


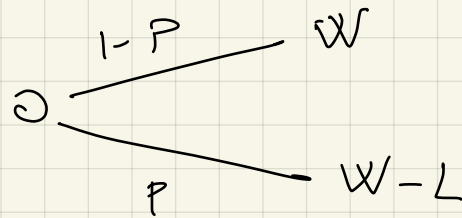
COMPETITIVE INSURANCE MARKETS



THE MARKET FOR INSURANCE.

M. Rothschild and J. Stiglitz, QJE (1976).

- A population of individuals face the risk of a wealth loss L with probability $p \in (0, 1)$; that is, face the lottery

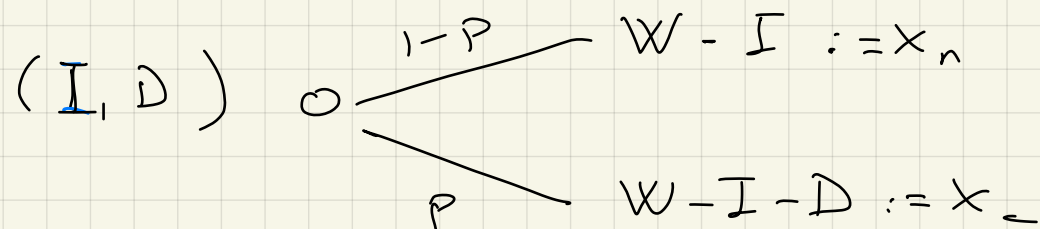


- Their preferences are represented by a Bernoulli utility function $u: \mathbb{R} \rightarrow \mathbb{R}$, such that $u' > 0$, $u'' < 0$.
- There is a competitive insurance market where firms offer policies (I, D) , where

I : Premium

D : Deductible

If an individual subscribes the policy (I, D) , then he faces the lottery



Hence

$$Eu(I, D) = (1-p) u(x_n) + p u(x_-) := U(x_n, x_-)$$

Examples:

- ① $(I, D) = (0, L)$ (No insurance)
- ② $(I, D) = (I, 0)$ (Full insurance)
- ③ $(I, D) = (pL, 0)$ (Full insurance at fair premium)
- ④ $(I, D) = (I, \frac{L}{2})$ (Partial insurance)

Typically, $D < 0$ is not allowed.

Preferences for insurance policies:

Let us calculate the $MRS(x_n, x_a)$:

$$U(x_n, x_a) = (1-p) u(x_n) + p u(x_a)$$

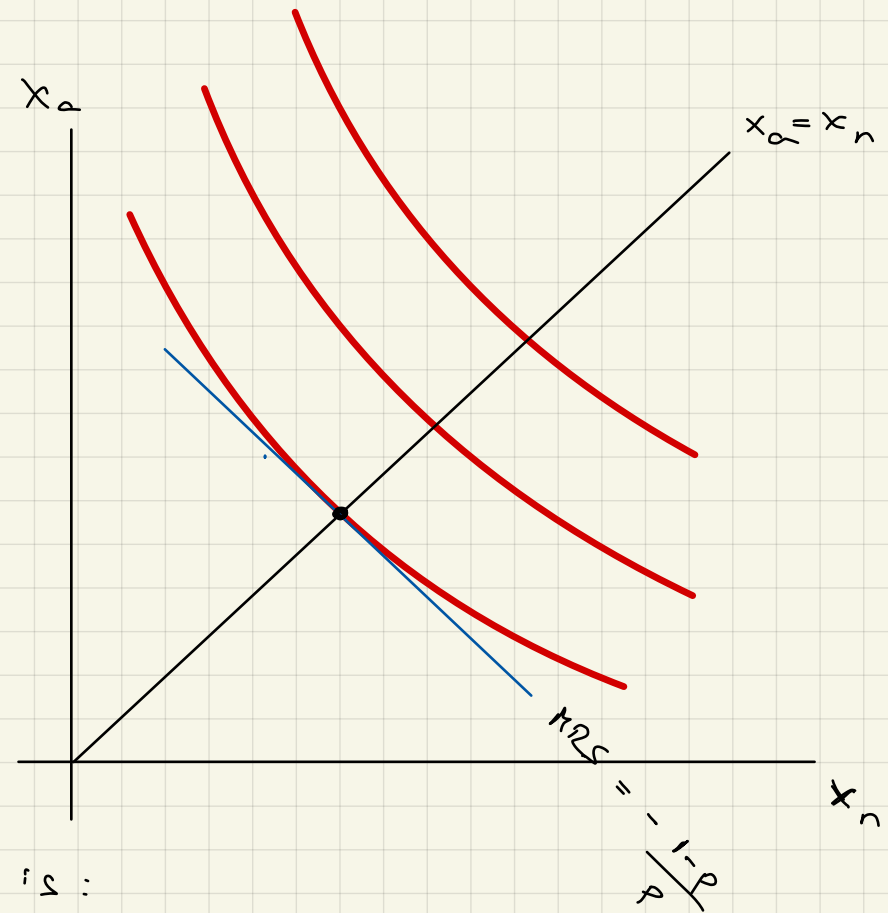
Hence

$$-\frac{dx_a}{dx_n} = MRS(x_n, x_a) = \frac{\frac{\partial U}{\partial x_n}}{\frac{\partial U}{\partial x_a}} = \frac{1-p}{p} \frac{u'(x_n)}{u'(x_a)}$$

