Midterm (Micro II, uc3m-Masters in Economics, March 19, 2024)

Exercise 1. In an economy that extends over two dates, today and tomorrow, there is a single perishable good, consumption. There is uncertainty about whether tomorrow the economy will be booming (B) or in a recession (R). There are two consumers with identical preferences for consumption today (x), consumption tomorrow if B (y) and consumption tomorrow if R (z), represented by the utility function $u(x, y, z) = x + \ln y + \ln z$. Consumers' endowments are $(\bar{x}_1, \bar{y}_1, \bar{z}_1) = (2, 2, 2)$ and $(\bar{x}_2, \bar{y}_2, \bar{z}_2) = (2, 4, 1)$, respectively.

(a) (20 points) Calculate the Arrow-Debreu competitive equilibrium prices and allocation. (Set $p_x = 1$.)

(b) (20 points) Assume that there are no contingent markets, but there are markets for credit and for a security operating today. The security pays 3 units of consumption when the economy is booming and nothing when it is in a recession. Calculate the CE security price q^* and interest rate r^* and how many units each consumer borrows/lends and buys/sells of the security. (Hint. You need not recalculate the CE. Instead, consolidate a consumer's budget constraints into a single constraint involving consumption goods, and use your results in part (a).)

Exercise 2. A competitive market provides insurance to a population of individuals with preferences represented by the Bernoulli utility function $u(x) = \ln x$, where x is the individual's disposable income. Everyone has an initial wealth $W = 5$ euros and faces the risk of a monetary lose $L = 4$ euros. For a fraction $\lambda \in (0, 1)$ of the individuals the probability of losing L is $p_h = 1/2$, whereas for the remaining fraction $1 - \lambda$ this probability is $p_l = 1/4$. This information is common knowledge to all market participants. Insurance providers ignore the type of an individual demanding insurance.

(a) (20 points) Identify the policies that will be offered in a competitive equilibrium and determine the values of λ for which such an equilibrium exists.

(b) (10 points) Assume that the government puts to a referendum a law making it mandatory for all individuals to have full insurance. For which values of λ will it be approved, and which individuals would be better off under this law?

Exercise 3. Robinson and Friday are the only inhabitants of an island. There is no food in the island, but it can be produced using labor (l) as input according to the production function $f(l) = 2\sqrt{l}$. Robinson is the owner of this technologyand manages it with the objective of maximizing his food consumption. Friday has no resources other than his time, and his preferences over food consumption (c) and labor (the counterpart of leisure) are represented by a utility function $u(l, c) = c - l^2/2$. Robinson supplies no labor – he is too busy watching over the sea to try to sight any ship that may navigate near the island. Set the price of consumption to 1 and denote by w the (real) wage.

(a) (15 points) Assuming that Robinson acts as a wage-taker, calculate the economy's *competitive* equilibrium wage and allocation.

(b) (15 points) Assuming that Robinson considers the impact of his labor decision on the wage, i.e., he acts as a monopsonist, calculate the *monopolistic* equilibrium wage and allocation. Is this allocation Pareto optimal? (To answer this question you must either proof that it is Pareto optimal or find a Pareto superior allocation.)

Solutions

Exercise 1. (a) Clearly both consumers will demand positive amounts of y and z. Since $RMS^i_{xy} = y$ and $RMS^i_{xz} = z$, consumer $i \in \{1,2\}$ demands of these goods are consumer $i \in \{1,2\}$ are

$$y_i(p_y, p_z) = \frac{1}{p_y},\ z_i(p_y, p_z) = \frac{1}{p_z}.$$

Hence market clearing requires

$$\begin{aligned} y_1(p_y,p_z) + y_2(p_y,p_z) &= \bar{y}_1 + \bar{y}_2 \Leftrightarrow \frac{2}{p_y} = 2 + 4 \\ z_1(p_y,p_z) + z_2(p_y,p_z) &= \bar{z}_1 + \bar{z}_2 \Leftrightarrow \frac{2}{p_z} = 2 + 1. \end{aligned}$$

Thus, $(p^*_y, p^*_z) = (1/3, 2/3)$. The Arrow-Debreu CE allocation is $y^*_i = y_i(1/3, 2/3) = 3$, $z^*_i = z_i(1/3, 2/3) = 3/2$, for $i \in \{1,2\}$ and

$$\begin{aligned} x^*_1 &= \bar{x}_1 - p^*_y(y^*_1 - \bar{y}_1) + p^*_z(z^*_1 - \bar{z}_1) = 2 + \frac{1}{3}(2-3) + \frac{2}{3}(2 - \frac{3}{2}) = 2 \\ x^*_2 &= \bar{x}_2 - p^*_y(y^*_2 - \bar{y}_2) + p^*_z(z^*_2 - \bar{z}_2) = 2 + \frac{1}{3}(4-3) + \frac{2}{3}(1 - \frac{3}{2}) = 2. \end{aligned}$$

