## Monopoly vs. Competition

## Theory of the Firm

## Monopolistic Markets

## Monopoly vs. Competition

## Monopoly

- One firm (market power)
- Barriers to entry

Perfect Competition

- Many price-taking firms
- Free entry/exit


## Perfect Competition



## Causes of Monopoly

- Natural

1. Scarcity of a resource: one firm owns a key resource (De Beers' Diamonds)
2. Technological reasons: existence of large economies of scale relative to the market size -- one firm may serve the demand more efficiently than several firms (water, telephone, ...)

- Legal

1. Licenses (taxis, pharmacies, notaries, ...)
2. Patents (medical drugs, intellectual property, ...)

## Natural Monopoly



## The Monopolist's Problem

The revenue function of the monopoly is

$$
R(q)=p(q) q,
$$

where $\mathrm{p}(\mathrm{q})$ is the INVERSE demand function.

Therefore the problem of the monopolistic firm is

$$
\operatorname{Max}_{\mathrm{q} \geq 0} \pi(\mathrm{q})=\mathrm{p}(\mathrm{q}) \mathrm{q}-\mathrm{C}(\mathrm{q}) .
$$

## Profit Maximization

- First Order Condition:

$$
\pi^{\prime}(q)=0 \Leftrightarrow M R(q)=M C(q)
$$

- Second Order Condition:

$$
2 p^{\prime}(q)+q p^{\prime \prime}(q)-M C^{\prime}(q) \leq 0
$$

- Shutdown Condition: $\pi\left(q_{M}\right) \geq \pi(0)$


## Profit Maximization



## Profit Maximization: MR

Marginal revenue is given by

$$
\operatorname{MR}(q)=\frac{d}{d q}(p(q) q)=\underset{(+)}{p(q)}+\underset{(-)}{p^{\prime}(q)} q
$$

The two terms in this expression have a very clear interpretation: the sale of one additional (infinitesimal) unit generates
(a) an increase of revenue equal to the price (quantity effect), and
(b) a decrease of revenue equal to the total output times the reduction in price necessary to generate the demand of this additional unit (price effect).

## Profit Maximization: MR



## Profit Maximization: MR



## Profit Maximization: SOC

The Second Orden Condition is:

$$
2 p^{\prime}(q)+q p^{\prime \prime}(q)-M C^{\prime} \leq 0
$$

Since $\mathrm{p}^{\prime}(\mathrm{q})<0$, the condition $\mathrm{C}^{\prime \prime}(\mathrm{q})>0$ (requiring that the firm has diseconomies of scale near the optimal level of output) is no longer required for profit maximization: at the optimal production level the firm may have economies of scale.

## Monopoly Equilibrium

## Monopoly



Surplus and Profits


## Lerner Index

First Order Condition for profit maximization is:

$$
M R(q)=M C(q)
$$

Marginal revenue is

$$
M R(q)=p+q p^{\prime}(q)=p+p\left(\frac{q}{p}\right) p^{\prime}(q) .
$$

## Lerner Index

We can write the monopoly price as a function of the demand price-elasticity. The demand price-elasticity is

Then,

$$
\varepsilon=q^{\prime}(p) \frac{p}{q}=\frac{p}{q p^{\prime}(q)}
$$

$$
M R=M C \Leftrightarrow p\left(1+\frac{1}{\varepsilon}\right)=M C
$$

## Lerner Index

If demand is very elastic ( $\varepsilon$ high), the margin will be small; and vice versa:



## Lerner Index

The Lerner index measures monopoly power: it explains the percentage of price that is not due to costs.

It is defined as

$$
L=\frac{p_{M}-M C\left(q_{M}\right)}{p_{M}}=-\frac{1}{\varepsilon} .
$$

Note that $0 \leq \mathrm{L} \leq 1$. ( $\mathrm{L}=0$ under perfect competition.)

## Example

A monopolist faces the demand function

$$
D(p)=\max \{12-p, 0\},
$$

and its cost function is

$$
C(q)=5+4 q
$$

We calculate the monopoly equilibrium, the consumer and producer surpluses and the deadweight loss, the monopoly profits, the Lerner index.

## Example

Monopoly Equilibrium. Revenue is

Hence

$$
R(q)=(12-q) q=12 q-q^{2}
$$

$$
M R(q)=12-2 q
$$

Since $M C(q)=4$, the equilibrium output is given by the equation

$$
M R(q)=M C(q) \Rightarrow 12-2 q=4
$$

Thus, $q_{M}=4$, and $p_{M}=12-q_{M}=12-4=8$.

