ECONOMETRIC IMPLICATIONS OF NON-EXACT PRESENT VALUE MODELS

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Abstract: One of the most commonly used and, at the same time, rejected models in finance and macroeconomics is the exact present value model (PVM), where a variable Y_t is expressed as the expected value at time t of the sum of discounted future values of another variable X_t. This paper extends the PVM by making it non-exact (NEPVM) in a simple way, allowing us to study situations where there are transitory deviations from the exact PVM. The proposed NEPVM is derived from similar non-arbitrage or equilibrium conditions the exact PVM comes from and it can explain some stylized economic facts that cannot be explained by exact PVMs. The rejection of the exact PVM produced by the standard volatility and cross-equation restriction tests is not enough to reject the NEPVM. We present the new variance bounds and cross-equation restrictions implied by the NEPVM and we show how to test them. The paper finishes by analyzing empirically the cases of stock prices and dividends, and short- and long-term interest rates. For these cases we are unable to reject a simple NEPVM. This fact, together with the theoretical results contained in the paper, suggests that the proposed NEPVM could be compatible with some of the empirical findings in the literature.

Key-Words: Cointegration, Cross-Equation Restrictions, Present Value Model, VAR, Volatility Tests.
JEL Classification: C3; C5; E4

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1 Introduction

Present value models have been extensively used to interpret the behavior of financial and macroeconomic time series. A present value relationship between two variables states that one of the variables (an endogenous variable) can be written as a linear function of the summed discounted value of expected future values of the other variable (a forcing variable). Let Y_t and X_t be an endogenous and a forcing variable, respectively. Then,

$$Y_t = \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i E_t X_{t+i}, \tag{1}$$

where θ is the coefficient of proportionality and $\delta < 1$ is the discount factor. E_t denotes mathematical expectation conditional on the full public information set I_t, which includes Y_t, X_t and their lagged values. For the sake of simplicity we do not add a constant term to the right hand side.

In finance, dynamic stochastic models like (1) have been used, for instance, to describe the expectations theory of the term structure, where Y_t is the long-term yield and X_t is the short-term yield (see e.g. Campbell and Shiller, 1987a, 1987b; Mattey and Meese, 1986); and to explain the behavior of stock prices and dividend payments (see e.g. Campbell and Shiller, 1987a, 1989; Bong-Soo Lee, 1995; West, 1987, 1988).

In macroeconomics, the present value model (PVM) (1) has been applied in such situations as the following: testing the validity of Cagan's model of hyperinflation (see Engsted, 1993); analyzing whether the conduct of US fiscal policy has been influenced by constraints on the accumulated stock of outstanding Federal debt (see Kremers, 1989; Hamilton and Flavin, 1986); and representing the permanent income theory of consumption. In the third case, equation (1) can be rearranged so that it becomes a statement about savings, by writing savings equals the expected present value of future declines in labor income (see e.g. Campbell, 1987; Campbell and Deaton, 1989; Flavin, 1981, 1993).

Despite the simplicity of its structure, or maybe as a consequence of it, there exists a high degree of controversy about the validity of this *exact* PVM (EPVM). In fact, the EPVM has been rejected very often in the applications reported above.

The principal goal of this paper is to present and analyze the econometric implications of a model that maintaining the fundamental aspects of the standard PVM it is more flexible, in

the sense of allowing Y_t to deviate transitorily from its fundamentals (the EPVM). This new model maintains all the essential features of the EPVM because it is derived from the same type of equilibrium conditions (see Section 2). Formally, it is given by the following expression,

$$Y_t = \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i E_t X_{t+i} + \epsilon_t, \tag{2}$$

where the additive component ϵ_t is an error term representing transitory deviations from the equilibrium conditions that generate a PVM like (1).

In the expectation theory of the term structure ϵ_t could represent a time-varying term premium. In the dividends-stock prices models ϵ_t could capture the influence of noise traders, the existence of fads, the risk-aversion of the agents or the non-constancy of the discount factor. In the permanent income theory of consumption, this disturbance term describes the transitory consumption component. More reasons for the existence of ϵ_t can be found in pages 93-109 of Salge (1997).

Opposite to what the literature has considered, ϵ_t in model (2) is neither a rational bubble nor a fad (see Section 2), but it is capable of describing some stylized facts that have been explained by EPVMs with bubbles or with fads.

The non-exact present value model (NEPVM) (2), changes the standard conditions (cross-equation restrictions and volatility conditions) used in the literature to test for the EPVM. This paper shows the new conditions and identifies the cases under which cointegration is the only testable econometric implication of the NEPVM.

The paper is organized as follows. In Section 2 we derive a NEPVM (2) from an equilibrium or non-arbitrage condition and show that the term ϵ_t does not constitute a bubble. As a consequence of that the EPVM and NEPVM imply the same type of long-run behavior. Section 3 introduces the new set of cross-equation restrictions, implied by the NEPVM, and shows how to test them for different stochastic structures of ϵ_t. Section 4 presents the volatility tests implied by both type of models. Section 5 analyzes empirically the cases of stock prices and dividends, and short- and long-term interest rates. Although the EPVM is not rejected for the interest rates for the first part of the sample (1952-1978), we are unable to reject the NEPVM for the stock prices example and for the interest rate case over the second part of the sample (1983-1991). These facts, together with the theoretical results contained in the paper, suggest that the proposed NEPVM could be compatible with some of the empirical findings in the

literature. The conclusions are found in Section 6. Proofs are provided in the appendix.

Throughout the paper, the variables involved in the PVM will be assumed to contain a random walk component (to be I(1)).

2 NEPVM and Cointegration

In this section the discussion is centered, without any loss of generality, on the model for dividends and stock prices. In this case the EPVM (1) is derived as a solution of the following equilibrium or non-arbitrage condition (the expected rate of return of a risky asset equals the return of a riskless asset)

$$E_t R_t \equiv \frac{E_t Y_{t+1} - Y_t}{Y_t} + \frac{X_t}{Y_t} = r_t = r. \tag{3}$$

R_t is the rate of return of the stock, X_t represents the dividends and $r_t = r$ is the constant rate of return of a riskless asset.