(b) Consumer i's budget constraints in this economy are

$$\begin{aligned} x_i &= \bar{x}_i + b - qw \\ y_i &= \bar{y}_i + 3w - (1+r)b \\ z_i &= \bar{z}_i - (1+r)b. \end{aligned}$$

Solving for b and w we may write the "consolidated" budget constraint as

$$x_i + \frac{q}{3}y_i + \left(\frac{1}{1+r} - \frac{q}{3}\right) z_i = \bar{x}_i + \frac{q}{3}\bar{y}_i + \left(\frac{1}{1+r} - \frac{q}{3}\right) \bar{z}_i.$$

The equilibrium interest rate and security price can be obtained by solving the system

$$\frac{q}{3} = p^*_y,\ \frac{1}{1+r} - \frac{q}{3} = p^*_z.$$

i.e., $(r^*, q^*) = (0, 1)$. Of course, the equilibrium allocation is the Arrow-Debreu CE allocation calculated in (a). Hence,

$$x_i = \bar{x}_i + b^*_i - q^* w^*_i \Leftrightarrow 2 = 2 + b_i - (1)\, w_i \Leftrightarrow b^*_i = w^*_i$$

and

$$\begin{aligned} z_1 &= \bar{z}_1 - (1+r^*)b^*_1 \Leftrightarrow \frac{3}{2} = 2 - b^*_1 \Leftrightarrow b^*_1 = w^*_1 = \frac{1}{2}, \\ z_2 &= \bar{z}_2 - (1+r^*)b^*_2 \Leftrightarrow \frac{3}{2} = 1 - b^*_2 \Leftrightarrow b^*_2 = w^*_2 = -\frac{1}{2}. \end{aligned}$$

Exercise 2. (a) As seen in class a competitive equilibrium, when it exists, is separating. In a separating equilibrium high risk agents get full insurance, that is $(I_H, D_H) = (p_H L, 0) = (2, 0)$. Hence the expected utility for a high risk individual is

$$u(W - p_H L) = \ln(W - p_H L) = \ln\left(5 - \left(\frac{1}{2}\right)4\right) = \ln 3$$

Low risk individuals get partial insurance $(I_L, D_L) = (p_L(L - D), D)$, where D must leave the high risk individuals indifferent between the policy (I_H, D_H) and the policy (I_L, D_L), i.e.,

$$p_H u(W - p_L(L - D) - D) + (1 - p_H)\, u(W - p_L(L - D)) = u(W - p_H L)$$

Substituting, this equation becomes

$$\frac{1}{2}\ln\left(5 - \frac{1}{4}(4 - D) - D\right) + \frac{1}{2}\ln(5 - \frac{1}{4}(4 - D)) = \ln 3.$$

Solving this equation we get

$$D_L = \frac{4}{3}(\sqrt{37} - 4).$$

Substituting these values we calculate the expected utility of the low risk type with this policy,

$$\begin{aligned} u_L^* &= \frac{1}{4}\ln\left(5 - \frac{1}{4}(4 - \frac{4}{3}(\sqrt{37} - 4)) - \frac{4}{3}(\sqrt{37} - 4)\right) \\ &\quad + (1 - \frac{1}{4})\ln\left(5 - \frac{1}{4}(4 - \frac{4}{3}(\sqrt{37} - 4))\right) \simeq 1.3225. \end{aligned}$$

These policies form a separating CE when the full insurance pooling policy $(\bar{p}L, 0)$, where

$$\bar{p} = \lambda p_H + (1 - \lambda)\, p_L = \frac{\lambda}{2} + \frac{1 - \lambda}{4} = \frac{1 + \lambda}{4},$$

