Endogenous Income

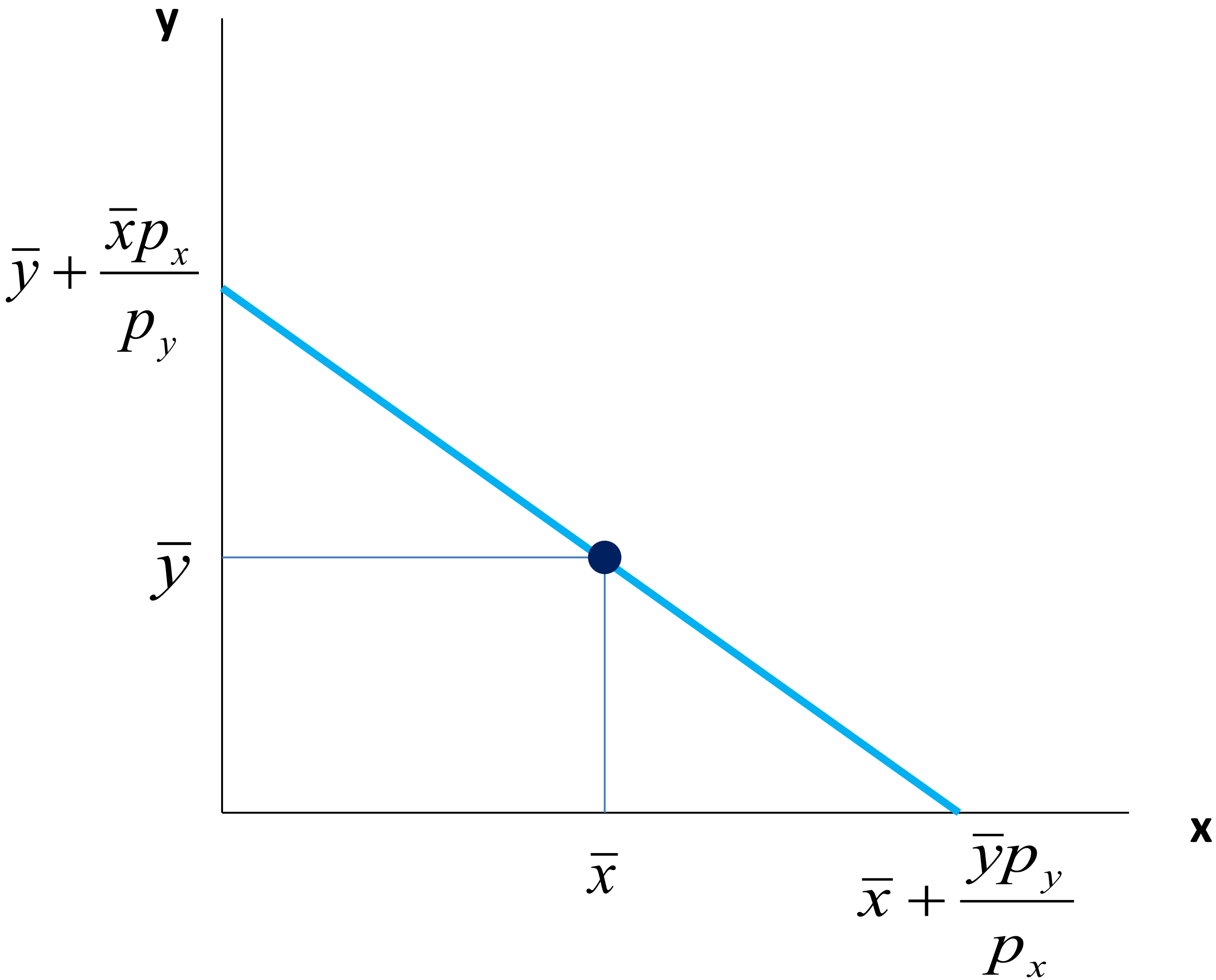
The consumption-leisure model

Modifying consumer's problem

- For the moment, assume there is no additional exogenous income
- Consumer's income is the market value of her initial endowment, $(\bar{x}, \bar{y})$
- Given market prices p_x and p_y, the consumer's budget constraint is

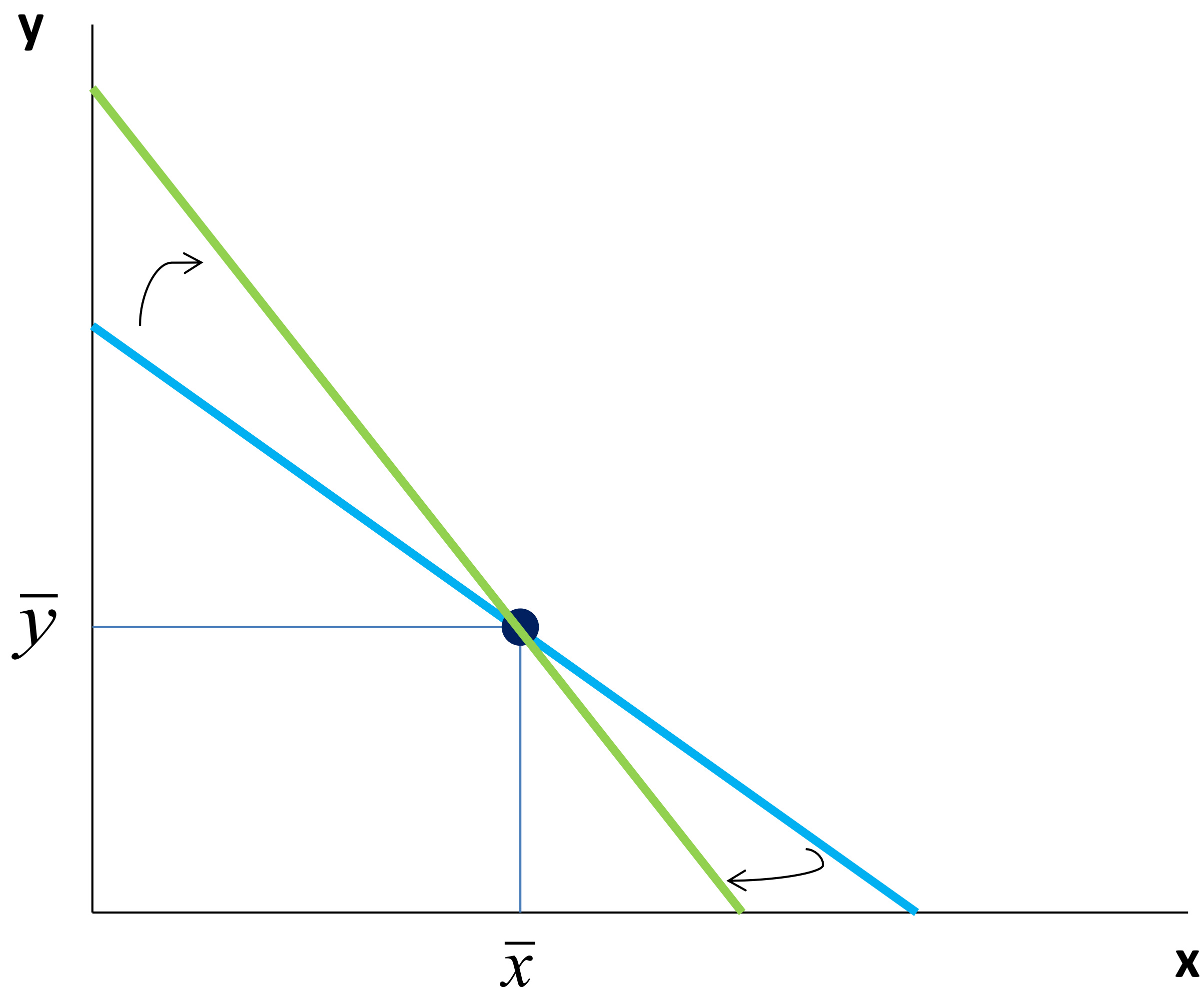
$$xp_x + yp_y \leq \bar{x}p_x + \bar{y}p_y$$

