CONSUMER THEORY

I. Preferences; indifference curves; utility functions

1. Graph indifference maps consistent with the preferences described:

(a) My welfare is larger the larger my income (x) and the smaller pollution (y).

(b) I need 1000 milligrams of Tylenol (x) to obtain the same relief to my pain I get with 500 milligrams of Aspirin (y).

(c) I like my martinis with one dose of vermouth (x) and 5 doses of gin (y).

(d) I like hamburgers (x) and beer (y), and I am always willing to exchange one beer for any amount of hamburgers.

(e) I am always willing to exchange two hamburgers for one beer.

(f) I always drink a beer when I have a hamburger.

(e) I like to drink a beer, but I am allergic to meet.

2. Identify the axioms of the consumer that imply the properties of the indifference maps described below:

(a) Indifference sets are curves (that is, they are not “thick”).

(b) Every bundle belongs to one indifference curve.

(c) Indifference curves do not cross.

(d) Indifference curves are downward sloping.

3. (a) Two items have been weighted. The first one weighs 50 Kilos and the second 55 Kilos. As 55/50=1.1, we say that the second item is 10% heavier than the first. Is that statement still true if the weight is measured in pounds?

(b). The temperature of 2 objects has been measured. The temperature of the first one is 50 degrees Fahrenheit and that of the second is 55 degrees Fahrenheit. We can say: “the second object is 10% hotter than the first”. Can we keep that statement if the measurement is made in Celsius? The temperature of a third object is 65 degrees Fahrenheit. We say: “the difference between the temperature of the third and the second object is twice the difference between the second and the first.” Is this true independently of the scale of measurement?

(c) A consumer whose preferences are represented by the utility function u, whose values we refer to as *utils*. Thus, if the consumption bundles A and B are worth 50 and 55 utils, respectively, we can say B *conveys a 10% more utility than* A. Is that statement still true if the (same) preferences are represented by the utility function $U = u^2$ (i.e., U is obtained from u the by squaring the values of u); that is, does B convey 10% more Utils than B.

(d) In the context of question (c), assume that the utility of a bundle C is 65 utils. Thus, $u(C) - u(B) = 2\,(u(B) - u(A))$. Does this equation hold for the function U? Is the statement: switching from bundle B to C consumer's welfare increases twice as much as switching from bundle A to B correct?

4. Calculate and represent the indifference curves that contain the bundles $(x, y) = (1, 1)$ and $(x, y) = (1, 2)$ of individuals with preferences represented by the utility functions: (a) $u(x, y) = \sqrt{xy}$. (b) $u(x, y) = xy/4$. (c) $u(x, y) = y + 2\ln x$. (d) $u(x, y) = 4(x + 2y)^2$. (e) $u(x, y) = \min\{x^2, 2y\}$.

Multiple Choice Questions

I.1. An individual's preferences over consumption bundles $(x, y) \in \mathbb{R}^2_+$ are complete and transitive (axioms $A.1$ and $A.2$). If the individual considers the consumption of good x to be detrimental to her welfare and the consumption of good y to be beneficial, then his indifference curves

☐ cross	☐ are increasing
☐ are concave	☐ have area.

I.2. If an individual's preferences over goods x and y are monotonic (axiom $A.3$), then its indifference curves

☐ do not cross	☐ are increasing
☐ are decreasing	☐ are convex.

I.3. Identify the axiom that guarantees that indifference curves do not cross.

☐ Completeness ($A.1$)	☐ Monotonicity ($A.3$)
☐ Transitivity ($A.2$)	☐ Convexity ($A.5$)

I.4. The Pareto preference $\succeq_P$, defined as $(x, y) \succeq_P (x', y')$ if $x \geq x'$ and $y \geq y'$,

☐ do not satisfy axiom $A.1$ (completeness)	☐ do not satisfy axiom $A.3$ (monotonicity)
☐ do not satisfy axiom $A.2$ (transitivity)	☐ satisfy axioms $A.1 - A.3$.

I.5. It is known that a consumer's preferences $\succeq$ satisfy axioms $A.1$, $A.2$ and $A.3$, and that $A = (0, 2) \succ B = (1, 1)$. Therefore, we can infer the following relation between these bundles and $C = (1, 2)$:

☐ $C \succ B$	☐ $C \backsim A$
☐ $C \backsim B$	☐ $C \succ A$.

I.6. A consumer's preferences for bundles $(x, y) \in \mathbb{R}^2_+$ are complete, transitive and monotonic (axioms $A.1$, $A.2$ and $A.3$). Then the preference relations between $A = (1, 1)$, $B = (2, 2)$, and $C = (1, 2)$ necesarily satisfy

☐ $C \succeq B$	☐ $C \succeq A$
☐ $C \succ A$	☐ $B \succ C$.

I.7. It is known that a consumer's preferences $\succeq$ satisfy axioms $A.1$, $A.2$ and $A.3$, and that the consumer is indifferent between the bundles $A = (1, 2)$ and $B = (2, 1)$. Therefore, we can infer the following relation between these bundles and $C = (1, 3)$:

☐ $C \succ B$	☐ $C \succeq B$
☐ $C \backsim B$	☐ $C \succ A$.

