"Stochastic Limit Theory" by James Davidson

Additions to the Mixing Nexus

Thm: Let $Y_t = g(X_t, X_{t-1}, \ldots, X_{t-\tau})$ be a measurable function, for finite τ.

If X_t is α-mixing (ϕ-mixing) of size $-\epsilon$, then Y_t is also.

The rate of convergence is often quantified by a summability criterion, that for some number $\epsilon > 0$, $\alpha_m \to 0$ sufficiently fast that

$$\sum_{m=1}^{\infty} \alpha_m^{1/\epsilon} < \infty.$$

Thm (Andrews 1984). Let $\{Z_t\}_{-\infty}^{\infty}$ be an independent sequence of Bernoulli r.v.s taking values 1 and 0 with fixed probabilities ρ and $1-\rho$. If

$X_t = \rho X_{t-1} + Z_t$ for $\rho \in (0, \frac{1}{2}]$,

$\{X_t\}_{-\infty}^{\infty}$ is **not** strong mixing

<u>Example</u> : Consider an AR(1) process with iid increments

$$X_t = \rho X_{t-1} + Z_t, \quad 0 < \rho < 1$$

in which the marginal distribution of Z_t has unbounded support. It can be shown that $\{X_t\}$ is not uniform mixing.

There are sufficient conditions for Strong and Uniform Mixing for LINEAR PROCESSES.