Budget constraint



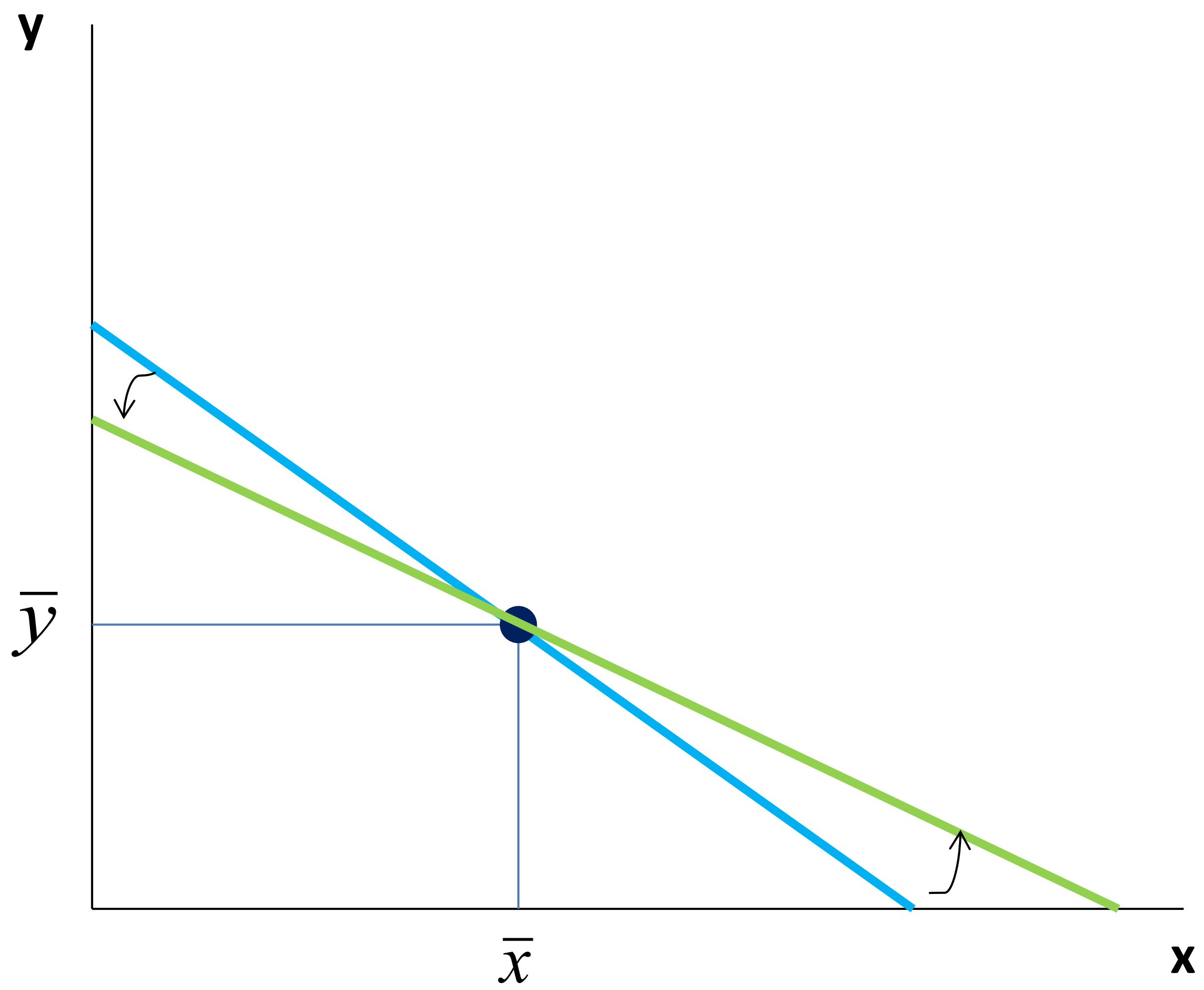
Changes in prices

An increase in p_x



Changes in prices

An increase in p_y



Changes in prices

- The initial endowment vector is on the consumer’s budget set whatever the prices – it is always feasible to not trade.
- The consumer’s budget set does not shrink when the price of a good increases: the rotation of the budget line around the initial endowment vector implies that the consumer can afford new bundles with more of the now relative cheaper good.
- If the consumer’s initial bundle contains a large amount of the good now relatively more expensive, then the consumer becomes *richer*.