II. The Marginal Rate of Substitution

5. The preferences of Juan and María over leisure activities differ significantly: Juan prefers going to the football stadium while María prefers to go to rock concerts.

(a) Graph indifference maps for Juan and María illustrating these differing preferences.

(b) Using the concept of marginal rate of substitution, explain the different curvature of their indifference curves.

6. The preferences of an individual over clothes (x) and food (y) are represented by the utility function $u(x, y) = x + 2\sqrt{y}$.

(a) What is the impact of an increase of the individual's consumption of clothing over his MRS?

(b) If we want to increase the individual MRS, should we increase the consumption of clothes or food?

(c) For which bundles is the MRS equal to 4?

7. Calculate the MRS for the preferences represented by the utility functions given in exercise 4.In each case, determine if an individual with 2 units of each good would be willing to give 1 infinitesimal unit of x in exchange of $1,5$ infinitesimal units of y. Would he be willing to exchange 1 unit of x for 1,5 units of y? Would he be willing to do either of these exchanges if he has two units of x and one unit of y?

8. Graph the indifference curve containing the bundle $(x, y) = (3, 3)$ and calculate the MRS at this bundle and a arbitrary bundle in this curve for individuals with preferences represented by the following utility functions: (a) $u(x, y) = \sqrt{xy}$. (b) $u(x, y) = 2\sqrt{xy}$. (c) $u(x, y) = 4 + 3\sqrt{xy}$. (d) $u(x, y) = xy$.

III. The consumer problem: budget set; interior and corner solutions

9. Assume that the price of natural gas is 0.05 euros/m^3 and the price of electricity is 0.06 euros/Kilowatt per hour. However, after buying 1000 Kilowatts per hour it falls to 0.03 euros. If the consumer has 120 euros at his disposal to spend on energy, draw his budget set.

10. In some autonomous communities, the water tariffs have a scheme as follows: in order to receive any water supply at all, the consumer must pay an initial tax T, which allows the individual to consume x_1 liters of water without extra costs. If water consumption is $x \in (x_1, x_2]$, then the water bill increases by $p(x - x_1)$ units. Finally, if consumption is $x > x_2$, then the water bill increases by $p(x_2 - x_1) + p'(x - x_2)$, where $p' > p$.

(a) Graph the corresponding budget constraint for water x and other goods (y, euros available for consumption of other goods) assuming that the consumer's income I satisfies $I > T + p(x_2 - x_1)$.

(b) If an individual with preferences satisfying axiom $A.3$ pays the connection tax T, would you expect that he consumes an amount of water less than x_1 under this scheme?

(c) Do you think it is possible that, under the axioms A1-A4, an individual be indifferent between two bundles that differ in water consumption?

11. A consumer preferences over x and y are described by the utility function $u(x, y) = 2x + y$. His monetary income is $I = 15$ euros. Calculate his optimal consumption bundle when prices of the goods are $(p_x, p_y) = (1, 2)$, $(p'_x, p'_y) = (3, 1)$ and $(p''_x, p''_y) = (2, 1)$.

12. A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = x + \sqrt{y}$. The prices are $p_x = 4$ and $p_y = 1$ euros per unit, respectively, and the consumer's income is $I = 10$ euros. Represent the consumer's budget set and calculate her optimal consumption bundle.

13. A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = \ln x + \ln y$. The prices are $p_x = 1$ and $p_y = 2$ euros per unit, respectively, and the consumer's income is $I = 10$ euros. Represent the consumer's budget set and calculate her optimal consumption bundle.

14. An individual owns an income $I = 200$ that she devotes to buying water (x) and food (y), whose prices are $p_x = 4$ and $p_y = 2$. Her preferences over these goods are represented by the utility function $u(x, y) = \min\{x, y\}$.

(a) Graph some of his/her indifference curves, the budget constraint and the optimal choice, and identify her optimal consumption bundle.

(b) Assume now that she must pay a tax $t = 1$ euro for every unit in excess of 10; that is, if she consumes 12 units of water, for example, the first 10 units are charged a price $p_x = 4$ euros per unit, and the 2 remaining units are charged $p_x + t = 5$ euros per unit. Repeat exercise a).

Multiple Choice Questions

III.1. If price of good y increases by 20%, then the budget line

☐ maintains its position ☐ rotates over its intersection with the y-axis
☐ shifts parallel towards the origin ☐ rotates over its intersection with the x-axis.

III.2. If prices increase by 20%, then the budget line

☐ translates parallel towards the origin ☐ rotates over its intersection with the x-axis
☐ translates parallel away from the origin ☐ maintains its position.

III.3. If a consumer's income increases by 10%, the price of good x increases by 5%, and the price of good y increases by 10%, then his budget line

☐ shifts parallel towards the origin ☐ rotates over its intersection on the y-axis
☐ shifts parallel away from the origin ☐ rotates over its intersection on the x-axis.

III.4. A consumer whose monetary income is $I = 4$ is considering buying the bundle $(2, 0)$. If $p_x = 2$, $p_y = 1$, and $MRS\,(2, 0) = 3$, then the consumer should

☐ buy more x and less y ☐ buy more x and y
☐ buy more y and less x ☐ buy the bundle $(2, 0)$.

