Time and Uncertainty in Competitive Economies

Diego Moreno

Masters in Economics - Micro II
Universidad Carlos III de Madrid

Time

Consider an economy

$$[Y_1, ..., Y_m, (u_1, \bar{x}_1, \theta_1), ..., (u_n, \bar{x}_n, \theta_n)]$$

that operates over T consecutive dates. Since in this economy the number of goods is lT, let us assume that $\mathbb{R}_+^{lT}$ is the *commodity space*, so that

$$Y_j \subset \mathbb{R}^{lT}$$

for all $j \in \{1, ..., m\}$, and

$$u_i : \mathbb{R}_+^{lT} \to \mathbb{R}, \ \bar{x}_i \in \mathbb{R}_+^{lT}$$

for all $i \in \{1, ..., n\}$.

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Issues:

$\triangleright$Are goods perishable, storable?

$\triangleright$Is T finite or infinite?

$\triangleright$Is there technology progress? (For example, $Y_j = Y_j^1 \times ... \times Y_j^T$, where $Y_j^t \subset Y_j^{t+1}$.)

$\triangleright$What are the dynamic features of preferences? If no assumptions are made, no conclusions about consumption smoothing, habits or persistence are derived. For example, what if

$$u(x) = \sum_{t=1}^{T} \delta^{t-1} v(x_t),$$

where $v : \mathbb{R}_+^l \to \mathbb{R}$.

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Consider a pure exchange economy that operates over T dates,

$$[(u_1, \bar{x}_1), ..., (u_n, \bar{x}_n)].$$

The definition of Arrow-Debreu competitive equilibrium (CE), which presumes that there are (future) markets for all goods, is the standard one.

In this economy, if prices are $p = (p_1, ...p_T)$, where $p_t \in \mathbb{R}^l_+$, then the budget set of a consumer endowed with $\bar{x} \in \mathbb{R}^{lT}_+$ is

$$B(p, \bar{x}) = \left\{ x \in \mathbb{R}^{lT}_+ \mid \sum_{t=1}^{T} p_t(x_t - \bar{x}_t) \leq 0 \right\}.$$

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An Arrow-Debreu CE is a collection (p^*, x^*), where

$$x^* = (x_1^*, ..., x_n^*) \in \mathbb{R}_+^{lTn},$$

satisfying:

1. For all $i \in \{1, ..., n\}$,

$$x_i^* \in x_i(p^*) := \arg\max_{x \in B(p^*, \bar{x}_i)} u_i(x).$$

2. For all (k, t),

$$\sum_{i=1}^{n} (x_{itk}^* - \bar{x}_{itk}) \leq 0 \text{ (with equality if } p_{tk}^* > 0).$$

Arrow-Debreu CE allocations are PO (FWT)

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In real economies there are future markets for some but not all goods. Moreover, (*spot*) markets for all real goods, as well as *credit* markets whereby consumers can borrow or lend, operate every date.

In the Arrow-Debreu static setting, after the operation of future markets no further transactions are needed. Thus, the theory leaves no role for spot or credit markets.

(But this observation is artificial since in real economies there are inflows and outflows of agents which may justify the existence of transactions in the spot and future markets every period.)

- What are the implications of missing (future) markets?
- What is the role of credit and spot market transactions in the intertemporal allocation of consumption?

Consider a (Radner) economy in which the primitives are the ones described above,

$$[(u_1, \bar{x}_1), ..., (u_n, \bar{x}_n)],$$

but in which there are no future markets. Instead, there are spot and credit markets operating each date.

In this economy, if the spot prices are $\hat{p} = (\hat{p}_1 ..., \hat{p}_T)$ and the interest rates are $r = (r_1 ..., r_{T-1})$, then the budget set of a consumer endowed with $\bar{x} \in \mathbb{R}^{IT}_+$ is

$$\begin{aligned} \hat{B}(\hat{p}, r, \bar{x}) = \quad & \{(x, b) \in \mathbb{R}^{IT}_+ \times \mathbb{R}^{T-1} \mid \\ & \hat{p}_1(x_1 - \bar{x}_1) \leq b_1, \\ & \hat{p}_t(x_t - \bar{x}_t) + (1 + r_{t-1})b_{t-1} \leq b_t, \text{ for } 1 < t < T, \\ & p_T(x_T - \bar{x}_T) + (1 + r_{T-1})b_{T-1} \leq 0\}. \end{aligned}$$

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A Radner CE is a collection

$$(\hat{p}^*, \hat{r}^*, (\hat{b}_1^*, ..., \hat{b}_n^*), (\hat{x}_1^*, ..., \hat{x}_n^*)) \in \mathbb{R}_+^{lT} \times \mathbb{R}^{T-1} \times \mathbb{R}^{(T-1)n} \times \mathbb{R}_+^{lTn}$$

satisfying:

1. For all $i \in \{1, ..., n\}$,
$$(\hat{x}_i^*, \hat{b}_i^*) \in \arg\max_{(x,b) \in \hat{B}(\hat{p}^*, \hat{r}^*, \bar{x}_i)} u_i(x).$$
2. For all (k, t),
$$\sum_{i=1}^{n} (\hat{x}_{itk}^* - \bar{x}_{itk}) \leq 0 \text{ (with equality if } \hat{p}_{tk}^* > 0).$$
3. For all $t < T$,
$$\sum_{i=1}^{n} \hat{b}_{it}^* = 0.$$

Are Radner CE allocations Pareto optimal?