Underneath (1) and (3) there are several crucial assumptions: individuals have rational expectations, individuals are risk neutral, and the expected returns are considered to be constant. There are many reasons why these assumptions may not hold. Expected equilibrium returns might vary either because subjective attitudes to risk versus return change or because changes in the risk-free rate of interest or because stocks are viewed as inherently more risky at certain periods. What type of present value model do we have in all these realistic situations? In general we do not know it. It will depend on the process the time varying expected returns follow, on the type of utility function agents have, ...etc. In this paper we conjecture that violations of any of the above assumptions can be handle by adding an extra stationary term (ν_t) to the equilibrium condition (3)

$$E_t R_t \equiv \frac{E_t Y_{t+1} - Y_t}{Y_t} + \frac{X_t}{Y_t} = r_t = r + \frac{\nu_t}{Y_t}. \tag{4}$$

The random term ν_t is divided by Y_t for mathematical convenience in order to avoid unnecessary non-linearities. As a by-product we will obtain an expression that has been proved in the literature to be very useful for testing the PVM (see the variable ξ_t in the next section). Both, the constant rate and the variable term are observed by private agents ($\nu_t \in I_t$), therefore there is no room to make any profit by arbitraging between the two assets. The equilibrium condition, equation (4), can be rearranged as

$$Y_t = \delta E_t Y_{t+1} + \delta X_t - \delta \nu_t, \tag{5}$$

where $\delta = (1+r)^{-1}$.

From (5), and assuming that the same transversality condition of the EPVM ($\lim_{i\to\infty} \delta^i E_t Y_{t+i} = 0$) is satisfied, it is straightforward to express Y_t as

$$Y_t = \theta(1-\delta) \sum_{i=0}^{\infty} \delta^i E_t X_{t+i} + \epsilon_t, \tag{6}$$

with $\theta = \delta/(1-\delta)$ and where

$$\epsilon_t = -\delta \sum_{i=0}^{\infty} \delta^i E_t \nu_{t+i}. \tag{7}$$

Model (6) together with condition (7) is what we define in this paper as NEPVM. Note that although the error term ϵ_t has the same spirit as what is normally defined as a fad in the literature (Summers (1986)), in our case it is not a fad. A fad, F_t, is a deviation between prices and intrinsic value that slowly reverts to its zero mean, $Y_t = \theta(1-\delta)\sum_{i=0}^{\infty} \delta^i E_t X_{t+i} + F_t$. The term F_t has to be stationary for being a fad and it has to be explosive in order to solve the equilibrium condition (3) (see Cameron (1989)). Therefore a PVM with a fad element can not solve (3), and this is the main difference with our NEPVM, that has explicitly been generated from a non-arbitrage condition and at the same time the term ϵ_t can be stationary.

From Gourieroux, Laffont and Monfort (1982) it is easily seen that if we assume a stationary ARMA process for ν_t, then ϵ_t will be another stationary ARMA process. This implies that if ν_t is stationary, then ϵ_t in (2) can not represent a rational bubble (see Cameron (1989)).

It is also interesting to observe that the NEPVM (6) can capture situations with time varying expected returns without loosing the linearity that characterizes the EPVM (1). The standard PVM with non-constant expected returns does not contain the error term ϵ_t, but instead of having $(1+r)^{-i}$ in the infinite sum, it has $\prod_{j=1}^{i}(1+r+\nu_{t+j})^{-1}$. This produces a very highly non-linear PVM. We avoid that non-linearity by simply considering the term ν_t/Y_t in the equilibrium condition (4), instead of only ν_t.

For simplicity in the notation, if nothing is said about ν_t it will be considered a m.d.s ($E_t v_{t+1} = 0$), and therefore $\epsilon_t = -\delta\nu_t$ will be a m.d.s too.

Next proposition shows that both type of models (EPVM and NEPVM) have the same long-run implications, and therefore will not be possible to discriminate between them with cointegration tests.

Proposition 1. *Let Y_t and X_t satisfy the present value relationship (2). Then, Y_t and X_t are cointegrated, with cointegrating vector* $(1, -\theta)$.

The above proposition provides another reason why an stationary ν_t can not generate a bubble, otherwise the variables $(Y_t,\ X_t)$ would not be cointegrated.

There is a very peculiar case where the EPVM implies a deeper level of cointegration: multicointegration (see Granger and Lee (1988) for its definition). This case is when X_t is a function of only its own past. In this situation multicointegration could be used to discriminate between the EPVM and the NEPVM. A simple test for multicointegration can be found in Engsted, Gonzalo and Haldrup (1997).

In the next section we present some orthogonality or cross-equation restrictions that help to differentiate between the EPVM and the NEPVM.

3 Cross-Equation Constraints

Since both models imply cointegration by the Granger Representation Theorem (Engle and Granger, 1987), Y_t and X_t obey an error-correction model. We approximate this representation by using a finite vector error correction model of order p,

$$W_t = C + \gamma\alpha' Z_{t-1} + \sum_{j=1}^{p} \Gamma_j W_{t-j} + \eta_t, \tag{8}$$

where $W_t = (\Delta Y_t, \Delta X_t)'$, $Z_{t-1} = (Y_{t-1}, X_{t-1})'$, and η_t is a vector white noise.

Following Campbell and Shiller (1987a), we define a limited information set H_t, observable to the econometrician, which includes current and lagged values of X_t and Y_t. Private agents generally have more information than econometricians ($H_t \subseteq I_t$). One of the differences of H_t with respect to the full market information set, I_t, comes through the amount of knowledge the econometrician has of the disturbance term. This paper reflects this fact by assuming that,

$$E(\epsilon_t|H_t) = \mu_t. \tag{9}$$

With this specification, we allow for the possibility that the econometrician observes the whole error term, in which case $\mu_t = \epsilon_t$; and for the opposite case, where the econometrician does not have any information about it (i.e. $\mu_t = 0$). The results obtained in this paper are not modified in any of these events.

Using (8) it is possible to write a VAR model for the stationary variables ΔX_t and S_t (with their means removed),

$$\begin{bmatrix} \Delta X_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}, \tag{10}$$

where $S_t = Y_t - \theta X_t$, $\Delta = 1 - L$ and the polynomials in the lag operator $a(L), b(L), c(L)$ and $d(L)$ are all of order p. To simplify notation, (10) can be written in first order form as $z_t = Az_{t-1} + v_t$, where z_t is the vector $[\Delta X_t, ..., \Delta X_{t-p+1}, S_t, ..., S_{t-p+1}]$ and A is the companion matrix of the VAR. Then for all i, $E(z_{t+i}|H_t) = A^i z_t$, where H_t contains contemporaneous and lag values of ΔX_t and S_t .

