Consumer Theory

Preferences

Indifference Curves

Utility Functions

Consumer Theory

A consumer decides how to spend his income or wealth to buy goods with the objective of maximizing his welfare.

Consumer Theory

How do *consumers* decide what to buy?

What determines the (individual, market) demands of goods and services?

How do the demands of goods and services depend on good prices, income, etc.?

Consumer Theory

In order to describe the consumer's problem we need to specify his:

- Preferences
  How are alternatives consumption bundles ordered?

- Constraints
  What is the set of feasible consumption bundles?

Consumer Theory

The consumer’s preferences and constraints determine his choice, that is,

the consumption bundle that maximizes the consumer’s welfare on the set of feasible consumption bundles.

Consumer Theory: Preferences

- List of specific quantities of distinct goods and services
- Example: Two goods x and y.

(x,y) = (quantity good x, quantity good y)

e.g. (x,y)=(coffee, shoes)

Consumer has to be able to rank all the bundles in order to identify which one he likes the most.

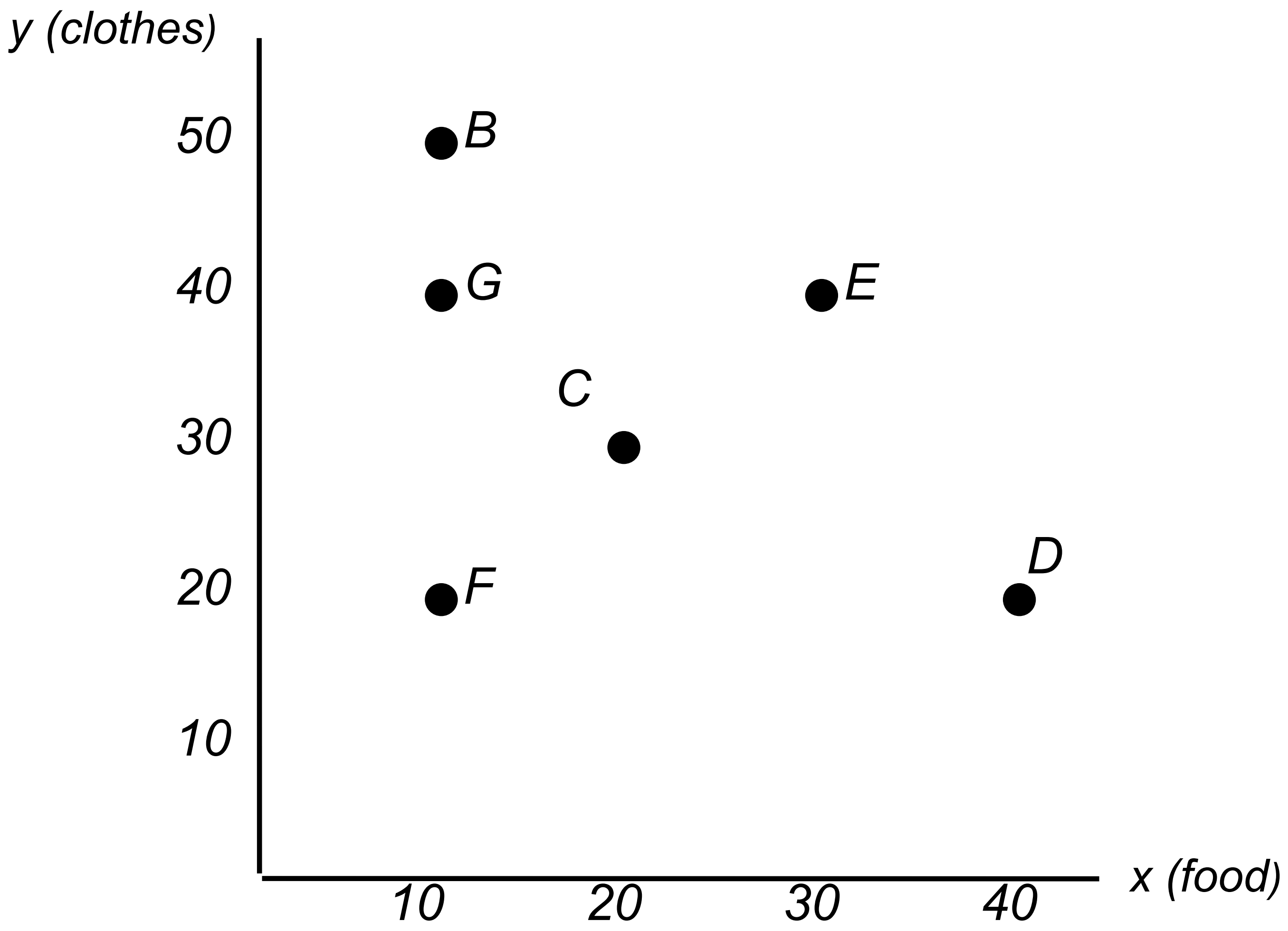
Consumer Theory: Preferences

We simplify the problem by assuming that there are only two goods, *x* and *y* (e.g., food and clothes).

Bundle	Units of food (x)	Units of clothes (y)
B	10	50
C	20	30
D	40	20
E	30	40
F	10	20
G	10	40

Consumer Theory: Preferences

We will assume that goods a perfectly divisible so that every point in the positive part of the real plane is a possible bundle.



Consumer Theory: Preferences

Let A=(x,y) and B=(x',y') be two bundles.

$\succsim$: preference relation;

A $\succsim$ B (A is preferred or indifferent to B).

$\succ$: strict preference relation;

A $\succ$ B (A is preferred to B) -- A $\succsim$ B, but not B $\succsim$ A.