Consumer Demand

- Assume $u(x, y)$ derivable and the system

$$xp_x + yp_y = \bar{x}p_x + \bar{y}p_y$$

$$MRS(x, y) = \frac{p_x}{p_y}$$

yields an interior solution to the consumer’s problem

$\tilde{x}(p_x, p_y)$, $\tilde{y}(p_x, p_y)$

- That is,

$$\tilde{x}(p_x, p_y) = x^*(p_x, p_y, \bar{x}p_x + \bar{y}p_y)$$
$$\tilde{y}(p_x, p_y) = y^*(p_x, p_y, \bar{x}p_x + \bar{y}p_y)$$

where the functions on the RHS are the ordinary demands.

Consumer Demand: example

$$u(x,y) = x\sqrt{y}\,; \quad (\bar{x},\bar{y}) = (2,1)$$

Calculate ordinary demands:

$$x^*(p_x, p_y, I) = \frac{2I}{3p_x}$$

$$y^*(p_x, p_y, I) = \frac{I}{3p_y}$$

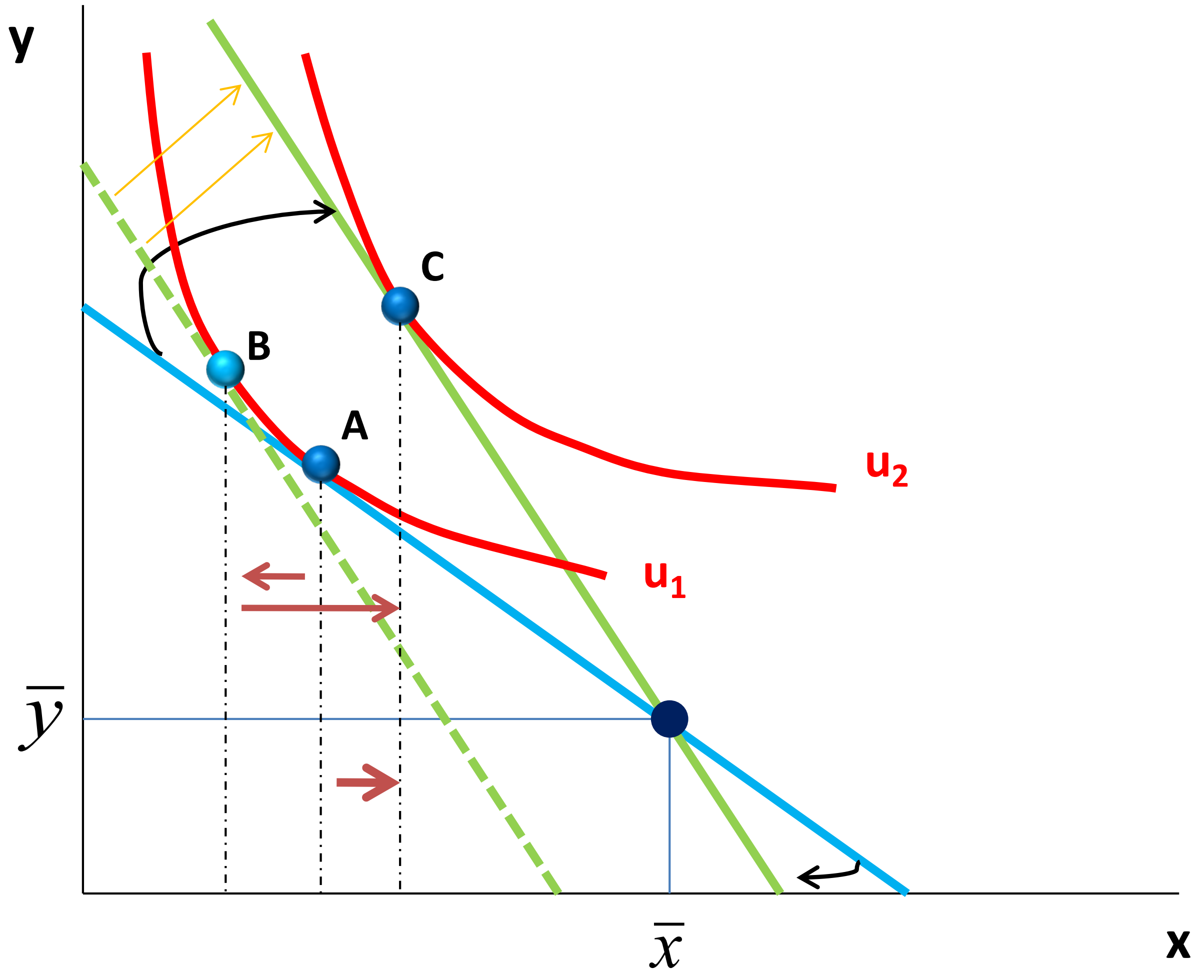
Hence,

$$\tilde{x}(p_x, p_y) = \frac{2(2p_x + p_y)}{3p_x} = \frac{4}{3} + \frac{2p_y}{3p_x}$$

$$\tilde{y}(p_x, p_y) = \frac{2p_x + p_y}{3p_y} = \frac{1}{3} + \frac{2p_x}{3p_y}$$

Income and substitution effects

- Study the effect of an increase in p_x for utility function $u(x, y)$ and endowment $(\bar{x}, \bar{y})$



Income and substitution effects

- Substitution effect is negative

$$SE = x_B - x_A < 0$$

- Income effect is positve

$$IE = x_C - x_B > 0$$

- In this case, income effect is larger than substitution effect, so the total effect is positive: an increase in p_x leads the consumer to increase its consumption of both goods

$$TE = SE + IE = x_C - x_A > 0$$

Income and substitution effects

Whether income is endogenous or exogenous, the substitution effect of an increase of the price of a good over its demand is negative.

However, when income is endogenous the income effect decomposes into two effects:

(1) *Ordinary income effect,* whose sign depends on whether the good is normal (N) or inferior (I).

(2) *Endowment effect,* whose sign depends on whether at the original prices the consumer was a net seller (S) or a net buyer (B) of the good.

The sign of the total income effect (TIE) is negative in the cases N-B and I-S, whereas it is ambiguous in the cases N-S and I-B.

Income and substitution effects

Total effect = substitution effect (SE) + (ordinary income effect + endowment effect)

= SE + TIE.

