Public Goods: An Example

Public Goods and Externalities: An Example

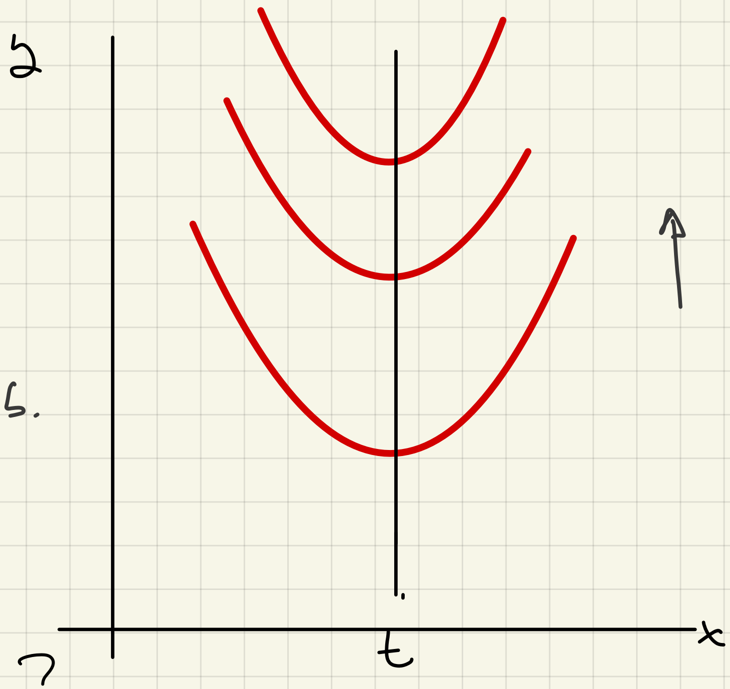
Ann and Bob share an apartment. Central heating is provided free of charge.

Their preferences for room temperature (x) and income (y) are represented by utility functions

$$u_i(x, y) = y - \alpha_i (t_i - x)^2,$$

where $t_A = 25$, $\alpha_A = 3/2$

$t_B = 20$, $\alpha_B = 1$.



Thus, the apartment's temperature is a public good to Ann and Bob.

At which temperature should be set the apartment's thermostat?

Let us try and identify "good outcomes". To begin, let us set the temperature to some intermediate value $t \in [t_B, t_A]$, e.g., $x = 21°$.

Is this temperature Pareto optimal?

