestingly, Figure 3 also shows that under both the consensus and the agenda-setter procedures, players' degree of polarization increases as $\Delta \to 0$, so there is strict status quo inertia in the limit.[23]

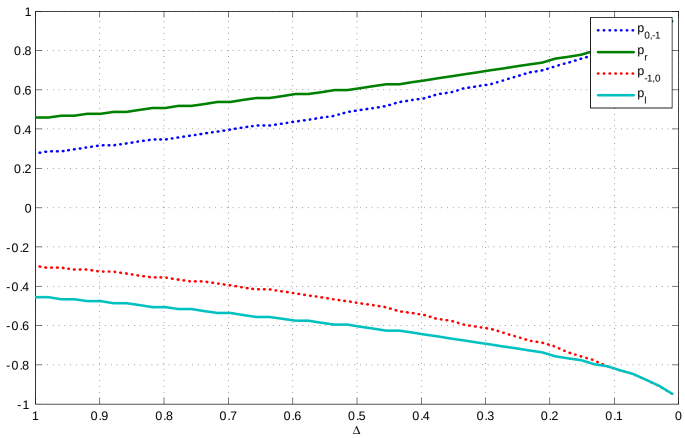


Figure 3: Strategic polarization in Γ_c^{en} and Γ_r^{en}.

4.3.2 General result: status quo inertia

We generalize the qualitative findings from Section 4.3.1 to a large class of within bargaining procedures $\wp$ and stochastic processes $\{\theta(t) : t \geq 0\}$.

The only restriction we impose on $\wp$ is that either player can unilaterally block any change away from the status quo. Formally, for all $\theta \in \Theta$ and $q \in X$, in any subgame perfect equilibrium of $\wp(\theta, q)$, with probability 1, both players weakly prefer the equilibrium outcome to the status quo q. Hence, $\wp$ can be a game form in which a state-dependent, randomly drawn proposer makes a take-it-or-leave-it offer to the other player, or the voting procedure used in Section 3.[24] It is

[22] For all parameters we used, we also obtain that there is no moderation effect between $q = 1$ and $q = 0$, i.e., $p_{0,1} = p_l$ and $p_{1,0} = p_r$. The intuition for this result is that with a rightist status quo, if both players wants to move the policy to the left but disagree by how much, under $\wp = r$, the player who wants the more drastic change, i.e. l, never has any bargaining power.

[23] All simulation results are available from the authors upon request.

[24] It is easy to see that the consensus procedure outlined in Section 4.2 satisfies this restriction in any pure strategy equilibrium: If the implemented policy left a player worse off than the status quo, this player would veto the first move away from the status quo. Under Assumption 1, our restriction is also satisfied for mixed-strategy equilibria.

worth noting that our conclusions also hold in the case in which the bargaining outcome within each period is determined by some cooperative bargaining solution such as the Nash bargaining solution, where players' payoffs are their continuation value from implementing a given alternative.

To capture the requirement that preferences evolve smoothly over time, we assume that the state space Θ is endowed with a topology such that for all $x \in X$, $u_k(\theta, x)$ is continuous in θ, and the process $\{\theta(t) : t \geq 0\}$ satisfies the following condition:

Assumption 2 *For all $\theta \in \Theta$, and all neighborhoods B of θ, if $\hat{t}_B$ denotes the first exit time of B, then $\Pr_{\theta(0)=\theta}\left(\hat{t}_B \leq t\right) = o(t)$ as $t \to 0$.*

Assumption 2 is satisfied for all standard continuous Markov processes such as the Brownian motion used in Section 4.3.1, and is violated for all standard discontinuous Markov processes such as the Poisson process used in 4.3.1.[25]

Let σ be a Markov strategy profile of $\Gamma_{\wp}^{en}(\Delta)$. For every (θ, q), the strategy profile σ induces a probability distribution over the possible outcomes of the bargaining stage-game, and we denote by $\phi_\sigma(\theta, q; x)$ the probability that x is implemented in that state. We call ϕ_σ the *outcome function* induced by the strategy profile σ.

In what follows, to characterize the equilibrium behavior as the bargaining friction Δ vanishes, we fix the within-period bargaining procedure $\wp$, and we consider a sequence $(\Delta_n)_{n\in\mathbb{N}}$ such that $\Delta_n \to 0$, and a sequence of strategy profiles $(\sigma_n)_{n\in\mathbb{N}}$ such that for all $n \in \mathbb{N}$, σ_n is a Markov perfect equilibrium of $\Gamma_{\wp}^{en}(\Delta_n)$. This sequence is fixed throughout, and we denote the corresponding sequence of outcome functions $\left(\phi_{\sigma_n}\right)_{n\in\mathbb{N}}$.

Definition 3 *An alternative x is stable at (q, θ) in the limit if there exists a neighborhood B of θ, some $\varepsilon > 0$, and some $N > 0$ such that for all $\zeta \in B$ and all $n \geq N$,*

(i) there exists a sequence of alternatives $x_1, ..., x_M$ with $x_1 = q$ and $x_M = x$ such that $\phi_{\sigma_n}(\zeta, x_m; x_{m+1}) > \varepsilon$ for all $m = 1, ..., M-1$;

(ii) $\phi_{\sigma_n}(\zeta, x; x) = 1$.

[25] See Remark 1 in the appendix for a formal proof of these claims. Note also that if $\{\theta(t) : t \geq 0\}$ is deterministic, then Assumption 2 is satisfied if and only if $\{\theta(t) : t \geq 0\}$ is continuous. When Θ is a metric space, it is satisfied if $\{\theta(t) : t \geq 0\}$ is Lipschitz continuous in t with probability 1.

Hence, if x is stable at (q,θ), then for Δ_n sufficiently small, with nonvanishing probability, the bargaining outcome moves from q to x with positive probability, possibly in several steps (condition (i) in Definition 3), and stays at x as long as $\theta(t)$ stays sufficiently close to θ (condition (ii) in Definition 3). The notion of stability is not a demanding one. To see this, note that for all (θ,q), conditions (i) and (ii) are always satisfied at $\zeta=\theta$: since X is finite and the bargaining outcome in state θ must always weakly improve players' intertemporal payoff relative to the status quo, if the state were to stay at θ for several periods, players would eventually reach a bargaining agreement that they would not want to revise.[26] Since X is finite, the path $x_1,...,x_M$ must be the same for some subsequence of $(\sigma_n)_{n\in\mathbb{N}}$. Definition 3 further requires that this path is the same for all states ζ sufficiently close to θ. Roughly speaking, this requirement is satisfied whenever σ_n does not change discontinuously in a neighborhood of θ for all n sufficiently large. For example, it is met for almost all θ if the equilibria $(\sigma_n)_{n\in\mathbb{N}}$ are in cutoff strategies as the equilibria analyzed in Sections 4.2.1 and 4.3.1.[27]

We are ready to state our result.

Proposition 8 *For all $\theta\in\Theta$ and all $q,x\in X$, if x is stable at (q,θ) in the limit, then for all $k\in\{l,r\}$, $u_k(\theta,x)\geq u_k(\theta,q)$.*

Proposition 8 states that when the negotiating parties can revise the bargaining agreement sufficiently frequently, the status quo q can be replaced by x in a neighborhood of some θ *only* if x Pareto improves on q in terms of the current preferences at θ. Note that with an exogenous status quo, the status quo q is replaced *exactly* when an alternative x Pareto improves on q in terms of the current preferences. Hence, Proposition 8 shows that if the endogenous status quo distorts the dynamics of the bargaining outcome relative to the exogenous status quo, this distortion can only take the form of status quo inertia.

[26] To be more precise, there might exists an equilibrium which prescribes players to cycle indefinitely between some subset of alternatives $X'\subset X$ in some state θ. However, in that case, both players must be indifferent between all alternatives in X', and one can avoid such cycles by requiring that for all $q,x\in X'$, whenever the equilibrium prescribes to move from q to x, one player blocks such a change and imposes to stay at q. The resulting strategy profile induces the same continuation value, and is thus still an equilibrium.

[27] To see this, note that these equilibria have the following properties: at each action node of the game $\Gamma_{\wp}^{en}$, players are prescribed pure action, and the set of states at which a given action is prescribed is an interval of $\Theta=\mathbb{R}$. Hence, for any sequence σ_n of such equilibria, there exists a subsequence in which these intervals converge, and in any state θ that does not coincide with the limit of the boundaries of these intervals, the actions prescribed by σ_n are locally constant in a neighborhood of θ for any status quo q.

The intuition for Proposition 8 is fairly simple. Consider player k deciding whether to agree to replace q with x at some bargaining time t. If the other player prefers such a change, then since preferences evolve smoothly, a small instant later she is still likely to prefer it. Hence, if q gives player k a higher current payoff than x, player k prefers to oppose the change in t and reconsider it an instant latter.

Since we impose minimal assumptions on the environment, the inertial effect of the endogenous status quo cannot be quantified. However, in the numerical example of Section 4.3.1, status quo inertia is strict as Δ tends to 0. The following intuition suggests that this is true in more general environments. Since the environment evolves smoothly, it is unlikely that players implement an alternative at some bargaining time, but an instant later both prefer a drastic policy change in the same direction. Therefore, the kind of disagreement that leads to moderation becomes unlikely (see the intuition provided in Sections 4.1 and 4.3.1). The kind of disagreement that leads to polarization, however, does not. To see this, note that when players implement an alternative at some bargaining time, typically the leftist player prefers a more leftist and the rightist player prefers a more rightist alternative than the agreement. Hence, since preferences evolve smoothly, an instant later r is likely to prefer to move the status quo to the right and l to the left, which is exactly the kind of disagreement that leads to polarization.[28]

5 Applications to legislative bargaining

The institutionally sparse model analyzed in this paper is not meant to be an accurate description of a particular institution. It is nonetheless informative to discuss how its assumptions and conclusions may apply to the U.S. Congress.

Most laws and policies enacted by the U.S. Congress are permanent: they remain in effect until

[28] Moreover, the degree of strategic polarization induced by this kind of disagreement has no reason to vanish as $\Delta \to 0$. To see this, consider the game $\Gamma_{\mathcal{G}}^{en}(\Delta)$ with $|X| = 2$, in which case only this kind of disagreement can occur. Since $\mathcal{G}$ can be assumed to be the voting protocol w.l.o.g. when $|X| = 2$, Proposition 4 applies, and by substituting $U_k(\theta, x) = E_{\theta(0)=\theta}\left[\int_0^{\Delta}(u_k(\theta(t), x))\rho e^{-\rho t}dt\right]$ into (3), we obtain that a gridlock equilibrium exists if and only if for all $\theta \in \Theta$,

$$E_{\theta(0)=\theta}\left[\int_0^{\infty} e^{-\rho t}(u_l(\theta(t), R) - u_l(\theta(t), L))\right] \leq 0 \leq E_{\theta(0)=\theta}\left[\int_0^{\infty} e^{-\rho t}(u_r(\theta(t), R) - u_r(\theta(t), L))\right].$$

Since Δ does not appear in the above expression, the condition under which the polarizing effect is strong enough to generate gridlock is independent of Δ.

a new legislative action is taken. As such, they fit the assumptions of our model. This is the case of mandatory spending policies, which include all entitlements and currently amount to 59% of total federal spending (Levit and Austin 2014), constitutional amendments, most statutes in the U.S. code, the Senate's rules of proceedings, or international treaties. Likewise, changes to the tax code are permanent unless legislators decide to attach a sunset provision, which historically has been the exception rather than the norm.[29]

Our model features two players but in Dziuda and Loeper (2014), we extend it to an arbitrary number of legislators and general voting rules. We show that if legislators' ideological positions can be ordered, the voting rule determines two pivotal players whose agreement, as in the current model, is necessary and sufficient for a policy change in any equilibrium. The assumption that legislators' ideological positions can be ordered is consistent with the finding in McCarthy et al. (2006) that almost all of the political conflict in Congress can be explained by the position of legislators on a one-dimensional liberal/conservative spectrum. As in the pivotal politics literature (Krebhiel 1998, Brady and Volden 2006), the leftist (rightist) player in our model can be interpreted as the most liberal (conservative) policy maker among the veto players.

5.1 Polarization and Gridlock

The finding of our paper that in a changing environment, the continuing nature of policies leads to polarization and gridlock resonates with the casual observations of the proceedings in the U.S. Congress. As a result of changing demographics, new social risks, increasing public debt and legislative complexity, there is a general consensus that important continuing policies such as entitlements, immigration laws, or the tax code are in need of a serious overhaul. Yet, legislators has repeatedly failed to act on them.

Since there is a lot of disagreement in the U.S. Congress also on temporary legislation (see, e.g., Keystone XL Pipeline), the endogenous status quo is admittedly not the only source of polarization and legislative inertia. As shown in the pivotal politics literature (Krehbiel 1999, Brady and Volden

[29] See, e.g., Posner and Verneule (2002, pages 1672, 1694, and 1701) on the permanent nature of statutes, the Senate's internal rules, or international treaties. As for tax legislation, prior to the Bush administration, sunsets applied mainly to relatively small provisions known as "tax extenders" (Gale and Orszag 2003). As Mooney (2004) puts it, "the use of sweeping sunset provisions in the tax code under the Bush Administration [referring to the sunset provisions in the Economic Growth and Tax Relief Reconciliation Act of 2001 and the Jobs and Growth Tax Relief Reconciliation Act of 2003] represents a massive departure from previous tax policy."

2006), supermajority requirements or veto points can create gridlock even for temporary legislation: as long as the status quo lies in-between the ideal points of the two pivotal players—the "gridlock interval"—any policy change will be vetoed. Our paper complements this literature by showing that in a dynamic setting, the continuing nature of policies makes these pivotal voters behave as if their ideal points were farther apart (see Sections 3.4 and 4.2.1), and thus increases the size of the gridlock interval. Indeed, it may be telling that many controversial but temporary legislations such as the Bush tax cuts of 2001 and 2003, the Patriot Act of 2001, the TARP plan, or Obama's fiscal stimulus in 2008 passed relatively swiftly, while the much needed reforms of the tax code or the entitlement system have not even reached the floor.

Our results (Proposition 4 and Corollary 2) show that strategic polarization and status quo inertia increase with legislators' discount rate. If legislators care about policies more when in office, their discount rate is linked to their reelection probability. In the case of the U.S. Congress, the incumbency reelection rate has been very high (above 80% in the Senate and 90% in the House since the 60s, see Friedman and Holden 2009, Figure 1), which suggests that the inertial effect of the endogenous status quo may be quite important. Our findings also suggest that political instability and term limits can be beneficial, which contrasts the existing dynamic political economy literature (Persson and Svensson 1989, Alesina and Tabellini 1990, Azzimonti 2011).

5.2 Legislators' ideology and polarization

Our paper adds to the discussion on legislators' ideology and polarization. Most of the literature measures legislators' ideal points on the left-right spectrum using NOMINATE scores derived from roll-call votes (Poole and Rosenthal 1997). The implicit premise of this empirical exercise is that legislators vote according to their true preferences. Our results imply that when voting on permanent policy changes, legislators may bias their voting behavior. As a result, NOMINATE scores may systematically overestimate legislators' ideological polarization.

The above observation resonates with the empirical findings in Bafumi and Herron (2010) that members of Congress are more extreme than the median voter of their respective constituency, where the former is measured by the voting record of the legislators and the latter by survey responses (see also Fiorina et al., 2006; Fiorina and Abrams, 2008; Krasa and Polborn, 2014). The difference between legislators' and voters' polarization is usually interpreted as a failure of political representation. Our paper suggests an alternative interpretation: even if legislators' preferences coincide with that of their median constituent, the voting records of forward-looking legislators may exhibit more polarization than the preferences expressed by their constituents in non-binding surveys.