## Example

We calculates the consumer and producer surplus and the monopoly's profit:
$C S=0.5^{*}\left(12-p_{M}\right){ }^{*} q_{M}=0.5^{*}(12-8) * 4=8 ;$
PS $=\left(\mathrm{p}_{\mathrm{M}}-\mathrm{CMa}\right)^{*} \mathrm{q}_{\mathrm{M}}=(8-4)^{*} 4=16$;
$\pi(q)=p_{M} q_{M}-C\left(q_{M}\right)=8 * 4-\left(5+4^{*} 4\right)=11$;

Since the competitive equilibrium is $p_{c}=4, q_{c}=8$, we have: DWL $=0.5^{*}\left(p_{M}-p_{c}\right)\left(q_{c}-q_{M}\right)=0.5^{*} 4^{*} 4=8$.

## Example

We calculate the Lerner index:
$\mathrm{LI}=\left(\mathrm{p}_{\mathrm{M}}-\mathrm{MC}\left(\mathrm{q}_{\mathrm{M}}\right)\right) / \mathrm{p}_{\mathrm{M}}=(8-4) / 8=0.5$


## Regulating a Monopoly

An unregulated monopoly generates a deadweight loss, DWL>0. Can this be avoided?

An obvious solution (if possible - that is, if we know the monopoly's cost function), is to impose a regulated price equal to marginal cost, thus generating the competitive outcome.

This solution may lead to monopoly's losses that would have to be subsidized.

When subsidizing is not possible or convenient (rising the necessary revenue may involve other inefficiencies), an alternative regulated price may be the average cost.

## Regulating a Monopoly



Regulating a Monopoly


In a monopoly, as in a competitive market, introducing a tax causes an increase in prices and a decrease in output, and therefore, a deadweight loss.

The proportion of the tax that is paid for by consumers depends, in a monopoly as well as in a competitive market, of the price elasticity of the demand.

## Price Caps

Recall that in a competitive market introducing a price cap (i.e., a maximum price at which the good may be traded), reduces the level of output (and creates an excess demand that requires rationing the demand), results in a deadweight loss (reduces the total surplus) and may or may not increase the consumer surplus.

In a monopoly, however, introducing a price cap below the equilibrium price may be an effective instrument to decrease the price and increase the output. Moreover, if the price cap is above marginal cost, then it does not create an excess demand, and results in a greater consumer and total surplus, and smaller deadweight loss.

## Taxes and Price Cap: an Example

Consider a monopolist whose cost function is

$$
C(q)=5+4 q
$$

facing the demand

$$
D(p)=\max \{12-p, 0\}
$$

The monopoly output is the solution to the equation

$$
12-2 q=4
$$

Therefore $\mathrm{q}_{\mathrm{M}}=4$, and $\mathrm{p}_{\mathrm{M}}=8$.

1. Suppose we introduce a tax $T=1 € /$ unit.
Demand:
Revenue: $D(p, T)=\max \{12-(p+T), 0\}$.
Marginal Revenue: $\quad \mathrm{MR}(\mathrm{q})=12-2 q-\mathrm{T}$. $R(q, T)=(12-q-T) q=12 q-q^{2}-T q$.

## Monopoly: Taxes

The equation $M R(q)=M C(q)$ is now

$$
12-2 q-T=4
$$

Hence output is $\mathrm{q}_{M}(\mathrm{~T})=4-\mathrm{T} / 2$, the consumer price is
$p_{M}(T)=8+T / 2$, and
monopoly price $=8+\mathrm{T} / 2-\mathrm{T}=8-\mathrm{T} / 2$.

That is, the tax burden is shared equally between the consumers and the monopoly.

## Monopoly:Taxes



Monopoly: Taxes
$M R(q)=M C(q) \Rightarrow 12-2 q-T=4 \Rightarrow q(T)=4-T / 2$.
With the taxe $T$, the price paid by the consumer and the price received by monopoly are not the same:

Price paid by consumer $=12-q(T)=8+T / 2$
Price received by monopoly $=8+\mathrm{T} / 2-\mathrm{T}=8-\mathrm{T} / 2$.
In this case, the tax burden is shared equally between the consumers and the monopoly.

## Taxes and Price Cap: an Example

2. Suppose we introduce a price cap $\mathrm{p}^{+}<8=\mathrm{p}_{\mathrm{M}}$.

The price cap changes de revenue function of the monopoly

$$
R\left(p^{+}, q\right)=\left\{\begin{array}{ll}
12-q & \text { if } 12-\mathrm{q} \leq \mathrm{p}^{+} \\
p^{+} q & \text { if } 12-q>p^{+}
\end{array}\right\}
$$

The Marginal Revenue is:

$$
\operatorname{MR}\left(p^{+}, q\right)=\left\{\begin{array}{ll}
12-2 q & \text { if } 12-\mathrm{q} \leq \mathrm{p}^{+} \\
p^{+} & \text {if } 12-q>p^{+}
\end{array}\right\}
$$

Para $p+=7$, the equilibrium is $q=5$ and $p=p^{+}=7$ (see the graph in the page).