~: indifference relation;

A ~ B (A is indifferent to B) -- A $\succsim$ B and B $\succsim$ A.

Consumer Theory: Preferences

Examples: Let A = (x,y) and B = (x',y') be two bundles.

1. Pareto:

$$A \gtrsim B \text{ if } x \geq x' \text{ and } y \geq y'.$$

2. Lexicographic:

$$A \gtrsim B \text{ if } x > x' \text{ or } [x = x' \text{ and } y \geq y'].$$

3. Goods and "Bads" (pollution, waste):

$$A \gtrsim B \text{ if } x - y \geq x' - y'.$$

Consumer Theory: Preferences

4. Perfect substitutes:

$$A \gtrsim B \text{ if } x + y \geq x' + y'.$$

5. Imperfect substitutes:

$$A \gtrsim B \text{ if } xy \geq x'y'.$$

6. Complements:

$$A \gtrsim B \text{ if } \min\{x,y\} \geq \min\{x',y'\}.$$

Consumer Theory: Preferences

I. Three basic axioms:

A.1. Preferences are *complete* if for all bundles A, B:

$A \succsim B$, or $B \succsim A$, or both.

Consumers can always compare any two bundles.

Consumer Theory: Preferences

I. Three basic axioms:

A.2. Preferences are *transitive* if for all bundles A, B, C:

$$A \succsim B \text{ and } B \succsim C \text{ implies } A \succsim C.$$

Consumer's preferences do not cycle, that is,

$$A \succ B \succ C \succ A$$

never holds.

Consumer Theory: Preferences

I. Three basic axioms:

A.3. Preferences are *monotone* if for all bundles A=(x,y) and B=(x',y'):

$$(x,y) \geq (x',y') \text{ implies } A \succsim B,$$

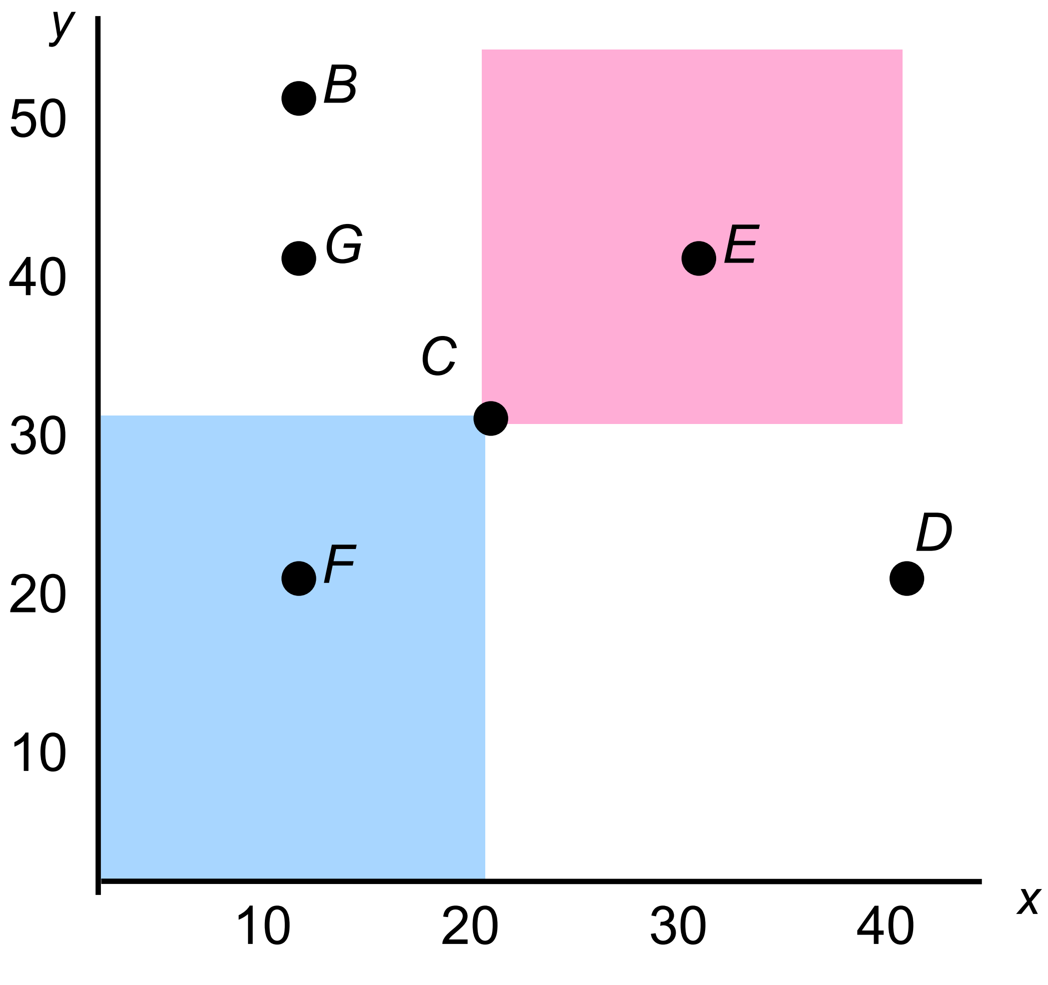
and

$$(x,y) >> (x',y') \text{ implies } A \succ B.$$

The more, the better!

Consumer Theory: Preferences

Axiom A.3 implies that *C* is preferred to F (and to all bundles in the blue area), while E (and all the bundles in the pink area), are preferred to *C.*



Consumer Theory: Preferences

II. Other Axioms:

A.4. Preferences are *continuous*:

If $A \gtrsim B(n)\ \forall\ n$ and $\{B(n)\} \rightarrow B$, then $A \gtrsim B$.

