Consumer Theory

The consumer's problem:
budget set,
interior and corner solutions.

The consumer's problem

The consumer chooses the consumption bundle that maximizes his welfare (that is, his utility) on the set of his feasible consumption bundles.

In most settings, the consumer's set of feasible bundles is determined by budget constraints: the consumer has a monetary income, denoted I, that is used to buy goods x and y at market prices, p_x and p_y.

The consumer's problem: Budget Set

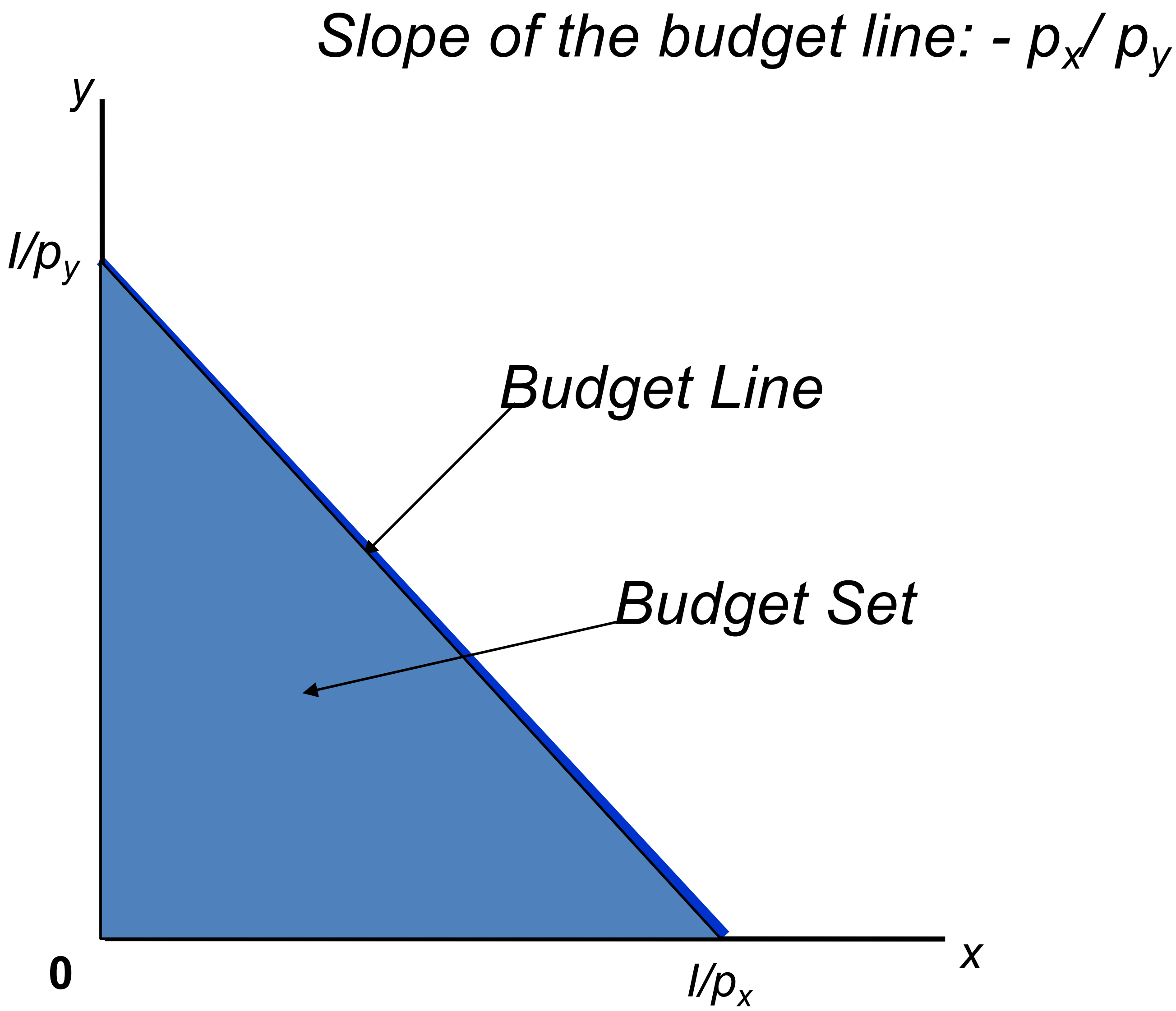
The budget set contains all bundles which cost is less than the given amount of income:

$$B(p_x,p_y,I) = \{(x,y) \mid p_x x + p_y y \leq I\}.$$

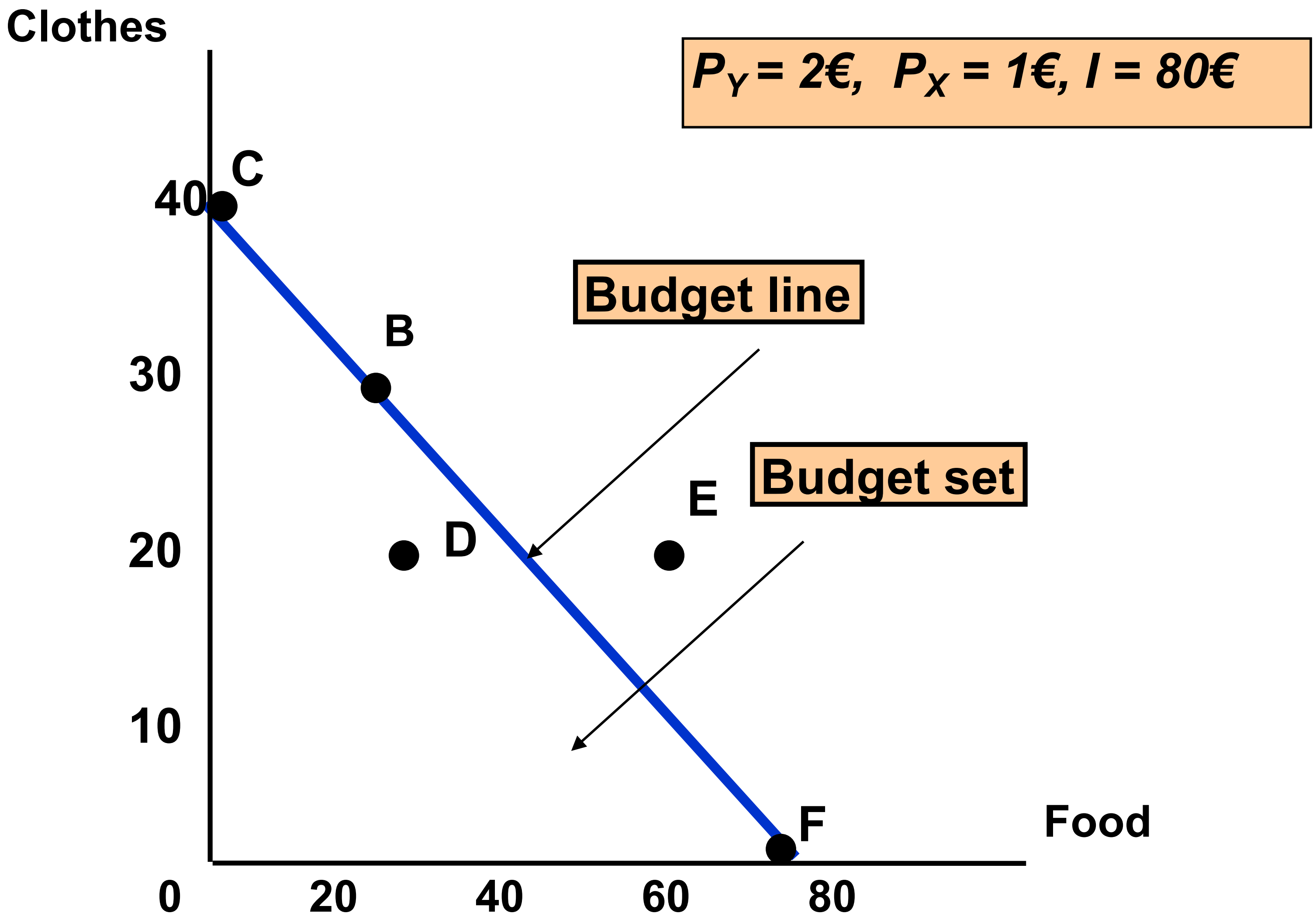
The budget line includes all bundles which cost is exactly the same as the given amount of income:

$$p_x x + p_y y = I.$$

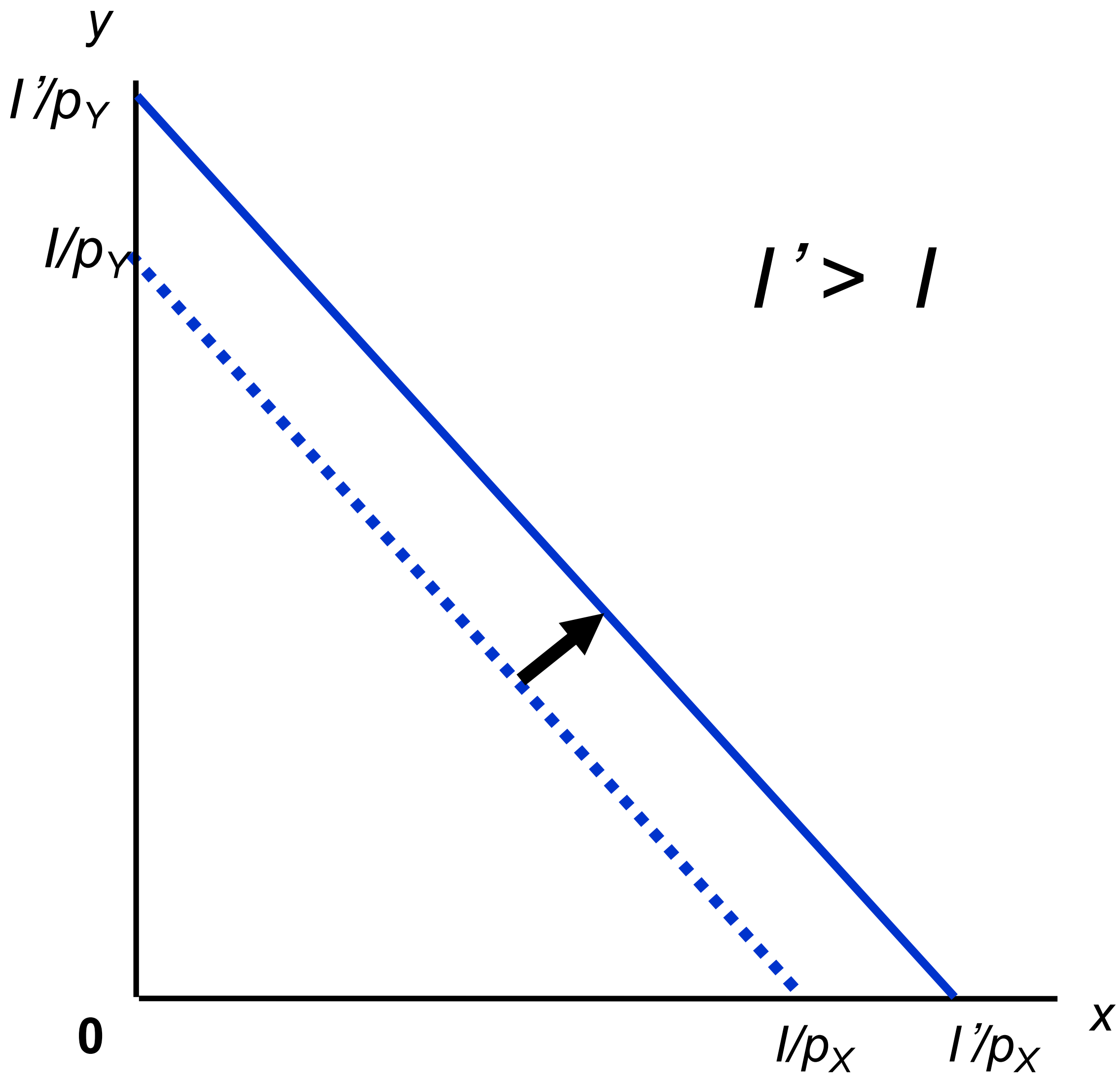
The consumer's problem: Budget Set



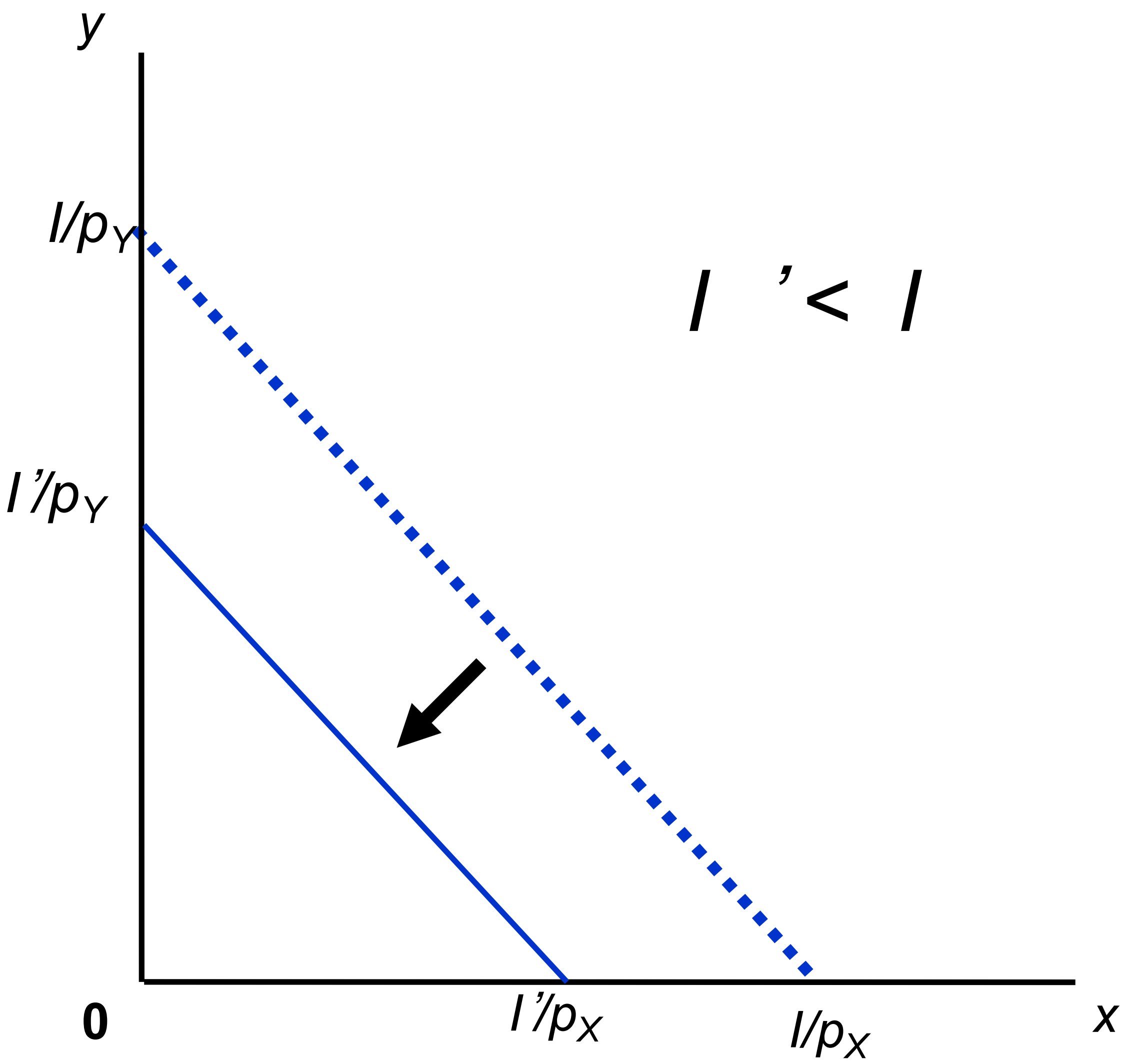
The consumer's problem: Budget Set



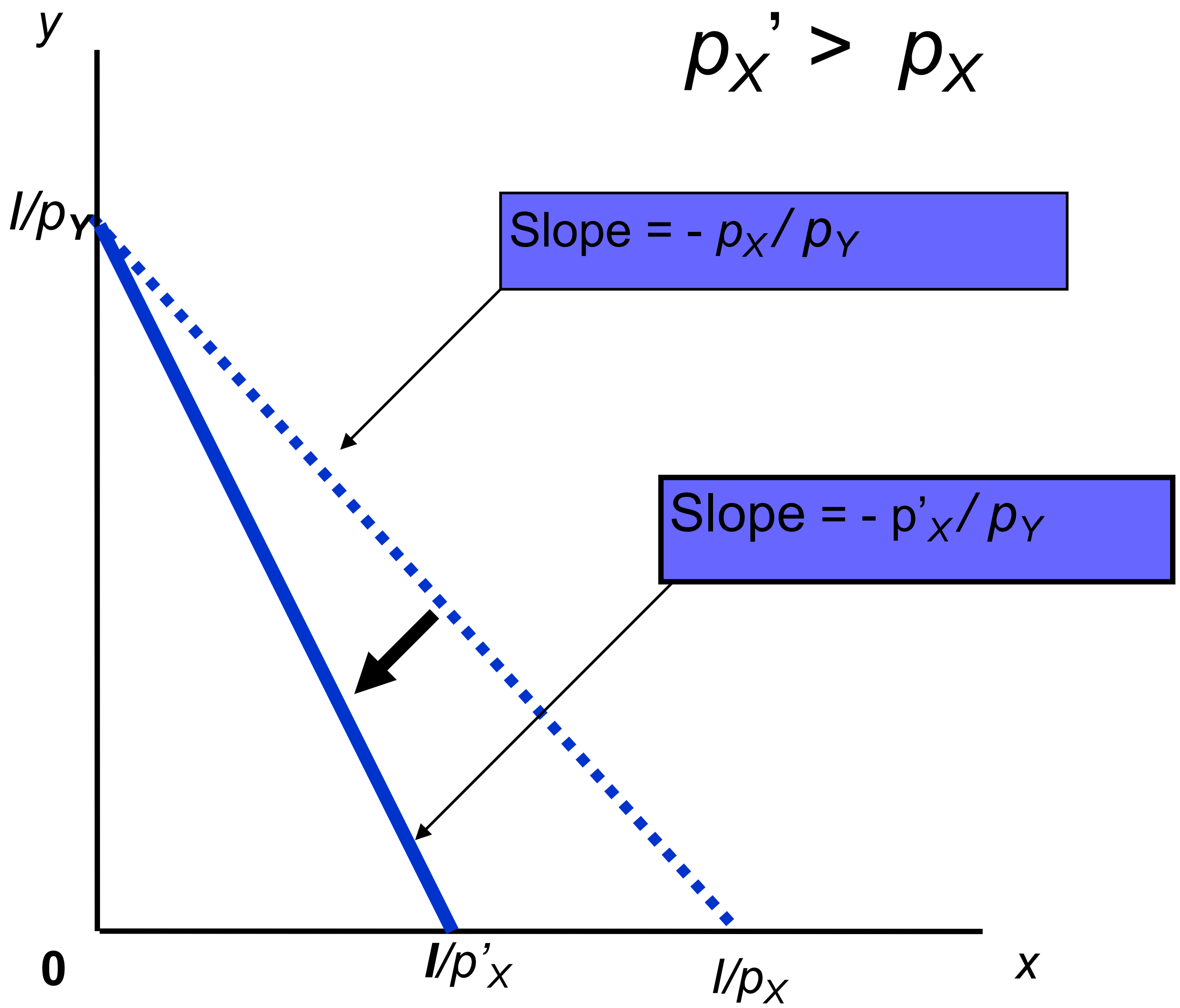
The consumer's problem: Budget Set



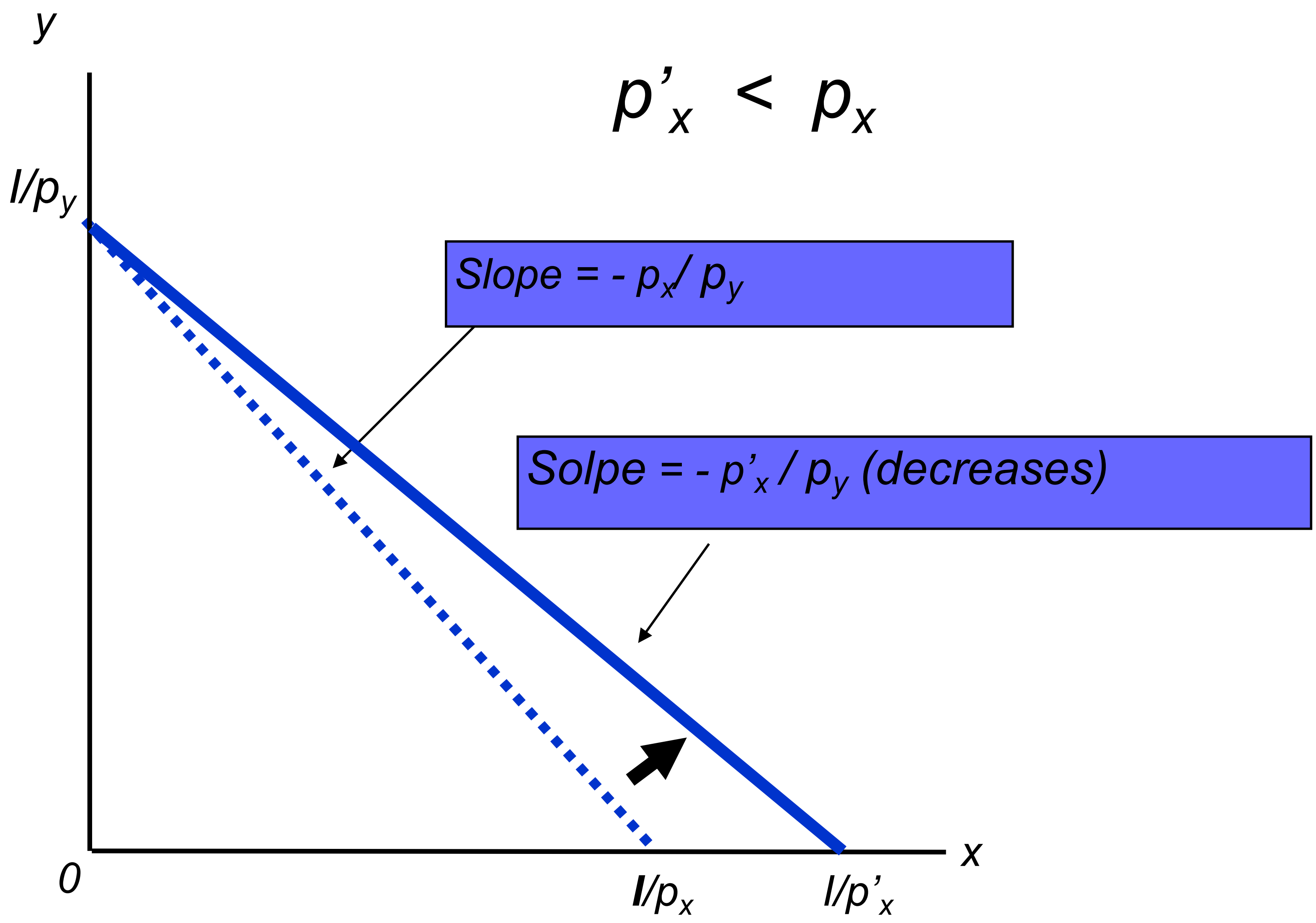
The consumer's problem: Budget Set



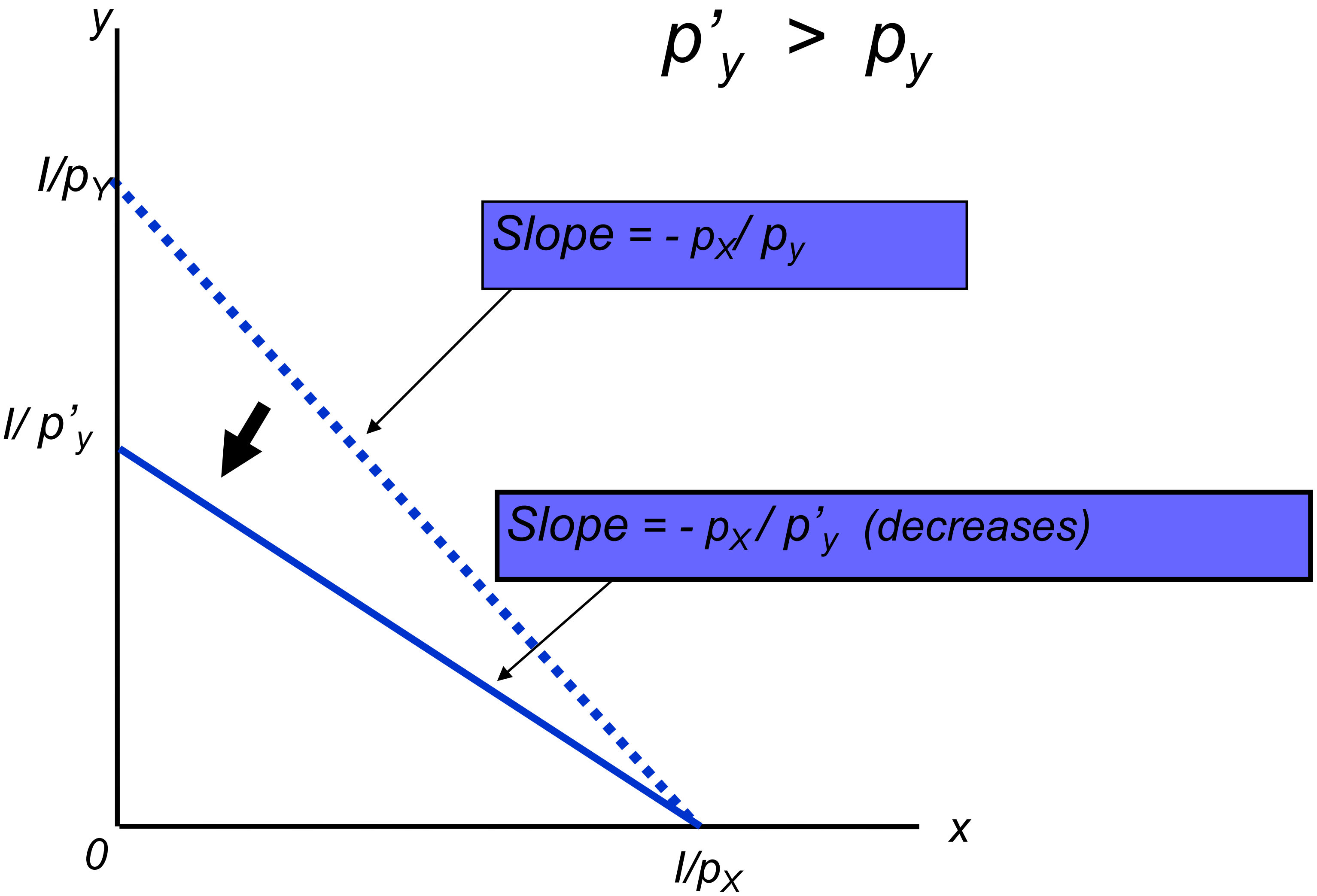
The consumer's problem: Budget Set



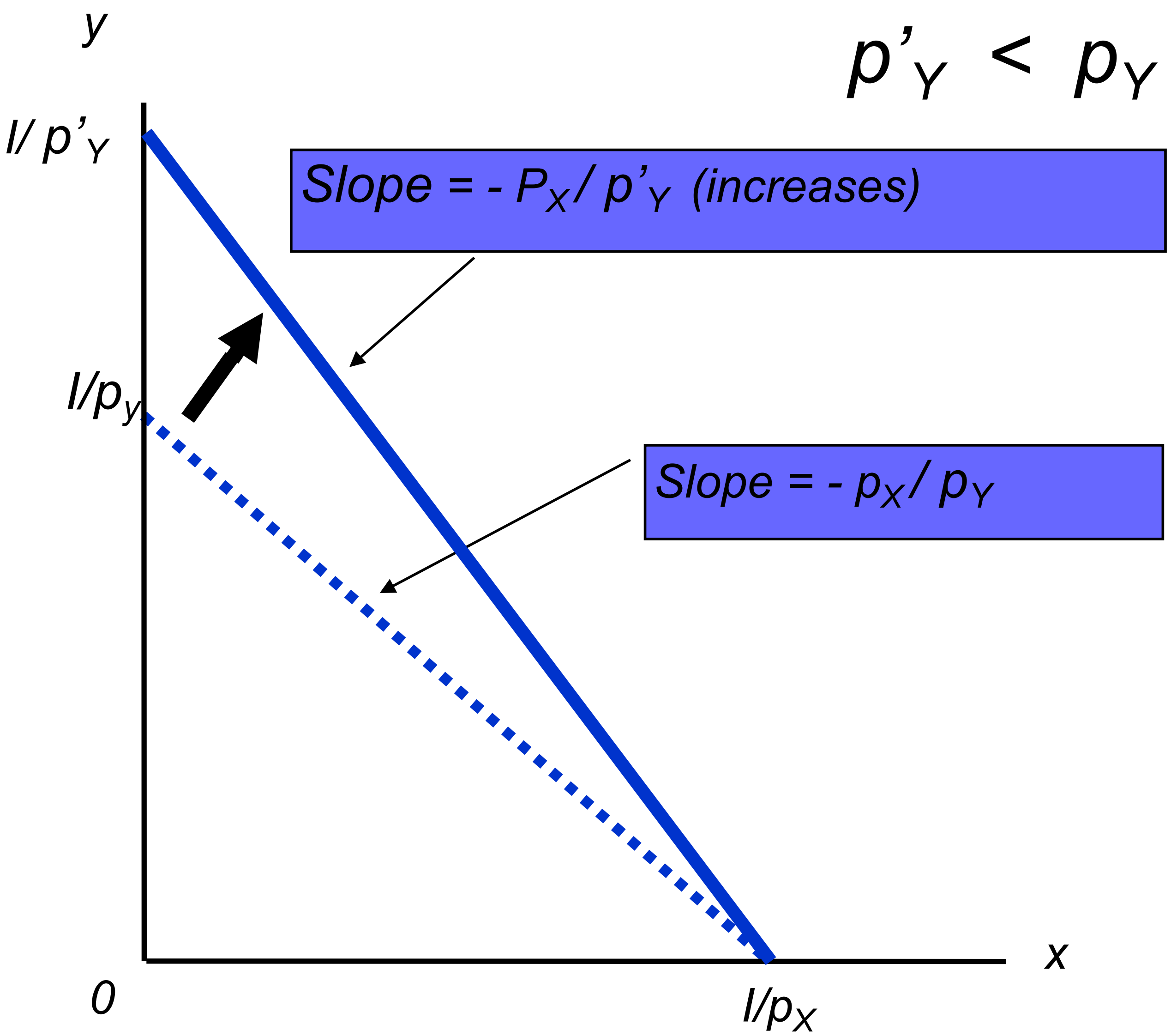