Since

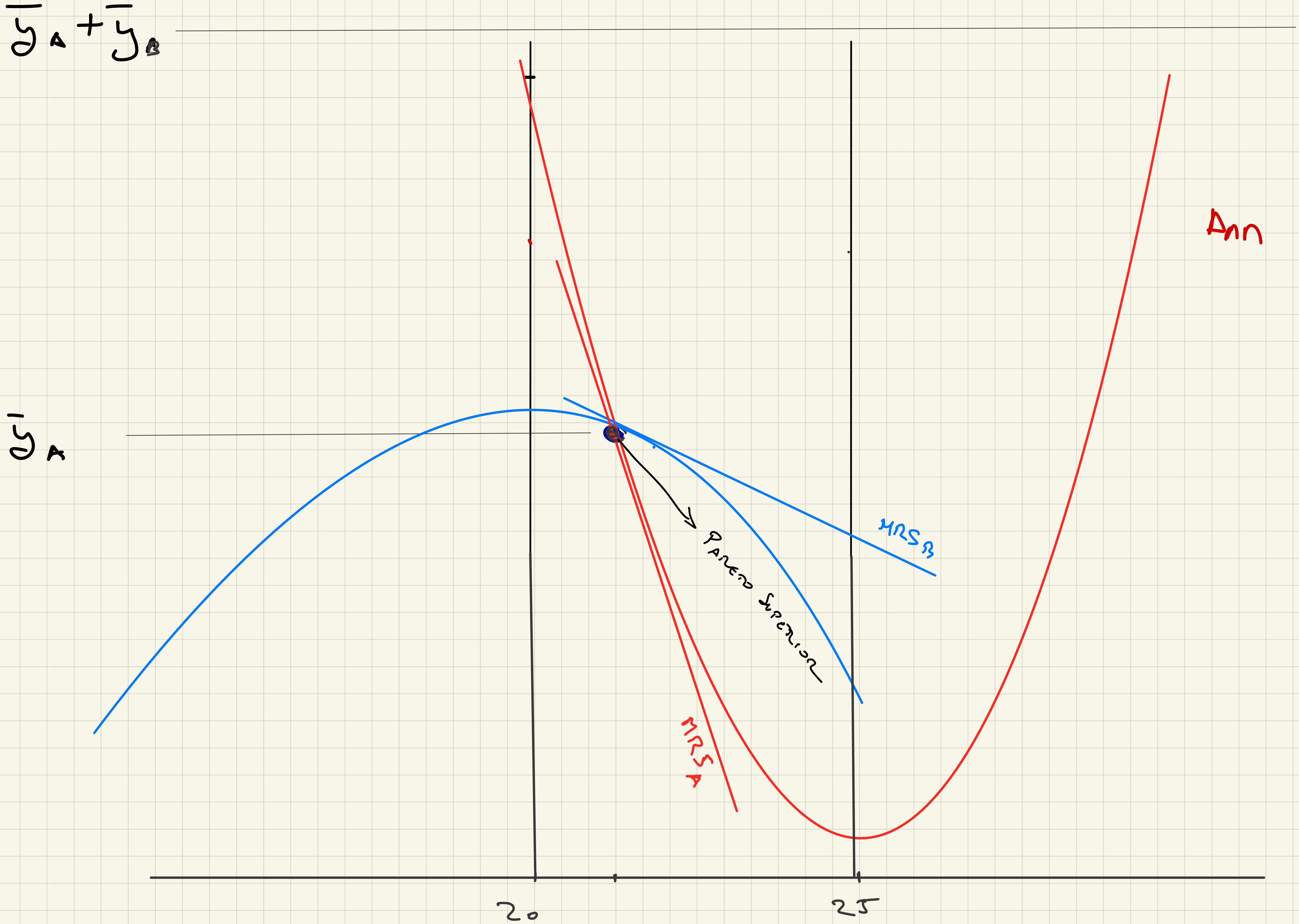
$$MRS_i(x, y) = 2\alpha_i(t_i - x),$$

we have

$$MRS_A(21, y) = 75 - 3(21) = 12$$

$$MRS_B(21, y) = 40 - 2(21) = -2.$$

How to interprete these numbers?



$\bar{y}_A + \bar{y}_B$
$\bar{y}_A$
Ann
MRS B
Pareto superior
MRS A
20
25

Ann proposes to increase the temperature by 1°, and offers Bob 5 euros as compensation

Would Bob accept ?

$$u_B(\bar{y}_B, 21) = \bar{y}_B - (20-21)^2 = \bar{y}_B - 1.$$

$$u_B(\bar{y}_B + 5, \underline{22}) = \bar{y}_B + 5 - (20-22)^2 = \bar{y}_B + 1.$$

Ann?

$$u_A(\bar{y}_A, 21) = \bar{y}_A - \frac{3}{2}(25-21)^2 = \bar{y}_A - 24.$$

$$u_A(\bar{y}_A - 5, 22) = \bar{y}_A - 5 - \frac{3}{2}(25-22)^2 = \bar{y}_A - 18.5$$

Both are better off !