The implication of our model that NOMINATE scores might be biased can be indirectly tested along the following lines. The voting behavior on permanent legislations of a given liberal (conservative) legislator is predicted to be more liberal (conservative) as her reelection probability increases and as the other legislators become more polarized. In contrast, these factors should not affect legislators' behavior if they vote on temporary legislations, or if they vote myopically as the NOMINATE methodology implicitly assumes. Such an analysis may not be straightforward, as many bills contain both types of policies, and legislators have some discretion about which policy changes are temporary or permanent, which may create an endogeneity problem. It is nevertheless interesting to note that the steady increase in the share of mandatory spending in the federal budget since the 60s (Levit and Austin 2014) has been accompanied by a sharp increase in polarization as measured by NOMINATE scores (McCarthy et al., 2006).

5.3 Temporary legislation and sunset provisions

This paper shows that linking today's agreement and tomorrow's status quo generates a detrimental polarizing effect. In the context of legislative bargaining, there are two natural ways to sever this link: the exogenous status quo, and sunset provisions.[30]

The exogenous status quo is arguably the most prevalent alternative to the endogenous status quo. For example, discretionary spending policies (e.g., federal wages, "pork barrel" projects) require annual appropriations, and are thus legislated under a fixed default of no spending (Levit and Austin 2014). However, the determination of the exogenous default policy can be controversial in many policy domains. For instance, there is arguably no natural default for income taxation or social security benefits. As a result, the exogenous status quo can hardly be advocated as a universal solution to the polarizing effect of the endogenous status quo. Moreover, the former may

[30] Automatic stabilizers can also affect the dynamic linkage of the endogenous status quo. An automatic stabilizer is a clause that triggers an automatic change in a continuing policy in response to a change in certain observable indicators (e.g., price levels, unemployment rate). In a related paper, Bowen et al. (2014b) show that the endogenous status can be an optimal rule when the status quo can be fully contingent on the state of nature. In practice, however, verifiability may be an issue.

not increase social welfare relative to the latter. The exogenous status quo is certain to be socially beneficial only if the default policy is set optimally in all states, but this may be hard to implement if the optimal default policy depends on the state and the latter is not verifiable.[31]

A sunset provision is a clause that specifies duration after which a legislative act automatically expires. Although sunset provisions have received little attention from economists, they have long been advocated by political theorists and legal scholars to address statute proliferation and obsolescence, and political entrenchment of regulations and agencies.[32] A formal analysis of a model with sunset provisions is beyond the scope of this paper, but our results suggest a novel rationale for sunsets: by allowing the legislators to change the law without affecting the future status quo, sunsets induce them to evaluate policies solely based on their adequacy to the current environment, and hence eliminate the polarizing effect.

In light of that rationale, one can ask why sunset provisions are not used more systematically. Legal scholars have pointed out that sunset provisions generate various inefficiencies.[33] Whenever these inefficiencies are large enough to cause legislators not to use sunsets, the positive conclusions of our model still apply.

It is also worth noting that in practice, sunsetted policies are typically extended multiple times or made permanent. Legal scholars have advanced various explanations for this pattern. Most prominently, the enactment of a sunsetted law encourages constituencies that benefit from it to organize and lobby for its extension.[34] As a result, even though sunsetted laws do not formally affect the status quo, they make it politically harder to revert to it. Whatever the explanation for this phenomenon, it suggests that the use of sunsets may not completely eliminate the perverse

[31] Since under the endogenous status quo, the default is determined by the equilibrium behavior, and hence evolves with the state, this protocol may dominate despite its polarizing effect. See Example 1 in Section 7.1.5 in the appendix for a formal proof of this claim.

[32] Jefferson (1789), Lowi (1969), and Calabresi (1982) famously supported the systematic use of sunset provision under various forms to avoid legislative obsolescence and entrenchment. Sunset provisions have also been argued to improve scrutiny over new policies with uncertain effects, and improve legislative response to temporary circumstances and new risks. See Gersen (2007) and the references therein.

[33] Most prominently, temporary legislations have been argued to generate legislative uncertainty, which is detrimental to economic activity, to increase compliance costs, and to diminish the incentive and preemptive effects of the law (Posner and Vermeule 2002 pp. 1670-2, Kysar 2011 pp. 1063-5, or Piqulliem and Riboni 2013a and 2013b). By requiring periodic reviews, automatic sunset clauses also increase administrative and bargaining costs (Kysar 2011 pp. 1056-9).

[34] It has also been argued that the enactment of a law generates electoral awareness and endowment effects that increase its popular support, even if the law is sunsetted. On these issues, see Mooney (2004), Kysar (2006), or Viswanathan (2007). Modelling entrenched interests formally in a dynamic setting is not a trivial exercise. Coate and Morris (1999) make progress on this front, but their model would not render permanent and temporary legislation identical in our model.

dynamic incentives highlighted in this paper.

Finally, legislators may not be willing to use sunsets for partisan reasons. Although we leave a formal analysis of this issue for future research, the following example illustrates this point. Suppose that a moderate tax cut is the only permanent change that is preferred by both pivotal legislators to the status quo, but a temporary and larger cut is statically Pareto better and would also be accepted. A rightist legislator may nevertheless prefer the former in order to secure a more favorable status quo for tomorrow. Hence, if she has the proposal power, she may not resolve the trade-off between the extent and the duration of a policy reform in a socially optimal way, and the implemented policy will be too close to the status quo.

6 Concluding remarks and directions for further research

Negotiations in a changing environment with an endogenous status quo are at the center of many economically relevant situations. They present the negotiating parties with a fundamental trade-off between responding adequately to the current environment and securing a favorable bargaining position for the future. In this paper, we show that this trade-off generates perverse incentives that have a detrimental impact on the efficiency of bargaining outcomes and their responsiveness to the environment.

7 Appendix

7.1 Proofs for the $2-$alternative model

7.1.1 Notations

Let m be a bound on $|U_k(\theta, x)|$ over all $k \in \{l, r\}$, $\theta \in \Theta$, and $x \in X$, and let $\mathcal{F}$ be the set of mappings from Θ into $\left[-\frac{m}{1-\delta}, \frac{m}{1-\delta}\right]$ that are measurable w.r.t. to the sigma algebra $\mathcal{S}$ of Θ. In what follows, $f = (f_l, f_r)$ denotes an arbitrary pair of such mappings, and $0^{\mathcal{F}}$ denotes the mapping that is equal to 0 for all $\theta \in \Theta$. For all $T \subseteq \Theta$ and $f \in \mathcal{F}^2$, denote

$$\begin{cases} D'(f) \mathrel{\hat{=}} \{\theta \in \Theta : f_l(\theta) < 0 \text{ and } f_r(\theta) > 0\} \\ D''(f) \mathrel{\hat{=}} \{\theta \in \Theta : f_l(\theta) > 0 \text{ and } f_r(\theta) < 0\} \\ I(f) \mathrel{\hat{=}} \{\theta \in \Theta : f_l(\theta) = 0 \text{ or } f_r(\theta) = 0\} \\ D(T, f) \mathrel{\hat{=}} D'(f) \cup D''(f) \cup (I(f) \cap T) \end{cases}. \quad (8)$$

For all $T \subseteq \Theta$, we define the mappings V and Ω^T as follows: for all $f \in \mathcal{F}^2$, $k \in \{l, r\}$, and $\theta \in \Theta$,

$$V_k(f_k, \theta) \mathrel{\hat{=}} U_k(\theta) + \delta f_k(\theta), \quad (9)$$

$$\Omega_k^T(f, \theta) \mathrel{\hat{=}} \int_{\zeta \in D(T,f)} f_k(\zeta)\, dP_\theta(\zeta), \quad (10)$$

where P_θ denote the probability distribution of $\theta(t+1)$, conditional on $\theta(t) = \theta$. When f is equal to the continuation value W^σ of some strategy profile σ as defined in Section 3.2, then $V_k(W_k^\sigma, \theta)$ is the relative gain in continuation payoff for player k of implementing alternative R instead of L in period t with state $\theta(t) = \theta$ in the game Γ^{en}, conditional on σ being played thereafter. Then, $D'(V(W^\sigma))$ and $D''(V(W^\sigma))$ are the set of states in which players would strictly prefer to implement different alternatives given the continuation play σ, and $I(V(W^\sigma))$ is the set of states in which some player is indifferent between the alternatives. Thus, the set $D(T, V(W))$ can be viewed as the set of states in which players' disagree, with the convention that they disagree when they strictly prefer implementing different alternatives (that is, when $\theta \in D'(V(W^\sigma)) \cup D''(V(W^\sigma))$), and when some player is indifferent and the state is in T. Hence, the set T can be viewed as an arbitrary tie breaking rule which specifies how players vote when they are indifferent. Finally, $\Omega_k^T(V(W^\sigma), \theta)$ is the expectation of the relative gain for player k of implementing alternative R instead of L in period $t+1$, conditional on $\theta(t) = \theta$, on continuation play σ, and on players disagreeing in $t+1$ in the above sense.

We denote by $(\leq, \geq)$ the partial order on $\mathcal{F}^2$ defined as follows: for all $f, g \in \mathcal{F}^2$, $f(\leq, \geq) g$ if for all $\theta \in \Theta$, $f_l(\theta) \leq g_l(\theta)$ and $f_r(\theta) \geq g_r(\theta)$. When f and g are continuation values of strategy profiles, this order corresponds to the polarization order on strategy profiles defined before Proposition 2.

7.1.2 Preliminary Lemmas

The following two lemmas shows that a strategy profile is an equilibrium if and only if its continuation value is a fixed point of the map $f \to \left(\Omega^T \circ V\right)(f)$, and establish that this map is monotonic in the order $(\leq, \geq)$.

Lemma 1 *If σ is an equilibrium of Γ^{en}, then there exists $T \subset \Theta$ such that W^σ is a fixed point of the map $f \to \left(\Omega^T \circ V\right)(f)$, and σ prescribes players' votes to disagree in state θ if and only if $\theta \in D(T, V(W^\sigma))$. Conversely, if W is a fixed point of $f \to \left(\Omega^T \circ V\right)(f)$ for some $T \subset \Theta$, then there exists an equilibrium σ of Γ^{en} such that $W^\sigma = W$ and σ prescribes players' votes to disagree in state θ if and only if $\theta \in D(T, V(W))$.*

Proof. Let σ be an equilibrium and W^σ be the corresponding continuation value. Recall that W_k^σ is the relative gain in continuation payoff for player k evaluated in t from having R instead of L as $t+1$ status quo. The status quo in $t+1$ affects the outcome in $t+1$ only if players' votes disagree in that period. By stage undomination, this happens when $\theta(t+1) \in D(T, V(W^\sigma))$ for some $T \subset \Theta$ (see the explanation in Section 7.1.1). In such states $\theta(t+1)$, either status quo stays in place, so the gain in continuation payoff of having status quo R relative to L is $V_k(W^\sigma, \theta(t+1))$. This means that W^σ is the expectation of $V_k(W^\sigma, \theta(t+1))$ over all disagreement states $\theta(t+1) \in D(T, V(W^\sigma))$, conditional on $\theta(t) = \theta$, so from (10), $W^\sigma = \Omega^T(V(W^\sigma))$.

Reciprocally, let $W \in \mathcal{F}^2$ be such that $W = \Omega^T(V(W))$ for some $T \subset \Theta$. Consider the following Markov strategy profile σ: for all $\theta \in \Theta$, (i) player k votes for R if $U_k(\theta) + \delta W_k(\theta) > 0$, for L when $U_k(\theta) + \delta W_k(\theta) < 0$, (ii) if $U_k(\theta) + \delta W_k(\theta) = 0$, then player k votes for a different alternative than her opponent if and only if $\theta \in T$. Hence, σ prescribes players to vote for different alternatives in state θ if and only if $\theta \in D(T, V(W))$. By definition, the corresponding continuation value W^σ is given by the expectation of $V(W, \theta(t+1))$ conditional on $\theta(t)$ over all the states in which players' votes disagree. Therefore, for all $\theta \in \Theta$,

$$W_k^\sigma(\theta) = \int_{\zeta \in D(T, V(W))} \left[U_k(\zeta) + \delta W_k^\sigma(\zeta)\right] dP_\theta(\zeta). \tag{11}$$

Equation (11) can be seen as a functional equation in W^σ, and since $W = \Omega^T(V(W))$, $W^\sigma = W$ is a solution to it. The right hand-side of (11) is a δ-contraction in W^σ for the *sup* norm. Therefore,

(11) has a unique solution, which implies that $W^\sigma = W$. So by construction of σ, σ prescribes each player k to vote for R when $U_k(\theta) + \delta W_k^\sigma(\theta) > 0$ and for L when $U_k(\theta) + \delta W_k^\sigma(\theta) < 0$, which means that both players use stage-undominated strategies. So σ is an equilibrium. ■

Lemma 2 *For all $T \subseteq \Theta$, and all $f, g \in \mathcal{F}^2$ such that $f(\leq, \geq) g (\leq, \geq) \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$, the map $\Omega^T \circ V$ is monotonic in the sense that $\left(\Omega^T \circ V\right)(f)(\leq, \geq)\left(\Omega^T \circ V\right)(g)$.*

Proof. Using the assumptions $U_l \leq U_r$ and $f(\leq, \geq) g (\leq, \geq) \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$ in (9), we obtain $V_l(f) \leq V_l(g) \leq V_r(g) \leq V_r(f)$. Using the latter inequalities in (8), we obtain that $D''(V(f)) = D''(V(g)) = \emptyset$ and that $D'(V(g)) \cup (I(g) \cap T)$ is included in $D'(V(f)) \cup (I(f) \cap T)$. From (10),

$$\left(\Omega_r^T \circ V\right)(f, \theta) - \left(\Omega_r^T \circ V\right)(g, \theta) = \int_{D'(V(f)) \cup (I(f) \cap T)} V_r(f_r, \zeta)\, dP_\theta(\zeta) - \int_{D'(V(g)) \cup (I(g) \cap T)} V_r(g_r, \zeta)\, dP_\theta(\zeta).$$

Since the first integral on the right-hand side of the above equality integrates larger and positive values over a larger set than the second integral, the left-hand side must be positive. An analogous proof shows that $\left(\Omega_l^T \circ V\right)(f, \theta) \leq \left(\Omega_l^T \circ V\right)(g, \theta)$. ■

7.1.3 Proofs for Section 3.2

Proof of Proposition 1. Let W be the continuation value of some equilibrium. From Lemma 1, $W = \Omega^T(V(W))$ for some $T \subset \Theta$. Using the notation (9) and (10), this means that for all $\theta \in \Theta$,

$$W_r(\theta) - W_l(\theta) = \int_{D(T, V(W))} \left(U_r(\zeta) - U_l(\zeta) + \delta\left(W_r(\zeta) - W_l(\zeta)\right)\right) dP_\theta(\zeta). \tag{12}$$

Using the assumption that $U_r - U_l \geq 0$, (12) implies that for all $\theta \in \Theta$, $W_r(\theta) - W_l(\theta) \geq 0$.[35] Substituting the last two inequalities in (9), we obtain $V_r(W_r) \geq V_l(W_l)$. From (8), this implies that $D''(V(W)) = \emptyset$, so $D(T, V(W)) = D'(V(W)) \cup (I(V(W)) \cap T)$. Moreover, from (8), for all $\zeta \in D'(V(W)) \cup (I(V(W)) \cap T)$, $V_l(W, \zeta) \leq 0 \leq V_r(W, \zeta)$. From (10), this shows that $\Omega^T(V(W))(\leq, \geq) 0$, and thus $W(\leq, \geq) 0$. ■

Proof of Proposition 2. Define the sequence $(W^z)_{z \in \mathbb{N}}$ recursively as follows: for all $\theta \in \Theta$,

[35] To see this, let $\underline{w} \doteq \inf_{\theta \in \Theta}\left(W_r(\theta) - W_l(\theta)\right)$ and let $(\theta_n)_{n \in \mathbb{N}}$ be a sequence of states such that $W_r(\theta_n) - W_l(\theta_n) \to \underline{w}$. Then from (12), for all $n \in \mathbb{N}$, $W_r(\theta_n) - W_l(\theta_n) \geq \delta \underline{w} P_{\theta_n}(D(T, V(W)))$. Since $0 \leq P_{\theta_n} \leq 1$, there exists a subsequence along which $\left(P_{\theta_n}(D(T, V(W)))\right)_{n \in \mathbb{N}}$ converges to some $p \in [0, 1]$. Taking the limit along this subsequence, we obtain $\underline{w} \geq \delta \underline{w} p$, so $\underline{w} \geq 0$, as needed.