III.5. If a consumer's marginal rate of substitution is $MRS(x, y) = 2$ (constant) and her income is $I = 8$, then at the prices $(p_x, p_y) = (1, 2)$ her optimal consumption bundle is

☐ $(2, 3)$ ☐ $(8, 0)$
☐ $(4, 2)$ ☐ $(2, 4)$

III.6. The optimal bundle (x^*, y^*) satisfies the budget constraint $p_x x^* + p_y y^* = I$ as a consequence of the axiom

☐ $A.1$ (completeness) ☐ $A.3$ (monotonicity)
☐ $A.2$ (transitivity) ☐ $A.5$ (convexity).

III.7. If a consumer's preferences are represented by the utility function $u(x, y) = (x + y)^2$, her income is $I = 4$, and prices are $(p_x, p_y) = (1, 2)$, then her optimal consumption bundle is

☐ $(0, 2)$ ☐ $(4, 0)$ ☐ $(2, 1)$ ☐ $(4, 1)$.

IV. Demand functions: the income and substitution effects

15. A consumer has preferences described by the utility function $u(x, y) = 2xy$.

(a) Compute the demand curve for good x. Is good x inferior or normal? Is x a Giffen-good? Which is the price-elasticity of this good? Represent the Engel-curve of x for $p_x = 2$ and $p_y = 3$.

(b) Determine and represent the optimal consumption bundle if his income is $I = 15$ and the prices of the goods are $p_x = 2$ and $p_y = 3$. Compute the income and substitution effects over good x of an increase to $p'_x = 3$.

16. A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = xy^2$. The prices are p_x and p_y euros per unit, respectively, and the consumer's income is I euros.

(a) Calculate the consumer's ordinary demands, $x(p_x, p_y, I)$ and $y(p_x, p_y, I)$.

(b) Assume that the consumer's income is $I = 12$ and prices are $(p_x, p_y) = (1, 2)$. In order to rise revenue to finance public expenditures, the government levies a 1 euro/unit sales tax on good x. Calculate the substitution and income effects of this tax on the demand of good x. If the government replaced this tax with an income tax that rises the same amount, would the consumer be better off or worse off than with the sales tax on good x?

17. A consumer's preferences over clothing (x) and food (y) are represented by the utility function $u(x, y) = x + 2\sqrt{y}$. The price of clothing is $p_x = 1$ euro per unit, the price of food is $p_y = p$ euros per unit and the consumer's income is I euros.

(a) Calculate her demand for food $y(p, I)$ for $I \geq 1/p$ and for $I < 1/p$.

(b) Represent graphically the consumer's budget set and calculate her optimal bundle and utility level for $p = 1/2$ and $I = 1$. Calculate the income and substitution effects over the demand for food of a unit tax of 50 cents on the price of food. Calculate the tax revenue.

18. A consumer preferences for goods x and y are represented by the utility function $u(x, y) = y + 2 \ln x$.

(a) Calculate her ordinary demand functions. (Use the notation p_x, p_y and I for prices and income, respectively.)

(b) Assume that the prices are $p_x = p_y = 1$ and the consumer's monetary income is $I = 30$, and that a sale tax of one euro per unit of x is introduced. Show that the revenue obtained with this sale tax is less than the revenue that can be obtained with a direct tax on monetary income, R, whose effect on the consumer welfare is identical to that of the sale tax.

19. A consumer's preferences over clothing (x) and food (y) are represented by the utility function $u(x, y) = 2 \ln x + \ln y$.

(a) Calculate her ordinary demand functions for food and clothing. Represent graphically the consumer's budget set and calculate her optimal bundle and utility level for $(p_x, p_y) = (2, 1)$ and income $I = 12$.

(b) Calculate the substitution and income effects over the demand of food (y) of a unit sale tax of one euro.

Multiple Choice Questions

IV.1. If a consumer's preferences are represented by the utility function $u(x, y) = \min\{x, y\}$, then a decrease in the price of x causes

☐ a decrease in the demand of x	☐ a negative income effect
☐ an ambiguous effect on the demand of x	☐ a substitution effect equal to zero.

IV.2. A consumer's preferences are represented by the utility function $u(x, y) = 2x+y$, and market prices are $p_x = p_y = 2$. A decrease in price of good x to $p'_x = 1$ causes

☐ an increase in the demand of y	☐ a substitution effect equal to 0
☐ an decrease in the demand of x	☐ an income effect equal to 0.

IV.3. If x is and inferior good, then a decrease of p_x

☐ increases the demand of x	☐ increases the demand of y
☐ decreases the demand of x	☐ has an ambiguous effect on the demand of x.

IV.4 If x is a giffen good, then the substitution (SE), income (IE) and total (TE) effects of an increase of its price p_x are:

☐ $SE < 0$, $IE > 0$, $TE < 0$	☐ $SE < 0$, $IE > 0$, $TE > 0$
☐ $SE < 0$, $IE < 0$, $TE < 0$	☐ $SE > 0$, $IE > 0$, $TE > 0$.