The cross-equation constraints implied by the EPVM (NEPVM) are given by subtracting θX_t from expression (1) (expression (2)) and projecting both sides of the resulting equation onto H_t. To do this, we define two vectors of $2p$ elements, g' and h', such that $g'z_t = S_t$ and $h'z_t = \Delta X_t$. These constraints are given, for the exact and non-exact PVM, in the following two propositions.

Proposition 2. (EPVM) *(Campbell and Shiller, 1987a) Let Y_t and X_t satisfy the exact present value relationship (1). Then, the true innovation at time t in Y_t, $\xi_t \equiv Y_t - \frac{1}{\delta}[Y_{t-1} - \theta(1-\delta)X_{t-1}]$, is unpredictable given information available at time t-1. This implication can be tested by regressing $\xi_t = S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t$ on information available at time t-1 or by testing the following cross-equation restrictions derived from the VAR model (10)*

$$g'(I - \delta A) = \theta h'\delta A. \tag{11}$$

Proposition 3. (NEPVM) *Let Y_t and X_t satisfy the non-exact present value relationship (2).*

Case 1: ϵ_t (or ν_t) is an $MA(q)$ error term.

ξ_{t+q+1}, is unpredictable given information available at time t-1. This implication can be tested by regressing $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$ onto information available at time t-1 or by testing the following cross-equation restrictions derived from the VAR model (10)

$$g'(I - \delta A)A^{q+1} = \theta\delta h' A^{q+2}. \tag{12}$$

Case 2: ϵ_t (or ν_t) is an $AR(p)$ error term.

Assume $E(\epsilon_t|H_t) = \mu_t \neq 0$. Then $\xi_{t+j}, j = 0, 1, 2, ...,$ is predictable given information available at time t-1. Therefore no cross-equation restrictions can be derived from a VAR model for (10)

The variable ξ_t has the economic interpretation of an asset return. In the application of the term structure it is the excess return on long bonds over short bills, while in the stock market it is the excess return on stocks over a constant mean, multiplied by the stock price. This multiplication by the stock price is another reason why in the equilibrium condition (4) we introduced the random term ν_t/Y_t instead of only ν_t. By doing that condition (4) can be re-written as $E_t(\xi_{t+1}) = \nu_t$.

It is important to notice that while the cross-equation restrictions (11) implied by the EPVM are linear, the ones (12) implied by the NEPVM are highly non-linear. For this reason we recommend to test these restrictions by testing the unpredictability of the variable ξ_t. This can be easily done by regressing leads of ξ_t onto the econometrician information set at time t-1. As it is shown in the appendix, these regressions contain autocorrelated error terms and therefore the regression tests have to be robust against that autocorrelation.

From Proposition 3, it is clear that, by testing the cross-equation restrictions of Proposition 2, the literature has only been testing the implications of the EPVM. Moreover, rejection of the standard cross-equation restrictions, given in Proposition 2, does not imply the rejection of the NEPVM. Take for instance, the case where dividends follow a random walk. If the EPVM (1) holds, stock prices must follow a random walk too and therefore the returns $(Y_t - Y_{t-1})$ will be uncorrelated. The introduction of the error term ϵ_t is one way to introduce serial correlation in the returns, something that is found in the empirical literature (see Fama (1991) for a review on

efficient capital markets). One of the purposes of this paper is to point out that we can reject the EPVM (1), because the returns are correlated, but still prices and dividends can follow a PVM (a form of market efficiency) that is compatible, for instance, with correlated returns.

Another consequence of Proposition 3 is that rejection of any of the above cross-equation restrictions does not invalidates the NEPVM with AR errors. This is a negative result that indicates that in this case, cointegration is the only testable econometric implication of the NEPVM. In this situation, from a practical point of view we recommend to analyze the autocorrelation structure of the variable ξ_t. It can be shown that $\xi_t = Y_t - E_{t-1}Y_t + \nu_t$ and therefore ξ_t has the same type of univariate model as the error terms ν_t or ϵ_t. Hence by analyzing this univariate structure we can get some knowledge about whether the cross-equation restrictions are not satisfied because ϵ_t follows an AR process or because some other reason.

Generalization of last proposition's results to an $ARMA(p,q)$ process for ϵ_t is straightforward. The case of fractional ϵ_t is under current investigation, and we expect equivalent conditions to those in proposition 3 (case 2) to hold.

4 Volatility Tests

The most common rejection of the PVM comes from the so-called volatility tests. These tests are designed to examine if the empirically observed volatility of the stock prices can be explained by the present discount value of dividends. In the following proposition we present the principal variance bound tests.

Proposition 4. (EPVM) *Let Y_t and X_t satisfy the present value relationship (1). Define Y_t^* as the "perfect foresight" or "ex-post rational price",*

$$Y_t^* = \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i X_{t+i},$$

and let Y_t^0 be some "naive forecast" of Y_t,

$$Y_t^0 = \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i F_t X_{t+i},$$

where $F_t X_{t+i}$ denotes a naive forecast of X_{t+i} made at time t. Rational agents at time t have

access to this naive forecast. Then,

1. *The variance of the ex-post rational price provides an upper bound to the variance of the observed Y_t,*

$$Var(Y_t^*) \geq Var(Y_t).$$

2. *The market price is a better forecast of the ex-post rational price in terms of the mean square error than the naive forecast price,*

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t^* - Y_t)^2.$$

3. *The ex-post rational price is more volatile around Y_t^0 than the market price.*

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t - Y_t^0)^2.$$

4. *Define S_t' as the unrestricted VAR forecast, given H_t, of the present value of all future changes in X_t, and ξ_t' as the innovation at time t in Y_t, given H_t. Then,*

$$Var(S_t)/Var(S_t') = 1,$$

and

$$Var(\xi_t)/Var(\xi_t') = 1.$$

The first statement of Proposition 4 was presented by Shiller (1981) in his seminal work on the volatility of stock markets. This statement is not valid when the variables are I(1). The second and third tests were developed by Mankiw, Romer and Shapiro (1985) and they do not suffer the I(1) problem of Shiller's test. The last statement of Proposition 4, presents two volatility tests under the VAR framework explained in the previous section. These tests are also valid with I(1) variables.

Proposition 5. (NEPVM) *Let Y_t and X_t satisfy the present value relationship (2). Define Y_t^* as the "perfect foresight" or "ex-post rational price" and let Y_t^0 be some "naive forecast" of Y_t as in Proposition 4. Then,*

1.
$$Var(Y_t^*) \geq Var(Y_t) \Longleftrightarrow Var(w_t) + Var(\epsilon_t) \geq 2Cov(Y_t, \epsilon_t),$$
where w_t is a forecast error equal to $Y_t^* - (Y_t - \epsilon_t)$.