Conjunto Presupuestario: reducciones del precio de *x*



The consumer's problem: Budget Set



The consumer's problem: Budget Set



The consumer's problem

The consumer's problem (CP) is:

$$\text{Max}_{x,y} \; u(x,y)$$
$$\text{s. t. } p_x x + p_y y \leq I$$
$$x \geq 0, \; y \geq 0.$$

Choice Variables: x, y.

Data: p_x, p_y, I.

The consumer's problem

Axioms A1, A2 and A4 imply that there is a utility function $u: \Re^2_+ \rightarrow \Re$ that represents the consumer's preferences. Moreover, the function u is continuous.

When prices are positive, the consumer's budget set is compact (that is, closed and bounded). Hence, Weierstrass' Theorem implies that the consumer's problem has a solution.

The consumer's problem: solution

- Axiom A3 implies that the function $u(x,y)$ is non decreasing in x and non decreasing in y; furthermore, it is increasing in (x,y).

  Hence a solution to the CP, (x^*, y^*), satisfies:

$$(1)\ p_x x^* + p_y y^* = I.$$

The consumer's problem: solution

Proof: If $p_x\, x + p_y\, y = I - \varepsilon < I$, then the bundle

$$(x + \varepsilon/(2p_x), y + \varepsilon/(2p_y))$$

is in the budget set, and is preferred to (x, y) by A.3.

The consumer's problem: solution

- Axiom A5 implies that u is concave.

  Hence a local maximum of the function u is a global maximum; that is, second order conditions need not be checked.