IV.5. An individual preferences are represented by the utility function $u(x, y) = x + 2y$, and the prices are $p_x = p_y = 2$. A decrease of the price of y to $p'_y = 1$ causes

☐ an increment of the demand of good x	☐ a substitution effect equal to zero
☐ a decrease of the demand of good y	☐ an income effect equal to zero.

V. Applications: the choice of consumption and leisure, labor supply

20. Esther receives a monthly assignment of M euros from her parents. In addition, her aunt offers her the possibility of day caring her cousins, Elena and Sara, during the weekend, paying her a daily wage of w euros. Esther's preferences for leisure during the weekend (h, measured in days) and consumption (c, measured in euros) are represented by the utility function $u(h, c) = \sqrt[3]{h^2 c}$. Esther is endowed with $H = 9$ weekend days. Describe Esther's problem and calculate her demand of consumption and leisure, and her "labor" supply (number of days she will offer to take care of her cousins) as a function of M and w. Represent Esther's budget set and calculate her optimal consumption-leasure bundle for $M = 120$ and $w = 40$. If $M = 120$, what is the lowest wage $\underline{w}$ for which Esther's labor supply is positive? If $w = 40$, what is the lowest monthly assignment $\underline{M}$ for which Esther's labor supply is zero?

21. A consumer-worker, who receives a non-wage rent of 360 euros every day, has the following preferences over consumption (c) and leisure (h), represented by the utility function $U(h, c) = c^3 h$.

(a) Determine which is the lowest wage per hour for which he is willing to work a positive amount of time.

(b) How many hours will he work at a wage of 4 euros per hour?

(c) How much time will he work at a wage of 9 euros per hour? And at 11.25 per hour?

(d) Determine the income and substitution effect of an increase in the wage per hour from 9 to 11.25 euros.

22. The preferences of a worker for leisure (h, measure in hours) and consumption (c, measured in euros) are represented by the utility function $u(h, c) = h + 2\sqrt{c}$. The worker has $H = 16$ hours to use as labor or leisure, and has no other sources of income than his labor income. Calculate and represent his labor supply. (Note that $p_c = 1$ since we are measuring consumption in euros). Assume that the wage is $w = 4$, and that there is an unemployment subsidy of S euros (that is received only by those workers who do not work at all). Calculate the leisure-consumption bundle of the worker assuming that $S = 5$. What is the smallest value of S for which the worker would choose not to work?

23. María has a daily endowment of 12 hours (to work or use for leisure activities) and a monetary (non-labor) income of M euros. Her preferences are represented by the utility function $u(h, c) = 2 \ln h + \ln c$, where h denotes the number of hours of leisure and c denotes her consumption. Set the price of consumption to $p_c = 1$, and denote by w the hourly wage.

(a) Describe María's problem, and calculate her demands of leisure and consumption, as well as her labor supply as functions of M and w.

(b) Using your results in (a), represent María's budget set and calculate her optimal consumption-leisure bundle for $M = 6$ and $w = 4$. Calculate income and substitution effects over the demand of leisure caused a 25% tax on labor income.

24. Ana is a student whose welfare depends on her average grade $m \in \mathbb{R}_+$ and her consumption $c \in \mathbb{R}_+$. (Assume that consumption is measured in euros, so that $p_c = 1$.) Her preferences are represented by the utility function $u(m, c) = \ln m + \ln c$. Ana has $H = 15$ hours that she can allocate to study and/or supply as labor. Ana's average grade m is determined by the number of hours she studies e according to the formula $m = \frac{2}{3}e$. The hourly wage is $w \geq 0$ euros, and Ana has no other income (besides her labor income).

(a) Describe Ana's budget constraint and graph her budget set in the plane (m, c). Calculate the number of hours she studies and works as a function of w. Assuming that $w = 4$, calculate Ana's optimal average grade and consumption (m^*, c^*) bundle, and represent it in the graph.

(b) Assume now that a program is introduced that rewards students with an average grade $\bar{m} = 7$ or above with $M = 10$ euros. Assuming that $w = 4$, graph Ana's new budget set and determine the impact of this change on her average grade and consumption.

25. Consider an individual about to retire with preferences for leisure (h, measured in hours) and consumption (c, measured in euros) represented by the utility function $u(h; c) = hc$, whose salary is 15 euros/hour, and has a total of 140 hours per month available to use as leisure or supply as labour. He is entitled to a monthly pension 1200 euros, but if he continues working, this pension would be reduced by $t(15l)$, where l is the number of hours he works and $t \in [0, 1/2]$. Write down the individual's budget constrain, draw his budget set and calculate his labor supply $l(t)$. For which values of t will he choose to retire altogether?

26. There are 10 consumer-workers with preferences over leisure (h) and consumption (c) described by the utility function $u(h, c) = h + \ln c$. Each individual has 1 unit of time (one day, for example) for leisure or labor activities, and a monetary income of M euros. The price of the consumption good is $p = 1$ and the wage is w.

(a) Derive the labor supply of each individual as a function of w and M.

(b) Compute the *aggregate labor supply*, assuming $M = 3$ for 5 individuals and $M = 0$ for the remaining 5 individuals. Calculate the equilibrium wage, the employment level and the worker surplus assuming that the labor demand is $L^D(w) = 20/w$. Graph the labor market demand and supply curves and identify the equilibrium.