Note $$MRS_A(21) + MRS_B(21) = 12 - 2 = 10 > 0$$

They should raise the temperature so long as

$$MRS_A(x) + MRS_B(x) > 0$$

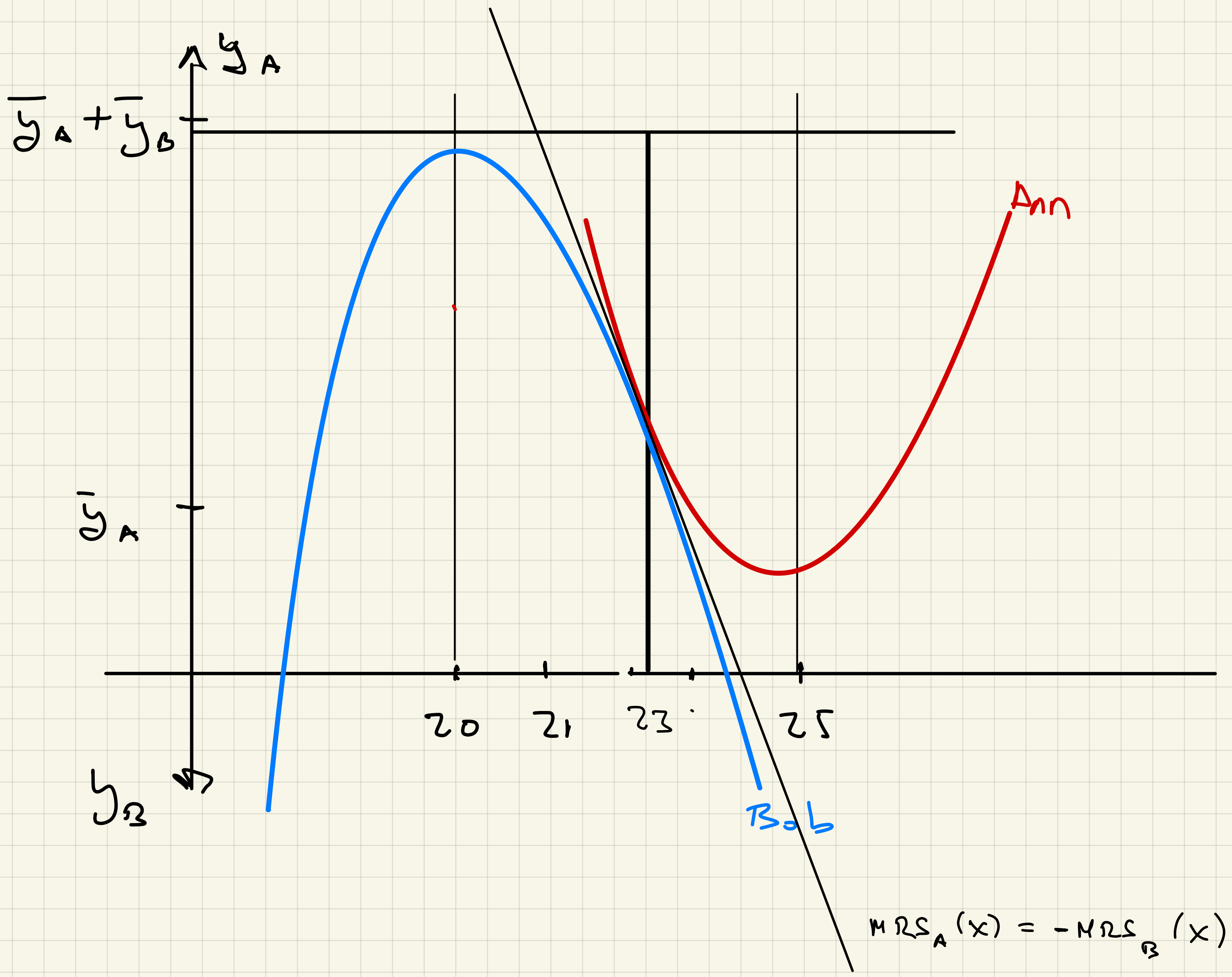
Likewise, if $MRS_A(x) + MRS(B) < 0$, they can both improve by reducing the temperature (and accord some compensations).