The consumer's problem: solution

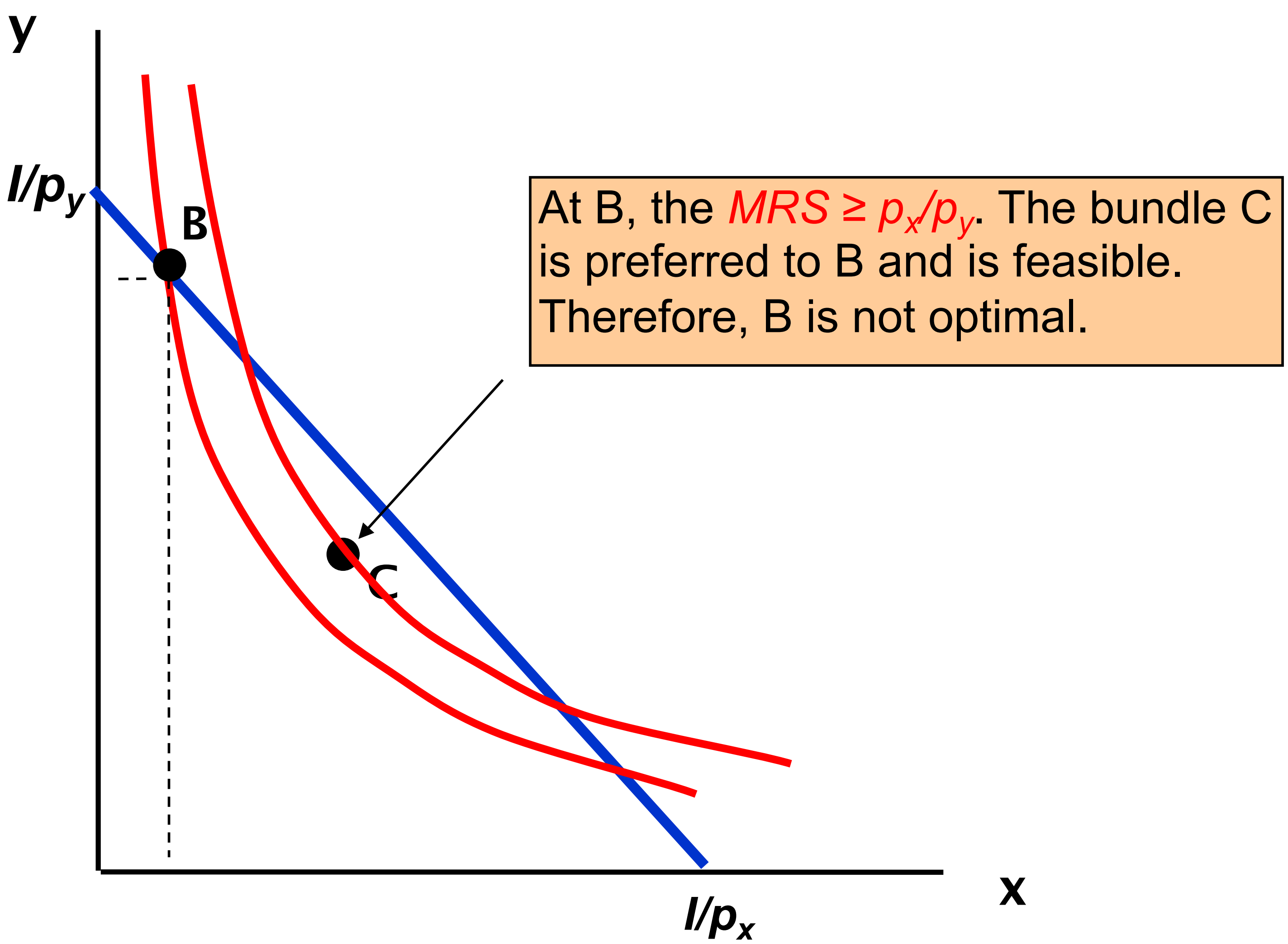
Characterizing a solution to the CP.

Let (x^*, y^*) be a solution to the CP. Then:

2.a. If $x^* > 0 \rightarrow MRS(x^*, y^*) \geq p_x / p_y$

2.b. If $y^* > 0 \rightarrow MRS(x^*, y^*) \leq p_x / p_y$

The consumer's problem: solution



The consumer's problem: solution

Interior solutions: $(x^*,y^*) >> (0,0)$

(1) $p_x\, x + p_y\, y = I$

(2) $MRS(x,y) = p_x/p_y$

The consumer's problem: solution

Corner solutions:

✓ Only good x is consumed: $x^* = I/p_x, \;\; y^* = 0$

(2) $MRS(I/p_x, 0) \geq p_x/p_y$

✓ Only good y is consumed: $x^* = 0, \;\; y^* = I/p_y$

(2) $MRS(0, I/p_y) \leq p_x/p_y$

The consumer's problem: solution

1. $u(x,y) = xy$; $p_x=1$, $p_y=2$, $I=80$.

We have $MRS(x,y) = y/x$.

Using (2) ($MRS(x,y) = p_x/p_y$) we have

$$y/x = 1/2 \rightarrow x = 2y$$

Substituting in (1) ($xp_x + yp_y = I$) we have

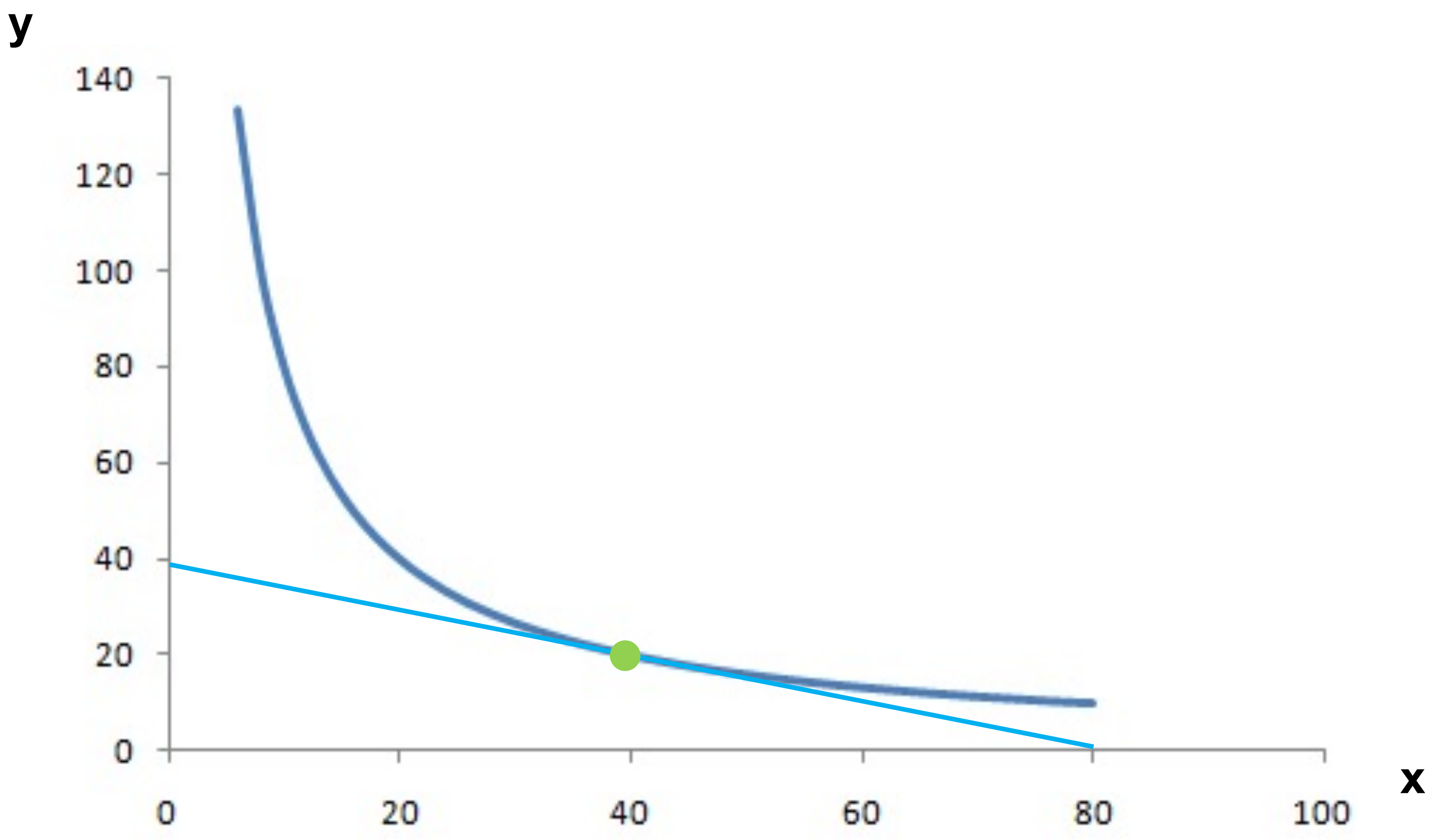
$$x+2y = 80 \rightarrow 2x=80.$$

That is, $x^*= 40$, $y^*= 20$, and $u^* = x^*y^* = 800$.