$W^0(\theta) = \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$, and for all $z \geq 0$, $W^{z+1} = \left(\Omega^{\emptyset} \circ V\right)(W^z)$. Below, we show that $(W^z)_{z\in\mathbb{N}}$ is monotonic, bounded, and thus converges (steps 1 and 2), its limit is the continuation value of a least polarized equilibrium (steps 3 to 6), and this equilibrium is Pareto best (step 7). Throughout, $\bar{W}$ is an arbitrary mapping such that $\bar{W}(\leq,\geq)\left(\Omega^{\emptyset} \circ V\right)\left(\bar{W}\right)$ and $\bar{W}(\leq,\geq)\left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$.[36]

Step 1: For all $z \in \mathbb{N}$, $\bar{W}(\leq,\geq) W^{z+1}(\leq,\geq) W^z$.

We prove this by induction on z. Since $W^0 = \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$, $V\left(W^0\right) = U$, and since $U_l \leq U_r$, $V_l\left(W^0\right) \leq V_r\left(W^0\right)$. Using the latter inequality in (8), we obtain $D''\left(V\left(W^0\right)\right) = \emptyset$, and thus $D\left(\emptyset, V\left(W^0\right)\right) = D'\left(V\left(W^0\right)\right)$, so for all $\theta \in D\left(\emptyset, V\left(W^0\right)\right)$, $V_l\left(W^0,\theta\right) \leq 0 \leq V_r\left(W^0,\theta\right)$. Using the latter inequality in (10), we obtain $\left(\Omega^{\emptyset} \circ V\right)\left(W^0\right)(\leq,\geq)\left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$, or equivalently, $W^1(\leq,\geq) W^0$. Moreover, since $\bar{W}(\leq,\geq)\left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right) = W^0$, Lemma 2 implies that $\left(\Omega^{\emptyset} \circ V\right)\left(\bar{W}\right)(\leq,\geq)\left(\Omega^{\emptyset} \circ V\right)\left(W^0\right)$, which, together with the assumption that $\bar{W}(\leq,\geq)\left(\Omega^{\emptyset} \circ V\right)\left(\bar{W}\right)$, implies that $\bar{W}(\leq,\geq) W^1$. This completes the proof of step 1 for $z = 0$. Suppose now that $\bar{W}(\leq,\geq) W^z(\leq,\geq) \ldots (\leq,\geq) W^0$ for some $z \geq 0$. Then Lemma 2 implies that $\left(\Omega^{\emptyset} \circ V\right)\left(\bar{W}\right)(\leq,\geq) W^{z+1}(\leq,\geq) W^z$, which, together with $\bar{W}(\leq,\geq)\left(\Omega^{\emptyset} \circ V\right)\left(\bar{W}\right)$, completes the induction argument.

Step 2: The sequence $(W^z)_{z\in\mathbb{N}}$ converges pointwise to some W^∞ such that $\bar{W}(\leq,\geq) W^\infty$.

From step 1, for all $\theta \in \Theta$, the sequence $(W^z_k)(\theta)$ is bounded and increasing (decreasing) in z for $k = r$ ($k = l$). As such, it converges to some $W^\infty_k(\theta)$. The inequality $\bar{W}(\leq,\geq) W^\infty$ follows directly from step 1.

Step 3: If 1_S denotes the indicator function of a set $S \subseteq \Theta$, then $1_{D(\emptyset,V(W^z))}$ is increasing in z and converges pointwise to $1_{D(\emptyset,V(W^\infty))}$.

Since $W^z(\leq,\geq) W^0$, the same reasoning as in step 1 shows that for all $z \in \mathbb{N}\cup\{\infty\}$, $D(\emptyset, V(W^z)) = D'(V(W^z))$ so $1_{D(\emptyset,V(W^z))} = 1_{D'(V(W^z))}$. From (9) and step 1, $V(W^z)$ is increasing in z for the order $(\leq,\geq)$, so from (8), $1_{D'(V(W^z))}$ is increasing. Now let θ be such that $1_{D'(V(W^\infty))}(\theta) = 1$. Then $V_l(W^\infty)(\theta) < 0 < V_r(W^\infty)$. Since V is continuous in W, for z sufficiently large, $V_l(W^z)(\theta) < 0 < V_r(W^z)(\theta)$, so $\lim_{z\to\infty} 1_{D'(V(W^z))}(\theta) = 1$. Reciprocally, suppose $\lim_{z\to\infty} 1_{D'(V(W^z))}(\theta) = 1$. Then for some z, $V_l(W^z)(\theta) < 0 < V_r(W^z)(\theta)$, and since $V(W^z)$ is increasing in z for the order $(\leq,\geq)$, $V_l(W^\infty)(\theta) < 0 < V_r(W^\infty)$, so $1_{D'(V(W^\infty))}(\theta) = 1$.

Step 4: W^∞ is a fixed point of $\Omega^{\emptyset} \circ V$.

[36] Such a mapping exists, e.g., the greatest element of $\mathcal{F}^2$ in the order $(\leq,\geq)$: for all $\theta \in \Theta$, $\bar{W}(\theta) \doteq \left(-\frac{m}{1-\delta}, \frac{m}{1-\delta}\right)$. The reason why we allow $\bar{W}$ to be any such mapping will be clear in the proof of Propositions 3 and 7.

For all $z \in \mathbb{N}$, $k \in \{l, r\}$ and $\theta \in \Theta$,

$$W_k^{z+1}(\theta) = \left(\Omega^{\emptyset} \circ V\right)_k (W^z)(\theta) = \int_{\zeta \in \Theta} 1_{D(\emptyset, V(W^z))}(\zeta) \times [U_k(\theta) + W_k^z(\zeta)]\, dP_\theta(\zeta).$$

From step 2, the first term of the above equation tends to $W^\infty(\theta)$ as $z \to \infty$. Using steps 2 and 3 together with Lebesgue's monotone convergence theorem, the last term tends to $\left(\Omega^{\emptyset} \circ V\right)(W^\infty)(\theta)$.

Step 5: for any equilibrium σ, $W^\sigma(\leq, \geq) W^\infty$.

From Lemma 1, there exists $T \subseteq \Theta$ such that $W^\sigma = \left(\Omega^T \circ V\right)(W^\sigma)$ and players' votes disagree in σ when $\theta \in D(T, V(W^\sigma))$. As shown in the proof of Proposition 1, in all disagreement states $\theta \in D(T, V(W^\sigma))$, $V_l(W^\sigma, \theta) \leq 0 \leq V_r(W^\sigma, \theta)$. Substituting this inequality and $D(\emptyset, V(W^\sigma)) \subseteq D(T, V(W^\sigma))$ into (10), we obtain $\left(\Omega^T \circ V\right)(W^\sigma)(\leq, \geq)\left(\Omega^{\emptyset} \circ V\right)(W^\sigma)$, and thus $W^\sigma(\leq, \geq)\left(\Omega^{\emptyset} \circ V\right)(W^\sigma)$. Hence, W^σ satisfies the condition for being $\bar{W}$. From step 2, $W^\sigma(\leq, \geq) W^\infty$.

Step 6: W^∞ is the continuation value of a least polarized equilibrium σ^∞ in which players' votes disagree in a smaller set of states (in the inclusion sense) than in any other equilibrium.

Step 4 and Lemma 1 imply that W^∞ is the continuation value of some equilibrium σ^∞, and from Step 5, σ^∞ is less polarized than any other equilibrium σ. Let $\theta \in \Theta$ be a state in which players' votes disagree in σ^∞. As established in the proof of Proposition 1, $V_l(W^\infty, \theta) \leq 0 \leq V_r(W^\infty, \theta)$, and from Lemma 1, $\theta \in D(\emptyset, V(W^\infty))$, so necessarily, $V_l(W^\infty, \theta) \leq 0 \leq V_r(W^\infty, \theta)$. Since $W^\sigma(\leq, \geq) W^\infty$, this implies that $V_l(W^\sigma, \theta) < 0 < V_r(W^\sigma, \theta)$, and stage undomination implies that players' votes also disagree in σ in state θ.

Step 7: σ^∞ is Pareto better than any other equilibrium σ.

Suppose that both players deviate from σ^∞ to the behavior prescribed by σ only in $t = 0$. From step 6, players disagree in more states under σ than under σ^∞. Therefore, the deviation from σ^∞ to σ in $t = 0$ can change the outcome at $t = 0$ only if under σ^∞ players agree to change the status quo while under σ, the status quo remains. Suppose w.l.o.g. that $q(0) = R$, and hence L is implemented under σ^∞ while R stays in place under σ. Since players play their equilibrium strategy σ^∞ in the subgame starting in $t = 1$, the net effect of this deviation for player k is $U_k(\theta(0)) + \delta W_k^\infty(\theta(0))$. Since both players vote for L under σ^∞, $U_k(\theta(0)) + \delta W_k^\infty(\theta(0))$ must be nonpositive for both $k \in \{l, r\}$, which implies that the deviation weakly decreases the payoff of both players. Consider now the strategy profile in which players play σ in $t=0$ and σ^{∞} afterwards, and let players deviate from that profile by playing according to σ also in $t=1$. The same reasoning as above shows that this deviation further decreases the payoffs of both players irrespective of the status quo distribution in $t=1$. By induction on the number of periods in which players deviate from σ^{∞} to σ, step 7 follows.

Step 8: A most polarized equilibrium exists and is Pareto worst.

The proof follows the same logic as the steps 1 to 7 with the sequence $(W^z)_{z\in\mathbb{N}}$ defined recursively as follows: W^0 is the greatest element of $\mathcal{F}$ (i.e., $W(\theta)=\left(-\frac{m}{1-\delta},\frac{m}{1-\delta}\right)$ for all $\theta\in\Theta$) and for all $z\geq 0$, $W^{z+1}=\left(\Omega^{\Theta}\circ V\right)(W^z)$. We omit the details for brevity. ■

7.1.4 Proofs for Section 3.3

Proof of Proposition 3. Let V and V' refer to the maps defined in (9) and let σ and σ' be a least polarized equilibria corresponding to (U,δ) and to $\left(U',\delta'\right)$, respectively, as constructed in the proof of Proposition 2. As shown in step 2 of that proof, $\bar{W}(\leq,\geq)W^{\sigma}$ for any $\bar{W}$ such that (i) $\bar{W}(\leq,\geq)\left(0^{\mathcal{F}},0^{\mathcal{F}}\right)$ and (ii) $\bar{W}(\leq,\geq)\left(\Omega^{\emptyset}\circ V\right)\left(\bar{W}\right)$. Hence, to prove Proposition 3, it suffices to show (i) and (ii) are satisfied for $\bar{W}=W^{\sigma'}$. Inequality (i) follows from Proposition 1. To prove (ii), note that as shown in the proof of Proposition 2, $W^{\sigma'}=\left(\Omega^{\emptyset}\circ V'\right)\left(W^{\sigma'}\right)$. Moreover, from (9), $V'\left(W^{\sigma'}\right)=V\left(\frac{\delta'}{\delta}W^{\sigma'}+\frac{1}{\delta}\left(U'-U\right)\right)$, and since $\delta'>\delta$ and $U'(\leq,\geq)U$, $\left(\frac{\delta'}{\delta}W^{\sigma'}+\frac{1}{\delta}\left(U'-U\right)\right)(\leq,\geq)W^{\sigma'}(\leq,\geq)\left(0^{\mathcal{F}},0^{\mathcal{F}}\right)$. Together with Lemma 2, this implies that

$$W^{\sigma'}=\left(\Omega^{\emptyset}\circ V'\right)\left(W^{\sigma'}\right)=\left(\Omega^{\emptyset}\circ V\right)\left(\frac{\delta'}{\delta}W^{\sigma'}+\frac{1}{\delta}\left(U'-U\right)\right)(\leq,\geq)\left(\Omega^{\emptyset}\circ V\right)\left(W^{\sigma'}\right).$$

The proof for the most polarized equilibrium follows the same steps and is omitted for brevity. ■

Proof of Proposition 4. Suppose σ_l^g is not a best response for player l to σ_r^g. From the one-step-deviation principle, this is true if and only if for some $\theta\in\Theta$, conditional on $\theta(0)=\theta$, player l is better-off voting for R than for L at $t=0$, given that she expects σ^g to be played in all future periods. By definition of σ^g, this is equivalent to saying that for $\theta(0)=\theta$ player l is better-off with alternative R in all periods, than with alternative L in all periods. Hence, σ_l^g is not a best response for player l to σ_r^g if and only if Condition (3) is violated for player l. A symmetric argument for player r completes the proof. ■

Proof of Corollary 2. Since $U_k(\theta)$ is bounded by m on Θ, for all $T > 0$,

$$\begin{aligned} E_{\theta(0)}\left[\sum_{t=0}^{\infty}\delta^t U_l(\theta(t))\right] &= E_{\theta(0)}\left[\sum_{t=0}^{T-1}\delta^t\left(U_l(\theta(t)) - E^*[U_l]\right) + \sum_{t=T}^{\infty}\delta^t\left(U_l(\theta(t)) - E\left[U_l\left(\tilde{\theta}\right)\right]\right)\right] + \frac{E\left[U_l\left(\tilde{\theta}\right)\right]}{1-\delta} \\ &\leq \frac{2m\left(1-\delta^T\right)}{1-\delta} + \sum_{t=T}^{\infty}\delta^t\left|E_{\theta(0)}\left[U_l(\theta(t))\right] - E\left[U_l\left(\tilde{\theta}\right)\right]\right| + \frac{E\left[U_l\left(\tilde{\theta}\right)\right]}{1-\delta}. \end{aligned}$$

Since $E\left[U_l\left(\tilde{\theta}\right)\right] \neq 0$, by definition of uniform ergodicity, one can choose T large enough such that for all $t \geq T$ and all $\theta \in \Theta$, $\left|E_{\theta(0)=\theta}\left[U_l(\theta(t))\right] - E\left[U_l\left(\tilde{\theta}\right)\right]\right| < \left|E\left[U_l\left(\tilde{\theta}\right)\right]\right|/2$. Moreover, there exists $\bar{\delta}_l < 1$ such that for all $\delta \in \left[\bar{\delta}_l, 1\right)$, $2m\left(1-\delta^T\right) < \left|E\left[U_l\left(\tilde{\theta}\right)\right]\right|/2$. Substituting these bounds in the above inequality, we obtain that for all $\delta \geq \bar{\delta}_l$ and all $\theta \in \Theta$,

$$E_{\theta(0)=\theta}\left[\sum_{t=0}^{\infty}\delta^t U_l(\theta(t))\right] < \frac{\left|E\left[U_l\left(\tilde{\theta}\right)\right]\right|}{2(1-\delta)} + \left(\sum_{t=T}^{\infty}\delta^t\right)\frac{\left|E\left[U_l\left(\tilde{\theta}\right)\right]\right|}{2} + \frac{E\left[U_l\left(\tilde{\theta}\right)\right]}{1-\delta} < 0.$$

A symmetric argument holds for the rightist player for some threshold $\bar{\delta}_r < 1$. Setting $\bar{\delta} = \max\left\{\bar{\delta}_l, \bar{\delta}_r\right\}$ completes the proof. ■

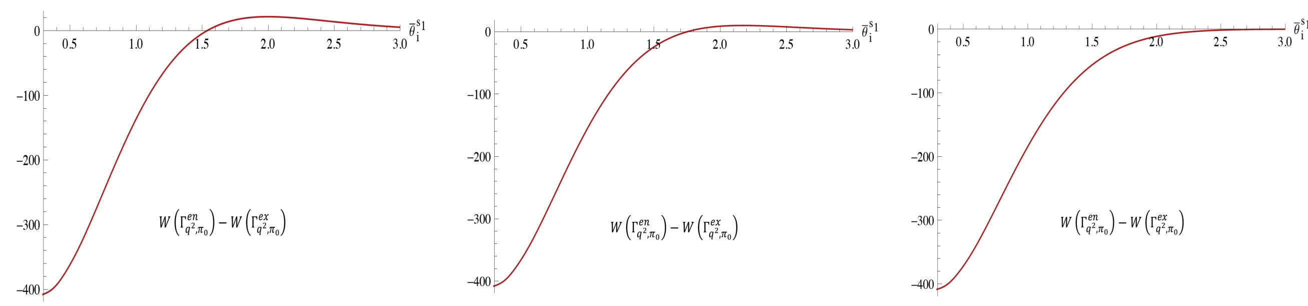
7.1.5 Example 1: Γ^{en} socially dominates Γ^{ex}

In this section, we discuss an example in which, as intuited in Section 5, the endogenous status quo yields higher social welfare than a fixed (and thus exogenous) status quo. As argued in that section, the strategic polarization induced by the endogenous status quo is socially detrimental, but the fact that the distribution of the status quo evolves over time with an endogenous status quo may more than compensate the welfare effect of strategic polarization.

