WORKSHEET 3 : Differentiation I

1. Find the points where the following functions have horizontal tangent.
   a) $f(x) = x^3 + 1$ b) $f(x) = 1/x^2$ c) $f(x) = x + sinx$
   d) $f(x) = \sqrt{x-1}$ e) $f(x) = e^x - x$ f) $f(x) = sinx + \cos x$

2. (*)Prove that the tangent lines to the graphs of $y = x$ and $y = 1/x$ in their points of intersection are perpendicular to each other.

3. In what point is the tangent to the curve $y^2 = 3x$ pararell to the line $y = 2x$?

4. (*)Calculate the intersection point with the x axis of the tangent line to the graph of $f(x) = x^2$ in the point $(1, 1)$.

5. Calculate a so that the tangent to the graph of $f(x) = a/x + 1$ in the point $(1, f(1))$ intersects the horizontal axis in $x = 3$.

6. (*)Find the tangent and normal lines to $f(x) = \arctan\left(\dfrac{sinx}{1+\cos x}\right)$ in $x = 0$.

7. Find the derivatives of the following functions.
   a) $f(x) = (sinx + \tan 3x)sin2x$ b) $f(x) = \dfrac{x\sqrt{x^2-1}}{2x+6}$
   c) $f(x) = 4x^{3/2}\cos 2x$ d) $f(x) = 5x\ln(8x + sin2x) + e^{\tan 5x}$

8. (*)Let $f(x) = 2[\ln(1 + g^2(x))]^2$. Using that $g(1) = g'(1) = -1$, calculate $f'(1)$.

9. (*)Using that $a^b = e^{b\ln a}$, differentiate $f(x) = x^{sinx}$ and $g(x) = (\sqrt{x})^x$.

10. (*)Let $f(x) = \ln(1 + x^2)$ and $g(x) = e^{2x} + e^{3x}$. Calculate $h(x) = f(g(x))$, $v(x) = g(f(x))$, $h'(0)$ and $v'(0)$.

11. Let $f : [-2, 2] \to [-2, 2]$ be continuous and bijective.

    a) Suppose that $f(0) = 0$ and $f'(0) = \alpha$, $\alpha \neq 0$. Find $\left(f^{-1}\right)'(0)$.
    b) Now suppose that $f(0) = 1$ and $f'(0) = \alpha$, $\alpha \neq 0$. Find $\left(f^{-1}\right)'(1)$.
    c) Now suppose that $f(1) = 0$ and $f'(1) = \alpha$, $\alpha \neq 0$. Find $\left(f^{-1}\right)'(0)$.

12. (*)Supposing that the following equations define y as a differentiable function of x, calculate y' in the given points:
    a) $x^3 + y^3 = 2xy$ in $(1, 1)$.
    b) $sinx = x(1 + \tan y)$ in $(\pi, 3\pi/4)$.
    c) $x^2 + y^2 = 25$ in $(3, 4), (0, 5)$ and $(5, 0)$.

13. Calculate the derivative of the following functions showing where they are not differentiable.

    a)(*) $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ 0 & \text{if } x > 0 \end{cases}$ b) (*)$g(x) = \begin{cases} 1/|x| & \text{if } x \leq -2 \\ (x+2)^2 & \text{if } -2 < x \leq 0 \\ 3 + sin(x + \frac{\pi}{2}) & \text{if } x > 0 \end{cases}$

    c) $h(x) = \begin{cases} \arctan^2 x & \text{if } x \leq 0 \\ sin^3 x & \text{if } 0 < x \leq 2\pi \\ sinx & \text{if } 2\pi < x \end{cases}$

14. (*)Find a and b so that the function $f(x) = \begin{cases} 3x + 2 & \text{if } x \geq 1 \\ ax^2 + bx - 1 & \text{if } x < 1 \end{cases}$ is differentiable.

15. Apply the mean value theorem to f in the given interval and find the c values of the thesis of the theorem.
    a) $f(x) = x^2$ in $[-2, 1]$ b) $f(x) = -2sinx$ in $[-\pi, \pi]$
    c) $f(x) = x^{2/3}$ in $[0, 1]$ d) $f(x) = 2sinx + sin2x$ in $[0, \pi]$

16. Let $f : [a, b] \longrightarrow [a, b]$ be a continuous function in $[a, b]$ and differentiable in (a, b). Prove that, if $f\,'(x) \neq 1$ in (a, b), then f has a unique fixed point in $[a, b]$.

17. Prove that the function f has a unique fixed point.

    a) $f(x) = 2x + \frac{1}{2} sinx$ b) $f(x) = 2x + \frac{1}{2}\cos x$

18. (*)Let $f(x) = x^3 - 3x + 3,\ f : [-3, 2] \rightarrow \mathbb{R}$. Determine the global extrema.

19. Let $f : [-5, 5] \rightarrow \mathbb{R}$ such that f reaches the maximum in $x = 2$ and the minumum in $x = -3$. Let $g(x) = -f(-x)$. What can be said about the maximum and the minimum of g?