2.
$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t^* - Y_t)^2 \Longleftrightarrow E(Y_t - Y_t^0)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)]$$

3.
$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t - Y_t^0)^2 \Longleftrightarrow E(Y_t^* - Y_t)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)].$$

4. *The variance ratio between the actual spread, S_t, and the theoretical spread, S_t', defined in the previous proposition, could be different from one. Also, the innovations variance ratio between ξ_t, and ξ_t', could be different from one.*

Proposition 5 shows the same tests as Proposition 4 but for the NEPVM. Again, the first statement is only valid when variables are stationary, while the last three tests are valid even when the variables are integrated of order one.

From the last proposition, it is clear that the variance bound tests which are valid for the exact PVM produce inconclusive results to reject the NEPVM. In other words, rejection of the EPVM by the standard volatility tests do not imply rejection of the NEPVM. This can be considered a negative result but it happens in other circumstances too. For instance in PVM with noise traders or with fads, the volatility tests also produce inconclusive results.

Implications of Proposition 5 do not change if the econometrician observes the perturbation term, $\mu_t = \epsilon_t$. If the econometrician does not have any information about it, $\mu_t = 0$, the only modification is that statement 4 in Proposition 5 is equivalent to the last statement in Proposition 4, that is, the variance ratios are equal to unity.

5 Applications

In this section we test the econometric implications of the present value model by applying the tests developed in this paper to two different cases: stock prices and dividends, and short- and long-term interest rates. Since, there is clear empirical evidence in the literature that the individual series are $I(1)$[1] these results are not reported here.

For each application, first we test for cointegration using Johansen's LR test, and second we test the cross-equation restrictions and volatility implications of the present value model (exact and non-exact).

In order to test the cross-equation restrictions $E_{t-1}\xi_{t+q+1} = 0$, the following strategy is used. First, we choose a maximum value for q and perform the test for that particular value. Second, if the cross-equation restrictions are not rejected, we keep performing the test for $q-1$ and so on, until a value of q is found, $q = k$, such that the cross-equation constraints are rejected. Then, the model selected will be a NEPVM with $MA(k)$ errors if $k \geq 0$ or an EPVM if $k = -1$. The reason to follow this strategy is that the cross-equation constraints are nested, in the sense that if they are satisfied for a given value of q they are also satisfied for a larger q. That is, if the restrictions obtained in Proposition 3, case 1, hold for an MA(q_1), then they will also hold for an MA(q_2), with $q_1 < q_2$.

For a given q, the cross-equation constraints derived in the paper can be tested, as it is shown in section 3, in two alternative ways: with a regression based test or with a VAR based test. We use the former because the constraints derived from the VAR model (8) are nonlinear and, as Gregory and Veall (1985) pointed out, in finite samples changing the form of a nonlinear restriction into a form which is algebraically equivalent under the null hypothesis, it will change the numerical value of the Wald test statistic. The regression based test consists on regressing ξ_{t+q+1} on information at $t-1$ and then testing that the coefficients of the variables reflecting this information are jointly zero. The errors from this regression are autocorrelated, therefore the test has to be robust against autocorrelation. For all the three cases, we use the correction suggested by Andrews (1991).

Two volatility tests, that are valid for I(1) variables, are conducted. We test $Var(S_t)/Var(S_t') = 1$ and $Var(\xi_t)/Var(\xi_t') = 1$. Standard errors and confidence intervals for these ratios are com-

[1] See Campbell and Shiller (1987a).

puted through bootstrapping (1000 samples). If volatility tests are rejected, we interpret this result as an implication against the EPVM model. If they are not rejected, the result is consistent with both type of present value models.

5.1 Stock Prices and Dividends

The data used in this application are the ones in Campbell and Shiller (1987a)[2]. The stock prices series is the Standard and Poor's composite index for January, divided by the January producer price index (1967=100), while the dividends series is a combination of series taken from Cowles (1939) up to 1925, and the dividends per share for the Standard and Poor's composite index from 1926 on. The whole sample goes from 1871 to 1985. In what follows, we use the dividends of *t-1* as a proxy for period t[3].

Figure 1 displays stock prices and dividends series.

FIGURE 1 ABOUT HERE

Table 1 shows Johansen's LR test results. The likelihood ratio test indicates the existence of cointegration at a 5% significance level.

TABLE 1 ABOUT HERE

Cross-equation restrictions are tested using a maximum value of $q = 4$. According to table 2 the EPVM is rejected, while the results are consistent with a NEPVM with an $MA(1)$ error term.

TABLE 2 ABOUT HERE

Table 3 checks the volatility conditions of both models showing mixed results for EPVM's implications. The estimated variance ratio between actual and theoretical spreads is 3.38 with a large standard error. The 90 % confidence interval rejects the hypothesis that the ratio equals unity. This fact goes against the EPVM. The estimated variance ratio between ξ_t and ξ'_t is 0.52, and the 90% confidence interval does not reject the hypothesis that this ratio equals unity. As proposition 5 states, both results are consistent with a NEPVM.

[2]We thank John Campbell for providing us with the data

[3]This approximation is usually made in the literature because stock prices are observed at the beginning of the period but dividends are paid some time during the same period.

TABLE 3 ABOUT HERE

Overall, the results reject the EPVM but are supportive of a NEPVM with MA(1) errors. This latter fact implies correlated market returns, something that does not necessarily go against the market efficiency hypothesis (see Fama, 1991).

5.2 Short- and Long-Term Interest Rates

We use the zero-coupon yield data set from McCulloch (1990), the same data Campbell and Shiller (1991) used to reject the exact version of the expectations hypothesis of the term structure of interest rates. The data are monthly and cover the period 1952:1 - 1991:2 of a 1-month yield and a 5-year yield.

Figure 2 displays the interest rate series.

FIGURE 2 ABOUT HERE

From the whole sample we have eliminated the period 1979-1982, that corresponds to the period where the Federal Reserve deviated from its usual practice of targeting interest rates and experimented using non-borrowed reserves as a new target instrument for monetary policy. This change of target produced a period of unprecedent interest rates volatility making very difficult to model interest rates with a linear model (see Gonzalez and Gonzalo (1999)). The subperiods analyzed go from January 1952 to July 1978 and from January 1983 to February 1991.