Exercise: Check that $u' > 0$, $u'' < 0$ imply

That indifference curves are convex, that is:

$$\frac{d^2 x_a}{d x_n^2} > 0.$$



Fair Odds Policies

$$I = p(L - D)$$

Fair Odds Line:

$$x_n = W - I \Leftrightarrow I = W - x_n$$

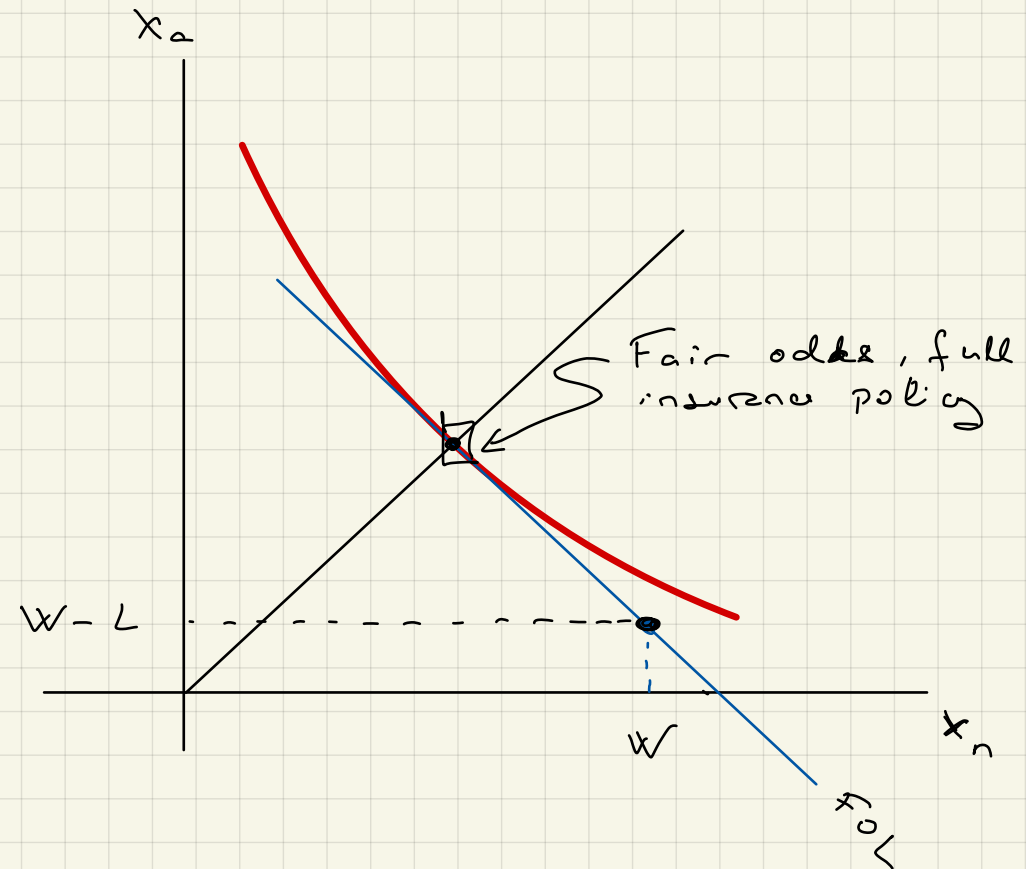
$$x_a = W - I - D = x_n - D \Leftrightarrow D = x_n - x_a$$

Hence

$$W - x_n = p[L - (x_n - x_a)]$$

i.e.,

$$x_a = \left(\frac{W}{p} - L \right) - \frac{1-p}{p} x_n$$



CE.