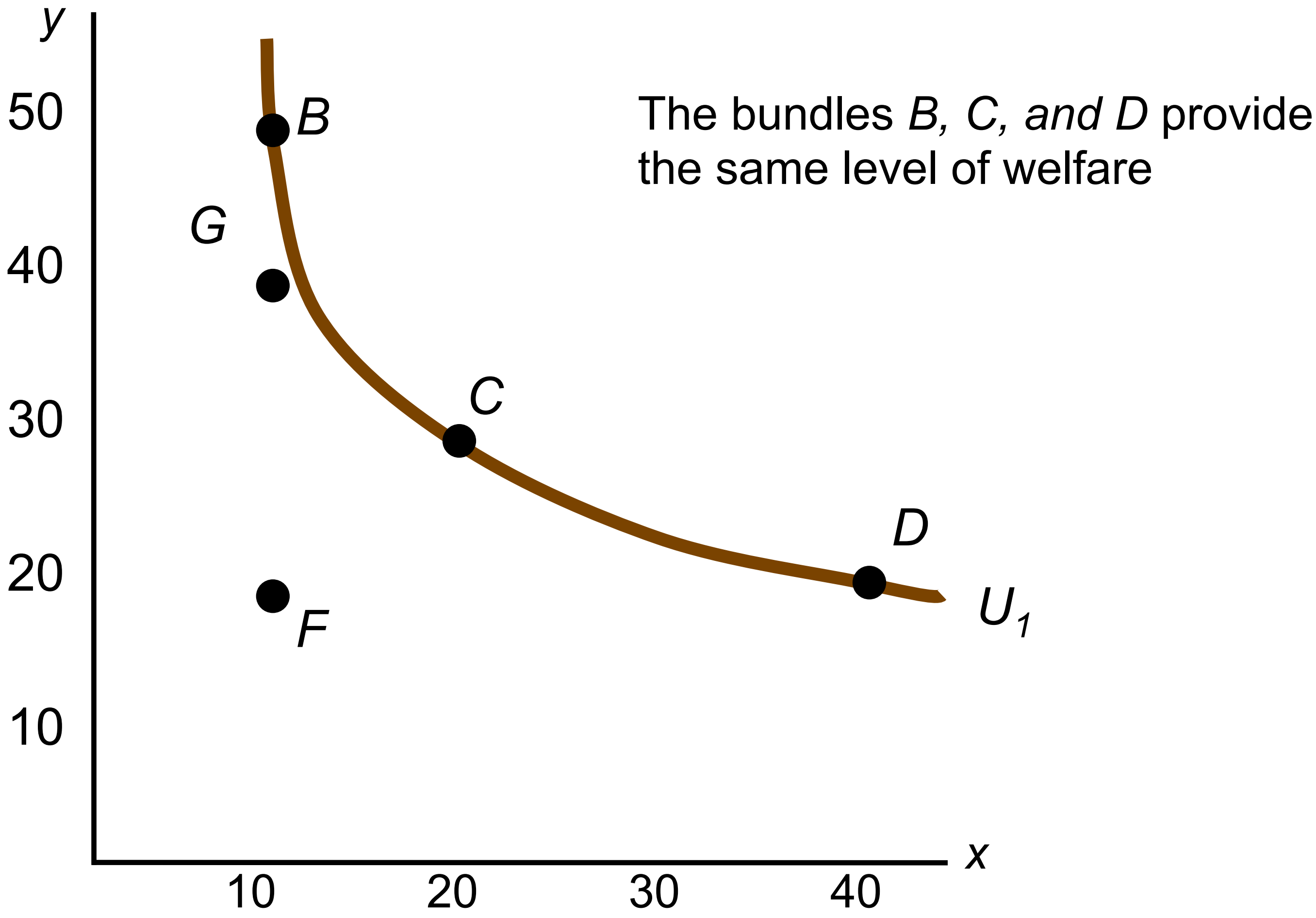
If $B(n) \gtrsim A\ \forall\ n$ and $\{B(n)\} \rightarrow B$, then $B \gtrsim A$.

A.5. Preferences are *convex:*

If $A \gtrsim B$ and $0 < \lambda < 1$, then $[\lambda A + (1-\lambda)B] \gtrsim B$.

Consumer Theory: Indifference Curves

A indifference set or indifference curve contains the bundles that provide the same level of satisfaction or welfare for a given individual.



Consumer Theory: Indifference Curves

Implications of axioms A1 to A3:

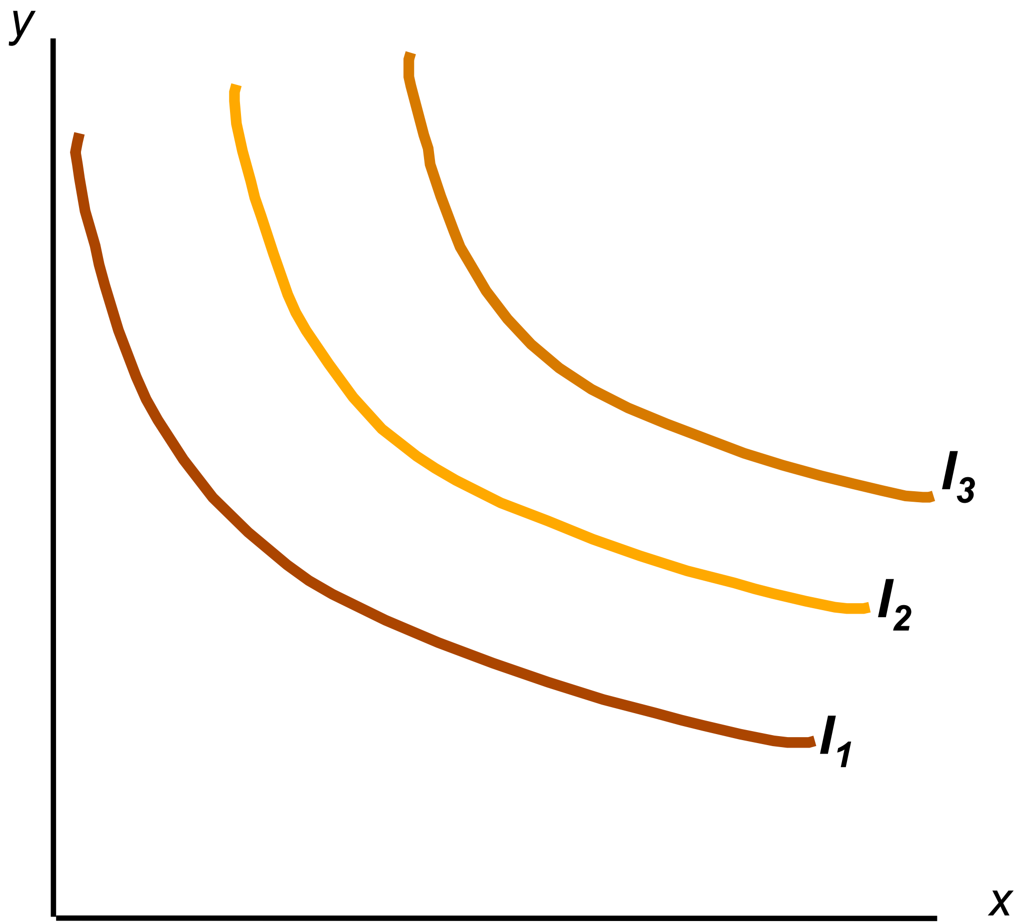
A.1. Every bundle is in some indifference curve.

A.2. Indifference curves cannot cross.

A3. Indifference curves are decreasing and have no area (that is, *are* curves).

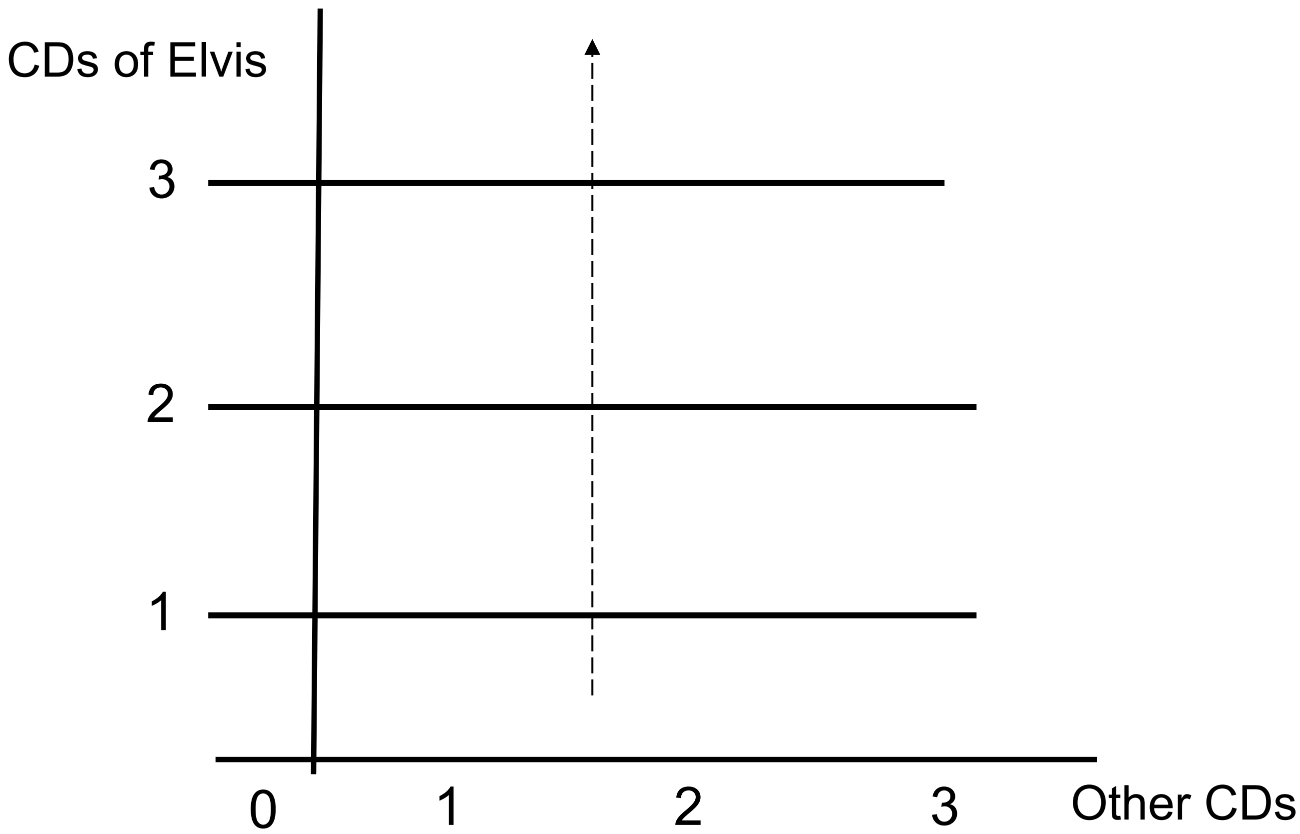
Consumer Theory: Indifference Curves

An indifference map provides a description of an individual's preferences by identifying his indifference curves.