The state space is $\Theta = \{(s,\varepsilon) : s \in \{s_1, s_2\}, \varepsilon \in \mathbb{R}\}$. The process $\{(s(t), \varepsilon(t)) : t \geq 0\}$ is defined as follows. The process $\{\varepsilon(t) : t \geq 0\}$ is i.i.d. over time and for each t, $\varepsilon(t) \sim N(0,1)$, the process $\{s(t) : t \geq 0\}$ is such that $\pi_{s_1,s_1} = \pi_{s_2,s_2} = \pi$ for some $\pi \geq \frac{1}{2}$, where $\left(\pi_{s,s'}\right)_{s,s' \in \{s_1,s_2\}}$ denotes the transition matrix of $\{s(t) : t \geq 0\}$, and the two processes $\{\varepsilon(t) : t \geq 0\}$ and $\{s(t) : t \geq 0\}$ are distributed independently of each other. Hence, $\{s(t) : t \geq 0\}$ captures the intertemporal correlation of the state, and $s(t)$ is somewhat persistent. With a slight abuse of language, in what follows, we shall refer to $s(t)$ simply as the state.

The payoff distribution is single-peaked (see Definition 2) with a bias that depends on the realization of s^t. That is, for all $k \in \{l, r\}$, $t \in \mathbb{N}$, and $(s(t), \varepsilon(t)) \in \Theta$, $U_k(s, \varepsilon) = \bar{\theta}_k^s + \varepsilon$. That is, conditional on s^t, players' preferences are perfectly correlated. The players' biases in s_1 are such that $\bar{\theta}_r^{s_1} > \bar{\theta}_l^{s_1}$, $\bar{\theta}_r^{s_1} + \bar{\theta}_l^{s_1} \geq 0$, and s_2 is the symmetric of s_1: $\bar{\theta}_r^{s_2} = -\bar{\theta}_l^{s_1}$ and $\bar{\theta}_l^{s_2} = -\bar{\theta}_r^{s_1}$. Hence, player r is more rightist than player l in both states, but in state s_1, when players' preferences disagree, R is socially better, so R is the socially optimal status quo in s_1. Conversely, L is the socially optimal status quo in s_2.

The following figure compares the utilitarian welfare (that is the sum of players' expected discounted payoff from $t = 0$, denoted by $W(\Gamma)$) in the least polarized equilibrium of Γ^{en} and Γ^{ex}, where $s^0 = s_1$ and $q(0) = L$. We fix players' ideological polarization $\bar{\theta}_r^{s_1} - \bar{\theta}_l^{s_1}$ at 0.5 and let $\bar{\theta}_r^{s_1}$ vary. Note that given our assumptions, when $\bar{\theta}_r^{s_1} = 0.25$, both states are identical. As $\bar{\theta}_r^{s_1}$ increases, the preferences of both players move to the right in s_1 and to the left in s_2: the states becomes less similar, but in each state, the probability of preference disagreement decreases. Each panel depicts $W(\Gamma^{en}) - W(\Gamma^{ex})$ as a function of $\bar{\theta}_r^{s_1}$ for $\pi = 0.99$, 0.8, and 0.5.



All panels show that the exogenous status quo dominates for small $\bar{\theta}_r^{s_1}$, but this can reverse itself for large $\bar{\theta}_r^{s_1}$. A comparison of panels A, B, and C reveals that the endogenous status quo is more likely to dominate as π increases. The intuition for this is as follows. When $\bar{\theta}_r^{s_1} = 0.25$, both states are identical, and the exogenous status quo dominates trivially because all status quos are optimal in Γ^{ex}, so the fact that the status quo's distribution is different in the two states in Γ^{en} does not give Γ^{en} any welfare advantage, but players' polarization in Γ^{en} leads to lower welfare. As $\bar{\theta}_r^{s_1}$ increases, the probability of disagreement in each state decreases; hence, defending the status quo becomes less important, and the degree of strategic polarization in Γ^{en} decreases. At the same time, both players become more likely to vote for L in s_1 and R in s_2, which guarantee an optimal status quo in the next period if the state does not change. The more persistent the states, the more beneficial this effect is.

7.2 Proofs for the $N-$alternative model

7.2.1 Proofs for Section 4.2

Notation 1 *The set of Markov states of the game* Γ_c^{en}, *denoted by* S, *is comprised of three kinds of states:* direction states, *in which the proposer must choose a direction,* voting states, *in which players vote simultaneously, and* payoff states, *in which players have no actions to choose, the current agreement is implemented, and players receive the corresponding payoff. A typical direction state and payoff state of* Γ_c^{en} *are denoted by* $\langle\theta, x\rangle$ *and* $\langle\langle\theta, x\rangle\rangle$, *respectively, where* $\theta \in \Theta$ *is the period-specific state of nature and* $x \in X$ *is the current agreement or the final agreement, respectively. A typical voting state is denoted by* $\langle\theta, x, x'\rangle$, *where* θ *and* x *have the same interpretation as in* $\langle\theta, x\rangle$, *and* x' *is the proposal under consideration.*
For any Markov strategy profile σ *and any state* $s \in S$, $V_k^\sigma(s)$ *denotes the expected continuation payoff of player* k *from state* s *given continuation play* σ, *and* $\phi^\sigma(s)$ *denotes the (possibly stochastic) outcome in a given period, starting from state* s *and given continuation play* σ.

The following Lemma formalizes the intuition laid out in Section 4.1 that the endogenous status quo has a polarizing effect whenever the bargaining procedure is such that in each period the outcome is monotonic in the status quo.

Lemma 3 *Let* σ *be an equilibrium of* Γ_c^{en}. *If for all* $\theta \in \Theta$ *and* $P_\theta-$*almost all* $\zeta \in \Theta$, *the distribution of* $(\phi^\sigma(\langle\zeta, q\rangle))_{q\in X}$ *is monotonic in* q *in the F.O.S.D. sense,*[37] *then* σ *is polarized.*

Proof. Let $\theta \in \Theta$. Under our assumptions, conditional on $\theta(t) = \theta$, for almost all realizations of $\theta(t+1)$, the distribution of the outcome in $t+1$, $x(t+1)$, is monotonic in the status quo in $t+1$, $q(t+1)$. Since $x(t+1)$ becomes $q(t+2)$, this implies that conditional on $\theta(t) = \theta$, for almost all realizations of $\theta(t+1)$ and $\theta(t+2)$, the distribution of $x(t+2)$ is monotonic in $x(t+1)$, and thus in $q(t+1)$. A straightforward induction shows then that conditional on $\theta(t) = \theta$, for all $t' > t$ and almost all realizations of $\theta(t')$, the distribution of $x(t')$ is monotonic in $q(t+1)$. Since U_r is more rightist than U_l, this implies that for all $\theta \in \Theta$ and all $x_n, x_{n+1} \in X$, $W_r^\sigma(\theta, x_{n+1}) -$

[37] That is, for all $q' > q$, and all $x \in X$, the probabilty that the outcome is to the right of x is weakly greater under $\phi^\sigma(\langle\zeta, q'\rangle)$ than under $\phi^\sigma(\langle\zeta, q\rangle)$.

$W_r^\sigma(\theta, x_n) \geq W_l^\sigma(\theta, x_{n+1}) - W_l^\sigma(\theta, x_n)$. Note that for any direction state $\langle\zeta, x\rangle$, $V^\sigma(\langle\zeta, x\rangle) = E[U(\phi^\sigma(\langle\zeta, q\rangle)) + \delta W^\sigma(\phi^\sigma(\langle\zeta, q\rangle))]$, where the expectation is taken w.r.t. ϕ^σ. Together with the previous inequality on W^σ and the assumption that U_r is more rightist than U_l, this implies that for all $\zeta \in \Theta$ such that $(\phi^\sigma(\langle\zeta, q\rangle))_{q\in X}$ is monotonic in q,

$$V_r^\sigma(\langle\zeta, x_{n+1}\rangle) - V_r^\sigma(\langle\zeta, x_n\rangle) \geq V_l^\sigma(\langle\zeta, x_{n+1}\rangle) - V_l^\sigma(\langle\zeta, x_n\rangle).$$

Below, we show that since σ is an equilibrium, $V_k^\sigma(\langle\zeta, x_{n+1}\rangle) - V_k^\sigma(\langle\zeta, x_n\rangle)$ cannot be of the same strict sign for both players. Together with the above inequality, this implies that $V_k^\sigma(\langle\zeta, x_{n+1}\rangle) - V_k^\sigma(\langle\zeta, x_n\rangle)$ is weakly positive (negative) for $k = r$ $(k = l)$. Since $W^\sigma(\theta, q) = \int_{\zeta\in\Theta} V_k^\sigma(\langle\zeta, q\rangle)\, dP_\theta$, this implies that $W_k^\sigma(\theta, q)$ is weakly increasing (decreasing) in q for $k = r$ $(k = l)$, as needed.

To conclude the proof, we show that $V_k^\sigma(\langle\zeta, x_{n+1}\rangle) - V_k^\sigma(\langle\zeta, x_n\rangle)$ cannot be of the same strict sign for both players. Suppose by contradiction that $V_k^\sigma(\langle\zeta, x_{n+1}\rangle) > V_k^\sigma(\langle\zeta, x_n\rangle)$ for both players, the argument in the other case is identical. Since the proposer is indifferent between any action played with positive probability in $\langle\zeta, x_{n+1}\rangle$, it must be that *(i)* $V_k^\sigma(\langle\zeta, x_{n+1}, x_{n+2}\rangle) > V_k^\sigma(\langle\zeta, x_n\rangle)$ for both players, or that *(ii)* $V_k^\sigma(\langle\zeta, x_{n+1}, x_n\rangle) > V_k^\sigma(\langle\zeta, x_n\rangle)$ for both players. In case *(i)*, since $V_k^\sigma(\langle\zeta, x_n\rangle) \geq V_k^\sigma(\langle\langle\zeta, x_n\rangle\rangle)$, the only stage-undominated action for both players in $\langle\zeta, x_n, x_{n+1}\rangle$ is to vote *yes*, so by playing *right* in $\langle\zeta, x_n\rangle$, the proposer can get $V_{pr}^\sigma(\langle\zeta, x_{n+1}, x_{n+2}\rangle)$, a contradiction with *(i)*. In case *(ii)*, since $V_{pr}^\sigma(\langle\zeta, x_n\rangle) \geq V_{pr}^\sigma(\langle\zeta, x_n, x_{n-1}\rangle)$, the proposer must vote *no* with probability 1 in $\langle\zeta, x_{n+1}, x_n\rangle$. Together with *(ii)*, this implies that for both players, $V_k^\sigma(\langle\langle\zeta, x_{n+1}\rangle\rangle) = V_k^\sigma(\langle\zeta, x_{n+1}, x_n\rangle) > V_k^\sigma(\langle\zeta, x_n\rangle) \geq V_k^\sigma(\langle\langle\zeta, x_n\rangle\rangle)$, so they must vote *yes* in $\langle\zeta, x_n, x_{n+1}\rangle$. By playing *right* in $\langle\zeta, x_n\rangle$, the proposer can then get at least $V_{pr}^\sigma(\langle\langle\zeta, x_{n+1}\rangle\rangle)$, which contradicts $V_{pr}^\sigma(\langle\langle\zeta, x_{n+1}\rangle\rangle) > V_{pr}^\sigma(\langle\zeta, x_n\rangle)$. ■

Proof of Proposition 5. For all Markov strategy profiles σ of Γ_c^{en}, $k \in \{l, r\}$, and $\theta \in \Theta$, let $\sigma_k(\theta)$ and $\sigma_k(-\theta)$ denote the restriction of σ_k to the Markov states in which the state of nature is θ, and to those in which the state of nature is different from θ, respectively, and let k' denote the opponent of k. For a given $\sigma_k(-\theta)$ and $\sigma_{k'}$, finding the optimal $\sigma_k(\theta)$ for player k in Γ_c^{en} is a standard Markov decision problem. Hence, there exists a stage-undominated best response $\sigma_k^*(\theta)$ to $\sigma_k(-\theta)$ and $\sigma_{k'}$. That is, $\sigma_k^*(\theta)$ prescribes stage-undominated actions in all Markov states with state of nature θ given continuation play $(\sigma_k^*(\theta), \sigma_k(-\theta), \sigma_{k'})$ in Γ_c^{en}. Let $\Sigma_k^\sigma(\theta)$ denote the set of such stage-undominated best responses, and let $\Sigma^{\sigma} \doteq \left(\Sigma_l^{\sigma}(\theta), \Sigma_r^{\sigma}(\theta)\right)_{\theta\in\Theta}$. By construction of Σ^{σ}, a strategy profile σ is an equilibrium of Γ_c^{en} if and only if $\sigma \in \Sigma^{\sigma}$.

If k is the proposer when the state of nature is θ, we say that $\sigma_k(\theta)$ is a monotonic, stage-undominated best response to $\sigma_k(-\theta)$ and $\sigma_{k'}$ if $\sigma_k(\theta) \in \Sigma_k^{\sigma}(\theta)$ and if $\left(\phi^{\sigma_k(\theta),\sigma_{k'}(\theta)}(\langle\theta, q\rangle)\right)_{q\in X}$ is monotonic in q in the sense of Lemma 3. Let $\hat{\Sigma}_k^{\sigma}(\theta) \subseteq \Sigma_k^{\sigma}(\theta)$ denote the set of such monotonic, stage-undominated best responses. If k is not the proposer in state θ, we simply set $\hat{\Sigma}_k^{\sigma}(\theta) \doteq \Sigma_k^{\sigma}(\theta)$. From Lemma 3, to prove the existence of a polarized equilibrium, it suffices to show that the correspondence $\sigma \to \hat{\Sigma}^{\sigma}$ has a fixed point. To do so, we will use the following theorem (Eilenberg and Montgomery 1946, Theorem 1):[38]

Theorem 1 *If C is a compact and convex subset of a Euclidean space and $F : C \to 2^C$ is an upper-hemicontinuous, contractible-valued correspondence, then F has a fixed point.*

In our case, C is the set of Markov strategy profiles of Γ_c^{en}. Since Θ is finite, it is a compact and convex subset of a Euclidean space. A set $D \subseteq C$ is contractible if there exists $\bar{a} > 0$, $x^0 \in D$, and a continuous function $f : [0, \bar{a}] \times D \to D$ such that for all $x \in D$, $f(0, x) = x$ and $f(\bar{a}, x) = x^0$. We then say that f is a contraction for D. In steps 1 to 6 below, we show that the correspondence $\sigma \to \hat{\Sigma}^{\sigma}$ satisfies the conditions of the theorem.

Step 1: $\sigma \to \hat{\Sigma}^{\sigma}$ is upper-hemicontinuous.