## Price Discrimination

So far, we have assumed that the monopoly charges the same price for each unit.

We now discuss how the monopoly may increase its profit by charging different prices to different consumers with different WILLINGNESS TO PAY.

## Price Discrimination

- First degree: the monopoly sells each infinitesimal unit at the maximum price a consumer is willing to pay. With this pricing policy we obtain
$C S=0, P S=T S=$ maximum surplus, $D W L=0$.
- Second degree: The monopoly uses NON-LINEAR pricing policies: for example, volume discounts. This type of discrimination is very common in water, electricity, telephone and internet supplies.


## Third Degree Price Discrimination

The monopoly tries to segment the market (by geographical criteria, by customers' features, etc.), and charges different prices to each "market segment."

Example: There are two groups of consumers, 1 and 2, whose demands are $D_{1}\left(p_{1}\right)$ and $D_{2}\left(p_{2}\right)$.

## Third Degree Price Discrimination

$$
\varepsilon_{2}>\varepsilon_{1}
$$



## Third Degree Price Discrimination

The monopolist chooses $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ to maximize the total profits:

```
\pi(\mp@subsup{q}{1}{},\mp@subsup{q}{2}{})=\mp@subsup{R}{1}{}(\mp@subsup{q}{1}{})+\mp@subsup{R}{2}{}(\mp@subsup{q}{2}{})-C}(\mp@subsup{q}{1}{}+\mp@subsup{q}{2}{}
    = p
```

FOC

$$
\begin{aligned}
& M R_{1}\left(q_{1}\right)=M C\left(q_{1}+q_{2}\right) \\
& M R_{2}\left(q_{2}\right)=M C\left(q_{1}+q_{2}\right) .
\end{aligned}
$$

## Third Degree Discrimination

In terms of elasticities:

$$
p_{1}\left(1+1 / \varepsilon_{1}\right)=p_{2}\left(1+1 / \varepsilon_{2}\right)=M C\left(q_{1}+q_{2}\right)
$$

Therefore

$$
\mathrm{p}_{1} / \mathrm{p}_{2}=\left(1+1 / \varepsilon_{2}\right) /\left(1+1 / \varepsilon_{1}\right)<1 ;
$$

and

$$
p_{1}<p_{2}
$$

That is, monopoly price is lower in the market with a more elastic demand.

## Third Degree Price Discrimination:

## Example

Suppose a monopoly produces a good with costs

$$
C(q)=q^{2} / 2,
$$

and faces two markets with demands

$$
\mathrm{D}_{1}\left(\mathrm{p}_{1}\right)=\max \left\{20-\mathrm{p}_{1}, 0\right\}
$$

and

$$
\mathrm{D}_{2}\left(\mathrm{p}_{2}\right)=\max \left\{60-2 \mathrm{p}_{2}, 0\right\} .
$$

Determine the quantities, prices and total profits under third degree price discrimination, and without price discrimination.

## Third Degree Price Discrimination: Example

(a) Third degree discrimination:
$M C\left(q_{1}+q_{2}\right)=q_{1}+q_{2}$
$R_{1}=20 q_{1}-q_{1}^{2} \Rightarrow M R_{1}=20-2 q_{1}$
$R_{2}=30 q_{2}-0.5^{*} q_{2}{ }^{2} \Rightarrow M R_{2}=30-q_{2}$

Equilibrium:
$20-2 q_{1}=30-q_{2}=q_{1}+q_{2}$
Solving, we obtain $q_{1}=2, q_{2}=14, p_{1}=18$ and $p_{2}=23$.
Monopoly profits are $\pi=18^{*} 2+14^{*} 23-C(2+14)=230$.

## Third Degree Price Discrimination:

## Example

(b) Without price discrimination: the aggregate demand is

$$
D(p)=\left\{\begin{array}{ll}
80-3 p & \text { if } \mathrm{p} \leq 20 \\
60-2 p & \text { if } 20<\mathrm{p} \leq 30 \\
0 & \text { if } \mathrm{p}>30
\end{array}\right\} \Rightarrow p(q)=\left\{\begin{array}{ll}
\frac{80-q}{3} & \text { if } 20 \leq \mathrm{q} \leq 80 \\
30-\frac{q}{2} & \text { if } \mathrm{q}<20 \\
0 & \text { if } \mathrm{q}>80
\end{array}\right\}
$$

Monopoly equilibrium: $\mathrm{MR}(\mathrm{q})=\mathrm{MC}(\mathrm{q})$
Solving, we obtain $\mathrm{q}_{\mathrm{M}}=15, \mathrm{p}_{\mathrm{M}}=22.5$.
Monopoly profits are: $\pi=84.375$.