There are no corner solutions since

$$u(x,0)=u(0,y)=0<u^*.$$

The consumer's problem: solution



The consumer's problem: solution

2. $u(x,y) = 2x + y$; $p_x=1$, $p_y=2$, $I=80$.

We have $MRS(x,y) = 2$.

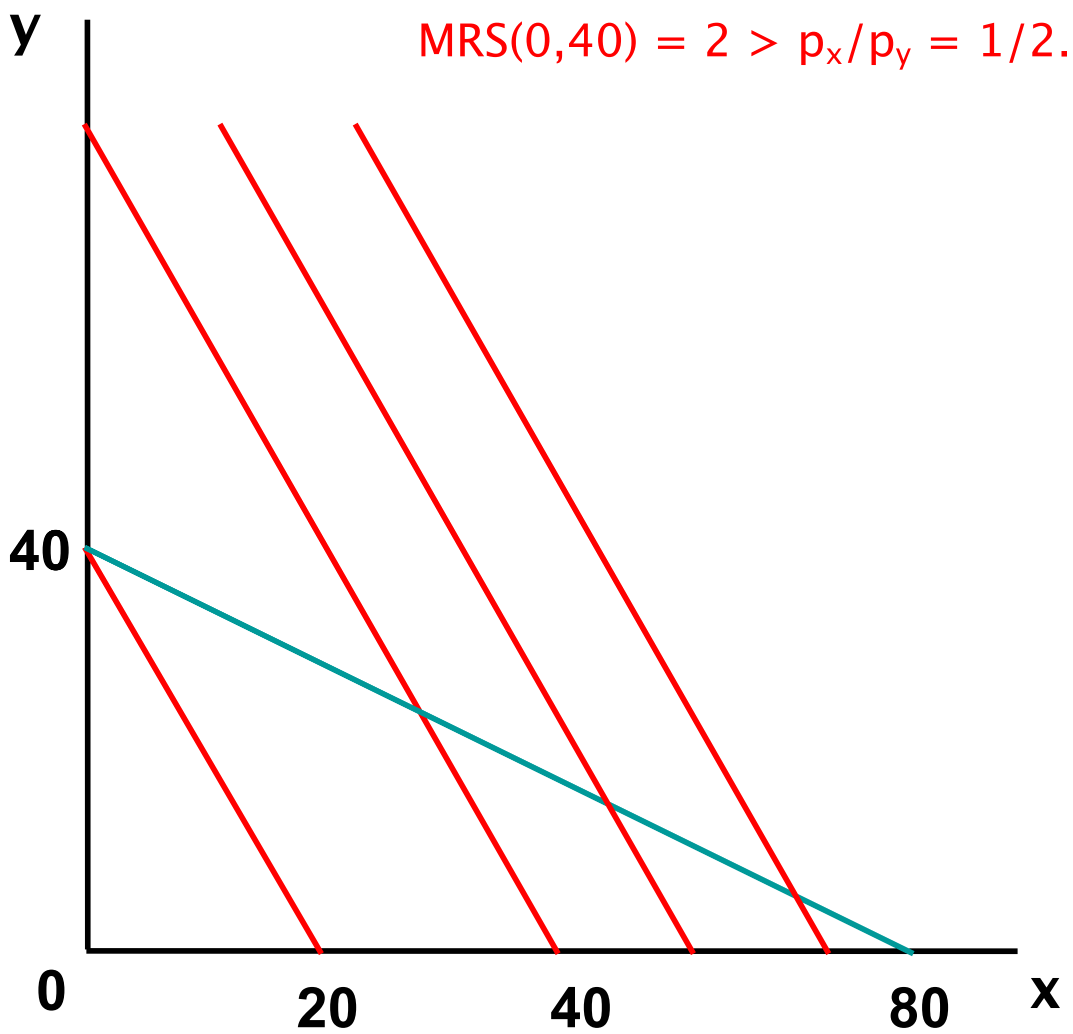
Interior solutions:

(1) $p_x\, x + p_y\, y = I \iff x + 2y = 80$
(2) $MRS(x,y) = p_x/p_y \iff 2= 1/2$??

Equation (2) is not satisfied. Hence there is no interior solution!

The consumer's problem: solution

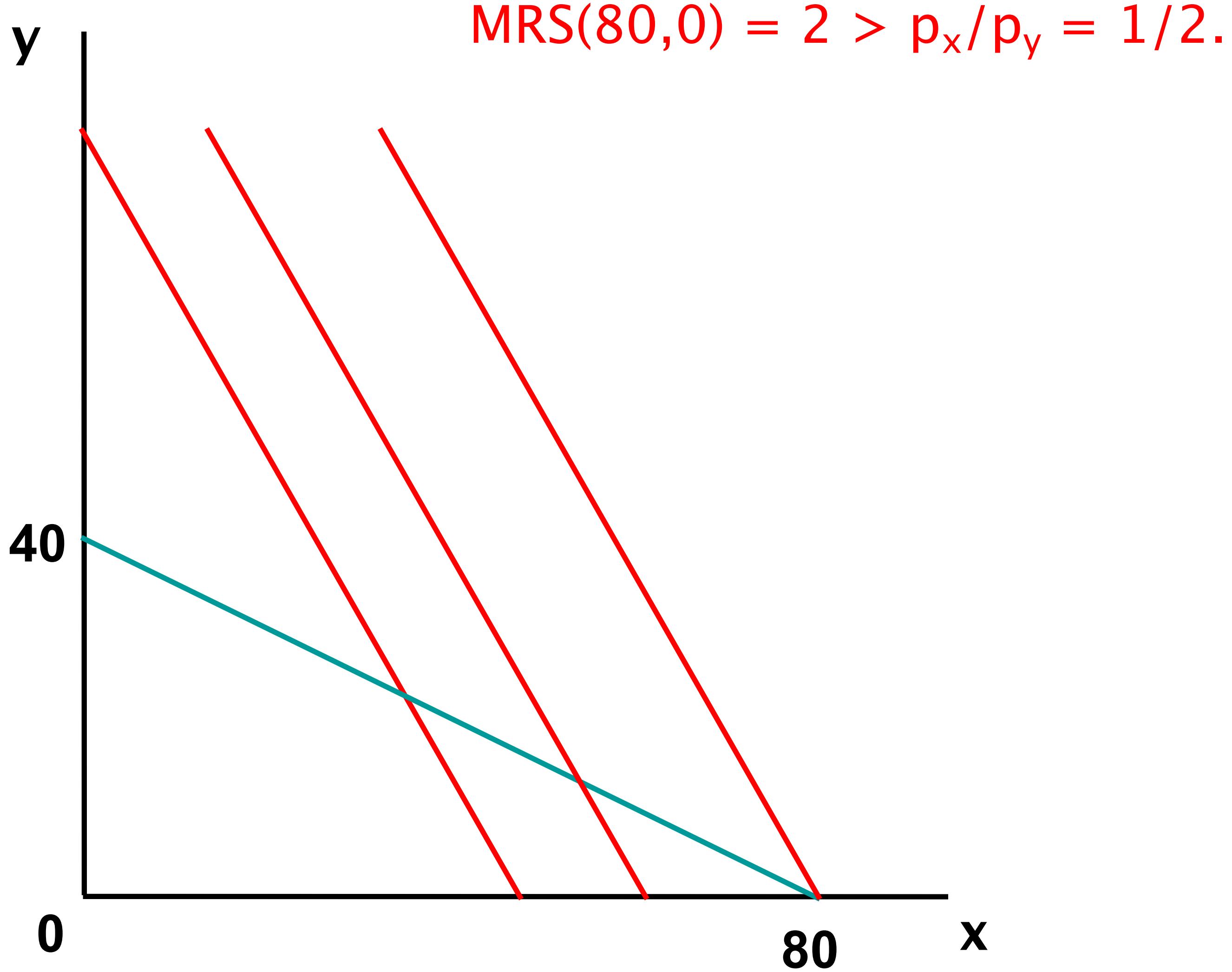
Corner solutions:



The bundle (0,40) is not a solution.

The consumer's problem: solution

Corner solutions:



The bundle (80,0) is a solution.

The consumer's problem: solution

3. $u(x,y) = \min\{x,2y\}$; $p_x=1$, $p_y=2$, $I=80$.

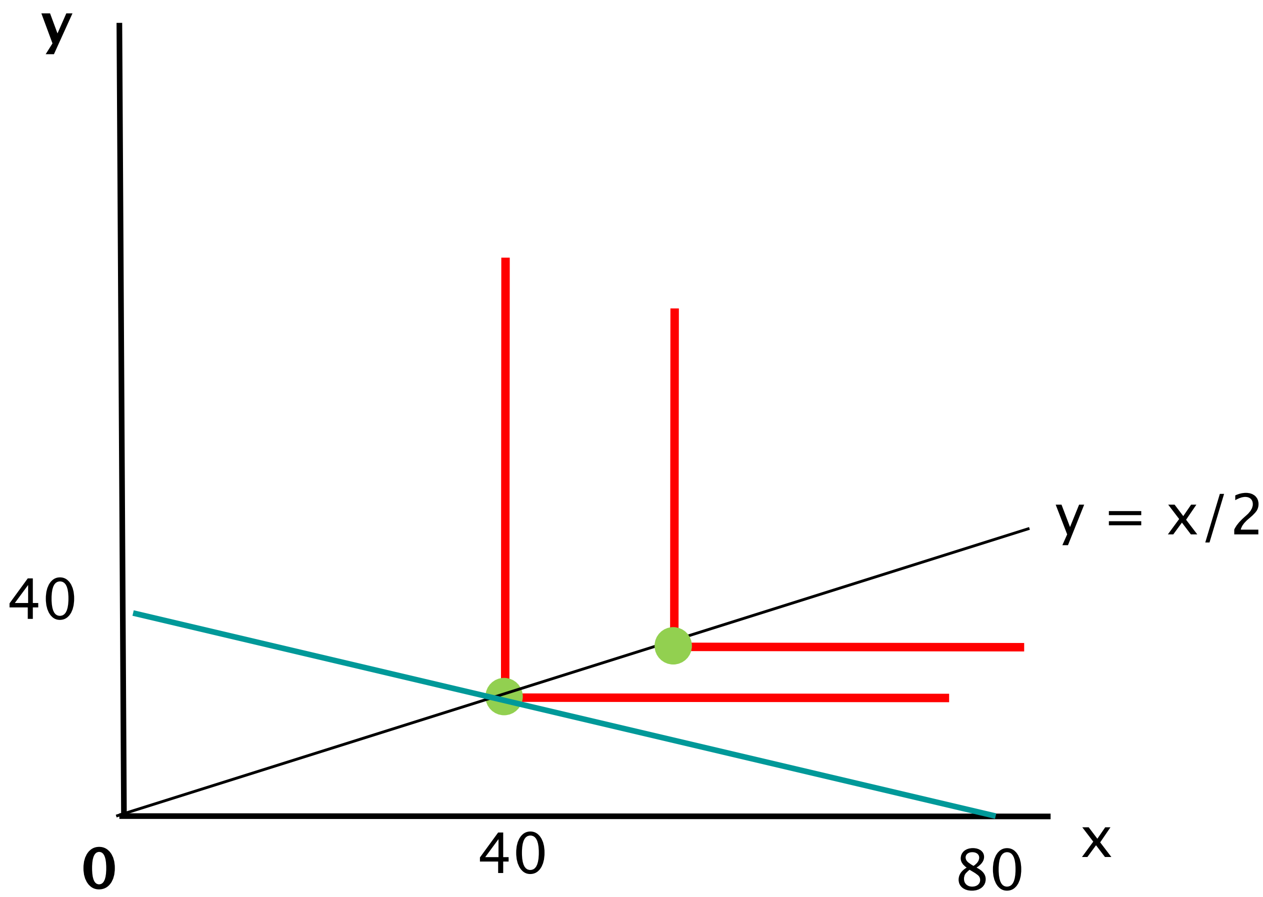
- $MRS(x,y) = 0$ if $y < x/2$ (the indifference curve is horizontal at these points).

- The $MRS(x,y)$ is not defined if $y \geq x/2$ (at these points, the indifference curve is vertical or it has several tangent lines).

The method discussed, which is based on the MRS, is not useful in solving this problem.

The consumer's problem: solution

Let's see that the solution is the bundle (40,20), like the graph below suggests.



The consumer's problem: solution

Let's suppose that (x^*,y^*) solves the CP.

a. If $y^* < x^*/2$, since $x^* + 2y^* = 80$, we have

$$y^* = (80 - x^*)/2 < 40 - y^* \rightarrow y^* < 20.$$

Therefore $u(x^*,y^*) = 2y^* < 40 = u(40,20)$.

b. If $y^* > x^*/2$, since $x^* + 2y^* = 80$, we have

$$x^* = 80 - 2y^* < 80 - x^* \rightarrow x^* < 40.$$

Therefore

$$u(x^*,y^*) = x^* < 40 = u(40,20).$$

(a) and (b) imply that $(x^*,y^*) = (40,20)$ is the solution to the CP.