Consumer Theory

Normative Aspects: Compensated and Equivalent Variations

If consumer's preferences are known, it is possible to provide a monetary measure of the impact on her welfare of variation in the prices of the goods?

Two alternative concepts:

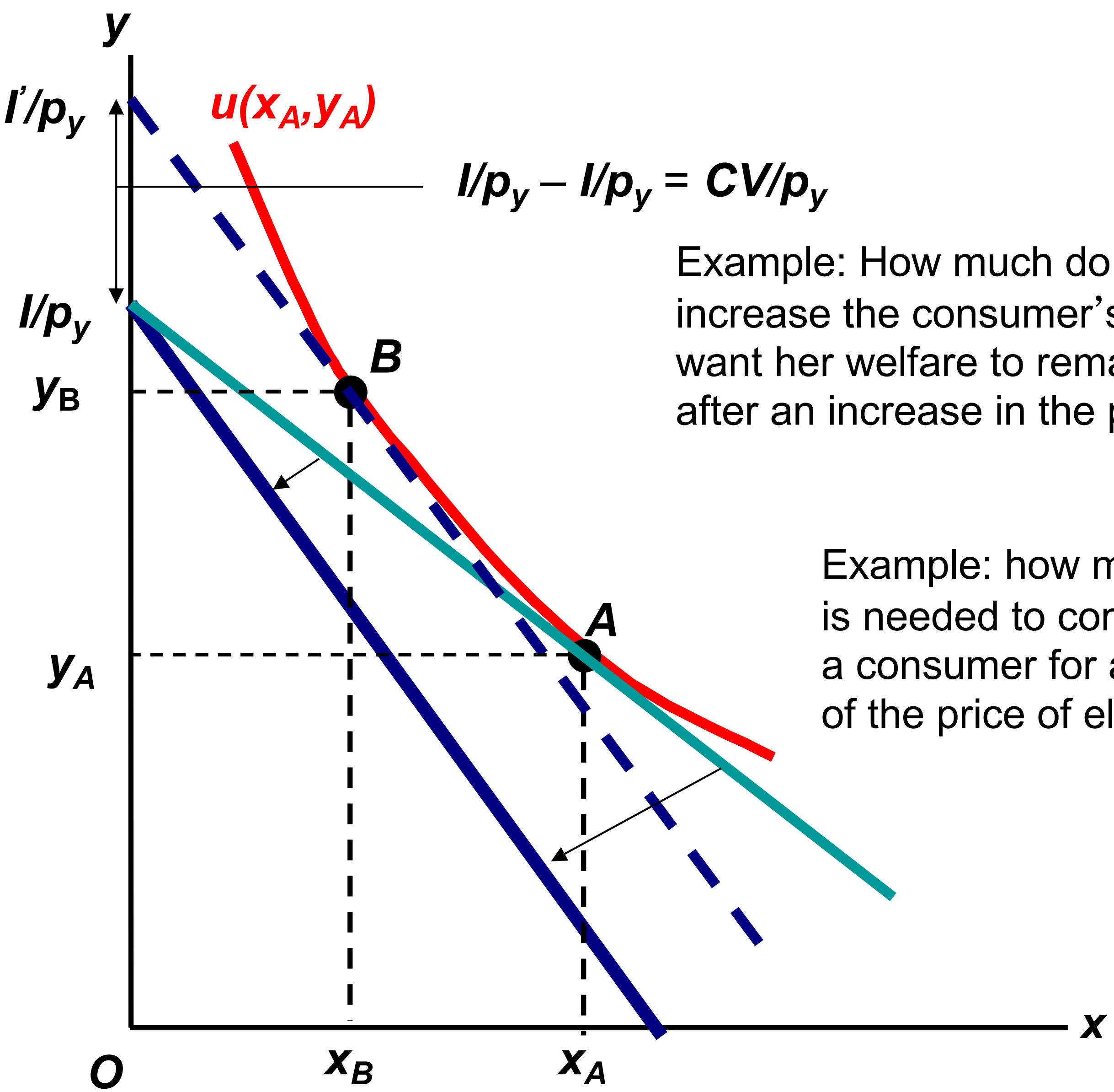
The Compensated Variation

How much do we have to increase/decrease the consumer' s income if we want her welfare to remain the same after a change in market prices?

The Equivalent Variation

How much can we increase/decrease the consumer' s income to induce the same welfare loss as a change in market prices?