Proposition. If $(\hat{p}^*, \hat{r}^*, \hat{b}^*, \hat{x}^*)$ is a Radner CE, then $(p^*, \hat{x}^*)$, where $p^*_T = \hat{p}^*_T$, and for $t < T$,

$$p^*_t = \hat{p}^*_t \prod_{s=t}^{T-1} (1 + r_s),$$

is an Arrow-Debreu CE. Hence a Radner CE allocation is Pareto optimal.

Proof. For simplicity, assume $T = 2$. We show that
(1) $\hat{x}_i^* \in B(p^*, \bar{x}_i)$, and
(2) $B(p^*, \bar{x}_i) \subset \hat{B}_x(\hat{p}^*, \hat{r}^*, \bar{x}_i)$, where

$$\hat{B}_x(\hat{p}^*, \hat{r}^*, \bar{x}_i) = \{x \in \mathbb{R}_+^{lT} \mid (x, b) \in \hat{B}(\hat{p}^*, \hat{r}^*, \bar{x}_i) \text{ for some } b \in \mathbb{R}^{T-1}\}.$$

Hence, since $(\hat{x}_i^*, \hat{b}_i^*)$ maximizes u_i on $\hat{B}(\hat{p}^*, \hat{r}^*, \bar{x}_i)$, $\hat{x}_i^*$ also maximizes u_i on $B(p^*, \bar{x}_i)$.

Thus, $\hat{x}_i^* \in x_i(p^*)$, and markets clear at prices p^*.

Therefore $(p^*, \hat{x}^*)$ is an Arrow-Debreu CE.

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(1) $\hat{x}_i^* \in B(p^*, \bar{x}_i)$. Since $\hat{p}_1^*(\hat{x}_{i1}^* - \bar{x}_{i1}) \leq b_i$,

$$\begin{aligned} p_1^*(\hat{x}_{i1}^* - \bar{x}_{i1}) + p_2^*(\hat{x}_{i2}^* - \bar{x}_{i2}) &= (1 + r_1^*)\hat{p}_1^*(\hat{x}_{i1}^* - \bar{x}_{i1}) + \hat{p}_2^*(\hat{x}_{i2}^* - \bar{x}_{i2}) \\ &\leq (1 + r_1^*)b_i - (1 + r_1^*)b_i \\ &= 0. \end{aligned}$$

(2) $B(p^*, \bar{x}_i) \subset \hat{B}(\hat{p}^*, \hat{r}^*, \bar{x}_i)$. Let $x_i \in B(p^*, \bar{x}_i)$. We show that $(x_i, b_i) \in \hat{B}(\hat{p}^*, \hat{r}^*, \bar{x}_i)$, where $b_i = \hat{p}_1^*(x_{i1} - \bar{x}_{i1})$.

$$\begin{aligned} \hat{p}_2^*(x_{i2} - \bar{x}_{i2}) + (1 + r_1^*)b_i &= \hat{p}_2^*(x_{i2}^* - \bar{x}_{i2}) + (1 + r_1^*)\hat{p}_1^*(x_{i1}^* - \bar{x}_{i1}) \\ &= p_1^*(x_{i1}^* - \bar{x}_{i1}) + p_2^*(x_{i2}^* - \bar{x}_{i2}) \\ &\leq 0. \ \square \end{aligned}$$

Time

Let us assume that u_i is increasing, so that budget constrains are binding. If $x \in B_i(\hat{p}^*, \hat{r}^*)$, then $\exists b \in \mathbb{R}$ such that

$$p_1(x_1 - \bar{x}_{i1}) = b$$
$$p_2(x_2 - \bar{x}_{i2}) + (1 + r_1)b = 0$$

which implies

$$(1 + r_1)p_1(x_1 - \bar{x}_{i1}) + p_2(x_2 - \bar{x}_{i2}) = 0,$$

i.e., future value of consumption equal to future value of endowments.
Also, since $1 + r_1 > 0$ (why?), we may write

$$p_1(x_1 - \bar{x}_{i1}) + \frac{p_2}{1 + r_1}(x_2 - \bar{x}_{i2}) = 0,$$

i.e., present value of consumption equal to present value of endowments.