Let $(\sigma^z)_{z\in\mathbb{N}}$ and $(\hat{\sigma}^z)_{z\in\mathbb{N}}$ be two sequences of strategy profiles that converge to some σ and $\hat{\sigma}$, respectively, and such that for all $z \in \mathbb{N}$, $\hat{\sigma}^z \in \hat{\Sigma}^{\sigma^z}$. Let $\theta \in \Theta$ and $k \in \{l, r\}$. By assumption, for all $z \in \mathbb{N}$, $\hat{\sigma}_k^z(\theta)$ prescribes stage-undominated actions given continuation play $\left(\hat{\sigma}_k^z(\theta), \sigma_k^z(-\theta), \sigma_{k'}^z\right)$. Since continuation payoffs in Γ_c^{en} are continuous in strategies, this implies that $\hat{\sigma}_k(\theta)$ prescribes stage-undominated actions given continuation play $(\hat{\sigma}_k(\theta), \sigma_k(-\theta), \sigma_{k'})$, so $\hat{\sigma}_k(\theta) \in \Sigma_k^{\sigma}(\theta)$. If k is not the proposer in state of nature θ, this means that $\hat{\sigma}_k(\theta) \in \hat{\Sigma}_k^{\sigma}(\theta)$, as needed. If she is, then by assumption, for all $z \in \mathbb{N}$, $\left(\phi^{\hat{\sigma}_k^z(\theta),\sigma_{k'}^z(\theta)}(\langle\theta, q\rangle)\right)_{q\in X}$ is monotonic in q, and since ϕ depends continuously on the strategy profile, so is $\phi^{\hat{\sigma}_k(\theta),\sigma_{k'}(\theta)}(\langle\theta, q\rangle)$, and thus $\hat{\sigma}_k(\theta) \in \hat{\Sigma}_k^{\sigma}(\theta)$.

For the rest of the proof, we fix a Markov strategy profile $\bar{\sigma}$, $\theta \in \Theta$, $k \in \{l, r\}$. Note that for all $\sigma_k(\theta), \sigma'_k(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$, $(\sigma_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$ and $(\sigma'_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$ yield the same continuation

[38] This result is actually a corollary from Eilenberg and Montgomery (1946, Theorem 1), see Reny (2011, p 503). In Reny (2011), C is an absolute retract, but any convex subset of a Euclidean space is an absolute retract, and F is also required to be nonempty-valued, but this is implied by F being contractible-valued.

payoff starting from any state (otherwise both could not be best responses). Thus, we can define $I_v^{\bar{\sigma}}(\theta)$ to be the set of voting states in which the state of nature is θ and both actions, *yes* and *no*, are stage-undominated for k given continuation play $(\sigma_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$ for any $\sigma_k(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$. Likewise, if k is the proposer in state of nature θ, we define $I_d^{\bar{\sigma}}(\theta)$ to be the set of direction states in which the state of nature is θ and both $left$ and $right$ are stage-undominated for k given continuation play $(\sigma_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$ for any $\sigma_k(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$. If k is not the proposer, we set $I_d^{\bar{\sigma}}(\theta) \doteq \emptyset$. When k is the proposer, we denote by $\sigma_k^{no,left}(\theta)$ the stage-undominated best response to $\bar{\sigma}_k(-\theta)$ and $\bar{\sigma}_{k'}$ that prescribes *no* with probability 1 in $I_v^{\bar{\sigma}}(\theta)$ and $left$ with probability 1 in $I_d^{\bar{\sigma}}(\theta)$. From what precedes, $\sigma_k^{no,left}(\theta)$ exists and is unique.

In what follows, to show that $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ is contractible when k is the proposer, we show that any two stage-undominated best responses $\sigma_k(\theta)$ and $\sigma'_k(\theta)$ differ only on $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$ (step 2), that $\sigma_k^{no,left}(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, and that for any other $\hat{\sigma}_k(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, one can continuously change the actions prescribed by $\hat{\sigma}_k(\theta)$ on $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$ so as to transform $\hat{\sigma}_k(\theta)$ into $\sigma_k^{no,left}(\theta)$ without violating stage undomination and monotonicity (step 3). The simpler case in which k is the proposer is treated in step 4.

Step 2: $\sigma_k(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$ *if and only if* $\sigma_k(\theta)$ *prescribes the same (pure) action as* $\sigma_k^{no,left}(\theta)$ *in all states that are not in* $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$.

Since $\sigma_k^{no,left}(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$, all actions available in $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$ yield the same continuation payoff given continuation play $\left(\sigma_k^{no,left}(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'}\right)$, so if $\sigma_k(\theta)$ prescribes the same action as $\sigma_k^{no,left}(\theta)$ in all other states, $(\sigma_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$ yields the same continuation payoff as $\left(\sigma_k^{no,left}(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'}\right)$ starting from any state. Hence, the set of stage-undominated actions must be the same in all states for the two continuation strategies. As a result, $\sigma_k(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$. Reciprocally, if $\sigma_k(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$, then $(\sigma_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$ must yield the same continuation payoff as $\left(\sigma_k^{no,left}(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'}\right)$ from any state, so $\sigma_k^{no,left}(\theta)$ and $\sigma_k(\theta)$ must prescribe the same pure action in any state in which only one action is stage-undominated, i.e., outside of $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$.

Step 3: If k *is the proposer in state* θ, $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ *is contractible.*

To show that $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ is contractible, we will construct a contraction (see the definition before step 1) which continuously shrink $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ into $\sigma_k^{no,left}(\theta)$ by first increasing the probability that *no* is played in $I_v^{\bar{\sigma}}(\theta)$ and then the probability that $left$ is played in $I_d^{\bar{\sigma}}(\theta)$, while preserving stage undomination and monotonicity.

Formally, for all $\sigma_k(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ and $a \in [0,1]$, let $f(a, \sigma_k(\theta))$ differ from $\sigma_k(\theta)$ only in that in any state $s \in I_v^{\bar{\sigma}}(\theta)$, $f(a, \sigma_k(\theta))$ prescribes *no* with probability $\max\{a, p_s^{no}\}$, where p_s^{no} is the probability with which $\sigma_k(\theta)$ prescribes *no* in s. For $a \in [1,2]$, $f(a, \sigma_k(\theta))$ differs from $f(1, \sigma_k(\theta))$ only in that in any state $s \in I_d^{\bar{\sigma}}(\theta)$, $f(a, \sigma_k(\theta))$ prescribes $left$ with probability $\max\left\{a-1, p_s^{left}\right\}$, where p_s^{left} is the probability with which $\sigma_k(\theta)$ prescribes $left$ in s.

Note that from step 2, for all $\sigma \in \Sigma_k^{\bar{\sigma}}(\theta)$, $f(2, \sigma_k(\theta)) = \sigma_k^{no,left}(\theta)$. To show that f is a contraction for $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, it remains to show that if $\sigma_k(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, then $f(a, \sigma_k(\theta)) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ for all $a \in [0,2]$ (step 3a), that $\sigma_k^{no,left}(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, i.e., $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ is nonempty (step 3b), and that $f(a, \sigma_k(\theta))$ is continuous in $(a, \sigma_k(\theta))$ (step 3c).

Step 3a: If $\sigma_k(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, then $f(a, \sigma_k(\theta)) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ for all $a \in [0,2]$.

Let $\sigma_k(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$. By definition, $f(a, \sigma_k(\theta))$ differs from $\sigma_k(\theta)$ only in $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$, so from step 2, $f(a, \sigma_k(\theta)) \in \Sigma_k^{\bar{\sigma}}(\theta)$. To prove that $f(a, \sigma_k(\theta)) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$, observe first that one cannot violate monotonicity by increasing the probability that a player votes *no* in some voting state.[39] Hence, for all $a \in [0,1]$, $f(a, \sigma_k(\theta))$ is monotonic. To conclude the proof of step 3a, we need to show that for all $a \in [1,2]$, $f(a, \sigma_k(\theta))$ is monotonic. Suppose this is not true. Since $f(1, \sigma_k(\theta))$ is monotonic and since $f(a, \sigma_k(\theta))$ differs from $f(1, \sigma_k(\theta))$ only in that it prescribes $left$ with greater probability in $I_d^{\bar{\sigma}}(\theta)$, monotonicity must be violated between states $\langle \theta, x_{n-1} \rangle$ and $\langle \theta, x_n \rangle$ for some $\langle \theta, x_n \rangle \in I_d^{\bar{\sigma}}(\theta)$. Moreover, $f(a, \sigma_k(\theta))$ must differ from $f(1, \sigma_k(\theta))$ in state $\langle \theta, x_n \rangle$, so $f(1, \sigma_k(\theta))$ must prescribe $left$ with probability at most $a-1$ in $\langle \theta, x_n \rangle$, and therefore $f(a, \sigma_k(\theta))$ prescribe $left$ with probability $a-1$ in $\langle \theta, x_n \rangle$. Suppose first that $\langle \theta, x_{n-1} \rangle \in I_d^{\bar{\sigma}}(\theta)$. In that case, by construction of f, $f(a, \sigma_k(\theta))$ prescribes left with probability at least $a-1$ in $\langle \theta, x_{n-1} \rangle$, and thus with a greater probability than in $\langle \theta, x_n \rangle$, so monotonicity cannot be violated between $\langle \theta, x_{n-1} \rangle$ and $\langle \theta, x_n \rangle$.

Suppose now that $\langle \theta, x_{n-1} \rangle \notin I_d^{\bar{\sigma}}(\theta)$. Then k is not indifferent between $right$ and $left$ in $\langle \theta, x_{n-1} \rangle$, and for monotonicity to be violated, she must strictly prefer to play $right$. Let $V_k(s)$ denote k's expected payoff from any state s given continuation play $(\sigma_k(\theta), \bar{\sigma}_k(-\theta), \bar{\sigma}_{k'})$. The preceding

[39] To see this, note that a violation of monotonicity implies that for some $x_n, x_{n+1} \in X$, the path of play from $\langle \theta, x_n \rangle$ reaches $\langle \theta, x_{n+1}, x_{n+2} \rangle$ with a greater probability than from $\langle \theta, x_{n+1} \rangle$, or that the path of play from $\langle \theta, x_{n+1} \rangle$ reaches $\langle \theta, x_n, x_{n-1} \rangle$ with a greater probability than from $\langle \theta, x_n \rangle$. If we keep the actions at the direction states unchanged, these paths of play are less likely to occur as we increase the probability that some voter votes *no* in $\langle \theta, x_n, x_{n+1} \rangle$ or in $\langle \theta, x_{n+1}, x_n \rangle$.

implies

$$V_k(\langle\theta, x_{n-1}, x_n\rangle) > V_k(\langle\theta, x_{n-1}, x_{n-2}\rangle). \tag{13}$$

Note that $V_k(\langle\theta, x_{n-1}, x_{n-2}\rangle) \geq V_k(\langle\langle\theta, x_{n-1}\rangle\rangle)$ and that $V_k(\langle\theta, x_{n-1}, x_n\rangle)$ is a convex combination of $V_k(\langle\langle\theta, x_{n-1}\rangle\rangle)$ and $V_k(\langle\theta, x_n, x_{n+1}\rangle)$, so (13) implies that $V_k(\langle\theta, x_n, x_{n+1}\rangle) > V_k(\langle\theta, x_{n-1}, x_{n-2}\rangle)$. Since $\langle\theta, x_n\rangle \in I_d^{\bar{\sigma}}(\theta)$, $V_k(\langle\theta, x_n, x_{n+1}\rangle) = V_k(\langle\theta, x_n, x_{n-1}\rangle)$, which with the previous inequality delivers $V_k(\langle\theta, x_n, x_{n-1}\rangle) > V_k(\langle\theta, x_{n-1}, x_{n-2}\rangle)$. This implies that k votes *no* with probability 1 in $\langle\theta, x_n, x_{n-1}\rangle$, and so $V_k(\langle\theta, x_n\rangle) = V_k(\langle\langle\theta, x_n\rangle\rangle)$. This, in turn, means that voting *no* in $\langle\theta, x_n, x_{n+1}\rangle$ is stage-undominated. By construction of f, for $a \geq 1$, $f(a, \sigma_k(\theta))$ prescribes *no* whenever voting *no* is stage-undominated, so k votes *no* in $\langle\theta, x_n, x_{n+1}\rangle$. So $\phi(\langle\theta, x_n\rangle) = x_n$ with probability 1 and $\phi(\langle\theta, x_{n-1}\rangle) \leq x_n$ with probability 1. So monotonicity is not violated.

Step 3b: $\sigma_k^{no,left}(\theta) \in \hat{\Sigma}_k^{\bar{\sigma}}(\theta)$.

Since $\sigma_k^{no,left}(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$, it suffices to show that $\sigma_k^{no,left}(\theta)$ is monotonic. Let $x_{n-1}, x_n \in X$, and suppose that monotonicity is violated between states $\langle\theta, x_{n-1}\rangle$ and $\langle\theta, x_n\rangle$. The same argument as in step 3a shows that this cannot happen if $\langle\theta, x_{n-1}\rangle \notin I_d^{\bar{\sigma}}(\theta)$ and $\langle\theta, x_n\rangle \in I_d^{\bar{\sigma}}(\theta)$ (we did not use the monotonicity of $f(1, \sigma_k(\theta))$ in that case). If $\langle\theta, x_{n-1}\rangle \in I_d^{\bar{\sigma}}(\theta)$, then by construction of $\sigma_k^{no,left}(\theta)$, k plays $left$ with probability 1 in $\langle\theta, x_{n-1}\rangle$, so monotonicity cannot be violated. Finally, suppose $\langle\theta, x_{n-1}\rangle \notin I_d^{\bar{\sigma}}(\theta)$ and $\langle\theta, x_n\rangle \notin I_d^{\bar{\sigma}}(\theta)$. Monotonicity can only be violated if k strictly prefers to play $right$ in $\langle\theta, x_{n-1}\rangle$ and $left$ in $\langle\theta, x_n\rangle$, which is impossible since $\sigma_k^{no,left}(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$.

Step 3c: f is continuous.

By construction of f, for all $a, a' \in [0, 2]$ and all $\sigma_k(\theta), \sigma_k'(\theta) \in \Sigma_k^{\bar{\sigma}}(\theta)$, the distance between $f(a, \sigma_k(\theta))$ and $f(a', \sigma_k(\theta))$ in the sup norm is smaller than $|a' - a|$, and from step 2, $f(a, \sigma_k(\theta))$ and $f(a, \sigma_k'(\theta))$ coincide outside $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$ and f can only decrease the difference between $\sigma_k(\theta)$ and $\sigma_k'(\theta)$ on $I_v^{\bar{\sigma}}(\theta) \cup I_d^{\bar{\sigma}}(\theta)$, so the distance between $f(a, \sigma_k(\theta))$ and $f(a, \sigma_k'(\theta))$ is smaller than the distance between $\sigma_k(\theta)$ and $\sigma_k'(\theta)$.

Step 4: If k is not the proposer in state θ, $\hat{\Sigma}_k^{\bar{\sigma}}(\theta)$ is contractible.