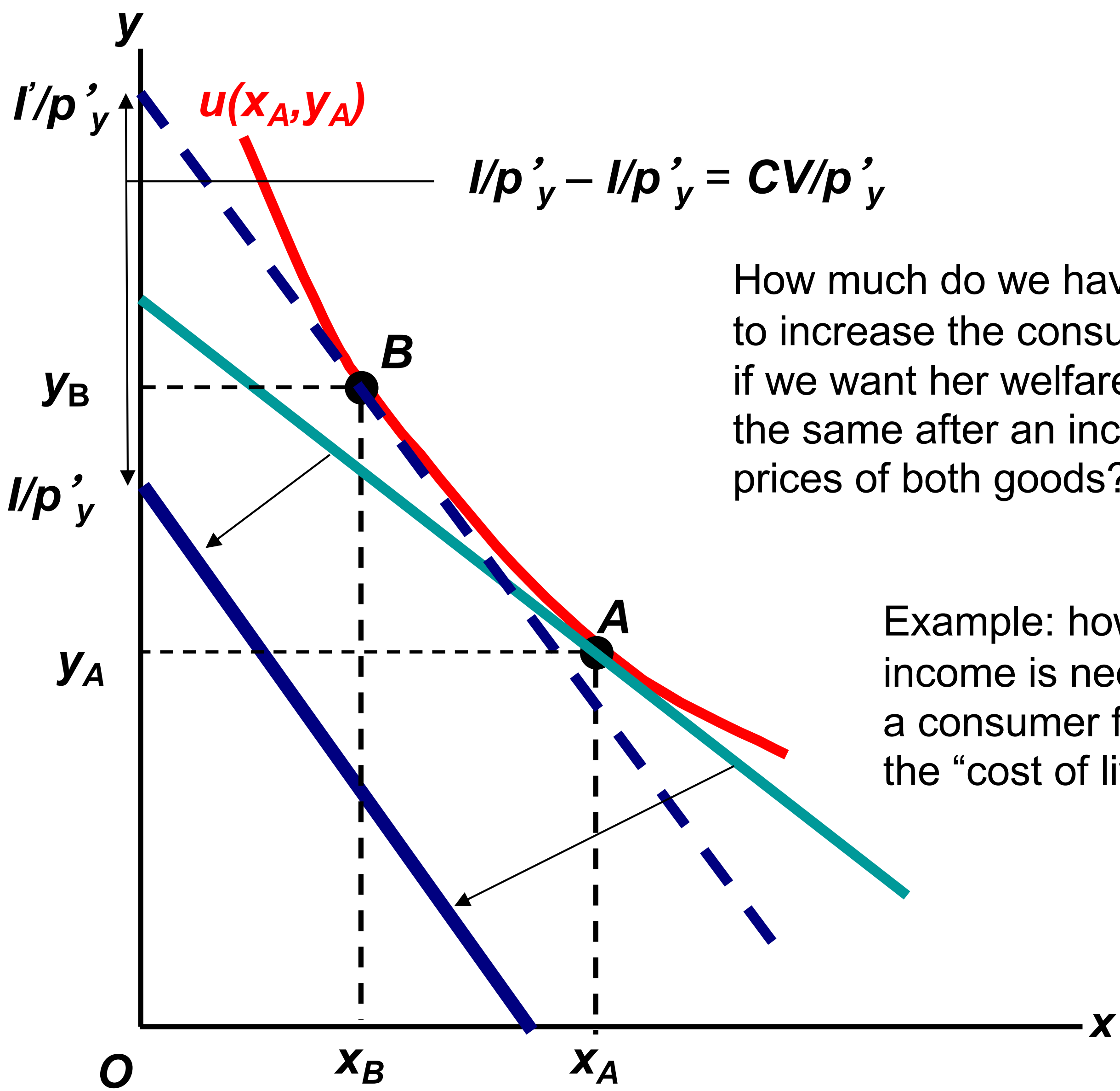
Normative Aspects: Compensated Variation



Example: How much do we have to increase the consumer's income if we want her welfare to remain the same after an increase in the prices of x?

Example: how much income is needed to compensate a consumer for an increment of the price of electricity?

Compensated Variation: Another Example

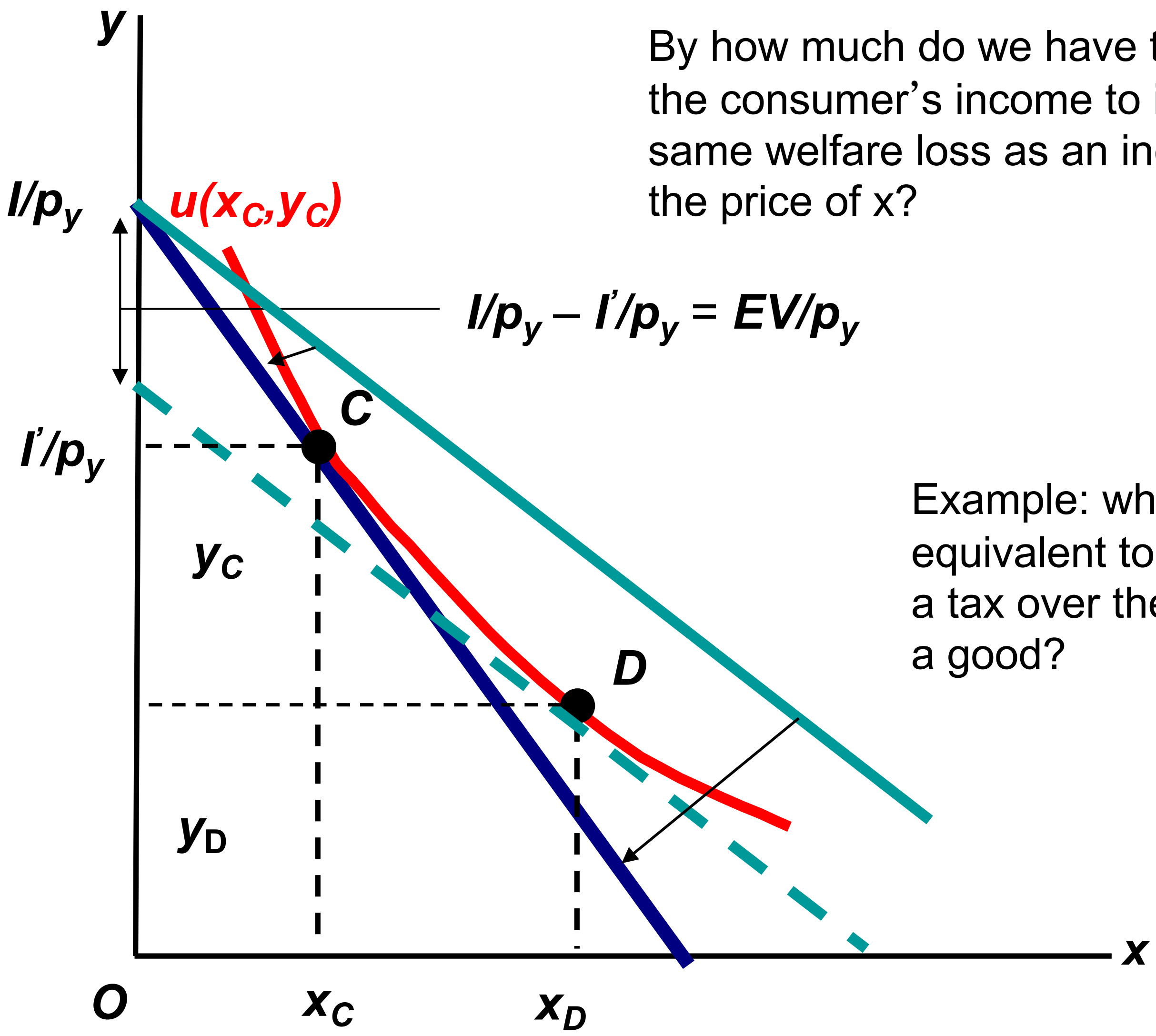


How much do we have to increase the consumer's income if we want her welfare to remain the same after an increase in the prices of both goods?