Johansen's cointegration test indicates cointegration at 5% significance level for both subperiods analyzed (Tables 4 and 7). For the subsample that goes from 1952 to 1978, the estimated cointegration vector is $(1, -0.92)$, which is not significantly different from $(1, -1)$ as the expectations theory of the term structure suggests.

TABLE 4 ABOUT HERE

Table 5 shows the cross-equation restrictions regression tests for the first subsample, for a maximum value of $q = 12$. These restrictions are satisfied for any value of q, indicating the non rejection of an EPVM. In other words we can not reject the EPVM implication of $E_{t-1}\xi_t = 0$[4].

[4]Following Campbell and Shiller (1987a) we computed ξ_t using a discount rate $\delta = \frac{1}{1+R}$ with R fixed at the mean of the five year rate over the corresponding sample period.

TABLE 5 ABOUT HERE

Table 6 presents the volatility tests results. The variance ratio between actual and theoretical spread is around one and the 90% confidence interval clearly non rejects the hypothesis that the ratio equals unity. The same occurs with the innovations ratio.

TABLE 6 ABOUT HERE

Overall, the results in this first subsample support the EPVM.

For the second period we re-computed the Johansen's cointegration test and the cross-equation restrictions tests using a maximum value of $q = 12$, as we did in the first subsample. These results are showed in Tables 7 and 8.

TABLE 7 ABOUT HERE

The estimated cointegration vector for the second period is $(1, -1.08)$, which is not significantly different from $(1, -1)$ as the expectations theory of the term structure suggests.

TABLE 8 ABOUT HERE

From table 8 is clear that the cross-equation restrictions tests reject the exact version of the PVM, but nevertheless the cross-equation implications of a NEPVM with $MA(1)$ or $MA(2)$ errors are not rejected.

Table 9 presents the volatility tests results. In both cases the variance ratios between actual and theoretical spread and, between ξ_t and ξ_t', are near 0.6. The 90% confidence intervals do not reject the hypothesis that both of them equal unity. This evidence supports the EPVM's volatility implications and at the same time it does not go against the NEPVM.

TABLE 9 ABOUT HERE

Overall, results in the second subsample reject the EPVM, but they can not reject the NEPVM with $MA(1)$ or $MA(2)$ errors (depending on the significant level we choose).

Summarizing, the analysis of these two sample periods seems to indicate, that it is more likely to reject the EPVM implications during periods of higher uncertainty or volatility like the second analyzed period. It is in this period where our NEPVM can have a room. Of course this empirical conjecture needs further future research.

6 Conclusion

The standard Present Value Model holds much theoretical attraction but it has been empirically rejected very often, as the literature on finance and macroeconomics has reported.

This paper shows that the version has been rejected in most cases is the *exact* version of the PVM, and that a very simple generalization of it, the NEPVM, while maintaining all the fundamental characteristics of the EPVM, has a different set of econometric implications. In fact, there are situations where cointegration is the only testable econometric implication from the non-exact PVM.

Further research should go in the direction of identifying the stochastic structure of ϵ_t. This is not an easy task as the literature on rational bubbles has already reported.

Appendix

PROOF: Proposition 1. Assuming ϵ_t is I(0), S_t will be I(0) too, since

$$S_t = Y_t - \theta X_t = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i} + \epsilon_t. \tag{13}$$

PROOF: Proposition 2. See Campbell and Shiller (1987a).

PROOF: Proposition 3.

Case 1. Assume, $\epsilon_t = \sum_{k=0}^{q} \alpha_k u_{t-k}$, $\alpha_0 = 1$. Define the variable,

$$\xi_t \equiv Y_t - \frac{1}{\delta}[Y_{t-1} - \theta(1-\delta)X_{t-1}]. \tag{14}$$

Replacing Y_{t-1} into (14) and rearranging we get,

$$\xi_t = Y_t - \theta(1-\delta)\sum_{i=0}^{\infty} \delta^i E_{t-1} X_{t+i} - \frac{1}{\delta}\epsilon_{t-1}, \tag{15}$$

or,

$$\xi_t = Y_t - E_{t-1}Y_t + \sum_{k=1}^{q} \alpha_k u_{t-k} - \frac{1}{\delta}\epsilon_{t-1}. \tag{16}$$

Writing (16) for period *t+q+1* and taking expectations conditional on information available at time *t-1*, it is clear that $E_{t-1}\xi_{t+1+q} = 0$. To show that this implication can be tested by using a regression of $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$ on information at time *t-1*, consider the following expression,

$$\begin{aligned}
\xi_{t+q+1} &= S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1} \\
&= \theta\sum_{i=1}^{\infty}\delta^i E_{t+q+1}\Delta X_{t+i+q+1} + \epsilon_{t+q+1} - \frac{1}{\delta}(\theta\sum_{i=1}^{\infty}\delta^i E_{t+q}\Delta X_{t+i+q} + \epsilon_{t+q}) + \theta\Delta X_{t+q+1} \\
&= Y_{t+q+1} - \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i E_{t+q}X_{t+i+q+1} - \frac{1}{\delta}\epsilon_{t+q} \\
&= Y_{t+q+1} - E_{t+q}Y_{t+q+1} + \sum_{k=1}^{q}\alpha_k u_{t-k+q+1} - \frac{1}{\delta}\epsilon_{t+q}.
\end{aligned} \tag{17}$$

Therefore the hypothesis $E_{t-1}\xi_{t+q+1} = 0$ can be tested by regressing $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$ on information available at time *t-1*, and then testing the hypothesis that the coefficients on the variables reflecting information at *t-1* are jointly equal to zero.

Alternatively, one can construct cross-equation restrictions from a VAR model for the spread and the change in X_t. To see this, subtract θX_t from both sides of (2), write the resulting expression for period *t+q+1* and take expectations conditional on information at time t,

$$E_t S_{t+q+1} = E_t[\theta \sum_{i=1}^{\infty} \delta^i E_{t+q+1}\Delta X_{t+i+q+1} + \epsilon_{t+q+1}]. \tag{18}$$

Rearranging equation (18),

$$E_t S_{t+q+1} = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i+q+1}, \tag{19}$$

and projecting both sides of (19) onto H_t, we get

$$g' A^{q+1} = \theta \sum_{i=1}^{\infty} \delta^i h' A^{i+q+1} = \theta\delta h'(I - \delta A)^{-1} A^{q+2}. \tag{20}$$