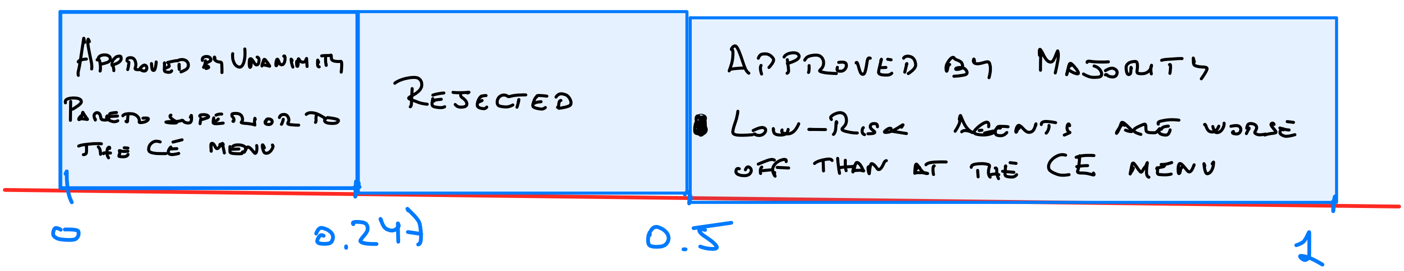
is not be preferred by a low risk individual to the policy (I_L, D_L), i.e.,

$$u(W - \bar{p}L) = \ln\left(5 - \frac{1}{4}(1 + \lambda)4\right) = \ln(4 - \lambda) \leq u_L^*,$$

which may be written as

$$\lambda \geq 4 - e^{u_L^*} := \underline{\lambda} \simeq 0.247.$$

(b) For the law to be approved it must be voted favorably by at least 50% of the drivers. If $\lambda < \underline{\lambda}$, then both high and low risk individuals are better off with the pooling policy and therefore it will be approved. If $\lambda \in (\underline{\lambda}, 1/2)$, then low risk individuals are worse off with the pooling policy and since they are a majority of the population the law would not be approved. If $\lambda \in (1/2, 1)$, then high risk individuals, who are better off with the pooling policy, are a majority of the population and therefore the law would be approved, with the consequence that low risk individual will end up worse off than in the competitive equilibrium.



Exercise 3. (a) Friday's problem is

$$\max_{c,l}\ c - l^2/2$$
$$s.t.\ c \leq wl.$$

His demand of consumption and supply of labor are then obtained by solving the system

$$\begin{aligned} MRS(l,c) &= l = w \\ c &= wl \end{aligned}$$

i.e., $l^s(w) = w$, $c(w) = w^2$.

Robinson's problem as wage-taker is

$$\max_{l\in\mathbb{R}_+}\ 2\sqrt{l} - wl.$$

Hence his demand of labor is obtained by solving the equation $1/\sqrt{l} = w$, *i.e.,* $l^d(w) = 1/w^2$.

Labor market clearing requires

$$l^s(w) = l^d(w) \Leftrightarrow w = \frac{1}{w^2} \Leftrightarrow w^* = 1 = l^*.$$

Hence the Robinson uses 1 *unit of labor, produces* $2\sqrt{1} = 2$ *units of food, and its profit (i.e., food consumption) is* $\pi^* = 2\sqrt{l^*} - w^*l^* = 2\sqrt{1} - 1^2 = 1$. *Friday supplies* 1 *unit of labor and consumes* $c^* = 1^2 = 1$ *units of food, and his utility is* $u^* = 1^2 - 1^2/2 = 1/2$.

(b) Robinson's problem as a monopsonist is

$$\max_{l\in\mathbb{R}_+}\ 2\sqrt{l} - w^s(l)l,$$

where $w^s(l) = l$ *is Friday's inverse labor supply. Hence the labor used by Robinson is obtained by solving the equation*

$$\frac{1}{\sqrt{l}} = 2l \Leftrightarrow l_M^* = 2^{-\frac{2}{3}} \simeq 0.63 = w_M^*.$$

Thus, Robinson produces $2\sqrt{2^{-\frac{2}{3}}} \simeq 1.59$ *units of food, and its profit (i.e., food consumption) is*

$$\pi_M^* = 2\sqrt{2^{-\frac{2}{3}}} - \left(2^{-\frac{2}{3}}\right)^2 \simeq 1.19061.$$

Friday supplies $2^{-\frac{2}{3}} \simeq 0.63$ *units of labor and consumes* $c_M^* \simeq (2^{-\frac{2}{3}})^2$ *units of food, and his utility is*

$$u_M = \left(2^{-\frac{2}{3}}\right)^2 - \left(2^{-\frac{2}{3}}\right)^2/2 \simeq 0.19845.$$

This allocation is not Pareto optimal. For example, if Friday supplies $l = 1$ *unit of labor and receives* $c = 4/5$ *units of food, then his utility is* $\hat{u} = 4/5 - (1)^2/2 = 0.3 > u_M$; *then Robinson receives* $2\sqrt{1} - 4/5 = 6/5 > \pi_M^*$ *units of food. This allocation is Pareto superior to the monopolistic equilibrium allocation since both Friday and Robinson are better off.*