VI. Applications: consumer surplus, compensated variation, equivalent variation, price indices

27. Assume that x and y represent housing services, measured in squared meters per year, and the rest of goods, respectively. A representative consumer has preferences for those goods represented by the utility function $u(x, y) = xy^2$. Prices are $p_x = 3$ and $p_y = 1$. The government proposes a subsidy of 1 euro per square meter consumed. The opposition complains arguing that the value of the subsidy to the individual is inferior to the cost incurred by the State. What would you recommend? Why?

28. Consider the following situation of a pensioner who consumes two goods, food (x) and clothes (y). When he got retired in 1997, the Social Security awarded a pension of 15000 euros to him. In that year, the prices of food and clothes were 8 euros and 50 euros, respectively. Suppose that the utility function of the pensioner is $u(x, y) = x\sqrt{y}$.

(a) Determine and represent the pensioner's choice under those conditions.

(b) Suppose that in 1998 the prices of food and clothes are 10 euros and 75 euros, respectively. Determine and represent the choice of the pensioner in case that his pension is not updated.

(c) Which pension should we give the pensioner for him to recover his initial utility level with the minimum cost for the social security?

29. The preferences of a consumer between 2 goods x and y are described by the utility function $u(x, y) = \ln x + \ln y$. The prices of these goods are $p_x = 1$ and $p_y = 1/2$.

(a) Determine the solution to the consumer's problem at those prices for any income I.

(b) Because of an ecological disaster, the supply of good x decreases and its price doubles. As a consequence, the welfare of the consumer decreases. Trying to mitigate the disaster, the local authority is willing to subsidize the consumer. Compute the monetary quantity S that must be given to the consumer to keep the same utility level he had before the disaster.

(c) If the authority did not know the preferences of the consumer but had observed the quantities of both goods consumed before the disaster, they could compensate the consumer giving him the income variation that would allow him to purchase those quantities at the new prices. Which would be the cheapest solution for the local authority?

30. János, the typical consumer from Hungary, only consumes two goods: paprika, x, and brandy, y. The preferences of János are represented by the utility function $u(x, y) = y + \ln x$. The price of paprika is $p_x = p$ euros and the price of brandy is $p_y = 1$ euro. János monetary income is I euros.

(a) Calculate János demand for paprika and brandy as a function of (p, I) for $I > 1$.

(b) Represent János' budget set for $(p, I) = (1/2, 10)$ and calculate his optimal bundle and utility level.

(c) Calculate the income and substitution effects on the demand of paprika of an increase of its price from $p = 1/2$ to $p' = 1$.

(d) How much is János willing to pay to avoid the price of paprika to increase from $p = 1/2$ to $p' = 1$?

31. Let's classify goods into two groups: clothes and shoes x and food y. The preferences of Manuel, who retired with a pension $I = 750$ euros, are represented by a utility function $u(x, y) = x^{2/5}y^{3/5}$. The prices the year he retired, which will be taken as a base year, were $p_0 = (1, 1)$. the current prices are $p_1 = (2, 3/2)$.

(a) Calculate the pension I_1 that would guarantee Manuel the same welfare level a the we had in his retirement year.

(b) Manuel's true consumer price index is $CPI^* = I_1/I_0$. Verify that the Laspeyres price index CPI_L is larger that CPI^*.

32. A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = x + \sqrt{y}$. The prices are p_x and p_y euros per unit, respectively, and the consumer's income is I euros.

(a) Calculate the consumer's ordinary demands, $x(p_x, p_y, I)$ and $y(p_x, p_y, I)$. Represent the consumer's budget set and calculate her optimal consumption bundle if her income is $I = 8$ and prices are $(p_x, p_y) = (4, 1)$.

(b) Assuming that $I = 8$ and $(p_x, p_y) = (4, 1)$, calculate the equivalent variation of a tax of 1 euro per unit of good y, and compare it to the tax revenue. How do you explain the difference?

Multiple Choice Questions

VI.1. Assume that in 2007 prices were $(p_x, p_y) = (3, 2)$, and in 2008 they were $(p'_x, p'_y) = (2, 4)$. If the bundle of the representative consumer is $(2, 2)$, and one measures the CPI (consumer price index) using a Laspeyres Index, then the percentage change in prices is

☐ 25% ☐ 20%
☐ 30% ☐ 50%.

VI.2. Assume that in 2007 prices were $(p_x, p_y) = (2, 3)$, and in 2008 they were $(p'_x, p'_y) = (3, 4)$. If the bundle of a consumer in 2007 was $(2, 2)$, then his *true* CPI index is

☐ less than 40% ☐ exactly 40%
☐ greater than 40% ☐ undetermined.

VI.3. Between 2009 and 2010 the prices of both goods, x and y, increased by 10%. If the consumer income increased by a rate equal to the consumer's *true* price index, then her budget line

☐ shifted parallel towards the origin ☐ rotated over its intersection with the x-axis
☐ shifted parallel away from the origin ☐ maintained its position.