We can rearrange (20) so that the cross-equation constraints are given by,

$$(\theta\delta h' A - g'(I - \delta A))A^{q+1} = 0. \tag{21}$$

To interpret these restrictions, write the VAR model (10) for period *t+q+1*. Multiplying the left hand side of (10) by $(\theta, 1 - \frac{1}{\delta}L)$ and writing the result in terms of (z_t, v_t), we have

$$\begin{aligned} g' z_{t+q+1} + \theta h' z_{t+q+1} - \frac{1}{\delta} g' z_{t+q} &= (g' A + \theta h' A - \frac{1}{\delta} g') z_{t+q} + (g' + \theta h') v_{t+q+1} \\ &= \frac{1}{\delta}(\theta\delta h' A - g'(I - \delta A)) A^{q+1} z_{t-1} \\ &+ \frac{1}{\delta}(\theta\delta h' A - g'(I - \delta A)) \sum_{j=0}^{q} A^j v_{t+q-j} \\ &+ (g' + \theta h') v_{t+q+1}. \end{aligned} \tag{22}$$

Notice that the left hand side is equal to ξ_{t+q+1}. Then, in order to get $E_{t-1}\xi_{t+q+1} = 0$ we need that $(\theta\delta h' A - g'(I - \delta A))A^{q+1} = 0$. These are the cross-equation constraints given by (21).

Therefore, testing $E_{t-1}\xi_{t+q+1} = 0$ by imposing the restrictions given by (21) in a VAR model for ΔX_t and S_t is equivalent to regressing $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$ on the information available at time *t-1*, and testing the null hypothesis that the coefficients of the variables reflecting information at time *t-1* are jointly equal to zero.

Case 2. Assume $\epsilon_t = \sum_{k=1}^{p} \rho_k \epsilon_{t-k} + u_t$. From (15) we have,

$$\xi_t = Y_t - \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i E_{t-1}X_{t+i} - \frac{1}{\delta}\epsilon_{t-1} = Y_t - E_{t-1}Y_t + \sum_{k=1}^{p}\rho_k\epsilon_{t-k} - \frac{1}{\delta}\epsilon_{t-1}. \tag{23}$$

From (23) it is clear that $E_{t-1}\xi_{t+j} \neq 0$ (j=0, 1, 2, ..., J), since $E_{t-1}\epsilon_{t+j} \neq 0$. In this case ξ_{t+j} (for a finite j) is predictable using information at time *t-1*. Therefore, it is not possible to get cross-equation restrictions from a VAR model for ΔX_t and S_t due to the presence of the autoregressive error term. The same result is obtained from,

$$E_t S_{t+j} = \theta\sum_{i=1}^{\infty}\delta^i E_t\Delta X_{t+i+j} + E_t\epsilon_{t+j}. \tag{24}$$

Since $E_t\epsilon_{t+j} \neq 0 \ \ \forall(finite)j$ in (24), we cannot derive any of the standard cross-equation restrictions.

PROOF: Proposition 4. For Statement 1, see Shiller (1981). For Statements 2 and 3, see Mankiw, Romer and Shapiro (1985). For Statement 4, see Campbell and Shiller (1987a).

PROOF: Proposition 5.

Statement 1. From the NEPVM we have

$$Y_t = E_t Y_t^* + \epsilon_t. \tag{25}$$

Then

$$Y_t^* = (Y_t - \epsilon_t) + w_t, \tag{26}$$

where w_t is a forecast error uncorrelated with information available at time *t*. From (26),

$$Var(Y_t^*) = Var(Y_t) + Var(\epsilon_t) + Var(w_t) - 2Cov(Y_t, \epsilon_t). \tag{27}$$

Therefore, it follows from (27) that

$$Var(Y_t^*) \geq Var(Y_t) \Longleftrightarrow Var(w_t) + Var(\epsilon_t) \geq 2Cov(Y_t, \epsilon_t). \tag{28}$$

Statements 2 and 3. Consider the following identity,

$$Y_t^* - Y_t^0 = (Y_t^* - Y_t) + (Y_t - Y_t^0), \tag{29}$$

and notice that $Y_t^* - Y_t = w_t - \epsilon_t$. Then

$$E_t[(Y_t^* - Y_t)(Y_t - Y_t^0)] = E_t[(w_t - \epsilon_t)(Y_t - Y_t^0)]. \tag{30}$$

Since $E_t[\epsilon_t(Y_t - Y_t^0)] \neq 0$ expression (30) is different from zero. Therefore,

$$E_t(Y_t^* - Y_t^0)^2 = E_t(Y_t^* - Y_t)^2 + E_t(Y_t - Y_t^0)^2 - 2E_t[\epsilon_t(Y_t - Y_t^0)]. \tag{31}$$

So

$$E_t(Y_t^* - Y_t^0)^2 \geq E_t(Y_t^* - Y_t)^2, \tag{32}$$

only if $E_t(Y_t - Y_t^0)^2 \geq 2E_t[\epsilon_t(Y_t - Y_t^0)]$.
Similarly, if $E_t(Y_t^* - Y_t)^2 \geq 2E_t[\epsilon_t(Y_t - Y_t^0)]$, expression (31) implies

$$E_t(Y_t^* - Y_t^0)^2 \geq E_t(Y_t - Y_t^0)^2. \tag{33}$$

Finally, the law of iterated projections allows us to replace expectations conditional on information available at time t with expectations conditional on information available prior to the beginning of the sample period. That is, letting E denote the expectation conditional on the initial conditions, we have

$$E(Y_t^* - Y_t^0)^2 = E(Y_t^* - Y_t)^2 + E(Y_t - Y_t^0)^2 - 2E[\epsilon_t(Y_t - Y_t^0)]. \tag{34}$$

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t^* - Y_t)^2 \Longleftrightarrow E(Y_t - Y_t^0)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)]. \tag{35}$$

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t - Y_t^0)^2 \Longleftrightarrow E(Y_t^* - Y_t)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)]. \tag{36}$$

Statement 4. Consider the non-exact present value model (2), adding and subtracting θX_t we get,

$$S_t = Y_t - \theta X_t = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i} + \epsilon_t. \tag{37}$$

Now, define the "theoretical spread", S'_t, as the optimal forecast, given the econometrician information set (H_t) of the present value of all future changes in X_t,

$$S'_t = \theta \sum_{i=1}^{\infty} \delta^i E\left(\Delta X_{t+i} | H_t\right). \tag{38}$$

