ADVANCED ECONOMETRICS I (Time Series)) Jesús Gonzalo

**Exercise 4: UNIT ROOT Processes**

Due day announced in the web page.

1. (This problem is adapted from Oxford M.Phil 1987)
Consider the following generation process for a random variable $y_t$:

$$y_t = \beta y_{t-1} + e_t \text{ where } e_t \sim IN(0,1),\ \beta = 1 \text{ and } y_o = 0.$$

- (a) Show that Var($y_t$)=t and $corr^2(y_t, y_{t-k})$=1-(k/t), and deduce the equivalent expression for $k < 0$.
- (b1) Derive the E[$\sum y_{t-1}e_t/T$]
- (b2) Derive the E[$\sum y_t^2/T$]
- (b3) Derive the E[$\sum y_{t-1}y_{t-1}/T$]
- (b4) Derive the Var[$\sum y_t^2/T$]
- (c) What is the order of integration of $y_t$?
- (d) Derive the limiting distribution of the sample mean.
- (e) Obtain the limiting distribution of the least squares estimator of $\beta$.
- (f) Derive the corresponding results in (a)-(e) when $|\beta| < 1$ and $y_o \sim N(0, (1-\beta^2)^{-1})$.
- (g) Undertake a Monte Carlo study of the finite sample behavior of the least squares estimator of $\beta$ for T=100 when (i) $\beta = 1.0$; (ii) $\beta = 0.9$; and (iii) $\beta = 0.5$. Pay attention to the distribution of the t-statistics in all the three cases. Do they follow a t-distribution (you can use a Q-Q plot)?

2. The *Brownian Bridge process* $B(t)$ on $[0,1]$ is defined from the Brownian process $W$ as

$$B(t) = W(t) - tW(1), t \in [0,1].$$

Let $\xi_n$ be an i.i.d. sequence with mean zero and variance 1, and define the array

$$\xi^*_{ni} = \xi_i - \frac{1}{n}\sum_{j=1}^{n} \xi_j, i = 1, ..., n, n = 1, 2, ...,$$

i.e., the deviation of $\xi_i$ from their sample mean for $n$ observations. Finally, define de process

$$X_n(t) = \frac{1}{\sqrt{n}} \sum_{i=1}^{[tn]} \xi^*_{ni}$$

in the functional space $D$; show the weak convergence of $X_n \to B$.

3. Generate the CVs $(1\%, 5\%, 10\%)$ of the t-stat DF test in the three cases discussed in class. For each case consider sample sizes of $T = 100$ and $T = 300$, initial conditions $Y_0 = 0$ and $Y_0 = 10$ and constant term values under the null of $\alpha = 0$ and $\alpha = 10$. Which conclusions do you obtain?

4. Generate a random walk $X_t$ (T=100) with no drift. Now regress $X_t$ on a deterministic trend. What do you find?

5. The same as problem (2) but now with drift.

6. Generate $X_t$= a + b t + $e_t$ where $e_t \sim IN(0,1)$. Regress $X_t$ on a constant and $X_{t-1}$. How many times do you reject that $X_t$ follows a random walk with drift (use the Cvs obtained in problem 2)?

7. Design a Monte-Carlos experiment to show that when the DGP is an AR(1) with a coefficient very close to one, and we want to use our model for forecasting it is better to assume that the coefficient is equal one.

8. Suppose $X_t$ is generated as $X_t = D_t + \epsilon_t$ with $t = 1, ..., 200$, $\epsilon_t \sim IN(0,1)$ and

$$D_t = \begin{cases} 1 & \text{if } t > 100, \\ 0 & \text{otherwise.} \end{cases}$$

Analyze the power of the DF t-test. What do you conclude?

**Note:** For all the Monte-Carlo experiments run 1000 simulations

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