VI.4. The prices of the goods x and y in the base period were $(p_x, p_y) = (2, 2)$, and the optimal bundle of the representative agent was $(x^*, y^*) = (2, 1)$. If the current prices are $(p'_x, p'_y) = (1, 4)$, then the Laspeyres CPI is

☐ $CPI_L = 1$ ☐ $CPI_L = 1,66$
☐ $CPI_L = 1,5$ ☐ $CPI_L = 0,8$.

VI.5. If a consumer's income is multiplied by the Laspeyres consumers price index, then

☐ the consumer receives exactly the compensated variation
☐ the consumer's budget set remains unchanged to be that of the base period
☐ the consumer's welfare increases relative to that of the base period
☐ the consumer's welfare remains unchanged to be that of the base period.

VI.6. A consumer's preferences are described by the utility function $u(x, y) = xy$. The prices in the base period were $(p_x, p_y) = (1, 1)$ and her optimal consumption bundle was $(x^*, y^*) = (1, 1)$. If the current prices are $(p'_x, p'_y) = (1, 4)$, then the *difference* between her Laspeyres consumer price index (CPI) and her true CPI is:

☐ 0,1 ☐ 0,2 ☐ 0,5 ☐ 1.

VI.7. If a consumer's preferences are represented by the utility function $u(x, y) = 2x + y$, her income is $I = 4$ and the prices are $p_x = p_y = 1$, then the *equivalent variation* of a sales tax on 1 euro per unit of good x is:

☐ 0 ☐ 1 ☐ 2 ☐ 4.

VII. Consumer Theory under uncertainty

33. A student has just graduated. He has just received an inheritance of 4 millions of euros and is considering whether to invest 2 millions of euros in a start up business. If the business is successful, he expects a gross profit of 6 millions of euros, but if it fails he will lose the investment. The probability of success is $p = 1/2$.

(a) Determine whether the student will make the investment if her preferences are represented by the Bernoulli utility function: (i) $u(x) = x$; $u(x) = x^2$; $u(x) = \sqrt{x}$?.

(b) Suppose now the student's preferences are represented by the utility function $u(x) = x^2$. A study that costs a millions of euros predicts with certainty if the investment will be lucky or not. Should the student buy the mentioned study if $a = 1$? And if $a = 0.5$?

(c) Suppose finally that the student's preferences are represented by the utility function $u(x) = \sqrt{x}$. Let's suppose that, to a this student, it is offered the possibility to implement the the previous investment if he receives a tax break of b millions of euros. Should the student accept this tax break if $b = 1$? And for $b \neq 1$?

34. Pedro Banderas has a wealth of 100 thousand euros and is considering whether to produce a movie whose budget is 250 thousand euros. A film company is willing to finance the movie but wants Pedro to share some of the risk (and profits); specifically it is willing to finance 80% of the budget. Assuming that the distributors like the movie, Pedro expects the movie to generate box office revenue of 250 thousand euros if the reviews are bad, and as much as 1,5 million euros it the reviews are good. It is known that distributors like 8 out of 10 movies that are produced, and that 1 out of 10 movies that are distributed get good reviews. Pedro's preferences are represented by the Bernoulli utility function $u(x) = \sqrt{x}$.

(a) Represent the decision problem and determine whether or not Pedro should produce the movie.

(b) Determine whether Pedro may be willing to finance 40% (instead of 20%) of the movies' budget.

35. The oil company Tibitrol has bought some deserted land in Monegros. The company's geologist estimates that the probability that they will find oil in this land is 0.2. The drilling of the land in order to check whether or not it really has oil costs 100 millions of euros. If they find oil, then the company will make revenues of 300 millions of euros. If they do not find oil, then the drilling will be completely useless. The company has to decide whether or not it will do the drilling. If the company is risk neutral what will it decide to do in order to maximize its expected utility? And if the company is risk averse?

36. An individual must decide whether to finance its house with a mortgage at a *fixed interest rate (FM),* or at a *variable interest rate (VM).* FM involves an annual payment of P thousands of euros, while a VM involves an annual payment of 17 thousand euros with probability $\frac{1}{2}$, 20 thousand euros with probability $\frac{1}{3}$, and 30 thousand euros with probability $\frac{1}{6}$. The individual's annual income is 50 thousand euros, and his welfare depends on his net income x, measured in thousands of euros, which is equal to his income minus his mortgage payment. The individual's preferences over lotteries are

represented by the Bernoulli utility function $u(x) = 2\sqrt{x}$. For which values of P would he prefer to finance his house with the FM mortgage?

37. A professor is preparing a multiple choice exam in which there are four possible answers to each question, of which only one is correct. A correct answer is worth 1 point. Students preferences satisfy the usual assumptions, and are described by a Bernoulli utility function $u(x)$, where x is the number of points obtained in the exam. Assume that when a student does not know the answer to a question, then he believes that all answers are equally likely to be correct.

(a) Assuming that there is no penalty for an incorrect answer – that is, an incorrect answer is worth zero points – describe the alternative decisions (lotteries) of a student who does not know the answer to a question. If a student is risk neutral, is it optimal to answer the question randomly? ¿What if she is risk averse?

(b) Assuming that the student is risk neutral, calculate the certainty equivalent of the lottery of responding to a question the student does not know the answer to. If it is known that students are either risk averse or risk neutral, which is the minimum penalty for each incorrect answer that would induce students not to respond to questions whose answers they do not know?

