WORKSHEET 5: Integration

1. (*) Calculate the following integrals:

$$
\begin{array}{lll}
a)\ \displaystyle\int \frac{x^2+x+1}{x\sqrt{x}}dx & b)\ \int xe^{-2x}dx & c)\ \int sin^{14}x\ \cos x\ dx \\
d)\ \int (x+1)(2-x)^{1/3}dx & e)\ \displaystyle\int \frac{x^4}{1+x^5}dx & f)\ \int (1+\frac{1}{x})^3\frac{1}{x^2}dx \\
g)\ \int sin^3x\ dx & h)\ \int xe^{ax^2}\ dx & i)\ \displaystyle\int \frac{1}{3+x^2}dx \\
j)\ \displaystyle\int \frac{\sqrt{x-1}}{1+\sqrt[3]{x-1}}dx & k)\ \displaystyle\int \frac{x}{\sqrt{16-x^2}}dx & l)\ \int x^4\ln x\ dx \\
m)\ \displaystyle\int \frac{dx}{\sqrt[4]{x^3}-\sqrt{x}} & n)\ \int (\ln x)^2dx & ñ)\ \displaystyle\int \frac{40x}{(x-1)^{40}}dx \\
o)\ \displaystyle\int \frac{4x+6}{(x^2+3x+7)^3}dx & p)\ \displaystyle\int \frac{2x-6}{(x-2)^2}dx & q)\ \displaystyle\int \frac{x^2+1}{x^3-4x^2+4x}dx \\
r)\ \displaystyle\int \frac{2x+1}{x^3+6x}dx & s)\ \displaystyle\int \frac{1}{\frac{x^2}{2}-2x+4}dx & t)\ \displaystyle\int \frac{x^4}{x^4-1}dx
\end{array}
$$

2. How many different intersection points can two different primitives of the same function have?

3. (*) Let $f : [0,2] \longrightarrow \mathbb{R}$ be continuous, increasing in $(0,1)$, decreasing in $(1,2)$ and, also, satisfying that: $f(0) = 3$, $f(1) = 5$ and $f(2) = 4$. Between which values can we guarantee that $\int_0^2 f(x)\,dx$ is located?

4. (*) Certain company has determined that its marginal cost is $\frac{dC}{dx} = 4(1+12x)^{-1/3}$.

   Find the cost function if $C = 100$ when $x = 13$.

5. (*) Given that the marginal cost of producing x units is $x+5$ and the average cost has a minimum in $x = 4$, find the fixed costs of the firm.

6. (*) Calculate $F'(x)$ in the following cases:

$$a)\ \int_x^{x^3} t\cos t\,dt \quad b)\ \int_1^{x^2} \sqrt{t^4+2t}\ dt \quad c)\ \int_1^{x^2} \left(t^2-2t+5\right)\ dt$$

7. Calculate $F'(x)$ in the following cases:

   (a) $\int_{-x}^{x^2} \tan^2 t\,dt$, supposing that $x^2 < \frac{\pi}{2}$.

   (b) $\int_{x^2}^{2x} f^2(2t)\,dt$, supposing that f is continuous.

8. (*) What are the values of x where $F(x) = \int_{-3}^{x} \frac{t^2-4}{3t^2+1}dt$ has a local maximum or minimum?

9. Let $F(x) = \int_{x^2}^{2x} f(t^2)dt$ be such that $f(1) = 1$, $f(2) = f(4) = 4$ and f is continuous. Calculate $F'(1)$.

10. (*) Calculate observing the symmetry of the functions:

$$a)\ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^{27}x\cos^{28}x\,dx \quad b)\ \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\sqrt[3]{x^5\cos 3x} + \cos\tfrac{x}{3} + \tan^3 x\,)dx$$

11. Let f be a function with period T, such that $\int_0^T f = b$. Find $\int_a^{a+nT} f$.

12. (*) Find the area located between the following curves:

$$\begin{array}{l} a)\ f(x) = x^2 - 4x + 3,\ g(x) = -x^2 + 2x + 3 \\ b)\ f(x) = (x-1)^3,\ g(x) = x - 1 \\ c)\ f(x) = x^4 - 2x^2 + 1,\ g(x) = 1 - x^2 \end{array}$$

13. (*) Graph the functions $y = 2e^{2x}$ and $y = 2e^{-2x}$. Calculate the area located between those graphs and the lines $x = -1$ and $x = 1$.

14. Let $f : [1, 3] \longrightarrow [2, 4]$ be increasing, continuous and bijective such that $\int_1^3 f\,dx = 5$. Calculate $\int_2^4 f^{-1}(x)\,dx$

15. (a) Given $f : [0, 4] \to \mathbb{R}$, convex and increasing with values $f(0) = 0$, $f(2) = \alpha$, $f'(2) = \beta$, $f(4) = 16$. Estimate as a function of α and β, the value of $\int_0^4 f(x)dx$.
    (b) Given $f : [0, 4] \to \mathbb{R}$, concave and increasing with values $f(0) = 0$, $f(2) = \alpha$, $f'(2) = \beta$, $f(4) = 2$. Estimate as a function of α and β, the value of $\int_0^4 f(x)dx$.

16. The sales of a product are given by the formula $S(t) = 10 + 5sin(\frac{\pi t}{6})$ where S is measured in thousands of units and time t in months. Calculate the average sales during the year $(0 \le t \le 12)$.

17. Calculate:

$$\begin{array}{lll} a)\ \displaystyle\int_0^1 \frac{1}{\sqrt{x}}dx & b)\ \displaystyle\int_0^3 \frac{1}{x^3}dx & c)\ \displaystyle\int_1^\infty \frac{1}{x^2}dx \\ d)\ \displaystyle\int_1^\infty e^{-x}dx & e)\ \displaystyle\int_{-\infty}^\infty \frac{dx}{1+x^2} & f)\ \displaystyle\int_{-2}^4 \frac{dx}{x^2} \end{array}$$

18. Calculate $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$