In a CE a policy (I, D) is subscribed only if

$$(1) \quad I = p(L - D)$$

$$(2) \quad \nexists (\tilde{I}, \tilde{D}) \text{ such that}$$

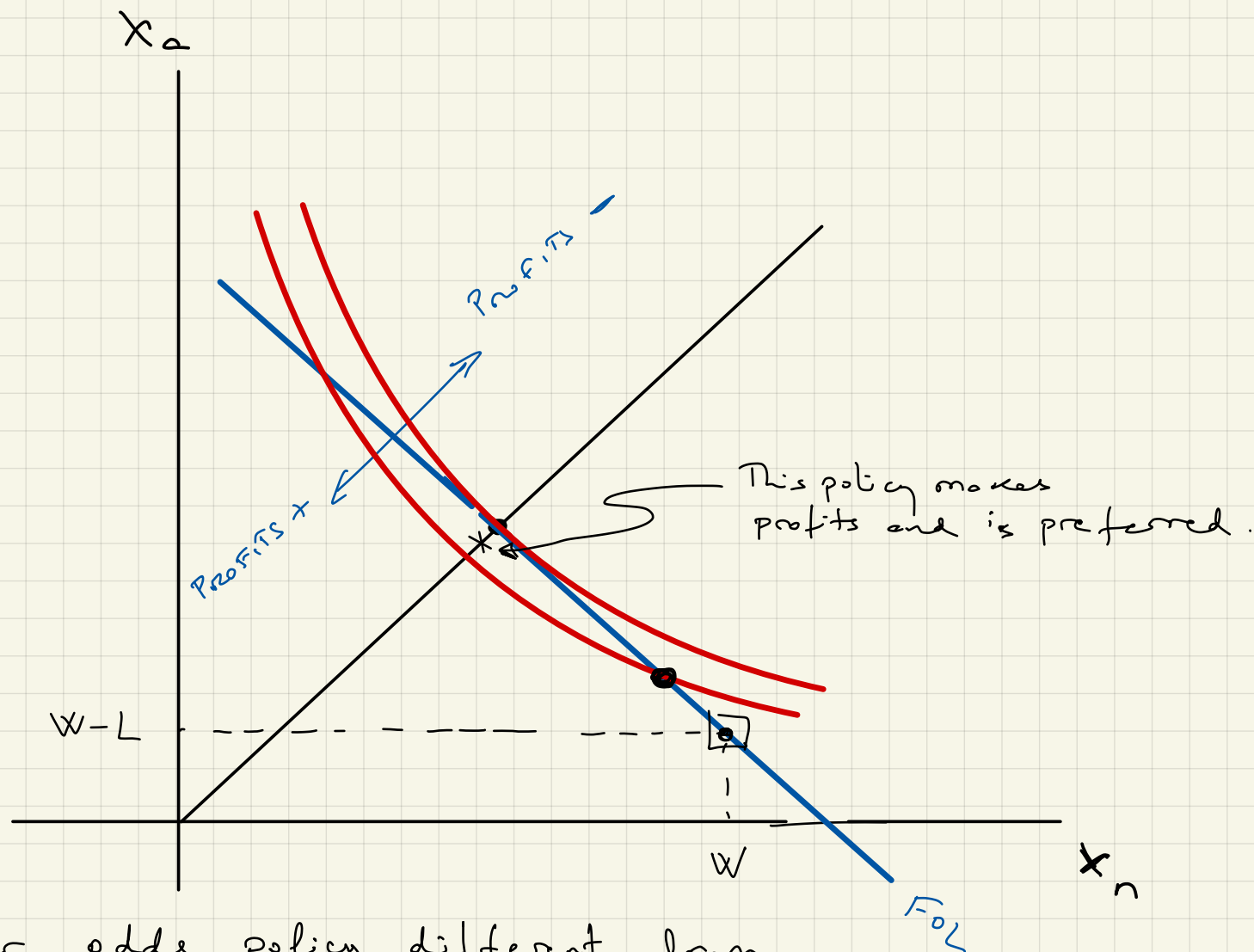
$$\bullet \tilde{I} > p(L - \tilde{D})$$

$$\bullet E_u(\tilde{I}, \tilde{D}) > E_u(I, D).$$

Proposition. In a CE all individuals subscribe the policy

$$(I^*, D^*) = (pL, 0).$$

Proof

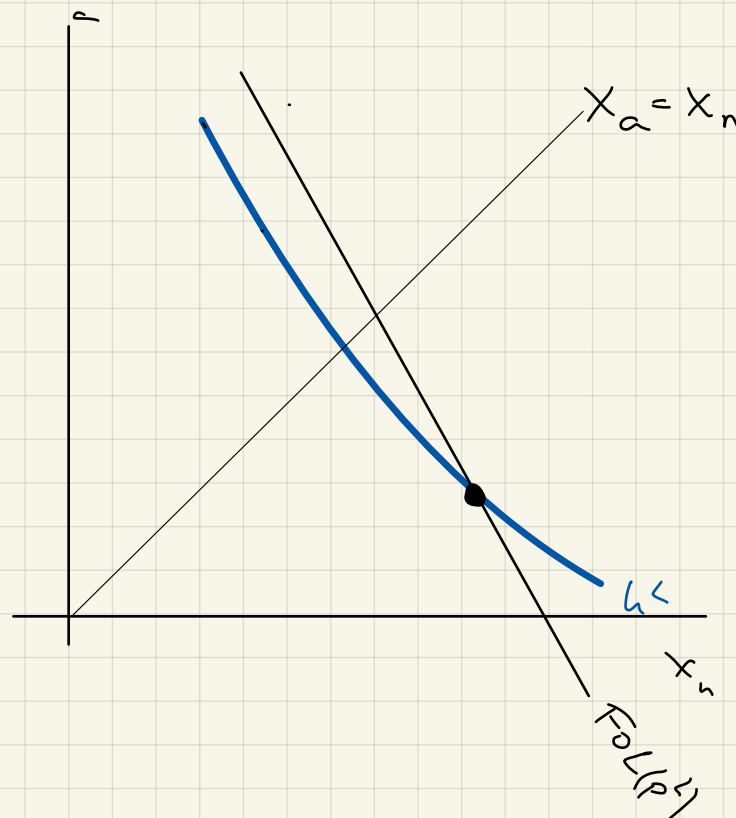
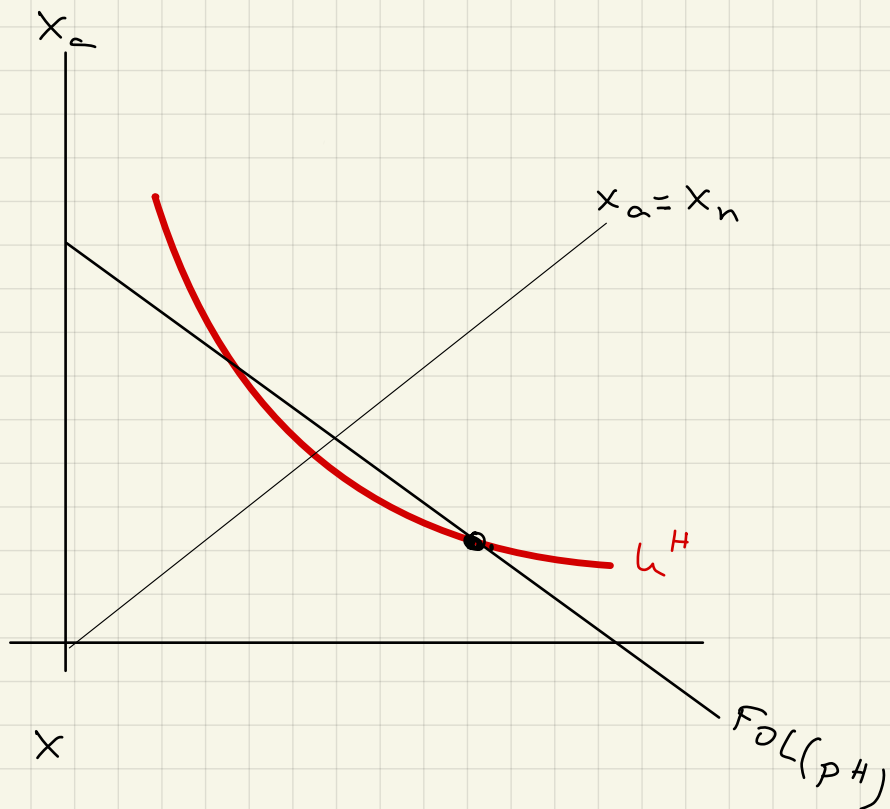