Subtracting (38) from (37),

$$\begin{aligned} S_t - S'_t &= \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i} + \epsilon_t - \theta \sum_{i=1}^{\infty} \delta^i E\left(\Delta X_{t+i} | H_t\right) \\ &= \theta \sum_{i=1}^{\infty} \delta^i E_t X_{t+i} - \theta \sum_{i=1}^{\infty} \delta^i E_t X_{t+i-1} + \epsilon_t - \theta \sum_{i=1}^{\infty} \delta^i E\left(\Delta X_{t+i} | H_t\right). \end{aligned} \tag{39}$$

Adding and subtracting θX_t we get,

$$S_t - S'_t = \theta(1-\delta) \textstyle\sum_{i=0}^{\infty} \delta^i E_t X_{t+i} + \epsilon_t - \theta(1-\delta) \sum_{i=0}^{\infty} \delta^i E\left(X_{t+i} | H_t\right).$$

Projecting both sides of equation (2) onto H_t we have,

$$E(Y_t | H_t) = Y_t = \theta(1-\delta) \sum_{i=0}^{\infty} \delta^i E\left(X_{t+i} | H_t\right) + \mu_t, \tag{40}$$

where $\mu_t = E(\epsilon_t | H_t)$. Then,

$$S_t - S'_t = Y_t - E\left(Y_t | H_t\right) + \mu_t = \mu_t.$$

Therefore,

$$Var(S_t) = Var(S'_t) + Var(\mu_t) + 2Cov(S'_t, \mu_t). \tag{41}$$

From (41) it is clear that the volatility test given by $Var(S_t)/Var(S'_t) = 1$ is not an implication of the NEPVM.

For the second volatility test we have,

$$\begin{aligned}\xi_t = S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t &= \theta\sum_{i=1}^{\infty}\delta^i E_t\Delta X_{t+i} + \epsilon_t - \frac{1}{\delta}(\theta\sum_{i=1}^{\infty}\delta^i E_{t-1}\Delta X_{t+i-1} + \epsilon_{t-1}) + \theta\Delta X_t \\ &= Y_t - \theta\sum_{i=0}^{\infty}\delta^i E_{t-1}\Delta X_{t+i} + \theta\sum_{i=2}^{\infty}\delta^{i-1}E_{t-1}\Delta X_{t+i-2} - \frac{1}{\delta}\epsilon_{t-1} \\ &= Y_t - \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i E_{t-1}\Delta X_{t+i} - \frac{1}{\delta}\epsilon_{t-1}. \qquad (42)\end{aligned}$$

From last expression and assuming, without any loss of generality, that ϵ_t follows an AR(p) process like in Proposition 3 (case 2), we obtain

$$S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t = Y_t - E_{t-1}Y_t + \sum_{k=1}^{p}\rho_k\epsilon_{t-k} - \frac{1}{\delta}\epsilon_{t-1}. \qquad (43)$$

Now, consider

$$\begin{aligned}\xi'_t = S'_t - \frac{1}{\delta}S'_{t-1} + \theta\Delta X_t &= \theta\sum_{i=1}^{\infty}\delta^i E(\Delta X_{t+i}|H_t) + \theta\Delta X_t - \frac{1}{\delta}\theta\sum_{i=1}^{\infty}\delta^i E(\Delta X_{t+i-1}|H_{t-1}) \\ &= Y_t - E(Y_t|H_{t-1}) + E(\epsilon_t|H_{t-1}) - \mu_t. \qquad (44)\end{aligned}$$

Then

$$\xi_t - \xi'_t = E(Y_t|H_{t-1}) - E_{t-1}Y_t + \mu_t - E(\epsilon_t|H_{t-1}) + \sum_{k=1}^{p}\rho_k\epsilon_{t-k} - \frac{1}{\delta}\epsilon_{t-1}.$$

Defining $\phi_t = \xi_t - \xi'_t$,

$$Var(\xi_t) = Var(\xi'_t) + Var(\phi_t) + 2Cov(\xi'_t, \phi_t). \qquad (45)$$

Again, it is clear from equation (45) that the second volatility test given by $Var(\xi_t)/Var(\xi'_t) = 1$ is not an implication of the NEPVM.

Table 1: Stock Prices and Dividends

Johansen's Cointegration Test		
Test Assumption: Linear deterministic trend in the data		
	Likelihood	5 Percent
Eigenvalue	Ratio	Critical value
0.118260	16.18557	15.41
0.018483	2.089499	3.76

Note: Johansen's LR testing of *r=1 vs r=2* and *r=0 vs r=2* was performed on the following model,
$\boldsymbol{W_t} = C + \Pi \boldsymbol{Z_{t-1}} + \Gamma \boldsymbol{W_{t-1}} + \boldsymbol{\eta_t}$, where $\boldsymbol{W_t} = (\Delta Y_t, \Delta X_t)'$, and $\boldsymbol{Z_{t-1}} = (Y_{t-1}, X_{t-1})'$.
The number of lags has been selected by the AIC criterion.

Table 2: Cross-Equation Constraints Regression Tests
Stock Prices and Dividends

Dependent Variable	ξ_t	ξ_{t+1}	ξ_{t+2}	ξ_{t+3}	ξ_{t+4}	ξ_{t+5}
F-statistic	2.297	4.770	0.269	1.057	0.718	0.722
p-Value	0.065	0.001	0.897	0.382	0.582	0.579

Note: Test procedures for NEPVM implications are robust to both heteroskedasticity and autocorrelation. The dependent variable is regressed in all cases on information available at time *t-1*.

Table 3: Volatility Tests
Stock Prices and Dividends

$Var(S_t)/Var(S_t')$	Standard Deviation	90% Confidence Interval	$Var(\xi_t)/Var(\xi_t')$	Standard Deviation	90% Confidence Interval
3.381	4.264	(1.53, 13.12)	0.527	0.332	(0.36, 1.27)

Note: Standard deviations and confidence intervals were obtained using 1000 bootstrapping samples.

Table 4: Short- and Long-Term Interest Rates.
Sample Period: 1952:01 - 1978:07

Johansen's Cointegration Test		
Test Assumption: Linear deterministic trend in the data		
	Likelihood	5 Percent
Eigenvalue	Ratio	Critical value
0.083149	27.18082	15.41
0.000307	0.095855	3.76

Note: Johansen's LR testing of *r=1 vs r=2* and *r=0 vs r=2*
was performed on the following model,
$\boldsymbol{W_t} = C + \Pi \boldsymbol{Z_{t-1}} + \sum_{j=1}^{6} \Gamma_j \boldsymbol{W_{t-j}} + \boldsymbol{\eta_t}$, where $\boldsymbol{W_t} = (\Delta Y_t, \Delta X_t)'$,
and $\boldsymbol{Z_{t-1}} = (Y_{t-1}, X_{t-1})'$.
The number of lags has been selected by the AIC criterion.