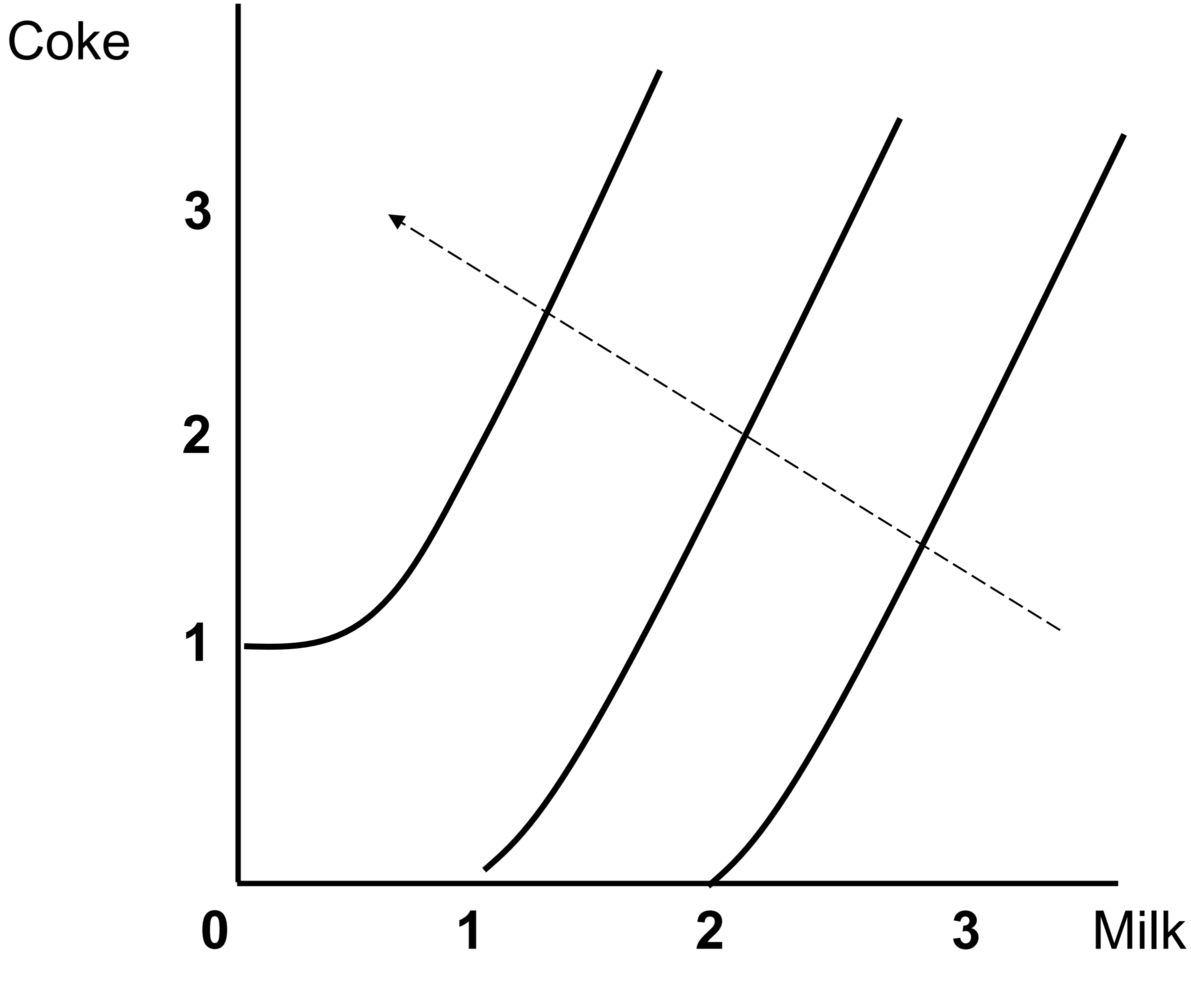
Consumer Theory: Indifference Curves

Indifference map: “I would not change a CD of Elvis for any other one.” Does this preference satisfy axiom A.3?