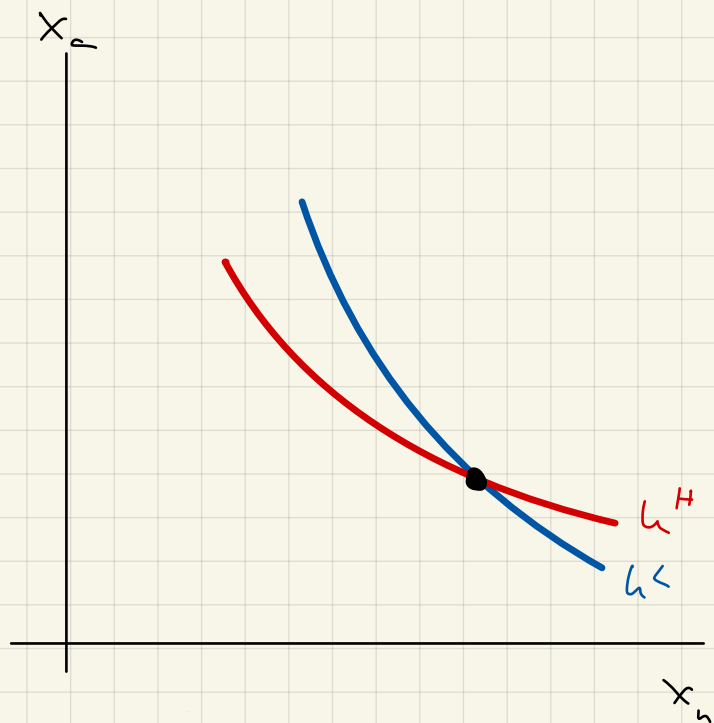
Any fair odds policy different from $(pL, 0)$ is upset by a policy $(pL + \epsilon, 0)$, $\epsilon > 0$, which makes profits.

ADVERSE SELECTION

Assume that for a fraction $\lambda \in (0, 1)$ of individuals the probability of the wealth loss is p^H , while the remaining fraction $1-\lambda$ of individuals the probability is p^L , where

$$0 < p^L < p^H$$





$$\begin{aligned}
 MRS_L(x_n, x_p) &= \frac{1-p^L}{p^L} \frac{u'(x_n)}{u'(x_p)} \\
 &> \frac{1-p^H}{p^H} \frac{u'(x_n)}{u'(x_p)} \\
 &= MRS_H(x_n, x_p)
 \end{aligned}$$

Note:

$$\frac{d}{dp} \left(\frac{1-p}{p} \right) = \frac{d}{dp} \left(\frac{1}{p} - 1 \right) = -\frac{1}{p^2} < 0.$$

If insurance companies could recognize the individual of either type, then in a CE they will offer to each type her full insurance fair-odds policy.

What if insurance companies cannot distinguish between high and low risk individuals?

Which policies will they offer in a CE?

Will they offer a single (pooling) policy to both types?

