WORKSHEET 4: Applications of the Derivative

1. (*)Calculate the second-order Taylor polynomial at a and find the approximate value of the function using the polynomial at $x = a + 0.1$.

a) $f(x) = e^x$ at $a = 0$ b) $f(x) = \dfrac{\ln x}{x}$ at $a = 1$

2. (*)Given the second-order Taylor polynomial of f at $a = 0$, find out if the function has a local maximum or minimum at the point $(0, f(0))$.

a) $P(x) = 1 + 2x^2$ b) $P(x) = 1 + x + x^2$ c) $P(x) = 1 - 2x^2$

3. Find the relative and absolute extrema of f in the given intervals:

a)(*) $f(x) = 3x^{2/3} - 2x$ in $[-1, 2]$ b) $f(x) = xe^{-x}$ in $[\frac{1}{2}, \infty)$, $[0, \infty)$ and $\mathbb{R}$

4. (*)Calculate the point of the graph of $y = -x^3 + 2x^2 + x + 2$ where its tangent line has the greatest slope.

5. (*)The figure A shows the graph of the derivative function of f. Determine the increasing/decreasing and concavity/convexity intervals of f, its local extrema and inflection points.

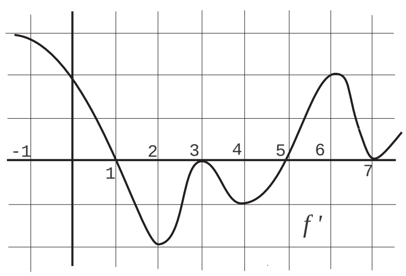

Figure A

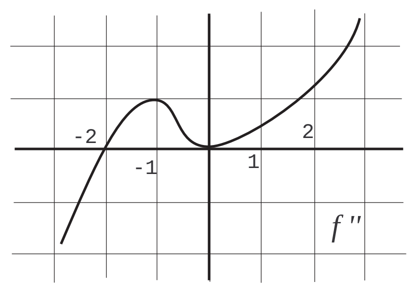

Figure B

6. The figure B shows the graph of the second derivative function of f. Determine concavity and convexity intervals of f and its inflection points. Determine the monotonicity and local extrema of f assuming that $f'(-3) = f'(0) = 0$.

7. (*)Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function, and $x > 0$. Check the following inequalities graphically:
$f(1) < \frac{1}{2}\left(f(1-x) + f(1+x)\right) < \frac{1}{2}\left(f(1-2x) + f(1+2x)\right)$

8. (*)Let $f : \mathbb{R} \to \mathbb{R}$ be a concave function, and $x > 0$. Check the following inequalities graphically:
$f(1) > \frac{1}{2}\left(f(1-x) + f(1+x)\right) > \frac{1}{2}\left(f(1-2x) + f(1+2x)\right)$

9. Let $f : [0, \infty] \to \mathbb{R}$ be a convex function such that $f'(1) = 0$

a) Find the local extrema of f.

b) What can be state about the global extrema of f?

c) Suppose now that $f : [0, n] \to \mathbb{R}$. What can be stated about the global extrema of f?

10. (*)Given the total cost function $C(x) = 4000 + 10x + 0.02x^2$ and the demand function $p(x) = 100 - \frac{x}{100}$, find the unitary price p that obtains the maximum benefit.

11. (*)Let $p(x) = x^2 - x + \frac{1}{3}$ be the sale price of one kilo of plutonium when x kilograms are sold. Taking into account that the firm sells a maximum of 2 kilograms on the market, find the value of x that maximizes the profits of the firm. We can assume that the Government pays all costs of the firm.

12. (*)Let $p(x) = 100 - \frac{x^2}{2}$ be the demand function of a product and $C(x) = 48 + 4x + 3x^2$ its cost function. What is the production x that minimizes the average cost?