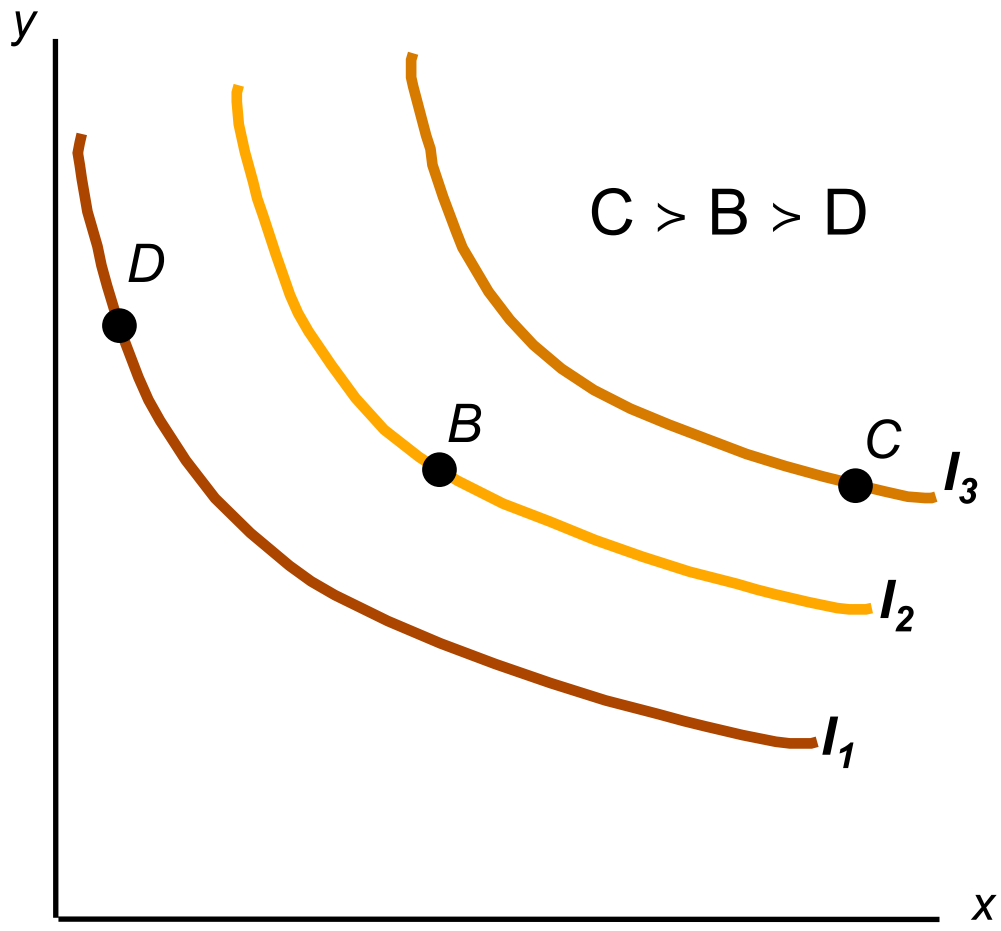
Consumer Theory: Indifference Curves

Indifference maps: “I drink coke but hate milk.” Does this preference satisfy axiom A.3?



Consumer Theory: Indifference Curves

Properties of indifference curves: axiom A.3 implies that indifference curves are decreasing: if (x,y) ~ (x',y'), then either x ≤ x' and y ≥ y', or x ≥ x' and y ≤ y'.

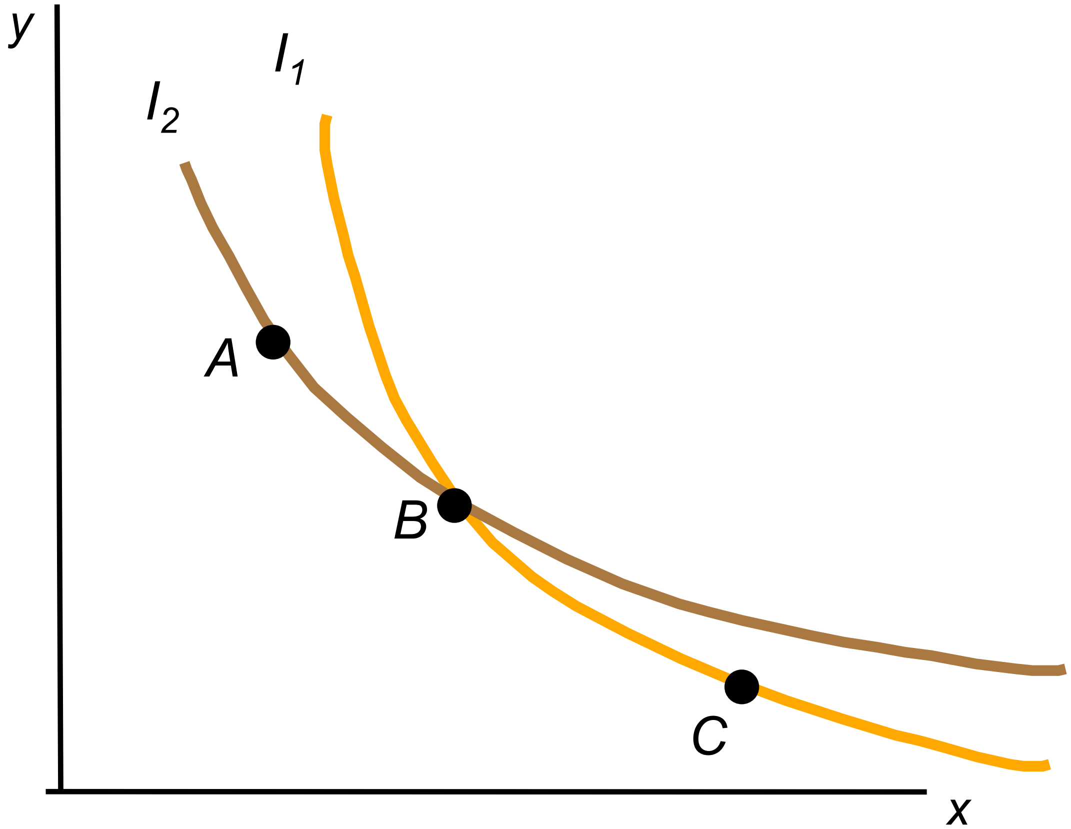


Consumer Theory: Indifference Curves

Axiom A.2 implies that indifference curves do not cross.

If A $\succ$ C, then A $\succ$ C $\succsim$ B implies A $\succ$ B (B does not belong to I_2)

If C $\succ$ A, then C $\succ$ A $\succsim$ B implies C $\succ$ B (B does not belong to I_1)



Consumer Theory: Utility Functions

The preferences of a consumer $\succcurlyeq$ can be described by an indifference map.

Can we find a function defined on the set of consumption bundles whose map of level curves coincide with this indifference map?