Example: how much aditional income is needed to compensate a consumer for the increment of the "cost of living"?

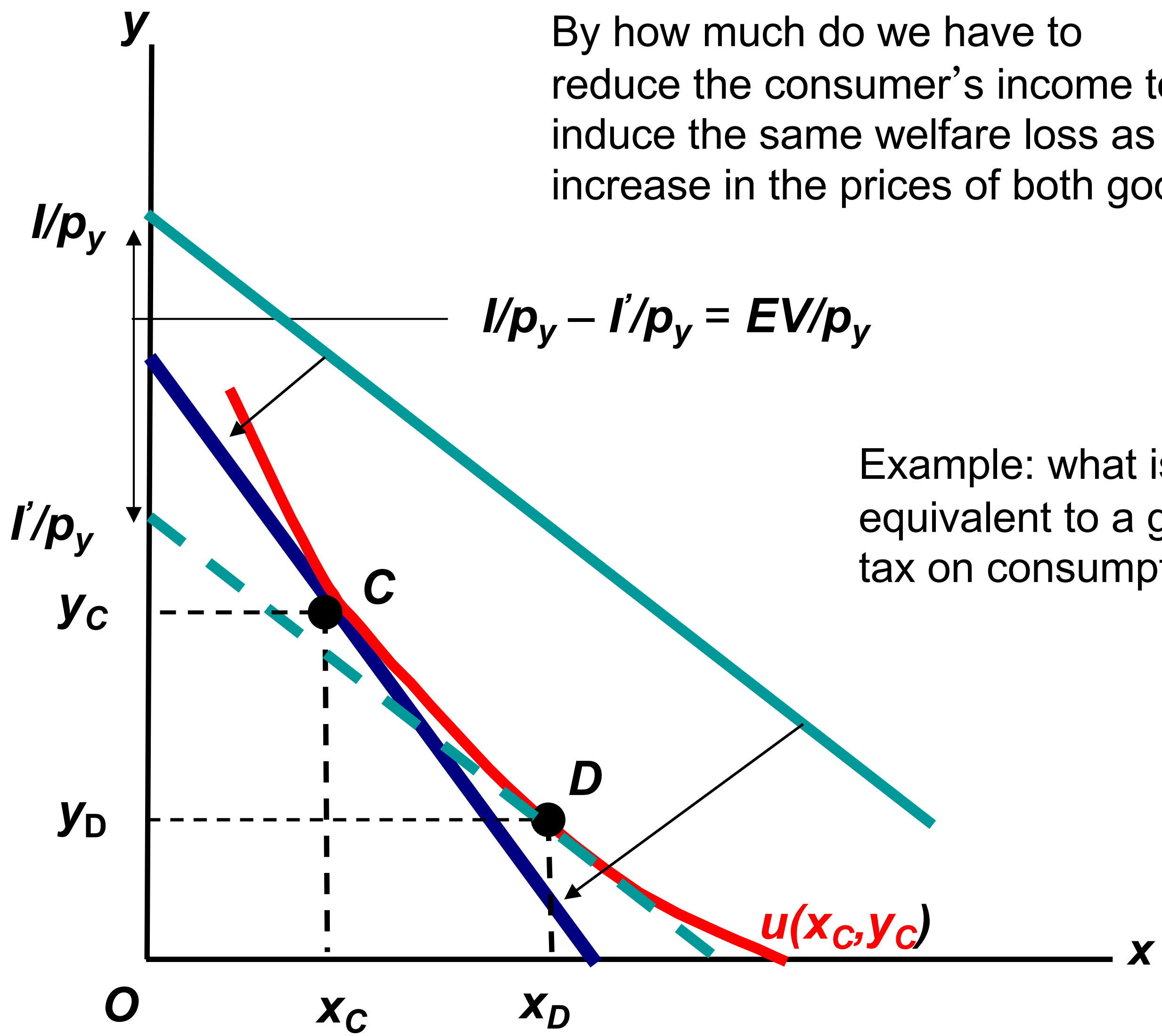
Normative Aspects : Equivalent Variation

By how much do we have to reduce the consumer's income to induce the same welfare loss as an increase in the price of x?



Example: what is the income equivalent to the introduction of a tax over the consumption of a good?

Equivalent Variation: Another Example



By how much do we have to reduce the consumer's income to induce the same welfare loss as an increase in the prices of both goods?

Example: what is the income equivalent to a generalized sale tax on consumption goods?

Compensated Variation: An Example

Assume: $u(x,y)=xy$, $(p_x,p_y)=(1,2)$, $I=12$, and $(p'_x,p'_y)=(3,3)$.

A.

$x_A + 2y_A = 12$

$y_A/x_A = 1/2.$

Solving: $(x_A,y_A)=(6,3)$, $u(x_A,y_A)=18$.

B.

$x_By_B = 18$

$y_B/x_B = 3/3.$

Solving: $(x_B,y_B)=(\sqrt{18},\sqrt{18})$, $I' = 3\sqrt{18} + 3\sqrt{18} = 25.456$.

$$CV = I' - I = 25.456 - 12 = 13.456.$$

Equivalent Variation: An Example

Assume: $u(x,y)=xy$, $(p_x,p_y)=(1,2)$, $I^0=12$, and $(p'_x,p'_y)=(3,3)$.