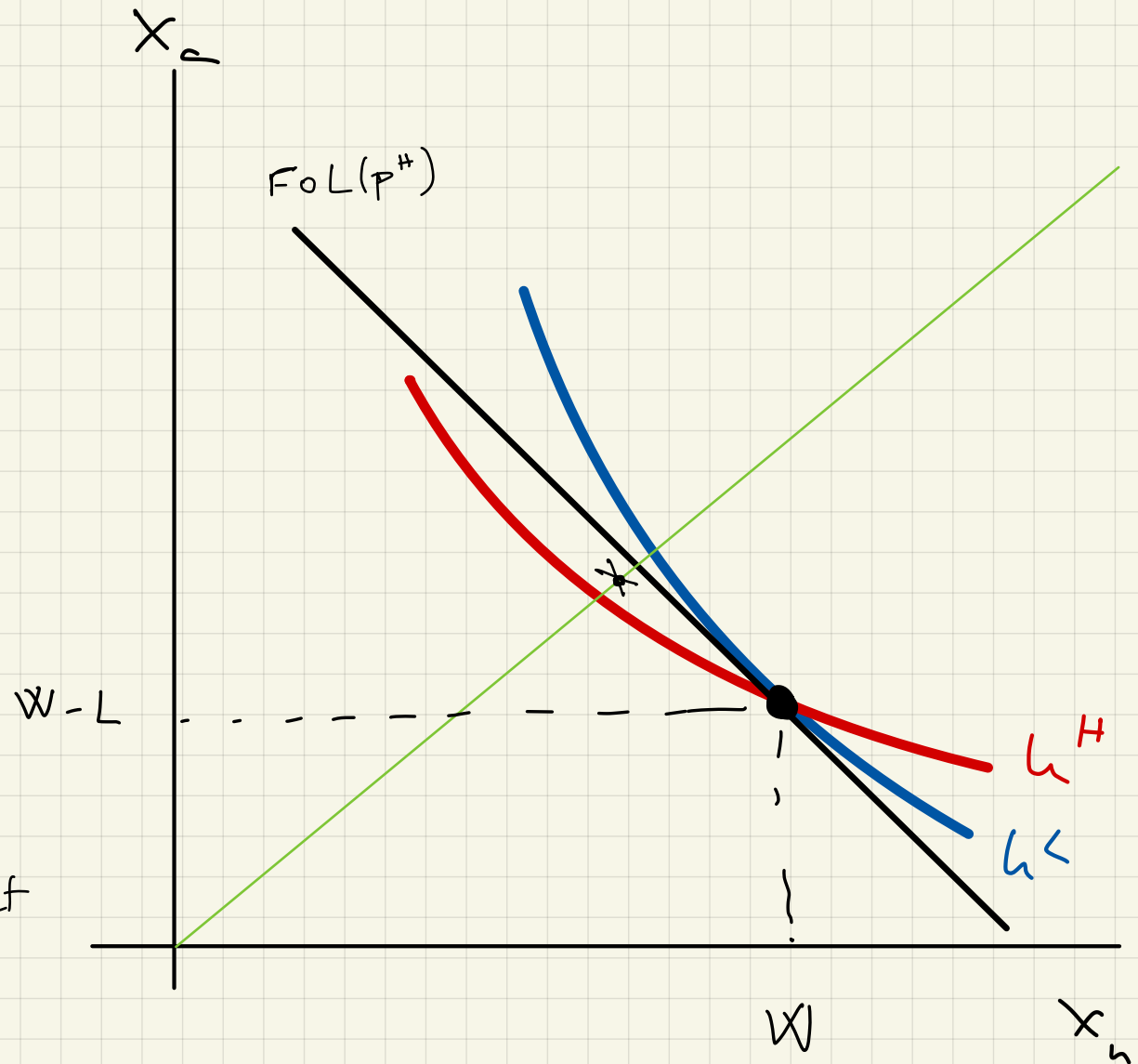
Proposition A CE in which the same policy is subscribed by both types (i.e., a pooling equilibrium) does not exist.

Proof

(a) Offering the single policy $(0, L)$
is not a CE.

A company offering the
policy $(p^H L + \varepsilon, 0)$
will attract all the
high risk individuals
and make positive
profits.