38. An entrepreneur has an income of 150 thousand euros and is considering introducing a new touristic product that requires an investment of 200 thousand euros. An investment fund offers him the possibility of financing 50% of the investment in exchange of 60% of the revenues it generates. The entrepreneur beliefs that if the weather is favorable during the tourist season, then the new product would generate a revenue of 200 thousand euros or 500 thousand euros with the same probability, while if the weather is unfavorable the product would be a fiasco (that is, it would generate no revenue). Historically, the region enjoys a favorable weather with probability 0.6. The preferences of the entrepreneur are represented by the Bernoulli utility function $u(x) = x^2$, where x is the net disposable income in thousands of euros. Determine whether the entrepreneur should introduce the new product (I), accepting the offer of the investment fund, or rather abstain form introducing the product (NI) maintaining his current income. Would the entrepreneur be willing to pay 25 thousand euros to know before making the decision whether the weather will be favorable during the tourist season? (Your answers would only be valid if they are supported by the appropriate correct calculations.)

39. Germán Cienfuegos is considering starting a business that requires an investment of 200 thousand euros, and that may yield a gross income of 300 thousand euros with probability p and of only 100 thousand euros with probability $1 - p$. (That is, the investment will result in gains or losses of 100 thousand euros with probabilities p and $1 - p$.) Germán only has 100 thousand euros available, but he may get a mortgage on his house (which has a value of 100 thousand euros) at an interest rate of 5%. Alternatively, he has a friend that seems willing to share equally (50%) risks and profits contributing with 100 thousand euros to complete the amount required to start the business. The problem is that it is unclear whether this friend is *cooperative* or *conflictive*. Germán believes that if he manages the firm himself (that is; if he is the only investor), then $p = 3/4$, while if he shares the investment and management

with his friend then this probability decreases to $p = 2/3$. Germán's Bernoulli utility is $u(x) = \sqrt{x}$, where x is his wealth (that is, the sum of the value of his house and his cash) measured in thousands of euros. Describe the problem that Germán faces (the set of lotteries he may choose from) and determine his optimal decision. Assuming that when his friend is cooperative the probability remains at $p = 3/4$, determine if Germán is willing to pay 5 thousand euros to know beforehand whether his friend is cooperative or conflictive.

40. In the market of car insurance, there are two kinds of drivers, good drivers (who have one accident per year with probability 0.1 and no accident with probability 0.9) and bad drivers (who have one accident per year with probability 0.1, two accident with probability 0.05 and no accident with probability 0.85). Repairing a car cost on average 2,000 euros. The proportion of good and to bad drivers is 2 to 1.

(a) Assume that the insurance companies are risk neutral and they cannot distinguish between good and bad drivers. What is the minimum price that these companies would be willing to offer in order to cover the risk of an accident?

(b) Imagine that the preferences of the drivers are represented by the utility function $u(x) = \sqrt{x}$, and that their initial wealth is 5,000 euros. Which type of drivers (good and/or bad) will subscribe to an insurance policy with the minimum price determined in part (a)?

41. A risk neutral person needs to put a mortgage on one of his buildings in order to get 200,000 euros. He has to pay back this amount in 2 annual payments of 100,000 euros, each one with the corresponding interest rate. The mortgage credits among which he could choose are: (1) Fixed interest rate: 10% per year; (2) Interest rate 9% in the first year which can increase to 14%, decrease to 8% or remain the same in the second year; (3) Interest rate 7% in the first year which can increase to 20%, decrease to 6% or remain the same in the second year.

(a) Determine the decision which maximizes the expected profits knowing that the interest rate increases with probability 0.6 and decreases with probability 0.2.

(b) How much is this person willing to pay in order to learn whether the interest rate will increase, decrease or remain the same?

42. A consumer must choose between buying an apartment in Madrid or a house in the suburb. Both choices would cost him 120,000 euros. He is indifferent between the two options, except for his expectation regarding revaluation. If the housing prices keep on increasing (event E_1), the price of the apartment will reach 140,000 euros, while the price of the house will reach 340,000 euros. The probability that this will happen is 0.3. If the opposite thing (decrease in the housing prices) happens (event E_2), the price of the apartment will be 70,000 euros and the price of the house 20,000. The preferences of the consumer are represented by the utility function $u(x) = \sqrt{x}$, where x is the wealth expressed in euros. The consumer's initial wealth is 140,000 euros.

(a) Represent the decision problem and determine whether the consumer should buy the house or the apartment.

(b) Should he pay 20,000 euros in order to learn whether the housing prices will decrease or increase?

43. The introduction of a new product in the market takes includes three stages: Design, Experimentation, and Production. 7 out of 10 products do not pass the design stage. From those that do pass it, only 10% pass the experimentation stage and are being produced. Only 1 out of 5 products produced has success in the market. For each new product the costs of each stage are 100,000, 20,000, and 200,000 euros, respectively. The expected profits from a product that passed successfully the three stages are 60 millions of euros.

(a) Which is the expected value of constructing a new product?