C.

$3x_C + 3y_C = 12$

$y_C/x_C = 3/3$.

Solving: $(x_C,y_C) = (2,2)$, $u(x_C,y_C) = 4$.

D.

$x_D y_D = 4$

$y_D/x_D = 1/2$.

Solving: $(x_D,y_D)=(2\sqrt{2}, \sqrt{2})$, $I^D= 4\sqrt{2} =5.656$.

$$EV = I – I' = 12 - 5.656 = 6,344.$$

Note. Assume that the increase in prices is due to a tax of 2€ on x and 1€ on y. The amount collected as tax is

$$2x_C + y_C = 6 < 6.344 = EV.$$

Price Indices

A price index provides a *summary statistics* of the changes observed on a set of prices between a base period 0 and a period t.

A *Laspeyres price index* identifies the change in the cost of a particular consumption bundle $x = (x_1, \ldots, x_I)$ between a base period 0 and a period t:

$$L_t = \sum_i p_{ti}\, x_i / \sum_i p_{0i}\, x_i.$$

Price Indices

Assume that Esther and Claudia, who are sisters, have the same preferences.

Esther started university in 2005 with a “discretionary”budget of €300.

In 2015, Claudia started university, and her parents promised her a “purchasing power equivalent budget”.

Price Indices: an Example

	Esther 2010	Claudia 2020
Price of food	10€/unit	15€/unit
Price of books	20€/book	40€/book
Quantity food	10	?
Quantity books	10	?
Expenditure	300€	€?

Price Indices: an Example

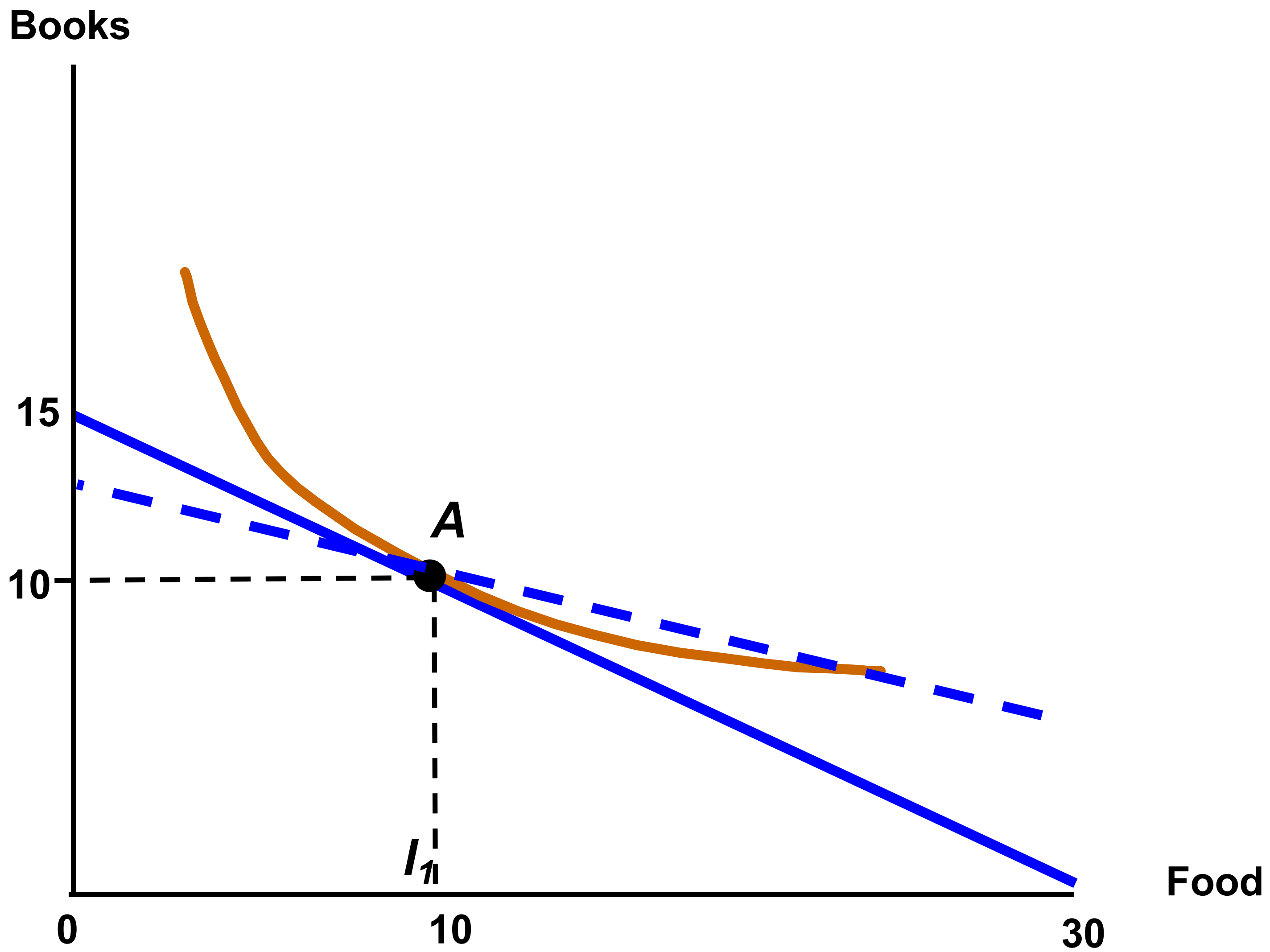
Laspeyres CPI: $LCPI_t$

Esther's expenditure: 300€ = 10 x 10 + 10 x 20

Claudia's expenditure: 550€ = 10 x 15 + 10 x 40

$$LCPI_t = 550 / 300 = 1,83.$$

Price Indices: an Example



Price Indices: an Example

If Esther and Claudia's parent know their preferences, then calculating Claudia's budget is simple.