<u>PO</u> requires:

$$MRS_A(x) + MRS_B(x) = 0$$

$$75 - 3x + 40 - 2x = 0$$

i.e., $$x^* = \frac{115}{5} = \underline{23^\circ}$$



y_A
$\bar{y}_A + \bar{y}_B$
$\bar{y}_A$
y_B
20
21
23
25
Ann
Bob
$MRS_A(x) = -MRS_B(x)$

... but how is the apartment's temperature decided ?

And how to solve this problem ?

BARGAINING

Set $x^* = 23°$. Bargain over the gains to be have.

Assume Ann owns the apartment and has the right to set the temperature.

Absent interactions:

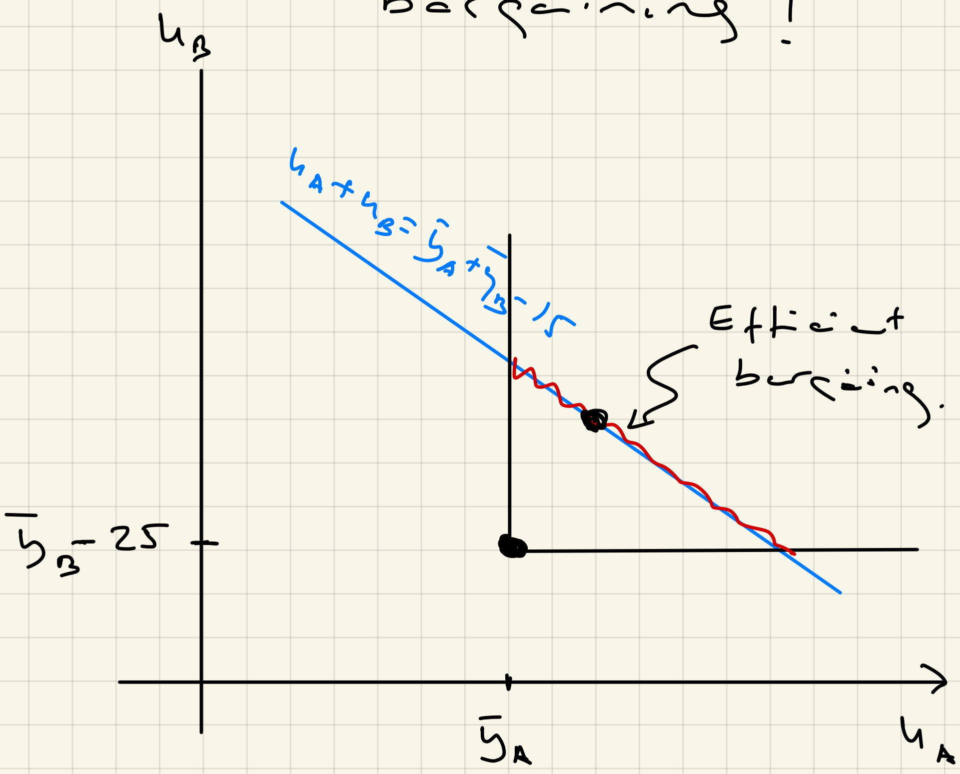
$\bar{x} = 25°$; $\bar{u}_A = \bar{y}_A$, $\bar{u}_B = \bar{y}_B - 25$.