Formally,

$$\frac{\partial \tilde{x}}{\partial p_x} = \frac{\partial x^*}{\partial p_x}\Big|_{u=cte} - x\frac{\partial x^*}{\partial I} + \bar{x}\frac{\partial x^*}{\partial I}$$

$$= \frac{\partial x^*}{\partial p_x}\Big|_{u=cte} - \frac{\partial x^*}{\partial I}(x - \bar{x})$$

Income and substitution effects

¿When is the TIE negative?

	Normal good $\left(\frac{\partial x^*}{\partial I} > 0\right)$	**Inferior good** $\left(\frac{\partial x^*}{\partial I} < 0\right)$
Net buyer $(x - \bar{x} > 0)$	$TIE < 0$	$TIE > 0$
Net seller $(x - \bar{x} < 0)$	$TIE > 0$	$TIE < 0$

- If TIE < 0, then SE + TIE < 0.
- If TIE > 0, then the sign of SE + TIE is ambiguous.

Income and substitution effects

- When is the total income effect positive (the consumer is richer)?

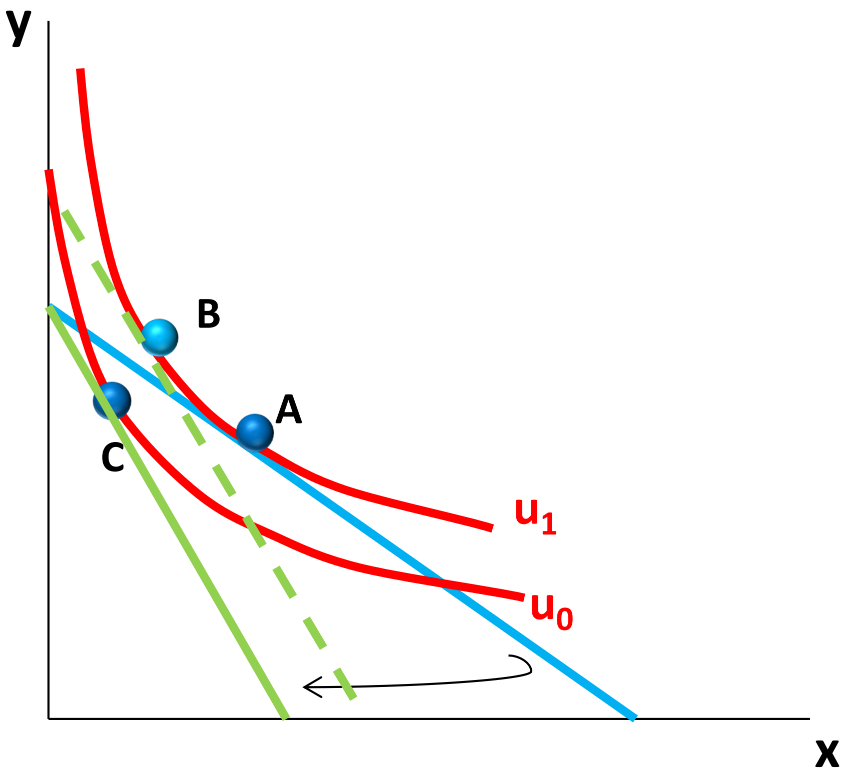
If the good is normal and the consumer is a net seller of the good

$$-\frac{\partial x^*}{\partial I}(x-\overline{x})>0 \quad if \quad \frac{\partial x^*}{\partial I}>0 \text{ and } x-\overline{x}<0$$

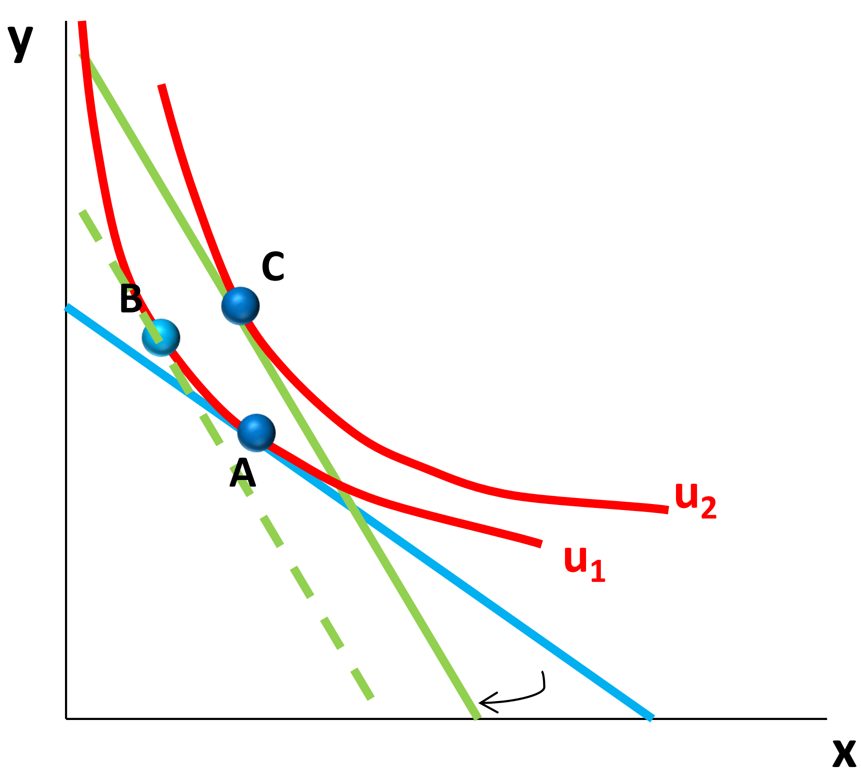
- In any case, notice that positive income effect (“richer consumer”) does not mean more consumption: we have to take into account the substitution effect

Income and substitution effects

Exogenous income



Endogenous income



The consumption-leisure model. Labor supply

- Two goods: leisure (x-axis) and consumption (y-axis)

- *Leisure*, denoted by h and measured in hours. The wage per hour (or price of leisure) is denoted by w.

- *Consumption*, denoted by c and measured in euros. The price of c is therefore $p_c = 1$).

- Initial endowment is (M,H), where:

- M : initial exogenous wealth (or non-labor income).

- H : number of hours available for leisure and work.