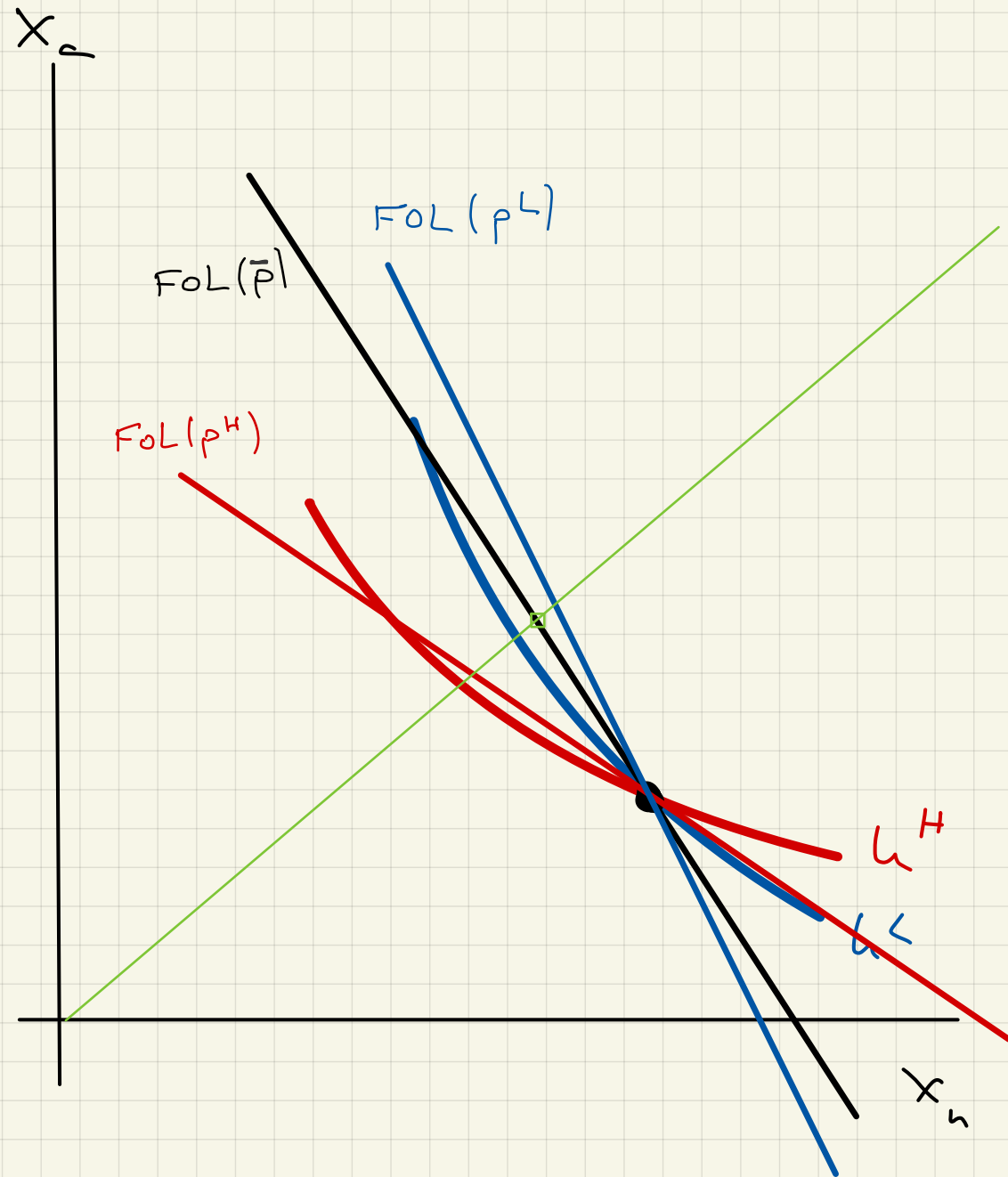
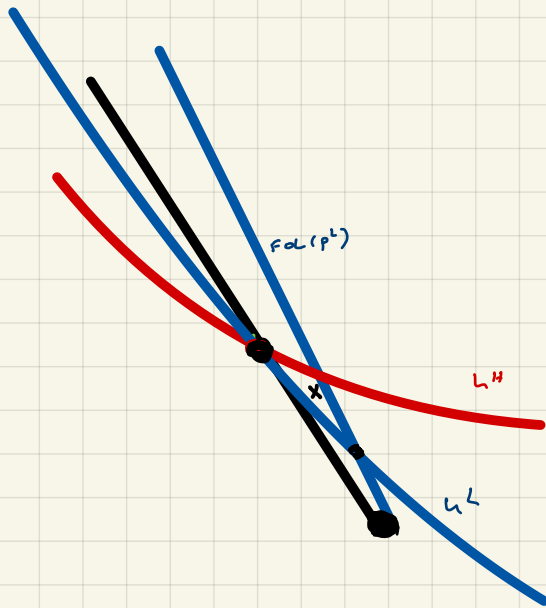
Note. It may also attract
the low risk individuals.