A function u with this property *represents* the consumer preferences.

Consumer Theory: Utility Functions

The following definition makes explicit the properties that a function *u* that represent some preferences must have.

A function $u: \Re^2_+ \to \Re$ *represents* the preferences of a consumer $\succcurlyeq$ if for any two consumption bundles*)* (x,y), $(x',y') \in \Re^2_+$

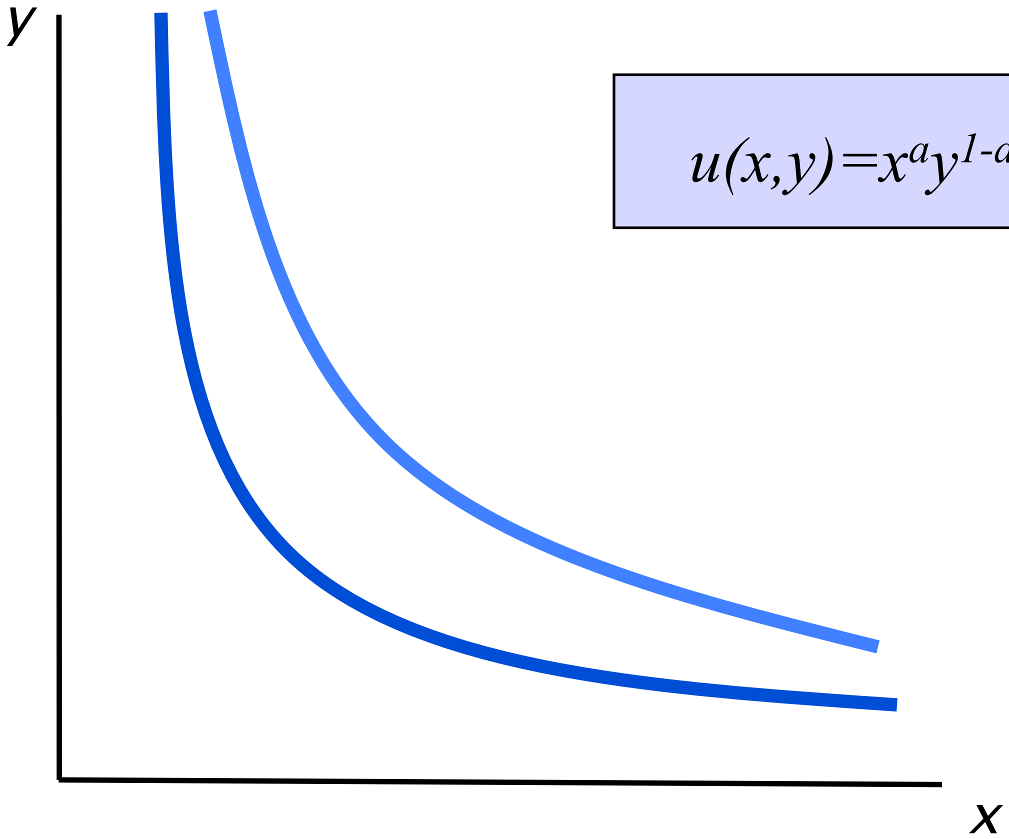
$$(x,y) \succcurlyeq (x',y') \Leftrightarrow u(x,y) \geq u(x',y').$$

Consumer Theory: Utility Functions

A utility function

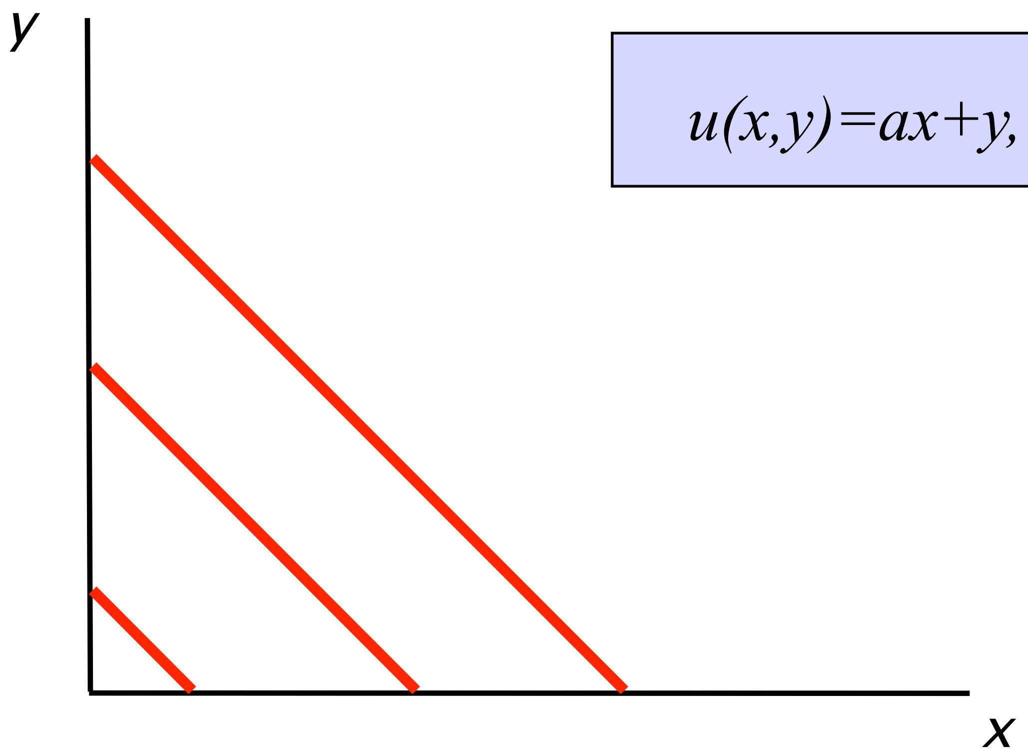
- provides a compact representation of the preferences of a consumer,
- does not allow to measure a consumer's welfare: differences in utility have an ordinal value (i.e., permit to order bundles of goods), but have no other meaning (i.e., larger differences in utility do not indicate larger changes of welfare).