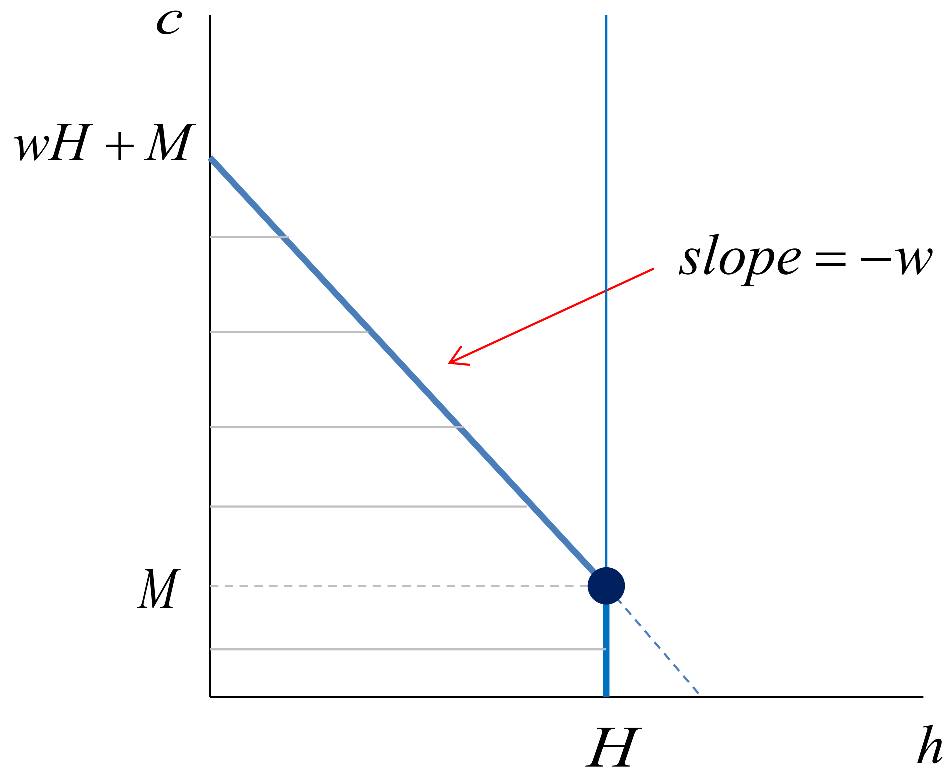
The consumption-leisure model. Labor supply

- Budget set (recall $p_c = 1$)

$$c + hw \leq wH + M$$

hw : expenditure on leisure

$wH + M$: monetary value of initial endowment



The consumption-leisure model. Labor supply

- Solve the problem as usual, but watch out for additional constraints

$$
\begin{aligned}
Max_{c,h} \quad & u(c,h) \\
st \quad & c + hw \leq wH + M \\
& 0 \leq h \leq H \\
& c \geq 0
\end{aligned}
$$

The consumption-leisure model. Labor supply. *Example*

$$Max_{c,h} \quad c + 2\ln h$$

$$st \quad c + wh = 16w + 4$$

$$0 \le h \le 16$$

$$c \ge 0$$

Interior solution requires

$$MRS(h,c) = w \Leftrightarrow \frac{2}{h} = w \Rightarrow h(w) = \frac{2}{w}$$

And

$$h(w) = \frac{2}{w} \ge 0 \Leftrightarrow \forall w > 0$$

$$h(w) = \frac{2}{w} \le 16 \Leftrightarrow w \ge \frac{1}{8}$$

The consumption-leisure model. Labor supply. *Example*

Therefore,

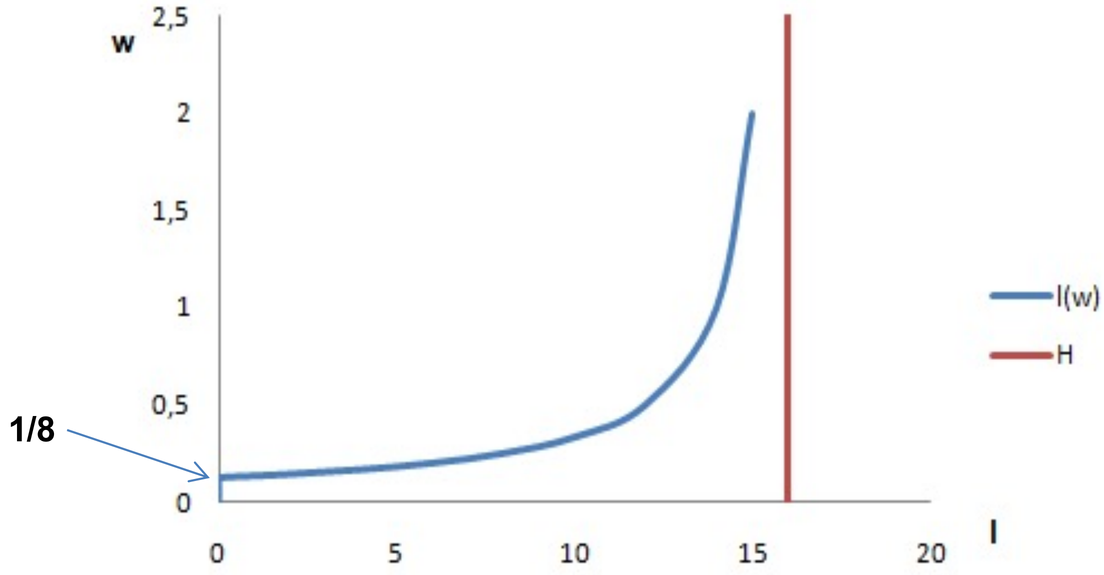
$$h(w) = \begin{cases} 16 & \text{if } w < 1/8 \\ \dfrac{2}{w} & \text{if } w \geq 1/8 \end{cases}$$

$$c(w) = \begin{cases} 4 & \text{if } w < 1/8 \\ 2 + 16w & \text{if } w \geq 1/8 \end{cases}$$

The consumption-leisure model. Labor supply. *Example*

And labor supply

$$l(w) = H - h(w) = \begin{cases} 0 & if \quad w < 1/8 \\ 16 - \dfrac{2}{w} & if \quad w \geq 1/8 \end{cases}$$