Assume that their preferences are represented by the utility function

$$u(x,y) = xy^2.$$

Then the utility of Esther in 2010 was

$$u(10,10) = 10(10)^2 = 10^3 .$$

Price Indices: an Example

A the prices of 2020 the cheapest consumption bundle that allows allow the welfare level of Esther in 2010 solves the system of equations

$$xy^2 = 10^3$$

$$y/2x = 15/40$$

The solution to this system is $(x_{2020}, y_{2020}) = (12{,}1,\ 9)$.

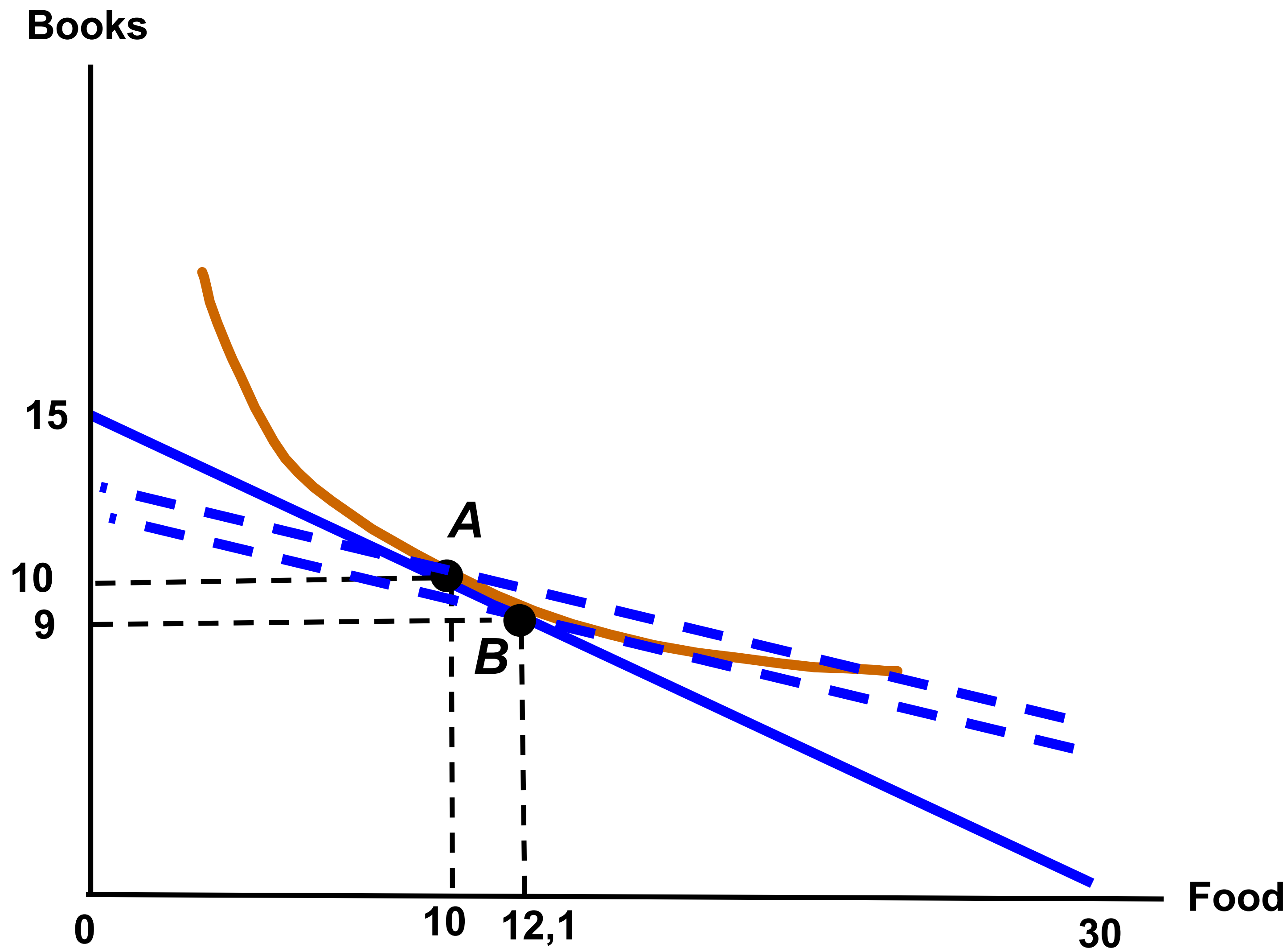
Hence the parent must assign Claudia an income

$$12{,}1 \text{ x } 15 + 9 \text{ x } 40 = 541{,}5\ €.$$

And the *true* consumer price index is

$$TCPI^*_t = 541{,}5 / 300 = 1{,}805.$$

Price Indices: an Example



Price Indices

The Laspeyres index *overestimates* the ideal index of the cost of living because it assumes that consumers do not react to price changes.

That is, it ignores that consumers will exploit the possibilities of *substitution* between goods, buying more (less) of the relatively cheaper (more expensive) goods.

Price Indices

The Spanish Instituto Nacional de Estadística (INE) publishes monthly a *CPI* obtained as a weighted average of the monthly variations of the prices of consumption goods,

$$\sum_i w_i \, (p_{ti} / p_{0i}),$$

where $w = (w_1, \ldots, w_l)$ are weights consistent with the consumption bundle of the representative of Spanish household.

How are the weights w_i calculated in practice?