Consumer Theory: Utility Functions



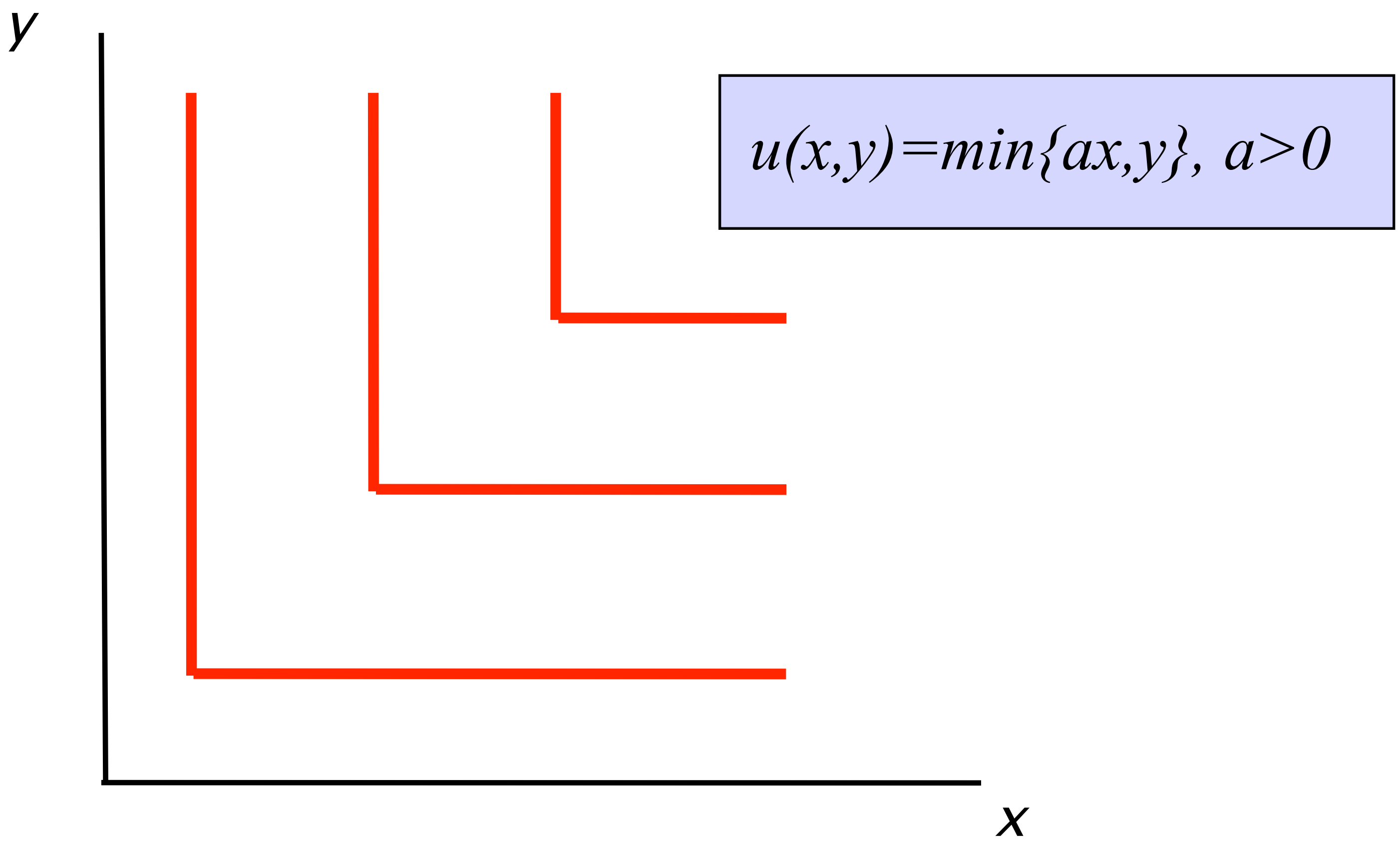
$u(x,y) = x^{a} y^{1-a},\ 0 < a < 1$

Consumer Theory: Utility Functions



$$u(x,y)=ax+y,\ a>0$$

Consumer Theory: Utility Functions



Consumer Theory: Utility Functions

The numerical values that a utility assigns to the alternative bundles do not have a meaning per se, other the allowing to order them according to the welfare they provide to the consumer:

If $u: \Re^2_+ \rightarrow \Re$ is a utility function and $g: \Re \rightarrow \Re$ is an increasing function, the the utility function $v: \Re^2_+ \rightarrow \Re$ given by $v(x,y)=g(u(x,y))$ represents the same preferences as u: simply, the map of level curves of v and u coincide.

Thus, we may say that a utility function provides an ordinal (not cardinal) representation of a consumer's preferences.

Consumer Theory: Utility Functions

Axioms A1, A2 and A4 imply the existence of a continuous utility function, $u: \Re^2_+ \rightarrow \Re$, that represents the consumer's preferences.

Axiom A3 implies that the function $u(x,y)$ is non-decreasing x and y, and is increasing in (x,y).

Axiom A5 implies that u es (quasi-)concave.