Since k is not the proposer, then by definition $\hat{\Sigma}_k^{\bar{\sigma}}(\theta) = \Sigma_k^{\bar{\sigma}}(\theta)$. From step 2, $\Sigma_k^{\bar{\sigma}}(\theta)$ is the set of strategies which prescribes the same pure action as $\sigma_k^{no,left}(\theta)$ outside $I_v^{\bar{\sigma}}(\theta)$, and any action on $I_v^{\bar{\sigma}}(\theta)$. So $\Sigma_k^{\bar{\sigma}}(\theta)$ is homeomorphic to the nonempty, convex set $[0,1]^{I_v^{\bar{\sigma}}(\theta)}$. As such, it is contractible (see, e.g., Reny 2011, p. 503). ■

Proof of Proposition 6. Let σ be an equilibrium of Γ_c^{en}. Assumption 1 (ii) implies that $W^\sigma(\theta, q)$ depends on θ only through θ_1. Since for all $\zeta \in \Theta$ and $x \in X$, $V(\langle\langle \zeta, x\rangle\rangle) = U(\zeta, x) + \delta W^\sigma(\zeta_1, q)$, Assumption 1 (i) implies that for all $\theta \in \Theta$ and for $P_\theta-$almost all ζ, the profile of ordinal preferences induced by $(V(\langle\langle \zeta, x\rangle\rangle))_{x\in X}$ is strict. Therefore, for all such ζ and all $q \in X$, the outcome $\phi^\sigma(\langle \zeta, q\rangle)$ is deterministic and monotonic in q. Determinism follows from a simple backward induction starting from the states $\langle \zeta, x_2, x_1\rangle$ and $\langle \zeta, x_{N-1}, x_N\rangle$. To show monotonicity, suppose by contradiction that for some $x_n, x_{n+1} \in X$, $\phi^\sigma(\langle \zeta, x_n\rangle) > \phi^\sigma(\langle \zeta, x_{n+1}\rangle)$. This implies that $\phi^\sigma(\langle \zeta, x_n\rangle) > x_n$ or $\phi^\sigma(\langle \zeta, x_{n+1}\rangle) < x_{n+1}$. Suppose the former, the proof in the latter case is identical. Then the proposer can obtain $\phi^\sigma(\langle \zeta, x_n\rangle)$ by playing *right* in $\langle \zeta, x_{n+1}\rangle$. Since she has strict preferences and σ is an equilibrium, this implies that $V_{pr}^\sigma(\langle\langle \zeta, \phi^\sigma(\langle \zeta, x_{n+1}\rangle)\rangle\rangle) > V_{pr}^\sigma(\langle\langle \zeta, \phi^\sigma(\langle \zeta, x_n\rangle)\rangle\rangle)$. But she can also obtain the outcome $\phi^\sigma(\langle \zeta, x_{n+1}\rangle)$ from state $\langle \zeta, x_n\rangle$ by voting *no* when the path of play reaches $\langle \zeta, x_{n+1}, x_{n+2}\rangle$ (if $\phi^\sigma(\langle \zeta, x_{n+1}\rangle) = x_{n+1}$) or by playing *left* in $\langle \zeta, x_n\rangle$ and σ thereafter (if $\phi^\sigma(\langle \zeta, x_{n+1}\rangle) < x_{n+1}$), a profitable deviation. Since for all $\theta \in \Theta$, $\phi^\sigma(\langle \zeta, q\rangle)$ is monotonic in q for $P_\theta-$almost all $\zeta \in \Theta$, Lemma 3 implies that σ is polarized.

Suppose now that Θ_1 is finite. Let $W \in \left[-\frac{m}{1-\delta}, \frac{m}{1-\delta}\right]^{\{l,r\}\times\Theta_1\times X}$, where m is the uniform bound on U. Let $\sigma(W)$ be a strategy profile such that for all $\theta \in \Theta$, the restriction of $\sigma(W)$ to the Markov states in which the state of nature is θ is an equilibrium of a one-period version of Γ_c^{en} in which the final payoff at any payoff state $\langle\langle \theta, x\rangle\rangle$ is $V^W(\langle\langle \theta, x\rangle\rangle) \doteq U(\theta, x) + \delta W(\theta_1, x)$. Let $W^{\sigma(W)}$ be the corresponding continuation value in Γ_c^{en}. Below, we show that irrespective of how $\sigma(W)$ is picked for each W, the map $W \to W^{\sigma(W)}$ is continuous. Since it maps $\left[\frac{-m}{1-\delta}, \frac{m}{1-\delta}\right]^{\Theta\times X}$ into itself, Brower's fixed point theorem implies that it has a fixed point W^*. By construction, the corresponding strategy profile $\sigma(W^*)$ is an equilibrium of Γ_c^{en}.

Let $(W^z)_{z\in\mathbb{N}}$ be a sequence that converges to some W and let $q \in X$. For all $z \in \mathbb{N}$, let

$$\Phi^z \doteq \left\{\zeta \in \Theta : \phi^{\sigma(W^z)}(\langle \zeta, q\rangle) = \phi^{\sigma(W)}(\langle \zeta, q\rangle)\right\}$$

and let 1_{Φ^z} denote its indicator function. Note that for all $\zeta \in \Phi^z$,

$$V_k^{W^z}\left(\zeta, \phi^{\sigma(W^z)}(\langle \zeta, q\rangle)\right) - V_k^W\left(\zeta, \phi^{\sigma(W)}(\langle \zeta, q\rangle)\right) = \delta\left(W_k^z\left(\zeta_1, \phi^{\sigma(W)}(\langle \zeta, q\rangle)\right) - W_k\left(\zeta_1, \phi^{\sigma(W)}(\langle \zeta, q\rangle)\right)\right).$$

So for all $\theta_1 \in \Theta_1$,

$$\begin{aligned} & W_k^{\sigma(W^z)}(\theta_1, q) - W_k^{\sigma(W)}(\theta_1, q) \qquad (14) \\ = \; & \delta \int \left[W_k^z \left(\zeta_1, \phi^{\sigma(W)}(\langle \zeta, q \rangle) \right) - W_k \left(\zeta_1, \phi^{\sigma(W)}(\langle \zeta, q \rangle) \right) \right] 1_{\Phi^z}(\zeta) \, dP_{\theta_1}(\zeta) \\ & + \int \left[V_k^{W^z} \left(\zeta, \phi^{\sigma(W^z)}(\langle \zeta, q \rangle) \right) - V_k^W \left(\zeta, \phi^{\sigma(W)}(\langle \zeta, q \rangle) \right) \right] 1_{(\Phi^z)^C}(\zeta) \, dP_{\theta_1}(\zeta) \, . \end{aligned}$$

Since $W^z \to W$, the dominated convergence theorem implies that the first integral on the right-hand side of (14) tends to 0. As for the second integral, observe that for all $\zeta_1 \in \Theta_1$, for almost all $U \in \mathbb{R}^{\{l,r\} \times X}$, the profile of payoff $(U_k(x) + \delta W_k(\zeta_1, x))_{k \in \{l,r\}, x \in X}$ induces strict ordinal preferences over X. Therefore, for any such U, for z sufficiently large, $(U_k(x) + \delta W_k^z(\zeta_1, x))_{k \in \{l,r\}, x \in X}$ induces the same strict ordinal preferences, and thus the same outcome. Together with Assumption 1 (i), this implies that for $P_{\theta_1}-$almost all ζ, for z sufficiently large, $\phi^{\sigma(W^z)}(\langle \zeta, q \rangle) = \phi^{\sigma(W)}(\langle \zeta, q \rangle)$, so $1_{(\Phi^z)^C}(\zeta) = 0$. The dominated convergence theorem implies then that the second integral on the right hand-side of (14) also tends to 0. ■

Proof of Proposition 7. For all $b \in \mathbb{R}^2$, let $\Gamma_c^{ex}(b)$ and $\Gamma_c^{en}(b)$ denote the game with an exogenous and endogenous status quo, respectively, in which players have quadratic preferences with biases b. For all $p_l \leq 0$ and $p_r \geq 0$, let $\sigma^{ex}(b+p)$ denote the equilibrium of $\Gamma_c^{ex}(b+p)$, as described in Section 4.2.1, and let $W^{\sigma^{ex}(b+p)}$ denote players' continuation value when $\sigma^{ex}(b+p)$ is played in $\Gamma_c^{en}(b)$. For all $x, y \in X$, $k \in \{l, r\}$, and $\theta \in \mathbb{R}$, denote

$$U_k^{x,y,b}(\theta) \mathrel{\hat{=}} -(y - \theta - b_k)^2 + (x - \theta - b_k)^2 = 2(y - x)\left(\theta - \left(\frac{x+y}{2} - b_k\right)\right). \qquad (15)$$

Step 1: For all $p_l \leq 0$ and $p_r \geq 0$, $\sigma^{ex}(b+p)$ is an equilibrium of $\Gamma_c^{en}(b)$ if for all $\theta \in \Theta$, $k \in \{l, r\}$, and for any two consecutive alternatives $x, y \in X$ such that $x < y$,

$$U_k^{x,y,b}(\theta) + \delta \left(W_k^{\sigma^{ex}(b+p)}(\theta, y) - W_k^{\sigma^{ex}(b+p)}(\theta, x) \right) \begin{cases} \geq 0 \text{ if } \theta > \frac{x+y}{2} - b_k - p_k \\ \leq 0 \text{ if } \theta < \frac{x+y}{2} - b_k - p_k \end{cases} . \qquad (16)$$

By construction of $\sigma^{ex}(b+p)$ (see Section 4.2.1), it should be clear that $\sigma^{ex}(b+p)$ is also an equilibrium of Γ_c^{ex} for any payoff profile U such that for all $\theta \in \theta$ and $k \in \{l, r\}$, $(U_k(\theta, x))_{x \in X}$ is weakly increasing to the left of the peak of $\left(-\left(x-\theta-b_k-p_k\right)^2\right)_{x\in X}$ and weakly decreasing to its right. From (15), this is satisfied if for any two *consecutive* alternatives $x,y\in X$, $U_k(\theta,y)-U_k(\theta,x)$ is weakly of the same sign as $U_k^{x,y,b+p}(\theta)$. Moreover, in any period of $\Gamma_c^{en}(b)$, players play the same game form as in $\Gamma_c^{ex}(b+p)$ with players' continuation payoff in $\Gamma_c^{en}(b)$ of implementing an alternative x being $-\left(x-\theta-b_k\right)^2+\delta W_k^{\sigma^{ex}(b+p)}(\theta,x)$. Therefore, using (15), $\sigma^{ex}(b+p)$ is an equilibrium of $\Gamma_c^{en}(b)$ if for any two consecutive alternatives $x,y\in X$, for all $\theta\in\theta$ and $k\in\{l,r\}$, $U_k^{x,y,b}(\theta)+\delta\left(W_k^{\sigma^{ex}(b+p)}(\theta,y)-W_k^{\sigma^{ex}(b+p)}(\theta,x)\right)$ is weakly of the same sign as $U_k^{x,y,b+p}(\theta)$. Step 1 follows then from (15).

Step 2: For all $p_l\le 0$, $p_r\ge 0$, and $k\in\{l,r\}$, for any two consecutive alternatives $x,y\in X$ such that $x<y$, the function $\theta\to W_k^{\sigma^{ex}(b+p)}(\theta,y)-W_k^{\sigma^{ex}(b+p)}(\theta,x)$ is uniquely characterized by the following condition: for all $\theta\in\Theta$,

$$\begin{aligned}&W_k^{\sigma^{ex}(b+p)}(\theta,y)-W_k^{\sigma^{ex}(b+p)}(\theta,x) \qquad (17)\\ =\;&\int_{\frac{x+y}{2}-b_r-p_r}^{\frac{x+y}{2}-b_l-p_l}\left[U_k^{x,y,b}(\zeta)+\delta\left(W_k^{\sigma^{ex}(b+p)}(\zeta,y)-W_k^{\sigma^{ex}(b+p)}(\zeta,x)\right)\right]f(\zeta-\theta)\,d\zeta.\end{aligned}$$

By construction of $\sigma^{ex}(b+p)$ (see Section 4.2.1), if x and y are consecutive alternatives, status quo x leads to a different outcome than status quo y under $\sigma^{ex}(b+p)$ in the game $\Gamma_c^{ex}(b+p)$ in a period with state ζ only if players disagree on how to rank x and y, that is, if $U_l^{x,y,b+p}(\zeta)<0$ and $U_r^{x,y,b+p}(\zeta)>0$.[40] From (15), this happens when $\zeta\in\left(\frac{x+y}{2}-b_r-p_r,\frac{x+y}{2}-b_l-p_l\right)$. In such states, either status quos $q\in\{x,y\}$ stays in place. So if $\sigma^{ex}(b+p)$ is played in $\Gamma_c^{en}(b)$, in such states, players' continuation payoff from status quo $q\in\{x,y\}$ is $-\left(q-\zeta-b_k\right)^2+\delta W_k^{\sigma^{ex}(b+p)}(\zeta,q)$. Using (15), this shows that (17) holds. Uniqueness follows from the fact that the right-hand side of (17) is a contraction in $\theta\to W_k^{\sigma^{ex}(b+p)}(\theta,y)-W_k^{\sigma^{ex}(b+p)}(\theta,x)$ for the sup norm.

Claim 1 *For all $x,y\in\mathbb{R}$ with $x<y$, let $\Gamma_{x,y}^{en}(b)$ denote the 2−alternative game defined in Section 3 in which players have quadratic preferences with bias b and $X=\{x,y\}$. This game has a least polarized equilibrium $\sigma^{x,y}(b)$ (in the sense of Proposition 2) and the corresponding continuation value $W^{\sigma^{x,y}(b)}$ satisfies the same condition as $\theta\to W^{\sigma^{ex}(b+p)}(\theta,y)-W^{\sigma^{ex}(b+p)}(\theta,x)$ in (16) and in (17) for some p^* that does not depend on $\{x,y\}$.*

[40] We ignore indifferences as they happen with probability 0.

Before we prove this claim, let us argue that together with step 1 and 2 above, it completes the proof. For any two consecutive alternatives $x, y \in X$, $W^{\sigma^{x,y}(b)}$ as constructed in Claim 1 satisfies (17) for $p = p^*$, so from Step 2, $\sigma^{ex}(b + p^*)$ is such that $W^{\sigma^{ex}(b+p^*)}(\theta, y) - W^{\sigma^{ex}(b+p^*)}(\theta, x)$ is equal to $W^{\sigma^{x,y}(b)}$. Claim 1 implies then that $W^{\sigma^{ex}(b+p)}(\theta, y) - W^{\sigma^{ex}(b+p)}(\theta, x)$ satisfies (16) for $p = p^*$, and since p^* does not depend on $\{x, y\}$, step 1 implies in turn that $\sigma^{ex}(b + p^*)$ is an equilibrium of $\Gamma_c^{en}(b)$ for any X. ■

Proof of Claim 1. Let $(W^z)_{z\in\mathbb{N}}$ be the sequence defined recursively by $W^0 = \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$ and $W^{z+1} = \left(\Omega^{\emptyset} \circ V\right)(W^z)$ as in the proof of Proposition 2 with current preferences $U^{x,y,b}$ as in (15).

Step A: $\sigma^{x,y}(b)$ exists, $(W^z)_{z\in\mathbb{N}}$ is increasing in the order $(\leq, \geq)$, and it converges to $W^{\sigma^{x,y}(b)}$. These results are established in the proof of Proposition 2. Note that this proof assumes that U is bounded over Θ, which is not true in $\Gamma_{x,y}^{en}(b)$ since $U = U_k^{x,y,b}$ and $\Theta = \mathbb{R}$. However, the boundedness of U is used only to establish the existence of a $\bar{W}$ such that (i) $\bar{W}(\leq, \geq)\left(\Omega^{\emptyset} \circ V\right)\left(\bar{W}\right)$ and (ii) $\bar{W}(\leq, \geq)\left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$. Consider the following candidate: for all $\theta \in \Theta$ and $k \in \{l, r\}$, $\bar{W}_k(\theta) \mathrel{\mathring{=}} E_{\theta(0)=\theta}\left[a_k \sum_{t=1}^{\infty} \delta^t \left|U_k^{x,y,b}(\theta(t))\right|\right]$, with $a_r = 1$ and $a_l = -1$.[41] Property (ii) is clearly satisfied. To prove (i), note that $\bar{W}_r(\theta) = \int_{\zeta\in\Theta}\left(\left|U_r^{x,y,b}(\zeta)\right| + \delta\bar{W}_r(\zeta)\right) f(\theta - \zeta)\, d\zeta$, so from (10),

$$\left(\Omega^{\emptyset} \circ V\right)_r \left(\bar{W}\right)(\theta) = \int_{\zeta\in D\left(\emptyset, V\left(\bar{W}\right)\right)} \left(\left|U_r^{x,y,b}(\zeta)\right| + \delta\bar{W}_r(\zeta)\right) f(\theta - \zeta)\, d\zeta \leq \bar{W}_r(\theta).$$

The proof for $\bar{W}_l$ is analogous.