Sadly, for $x = 23$,

$$u_A + u_B = \bar{y}_A - \frac{3}{2}(25-23)^2 + \bar{y}_B - (20-23)^2$$

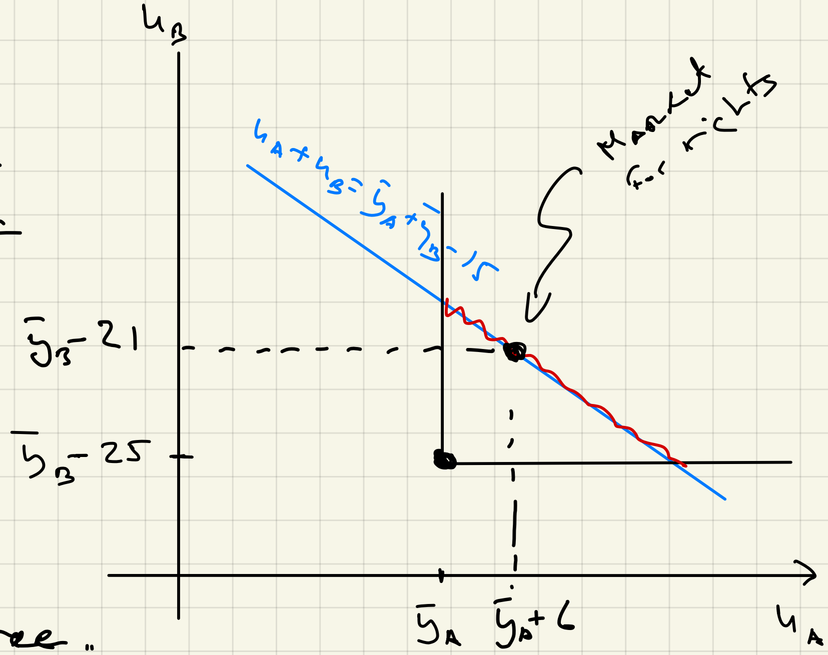
$$= \bar{y}_A + \bar{y}_B - 15 > \bar{y}_A + \bar{y}_B - 25 = \bar{u}_A + \bar{u}_B.$$

Perhaps a compromise may be reached by bargaining !



A Market Solution

Assume instead that starting from Ann's ideal temperature (25°), a market is created whereby Ann and Bob may supply (demand) "rights" to lower the temperature.



p : price of lowering the temperature 1 degree.

Ann $\quad \max\limits_{r} \; \bar{y}_A + pr - \frac{3}{2} r^2 \implies r_A(p) = \frac{p}{3}$

Bob $\quad \max\limits_{r} \; \bar{y}_B - pr - (20-25-r)^2 \implies r_B(p) = 5 - \frac{p}{2}$

Market Clearing. $\frac{p}{3} = 5 - \frac{p}{2} \iff p^* = 6 \; ; \; r^* = 2.$

Market Outcome: $x^* = 23°$; $y_A^* = \bar{y}_A + 12$, $u_A^* = \bar{y}_A + 6$.

$y_B^* = \bar{y}_B - 12$, $u_B^* = \bar{y}_B - 21$.

Exercises:

1. If there is another apartment's resident, Conrad, whose preferences parameters are $\alpha_c = 1$, $t_c = 22$, what is the apartment's optimal temperature?

2. If the cost of maintaining the temperature at x degrees is $C(x) = 2x$, what is the apartment's optimal temperature?

3. What would be the temperature if Ann, Bob and Conrad vote, and the thermostat is set at the median temperature?

Restaurant Example (Exercise 2.7)

① Separate Checks

$$\max_{x \geq 0} \quad \bar{y}_i - px + \ln x$$

F.O.C. $\quad -p + \frac{1}{x} = 0 \implies \boxed{x_i(p) = \frac{1}{p}}, \quad \boxed{n\, x_i(p) = \frac{n}{p}}$

② Common Equally Divided Check.

$$\max_{x_i} \quad \bar{y}_i - \frac{p}{n}\Big(\sum_{j \neq i} x_j + x_i\Big) + \ln x_i$$

$$-\frac{p}{n} + \frac{1}{x_i} = 0 \implies \boxed{\tilde{x}_i(p) = \frac{n}{p}.} \quad \boxed{n\,\tilde{x}_i(p) = \frac{n^2}{p}}$$

Yes, more food is consumed under this scheme!