Endogenous Income

The consumption-leisure model
Modifying consumer’s problem

• For the moment, assume there is no additional exogenous income

• Consumer’s income is the market value of her initial endowment, \((\bar{x}, \bar{y})\)

• Given market prices \(p_x\) and \(p_y\), the consumer’s budget constraint is

\[
xp_x + yp_y \leq \bar{x}p_x + \bar{y}p_y
\]
Budget constraint

\[ \bar{y} + \frac{x p_x}{p_y} \]

\[ \bar{y} \]

\[ \bar{x} \]

\[ \bar{x} + \frac{\bar{y} p_y}{p_x} \]
Changes in prices

An increase in $p_x$
Changes in prices

An increase in $p_y$
Changes in prices

• The impact of price changes in the consumer’s budget set is now more subtle: the increase of a price may make the consumer “relatively richer” if she is “relatively rich” in that good (i.e., if her endowment of this good is large)

• Notice that, independently of market prices, the endowment is always a feasible bundle (consumer always can avoid trade and consume her own endowment)
Consumer Demand

• Assume \( u(x, y) \) derivable and the system

\[
x p_x + y p_y = x p_x + y p_y
\]

\[
MRS(x, y) = \frac{p_x}{p_y}
\]

yields an interior solution to the consumer’s problem

\( \tilde{x}(p_x, p_y), \tilde{y}(p_x, p_y) \)

• That is,

\[
\tilde{x}(p_x, p_y) = x^*(p_x, p_y, x p_x + y p_y)
\]

\[
\tilde{y}(p_x, p_y) = y^*(p_x, p_y, x p_x + y p_y)
\]

where the functions on the RHS are the ordinary demands.
Consumer Demand: example

\[ u(x, y) = x\sqrt{y}; \quad (\bar{x}, \bar{y}) = (2,1) \]

Calculate ordinary demands:

\[ x^*(p_x, p_y, I) = \frac{2I}{3p_x} \]
\[ y^*(p_x, p_y, I) = \frac{I}{3p_y} \]

Hence,

\[ \bar{x}(p_x, p_y) = \frac{2(2p_x + p_y)}{3p_x} = \frac{4 + 2p_y}{3p_x} \]
\[ \bar{y}(p_x, p_y) = \frac{2p_x + p_y}{3p_y} = \frac{1 + 2p_x}{3p_y} \]
Income and substitution effects

- Study the effect of an increase in $p_x$ for utility function $u(x, y)$ and endowment $(\bar{x}, \bar{y})$
Income and substitution effects

• Substitution effect is negative

\[ SE = x_B - x_A < 0 \]

• Income effect is positive

\[ IE = x_C - x_B > 0 \]

• In this case, income effect is larger than substitution effect, so the total effect is positive: an increase in \( p_x \) leads the consumer to increase its consumption of both goods

\[ TE = SE + IE = x_C - x_A > 0 \]
Income and substitution effects

Suppose an increase in $p_x$.

The substitution effect is the same as in the case with exogenous income.

The income effect of a price increase now involves two components:

1. It makes the consumer poorer as it becomes more expensive buying the optimal amount of the good (%primary_language:en%ordinary income effect), and

2. It modifies the value of the consumer’s endowment (%primary_language:en%endowment effect).

The sign of the (total) income effect depends on whether the consumer is a net seller (negative) or a net buyer (positive) of the good whose price has increased.
Income and substitution effects

Total effect = substitution effect + ordinary income effect + endowment effect
= substitution effect + (total) income effect

Formally,

\[
\frac{\partial x}{\partial p_x} = \frac{\partial x^*}{\partial p_x} \bigg|_{u=cte} - x \frac{\partial x^*}{\partial I} + \bar{x} \frac{\partial x^*}{\partial I}
\]

\[
= \frac{\partial x^*}{\partial p_x} \bigg|_{u=cte} - \frac{\partial x^*}{\partial I}(x - \bar{x})
\]
Income and substitution effects

• When is the total income effect positive (the consumer is richer)?

If the good is normal and the consumer is a net seller of the good

\[-\frac{\partial x^*}{\partial I}(x - \bar{x}) > 0 \quad if \quad \frac{\partial x^*}{\partial I} > 0 \quad and \quad x - \bar{x} < 0\]

• In any case, notice that positive income effect (“richer consumer”) does not mean more consumption: we have to take into account the substitution effect
Income and substitution effects

Exogenous income

Endogenous income
The consumption-leisure model.
Labor supply

• Two goods: leisure (x-axis) and consumption (y-axis)
  - *Leisure*, denoted by \( h \) and measured in hours. The wage per hour (or price of leisure) is denoted by \( w \).
  - *Consumption*, denoted by \( c \) and measured in euros. The price of \( c \) is therefore \( p_c = 1 \).