Step B: There exists a negative and decreasing sequence $(p_l^z)_{z\in\mathbb{N}}$ and a positive and increasing sequence $(p_r^z)_{z\in\mathbb{N}}$ such that for all $z \in \mathbb{N}$, $k \in \{l, r\}$ and $\theta \in \mathbb{R}$,

$$U_k^{x,y,b}(\theta) + \delta W_k^z(\theta) \begin{cases} > 0 \text{ if } \theta > \frac{x+y}{2} - b_k - p_k^z \\ < 0 \text{ if } \theta < \frac{x+y}{2} - b_k - p_k^z \end{cases}, \tag{18}$$

[41] Note that under our assumptions, $\bar{W}$ is well defined, that is, conditional on $\theta(0)$, $\sum_{t=1}^{\infty} \delta^t \left|U_k^{x,y,b}(\theta(t))\right|$ is integrable. To see this, note that from (15), $U_k^{x,y,b}(\theta)$ is linear in θ, so it suffices to prove that conditional on $\theta(0)$, $\sum_{t=1}^{\infty} \delta^t |\theta(t) - \theta(0)|$ is integrable. Note that $\theta(t) - \theta(0) = \sum_{t'=1}^{t} v(t')$, so for all $T \in \mathbb{N}$,

$$\sum_{t=1}^{T} \delta^t |\theta(t) - \theta(0)| \leq \sum_{t=1}^{T} \delta^t \sum_{t'=1}^{t} |v(t')| = \sum_{t'=1}^{T} |v(t')| \sum_{t=t'}^{T} \delta^t \leq \sum_{t'=1}^{T} |v(t')| \sum_{t=t'}^{\infty} \delta^t = \sum_{t'=1}^{T} |v(t')| \frac{\delta^{t'}}{1-\delta}.$$

Note that $\sum_{t'=0}^{T} E[|v(t')|] \frac{\delta^{t'}}{1-\delta}$ converges to a finite limit as $T \to \infty$ since by assumption, $E[|v(t')|]$ is constant and finite. Therefore, the monotone convergence theorem implies that $\sum_{t'=1}^{\infty} |v(t')| \frac{\delta^{t'}}{1-\delta}$ is integrable, and from the above inequality, so is $\sum_{t=1}^{\infty} \delta^t |\theta(t) - \theta(0)|$.

and the sequence $(W^z)_{z\in\mathbb{N}}$ *satisfies*

$$W_k^{z+1}(\theta) = \int_{\frac{x+y}{2}-b_r-p_r^z}^{\frac{x+y}{2}-b_l-p_l^z} \left[U_k^{x,y,b}(\zeta) + \delta W_k^z(\zeta)\right] f(\zeta-\theta)\, d\zeta. \tag{19}$$

We prove step B by induction on z. Since $W^0 = \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$, from (15), (18) is satisfied for $z = 0$ and $p^0 = (0,0)$. Suppose now that (18) holds for some $z \in \mathbb{N}$ and some $p_l^z \leq 0$, and $p_r^z \geq 0$. From (8) and (18), $D(\emptyset, V(W^z)) = \left(\frac{x+y}{2} - b_l - p_l^z, \frac{x+y}{2} - b_r - p_r^z\right)$. Substituting this expression for $D(\emptyset, V(W^z))$ and (10) in $W^{z+1} = \left(\Omega^{\emptyset} \circ V\right)(W^z)$, we obtain (19).

To complete the induction argument, we need to show that if (18) and (19) holds for some $z \in \mathbb{N}$ and some $p_l^z \leq 0$, and $p_r^z \geq 0$, then (18) holds for $z+1$ and some $p_l^{z+1} \leq p_l^z$, and $p_r^{z+1} \geq p_r^z$. Since f is single-peaked, for all $\zeta, \theta \in \mathbb{R}$ such that $\theta \leq \frac{x+y}{2} - b_r - p_r^z \leq \zeta$, $f(\zeta - \theta)$ is increasing in θ. Hence, (18) and (19) imply that for all $\theta \leq \frac{x+y}{2} - b_r - p_r^z$, $W_r^{z+1}(\theta)$ is increasing in θ, and from (15), $U_r^{x,y,b}(\theta) + \delta W_r^{z+1}(\theta)$ is strictly increasing in θ. Since f is single-peaked, as $\theta \to -\infty$, $f(\zeta - \theta) \to 0$, so from (19), $W_r^{z+1}(\theta) \to 0$, whereas from (15), $U_r^{x,y,b}(\theta) \to -\infty$, so $U_r^{x,y,b}(\theta) + \delta W_r^{z+1}(\theta) \to -\infty$. Moreover, from step A, $W_r^{z+1} \geq W_r^z$. Together with the induction hypothesis (18), this implies that for all $\theta > \frac{x+y}{2} - b_r - p_r^z$,

$$U_r^{x,y,b}(\theta) + \delta W_r^{z+1}(\theta) \geq U_r^{x,y,b}(\theta) + \delta W_r^z(\theta) > 0.$$

Given the above properties of $\theta \to U_r^{x,y,b}(\theta) + \delta W_r^{z+1}(\theta)$, there exist $\theta^* \leq \frac{x+y}{2} - b_r - p_r^z$ such that $U_r^{x,y,b}(\theta) + \delta W_r^{z+1}(\theta) \gtreqless 0$ when $\theta \gtreqless \theta^*$. Setting $p_r^{z+1} = \frac{x+y}{2} - b_r - \theta^*$, we obtain $p_r^{z+1} \geq p_r^z$ and (18) for $z+1$. The proof for l is analogous.

Step C: $(p^z)_{z\in\mathbb{N}}$ *converges to some finite* p^* *such that* $p_l^* < 0 < p_r^*$ *and* $W_k^{\sigma^{x,y}(b)}$ *satisfies the same condition as* $\theta \to W_k^{\sigma^{ex}(b+p)}(\theta, y) - W_k^{\sigma^{ex}(b+p)}(\theta, x)$ *in (17) and (16) for* $p = p^*$.

By step B, $(p^z)_{z\in\mathbb{N}}$ tends to some $p^* \in [-\infty, 0] \times [0, +\infty]$, and $\left(\frac{x+y}{2} - b_r - p_r^z, \frac{x+y}{2} - b_l - p_l^z\right)$ is increasing in z in the inclusion sense. Therefore, if we let $z \to \infty$ in (19), the monotone convergence theorem implies that $W_k^{\sigma^{x,y}(b)}(\theta)$ satisfies the same condition as $\theta \to W_k^{\sigma^{ex}(b+p)}(\theta, y) - W_k^{\sigma^{ex}(b+p)}(\theta, x)$ in (17) for $p = p^*$.

To show that $p_r^* \in (0, +\infty)$, suppose first that $p_r^* = 0$. Then since $p_r^* \geq p_r^1 \geq 0$, $p_r^1 = 0$. Since $W^0 = \left(0^{\mathcal{F}}, 0^{\mathcal{F}}\right)$, from (19), W^1 is continuous, so (18) implies that $U_r^{x,y,b}\left(\frac{x+y}{2} - b_r\right) + \delta W_r^1\left(\frac{x+y}{2} - b_r\right) = 0$, which, from (19) and (15), implies that

$$0=\delta 2\left(y-x\right)\int_{\frac{x+y}{2}-b_{r}}^{\frac{x+y}{2}-b_{l}}\left(\zeta-\left(\frac{x+y}{2}-b_{r}\right)\right)f\left(\zeta-\left(\frac{x+y}{2}-b_{r}\right)\right)d\zeta,$$

which is impossible since the above integral is strictly positive, so $p_r^*>0$. Suppose now that $p_r^*=+\infty$. As argued above, if we let $z\to\infty$ in (19), we get

$$W_r^{\sigma^{ex}(b+p)}\left(\theta\right)=\int_{-\infty}^{\frac{x+y}{2}-b_l-p_l^*}\left[U_k^{x,y,b}\left(\zeta\right)+\delta W_r^{\sigma^{ex}(b+p)}\left(\zeta\right)\right]f\left(\zeta-\theta\right)d\zeta.$$

The above equation means that $W_r^{\sigma^{ex}(b+p)}\left(\theta\right)=E_{\theta(0)=\theta}\left[\sum_{t=1}^{\hat{t}-1}U_k^{x,y,b}\left(\theta\left(t\right)\right)\right]$ where $\hat{t}$ is the first period $t\geq 1$ such that $\theta\left(t\right)>\frac{x+y}{2}-b_l-p_l^*$. Since $\left\{\theta\left(t\right):t\geq 0\right\}$ is a random walk and $U_r^{x,y,b}\left(\theta\right)\to -\infty$ as $\theta\to-\infty$, then clearly, $W_r^{\sigma^{ex}(b+p)}\left(\theta\right)<0$ as $\theta\to-\infty$, which is impossible since from step A, $W_r^{\sigma^{ex}(b+p)}\left(\theta\right)\geq\bar{W}^0=0^{\mathcal{F}}$. The proof that $p_l^*\in\left(-\infty,0\right)$ is analogous. Finally, if we let $z\to\infty$ in (18), we obtain that $W_k^{\sigma^{x,y}(b)}\left(\theta\right)$ satisfies the same condition as $W_k^{\sigma^{ex}(b+p)}\left(\theta,y\right)-W_k^{\sigma^{ex}(b+p)}\left(\theta,x\right)$ in (16) for $p=p^*$.

Step D: p^* does not depend on x,y.

Let $x',y'\in\mathbb{R}$, and let $\left(W'^z\right)_{z\in\mathbb{N}}$ and $\left(p'^z\right)_{z\in\mathbb{N}}$ be the sequences defined in step B for the game $\Gamma_{x',y'}^{en}\left(b\right)$. We show by induction on z that for all $z\in\mathbb{N}$, $p'^z=p^z$ and for all $\theta\in\Theta$, $W'^z\left(\theta\right)=\frac{y'-x'}{y-x}W^z\left(\theta+\frac{x+y}{2}-\frac{x'+y'}{2}\right)$. Since $W'^0=W^0=\left(0^{\mathcal{F}},0^{\mathcal{F}}\right)$, from (18), $p'^0=p^0=\left(0,0\right)$, and the property is true for $z=0$. Suppose it is true for some $z\in\mathbb{N}$. From (15), $U_k^{x',y',b}\left(\theta\right)=\frac{y'-x'}{y-x}U_k^{x,y,b}\left(\theta+\frac{x+y}{2}-\frac{x'+y'}{2}\right)$. Using successively (19) for $\left(W'^z\right)_{z\in\mathbb{N}}$, the latter equality, the induction hypothesis, and the change of variable $\zeta'=\zeta+\frac{x+y}{2}-\frac{x'+y'}{2}$, we obtain

$$\begin{aligned}
W_k'^{z+1}\left(\theta\right) &= \int_{\frac{x'+y'}{2}-b_r-p_r'^z}^{\frac{x'+y'}{2}-b_l-p_l'^z}\left[U_k^{x',y',b}\left(\zeta\right)+\delta W_k'^z\left(\zeta\right)\right]f\left(\zeta-\theta\right)d\zeta \\
&= \frac{y'-x'}{y-x}\int_{\frac{x+y}{2}-b_r-p_r^z}^{\frac{x+y}{2}-b_l-p_l^z}\left[U_k^{x,y,b}\left(\zeta'\right)+\delta W_k^z\left(\zeta'\right)\right]f\left(\zeta'-\left(\theta+\frac{x+y}{2}-\frac{x'+y'}{2}\right)\right)d\zeta' \\
&= \frac{y'-x'}{y-x}W_k^{z+1}\left(\theta+\frac{x+y}{2}-\frac{x'+y'}{2}\right).
\end{aligned}$$

Thus, $U^{x',y',b}\left(\theta\right)+\delta W'^{z+1}\left(\theta\right)=\frac{y'-x'}{y-x}\left(U^{x,y,b}\left(\theta+\frac{x+y}{2}-\frac{x'+y'}{2}\right)+\delta W^{z+1}\left(\theta+\frac{x+y}{2}-\frac{x'+y'}{2}\right)\right)$, so one can easily see from (18) that necessarily, $p'^{z+1}=p^{z+1}$. ■

7.2.2 Proofs for Section 4.3

For all $\theta \in \Theta$, $x \in X$, and $n \in \mathbb{N}$, let $V_k^n(\theta, q)$ be the continuation value for player k in the game $\Gamma_{\wp,\Delta_n}^{en}$ of implementing alternative $q \in X$ at some bargaining time t such that $\theta(t) = \theta$, and conditional on players playing σ_n from $t + \Delta_n$ onwards:

$$V_k^n(\theta, q) = E_{\theta(0)=\theta}\left[\int_0^{\Delta_n} (u_k(\theta(t), q))\,\rho e^{-\rho t} dt\right] + e^{-\rho\Delta_n} E_{\theta(0)=\theta}\left[\sum_{y\in X} V_k^n(\theta(\Delta_n), y)\,\phi_{\sigma_n}(\theta(\Delta_n), q; y)\right].$$

Since σ_n is a Markov perfect equilibrium, our assumption on the bargaining stage game $\wp$ implies that for all $\theta \in \Theta$, $q, x \in X$, and $k \in \{l, r\}$,

$$\phi_{\sigma_n}(\theta, q; x) > 0 \Rightarrow V_k^n(\theta, x) \geq V_k^n(\theta, q). \tag{20}$$

For all n, let $\{e_q^n(t) : t \geq 0\}$ be the continuous-time process equal to the path of the bargaining outcome in the game $\Gamma_{\wp,\Delta_n}^{en}$ when players play σ_n and the initial status quo is q. For all $p > 0$, let $\hat{e}^n(\theta, q, p)$ denote the random variable on X that is equal to the bargaining outcome under σ_n at a bargaining time $p\Delta_n$ if the state and status quo at that time are θ and q.

Definition 4 *For all $n \in \mathbb{N}$ and all $q \in X$, a continuous time stochastic process $\{d^n(t) : t \geq 0\}$ is a deviation from $\{e_q^n(t) : t \geq 0\}$ if*

(i) for all integer $p \geq 0$, $d^n(t)$ is constant on $t \in [p\Delta_n, (p+1)\Delta_n)$,

(ii) $d^n(0) = q$,

(iii) for all integer $p > 0$, $d^n(p\Delta_n)$ is such that $d^n(p\Delta_n)$ is either equal to $\hat{e}^n(\theta(p\Delta_n), d^n((p-1)\Delta_n), p)$ or to the status quo $d^n((p-1)\Delta_n)$.

Hence, $\{d^n(t) : t \geq 0\}$ is a deviation of $\{e_q^n(t) : t \geq 0\}$ if d departs from the equilibrium transition $\hat{e}^n$ only by staying at the status quo. If $\{d^n(t) : t \geq 0\}$ is an continuous time processes on X that is measurable with probability 1, we denote the (random) discounted payoff of player k from d^n as follows:

$$\Pi_k(d) = \int_0^{\infty} u_k(\theta(t), d^n(t))\,\rho e^{-\rho t} dt.$$

Note that if m denotes a uniform bound on flow payoffs, for any two such processes d^n and $d^{n\prime}$,

$$\left|\Pi_k\left(d^n\right) - \Pi_k\left(d^{n\prime}\right)\right| \leq 2m. \tag{21}$$

Note also that $E_{\theta(0)=\theta}\left[\Pi_k\left(e_q^n\right)\right] = V_k^n\left(\theta, q\right)$.