The consumption-leisure model. Labor supply. *Effect of changes in wages*

- Assume $w' < w$

- For interior solutions, the consumer is a net supplier of leisure

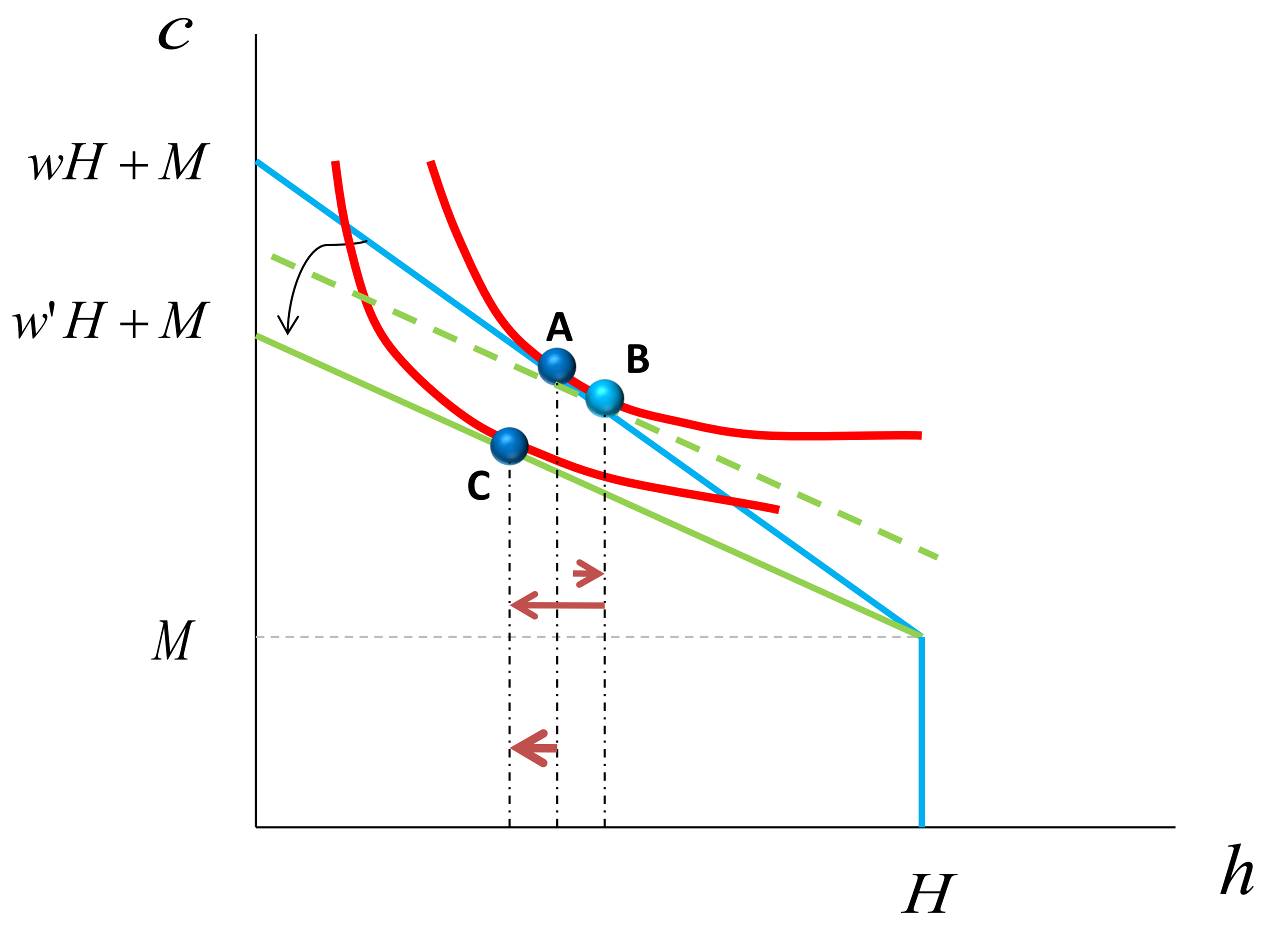
- Total income effect: <u>if</u> leisure is a normal good, $\partial h / \partial I > 0$, TIE is positive, leading the consumer to demand less leisure (or supply more labor)

$$-\underbrace{\frac{\partial h}{\partial I}}_{>0}\underbrace{(l - H)}_{<0} > 0$$

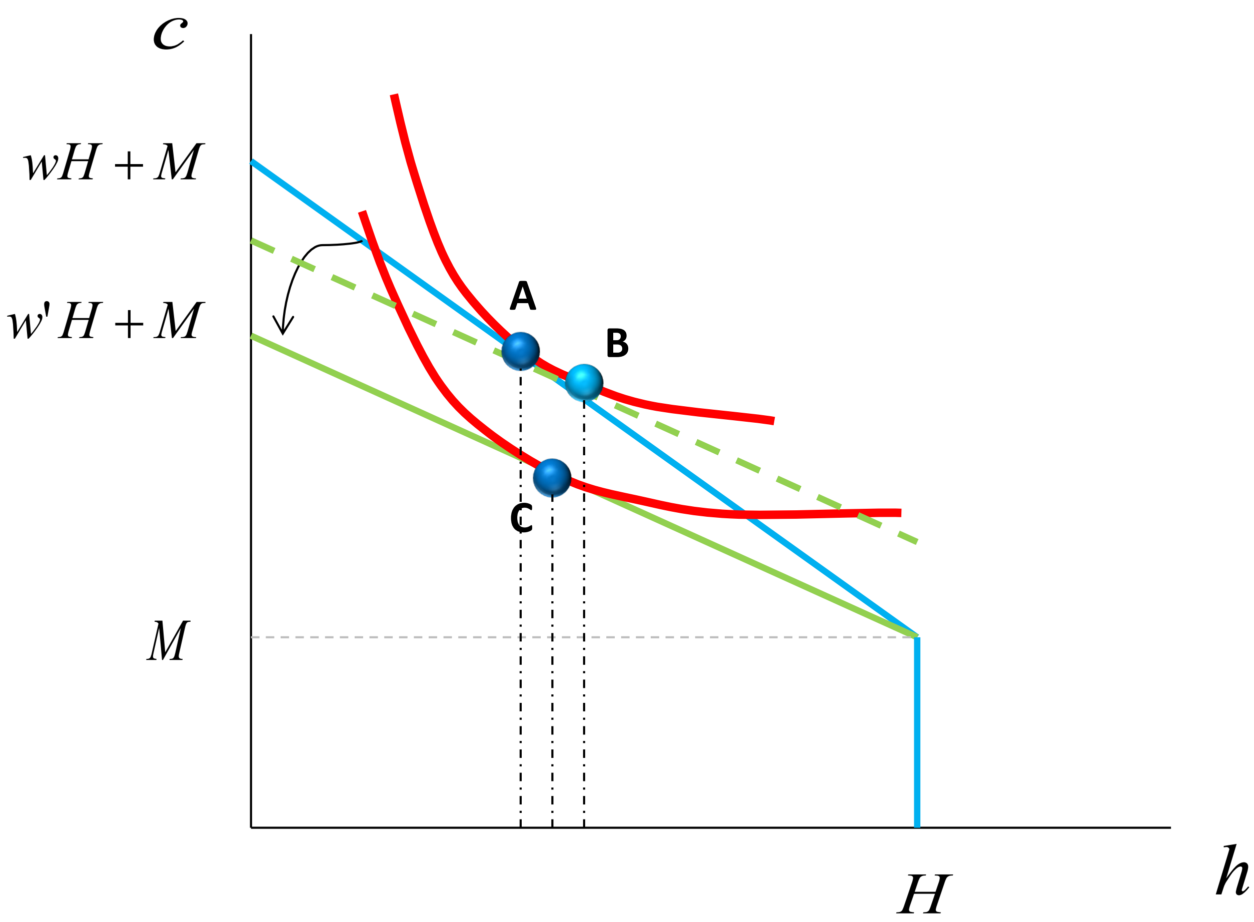
- Substitution effect : is always non-positive. Since leisure is cheaper, this effect leads the consumer to demand more leisure (or supply less labor)