• Initial endowment is \( (M,H) \), where:
  - \( M \) : initial exogenous wealth (or non-labor income).
  - \( H \) : number of hours available for leisure and work.
The consumption-leisure model.
Labor supply

• Budget set (recall $p_c = 1$)

$$c + hw \leq wH + M$$

- $hw$: expenditure on leisure
- $wH + M$: monetary value of initial endowment

[Diagram of budget line with slope $-w$ and intercepts at $M$ and $H$.]
The consumption-leisure model.
Labor supply

- Solve the problem as usual, but watch out for additional constraints

\[
\begin{align*}
\text{Max}_{c,h} & \quad u(c,h) \\
\text{st} & \quad c + hw \leq wH + M \\
& \quad 0 \leq h \leq H \\
& \quad c \geq 0
\end{align*}
\]
The consumption-leisure model.
Labor supply. *Example*

\[\max_{c,h} \quad c + 2 \ln h\]

\[s.t.\quad c + wh = 16w + 4\]
\[0 \leq h \leq 16\]
\[c \geq 0\]

Interior solution requires

\[MRS(h, c) = w \iff \frac{2}{h} = w \Rightarrow h(w) = \frac{2}{w}\]

And

\[h(w) = \frac{2}{w} \geq 0 \iff \forall w > 0\]
\[h(w) = \frac{2}{w} \leq 16 \iff w \geq \frac{1}{8}\]
The consumption-leisure model. Labor supply. Example

Therefore,

\[ h(w) = \begin{cases} 16 & \text{if } w < 1/8 \\ \frac{2}{w} & \text{if } w \geq 1/8 \end{cases} \]

\[ c(w) = \begin{cases} 4 & \text{if } w < 1/8 \\ 2 + 16w & \text{if } w \geq 1/8 \end{cases} \]
The consumption-leisure model.

Labor supply. *Example*

And labor supply

\[ l(w) = H - h(w) = \begin{cases} 
0 & \text{if } w < 1/8 \\
16 - \frac{2}{w} & \text{if } w \geq 1/8 
\end{cases} \]
The consumption-leisure model. Labor supply. *Effect of changes in wages*

- Assume $w' < w$

- For interior solutions, the consumer is a net supplier of leisure

- Total income effect: if leisure is a normal good, $\frac{\partial h}{\partial I} > 0$, TIE is positive, leading the consumer to demand less leisure (or supply more labor)

\[
-w \frac{\partial h}{\partial I} (l - H) > 0
\]

- Substitution effect: is always non-positive. Since leisure is cheaper, this effect leads the consumer to demand more leisure (or supply less labor)

- Total effect is ambiguous (it depends on the shape of the utility function)
The consumption-leisure model. Labor supply. *Effect of changes in wages*
The consumption-leisure model.
Labor supply. *Effect of changes in wages*
The consumption-leisure model.

Labor supply. *Effect of changes in wages*

For \( w \in (0,10) \), SE dominates (leisure is more expensive and consumer offers more labor)

For \( w \in (10,20) \), TIE dominates (consumer is richer and does not need to work as much as before)
Application: a tax on labor income

- Impose $t \in [0,1]$
- The new budget constraint is
  
  $$c + (1-t)wh \leq (1-t)wH + M$$

- The tax is equivalent to a reduction of wage: its impact on leisure consumption (or labor supply) is ambiguous

- Its impact on welfare is unambiguous. Of course, tax policies have other objectives we are not considering here.
Application: a tax on labor income

\[
\begin{align*}
\text{(1-}t)wH + M \\
wH + M \\
c^* \\
M
\end{align*}
\]
Application: a tax on labor income

- Alternative: a non-labor income tax $T$.
- The new budget constraint is
  \[ c + wh \leq wH + (M - T) \]
- If both goods are normal, then the introduction of $T$ reduces their demands (increases labor supply, in particular)
Application: a tax on labor income
Application: a tax on labor income

• Exercise: what if $T = tw(H-h^*)$?

• Hint: is $(c^*, h^*)$ optimal for $T$?