Lemma 4 *If x is stable at (q,θ) in the limit, then there exists a neighborhood B_θ of θ such that for all $\zeta \in B_\theta$,*

$$V_k^n\left(\zeta, q\right) - V_k^n\left(\zeta, x\right) \rightarrow_{n\rightarrow\infty} 0. \tag{22}$$

Proof. Property (i) in Definition 3 implies that there exists a neighborhood B_θ of θ, some $\varepsilon > 0$, some $N > 0$, and a sequence of alternatives $x_1, ..., x_M$ with $x_1 = q$ and $x_M = x$ such that for all $m = 1, ..., M-1$, all $\zeta \in B_\theta$, and all $n \geq N$, $\phi_{\sigma_n}\left(\zeta, x_m; x_{m+1}\right) > \varepsilon$. For all $n \in \mathbb{N}$, Let $\left\{d_q^n\left(t\right) : t \geq 0\right\}$ be a deviation from $\left\{e_q^n\left(t\right) : t \geq 0\right\}$ such that d_q^n either stays at status quo or moves from x_m to x_{m+1} whenever the equilibrium transition $\hat{e}^n$ does (point (i) below), and after reaching x, d_q^n follows the equilibrium transition $\hat{e}^n$ (point (ii) below). Formally, for all $p > 0$,
(i) if for all $p' < p$, $d_q^n\left(p'\Delta_n\right) \neq x$, and if $d_q^n\left(\left(p-1\right)\Delta_n\right) = x_m$ for some $m < M$, then $d_q^n\left(p\Delta_n\right) = x_{m+1}$ if $\hat{e}^n\left(\theta\left(p\Delta_n\right), d^n\left(\left(p-1\right)\Delta_n\right), p\right) = x_{m+1}$ and $d_q^n\left(p\Delta_n\right) = d_q^n\left(\left(p-1\right)\Delta_n\right)$ otherwise,
(ii) if for some $p' < p$, $d_q^n\left(p'\Delta_n\right) = x$, then $d_q^n\left(p\Delta_n\right) = \hat{e}^n\left(\theta\left(p\Delta_n\right), d_q^n\left(\left(p-1\right)\Delta_n\right), p\right)$.

Since d_q^n is a deviation of e_q^n, then at all bargaining times d_q^n either follows the equilibrium path or deviates by staying at the status quo. Therefore, individual rationality implies that for all k and all $\zeta \in \Theta$,

$$E_{\theta(0)=\zeta}\left[\Pi_k\left(d_q^n\right)\right] \leq E_{\theta(0)=\zeta}\left[\Pi_k\left(e_q^n\right)\right] = V_k^n\left(\zeta, q\right).$$

Property (i) in Definition 3 together with (20) imply that for all $k \in \{l, r\}$ and all $\zeta \in B_\theta$,

$$V_k^n\left(\zeta, q\right) = V_k^n\left(\zeta, x_1\right) \leq V_k^n\left(\zeta, x_2\right) \leq ... \leq V_k^n\left(\zeta, x_M\right) = V_k^n\left(\zeta, x\right).$$

The two inequalities above imply that

$$E_{\theta(0)=\zeta}\left[\Pi_k\left(d_q^n\right)\right] \leq V_k^n\left(\zeta, q\right) \leq V_k^n\left(\zeta, x\right) = E_{\theta(0)=\zeta}\left[\Pi_k\left(e_x^n\right)\right]. \tag{23}$$

In the rest of the proof, we show that $\liminf_{n\rightarrow\infty} E_{\theta(0)=\zeta}\left[\Pi_k\left(d_q^n\right) - \Pi_k\left(e_x^n\right)\right] \geq 0$, which, together with (23), implies (22). Let $\hat{t}_{B_\theta}$ be the first exit time of $\{\theta(t) : t \geq 0\}$ from B_θ. Property (ii) in Definition 3 implies that for all $n \geq N$ and all $t \leq \hat{t}_{B_\theta}$, $e_x^n(t) = x$. Let $\hat{p}^n$ be the first period p such that $d_q^n(p\Delta_n) = x$. If the deviation d_q^n reaches x before the state exits B_θ, that is, if $\Delta_n \hat{p}^n \leq \hat{t}_{B_\theta}$, then at any time $t < \Delta_n \hat{p}^n$, the flow payoff from $e_x^n(t)$ is $u_k(\theta(t), x)$ while by definition of m, the flow payoff from $d_q^n(t)$ is no smaller than $u_k(\theta(t), x) - 2m$, and at any time $t \geq \Delta_n \hat{p}^n$, by point (ii) in the definition of d_q^n, the deviation follows the equilibrium path, that is, $d_q^n(t) = e_x^n(t)$. If the state exits B_θ before the deviation d_q^n reaches x, that is, if $\Delta_n \hat{p}^n > \hat{t}_B$, then from (21), we have $\Pi_k\left(d_q^n\right) \geq \Pi_k\left(e_x^n\right) - 2m$. Therefore,

$$\begin{aligned} & E_{\theta(0)=\zeta}\left[\Pi_k\left(d_q^n\right) - \Pi_k\left(e_x^n\right)\right] \qquad (24) \\ \geq\ & E_{\theta(0)=\zeta}\left[1_{\Delta_n\hat{p}^n \leq \hat{t}_{B_\theta}}\left(\left(1 - e^{-\rho\Delta_n\hat{p}^n}\right)(-2m) + e^{-\rho\Delta_n\hat{p}^n} \times 0\right)\right] + \Pr_{\theta(0)=\zeta}\left(\Delta_n\hat{p}^n > \hat{t}_{B_\theta}\right)(-2m). \end{aligned}$$

By definition of d_q^n and from property (i) in Definition 3, as long as θ stays in B_θ, the probability that d_q^n moves from some x_m to some x_{m+1} is at least ε. Therefore, conditional on d_q^n reaching x before $\hat{t}_{B_\theta}$ (i.e., conditional on $\Delta_n\hat{p}^n \leq \hat{t}_{B_\theta}$), the probability that it does so in more than p rounds is smaller than the probability of having less than M successes out of p draws in a Bernoulli trial with success probability ε . Hence, $\Pr\left(\hat{p}^n \geq p | \Delta_n\hat{p}^n \leq \hat{t}_{B_\theta}\right) \leq \sum_{m<M} \binom{p}{m} \varepsilon^m (1-\varepsilon)^{p-m}$. So, for all $p \geq 0$,

$$\begin{aligned} E_{\theta(0)=\zeta}\left[1_{\Delta_n\hat{p}^n \leq \hat{t}_{B_\theta}}\left(1 - e^{-\rho\Delta_n\hat{p}^n}\right)\right] &\leq E_{\theta(0)=\zeta}\left[1_{\Delta_n\hat{p}^n \leq \hat{t}_{B_\theta}}\left(1 - e^{-\rho\Delta_n p}\right)1_{\hat{p}^n<p} + 1_{\Delta_n\hat{p}^n \leq \hat{t}_{B_\theta}}1_{\hat{p}^n \geq p}\right] \\ &\leq 1 - e^{-\rho\Delta_n p} + \sum_{m<M}\binom{p}{m}\varepsilon^m(1-\varepsilon)^{p-m}. \end{aligned}$$

Likewise, for all $p \in \mathbb{N}_+$,

$$\begin{aligned} \Pr_{\theta(0)=\zeta}\left(\hat{t}_{B_\theta} < \Delta_n\hat{p}^n\right) &\leq \Pr_{\theta(0)=\zeta}\left(\hat{t}_{B_\theta} < p\Delta_n\right) + \Pr_{\theta(0)=\zeta}\left(\hat{t}_{B_\theta} \geq p\Delta_n \ \&\ \hat{p}^n > p\right) \\ &\leq \Pr_{\theta(0)=\zeta}\left(\hat{t}_{B_\theta} < p\Delta_n\right) + \sum_{m<M}\binom{p}{m}\varepsilon^m(1-\varepsilon)^{p-m}. \end{aligned}$$

From Assumption 2, for all $\zeta \in B_\theta$, $\Pr_{\theta(0)=\zeta}\left(\hat{t}_{B_\theta} < p\Delta_n\right)$ tends to 0 as $p\Delta_n \to 0$. Also, $\binom{p}{m}\varepsilon^m(1-\varepsilon)^{p-m}$ tends to 0 as $p \to \infty$. Therefore, if we take $p = \left\lfloor \Delta_n^{-1/2} \right\rfloor$ in the two inequalities above, we obtain that $E_{\theta(0)=\zeta}\left[1_{\Delta_n\hat{p}^n \leq \hat{t}_B}\left(1 - e^{-\rho\Delta_n\hat{p}^n}\right)\right]$ and $\Pr_{\theta(0)=\zeta}\left(\hat{t}_B < \Delta_n\hat{p}^n\right)$ both tend to 0 as $n \to \infty$. This shows that the right-hand side of (24) also tends to 0 as $n \to \infty$, which completes the proof. ■

Proof of Proposition 8. For all $s > 0$ and all $n \in \mathbb{N}$, let $\{d_q^{n,s}(t) : t \geq 0\}$ be a deviation from $\left\{e_q^n(t) : t \geq 0\right\}$ (see Definition 4) such that $d_q^{n,s}(t)$ stays at the status quo before s/Δ_n (point (i) below) and $d_q^{n,s}$ follows the equilibrium transition $\hat{e}^n$ after s/Δ_n (point (ii) below). Formally, for all $p \in \mathbb{N}_+$,

(i) if $p \leq s/\Delta_n$, $d_q^{n,s}(p\Delta_n) = q$,

(ii) if $p > s/\Delta_n$, $d_q^{n,s}(p\Delta_n) = \hat{e}^n(\theta(p\Delta_n), d^{n,s}((p-1)\Delta_n), p)$.

Using the same argument as in the proof of Lemma 4, property (i) in Definition 3 together with (20) imply that

$$E_{\theta(0)=\theta}\left[\Pi_k\left(d_q^{n,s}\right)\right] \leq V_k^n(\theta, q) \leq V_k^n(\theta, x) = E_{\theta(0)=\theta}\left[\Pi_k\left(e_x^n\right)\right] \tag{25}$$

In the rest of the proof, we show that if by contradiction of Proposition 8, $u_k(\theta, q) > u_k(\theta, x)$, then for n sufficiently large,

$$E_{\theta(0)=\theta}\left[\Pi_k\left(d_q^{n,s}\right)\right] > E_{\theta(0)=\theta}\left[\Pi_k\left(e_x^n\right)\right], \tag{26}$$

which contradicts (25).

Suppose that $u_k(\theta, q) > u_k(\theta, x)$. Since u_k is continuous in θ, there exists a neighborhood B'_θ of θ such that for all $\zeta \in B'_\theta$, $u_k(\zeta, q) \geq u_k(\zeta, x) + \epsilon$ for some $\epsilon > 0$. Property (ii) in Definition 3 implies that there exists $N > 0$ and a neighborhood B''_θ of θ such that for all $n \geq N$ and all $\zeta \in B''_\theta$, $\phi_{\Delta_n}(\zeta, x; x) = 1$. Let $B_\theta = B'_\theta \cap B''_\theta$ and let $\hat{t}_{B_\theta}$ be the first exit time of $\{\theta(t) : t \geq 0\}$ from B_θ. For all $n \geq N$ and all $t \leq \hat{t}_{B_\theta}$, $\theta(t) \in B''_\theta$ so $e_x^n(t) = x$. Therefore, conditional on $\hat{t}_{B_\theta} > s$, for all $t \leq s$, the flow payoff from e_x^n is $u_k(\theta(t), x)$ while the flow payoff from $d_q^{n,s}$ is $u_k(\theta(t), q)$. Conditional on $\hat{t}_{B_\theta} \leq s$, inequality (21) implies that $\Pi_k(d_q^{n,s}) \geq \Pi_k(e_x^n) - 2m$. Therefore, if $s^n = \Delta_n\lfloor s/\Delta_n\rfloor$ denotes the last bargaining time before s,

$$\begin{aligned} & E_{\theta(0)=\theta}\left[\Pi_k\left(d_q^{n,s}\right) - \Pi_k\left(e_x^n\right)\right] \\ \geq\ & -2m \Pr_{\theta(0)=\theta}\left(\hat{t}_{B_\theta} \leq s\right) + E_{\theta(0)=\theta}\left[1_{\hat{t}_{B_\theta}>s}\int_0^{s^n}\left(u_k(\theta(t), q) - u_k(\theta(t), x)\right)\rho\exp(-\rho t)\,dt\right] \\ & + E_{\theta(0)=\theta}\left[1_{\hat{t}_{B_\theta}>s}e^{-\rho s^n}\left(V_k^n(\theta(s^n), q) - V_k^n(\theta(s^n), x)\right)\right] \end{aligned} \tag{27}$$

Since $s^n \leq s$, conditional on $\hat{t}_{B_\theta} > s$, $\theta(s^n) \in B''_\theta$. Moreover, V_k^n is normalized to the units of the flow payoffs and is thus bounded by m. Therefore, Lemma 4 together with the bounded convergence theorem imply that the last term on the right-hand side of (27) tend to 0 as Δ_n tends to 0. Therefore, letting $n \to \infty$ in (27) we obtain

$$\begin{aligned}
& \liminf_{n\to\infty} \left(E_{\theta(0)=\theta} \left[\Pi_k \left(d_q^{n,s} \right) - \Pi_k \left(e_x^n \right) \right] \right) \qquad (28) \\
\geq & \ -2m \Pr_{\theta(0)=\theta} \left(\hat{t}_{B_\theta} \leq s \right) + E_{\theta(0)=\theta} \left[1_{\hat{t}_{B_\theta} > s} \int_0^s \left(u_k \left(\theta(t), q \right) - u_k \left(\theta(t), x \right) \right) \rho \exp(-\rho t) \, dt \right] \\
\geq & \ -2m \Pr_{\theta(0)=\theta} \left(\hat{t}_{B_\theta} \leq s \right) + E_{\theta(0)=\theta} \left[1_{\hat{t}_{B_\theta} > s} \right] \epsilon \left(1 - e^{-\rho s} \right) \\
\geq & \ -(2m + \varepsilon) \Pr_{\theta(0)=\theta} \left(\hat{t}_{B_\theta} \leq s \right) + \epsilon \left(1 - e^{-\rho s} \right)
\end{aligned}$$

From Assumption 2, $\Pr_{\theta(0)=\theta} \left(\hat{t}_{B_\theta} \leq s \right) = o(s)$, and since $1 - e^{-\rho s} \sim \rho s$ as $s \to 0$, the right-hand side of the above inequality is strictly positive for some $s > 0$. This shows that for some $s > 0$, and for n sufficiently large, (26) must hold. ■

Remark 1 *Assumption 2 is satisfied for Brownian processes. It is violated for Poisson processes.*

Proof. Let $\{\theta(t) : t \geq 0\}$ is a standard Brownian motion, and let $M(t) = \max_{s\in[0,t]} (\theta(s) - \theta(0))$ denote its running maximum. As is well known, $M(t)/\sqrt{t}$ has a normal distribution truncated at 0. By symmetry, $m(t) = \min_{s\in[0,t]} (\theta(s) - \theta(0))$ has the same distribution (more precisely, $-m(t)$ has the same distribution). If $D(t) = \max_{s\in[0,t]} |\theta(s) - \theta(0)|$ denotes the running maximal deviation, $\frac{D(t)}{\sqrt{t}}$ is equal to $\max\left(\frac{M(t)}{\sqrt{t}}, -\frac{m(t)}{\sqrt{t}}\right)$, so $D(t)$ is first order stochastically dominated by twice a truncated normal. If f denotes the p.d.f. of the normal distribution, and if τ_B denotes the first exit time from $[\theta - b, \theta + b]$ conditional on $\theta(0) = \theta$,

$$\Pr(\tau_B \leq t) = \Pr\left(\frac{D(t)}{\sqrt{t}} \geq \frac{b}{\sqrt{t}}\right) \leq \int_{\frac{b}{\sqrt{t}}}^{\infty} 2f(\mu) \, d\mu = 2\frac{t}{b^2} \int_{\frac{b}{\sqrt{t}}}^{\infty} \frac{b^2}{t} f(\mu) \, d\mu \leq 2\frac{t}{b^2} \int_{\frac{b}{\sqrt{t}}}^{\infty} \mu^2 f(\mu) \, d\mu.$$

Since the normal distribution has a finite second moment, $\int_{\frac{b}{\sqrt{t}}}^{\infty} \mu^2 f(\mu) \, d\mu \to 0$ as $t \to 0$. So the above inequality shows that $\Pr(\tau_B \leq t) = o(t)$ as $t \to 0$.

Let $\{\theta(t) : t \geq 0\}$ be a process in which at any t, $\theta(t)$ either stays constant or is redrawn according to some given distribution, and the occurrence of such draws is Poisson distributed with parameter λ. Note that for B sufficiently small, the process exits B as soon as a Poisson shock occurs, so τ_B is approximately the occurrence of the first shock, and $\Pr_{\theta(0)=\theta}(\tau_B \leq t) \sim \lambda t$ as $t \to 0$. ∎

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