Table 5: Cross-Equation Constraints Regression Tests
Short- and Long-Term Interest Rates. Sample Period: 1952:01 - 1978:07

Dependent Variable	ξ_t	ξ_{t+1}	ξ_{t+2}	ξ_{t+3}	ξ_{t+4}	ξ_{t+11}	ξ_{t+12}
F-statistic	1.487	0.768	0.881	1.250	1.338	0.796	0.852
p-Value	0.128	0.683	0.567	0.249	0.196	0.655	0.596

Note: Test procedures for NEPVM are robust to both heteroskedasticity and autocorrelation. The dependent variable is regressed in all cases on information available at time *t-1*.

Table 6: Volatility Tests
Short- and Long-Term Interest Rates. Sample Period: 1952:01 - 1978:07

$Var(S_t)/Var(S_t')$	Standard Deviation	90% Confidence Interval	$Var(\xi_t)/Var(\xi_t')$	Standard Deviation	90% Confidence Interval
1.849	0.742	(0.395, 3.303)	0.589	0.540	(0.050, 1.129)

Note: Standard deviations and confidence intervals were obtained using 1000 bootstrapping samples.

Table 7: Short- and Long-Term Interest Rates.
Sample Period: 1983:01 - 1991:02

Johansen's Cointegration Test		
Test Assumption: No deterministic trend in the data		
	Likelihood	5 Percent
Eigenvalue	Ratio	Critical value
0.136166	14.35813	12.53
0.000136	0.013366	3.84

Note: Johansen's LR testing of *r=1 vs r=2* and *r=0 vs r=2*
was performed on the following model,
$\boldsymbol{W_t} = C + \Pi \boldsymbol{Z_{t-1}} + \sum_{j=1}^{1} \Gamma_j \boldsymbol{W_{t-j}} + \boldsymbol{\eta_t}$, where $\boldsymbol{W_t} = (\Delta Y_t, \Delta X_t)'$,
and $\boldsymbol{Z_{t-1}} = (Y_{t-1}, X_{t-1})'$.
The number of lags has been selected by the AIC criterion.

Table 8: Cross-Equation Constraints Regression Tests
Short- and Long-Term Interest Rates. Sample Period: 1983:01 - 1991:02

Dependent Variable	ξ_t	ξ_{t+1}	ξ_{t+2}	ξ_{t+3}	ξ_{t+4}	ξ_{t+11}	ξ_{t+12}
F-statistic	2.293	2.158	1.476	1.376	1.425	0.832	0.713
p-Value	0.017	0.025	0.078	0.201	0.178	0.618	0.733

Note: Test procedures for NEPVM are robust to both heteroskedasticity and autocorrelation. The dependent variable is regressed in all cases on information available at time *t-1*.

Table 9: Volatility Tests
Short- and Long-Term Interest Rates. Sample Period: 1983:01 - 1991:02

$Var(S_t)/Var(S_t')$	Standard Deviation	90% Confidence Interval	$Var(\xi_t)/Var(\xi_t')$	Standard Deviation	90% Confidence Interval
0.666	0.265	(0.146, 1.186)	0.627	0.293	(0.053, 1.200)

Note: Standard deviations and confidence intervals were obtained using 1000 bootstrapping samples.

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Figure 1
Stock Prices and Dividend Series

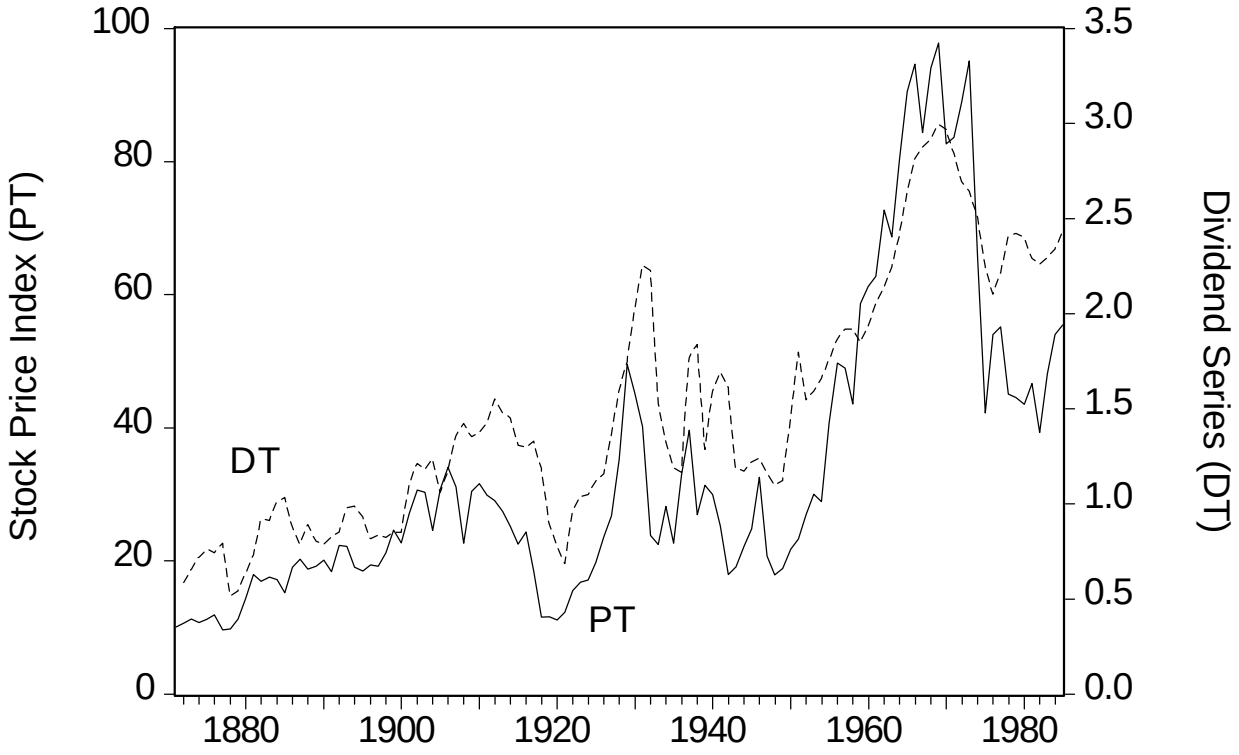

Figure 2
Monthly Interest Rates (1-month and 5-year yields)