# WORKSHEET 3: Differentiation of functions of one variable

**1.** Find all points where the graph of the following functions have a horizontal tangent line.

a) $f(x) = x^3 + 1$ b) $f(x) = 1/x^2$ c) $f(x) = x + \operatorname{sen} x$
d) $f(x) = \sqrt{x-1}$ e) $f(x) = e^x - x$ f) $f(x) = \operatorname{sen} x + \cos x$

**2.** (*)Prove that the tangent lines to the graphs of $y = x$ and $y = 1/x$ at their intersection points are perpendicular to each other.

**3.** In which point is the tangent line to the curve $y^2 = 3x$ parallel to the straight line $y = 2x$?

**4.** (*)Calculate the intersection point with the x-axis of the tangent line to the graph of $f(x) = x^2$ at the point $(1, 1)$.

**5.** Calculate the value of $a$ so that the tangent to the graph of $f(x) = \frac{a}{x} + 1$ at the point $(1, f(1))$ intersects the horizontal axis at $x = 3$.

**6.** Calculate the angle of intersection of the curves $y = \frac{1}{2}(x^2 - 1)$ and $y = \frac{1}{2}(x^3 - x)$.

**7.** (*)Given $f(x) = 2[\ln(1 + g^2(x))]^2$, use $g(1) = g'(1) = -1$, to find $f'(1)$.

**8.** (*)Knowing that $a^b = e^{b \ln a}$, calculate the derived function of $f(x) = x^{\operatorname{sen} x}$ and $g(x) = (\sqrt{x})^x$.

**9.** (*)Let $f(x) = \ln(1+x^2)$ and $g(x) = e^{2x} + e^{3x}$ be two real functions. Calculate $h(x) = f(g(x))$, $v(x) = g(f(x))$, $h'(0)$ and $v'(0)$.

**10.** Let $f : [-2, 2] \to [-2, 2]$ be a continuous and bijective function.

a) Suppose that $f(0) = 0$ and $f'(0) = \alpha$, $\alpha \neq 0$. Find $\left(f^{-1}\right)'(0)$.

b) Suppose now that $f(0) = 1$ and $f'(0) = \alpha$, $\alpha \neq 0$. Find$\left(f^{-1}\right)'(1)$.

c) Finally, suppose that $f(1) = 0$ and $f'(1) = \alpha$, $\alpha \neq 0$. Find$\left(f^{-1}\right)'(0)$.

**11.** (*)Supposing that the following equations define $y$ as an implicit, differentiable function of $x$, find $y'$ at the given points:

a) $x^3 + y^3 = 2xy$ at $(1, 1)$.
b) $x^2 + y^2 = 25$ at $(3, 4), (0, 5)$ y $(5, 0)$.

**12.** (*)Find $a$ and $b$ so that the function $f(x) = \begin{cases} 3x + 2 & \text{if } x \geq 1 \\ ax^2 + bx - 1 & \text{if } x < 1 \end{cases}$ is differentiable everywhere.

**13.** Apply the Mean Value Theorem (Lagrange's Theorem) to $f$ in the given interval and find the x-coordinate values of the points that satisfy the thesis of the theorem.

a) $f(x) = x^2$ in $[-2, 1]$ b) $f(x) = -2 \operatorname{sen} x$ in $[-\pi, \pi]$
c) $f(x) = x^{\frac{2}{3}}$ in $[0, 1]$ d) $f(x) = 2 \operatorname{sen} x + \operatorname{sen} 2x$ en $[0, \pi]$

**14.** (*)Let $f(x) = x^3 - 3x + 3$, $f : [-3, 2] \to \mathbb{R}$. Find the globlal extrema.

**15.** Calculate the following limits:

a)(*) $\lim_{x\to\infty} (1 + x)^{1/x}$ b) $\lim_{x\to 0^+} x \ln x$ c)(*) $\lim_{x\to\infty} x^{1/x}$ d)(*) $\lim_{x\to 1^+} \left(\frac{1}{\ln x} - \frac{2}{x-1}\right)$

**16.** Find all the asymptotes of the following functions:

a)(*) $f(x) = \dfrac{2x^3 - 3x^2 - 8x + 4}{x^2 - 4}$ b) $f(x) = \dfrac{x^3}{x^3 + x^2 + x + 1}$ c)(*) $f(x) = 2x + e^{-x}$

d) $f(x) = \dfrac{\operatorname{sen} x}{x}$ e)(*) $f(x) = \dfrac{x - 2}{\sqrt{4x^2 + 1}}$ f) $f(x) = \dfrac{3x^2 - x + 2 \operatorname{sen} x}{x - 7}$

g)(*) $f(x) = \dfrac{e^x}{x}$ h)(*) $f(x) = xe^{1/x}$ i)(*) $f(x) = \dfrac{x}{e^x - 1}$