Price Indices

Using data from the Encuesta de Presupuestos Familiares (EPF). Each household $h \in \{1,\ldots,H\}$ in the EPF reports its expenditure on each good $i \in \{1,\ldots,I\}$. Denote by

x^h_i : *consumption of household* h *good* i,

$x_i = \sum_h x^h_i$: *total consumption of good* i *by all* h,

$g_i = p_{oi}\, x_i$: total expenditure in good i,

$g = \sum_i p_{oi}\, x_i$: total expenditure.

Price Indices

In the INE's CPI, $w_i = g_i \,/\, g.$

Is this a Laspeyres index?

Yes:

$$\begin{aligned}
\textstyle\sum_i w_i\,(p_{ti}\,/p_{0i}) &= \textstyle\sum_i (g_i/g)(p_{ti}\,/p_{0i}) \\
&= (\textstyle\sum_i p_{0i}\,x_i\,(p_{ti}\,/p_{0i}))\,/\,\textstyle\sum_i p_{0i}\,x_i \\
&= \textstyle\sum_i p_{ti}\,x_i / \textstyle\sum_i p_{0i}\,x_i \\
&= LCPI_t.
\end{aligned}$$

Price Indices

Boskin's Committee estimated an upwards bias of the USA CPI (around 1,1% on 1995) of about 0,40

The *substitution effect* accounted for only 0.25% of this bias. The remaining 0.15% was due to the procedure used in sampling prices, which ignores consumers' *arbitrage*.

Boskin's Committee identified other biases arising from changes in the quality of the goods, which the CPI does not account for.

Price Indices

The estimated the CPI and its use in practice (e.g., to revise social security pensions or wages) may have significant consequences on *income distribution*.

The CPIs of the household h in the EPF sample is:

$$LCPI_t^h = \sum_i w_i^h \,(p_{ti} / p_{0i}) = \sum_i (g_i^h / g^h)\,(p_{ti} / p_{0i}),$$

where

$g^h_i = p_{oi}\, x^h_i$: *expenditure household* h *in good* i,

$g^h = \sum_i g^h_i$: *total expenditure household* h.

Price Indices

$$LCPI_t = \sum_i w_i\,(p_{ti} / p_{0i})$$

$$= \sum_i (g_i/g)(p_{ti} / p_{0i})$$

$$= \sum_i (\sum_h g_i{}^h/g)(p_{ti} / p_{0i})$$

$$=\sum_h \sum_i (g^h/g)(g_i{}^h/g^h)(p_{ti} / p_{0i})$$

$$=\sum_h (g^h/g)\sum_i (g_i{}^h/g^h)(p_{ti} / p_{0i})$$

$$=\sum_h (g^h/g)LCPI_t^h$$

Price Indices

Therefore, the LCPI follows more closely the consumption habits of the richer households in the sample that those of the poorer households.

For this reason, Laspeyres consumer price indices are often referred to as *plutocratic indices*.

Price Indices

However, there is not difficulty in estimating a *democratic* CPI; that is, an index in which every family has the same weight :

$$DCPI_t = \sum_h cpi^h_t \,/\, H.$$

Price Indices

The difference between these two indices,

$$LCPI_t - DCPI_t,$$

which is known as the *plutocratic gap*, has an interesting interpretation: when it is positive, the prices of the goods consumed more by the richer households (e.g., luxury goods) increase more than those consumed more by poorer households (e.g., normal or inferior goods), then the *plutocratic* LCPI overestimates the CPI of the poorer households.

Price Indices

When the cpi^h_t of the richer households are greater than the ones of the poorer households, since the weights a_h of richer households are also greater than the ones of the poorer households, the *plutocratic gap* is positive,

$$LCPI_t - DCPI_t > 0.$$

(Of course, the effect would be opposite when the prices of luxury goods increase less than those of the normal and/or inferior goods.)

Price Indices

Ruiz-Castillo, Ley e Izquierdo (2003), estimates that the plutocratic gap in Spain was 0.234% for the period 1973- 1981, 0.091% for the period 1981-1991, and 0.055% for the period 1991-1998.

(Compare to the substitution bias, which is estimated at 0,25% a year.)

Price Indices

To judge the importance of this gap is enough to relate it with the usually accepted estimation of the magnitude of the CPI bias due to substitution effect: 0.25% per year.