- Total effect is ambiguous (it depends on the shape of the utility function)

The consumption-leisure model.
Labor supply. *Effect of changes in wages*

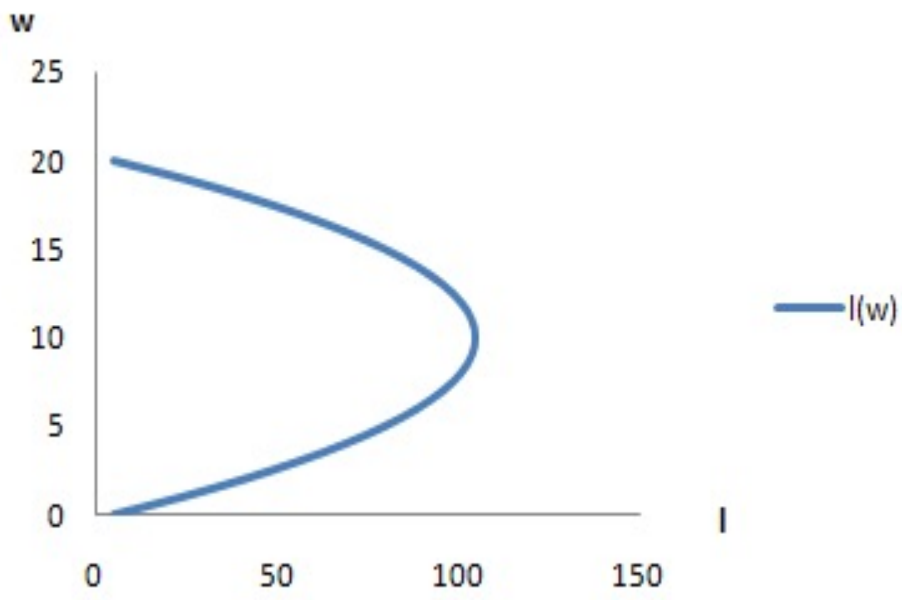


The consumption-leisure model. Labor supply. *Effect of changes in wages*



The consumption-leisure model. Labor supply. *Effect of changes in wages*

For $w \in (0,10)$, SE dominates (leisure is more expensive and consumer offers more labor)



For $w \in (10,20)$, TIE dominates (consumer is richer and does not need to work as much as before)

Application: a tax on labor income

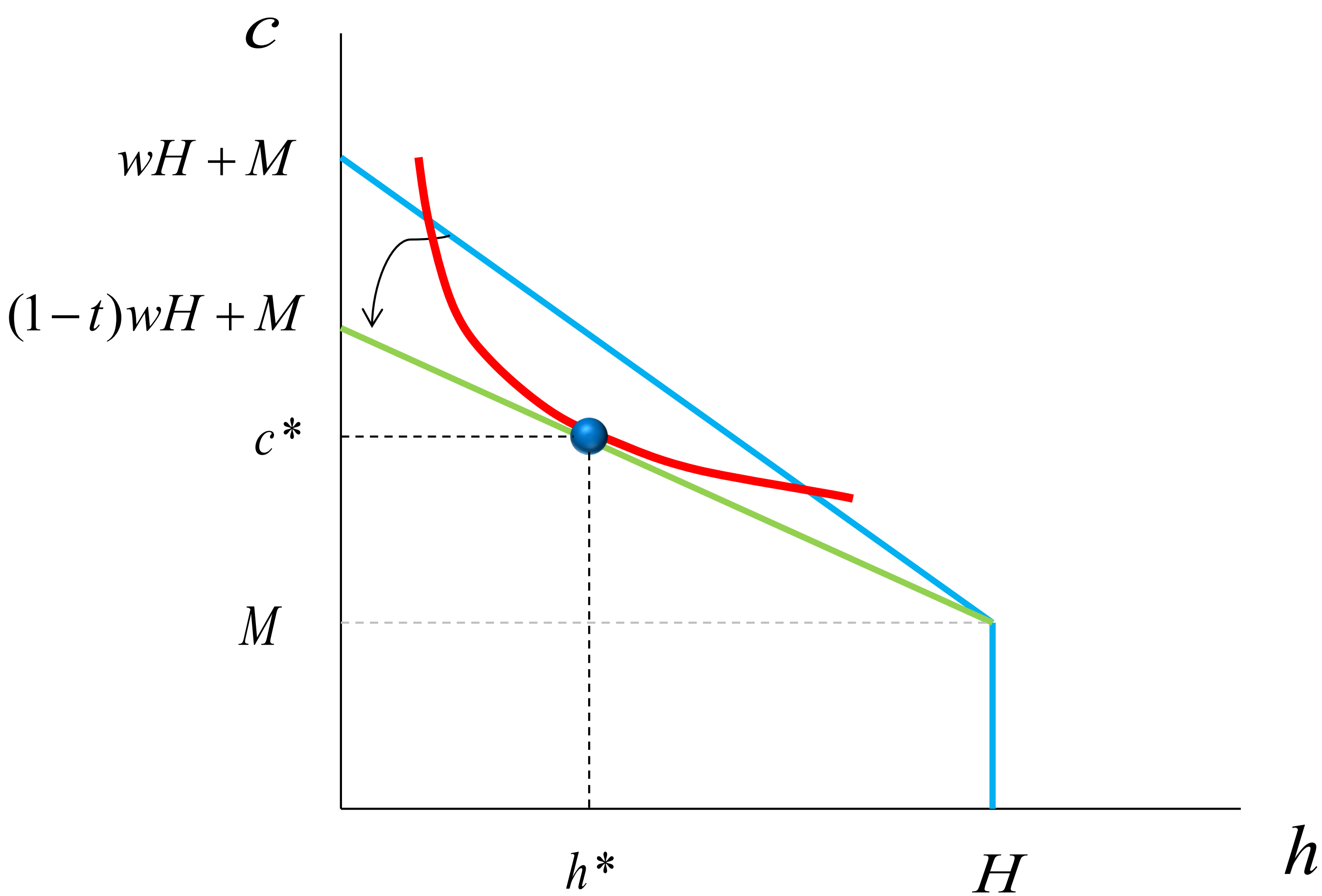
- Impose $t \in [0,1]$
- The new budget constraint is