(b) Any other fair odds
policy $(\bar{p}(L-D), D)$, with

$$\bar{p} = \lambda p^H + (1-\lambda)p^L,$$

will be upset by a
policy aimed to attracting
low risk individuals

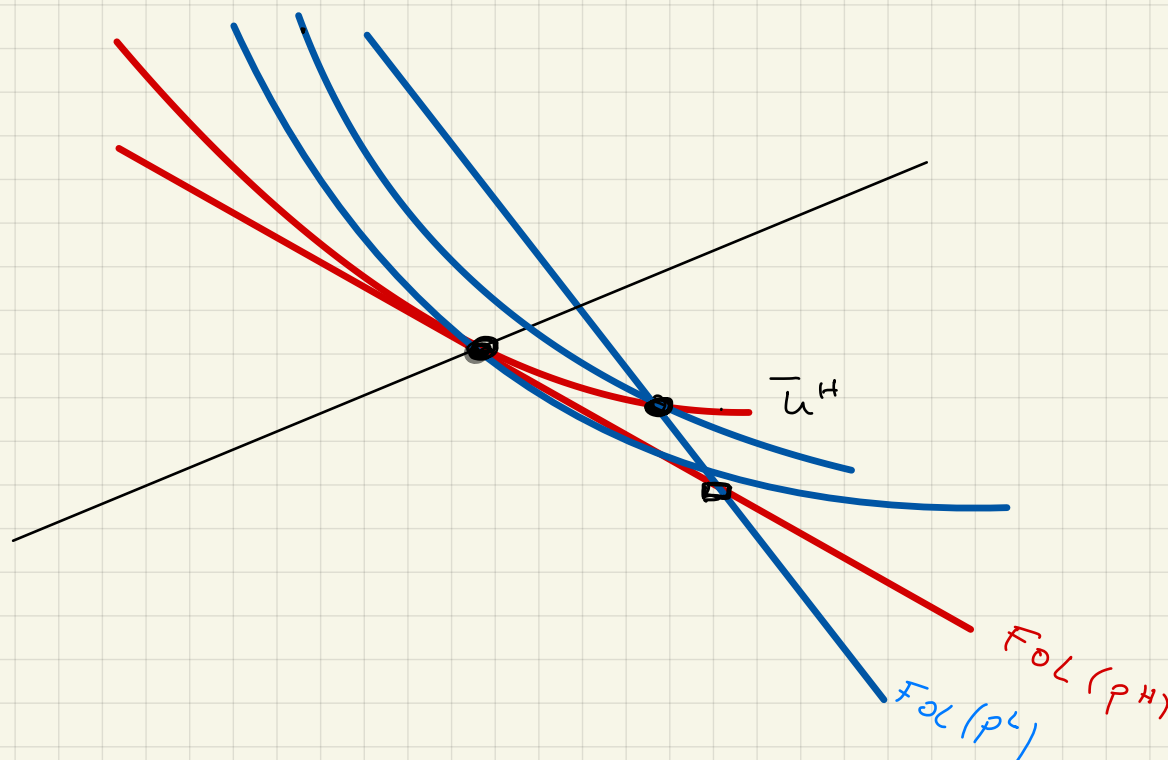


A SEPARATING EQUILIBRIUM

$$\{ (I^H, D^H), (I^L, D^L) \}, \quad (1) \quad I^H = p^H L, \quad D^H = 0$$

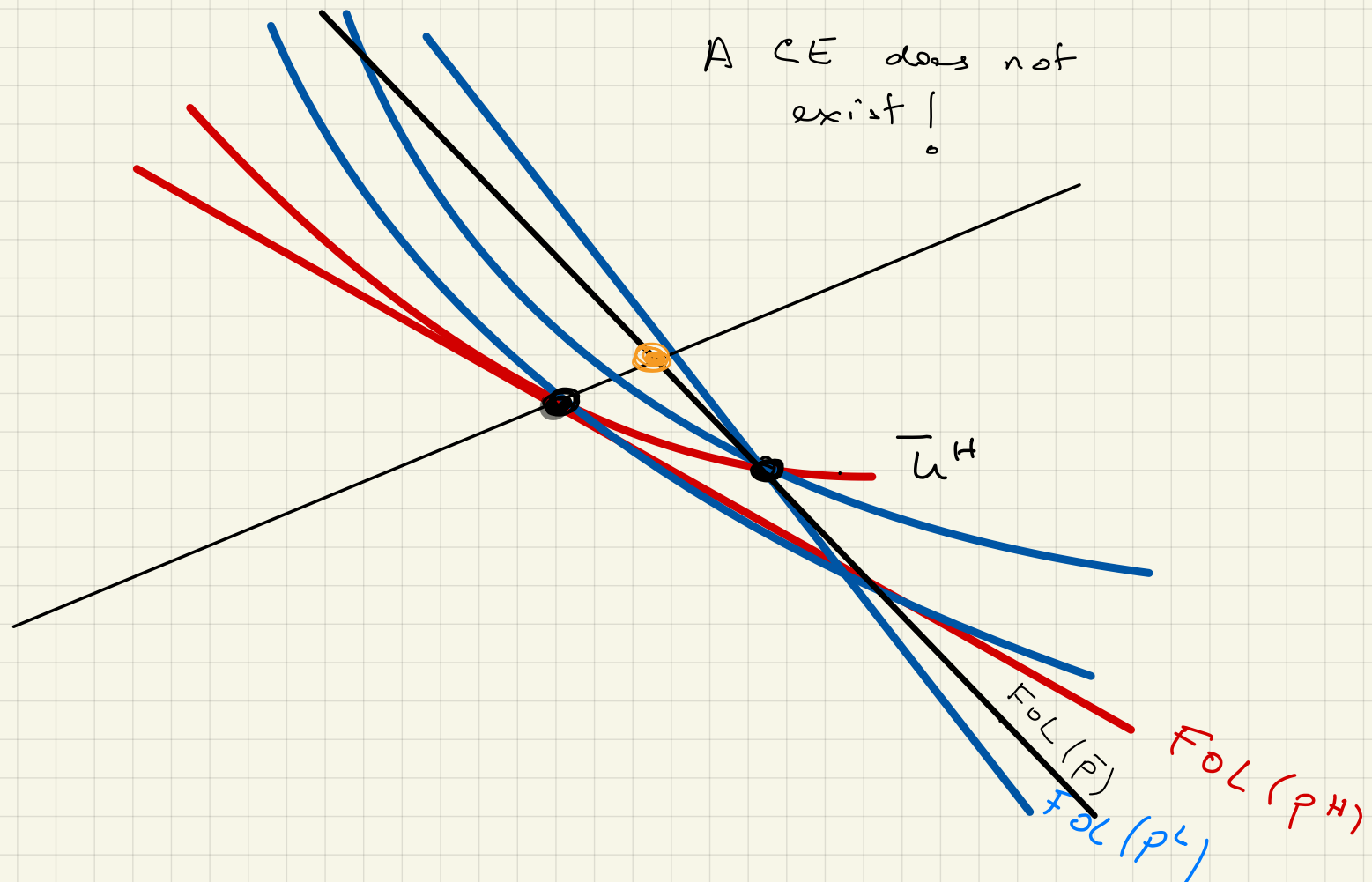
$$(2) \quad I^L = p^L (L - D^L)$$

$$E_H u(I^L, D^L) := p^H u(W - I^L - D^L) + (1 - p^H) u(W - I^L) = E_H u(I^H, D^H) = u(W - I^H).$$



For this menu to be a CE, λ must be sufficiently large that.

$$u(W - \bar{p}L) \leq p^L u(W - I^L - D^L) + (1 - p^L) u(W - I^L)$$



$$W = L = 1, \quad u(x) = \sqrt{x}, \quad p^L = \frac{1}{4}, \quad p^H = \frac{1}{2}, \quad \lambda \in (0, 1).$$