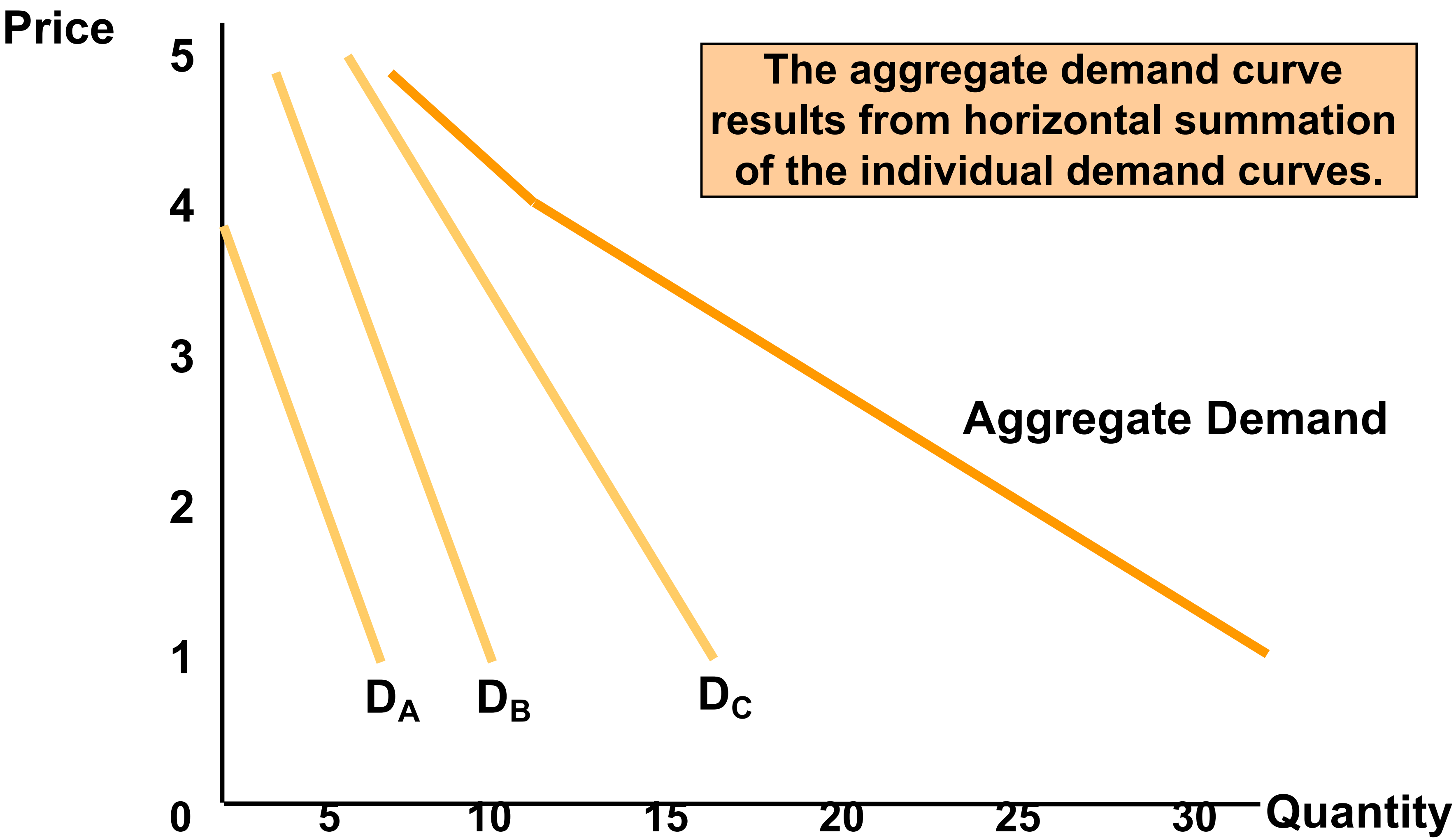
Aggregate Demand

- Relates the *aggregate* demand of all consumers for a certain good with its price (Market demand).
- Summation of the individual demand curves.

Aggregate Demand

Price	Consumer *A*	Consumer B	Consumer C	Market
1	**6**	**10**	**16**	**32**
2	**4**	**8**	**13**	**25**
3	**2**	**6**	**10**	**18**
4	**0**	**4**	**7**	**11**
5	**0**	**2**	**4**	**6**

Aggregate Demand



Aggregate Demand

Example: There are three consumers with preferences represented by the utility function *$u(x,y)=xy$*, and incomes $I_1 = 20$, $I_2 = 100$, and $I_3 = 160$.

Individual demand**:**

$x_i\,(p_x, p_y, I) = I\,/\,(2p_x)$.

Aggregate demand:

$$X(p_x, p_y, I_1, I_2, I_3) = (I_1+I_2+I_3)\,/\,(2p_x).$$

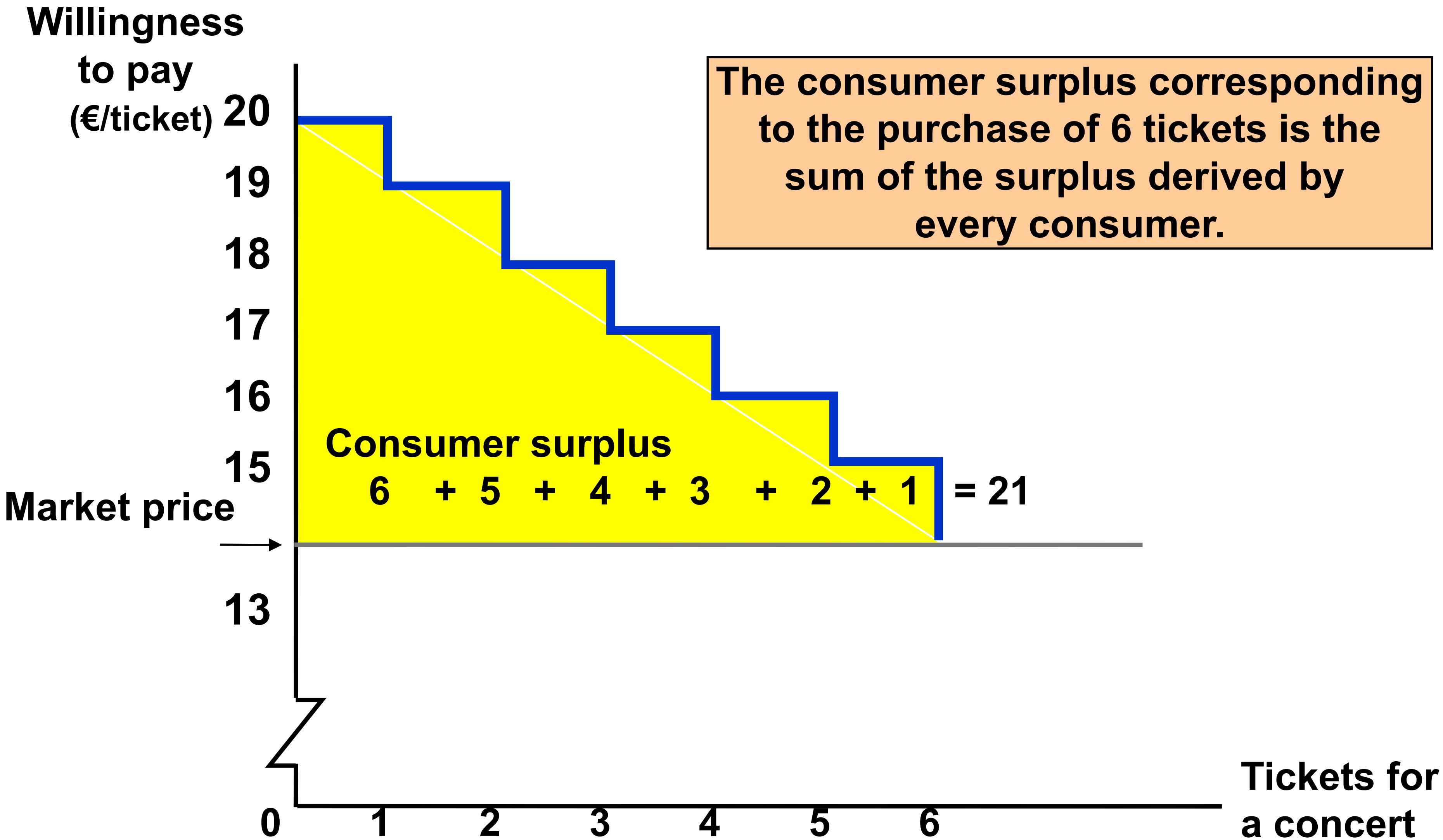
Consumer Surplus

Measures the benefits consumers derive from participating in the market

- Willingness to pay: maximum price a consumer is willing to pay for a good

- Consumer surplus: difference between a buyers’ willingness to pay and the price he actually pays

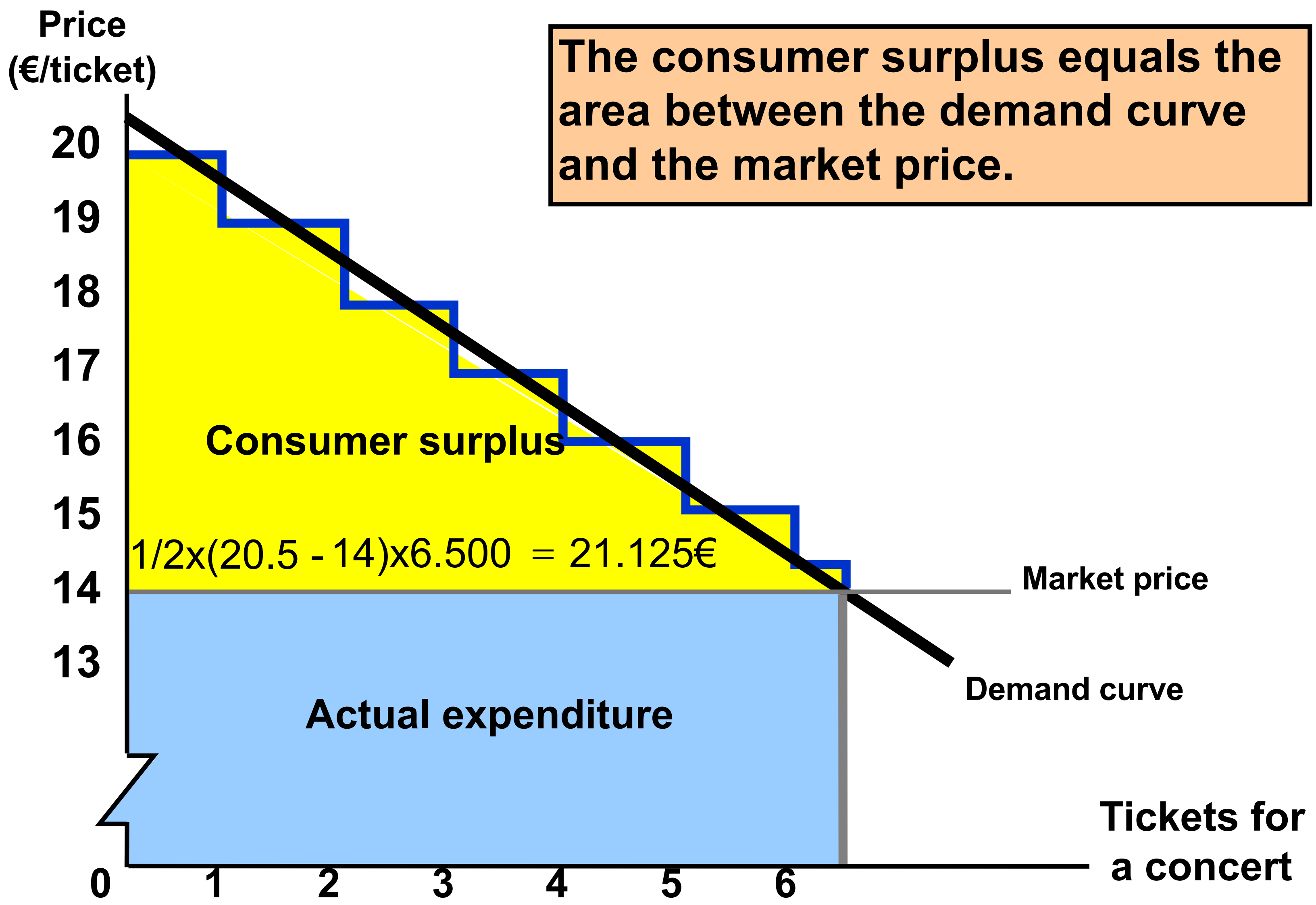
Consumer Surplus



The consumer surplus corresponding to the purchase of 6 tickets is the sum of the surplus derived by every consumer.

Consumer Surplus

The consumer surplus equals the area between the demand curve and the market price.



Consumer Surplus

Example: Suppose aggregate demand is Q = 10 - 2P. Calculate total expenditure and consumer surplus when price is P = 1.

Q = 10 - 2 = 8.   Total expenditure = 8.

CS = 1/2 (5-1) 8 = 16

Calculate the loss in consumer surplus when the price increases to P = 2.

Q = 10 - 4 = 6   CS = 1/2 (5-2) 6 = 9

Loss in consumer surplus = 7