WORKSHEET 4 : Differentiation II

1. Compute the following limits:

   a)(*) $\lim_{x\to\infty}(1+x)^{1/x}$ b) $\lim_{x\to 0^+} x\ln x$ c)(*) $\lim_{x\to\infty} x^{1/x}$

   d)(*) $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{2}{x-1}\right)$ e) $\lim_{x\to\infty} x\tan(1/x)$ f) $\lim_{x\to 0}\frac{arcsin x-\arctan x}{x}$

   g) $\lim_{x\to 1/2}(4x^2-1)\tan(\pi x)$

2. Compute the asymptotes of the following functions:

   a)(*) $f(x)=\frac{2x^3-3x^2-8x+4}{x^2-4}$ b) $f(x)=\frac{x^3}{x^3+x^2+x+1}$ c)(*) $f(x)=2x+e^{-x}$

   d) $f(x)=\frac{sinx}{x}$ e)(*) $f(x)=\frac{x-2}{\sqrt{4x^2+1}}$ f) $f(x)=\frac{3x^2-x+2\sin x}{x-7}$

   g)(*) $f(x)=\frac{e^x}{x}$ h)(*) $f(x)=xe^{1/x}$ i)(*) $f(x)=\frac{x}{e^x-1}$

3. (*)Find the Taylor polynomium of order 2 in a and, using that polynomium, compute the approximate value of the function on $x=a+0.1$.

   a) $f(x)=e^x$ in $a=0$ b) $f(x)=\sin x$ in $a=0$ c) $f(x)=\frac{\ln x}{x}$ in $a=1$

4. (*)Given the Taylor polynomium of order 2 in $a=0$ of f , determine if the function has a local maximum or minimum at the point $(0,f(0))$.

   a) $P(x)=1+2x^2$ b) $P(x)=1+x+x^2$ c) $P(x)=1-2x^2$

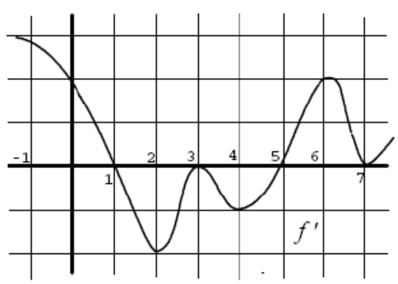
5. Compute the (absolute and local) maxima and minima of f in the given intervals:

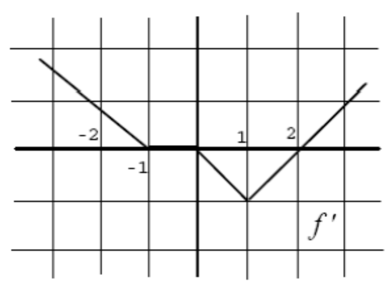
   a)(*) $f(x)=3x^{2/3}-2x$ in $[-1,2]$.

   b) $f(x)=xe^{-x}$ in $[1/2,\infty)$, $[0,\infty)$ and $I\!R$.

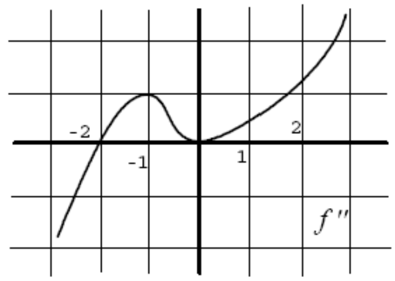
6. (*)Compute in which point the slope of the tangent line to the graph of the function $f(x)=-x^3+2x^2+x+2$ takes its maximum value.

7. The first (*) and second drawings show the graphs of the derivatives of different functions f. Determine the increasing/decreasing, concavity/convexity intervals of f , and its local extreme and inflection points.





8. The following drawing shows the graph of the second derivative of f. Determine the convexity intervals of f and the inflection points. Determine where the function is increasing and decreasing and the relative extrema of f assuming that $f\prime(-3)=f\prime(0)=0$.



9. Let $f(x) = \begin{cases} x^{\alpha} \text{ if } 0 \leq x \leq 1 \\ x^{\beta} \text{ if } 1 \leq x \end{cases}$ Discuss, depending on the values of α y β,when f is concave or convex.

10. (*)Let $f : \mathbb{R} \to \mathbb{R}$ convex, and let $x > 0$. Check graphically the following inequalities:

$$f(1) < \frac{1}{2}(f(1-x) + f(1+x)) < \frac{1}{2}(f(1-2x) + f(1+2x))$$

11. (*)Let $f : \mathbb{R} \to \mathbb{R}$ concave, and let $x > 0$. Check graphically the following inequalities:

$$f(1) > \frac{1}{2}(f(1-x) + f(1+x)) > \frac{1}{2}(f(1-2x) + f(1+2x))$$

12. (*)Let $f : [0, \infty) \to \mathbb{R}$, convex, such that $f'(1) = 0$.

a) Find the local extrema of f.
b) What can be said of the global extrema of f?
c) Consider now $f : [0, n] \to \mathbb{R}$. What can be said of the global extrema of f ?

13. (*)Let $f : [0, \infty) \to \mathbb{R}$, concave, such that $f'(1) = 0$.

a) Find the local extrema of f.
b) What can be said of the global extrema of f?
c) Consider now $f : [0, n] \to \mathbb{R}$. What can be said of the global extrema of f?

14. Study and graph the following functions:

a) $f(x) = x + \cos x$ b) $f(x) = \dfrac{e^{2x}}{e^x - 1}$ c) $f(x) = \dfrac{x}{\ln x}$ d) $f(x) = \sqrt{|x-4|}$

15. (*)Given the cost function $C(x) = 4000 + 10x + 0.02x^2$ and the demand function $p(x) = 100 - (x/100)$, find the price p per unit that gives the maximum benefit.

16. (*)Let $p(x) = x^2 - x + 1/3$ be the sale price of 1 kilo of plutonium when x units are sold. Taking into account that the firm sells in the market a maximum of 2 kilos, find the value of x that maximizes the profits of the firm. We can assume that the Government pays all the costs of the firm.

17. (*)Let $p(x) = 100 - x^2/2$ be the demand function of a product and $C(x) = 48 + 4x + 3x^2$ its cost function. What is the production x that minimizes the average cost? And if there exists a maximum production $\hat{x}$?

18. A firm that has a cost function $c(x) = x^2 + 1$ faces a demand given by the function $p(x) = \begin{cases} 10 & \text{si } 0 \leq x \leq 1 \\ 1 & \text{si } 1 < x \leq 10 \end{cases}$. Find the production that gives the maximum profit.

19. (*)A manufacturer sells 5000 units per month for 100 euros per unit and he believes that his sales would increase by 500 units for each 5 euros of decrease on the unitary price.

a) Find the demand, revenues and marginal revenues functions.

b) If the cost of production of x units is $C(x) = 1000 + 0.12x$, find the marginal profit function.