With three possible observables, the CE policies are:

$$(\bar{I}^H, \bar{D}^H) = \left(\frac{1}{2}, 0\right), \quad (\bar{I}^L, \bar{D}^L) = \left(\frac{1}{4}, 0\right)$$

The pooling policy is $(\bar{I}, \bar{D}) = (\bar{p}(\lambda)L, 0)$,

where:

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{(1-\lambda)}{4} = \frac{1+\lambda}{4}.$$

For $\lambda = 1/4$, for example, $\bar{p}(1/4) = \frac{5}{16}$, and

$$(\bar{I}, \bar{D}) = \left(\frac{5}{16}, 0\right)$$

With this policy, the expected utilities of agents are

$$\bar{U}_H = \bar{U}_L = \frac{\sqrt{41}}{4} \approx .83$$

BUT THIS POLICY IS "DESTABILIZED" RS, E.G., THE POLICY

$$(\tilde{I}^L, \tilde{D}^L) = \left(\frac{1}{8}, \frac{1}{2} \right).$$

THE EXPECTED UTILITIES OF AGENTS THAT SUBSCRIBE THIS POLICY ARE

$$\tilde{U}_L = \frac{3}{4} \sqrt{\frac{7}{8}} + \frac{1}{4} \sqrt{\frac{3}{8}} \approx .854$$

$$\tilde{U}_H = \frac{1}{2} \sqrt{\frac{7}{8}} + \frac{1}{2} \sqrt{\frac{3}{8}} \approx .773.$$

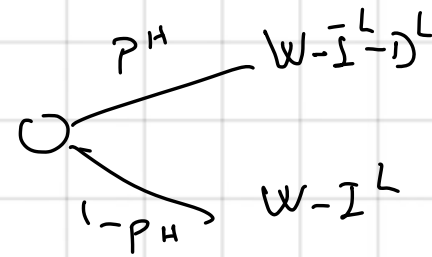
HENCE LOW RISK INDIVIDUALS PREFER THIS POLICY TO THE POOLING POLICY, WHILE HIGH RISK INDIVIDUALS PREFER THE POOLING POLICY.

THE SEPARATING POLICIES ARE :

$$(I^H, D^H) = \left(\frac{1}{2}, 0\right), \quad (I^L, D^L) = \left(p^L(L - D^*), D^*\right)$$

SOLVES :

$$\frac{1}{2} \sqrt{1 - \frac{1-D}{4}} + \frac{1}{2} \sqrt{1 - \frac{1-D}{4} - D} = \frac{1}{\sqrt{2}},$$



WHERE $\frac{1}{\sqrt{2}} = U_H^S$ IS THE EXPECTED UTILITY OF A

HIGH RISK INDIVIDUAL WHO SUBSCRIBES THE POLICY

(I^H, D^H) . SOLVING THIS EQUATION WE GET

$$D^* = \sqrt{3} - 1 \approx .732.$$

$$I^L = \frac{1-D^*}{4} = \frac{1-(\sqrt{3}-1)}{4} = \frac{2-\sqrt{3}}{4}$$

THE SEPARATING "MENU" FORMS A CE PROVIDED THE (EXP.)
 UTILITY OF LOW RISK INDIVIDUALS OF THE POOLING
 POLICY, $(\bar{p}(\lambda) L, 0)$, WHERE

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{1-\lambda}{4} = \frac{1+\lambda}{4},$$

WHICH IS

$$u\left(1 - \frac{1+\lambda}{4}\right) = u\left(\frac{3-\lambda}{4}\right) = \frac{\sqrt{3-\lambda}}{2},$$

IS LESS THAN THE EXPECTED UTILITY OF THE POLICY (I^L, D^L) ,

$$\begin{aligned} E u(I^L, D^L) &= p^L u(1 - I^L - D^L) + (1 - p^L) u(1 - I^L) \\ &= \frac{1}{4} \sqrt{1 - \frac{2-\sqrt{3}}{4} - (\sqrt{3}-1)} + \frac{3}{4} \sqrt{1 - \frac{2-\sqrt{3}}{4}} \\ &= \frac{\sqrt{6}}{8} (\sqrt{3} + 1). \end{aligned}$$

THAT IS

$$\frac{\sqrt{6}}{8} (\sqrt{3} + 1) \geq \frac{\sqrt{3-\lambda}}{2} \Leftrightarrow \lambda \geq \frac{3}{4} (2 - \sqrt{3}) \approx 0.2$$

NOTE: IF THE POOLING POLICY (WHICH IS PREFERRED TO THE SEPARATING POLICIES BY THE HIGH RISK INDIVIDUALS) IS PREFERRED TO THE SEPARATING POLICY BY THE LOW RISK INDIVIDUAL (AS WELL) THEN:

- THE POOLING POLICY IS PARETO SUPERIOR TO THE SEPARATING POLICIES, AND

- THE POLICY $(\bar{p}(\lambda) + \epsilon, 0)$, FOR $\epsilon > 0$ SMALL, DE-STABILIZES THE SEPARATING POLICIES — THAT IS, THE SEPARATING POLICIES DO NOT FORM A CE.

... AND THE

Thus, when the separating menu is NOT a CE ---

making the pooling policy MANDATORY makes everyone

better off.

(Ex. 3 in List 2 is analogous.)