(b) For 15,000 euros a consultant can predetermine (without any uncertainty) whether or not a product that has already passed the design stage will pass the experimentation stage. What is the value of the consultant's services, assuming the entrepreneur is risk neutral?

44. The marketing chief of a big computer producer has to decide whether to launch a new campaign before (d_1) or after the month of May (d_2). If he does before, he will manage to obtain 100 millions Euros of sales. If he does after, there is a risk that its competitor launches its own campaign before (C), which will occur with probability 0.4. Moreover, the sales also depend on the predictions of the state of the economy, which can be good (A) with probability 0.5, stable (E) with probability 0.3, and bad (R). If the economy is good, and the competitor has not launched its campaign, sales can reach 150 millions Euros, and if its competitor did launch its campaign, sales would reach a value of 120 millions Euros. If the economy is stable, sales would reach 90 millions of Euros if the competitor launches its campaign and 110 millions if it does not. Finally, when the economy is bad, and if the competitor launches its campaign, sales will reach 70 millions Euros while they would go up to 80 millions Euros if the competitor does not. Assuming that the producer is risk-neutral, what is the best decision? How much would be the marketing chief ready to pay in order to know with certainty all the uncertain variables of the problem? How much would he be willing to give to an industrial spy who would tell him with certainty whether the competitive firm will launch its own campaign or not?

45. An individual has an initial wealth of 500 thousand euro and an annual income of 250 thousand euro. The income tax rate is 50%. He is considering whether to declare his full income, only half of his income, or nothing at all. It is known that the probability that he is inspected by *Hacienda* is 0.1. If the inspection detects that he misdeclared his income, he will have to pay the missing taxes plus an identical amount as a fine. His preferences are represented by the Bernoulli utility function $u(r) = 2\sqrt{x}$, where x is wealth plus his income net of taxes and/or fines.

(a) Describe his decision problem and identify the optimal decision.

In parts (b) and (c) suppose that the individual decides not to files a tax form, but now worries about the possibility that he may be inspected by the *Hacienda* authorities.

(b) The individual consults with an expert lawyer that tells him that the solution involves filing a voluntary tax return and facing a fine of m thousand euro. For which values of m will he be willing to adopt this solution?

(c) Alternatively, assume that someone offers to inform him whether or not he is in the list of citizens *Hacienda* plans to inspect. Obtain the equation identifying the value of this information to the individual. Will he pay 20 euro for this information?

Multiple Choice Questions

VII.1. An individual's risk premium for the lottery $l = (x, p)$, that pays $x = (0, 4, 16)$ with probabilities $p = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$, is $RP(l) = 2$. Hence his certainty equivalent is

☐ $CE(l) = 2$ ☐ $CE(l) = 1$
☐ $CE(l) = 3$ ☒ $CE(l) = 4$.

VII.2. If the certainty equivalent of a lottery l, which pays 20 thousand euros or 10 thousand euros with the same probability, is 14 thousand euros, then the individual

☐ is risk loving ☐ is risk neutral
☒ is risk averse ☐ has an indeterminate risk attitude.

VII.3. If an individual's certainty equivalent of lottery $l = (0, 2, 10; \frac{3}{10}, \frac{1}{2}, \frac{1}{5})$ is $CE(l) = 2$, then his risk premium is

☐ $RP(l) = -1$ ☐ $RP(l) = 1$
☐ $RP(l) = 2$ ☐ $RP(l) = 0$.

VII.4. The risk premium of the lottery $l = \left(0, 8; \frac{1}{4}, \frac{3}{4}\right)$ for an individual A whose preferences are represented by the Bernoulli utility function $u_A(x)$ is $RP_A(l) = 2$. If the preferences of an individual B are represented by the Bernoulli utility function $u_B(x) = \frac{1}{3}u_A(x)$, then her certainty equivalent of the lottery l is:

☐ $CE_B(l) = 2$ ☐ $CE_B(l) = 6$ ☒ $CE_B(l) = 4$ ☐ $CE_B(l) = 0$.

VII.5. Identify the certainty equivalent and risk premium of the lottery $l = (x, p)$ that pays $x = (0, 2, 4)$ with probabilities $p = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$ for an individual with preferences represented by the Bernoulli utility function $u(x) = x^2$.

☐ $CE(l) = 2$, $RP(l) = \sqrt{6} - 2$ ☐ $CE(l) = 2$, $RP(l) = 2 - \sqrt{6}$
☒ $CE(l) = \sqrt{6}$, $RP(l) = 2 - \sqrt{6}$ ☐ $CE(l) = \sqrt{6}$, $RP(l) = \sqrt{6} - 2$.

VII.6. A consumer preferences over lotteries are represented by the Bernoulli utility function $u(x) = \sqrt{x}$. Identify the expected utility and certainty equivalent of the lottery $l = (x, p)$ that pays $x = (0, 4, 9)$ with probabilities $p = (\frac{1}{6}, \frac{1}{2}, \frac{1}{3})$.

☐ $Eu(l) = 1$, $CE(l) = 1$ ☐ $Eu(l) = 4$, $CE(l) = 2$
☐ $Eu(l) = 2$, $CE(l) = 2$ ☒ $Eu(l) = 2$, $CE(l) = 4$.