$$c + (1-t)wh \leq (1-t)wH + M$$

- The tax is equivalent to a reduction of wage: its impact on leisure consumption (or labor supply) is ambiguous

- Its impact on welfare is unambiguous. Of course, tax policies have other objectives we are not considering here.

Application: a tax on labor income



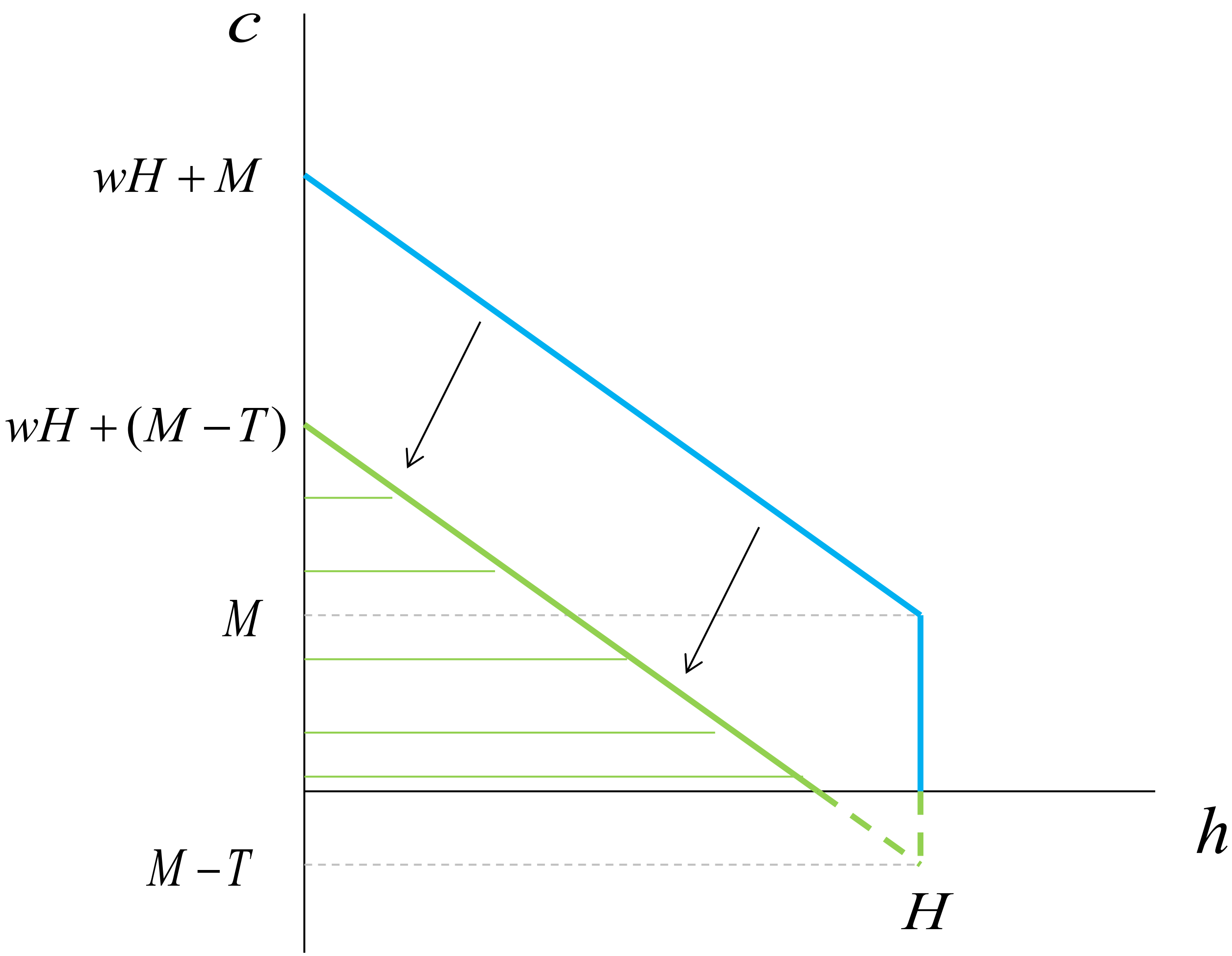
Application: a tax on labor income

- Alternative: a non-labor income tax *T.*
- The new budget constraint is

$$c + wh \leq wH + (M - T)$$

- If both goods are normal, then the introduction of *T* reduces their demands (increases labor supply, in particular)

Application: a tax on labor income



Application: a tax on labor income

- Exercise: what if $T = tw(H\text{-}h^*)$?
- Hint: is (c*,h*) optimal for *T*?